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# A Realistic Understanding of the Concept of Operation in Cavaillès's Philosophy

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# Abstract

At the root of Cavaillès' mathematical philosophy is the notion of *operation*. How we interpret it is therefore the major problem we face. In this article, we insist that it is possible to interpret this notion in a realistic meaning and that our explanation can still be well accepted. To this end, in Section 1, we attempt to analyse the concept of operation, that is common to both realists and idealists. In Section 2, we present the idealist argument of Cassou-Noguès [2001], which contradicts our interpretation developed in Part 3. Finally, in Section 3, in defence of the realist interpretation of the concept of operation, we confirm that the instance of the problem is indispensable to the genetic process of Truth and that this instance relates to the place that Cavaillès calls 'outside'.

Key words: Cavaillès; Operation; Problem

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The concept of *operation* consistently plays an important part in the works of Cavaillès and is essential for understanding the "philosophy of the concept" present from his earliest to his latest works. In terms of interpreting this concept of "operation," two positions exist: idealism and realism. The juxtaposition of these positions does not cause any intrinsic conflict as long as the focus is limited to the executive aspect of "operation," as discussed in Section 1. However, regarding the larger framework explaining the creation of mathematics and the process of the historical development of truth—the most important claim in the philosophy of the concept—this difference in position becomes a clear turning point that determines the interpretation of the philosophy of the concept as a whole.

First, the following must be clarified. Apart from its complex connotations in philosophy, the term idealism, as used in this paper, describes the explanatory basis for the process of the historical development of truth, whether transcendent or phenomenal, in the position sought in the consciousness of the human engaging in mathematics. Conversely, realism asserts that the cause of the process of the historical progression of truth is independent of the consciousness that conducts the "operation" but is related to some entity exterior to consciousness. The difference between these positions is not a superficial one that arises from different styles of explaining the historical development of mathematical truth; rather, it is a substantial difference in how one understands the essence of truth, as this theory defines. Idealism seeks evidence in the presence of truth vis-à-vis consciousness and the mechanism by which it appears, whereas realism seeks it in the "problem" provoked by inexhaustive rules excluded as "other" and concurrently defined as the essence of truth by concepts. Thus, this paper, which defends the superiority of the realistic interpretation over the idealistic interpretation of the concept of "operation," argues that the trial of "problems" must function essentially in the self-transcendent generation of truth.

Section 1 explores the essential characteristics of the concept of "operation" in Cavaillès's "philosophy of the concept" as common to the idealistic and realistic interpretations of the self-transcendent generation of truth. Section 2 presents the idealistic interpretation of the creation of mathematics provided by Cassou-Noguès and the interpretation of the concept of "operation" that involves Merleau-Ponty's phenomenology. Further, Section 3 identifies the indispensability of the trial of "problems" in the self-transcendent process of generation of truth and describes the relationship between this evaluation and the place that Cavaillès calls "outside" to defend the realistic interpretation of the concept of "operation." Finally, the conclusion distinguishes the term "real" used in this paper from its meaning in general and mathematical realism.

## Section 1: Common characteristics of "operation" in idealistic and realistic interpretations

Cavaillès consistently emphasizes the concept of "operation"; however, this emphasis warrants an explanation. Cavaillès (1939) stated, "Mathematics is uniquely generative," "the generation of mathematics is autonomous," and it "expands itself as a genuine generation; it is unpredictable" (pp. 600–601). One of the most fundamental beliefs of Cavaillès's philosophy of the concept is that the

history of mathematics comprises the process by which mathematical truths are self-transcending.

Two opposing positions to his views are logicism, which replaces common mathematics with logical definitions, and radical formalism, which replaces common mathematics with one or more formal systems and procedures to derive theorems. They are opposed to Cavaillès because these positions exclude the constitutive process (a type of synthesis) implied by the concept of "operation." However, these positions cannot be maintained due to mathematical combinations such as Gödel's two incompleteness theorems. Therefore, Cavaillès's (1939) conception of "operation" as a constitutive process merits further attention.

