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# THREE DUALITIES ON THE INTEGRAL HOMOLOGY OF INFINITE CYCLIC COVERINGS OF MANIFOLDS

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## 0. Statement of main result

We consider a compact oriented *topological* $n$-manifold $M$. Let $\gamma$ be an element of the first integral cohomology $H^1(M)$ and $\tilde{M}$ be the infinite cyclic covering space of $M$ associated with $\gamma$. The covering transformation group is infinite cyclic and denoted by $\langle t\rangle$ with a generator $t$, specified by $\gamma$. A *subboundary*, $A$, of $M$ is $\emptyset$ or a compact $(n-1)$-submanifold of the boundary $\partial M$ such that $A'=cl_{\partial M}(\partial M-A)$ is $\emptyset$ or a compact $(n-1)$-submanifold of $\partial M$. The pair $(A, A')$ is called a *splitting* of $\partial M$. Let $\tilde{A}$ be the lift of $A$, i.e., the preimage of $A$ under the covering $\tilde{M}\to M$. Let $\Lambda$ be the integral group ring of $\langle t\rangle$. The integral homology $H_*(\tilde{M}, \tilde{A})$ forms a *finitely generated* $\Lambda$-module, because by [K/S] $(M, A)$ is homotopy equivalent to a compact polyhedral pair and $\Lambda$ is Noetherian. For an abelian group $H$, let $e^iH=\mathrm{Ext}^i_Z(H; Z)$ (so that $e^iH=0$ for $i\geq 2$ and $\mathrm{Hom}_Z(H,Z)=e^0H$), $tH=$the $Z$-torsion part of $H$ and $bH=H/tH$. When $H$ is a $\Lambda$-module, let $E^iH=\mathrm{Ext}^i_\Lambda(H, \Lambda)$ (so that $\mathrm{Hom}_\Lambda(H, \Lambda)=E^0H$) and $TH=$ the $\Lambda$-torsion part of $H$ and $BH=H/TH$. Since $\Lambda$ has the global dimension 2 (cf. MacLane [Ma, p. 205]), we have $E^iH=0$ for $i\geq 3$. The following $\Lambda$-submodule, $DH$, of $H$ was introduced by Blanchfield [*B*]:

$$DH = \{x\in H \mid \exists \text{ coprime } \lambda_1, \lambda_2, \cdots, \lambda_m\in\Lambda(m\geq 2) \text{ with } \lambda_i x = 0,\ \forall i\}\ .$$

If $H$ is finitely generated over $\Lambda$, then we see that $DH$ is the (unique) maximal finite $\Lambda$-submodule of $H$ and there are natural $\Lambda$-isomorphisms $DH\cong E^2E^2H$ and $TH/DH\cong E^1E^1H$. Further, $E^0H$ is $\Lambda$-free and there is a natural $\Lambda$-monomorphism $BH\to E^0E^0H$ whose cokernel is finite. The purpose of this paper is to establish the Zeroth, First and Second Duality Theorems giving dual structures between $E^iE^iH_p(\tilde{M}, \tilde{A})$ and $E^iE^iH_{n-p-i}(\tilde{M}, \tilde{A}')$ for $i=0$, 1 and 2, respectively. It turns out that the first two are similar to the Blanchfield Dualities [B] and the third, the Farber/Levine Duality [F], [L]. Let $f$: $(M_1; A_1, A_1')\to(M_2; A_2, A_2')$ be a proper oriented homotopy equivalence (on each of $M_1$, $A_1$ and $A_1'$) with $f^*(\gamma_2)=\gamma_1$ for compact oriented $n$-manifolds $M_i$ with splittings $(A_i, A_i')$ of $\partial M_i$ and $\gamma_i\in H^1(M_i)$, $i=1, 2$. For the covering spaces $\tilde{M}_i$ of $M_i$ associated with $\gamma_i$, $f$ lifts to a proper homotopy equivalence $\tilde{f}: (\tilde{M}_1; \tilde{A}_1, \tilde{A}'_1) \to (\tilde{M}_2; \tilde{A}_2, \tilde{A}'_2)$, which induces $\Lambda$-isomorphisms $E^iE^iH_*(\tilde{M}_1, \tilde{A}_1) \cong E^iE^iH_*(\tilde{M}_2, \tilde{A}_2)$ and $E^iE^iH_*(\tilde{M}_1, \tilde{A}'_1) \cong E^iE^iH_*(\tilde{M}_2, \tilde{A}'_2)$ denoted by $\tilde{f}_*$.

**The Zeroth Duality Theorem.** *For a compact oriented n-manifold $M$ with $\gamma \in H^1(M)$ and a splitting $(A, A')$ of $\partial M$ and integers $p, q$ with $p+q=n$, there is a pairing*

$$S: E^0E^0H_p(\tilde{M}, \tilde{A}) \times E^0E^0H_q(\tilde{M}, \tilde{A}') \to \Lambda$$

*such that*

(1) (*Homotopy invariance*) *A proper oriented homotopy equivalence $f: (M_1; A_1, A'_1) \to (M_2; A_2, A'_2)$ with $f^*(\gamma_2)=\gamma_1$ induces the identity $S(\tilde{f}_*(x), \tilde{f}_*(y))=S(x, y)$ for $x \in E^0E^0H_p(\tilde{M}_1, \tilde{A}_1)$ and $y \in E^0E^0H_q(\tilde{M}_1, \tilde{A}'_1)$,*

(2) *$S$ is sesquilinear, i.e., $\lambda S(x, y)=S(\bar{\lambda}x, y)=S(x, \lambda y)$ for $x \in E^0E^0H_p(\tilde{M}, \tilde{A})$, $y \in E^0E^0H_q(\tilde{M}, \tilde{A}')$ and $\lambda \in \Lambda$, where—denotes the involution on $\Lambda$ sending $t$ to $t^{-1}$,*

(3) *$S$ is $\varepsilon(pq)$-Hermitian, i.e., $S(x, y)=\varepsilon(pq)\overline{S(y, x)}$ for $x \in E^0E^0H_p(\tilde{M}, \tilde{A})$ and $y \in E^0E^0H_q(\tilde{M}, \tilde{A}')$, where $\varepsilon(m)=(-1)^m$,*

(4) *$S$ is non-singular, i.e., $S$ induces a t-anti $\Lambda$-isomorphism*

$$E^0E^0H_p(\tilde{M}, \tilde{A}) \cong \mathrm{Hom}_\Lambda(E^0E^0H_q(\tilde{M}, \tilde{A}'), \Lambda).$$

In fact, we construct $S$ by extending the $\Lambda$-intersection pairing $\tilde{Int}: BH_p(\tilde{M}, \tilde{A}) \times BH_q(\tilde{M}, \tilde{A}') \to \Lambda$. Blanchfield [B] has formulated a similar duality over local rings of $\Lambda$. Let $Q(\Lambda)$ be the quotient field of $\Lambda$.

