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Bifurcation angle distribution in the Japanese river network

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In this study, the distribution of a bifurcation angle in the Japanese river network is investigated. The resultant distribution has a peak at approximately 72° obtained via Gaussian fitting. This characteristic value is consistent with the one obtained in a field observation and theoretically predicted in previous studies for the bifurcation caused by groundwater seepage erosion process. In addition, we investigate the distributions of the remaining two angles, called bending angles, at a junction. This distribution has an anomalous peak of approximately 180°. The conditional distribution of the bifurcation angle given the bending angle shows that bifurcation and bending angles have no correlations.

1. Introduction

Rain water drains through a land drainage system. In this drainage system (mainly a river network), many physically interesting statistical properties are observed. Hack's law[1)] is a fundamental one. It describes a relation between the basin area of a river network and the length measured along the longest stream in the network as

$$L \simeq A^a \quad , \tag{1}$$

where L is the length of the longest stream and A is the area of the corresponding basin. If the exponent a equals 0.5, this relation shows a trivial area-length scaling. However, in the actual river network, the exponent a takes $0.6 \sim 0.7$, indicating that the drainage network has fractal nature. This scaling law holds from $L \sim 10^1$ to 10^3 km scales.[1–5)]

Horton's law[6,7)] is also a fundamental statistical property of a river network. Horton defined the branch order of a river network to clarify its hierarchical structure. The order represents the significance of a stream for a drainage system. Most upstream branches have the smallest ordering number $n = 1$, and the main stream has the largest ordering number which value depends on the number of branching. Horton claimed that a population of branch that

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has an ordering number n is exponentially dependent on n, that is,

$$h_n \sim r^{\omega - n} \ , \tag{2}$$

where h_n is the population of the stream with the order n and ω is the order of the main stream. Here, the bifurcation ratio r of the stream is practically almost constant, which is independent of the order n, and a river network is similar to a tree with an almost constant branching ratio.

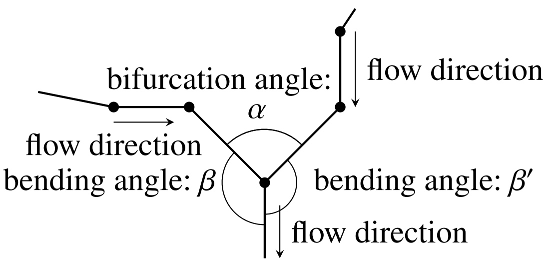


Fig. 1. Schematic picture of a river near a confluence and definition of a bifurcation angle α and two bending angles β and β'. A river stream is represented by a set of line segments in the data used by the present study. The bifurcation angle α and bending angles β and β' are defined from the nearest two line segments connected to the confluence.

Recently, another statistical property is identified by Devauchelle *et al.* in the bifurcation angle distribution of the Apalachicola River on a sandy terrain of the Florida Panhandle.[8)] A bifurcation angle is an angle between two incoming segments of streams at a confluence in a river (Fig. 1). The researchers found that the bifurcation angle distribution is the Gaussian distribution with a particular mean value $72^\circ = 2\pi/5$. In addition, this angle can be analytically obtained in a special condition of a sandy terrain.[8,9)]

The formation of a river in a sandy terrain is mainly caused by seepage erosion of groundwater at springs. In this case, the effects of surface runoff are negligibly small. Thus, only groundwater dynamics should be considered. In several assumptions (e.g., Darcy's law and Dupuit assumption), we can derive the Laplace equation for the square of water table height,[9,10)] where the groundwater flux aligns with the gradient of the water table height. In their theoretical model, a river is regarded as a sink or an electrode with the terminology of a problem of calculating the electrostatic potential. In addition, a growing direction of stream, which is a moving direction of a spring owing to erosion, is equivalent to a direction of static electric fields at the tip of the electrode. The researchers calculated the growing direction of the Y-shaped sink, which corresponds to a river near a confluence, on a two dimensional plane by using a conformal mapping method. The analytical solution of this case provides the characteristic angle $2\pi/5$ for a stable straight growth of two branches. Their theory did not explain how bifurcation occurs, but the existence of the special characteristic angle at the Y-shaped confluence.

