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# The interaction between personalized pricing and multi-item purchases: A random utility model approach ☆

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ABSTRACT

We construct a duopolistic random utility model to investigate the effect of personalized pricing on consumers and firms, allowing consumers to purchase from both firms. Under an exponential distribution, personalized pricing always benefits firms but can either benefit or harm consumer welfare.

## 1. Introduction

Personalized pricing captivates researchers in fields such as industrial organization, marketing, and competition law due to its characteristics, including exploitability, targetability, and potential unfairness. Many papers analytically investigate competitive personalized pricing, such as Thisse and Vives (1988) in economics and Shaffer and Zhang (2002) in marketing.[1] Following the trend, the policy circle discusses the pros and cons of personalized pricing (*e.g.*, European Commission, 2018; Ofcom, 2020).

We contribute to the literature on competitive personalized pricing by studying a random utility model where consumers can purchase from both firms, the combination of the two factors being the main innovation of our paper. Multi-item purchasing is a natural consumption behavior in online businesses thanks to the low cost of visiting online retailers (Lu and Matsushima, 2024). Typical examples of such multi-item purchasing are the markets for subscription video on demand and online games (Lu and Matsushima, 2024, Section 1).

We compare equilibrium outcomes under uniform pricing and personalized pricing, using the framework in Perloff and Salop (1985), as in a recent paper, Rhodes and Zhou (2024). In the framework, each consumer independently draws the valuation for each product, which follows a cumulative distribution function (CDF). We assume that the gain from the second product is reduced by an exogenous value, capturing a utility discount on the gain from the second product.

We obtain the following results. When the CDF follows an exponential function, personalized pricing always benefits firms, which contrasts with Lu and Matsushima (2024) but aligns with Rhodes and Zhou (2024). Also, personalized pricing harms consumer surplus if and only if the utility discount on the second item are sufficiently small or sufficiently large in cases where some consumers do not purchase any product under uniform pricing. This non-monotonic effect of personalized pricing does not appear in previous papers.

Many papers in this literature use the standard Hotelling model with purchasing from only one of the firms (*e.g.*, Thisse and Vives, 1988; Shaffer and Zhang, 2002; Choe et al., 2018). They commonly show that personalized pricing tends to increase competition and improve consumer welfare.

There are several static symmetric Hotelling duopoly models in which personalized pricing benefits firms but harms consumer welfare (Esteves, 2022; Esteves and Shuai, 2022; Matsushima et al., 2023). Esteves (2022) and Matsushima et al. (2023) consider two types of consumers with different price elasticity. Esteves and Shuai (2022)

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[1] Furthermore, (Townley et al., 2017) express concerns over the exploitative potential of personalized pricing from the viewpoint of competition law.

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incorporate elastic demands of consumers, as in Gu and Wenzel (2009). Also, Chen et al. (2020) consider a static Hotelling duopoly model in which each firm can offer personalized prices over a particular range. They show situations in which personalized pricing can harm consumers and benefit firms.

Rhodes and Zhou (2024) and Lu and Matsushima (2024) are closed related to ours. Rhodes and Zhou (2024) discuss generalized oligopoly models based on Perloff and Salop (1985) to investigate the effects of personalized pricing on profits and welfare. Each consumer chooses no more than one of the firms, contrasting to our model. They generalize the above-mentioned result in Thisse and Vives (1988), showing that personalized pricing improves consumer welfare but reduces profits. They also show that the effects reverse when firms have some consumers that do not consider other firms as good alternatives.

Lu and Matsushima (2024) discuss a Hotelling duopoly model where consumers can purchase from both firms. We replace their framework with a random utility model on Perloff and Salop (1985) and derive both similar and different results compared to those in Lu and Matsushima (2024).

## 2. A random utility model

Two firms, firms 1 and 2, supply products whose valuations differ among consumers, without incurring any cost. Each consumer independently draws the valuation for the product of firm $i$, $w_i(>0)$, which follows the cumulative distribution function (CDF) $F(w_i)$ on $[\underline{w}, \overline{w}]$, with the probability density function (PDF) $f(w_i)$, where $\underline{w}$ and $\overline{w}$ are positive constants and $0 < \underline{w} < \overline{w}$. Each consumer's $w_1$ and $w_2$ are independently drawn. Each consumer can purchase from one or both of the firms. The utility of a consumer with $w_i$ from firm $i$'s product is $w_i - p_i$, where $p_i$ is firm $i$'s price. The utility of a consumer with $w_1$ and $w_2$ from purchasing both firms' products is $w_1 + w_2 - \delta - p_1 - p_2$, where $\delta(>0)$ is the utility discount on the gain from the second item.