**The First Duality Theorem.** *For a compact oriented n-manifold $M$ with $\gamma \in H^1(M)$ and a splitting $(A, A')$ of $\partial M$ and integers $p, r$ with $p+r+1=n$, there is a pairing*

$$L: E^1E^1H_p(\tilde{M}, \tilde{A}) \times E^1E^1H_r(\tilde{M}, \tilde{A}') \to Q(\Lambda)/\Lambda$$

*such that*

(1) (*Homotopy invariance*) *A proper oriented homotopy equivalence $f: (M_1; A_1, A'_1) \to (M_2; A_2, A'_2)$ with $f^*(\gamma_2)=\gamma_1$ induces the identity $L(\tilde{f}_*(x), \tilde{f}_*(y))=L(x, y)$ for $x \in E^1E^1H_p(\tilde{M}_1, \tilde{A}_1)$ and $y \in E^1E^1H_r(\tilde{M}_1, \tilde{A}'_1)$,*

(2) *$L$ is sesquilinear, i.e., $\lambda L(x, y)=L(\bar{\lambda}x, y)=L(x, \lambda y)$ for $x \in E^1E^1H_p(\tilde{M}, \tilde{A})$, $y \in E^1E^1H_r(\tilde{M}, \tilde{A}')$ and $\lambda \in \Lambda$,*

(3) *$L$ is $\varepsilon(pr+1)$-Hermitian, i.e., $L(x, y)=\varepsilon(pr+1)\overline{L(y, x)}$ for $x \in E^1E^1H_p(\tilde{M}, \tilde{A})$ and $y \in E^1E^1H_r(\tilde{M}, \tilde{A}')$,*

(4) *$L$ is non-singular, i.e., $L$ induces a t-anti $\Lambda$-isomorphism*

$$E^1E^1H_p(\tilde{M}, \tilde{A}) \cong \mathrm{Hom}\,(E^1E^1H_r(\tilde{M}, \tilde{A}'), Q(\Lambda)/\Lambda).$$

When $M$ is triangulated, we can see that our pairing $L$ is essentially the same as (precisely, the $t$-conjugate of) a pairing of Blanchfield [B] (cf. Remark

5.5). Our next plan is to give a dual structure between $E^2E^2H_p(\tilde{M}, \tilde{A})$ and $E^2E^2H_s(\tilde{M}, \tilde{A}')$ with $p+s+2=n$, but it turns out that there is not in general any non-singular pairing on these whole modules [In fact, $E^2E^2H_p(\tilde{M}, \tilde{A}) \not\cong E^2E^2H_s(\tilde{M}, \tilde{A}')$ as abelian groups in general]. For this reason, we construct (in **6**) a $t$-anti $\Lambda$-epimorphism $\theta\colon E^2E^2H_p(\tilde{M}, \tilde{A}) \to E^1BH_{s+1}(\tilde{M}, \tilde{A}')$ which is invariant under a proper oriented homotopy equivalence $f\colon (M_1; A_1, A_1') \to (M_2; A_2, A_2')$ with $f^*(\gamma_2) = \gamma_1$. Let $E^2E^2H_p(\tilde{A}, \tilde{M})^\theta$ be the kernel of $\theta$. Similarly, $E^2E^2H_s(\tilde{M}, \tilde{A}')^\theta$ for the kernel of $\theta\colon E^2E^2H_s(\tilde{M}, \tilde{A}') \to E^1BH_{p+1}(\tilde{M}, \tilde{A})$.

**The Second Duality Theorem.** *For a compact oriented $n$-manifold $M$ with $\gamma \in H^1(M)$ and a splitting $(A, A')$ of $\partial M$ and integers $p, s$ with $p+s+2=n$, there is a pairing*

$$l\colon E^2E^2H_p(\tilde{M}, \tilde{A})^\theta \times E^2E^2H_s(\tilde{M}, \tilde{A}')^\theta \to Q/Z$$

*such that*

(1) *(Homotopy invariance) A proper oriented homotopy equivalence $f\colon (M_1; A_1, A_1') \to (M_2; A_2, A_2')$ with $f^*(\gamma_2) = \gamma_1$ induces the identity $l(\tilde{f}_*(x), \tilde{f}_*(y)) = l(x, y)$ for $x \in E^2E^2H_p(\tilde{M}_1, \tilde{A}_1)^\theta$ and $y \in E^2E^2H_s(\tilde{M}_1, \tilde{A}_1')^\theta$,*

(2) *$l$ is $t$-isometric, i.e., $l(tx, ty) = l(x, y)$ for $x \in E^2E^2H_p(\tilde{M}, \tilde{A})^\theta$ and $y \in E^2E^2H_s(\tilde{M}, \tilde{A}')^\theta$,*

(3) *$l$ is $\varepsilon(ps+1)$-symmetric, i.e., $l(x, y) = \varepsilon(ps+1)l(y, x)$ for $x \in E^2E^2H_p(\tilde{M}, \tilde{A})^\theta$ and $y \in E^2E^2H_s(\tilde{M}, \tilde{A}')^\theta$,*

(4) *$l$ is non-singular, i.e., $l$ induces a $t$-anti $\Lambda$-isomorphism*

$$E^2E^2H_p(\tilde{M}, \tilde{A})^\theta \cong \mathrm{Hom}_Z(E^2E^2H_s(\tilde{M}, \tilde{A}')^\theta, Q/Z)\,.$$

Since a finitely generated torsion-free $\Lambda$-module $H$ is $\Lambda$-free if and only if $E^1H=0$ (cf. **3**), it follows that $BH_{p+1}(\tilde{M}, \tilde{A})$ and $BH_{s+1}(\tilde{M}, \tilde{A}')$ are $\Lambda$-free if and and only if $l$ defines a pairing

$$E^2E^2H_p(\tilde{M}, \tilde{A}) \times E^2E^2H_s(\tilde{M}, \tilde{A}') \to Q/Z\,.$$

Hence we see that $tH_p(\tilde{M}, \tilde{A})$, $tH_s(\tilde{M}, \tilde{A}')$ are finite and $BH_{p+1}(\tilde{M}, \tilde{A})$, $BH_{s+1}(\tilde{M}, \tilde{A}')$ are $\Lambda$-free if and only if $l$ defines a pairing

$$tH_p(\tilde{M}, \tilde{A}) \times tH_s(\tilde{M}, \tilde{A}') \to Q/Z\,,$$

since a finitely generated $\Lambda$-module $H$ has $tH = DH (\cong E^2E^2H)$ if and only if $tH$ is finite. Farber [4] and Levine [L] constructed the same pairing when $tH_p(\tilde{M}, \tilde{A})$, $tH_s(\tilde{M}, \tilde{A}')$ are finite and $BH_{p+1}(\tilde{M}, \tilde{A}) = BH_{s+1}(\tilde{M}, \tilde{A}') = 0$.*) Therefore, our pairing $l$ may be considered as an extreme generalization of their pairing. A basic idea of proving these Duality Theorems is to examine a universal coefficient exact sequence for cohomology over $\Lambda$, which has been done by

*) They also assumed that manifolds are piecewise-linear.

Levine [L] in an important special case (cf. Corollary 1.3).

In §1 we construct a universal coefficient exact sequence for chomology over a ring of global dimension $\leq 2$. In §2 we describe the Reidemeister duality on a regular covering of a (topologiacl) manifold. In §3 we note several properties of $\Lambda$-modules needed for our purpose. In §4, 5 and 6 we prove the Zeroth, First and Second Duality Theorems, respectively.