This derivation of the characteristic angle substantially depends on geological conditions. Thus, the angle may not be considered as a universal one. In this paper, we investigate the bifurcation angle distribution in Japan to clarify whether the characteristic angle is universal or not. The geological map of Japan is not simple compared with the sandy terrains. Volcanic or plutonic, as well as sedimentary, rock regions refer to the geological map of Japan in Ref. 11 are present. In other types of rock regions, river developing processes are different from that of the sandy terrains. Runoff water is also important in developing the network. Therefore, we can discuss the universality based on a study of the Japanese drainage network.

The paper is organized as follows. Section 1 provides the introduction. Section 2 explains the data by using this study. Section 3 discusses the results. Finally, Section 4 presents the summary and discussion.

2. Data

In this study, we use the numerical digital data of Japanese drainage systems from the National Land Numerical Information download service provided by the National Land Information Division, National Spatial Planning and Regional Policy Bureau, Ministry of Land, Infrastructure, Transport and Tourism of Japan.[14] The numerical data are assembled from several analog and digital maps on a scale of 1:25000, such as river infrastructure, digital, and river management section maps, all of which are controlled by the government. The data are written in an XML format in which the river is represented by line segments. The end points of the line segments are placed at the center line of the river. The coordinates of points have an accuracy of less than ±8.75 m guaranteed in the data requirement specification. The curvature of actual river is also represented within this accuracy. The data have confluence coordinates with altitudes and flow directions.

Moreover, we can reconstruct a river network from the data, e.g., drainage network in northern part of the Osaka prefecture, Japan, as shown in Fig. 2. We only plot the end points of the line segment. Some sparse streams exist, but number of the points is sufficient in representing the river network, including the bifurcation of streams.

To analyze the bifurcation angle, we pick up a confluence where two incoming river streams merge into one outgoing stream from the data. We call this type of confluence as 2in/1out type. The bifurcation angle α is defined using two incoming line segments as shown

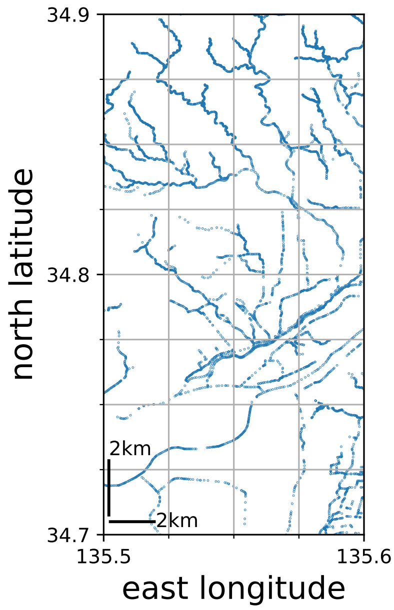


Fig. 2. Example drainage network in northern part of the Osaka prefecture, Japan. Horizontal and vertical axes represent the east longitude and north latitude, respectively. Meanwhile, horizontal and vertical lines shown in the lower left corner correspond to a 2 km scale. A river is represented by line segments. Some disconnected streams are present because only the end points of the line segments are plotted here.

in Fig. 1. In this study, we also focus on other two angles, which are called as bending angles herein. Their definitions presented in Fig. 1.