Next, let us take a look at how Cavaillès defines this concept of "operation." Kant's idea of "construction [of objects] in pure intuition" can be considered the first philosophical attempt at the "operations" of mathematics. Cavaillès (1938a) highly regards the intuitionism of Brouwer, who revived this idea of "construction in pure intuition" into modern mathematics as the concept of "positive construction" for proofs (pp. 32–44). Conversely, from Cavaillès's (1938a) perspective, the essential weakness of intuitionism is that Brouwer's insistence on "positive construction" overlooks the role of impredicative definitions that the axiomatic method presents (pp. 43–44). In other words, the essential difference between Cavaillès's position and Brouwer's intuitionism lies in whether they actively acknowledge the role of "concepts" in extending "operations."

The essential role of "concepts" that distinguishes it from intuitionism is discussed in detail in Section 3. Cavaillès's concept of "operations" is expounded upon by touching on Kant's philosophy and Brouwer's intuitionism. The basic characteristics of Cavaillès's concept of "operations" can be summarized using the following ideas:

- 1. The "operation" conducted is clearly present in the consciousness. Cavaillès described the certainty of this clear presence of consciousness using the term "effective." However, "effective" does not refer to an impression of operations being understood as a subjective matter but the concrete experience of "operations" being performed according to a given set of rules. In other words, it refers to the unique experience in mathematics of adding 7 to 5 to obtain 12 and performing an "operation" according to an explicit procedure in reality to gain a result.<sup>1)</sup>
- 2. Effectively executed "operations" are intuitive or constitutive. As such, operations are understood as a process of producing mathematical objects.
- 3. Based on 2, the existence of a mathematical object is observed only in correlation with the effective and constitutive process of "operations." Therefore, the existence of fictional objects that do not involve the performance of effective "operations" (e.g., "circular squares") cannot be recognized, regardless of logical contradiction.

The above characteristics of the concept of "operation" are interconnected, and even one missing element would prevent a complete discussion of the generation of mathematics. Moreover, the characteristics of these "operational" concepts serve as restrictions for the mathematical truths that should be recognized. Without these restrictions, all mathematical truths would be immediately obvious (i.e., mathematical truths would be likened to geographical discoveries) and would prevent considering

the generation of mathematics.

## Section 2: Potential idealistic interpretation of operations and generation of mathematics

Based on the characteristics of the concept of "operation" described above, the author explores Cassou-Noguès's argument that explains the generation of mathematics (i.e., the generation of truth), as advocated by Cavaillès, from an idealistic perspective. The ultimate stake in his attempt is to extend Cavaillès's "philosophy of the concept" into phenomenology or transform phenomenology into the "philosophy of the concept."

Cassou-Noguès (2001) analyzed Cavaillès's concepts of "operation" using three criteria. First, operation is generally defined as a "geste"—an experience on symbols executed by mathematicians in correlation with rules. Gestes are classified into three types: (1) sensitive, (2) associative, and (3) operative.

Sensitives gestes signify the "handling of symbols regarded as merely sensitive objects" (Cassou-Noguès, 2001, p. 12) and correspond to the role of symbols, as noted by Hilbert. This is evaluated exclusively as a preparatory stage before gestes (2) and (3) and does not have philosophical importance.

Geste (2) is defined using Cavaillès's unique concept, "*espace combinatoire*." This concept is (a) illustrated using "symbols" regarded as ideal unity rather than sensitive objects as well as "symbols comprising 'used rules' that define the manners of use" (i.e., "regular synthesis") and is (b) the abstract space in which the "regular synthesis" is written. This abstract space is made possible by the procedure of "formalization," which is based on Hilbert's axiomatic method and research on formal systems. Interestingly, Cassou-Noguès (2001) did not analyze the function of "concept" that should also play an important role here. Accordingly, analyses of the requirements associated with formalization and its various effects are not discussed.