## 1. A universal coefficient exact sequence for chomology over a ring of global dimension $\leq 2$

Let $\Gamma$ be a ring with unit. Let $C=\{C_q, \partial\}$ be a left $\Gamma$-projective chain complex and $F$, a left $\Gamma$-module. In general, $H^*(C; F)=H^*(\mathrm{Hom}_\Gamma(C, F))$ and $\mathrm{Ext}^i_\Gamma(H_*(C), F)$ are abelian groups, but when $\Gamma$ is commutative, they can be considered as $\Gamma$-modules. Let $h: H^*(C; F) \to \mathrm{Hom}_\Gamma(H_*(C), F)$ be the homomorphism defined by $h(\{f\})(\{z\})=f(z)$ for $\{f\} \in H^*(C; F)$ and $\{z\} \in H_*(C)$. Let $K^*(C; F)$ be the kernel of $h$. We assume that $\Gamma$ has the left global dimension$\leq 2$. Then $\mathrm{Ext}^i_\Gamma(H_*(C),F)=0$ for $i\geq 3$ and we obtain the following *Universal Coefficient Exact Sequence,* referred to as UCES:

**Theorem 1.1.** *For all q, there is a natural exact sequence*

$$0 \to K^q(C; F) \overset{\subset}{\to} H^q(C; F) \overset{h}{\to} \mathrm{Hom}_\Gamma(H_q(C), F) \overset{\eta}{\to} \mathrm{Ext}^2_\Gamma(H_{q-1}(C), F)$$
$$\overset{\Delta}{\to} K^{q+1}(C; F) \overset{\rho}{\to} \mathrm{Ext}^1_\Gamma(H_q(C), F) \to 0\,.$$

The proof is quite elementary. The following corresponds to the usual universal coefficient theorem:

**Corollary 1.2.** *If $\mathrm{Ext}^2_\Gamma(H_*(C),F)=0$, then for all q there is a natural short exact sequence*

$$0 \to \mathrm{Ext}^1_\Gamma(H_{q-1}(C), F) \overset{\rho^{-1}}{\to} H^q(C; F) \overset{h}{\to} \mathrm{Hom}_\Gamma(H_q(C), F) \to 0\,.$$

The following corresponds to the case considered by Levine [L]:

**Corollary 1.3.** *If $\mathrm{Hom}_\Gamma(H_*(C),F)=0$, then for all q there is a natural short exact sequence*

$$0 \to \mathrm{Ext}^2_\Gamma(H_{q-2}(C), F) \overset{\Delta}{\to} H^q(C; F) \overset{\rho}{\to} \mathrm{Ext}^1_\Gamma(H_{q-1}(C), F) \to 0\,.$$

1.4 Proof of Theorem 1.1. For all $q, B_q(C)=\partial C_{q+1}$ has the $\Gamma$-projective dimension$\leq 1$, since $C_q$ is $\Gamma$-projective and $0 \to B_q(C) \to C_q \to C_q/B_q(C) \to 0$ is $\Gamma$-exact and $C_q/B_q(C)$ has the $\Gamma$-projective dimension$\leq 2$. So, $Z_q(C)=\mathrm{Ker}(\partial\colon C_q \to C_{q-1})$ is $\Gamma$-projective by the short $\Gamma$-exact sequence $0\to Z_q(C)\to C_q\overset{\partial}{\to}B_{q-1}(C)\to 0$. This sequence also induces an exact sequence

$$0 \to B^{q-1}(C) \to C^q \xrightarrow{j^q} Z^q(C) \to \mathrm{Ext}^1_\Gamma(B_{q-1}(C), F) \to 0\,,$$

where $B^{q-1}(C)=\mathrm{Hom}_\Gamma(B_{q-1}(C), F)$, $C^q=\mathrm{Hom}_\Gamma(C_q, F)$ and $Z^q(C)=\mathrm{Hom}_\Gamma(Z_q(C), F)$ and $j^q$ is the map induced from the inclusion $Z_q(C)\subset C_q$. Let $Z^q_1(C)=\mathrm{Im}\, j^q$. Then we have an exact sequence $0\to B^{q-1}(C)\to C^q \to Z^q_1(C)\to 0$ and an isomorphism $Z^q(C)/Z^q_1(C)\cong \mathrm{Ext}^1_\Gamma(B_{q-1}(C), F)$. Regarding $B^{*-1}(C)=\{B^{q-1}(C)\}$ and $Z^*_1(C)=\{Z^q_1(C)\}$ as cochain complexes with trivial coboundary operators, we obtain from the short exact sequence $0\to B^{*-1}(C)\to C^* \to Z^*_1(C)\to 0$ a long cohomology exact sequence

$$\to H^{q-1}(Z^*_1(C)) \xrightarrow{\delta} H^q(B^{*-1}(C)) \to H^q(C; F) \to H^q(Z^*_1(C)) \xrightarrow{\delta} H^{q+1}(B^{*-1}(C)) \to .$$

Note that the coboundary map $\delta\colon H^q(Z^*_1(C))\to H^{q+1}(B^{*-1}(C))$ is identical with the restriction $i^q_1\colon Z^q_1(C)\to B^q(C)$ of the map $i^q\colon Z^q(C)\to B^q(C)$, induced from the inclusion $B_q(C)\subset Z_q(C)$. We have the following four short exact sequences.

$$0 \to \mathrm{Coker}\, i^{q-1}_1 \to H^q(C; F) \to \mathrm{Ker}\, i^q_1 \to 0\,,$$
$$0 \to \mathrm{Ker}\, i^q_1 \xrightarrow{\subset} \mathrm{Ker}\, i^q \to \mathrm{Ker}\, i^q/\mathrm{Ker}\, i^q_1 \to 0\,,$$
$$0 \to (\mathrm{Ker}\, i^q + Z^q_1(C))/Z^q_1(C) \xrightarrow{\subset} Z^q(C)/Z^q_1(C) \to Z^q(C)/(\mathrm{Ker}\, i^q + Z^q_1(C)) \to 0\,,$$
$$0 \to \mathrm{Im}\, i^q/\mathrm{Im}\, i^q_1 \to \mathrm{Coker}\, i^q_1 \to \mathrm{Coker}\, i^q \to 0\,.$$

Using the isomorphisms $\mathrm{Ker}\, i^q/\mathrm{Ker}\, i^q_1 \cong (\mathrm{Ker}\, i^q + Z^q_1(C))/Z^q_1(C)$ and $Z^q(C)/(\mathrm{Ker}\, i^q + Z^q_1(C))\cong \mathrm{Im}\, i^q/\mathrm{Im}\, i^q_1$, we can construct an exact sequence

$$0 \to \mathrm{Coker}\, i^{q-1}_1 \xrightarrow{\alpha_1} H^q(C; F) \xrightarrow{\alpha_2} \mathrm{Ker}\, i^q \xrightarrow{\alpha_3} Z^q(C)/Z^q_1(C) \xrightarrow{\alpha_4} \mathrm{Coker}\, i^q_1 \xrightarrow{\alpha_5} \mathrm{Coker}\, i^q \to 0\,.$$