The calculation procedure of the bifurcation angle is as follows. First, we pick up the coordinates of three points of two incoming line segments corresponding to a 2in/1out-type confluence. These coordinates are expressed by the east longitude ϕ (degree) and north latitude θ (degree). Here, we denote that (θ_0, ϕ_0) is for the coordinate of the confluence and (θ_1, ϕ_1) and (θ_2, ϕ_2) are for two upstream points. Subsequently, we calculate the angle according to spherical trigonometry.[12] Assuming that Earth is a sphere, we define the three positional vectors from the center of the Earth by using spherical coordinate. For the confluence, we obtain

$$\mathbf{n}_0 = \begin{pmatrix} \sin(\pi/2 - \tilde{\theta}_0)\cos\tilde{\phi}_0 \\ \sin(\pi/2 - \tilde{\theta}_0)\sin\tilde{\phi}_0 \\ \cos(\pi/2 - \tilde{\theta}_0) \end{pmatrix} = \begin{pmatrix} \cos\tilde{\theta}_0\cos\tilde{\phi}_0 \\ \cos\tilde{\theta}_0\sin\tilde{\phi}_0 \\ \sin\tilde{\theta}_0 \end{pmatrix} \tag{3}$$

and two points of segments as

$$\mathbf{n}_1 = \begin{pmatrix} \cos\tilde{\theta}_1\cos\tilde{\phi}_1 \\ \cos\tilde{\theta}_1\sin\tilde{\phi}_1 \\ \sin\tilde{\theta}_1 \end{pmatrix} \quad \text{and} \quad \mathbf{n}_2 = \begin{pmatrix} \cos\tilde{\theta}_2\cos\tilde{\phi}_2 \\ \cos\tilde{\theta}_2\sin\tilde{\phi}_2 \\ \sin\tilde{\theta}_2 \end{pmatrix}, \tag{4}$$

Type of junction	2in/1out type	Other Nin/Mout types
Whole Japan	117865	10493
Hokkaido island	22202	583
Honshu main island	75592	8323
highlands (> 500 m)	20516	2695

Table I. Statistical summary of the data. The numbers of 2in/1out-type and other Nin/Mout-type junctions are summarized for several regions and conditions. In the present study, we only use the 2in/1out-type confluences.

where $\tilde{}$ shows a radian representation of the angle. By using these vectors, the (cosine of) bifurcation angle α is calculated as[12)]

$$\cos\alpha = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2 - (\mathbf{n}_1 \cdot \mathbf{n}_0)(\mathbf{n}_2 \cdot \mathbf{n}_0)}{\sqrt{\left(1 - (\mathbf{n}_1 \cdot \mathbf{n}_0)^2\right)\left(1 - (\mathbf{n}_2 \cdot \mathbf{n}_0)^2\right)}} \quad . \tag{5}$$

Notably, this calculation gives an angle defined on the sphere. There is a curvature effect on this angle. However, such effect is very minimal in this case because the mean single segment length is approximately several 10 m against the mean radius of the Earth. Therefore, this definition is almost equivalent to another definition of the angle by projection on the plane:

$$\cos\alpha' = \frac{(\mathbf{n}_1 - \mathbf{n}_0) \cdot (\mathbf{n}_2 - \mathbf{n}_0)}{|\mathbf{n}_1 - \mathbf{n}_0||\mathbf{n}_2 - \mathbf{n}_0|} \quad . \tag{6}$$

The gradient of the plane composed of $\mathbf{n}_1, \mathbf{n}_2$, and $\mathbf{n}_3$ may affect the bifurcation angle definition. However, we ignore such dependence herein.

3. Results

Prior to the analysis, the statistical parameters are summarized in Table I. The number of 2in/1out-type confluence and other Nin/Mout types in several regions are shown. The labels in left columns represent the regions where the data are obtained. "Whole Japan" indicates that all data are obtained from the country. "Hokkaido island" is the most northern island of the four main islands in Japan, which has less population density than the largest island of the country (i.e., "Honshu main island"). The data collected from Hokkaido island are used in evaluating the effects of civil engineering for the drainage system. "Highlands" indicates that the data are obtained from higher altitude (> 500 m) lands in the entire Japan. In this region, natural river network is well preserved because of very low population density. Furthermore, the data from the highlands are used for clarifying the artificial constructions. A minimum of