According to Cassou-Noguès (2001), "espace combinatoire" is a space in which external restrictions, such as pure understanding and intuition, are removed from Kant's "constitution by intuition," allowing constituents that conform to internal conditions alone. "Espace combinatoire" then realizes prolongation of experience (Cassou-Noguès, 2001, pp. 12, 106–112), allowing the execution of explicit "operation," as described in the previous section.

Operative gestes refer to the effect of a mathematician's thought process and "intellectual activity" (Cassou-Noguès, 2001, p. 12). They are also defined as the experience of dealing with a standard of "significance" distinguished from "symbols" as ideal unities. In other words, they are the "horizon" of the operating system the entire "operation" reveals and the semantic unity of the entire "operation."

Cassou-Noguès (2001) calls this horizontal standard opened by operative gestes, which are distinguished from associative gestes, as "significance" in the primary sense of the word (apparently referring to the relationship with the phenomenological concept of "horizon") and states that this cannot be explained by Cavaillès's concept of "operations" (p. 187).

Following this discussion, Cassou-Noguès connects this to the phenomenology of the "philosophy

of the concept" as a reason for complementing this weakness. However, he stops at saying that it does not connect directly to Husserl's discussion of "intentionality" but is involved in this orientation (Cassou-Noguès, 2001, p. 210). Instead, Cassou-Noguès connects to Merleau-Ponty's vertical theory.<sup>2</sup>) The important concepts here are "*empiétement*," "expression," and "consciousness not transparent to oneself."

This is simplified to provide an explanation. For instance, let us assume that a set theoretical axiom system exists, and a person who has studied it explicitly performs set theorical "operations" conforming to symbolic rules. This corresponds to the "associative geste." However, in the process of executing this "operation," people find that the "semantic unity that the entire operation of the set theory presupposes" is hidden behind the symbol being "operated" that has not yet been properly symbolized. Consequently, people begin to experience the horizon of meaning of "this semantic unity." This horizon of meaning is not realized yet as an "associative geste" in the "espace combinatoire," and the non-original meaning alone is expressed in the "espace combinatoire." This is called latency-"consistent manifold" assumed by Cantor. To integrate these non-originally "expressed" ideas, processes of thought, such as rewriting or violating existing symbolic rules, occur (e.g., Zermelo's "posing of axiom of choice."). This is what Cassou-Noguès referred to as "empiétement." This is incarnated as explicit truth in the "espace combinatoire," composed of occasionally set symbols and symbolic rules using latent meanings expressed non-originally—as a semantic unity of an operative system. The "opaque reflexivity" of consciousness makes this latency of the horizon of meaning possible. Consciousness makes truth explicit through the "espace combinatoire"; nonetheless, all truth cannot be grasped as explicit in one stroke due to the inevitable horizontal structure of the intentionality of consciousness. Thus, from an idealistic standpoint, one may explain the generation of truth as being realized by the "opaque reflexivity" of consciousness that causes "empiétement" (Cassou-Noguès, 2001, pp. 174-197).

Thus, by connecting Cavaillès's "philosophy of the concept" to Merleau-Ponty's phenomenology, Cassou-Noguès approaches the internal or transcendental mechanism of consciousness from an idealistic standpoint—the generation of mathematics is clarified by defining the horizon of meaning through its symbolic expression.

# Section 3: Trial of the "problem" in mathematics and the dialectical process of responding by "solving" the problem

Cassou-Noguès's interpretation makes it possible to connect Cavaillès's argument to modern phenomenological discussion and can demystify Cavaillès's "philosophy of the concept."

Nevertheless, the author believes that the limitations outweigh the benefits described above. The following limitations are identified:

 Cassou-Noguès's interpretation (Cassou-Noguès, 2001) does not adequately explain the tension inherent in the history of mathematics maintained by the two opposing characteristics of "unpredictability" and "internal necessity," which are considered essential characteristics of the generation of mathematics.