Since $Z_q(C)$ is $\Gamma$-projective, the short exact sequence $0\to B_q(C)\to Z_q(C)\to H_q(C)\to 0$ induces an isomorphism $\mathrm{Ext}^1_\Gamma(B_q(C),F)\cong \mathrm{Ext}^2_\Gamma(H_q(C), F)$ and an exact sequence $0 \to \mathrm{Hom}_\Gamma(H_q(C), F) \to Z^q(C) \xrightarrow{i^q} B^q(C) \to \mathrm{Ext}^1_\Gamma(H_q(C), F) \to 0$, so that $\mathrm{Hom}_\Gamma(H_q(C),F)\cong \mathrm{Ker}\, i^q$ and $\mathrm{Coker}\, i^q \cong \mathrm{Ext}^1_\Gamma(H_q(C),F)$. Note that the composite $H^q(C; F) \xrightarrow{\alpha_2} \mathrm{Ker}\, i^q \cong \mathrm{Hom}_\Gamma(H_q(C), F)$ is given by $h$. So, $\alpha_1$ induces an isomorphism $\mathrm{Coker}\, i^{q-1}_1 \cong K^q(C; F)$. Let $\eta$ be the composite $\mathrm{Hom}_\Gamma(H_q(C), F)\cong \mathrm{Ker}\, i^q \xrightarrow{\alpha_3} Z^q(C)/Z^q_1(C) \cong \mathrm{Ext}^1_\Gamma(B_{q-1}(C), F) \cong \mathrm{Ext}^2_\Gamma(H_{q-1}(C), F)$ and $\Delta$, the composite $\mathrm{Ext}^2_\Delta(H_{q-1}(C), F)\cong \mathrm{Ext}^1_\Gamma(B_{q-1}(C), F)\cong Z^q(C)/Z^q_1(C) \xrightarrow{\alpha_4} \mathrm{Coker}\, i^q_1 \cong K^{q+1}(C; F)$ and $\rho$, the composite $K^{q+1}(C; F)$ $\mathrm{Coker}\, i^b_1 \xrightarrow{\alpha_5} \mathrm{Coker}\, i^q \cong \mathrm{Ext}^1_\Gamma(H_q(C), F)$, where $\cong$ denotes one of the isomorphisms constructed above or its inverse. Then we obtain the exact sequence stated in Theorem 1.1. It is easy to check from construction that a $\Gamma$-chain map between left $\Gamma$-projective chain complexes induces homomorphisms commuting the resulting two exact sequences. It is similar for a $\Gamma$-homomorphism between coefficient left $\Gamma$-modules. This completes the proof.

## 2. The Reidemeister duality on a regular covering of a manifold

Let $X$ be an oriented (possibly, non-compact) $n$-manifold and $\partial_i X$, $i=1, 2$, be $\emptyset$ or $(n-1)$-submanifolds of $\partial X$ with $\partial_1 X=cl_{\partial X}(\partial X-\partial_2 X)$ and $\partial_2 X=cl_{\partial X}(\partial X-\partial_1 X)$. By Spanier [Sp, p. 301] the orientation of $X$ determine determines a unique element of $H_n^c(X, \partial X)=\lim_{\leftarrow}\{H_n(X, (X-K)\cup\partial X)\,|\,K\subset X,\ \text{compact}\}$, which we call the *fundamental class* of $X$ and denote by [X]. For integers $p$, $q$ with $p+q=n$ the map $\cap[X]$: $H_c^p(X, \partial_1 X)\to H_q(X, \partial_2 X)$ is well defined by taking the limit of $\cap[X]_K$: $H^p(X, (X-K)\cup\partial_1 X)\to H_q(X, \partial_2 X)$ for all $K$, where $[X]_K\in H_n(X, (X-K)\cup\partial X)$ denotes the projection image of $[X]$.

2.1. **The Poincaré duality theorem.** *The map* $\cap[X]$: $H_c^p(X, \partial_1 X)\to H_q(X, \partial_2 X)$ *is an isomorphism.*

This is known (cf., for example, [Ka$_3$, Appendix A] for an outlined proof). Let $(\tilde{M}, \tilde{A})$ be a regular covering space over a compact pair $(M, A)$ with covering transformation group $G$. The singular chain complex $\Delta_\sharp(\tilde{M}, \tilde{A})$ forms a left $ZG$-free chain complex. $H_c^*(\tilde{M}, \tilde{A})$ is the cohomology of the complex $\Delta_c^\sharp(\tilde{M}, \tilde{A})$ of all singular cochains with compact supports. Let $H_{ZG}^*(\tilde{M}, \tilde{A})$ be the cohomology of $\Delta_{ZG}^\sharp(\tilde{M}, \tilde{A})=\mathrm{Hom}_{ZG}(\Delta_\sharp(\tilde{M}, \tilde{A}), ZG)$. We define a cochain map

$$\phi\colon \Delta_c^\sharp(\tilde{M}, \tilde{A}) \to \Delta_{ZG}^\sharp(\tilde{M}, \tilde{A})$$

by the identity $\phi(f)(x)=\sum_{g\in G} f(gx)g^{-1}$ for $f\in\Delta_c^\sharp(\tilde{M}, \tilde{A})$ and $x\in\Delta_\sharp(\tilde{M}, \tilde{A})$, where the sum is easily checked to be a finite sum.

**Lemma 2.2.** *If $(M, A)$ is homotopy equivalent to a compact polyhedral pair, then the induced map $\phi^*$: $H_c^*(\tilde{M}, \tilde{A})\to H_{ZG}^*(\tilde{M}, \tilde{A})$ is an isomorphism.*

Proof. Since $H_c^*(\tilde{M}, \tilde{A})$ and $H_{ZG}^*(\tilde{M}, \tilde{A})$ are proper $G$-homotopy type invariants and $\phi$ commutes with proper $G$-maps, it suffices to show that $\phi^*$ is an isomorphism when $(M, A)$ is a compact polyhedral pair. Let $(M^*, A^*)$ be a triangulation of $(M, A)$ and $(\tilde{M}^*, \tilde{A}^*)$ be its lift. For a subcomplex $N^*$ of $\tilde{M}^*$, let $\Delta_\sharp(\tilde{M}^*, \tilde{A}^*\cup N^*)$ (or $C_\sharp(\tilde{M}^*, \tilde{A}^*\cup N^*)$, resp.) be the ordered (or oriented, resp.) chain complex. Let $k_1$: $\Delta_\sharp(\tilde{M}^*, \tilde{A}^*\cup N^*)\to\Delta_\sharp(\tilde{M}, \tilde{A}\cup N)$, $N=|N^*|$, and $k_2$: $\Delta_\sharp(\tilde{M}^*, \tilde{A}^*\cup N^*)\to C_\sharp(\tilde{M}^*, \tilde{A}^*\cup N^*)$ be the natural chain equivalences (cf. [Sp, 4.3.8 and 4.6.8]). Let $\Delta_f^\sharp(\tilde{M}^*, \tilde{A}^*)$ (or $C_f^\sharp(\tilde{M}^*, \tilde{A}^*)$, resp.) be the complex of all finite ordered (or oriented, resp.) cochains. Let $\Delta_{ZG}^\sharp(\tilde{M}^*, \tilde{A}^*)=\mathrm{Hom}_{ZG}(\Delta_\sharp(\tilde{M}^*, \tilde{A}^*), ZG)$ and $C_{ZG}^\sharp(\tilde{M}^*, \tilde{A}^*)=\mathrm{Hom}_{ZG}(C_\sharp(\tilde{M}^*, \tilde{A}^*), ZG)$. We have the following commutative diagram:

$$\begin{array}{ccccc} \Delta_c^\sharp(\tilde M, \tilde A) & \xrightarrow{k_1^\sharp} & \Delta_f^\sharp(\tilde M^*, \tilde A^*) & \xleftarrow{k_2^\sharp} & C_f^\sharp(\tilde M^*, \tilde A^*) \\ \downarrow \phi & & \downarrow \phi_1 & & \downarrow \phi_2 \\ \Delta_{ZG}^\sharp(\tilde M, \tilde A) & \xrightarrow{k_1^\sharp} & \Delta_{ZG}^\sharp(\tilde M^*, \tilde A^*) & \xleftarrow{k_2^\sharp} & C_{ZG}^\sharp(\tilde M^*, \tilde A^*)\,, \end{array}$$

where $\phi_i$ are defined by the same rule as $\phi$. Note that all of the $k_i^\sharp$'s in this diagram induce isomorphisms in cohomology. In fact, for the upper $k_i^\sharp$, it can be seen by taking the limit of the sequence

$$\Delta^\sharp(\tilde M, \tilde A \cup N) \xrightarrow{k_1^\sharp} \Delta^\sharp(\tilde M^*, \tilde A^* \cup N^*) \xleftarrow{k_2^\sharp} C^\sharp(\tilde M^*, \tilde A^* \cup N^*)$$

for all cofinite subcomplexes $N^*$ of $\tilde M^*$, and for the lower $k_i^\sharp$, Eilenberg [E, p. 392] proved it. Since $C_\sharp(\tilde M^*, \tilde A^*)$ is $ZG$-free of finite rank, we see that $\phi_2$ is bijective. Hence we have the isomorphism $\phi^*: H_c^*(\tilde M, \tilde A) \cong H_{ZG}^*(\tilde M, \tilde A)$, completing the proof.

2.3. **The Reidemeister duality theorem.** *For a compact oriented $n$-manifold $M$ and a splitting $(A, A')$ of $\partial M$ and integers $p$, $q$ with $p+q=n$, there is an isomorphism $\tilde D$: $H_{ZG}^p(\tilde M, \tilde A) \cong H_q(\tilde M, \tilde A')$.*

Proof. By [K/S] $(M, A)$ is homotopy equivalent to a compact polyhedral pair. So, by Lemma 2.2 $\phi^*: H_c^*(\tilde M, \tilde A) \to H_{ZG}^*(\tilde M, \tilde A)$ is an isomorphism. We take as $\tilde D$ the composite $H_{ZG}^p(\tilde M, \tilde A) \overset{\phi^{*-1}}{\cong} H_c^p(\tilde M, \tilde A) \overset{\cap[\tilde M]}{\cong} H_q(\tilde M, \tilde A')$, where the later denotes the Poincaré duality. This completes the proof.

This duality is due to Reidemeister when $M$ is triangulated (cf. Milnor [$\mathrm{Mi}_1$]). Wall [W] also considered it from a different viewpoint. We can always give $H_{ZG}^*(\tilde M, \tilde A)$ a left $ZG$-module structure so that $\tilde D$ is a $ZG$-isomorphism (cf. [$\mathrm{Mi}_1$]), but in this paper we never use it to avoid making a confusion. When $G$ is abelian, $H_c^*(\tilde M, \tilde A)$ and $H_{ZG}^*(\tilde M, \tilde A)$ form $ZG$-modules by the action of $G$, so that $\phi^*$ is a $ZG$-isomorphism and $\tilde D$ is a $g$-anti map, i.e., $g^{-1}\tilde D = \tilde D g$, for all $g \in G$. Here we used the identity $g[\tilde M] = [\tilde M]$. The following chain level version of this identity is used in **5** and **6**:

**Lemma 2.4.** *For a splitting $(A, A')$ of $\partial M$, there is a cycle $\tilde z$ in $\Delta_n^c(\tilde M; \tilde A, \tilde A')$ $= \varprojlim \{\Delta_n(\tilde M)/(\Delta_n(\tilde M - K) + \Delta_n(\tilde A) + \Delta_n(\tilde A')) \mid K \subset \tilde M,$ compact$\}$ representing $[\tilde M] \in H_n^c(\tilde M, \partial \tilde M)$ such that $g\tilde z = \tilde z$ for all $g \in G$.*

Proof. Let $z = \sum_{i=1}^m n_i \sigma_i \in \Delta_n(M)$ represent an element of $H_n(\Delta_\sharp(M)/(\Delta_\sharp(A) + \Delta_\sharp(A')))$ coresponding to $[M] \in H_n^c(M, \partial M)$ under the natural isomorphisms $H_n(\Delta_\sharp(M)/(\Delta_\sharp(A) + \Delta_\sharp(A'))) \cong H_n(M, \partial M) \cong H_n^c(M, \partial M)$ (cf. [Sp, 6.3.7]). Let $\sigma_{i,j}$, $j \in J$, be the lifts of the singular $n$-simplex $\sigma_i$ to $\tilde M$. For any compact $K \subset \tilde M$, $\sigma_{i,j}$ are in $\Delta_n(\tilde M - K)$ except a finite number of $j$ and we let $\tilde z_K = \sum_{i=1}^m n_i \sum_{j \in J} \sigma_{i,j}$ $\in \Delta_n(\tilde M)/(\Delta_n(\tilde M - K) + \Delta_n(\tilde A) + \Delta_n(\tilde A'))$. Then we see that $\tilde z_K$ is a cycle and

$\{\tilde{z}_K\}_K$ determines a cycle $\tilde{z}$ in $\Delta_n^c(\tilde{M};\tilde{A},\tilde{A}')$ with $g\tilde{z}=\tilde{z}$ for all $g\in G$. Take $\tilde{x}\in\tilde{V}\subset\tilde{M}$ so that $\tilde{V}$ is an open ball and the projection $\tilde{M}\to M$ sends $(\tilde{V},\tilde{x})$ to a pair $(V,x)$ homeomorphically. For any cycle $z'_x=\sum_{i=1}^{m'}n'_i\sigma'_i\in\Delta_n(M,M-x)$ with $\{z'_x\}=\{z\}$ in $H_n(M,M-x)$, let $\tilde{z}'_{\tilde{x}}=\sum_{i=1}^{m'}n'_i\sum_{j\in J}\sigma'_{i,j}\in\Delta_n(\tilde{M},\tilde{M}-\tilde{x})$. Then $\tilde{z}'_{\tilde{x}}$ is a well-defined cycle with $\{\tilde{z}'_{\tilde{x}}\}=\{\tilde{z}_{\tilde{x}}\}$ in $H_n(\tilde{M},\tilde{M}-\tilde{x})$. Let $z'_x$ be in $\Delta_n(V,V-x)$. Since $\tilde{V}\cap g\tilde{V}=\emptyset$ for $g\neq 1$, we see from the isomorphisms

$$H_n(\tilde{M},\tilde{M}-\tilde{x})\overset{\cong}{\leftarrow}H_n(\tilde{V},\tilde{V}-\tilde{x})\overset{\cong}{\rightarrow}H_n(V,V-x)\overset{\cong}{\rightarrow}H_n(M,M-x)$$

that $\tilde{z}'_{\tilde{x}}$ represents $[\tilde{M}]_{\tilde{x}}$ so that $\tilde{z}$ represents $[\tilde{M}]$ (cf. [Sp, 6.3.3]). This completes the proof.