We consider two types of pricing competition: (i) competing in uniform pricing; (ii) competing in personalized pricing. When they compete in personalized pricing, they know consumers' exact valuations for their products and then prices are a function of $w_1$ and $w_2$, $p_i(w_1, w_2)$.

## 3. Analysis

### 3.1. Uniform pricing

First, we study competition in uniform pricing. The probability that a consumer purchases only from firm $i$ is:

$$\begin{aligned}
&\Pr[w_i - p_i \geq \max\{w_i + w_j - \delta - p_i - p_j, w_j - p_j, 0\}] \\
&= \Pr[w_j \leq \min\{\delta + p_j, w_i - p_i + p_j\} \;\&\; w_i \geq p_i] \\
&= \int_{p_i}^{\delta+p_i} \int_{\underline{w}}^{w_i - p_i + p_j} dF(w_j) dF(w_i) + \int_{\delta+p_i}^{\overline{w}} \int_{\underline{w}}^{\delta+p_j} dF(w_j) dF(w_i) \\
&= \int_{p_i}^{\delta+p_i} F(w_i - p_i + p_j) dF(w_i) + F(\delta + p_j)[1 - F(\delta + p_i)],
\end{aligned} \tag{1}$$

where $j \neq i$. The probability that a consumer purchases from both firms is:

$$\begin{aligned}
&\Pr[w_i + w_j - \delta - p_i - p_j > \max\{w_i - p_i, w_j - p_j, 0\}] \\
&= \Pr[w_j > \delta + p_j \;\&\; w_i > \delta + p_i] \\
&= [1 - F(\delta + p_i)][1 - F(\delta + p_j)].
\end{aligned} \tag{2}$$

From (1) and (2), firm $i$'s demand and profit, $D_i$ and $\pi_i$, are:

$$\begin{aligned}
D_i &= \int_{p_i}^{\delta+p_i} F(w_i - p_i + p_j) dF(w_i) + [1 - F(\delta + p_i)], \\
\pi_i &= p_i D_i.
\end{aligned} \tag{3}$$

Using (3), we derive the first-order derivative with respect to $p_i$:

$$\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= D_i + \frac{\partial D_i}{\partial p_i} p_i \\
&= \int_{p_i}^{\delta+p_i} F(w_i - p_i + p_j) dF(w_i) + [1 - F(\delta + p_i)] \\
&\quad - p_i \left\{ \int_{p_i}^{\delta+p_i} f(w_i - p_i + p_j) dF(w_i) \right. \\
&\qquad \left. + f(p_i)F(p_j) + f(\delta + p_i)[1 - F(\delta + p_j)] \right\}.
\end{aligned} \tag{4}$$

We focus on a symmetric pure-strategy equilibrium, which means $p_i = p_j = p$ in equilibrium. In relation to (4), to ensure the existence of equilibrium and its uniqueness, we impose the following assumption (we apply integration by parts to the first term in (4)):

**Assumption 1.** The following function is non-increasing in $p$:

$$G(p) \equiv \frac{\frac{1}{2}\left(F(\delta + p)^2 - F(p)^2\right) + 1 - F(\delta + p)}{\int_p^{\delta+p} f(w) dF(w) + f(p)F(p) + f(\delta + p)(1 - F(\delta + p))}. \tag{5}$$

Also, for $p = 0$, $G(0) > 0$, that is,[2]

$$G(0) \equiv \frac{\frac{F(\delta)^2}{2} + 1 - F(\delta)}{\int_{\underline{w}}^{\delta} f(w) dF(w) + f(\delta)(1 - F(\delta))} > 0. \tag{6}$$