2. The role of "concepts" is unjustifiably neglected, and its analysis is insufficient.

Particularly from the standpoint of the general philosophy of mathematics, the second limitation is highly significant. The role of "concepts" being unjustifiably neglected precludes the discussion of theoretical problems important to mathematical definitions and axioms, intentionality of axioms, and impredicative definitions. However, Cavaillès values the role of "impredicative definitions" in setting axiom systems as a fundamental difference from intuitionism described in Section 1.<sup>3</sup>) Therefore, the author believes that comprehension of Cavaillès's reasons to contrapose the role of "concepts" complementarily with the concept of "operations" is essential for understanding the distinction from intuitionism. Furthermore, the first limitation is fundamental to interpreting Cavaillès's "philosophy of the concept," as Cavaillès characterizes the "unpredictable necessity" as the essence of the generation of mathematics. Moreover, the author believes that the <u>paradox that problems that cannot be solved by principles can be resolved despite this</u> must be approached to understand this "unpredictable necessity."

Therefore, the author attempts to show that investigating the relationships with terms used by Cavaillès, such as "*concept*," "*règle* (rule)," "*problème* (problem)," "*solution*," and "*dialectique* (dialectic)," in addition to "operation," will provide an explanation of the generation of mathematics in "philosophy of the concept" or the self-transcendental generation of truth.

In the author's interpretation, the term "problem" plays a highly important role. Cavaillès (1939) explains his concept of "problem" in the context of exchanges at the end of the Société Française de Philosophie in 1939, particularly those between Paul Lévy and Albert Lautman, as follows:

Incidental mathematicians in history can quit when they get tired. However, problems continue to demand mathematicians to take gestes to solve them. I think it is safe to say that this is what I was trying to show by the <u>reality of cognition</u>. From an anthropological perspective or standpoint of the philosophy of human development, this may be a miracle that deviates from human destiny. Nevertheless, <u>some problems demand solutions by an internal necessity independent</u> of life in the lifeworld and which lead us exterior to what exists now (p. 629).

In this quote, Cavaillès seems to understand the autonomy of the historical generation of mathematics as a movement of providing a "solution" to the demands of the "problems." As described in Section 1, Cavaillès insisted on using the term "operation," which refers to an effective and constitutive mathematical experience, likely because he believed in thinking of the trial of the "problem" as a serious activity. Accurately understanding how these "problems lead us exterior to what exists now" is a fundamental challenge here.

In other words, the goal is to obtain a concrete understanding of "what exists now" and what corresponds to its "exterior" in actual mathematic examples. Let us look at some examples to explore this.

We deal with an example of the "operation" of counting here. As this operation is effective and

constitutive—explicit—the object called numbers can only be constituted by counting. What falls into the category of natural numbers are constituted sequentially by starting from 0 and adding 1 each time. If, hypothetically, counting numbers <u>could be completed exhaustively</u> by counting the numbers constituted by counting from 0 and sequentially, a "problem" would not occur. However, the reality is the opposite. As revealed by Cantor, numbers are not composed of a set corresponding one-to-one to the elements of the set, even if the set of natural numbers is hypothetically found. If we accept that analytics of the real number continuum hold true, the real number continuum cannot be counted even with this set of all natural numbers—their one-to-one correspondence cannot be found. This is what poses the "problem" that <u>cannot be solved</u> by the operation of counting numbers with constitutive features: how should <u>one count things that cannot be counted</u> in the "operation" of counting known numbers (constructive objects that are included in a set of natural numbers, or creating a one-to-one correspondence with that set)? To rephrase, the "problem" proposed is how to think about a sequence that includes the real number continuum.

The detailed history of the "solution" to this "problem" cannot be described here; however, the formal conditions for the "solution" to be presented can be shown.<sup>4)</sup> This section discusses transfinite ordinal numbers defined recursively. Cavaillès focuses on the function of "concepts" to expand effective and constitutive "operations."<sup>5)</sup> Concepts help explicitly universalize "rules" implicitly (or partially) realized through the performance of "operations" For instance, the "rule" that the "operation" of counting up from 0 to 1 follows is made explicit as a recursive definition by the successor function "S."