## 3. Several properties of Λ-modules

Let $\Lambda_0=\Lambda\otimes_Z Q$ and for the field $Z_p$ of prime order $p$, $\Lambda_p=\Lambda\otimes_Z Z_p$. For any finitely generated Λ-module $H$, note that $E^2H$ is $Z$-torsional and $E^1H$ is Λ-torsional, since $E^2H\otimes_Z Q=E^1H\otimes_\Lambda Q(\Lambda)=0$. Let $H^{(p)}=\{x\in H\,|\,px=0\}$. $H^{(p)}$ is a $\Lambda_p$-module.

**Lemma 3.1.** *$\Lambda/(m,\lambda_1,\cdots,\lambda_r)$ is a finite Λ-module for coprime non-zero $m$, $\lambda_1,\cdots,\lambda_r\in\Lambda$ $(r\geq 1)$ with $m$ an integer.*

Proof. Let $m=\pm p_1p_2\cdots p_s$ be a prime decomposition. $\Lambda/(p_s,\lambda_1,\cdots,\lambda_r)=\Lambda_{p_s}/(\lambda_1,\cdots,\lambda_r)$ is finite. Since

$$\Lambda/(p_1\cdots p_{s-1},\lambda_1,\cdots,\lambda_r)\overset{p_s}{\rightarrow}\Lambda/(m,\lambda_1,\cdots,\lambda_r)\rightarrow\Lambda/(p_s,\lambda_1,\cdots,\lambda_r)$$

is exact, the induction on $s$ shows that $\Lambda/(m,\lambda_1,\cdots,\lambda_r)$ is finite, completing the proof.

**Corollary 3.2.** *A finitely generated Λ-module $H$ has $mH=(t^{m'}-1)H=0$ for some non-zero integers $m$, $m'$ if and only if $H$ is finite*

Proof. The "if" part is easy. The "only if" part follows from Lemma 3 1, since $H$ is a quotient of a direct sum of finite copies of $\Lambda/(m,t^{m'}-1)$. This completes the proof.

**Corollary 3.3.** *For any Λ-module $H$, $DH$ is the smallest Λ-submodule of $H$ containing all finite Λ-submodules. Further, if $H$ is finitely generated over Λ, then $DH$ is finite.*

Proof. By Corollary 3.2 $DH$ contains all finite Λ-submodules. For $x\in DH$ let $\lambda_1,\cdots,\lambda_r\in\Lambda$ $(r\geq 2)$ be non-zero coprime elements with $\lambda_i x=0$ for all $i$. Since $\Lambda_0$ is PID, there are $\lambda'_1,\cdots,\lambda'_r\in\Lambda$ and non-zero $m\in Z$ such that $\lambda_1\lambda'_1+\cdots+\lambda_r\lambda'_r=m$. Then $mx=0$ and $x$ is in the image of a Λ-homomorphism $\Lambda/(m,\lambda_1,\cdots,$

$\lambda_r) \to H$. Since $m, \lambda_1, \cdots, \lambda_r$ are coprime, we see from Lemma 3.1 that $x$ is in a finite $\Lambda$-submodule of $H$, showing the first half. If $H$ is finitely generated over $\Lambda$, so is $DH$. Then $DH$ is a quotient of a direct sum of a finite number of finite $\Lambda$-modules and hence is finite. This completes the proof.

**Lemma 3.4.** *For a finitely generated $\Lambda$-module $H$, $E^2H$ is finite and there are natural isomorphisms $E^2H \cong E^2DH$ and $DH \cong E^2E^2H$.*

Proof. Since $E^2bH$ is $Z$-torsional and finitely generated over $\Lambda$, there is an integer $m \neq 0$ with $mE^2bH = 0$. By the short exact sequence $0 \to bH \xrightarrow{m} bH \to bH/mbH \to 0$, we have $E^2bH = mE^2bH = 0$. So, $E^2H \cong E^2tH$ by the short exact sequence $0 \to tH \to H \to bH \to 0$. Let $H_p$ be the $p$-component of $tH$. We show that $E^2H_p$ is finite by induction on $n \geq 0$ with $p^nH_p = 0$. The short exact sequence $0 \to pH_p \xrightarrow{\subset} H_p \to H_p/pH_p \to 0$ induces an exact sequence $E^2(H_p/pH_p) \to E^2H_p \to E^2(pH_p)$. $H_p/pH_p$ is a finitely generated $\Lambda_p$-module and splits into a free $\Lambda_p$-module and a torsion (i.e., finite) $\Lambda_p$-module $T_p$, so that $E^2(H_p/pH_p) \cong E^2T_p$ is finite (by Corollary 3.2). By the inductive hypothesis, $E^2(pH_p)$ is finite. Hence $E^2H_p$ is finite. Since $tH$ is finitely generated over $\Lambda$, $H_p = 0$ except a finite number of $p$. Therefore, $E^2H \cong E^2tH \cong \oplus_p E^2H_p$ is finite. Next, let $H' = tH/DH$. Take an integer $m' \neq 0$ with $(t^{m'}-1)E^2H' = 0$. Since $0 \to H' \xrightarrow{t^{m'}-1} H' \to H'/(t^{m'}-1)H' \to 0$ is exact, $t^{m'}-1\colon E^2H' \to E^2H'$ is onto, so that $E^2H' = 0$ and $E^2tH \cong E^2DH$. Thus, $E^2H \cong E^2tH \cong E^2DH$. Since $DH$ is finite and $E^1DH \cong \mathrm{Hom}_\Lambda(DH, Q(\Lambda)/\Lambda) = 0$, we see from [$L$, (3.6)] that $DH \cong E^2E^2DH$. Using $E^2H \cong E^2DH$, we complete the proof.

**Lemma 3.5.** *For a finitely generated $\Lambda$-module $H$, there are a natural short exact sequence $0 \to E^1BH \to E^1H \to E^1(TH/DH) \to 0$ and natural isomorphisms $E^1BH \cong DE^1H$ and $TH/DH \cong E^1E^1H$.*

Proof. By Lemma 3.4, $E^2BH = 0$. The short exact sequence $0 \to TH \to H \to BH \to 0$ induces an exact sequence $(S)$ $0 \to E^1BH \to E^1H \to E^1TH \to 0$. Since $E^0DH = E^1DH = 0$, $E^1(TH/DH) \cong E^1TH$. Combining it with $(S)$, we obtain a desired sequence. Directly, $DE^1TH \cong D\,\mathrm{Hom}_\Lambda(TH, Q(\Lambda)/\Lambda) = 0$. By $(S)$, $DE^1BH \cong DE^1H$. For a free $\Lambda$-module $F$ of finite rank containing $BH$ (cf. Cartan/Eilenberg [C/E, p. 131]), we have $E^1BH \cong E^2(F/BH)$. By Lemma 3.4, $E^1BH$ is finite and $E^1BH = DE^1BH \cong DE^1H$. Then $E^1E^1TH \cong E^1E^1H$ by $(S)$. Since $E^2(TH/DH) = 0$ by Lemma 3.4, $TH/DH$ has the projective dimension $\leq 1$ by [L, (3.5)]. By (L, (3.6)], we have $TH/DH \cong E^1E^1(TH/DH)$. Since $E^1(TH/DH) \cong E^1TH$, the proof is completed.