Let $p_i = p_j = p^*$ be the prices $\partial \pi_i / \partial p_i = 0$. Then, $p^*$ is implicitly defined by[3]

$$p^* = G(p^*) = \frac{\frac{1}{2}\left(F(\delta + p^*)^2 - F(p^*)^2\right) + 1 - F(\delta + p^*)}{\int_{p^*}^{\delta+p^*} f(w) dF(w) + f(p^*)F(p^*) + f(\delta + p^*)(1 - F(\delta + p^*))}. \tag{7}$$

All consumers purchase from both firms when $1 - F(\delta + p) \geq 1$, and some consumers purchase multiple items when $1 - F(\delta + p) > 0$ (see (2)). Also, some consumers do not purchase from any firm if and only if $p > \underline{w}$. Substituting $p^*$ into these inequalities, we have:

**Proposition 1.** *There are three borderlines on $p^*$: First, at least some consumers purchase from both firms if $p^* < \overline{w} - \delta$ (equivalently, $\delta < \overline{w} - p^*$). Second, all consumers purchase from both firms if $p^* \leq \underline{w} - \delta$. Finally, some consumers do not purchase if $p^* > \underline{w}$. By combining these three borderlines, we find that the market may exhibit five different cases (the detail is in Appendix B).*

We check how a marginal increase in $\delta$ influences the equilibrium price (the proof is available in Appendix C). $p^U$ is non-increasing in $\delta$ except for the fourth case in Appendix B under a general $F(\cdot)$. If we specify the form of $F(\cdot)$, we find that $p^U$ decreases in $\delta$ if $F(\cdot)$ is uniform; $p^U$ is independent of $\delta$ if $F(\cdot)$ is exponential.

Related to the five possible cases, we also derive the consumer surplus. We include the derived outcomes in Appendix D.

### 3.2. Personalized pricing

We consider personalized pricing by checking two cases: firm $i$ sell its product to consumers with $w_i$ given that (i) firm $j$ sells to those

[2] We derive the second-order condition in Appendix A. To satisfy both Assumption 1 and the second-order condition, $f'(p)$ should be greater than a negative value.

[3] Under the exponential function used in this paper, $F(w) = 1 - e^{-(w-\underline{w})}$ and $f(w) = 1 - F(w)$. Using the fact and integration by parts, we find that the denominator of (7) equals the numerator, that is, $G(p) = 1$ for any $p$. Thus, the exponential function satisfies Assumption 1. Meanwhile, under uniform distribution functions, $G(p)$ is non-increasing in $p$ if and only if $\delta$ is not large enough.

consumers; (ii) firm $j$ does not sell to those consumers. In case (i), the price of firm $i$ should satisfy $w_i+w_j-\delta-p_i-p_j \geq w_j-p_j$, so the highest price firm $i$ can set is $p_i = w_i-\delta$. In case (ii), the price of firm $i$ should satisfy $w_i - p_i \geq w_j$, so the price is $p_i = w_i - w_j$. Consumers purchase from both firms if $\delta \leq w_i$ and $\delta \leq w_j$, and they purchase from only firm $i$ if $w_i \geq w_j$ and $\delta > w_j$. The personalized prices of firm $i$ are:

$$p_i = \begin{cases} w_i-\delta, & \delta \leq w_i \text{ and } \delta \leq w_j, \\ w_i - w_j, & w_i \geq w_j \text{ and } \delta > w_j, \\ 0, & otherwise. \end{cases} \quad (8)$$

From (8), we find several results. First, a decrease in $\delta$ increases the monopoly power of the firms over consumers with $w_i(\geq \delta)$ and $w_j(\geq \delta)$, as in Lu and Matsushima (2024). Second, since $\underline{w} > 0$, all consumers purchase from at least one of the firms. Furthermore, we have the following proposition:

**Proposition 2.** *We have one of the following cases in equilibrium:*

1. *If $\delta \leq \underline{w}$, all consumers purchase from both firms.*
2. *If $\underline{w} < \delta < \overline{w}$, some consumers purchase from both firms, and others purchase from one of the firms.*
3. *If $\overline{w} \leq \delta$, all consumers purchase from one of the firms.*

*Some consumers purchase from both firms if and only if $\delta < \bar{w}$.*

From the first borderline in Proposition 1 (note that $p^* > 0$) and Proposition 2, we have (the proof is available in Appendix F):