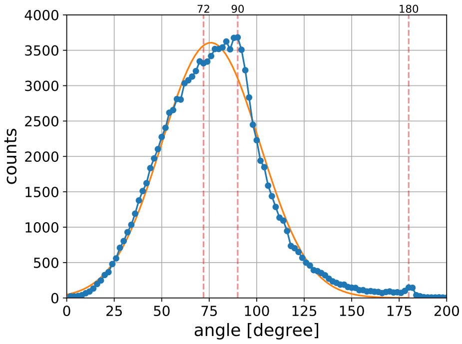
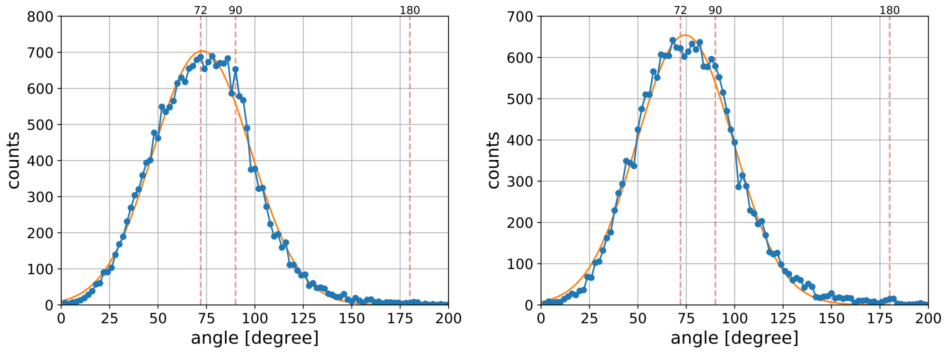


Fig. 3. Distribution of the bifurcation angle α in the whole Japan. The total count of the junctions is presented in Table I. The distribution is denoted by blue dots. The red curve shows the result of Gaussian fitting in the interval $[0°, 200°]$, which has the average and variance as 75.8(3)[deg] and 665(17)[deg]^2, respectively. Note that the distribution is not a Gaussian distribution and it has peaks at approximately 90° and 180°. The skewness and kurtosis of the original data in the interval $[0°, 200°]$ are 0.50, and 0.90, respectively.

20,000 junctions are presented in each region. These junctions are sufficient in evaluating the statistical properties of the angle distribution.

First, we establish the distribution of the bifurcation angle for the data obtained from the whole Japan as shown in Fig. 3. Several guided lines for 72°, 90°, and 180° are shown in the figure. The total count of the junctions is listed in Table I. The distribution is skewed and it has a peak at approximately 90°. Moreover, the resultant curve of the Gaussian fitting is shown in the figure as a guide, which is a red one with the average 75.8(3)[deg] and variance 665(17)[deg]^2 in the interval $[0°, 200°]$. This result does not fit well with the data. We can observe the small peak near 180°. The peaks of 90° and 180° considerably come from artificial constructions of the river. Thus, we cannot discuss the characteristic bifurcation angle with the data taken from the whole Japan. We should use a river network that preserving the original shape from the data. To eliminate the artificial effect, we consider filtering the data.

Here, we use population density and altitude as filters. Figure 4(a) shows the bifurcation angle distribution in Hokkaido island. The island has less population density than that of Honshu main island. Thus, civil engineering for the drainage system has less influence on the drainage network. By comparing the distribution for the whole Japan, the 90° and 180° peaks become much smaller. This trend is much clearly observed in the second filtering from an altitude. We present the distribution of the bifurcation angle in higher altitude region than 500 m in Fig. 4(b). To discuss the characteristic angle of the bifurcation, we estimate the characteristic angle by using the peak angle of the distribution. The peak value itself is obtained by the Gaussian fitting of the distribution. In the Hokkaido island case, Gaussian fitting provides