**Lemma 3.6.** *For a finitely generated $\Lambda$-module $H$, $E^0H$ is $\Lambda$-free and there is a natural exact sequence $0 \to BH \to E^0E^0H \to E^2E^1BH \to 0$.*

Proof. Since $E^0BH=E^0H$, we may assume that $H=BH$. Then $H$ has the projective dimension $\leq 1$, for there is a $\Lambda$-free module $F$ containing $H$ and $F/H$ has the projective dimension $\leq 2$. A $\Lambda$-projective (i.e., $\Lambda$-free by [Se]) resolution $0\to F^1\to F^0\to H\to 0$ of $H$ with $F^i$ of finite rank induces an exact sequence $(S^*)$ $0\to E^0H\to E^0F^0\to E^0F^1\to E^1H\to 0$. Since $E^1H$ has the projective dimension $\leq 2$ and $E^0F^i$ are $\Lambda$-free, $E^0H$ is $\Lambda$-projective that is $\Lambda$-free by [Se]. By Lemma 3.5, $E^1E^1H=0$. Then $(S^*)$ induces an exact sequence $0\to E^0E^0F^1\to E^0E^0F^0\to E^0E^0H\to E^2E^1H\to 0$. Using $F^i\cong E^0E^0F^i$ and the natural injection $H\to E^0E^0H$, we obtain a natural short exact sequence $0\to H\to E^0E^0H\to E^2E^1H\to 0$. This completes the proof.

The following is obtained from Lemmas 3.4, 3.5 and 3.6:

**Corollary 3.7.** *A finitely generated $\Lambda$-module $H$ is $\Lambda$-free if and only if $E^1H=E^2H=0$.*

**Corollary 3.8.** *The following conditions on a finitely generated $\Lambda$-module $H$ are equivalent*:

(1) $E^2H=0$,

(2) $DH=0$,

(3) $H^{(p)}$ *is $\Lambda_p$-free for all prime $p$,*

(4) $H$ *has the projective dimension $\leq 1$.*

Proof. Take a short exact sequence $0\to H'\to F\to H\to 0$ with $F$, $\Lambda$-free of finite rank. Assuming (1), $E^1H'\cong E^2H=0$. By Lemma 3.6, $H'\cong E^0E^0H'$ is $\Lambda$-free, showing (1) $\Rightarrow$ (4). The others are trivial or follow from Lemma 3.4. This completes the proof.

Corollary 3.8 generalizes [L, (3.5)] and implies that a self-reciprocal $\Lambda$-module in [$\mathrm{Ka}_2$] has the $\Lambda$-projective dimension $\leq 1$. The following observation is originally due to Kervaire [Ke] (when $\lambda=t-1$):

**Corollary 3.9.** *Let $\lambda\in\Lambda$ be no unit in $\Lambda_p$ for all prime $p$. If a finitely generated $\Lambda$-module $H$ has $\lambda H=H$, then $\lambda\colon H\cong H$, $H=TH$ and $tH$ is finite.*

Proof. The Noetherian property gives $\lambda\colon H\cong H$ (cf. Shinohara/Sumners [S/S]). $E^0H$ is $\Lambda$-free by Lemma 3.6 and $\lambda\colon E^0H\cong E^0H$, meaning that $E^0H=0$, i.e., $H=TH$. If $tH/DH\neq 0$, then there is a prime $p$ with $(tH/DH)^{(p)}\neq 0$. $(tH/DH)^{(p)}$ is $\Lambda_p$-free by Corollary 3.8 and $\lambda\colon (tH/DH)^{(p)}\cong(tH/DH)^{(p)}$, meaning that $(tH/DH)^{(p)}=0$, a contradiction. Hence $tH=DH$, which is finite by Corollary 3.3. This completes the proof.

## 4. Proof of the Zeroth Duality Theorem

For a $\Lambda$-projective chain complex $C$ with $H_*(C)$ finitely generated over $\Lambda$, we see from UCES that $TH^*(C;\Lambda)=K^*(C;\Lambda)$ and $h\colon H^*(C;\Lambda)\to E^0H_*(C)$ induces a monomorphism $BH^*(C;\Lambda)\to E^0H_*(C)$, also denoted by $h$. We now return to **0** where $M$ is a compact oriented $n$-manifold and $(A, A')$ is a splitting of $\partial M$ and $(\tilde{M};\tilde{A},\tilde{A}')$ is an infinite cyclic covering of $(M;A,A')$, associated with $\gamma\in H^1(M)$. We denote by $\varepsilon_{\tilde{M}}$ the augmentation map $H_0(\tilde{M};G)\to G$ for any (untwisted) coefficient group $G$.

For integers $p, q$ with $p+q=n$, the $Z$-intersection pairing

$$\mathrm{Int}\colon H_p(\tilde{M},\tilde{A})\times H_q(\tilde{M},\tilde{A}')\to Z$$

is given by the identity $\mathrm{Int}(x,y)=\varepsilon_{\tilde{M}}((u\cup v)\cap[\tilde{M}])=\varepsilon_{\tilde{M}}(u\cap y)$ for $x\in H_p(\tilde{M},\tilde{A})$, $y\in H_q(\tilde{M},\tilde{A}')$, $u\in H_c^q(\tilde{M},\tilde{A}')$, $v\in H_c^p(\tilde{M},\tilde{A})$ with $x=u\cap[\tilde{M}]$, $y=v\cap[\tilde{M}]$ (cf. [Ka$_3$, Appendix A]). Then the $\Lambda$-intersection pairing

$$\widetilde{\mathrm{Int}}\colon H_p(\tilde{M},\tilde{A})\times H_q(\tilde{M},\tilde{A}')\to\Lambda$$

is given by the identity $\widetilde{\mathrm{Int}}(x,y)=\sum_{i=-\infty}^{+\infty}\mathrm{Int}(x,t^iy)t^{-i}$. By $\Lambda$-sesquilinearity of $\widetilde{\mathrm{Int}}$, $\widetilde{\mathrm{Int}}$ induces a pairing

$$\widetilde{\mathrm{Int}}_B\colon BH_p(\tilde{M},\tilde{A})\times BH_q(\tilde{M},\tilde{A}')\to\Lambda\,.$$

Let $\beta$ be the composite $t$-anti $\Lambda$-homomorphism

$$H_p(\tilde{M},\tilde{A})\overset{\tilde{D}^{-1}}{\cong}H_\Lambda^q(\tilde{M},\tilde{A}')\overset{h}{\to}E^0H_q(\tilde{M},\tilde{A}')\,,$$

where $\tilde{D}$ denotes the Reidemeister duality in **2**.