Cavaillès refers to the process of making rules explicit by a concept as "*idéalisation* (idealization)." Idealization allows (1) the separation of the "operation" from material restrictions and (2) the axiomatical definition of "operations" that go beyond the constitutive scope, such as limit transitions. However, two conditions exist:

- 1. "Concepts" that have been "idealized" cannot produce subjects without the execution of the "operation."
- 2. Such "concepts" are made by universalizing the "rule" that was realized by the "operation." The essential difference between this and the intuition-based approach described in Section 1 arises from its active recognition of the "impredicative definition" by "idealization," as described above.

By the second effect of "idealization," the "operation" of the limit transition is defined axiomatically as a limit of the successor function. Thus, " $\omega$ " as a set of all natural numbers is defined as the limit of the "operation." Furthermore, " $\omega$ " becomes the origin of the "operation" "S" by the first effect of "idealization"—"thematization." Cavaillès refers to this function of the "concept" of making an existing "operation" that base or origin of a new "operation" as "thematization."<sup>6</sup>

The detailed discussion of the limiting conditions for "idealization" and "thematization" using "concepts" is a topic for another paper; however, the relationship between "operations," "rules," "problems," and "concepts" is essential here.

As discussed in Section 1, an "operation" is executed constitutively according to a given "rule."

That "rule" is understood only within the scope that can be shown by the execution. Therefore, the "rule" in an unconditional sense is unknown. One of the first roles of "concepts" is to make the "rule" explicit and recognizable. The recursive definition of a finite order by the successor function "S" can be understood as making explicit the specific "rules" that the "operation" of counting numbers within a finite range follows. However, a "rule" of a recognized finite order is only a part of an entire "rule" (transfinite order). This is another role of "concepts" that made this extension of a rule possible. Expansion of "rules" following an "operation" of the transfinite order is made explicit in this way by "concepts." Nevertheless, the establishment of this "concept" and subsequent new "problems" (e.g., the continuum hypothesis) have revealed that <u>regardless</u>, not everything about counting is known. Nonetheless, what distinguishes this conceptual definition of transfinite order from mere fabrication is that it solves the original "problem" of how to count numbers beyond countable infinity and that a conceptual definition enables solving the problem mathematically, at least partially.<sup>7</sup>

The trial of "problems" that inevitably occur in mathematics can be understood as the inverse of "*inexhaustivité* (inexhaustivity)," which can be called the latent reality of "rules" followed by effective and constitutive "operations" and can be understood as the inverse manifestation of the problem. A "problem" disappearing completely in mathematics signifies that the entirety of mathematics can be completely formalized; however, as Gödel's proof showed, this does not hold true. Thus, <u>the trial of the</u> "problem" will never disappear.<sup>8)</sup>

The new "problem" is inevitably generated from the nature of mathematics and requires generating a "solution" in the form of a proof using the force of "concepts." Conversely, this "concept" raises a new "problem," which, in turn, demands a "solution" by a new "operation" and "concept." This series of "dialectical" processes between "problems" and "solutions" is what Cavaillès expressed as going "*dehors* (outside)" to what is in the present. A "problem" in mathematics <u>cannot be solved in principle</u> using the "explicit" procedure given at the time. In other words, because it lies "outside" of what is, expanding the "operation" by "concepts" in an occasion that will allow it to be solved—taking it "outside"—is required. Thus, the <u>presence of unsolvable "problems" lies outside the explicit consciousness in a latent way</u>.