**Corollary 1.** *Compared with uniform pricing, more consumers purchase from both firms under personalized pricing.*

Related to the three cases in Proposition 2, we also derive the consumer surplus in each case (available in Appendices D and E). We also check the property of the consumer surplus and obtain the following result (the proof is in Appendices D and E):

**Proposition 3.** *Under personalized pricing, the consumer surplus is increasing in $\delta$ whenever some consumers purchase from both firms.*

The result aligns with that in a Hotelling model (Lu and Matsushima, 2024, Section 3.2). As mentioned in the first line in (8), firms fully extract the additional utility from the second item, $w_i - \delta$, when consumers purchase from both firms. The smaller $\delta$ is, the greater the exploitation.

## 4. Example: Exponential distribution

Suppose that $w_i$'s are independently and exponentially distributed on $[\underline{w}, \infty)$. The CDF $F(w)$ is $1 - e^{-(w-\underline{w})}$, and the PDF $f(w)$ is $e^{-(w-\underline{w})}$. This assumption on the distribution function follows that in Rhodes and Zhou (2024, p. 2154). We substitute the CDF and PDF into the general form derived in Section 3.[4]

In Proposition 1, $p^* = \underline{w} - \delta$, which is the upper bound under the second borderline (a corner solution) or $p^* = G(p^*) = 1$ (an interior solution) (see (7) and footnote 3):

$$p^U = \begin{cases} \underline{w}-\delta, & \text{if } 1+\delta \leq \underline{w}, \\ 1, & \text{otherwise.} \end{cases} \quad (9)$$

In the first case of (9), all consumers purchase from both firms, so $D_i = 1$ for both firms. In the second case of (9), when $\delta$ is small, consumers with valuation $w_i$ higher than $\delta + 1$ for both firms purchase from both firms, while others purchase from one of the firms, so $D_i = \left(1 + e^{2(\underline{w}-\delta-1)}\right)/2$. When $\delta$ is large, consumers with valuation $w_i$ lower than 1 for both firms do not purchase, and $D_i = e^{\underline{w}-2}\left(e^{\underline{w}-2\delta} - e^{\underline{w}} + 2e\right)/2$.[5] Given $\pi_i = D_i p_i$, profits under uniform pricing are

$$\pi^U = \begin{cases} \underline{w}-\delta, & \text{if } 1+\delta \leq \underline{w}, \\ (1+e^{2(\underline{w}-\delta-1)})/2, & \text{if } 1 \leq \underline{w} < 1+\delta, \\ e^{\underline{w}-2}(e^{\underline{w}-2\delta} - e^{\underline{w}} + 2e)/2, & \text{if } \underline{w} < 1. \end{cases} \quad (10)$$

Also, the consumer surplus under uniform pricing.

$$CS^U = \begin{cases} 2+\delta, & \text{if } 1+\delta \leq \underline{w}, \\ -2(\delta+1)e^{-\delta+\underline{w}-1} + (2\delta+5/2)\, e^{-2(\delta-\underline{w}+1)} + \underline{w} + 1/2, & \text{if } 1 \leq \underline{w} < 1+\delta, \\ e^{-2\delta+\underline{w}-2}(4e^{2\delta+1} - e^{2\delta+\underline{w}} - (\delta+1)4e^{\delta+1} + (4\delta+5)e^{\underline{w}})/2, & \text{if } \underline{w} < 1. \end{cases}$$

Personalized prices are available in (8). The profits under personalized pricing are

$$\pi^P = \begin{cases} \underline{w}-\delta+1, & \text{if } \delta \leq \underline{w}, \\ (1+e^{2(\underline{w}-\delta)})/2, & \text{if } \underline{w} < \delta. \end{cases} \quad (11)$$

Also, consumer surplus under personalized pricing is

$$CS^P = \begin{cases} \delta, & \text{if } \delta \leq \underline{w}, \\ (1-e^{-2(\delta-\underline{w})})/2 + \underline{w}, & \text{if } \underline{w} < \delta. \end{cases}$$

For profits, we analytical obtain the following result (the proof is in Appendix G):

**Proposition 4.** *Suppose that the CDF, $F(w)$, is $1 - e^{-(w-\underline{w})}$ on $[\underline{w}, \infty)$. Personalized pricing always benefits firms.*

The result aligns with Proposition 2 in Rhodes and Zhou (2024) because personalized pricing allows firms to exploit sufficient surpluses from consumers with very high $w_i$ and low $w_j$ in contrast with $p^U = \max\{\underline{w}-\delta, 1\}$ (see (9)). In addition to this effect, the market expansion through personalized pricing also benefits firms.