(a) Hokkaido island

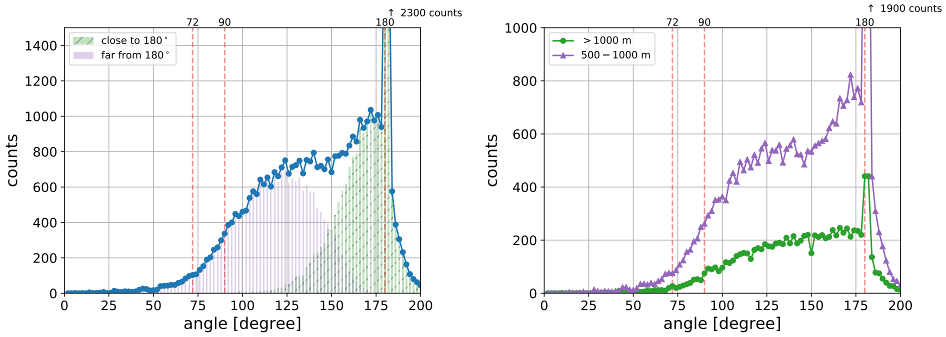
(b) Highlands

Fig. 4. Distributions of the bifurcation angle α in Hokkaido island and the highlands ($>$ 500 m). The total counts of the junctions are presented in Table I. The distribution is denoted by blue dots. A red curve is the result of Gaussian fitting in the interval $[0°, 200°]$. The fitting results of the average and variance are 72.8(3)[deg], 635(13)[deg]^2 for Hokkaido island, and 74.4(2)[deg], 619(12)deg]^2 for the highlands, respectively. The original distributions in the interval $[0°, 200°]$ have skewness of 0.38 and kurtosis of 0.59 for the Hokkaido case and skewness of 0.50 and kurtosis of 0.81 for the highland case.

the average 72.8(3)[deg] and variance 635(13)[deg]^2. Meanwhile, in the highland case, the average and variance are 74.4(2)[deg] and 619(12)[deg]^2, respectively.

Based on these results, we conclude the existence of characteristic angle $72° = 2\pi/5$ in the Japanese drainage network that preserves the form of the natural network. Several geological regions are present in Japan. Some of them may not be consistent with the assumptions in the previous theoretical study. Therefore, the existence of the characteristic angle is of universal nature beyond the assumptions in the previous study based on the representation with the Laplace equation.

In the 2in/1out-type confluence, other two angles except for the bifurcation angle exist (Fig. 1). Here, we call these two angles as bending angles. We also study the statistical distributions of bending angles. As shown in Fig. 5(a), the distribution of the bending angles β and β' in the highland region is the blue points. We can clearly see a peak at approximately 180°, indicating that the shape of the junction is mainly asymmetric "y" shape and not symmetric "Y" type; that is, the tributary stream of river merges into a straight main stream. This distribution is composed of two bending angles β and β'. Thus, we decompose the distribution into two histograms, denoted by purple and green colors in the figure. The purple histogram is composed of an angle far from 180° and the green one is of the other angle close to 180°. These decomposed histograms have Gaussian-like shape with anomalous peak at 180°. The form of the bending angle distribution is not strongly dependent on the altitude. As shown in



(a) Bending angle distribution

(b) Altitude dependence

Fig. 5. (a) Distribution of the bending angles β and β' defined in Fig. 1 is denoted by blue points. This distribution is obtained from the highlands (> 500 m) shown in Fig. 4(b). Purple and green histograms are the decomposition of the bending angle distribution into two distributions, which are close to 180° and far from 180°, respectively. The total distribution has a skewed shape, but both the decomposed distributions are in Gaussian-like shape with anomalous peak. (b) Altitude dependence of bending angle distribution. The distribution represented by purple points corresponds to the data obtained between the 500 and 1000 m altitudes, and the green one taken from higher than 1000 m altitude. The sum of these two distributions is identical to that shown in (a).