We cannot understand that as long as the standard of truth is "evidence" as it appears in consciousness and idealism, truth is connected to the "outside" of operation as "evidence" and that only transcending and redefining the scope of "evidence" allows "truth" to descend back to the scope of effective "evidence." Herein lies the reason that demanding the explanation of the process of historical evolution of "truth" in the conscious dimension alone is impossible. The latent reality outside the consciousness must be acknowledged to understand, at the least, the dialectic process of responding to a trial of a "problem" in mathematics with a "solution."<sup>9</sup>

## Conclusion: Significance of "reality" in the "philosophy of concept"

In summary, the trial of "problems" is essential for generating mathematics, and the nature of the

"problem" cannot be solved using "evidence" of a given "operation." Thus, taking it "outside" of that "evidence" and understanding the "rule" that defines the essence of the "operation" is necessary to reassess the "rule" using a new concept. This dialectical process between "problems" and "solutions" makes it possible to understand the tension comprised of the contradiction between the "unpredictable" and "inevitable" generation of mathematics.

Therefore, the author notes that precisely understanding that "<u>problems are essentially unsolvable</u>" through an idealistic interpretation is impossible. The author also uses the concept of "reality" merely to describe what is "outside" of the conscious strictly in the context of the unsolvable problem. Hence, the realist position that the author has discussed differs completely from ordinary realism.

Ordinary realism is a perspective in epistemology—it claims that a subject lies independent of the details of consciousness by presupposing the dichotomy between the content and subject of recognition. This is often modeled on daily perceptual experience.

Ordinary mathematical realism is also founded on this analogy of daily perceptual experience. It is the idea that the extensions of what is signified by mathematical concepts and propositions somehow exist independent of consciousness or proof and that the significance of concepts and propositions are extensions of external realities they imply.<sup>10</sup>

Conversely, mathematical objects in the "philosophy of the concept" are only recognized as correlations of the execution of "operations." This stance resembles the position referred to as antirealism or constructivism in the common usages of the terms. However, the difference from these stances is that the "philosophy of the concept" allows a partial overview—a conditional use with the impredicative definition. In other words, the realist interpretation in the "philosophy of the concept" differs from realism in the ordinary sense and mathematical realism and is also opposed to them.

The author's use of the term "realistic" referring to the position described in this paper is rooted in the assumption that acknowledging the "outside" of the consciousness shown by the trial of "problems" is necessary to understand the generation of mathematics as claimed by the "philosophy of the concept" and that the term "reality" is understood only in the sense of what is presented "outside" of it.

### Notes

- 1) Cavaillès (1939) refers to this as the "mathematical experience" (p. 601).
- The empirical fact that Cavaillès had regular contact with Merleau-Ponty from 1940 to 1942 is cited as the basis for the validity of the connection to Merleau-Ponty's argument (Cassou-Noguès, 2001, p. 319, note).
- 3) Cavaillès (1938a) states, "The separation between intuitionistic and classical mathematics appears only with analysis and impredicative definitions (i.e., it is a definition that takes an infinite system as its starting point, in which the law of excluded middle does not yet play any role)" (p. 155).
- 4) See Kondo (2008) for details of the definition of transfinite ordinal numbers, historical circumstances surrounding it, its background, and its relationship to Cavaillès's philosophy.

- Brouwer's (1930) solution to this problem is the effects of will and free selection, whereas Cavaillès (1938a) criticizes this intuitionistic solution and presents a resolution through "concepts" (pp. 176–179).
- 6) More suitable examples of "thematization" include homotopy, which deals with topological relationships between transformations, and functional space, which uses functions as elements and defines norms between them.
- Furthermore, the proof of consistency of Gentzen's arithmetic system using the transfinite order and relative consistency of Gödel's continuum hypothesis demonstrate that the structure of the transfinite order is not arbitrary.
- 8) See Kondo (2009) for the explanation of this problem and its relationship with the generation of mathematics.
- 9) The reason for "potentially existing" rather than simply "existing" is to differentiate it from "existing in reality." Existing in reality indicates that this involves "operations" in a constitutive stance such as the "philosophy of the concept." Therefore, this becomes a "solution" and "existence," disappearing as a problem.
- 10) This is a position often seen in mathematicians; however, this was explicitly advocated by Gödel.

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