Because we cannot derive a clear cut analytical result about consumer surplus, we use two numerical examples. Fig. 1(a) shows a parametric example under the exponential distribution with a high $\underline{w}$ such that all consumers purchase from at least one firm.

We have the following numerical result by controlling two parameters $\underline{w}$ and $\delta$:

**Proposition 5.** *When $\underline{w}$ is large such that all consumers purchase from at least one firm, we numerically obtain two cases: (i) personalizing pricing harms consumer surplus if $\delta$ is small; (ii) personalizing pricing improves consumer surplus if $\delta$ is large.*

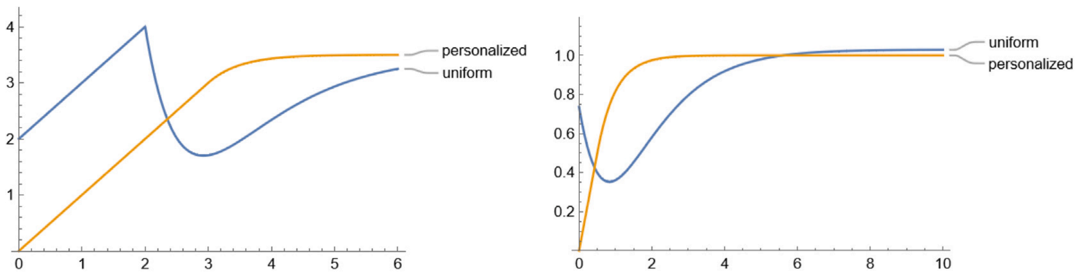
As mentioned in Proposition 3, as the value of $\delta$ decreases, firms exploit the gains from consumers' second-item purchases more. For low $\delta$, each firm is essentially a monopolist because consumers basically evaluate each product on its own merits; it must then follow that personalized pricing harms consumers. In the extreme, they completely exploit those gains when $\delta$ is zero (see around the origin in Fig. 1(a)). When the value of $\delta$ is large, as mentioned in Corollary 1, personalized pricing is useful to expand consumer demands. Also, exploitation is less likely to happen when $\delta$ is large because consumers are more likely to purchase from one of the firms. Because of the demand expansion, personalized pricing improves consumer surplus.

The result contrast with those in Rhodes and Zhou (2024). Proposition 2 in Rhodes and Zhou (2024) shows that personalized pricing has no impact on firms' profits and consumer surplus if all consumers purchase under uniform pricing.

Fig. 1(b) shows a parametric example under the exponential distribution with a low $\underline{w}$ such that some consumers do not purchase from

[4] The profit function is log-concave in this case (see Appendix A).

[5] These three cases correspond to cases 1, 2, and 4 in Appendix B.




(a) Consumer surplus ($\underline{w} = 3$) (b) Consumer surplus ($\underline{w} = 1/2$)

**Fig. 1.** The comparison of the two pricing policies: Exponential distribution.

any firm. We have the following numerical result by controlling two parameters $\underline{w}$ and $\delta$:

**Proposition 6.** *When $\underline{w}$ is small such that some consumers do not purchase from any firm, we numerically obtain two cases: (i) personalizing pricing harms consumer surplus if $\delta$ is small enough or large enough; (ii) personalizing pricing improves consumer surplus if $\delta$ is neither small enough nor large enough.*

We explain the intuition behind the non-monotonic relationship between $\delta$ and the property of consumer surplus. For sufficiently small $\delta$, the logic is the same as that in Proposition 5. For the intermediate range of $\delta$, market expansion through personalized pricing is significant and improves consumer surplus. For large $\delta$, the competitive environment is similar to that in Rhodes and Zhou (2024, Proposition 2) because almost all consumers purchase one unit only. The non-monotonic relationship is contrast with the results in Rhodes and Zhou (2024, Proposition 2) and Lu and Matsushima (2024).

## Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2024.112113.

## Data availability

No data was used for the research described in the article.

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