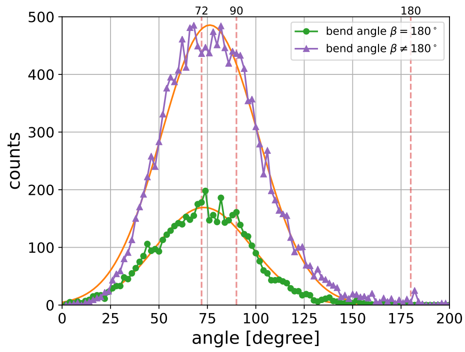


Fig. 6. Conditional distribution of the bifurcation angle on the bending angle. Green points represent the conditional distribution of the bifurcation angle in the case where the bending angle is in 180° ± 2° range, whereas blue shows the opposite case. Fitting via Gaussian distribution provides a peak value of 73.1(4)[deg] in the case of 180° ± 2° and 76.2(3)[deg] otherwise.

Fig. 5(b), we decompose the distribution shown in Fig. 5(a) into two distributions based on the altitude. The data represented by purple points correspond to the altitude of 500-1000 m interval and green ones are that higher than 1000 m.

The bending angle also has a characteristic value of 180°. Therefore, the dependence of the bifurcation angle on the bending angle is of interest. To clarify this dependence, we establish a conditional distribution of the bifurcation angle on the bending angle. Figure 6 shows the conditional distribution of the bifurcation angle, where blue denotes one of the case that the bending angle is not in $180° \pm 2°$ and green one is in $180° \pm 2°$. Both distributions have a peak of approximately 72° although the total counts are vary. Thus, the bifurcation angle does not depend on the bending angle. This information is important in the modeling of the bifurcation process of river theoretically.

4. Summary and Discussion

In this article, we investigated the bifurcation angle distribution in Japan. The data distribution collected from the whole Japan has a peak of approximately 90°. Meanwhile, the peak of 72° appears in the distributions for Hokkaido island and the highlands, where we consider that the natural form of drainage network is well preserved. The peak of 90° comes from the civil engineering influence on the river network. Hence, the characteristic angle $2\pi/5$ for the bifurcation of the river network presented in the recent studies[8,9] is also observed in the drainage network of Japan. The geological conditions of Japan are not the same as that assumed in the previous study. Thus, our result shows that another universal mechanics is present beyond the harmonic model analyzed by the previous study.

Apart from the bifurcation angle, we also evaluated the distributions of the remaining two angles, that is, the bending angles. The distribution of the bending angle is basically characterized by two Gaussian-like distributions, one of which has an anomalous peak of approximately 180°, indicating the presence of a hierarchical structure in merging two streams: A tributary stream of river merges into a straight main stream. This structure may be developed in a step-by-step way. After the main stream grows, the river bifurcates at the mid point of the main stream. Noted that this development history differs from that assumed by the previous theoretical study. The fact that the bending angle distribution has the anomalous peak leads to a new question (i.e., whether the bifurcation angle depends on the bending angle or not). Thus, we investigate the conditional distribution of the bifurcation angle on the bending angle. The result is the bifurcation angle is independent of the bending angle.

These results are significant for the theoretical bifurcation modeling a river network. In the previous study, it was assumed that two merging streams are geometrically and hierarchically symmetric. A theoretical calculation was performed in this symmetric case. The prediction of the characteristic angle might be changed by the asymmetric case calculation. However, we still need additional theoretical studies.

Recently, another work that measured the bifurcation angle in the United States was published.[13] In this study, the correlation between the mean bifurcation angle and aridity was investigated. Their conclusion indicated that the river network in a humid landscape has a characteristic angle of 72° and that in an arid landscape has 45°. In addition, they indicated that the correlation between the characteristic angle and mean slope of the river or a network stream concavity is much smaller than that with aridity. These results are consistent with the present results from climatic perspective. However, the relation between the theoretical model and actual river network formation in a humid landscape remains unclear.

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