**Lemma 4.1.** *For $x\in H_p(\tilde{M},\tilde{A})$ and $y\in H_q(\tilde{M},\tilde{A}')$. we have $\beta(x)(y)=\widetilde{\mathrm{Int}}(x,y)$.*

Proof. For $u_x=\{f_x\}\in H_c^q(\tilde{M},\tilde{A}')$ with $x=u_x\cap[\tilde{M}]$ and $y=\{c_y\}$, $\beta(x)(y)=\phi(f_x)(c_y)=\sum_{i=-\infty}^{+\infty}f_x(t^ic_y)t^{-i}=\sum_{i=-\infty}^{+\infty}\varepsilon_{\tilde{M}}(u_x\cap t^iy)t^{-i}=\widetilde{\mathrm{Int}}(x,y)$, as desired.

4.2 Proof of the Zeroth Duality Theorem. Let $\beta_B$ be the composite $t$-anti $\Lambda$-monomorphism

$$BH_p(\tilde{M},\tilde{A})\overset{\tilde{D}^{-1}}{\cong}BH_\Lambda^q(\tilde{M},\tilde{A}')\overset{h}{\to}E^0H_q(\tilde{M},\tilde{A}')=E^0BH_q(\tilde{M},\tilde{A}')$$

induced from $\beta$. By UCES and Lemma 3.4, the cokernel of $\beta_B$ is a finite $\Lambda$-module. By Lemma 3.5, $\beta_B$ induces a $t$-anti $\Lambda$-isomorphism $\beta_B^*\colon E^0E^0BH_q(\tilde{M},\tilde{A}')\cong E^0BH_p(\tilde{M},\tilde{A})$ and hence a $t$-anti $\Lambda$-isomorphism $\beta_B^{**}\colon E^rE^0BH_p(\tilde{M},\tilde{A})\cong E^0E^0E^0BH_q(\tilde{M},\tilde{A}')$. Regard $BH_p(\tilde{M},\tilde{A})\subset E^0E^0BH_p(\tilde{M},\tilde{A})$ and $BH_q(\tilde{M},\tilde{A}')\subset E^0E^0BH_q(\tilde{M},\tilde{A}')$ in a natural way. We can see from Lemmas 3.4, 3.5 and 3.6 that $\beta_B^{**}\,|\,BH_p(\tilde{M},\tilde{A})=\beta_B$ under the identification $E^0E^0E^0BH_q(\tilde{M},\tilde{A})=E^0BH_q(\tilde{M},\tilde{A})$. We define a pairing

$$S\colon E^0E^0H_p(\tilde{M}, \tilde{A})\times E^0E^0H_q(\tilde{M}, \tilde{A}') = E^0E^0BH_p(\tilde{M}, \tilde{A})\times E^0E^0BH_q(\tilde{M}, \tilde{A}') \to \Lambda$$

by $S(x,y)=\beta_B^{**}(x)(y)$. By Lemma 4.1, $S$ is an extension of the pairing $\widetilde{\mathrm{Int}}_B$: $BH_p(\tilde{M}, \tilde{A})\times BH_q(\tilde{M}, \tilde{A}')\to\Lambda$. From construction, (2) and (4) are satisfied. To see (1), let $f\colon (M_1; A_1, A_1')\to(M_2; A_2, A_2')$ be a proper oriented homotopy equivalence with $f^*(\gamma_2)=\gamma_1$. The lift $\tilde{f}\colon (\tilde{M}_1; \tilde{A}_1, \tilde{A}_1')\to(\tilde{M}_2; \tilde{A}_2, \tilde{A}_2')$ induces $\Lambda$-isomorphisms $\tilde{f}_*\colon (E^0E^0BH_p(\tilde{M}_1, \tilde{A}_1), BH_p(\tilde{M}_1, \tilde{A}_1))\cong(E^0E^0BH_p(\tilde{M}_2, \tilde{A}_2), BH_p(\tilde{M}_2, \tilde{A}_2))$ and $\tilde{f}_*\colon (E^0E^0BH_q(\tilde{M}_1, \tilde{A}_1'), BH_q(\tilde{M}_1, \tilde{A}_1'))\cong(E^0E^0BH_q(\tilde{M}_2, \tilde{A}_2'), BH_q(\tilde{M}_2, \tilde{A}_2'))$. For $x\in E^0E^0BH_p(\tilde{M}_1, \tilde{A}_1)$, $y\in E^0E^0BH_q(\tilde{M}_1, \tilde{A}_1')$, there are non-zero integers $m, m'$ such that $mx=x'\in BH_p(\tilde{M}_1, \tilde{A}_1)$, $m'y=y'\in BH_q(\tilde{M}_1, \tilde{A}_1')$, by Lemmas 3.4 and 3.6. Since $\tilde{f}_*[\tilde{M}_1]=[\tilde{M}_2]$, it is easily proved that $\widetilde{\mathrm{Int}}_B(\tilde{f}_*(x'), \tilde{f}_*(y'))=\widetilde{\mathrm{Int}}_B(x', y')$. Then $mm'S(\tilde{f}_*(x), \tilde{f}_*(y))=S(\tilde{f}_*(x'), \tilde{f}_*(y'))=\widetilde{\mathrm{Int}}_B(\tilde{f}_*(x'), \tilde{f}_*(y'))=\widetilde{\mathrm{Int}}_B(x', y')=S(x', y')=mm'S(x, y)$. That is, $S(\tilde{f}_*(x), \tilde{f}_*(y))=S(x,y)$, showing (1). To see (3), let $x\in E^0E^0BH_p(\tilde{M}, \tilde{A})$ and $y\in E^0E^0BH_q(\tilde{M}, \tilde{A}')$. For $x'=mx\in BH_p(\tilde{M}, \tilde{A})$ and $y'=m'y\in BH_q(\tilde{M}, \tilde{A}')$ with $mm'\neq 0$, we have $mm'S(x,y)=S(x',y')=\widetilde{\mathrm{Int}}_B(x', y')=\varepsilon(pq)\overline{\widetilde{\mathrm{Int}}_B(y', x')}=\varepsilon(pq)\overline{S(y', x')}=mm'\varepsilon(pq)\overline{S(y, x)}$, i.e., $S(x,y)=\varepsilon(pq)\overline{S(y, x)}$. This completes the proof.

## 5. Proof of the First Duality Theorem

For a $\Lambda$-module $H$, we have a $\Lambda$-exact sequence $\mathrm{Hom}_\Lambda(H, Q(\Lambda)) \xrightarrow{\nu_\#} \mathrm{Hom}_\Lambda(H, Q(\Lambda)/\Lambda)\to E^1H\to 0$ induced from the short exact sequence $0\to\Lambda \to Q(\Lambda)\xrightarrow{\nu} Q(\Lambda)/\Lambda\to 0$, by which we identify $E^1H$ with the cokernel of $\nu_\#$. Let $C$ be a projective $\Lambda$-chain complex with $H_*(C)$ finitely generated over $\Lambda$. For $u=\{f_u\}\in TH^{q+1}(C; \Lambda)$ we have a non-zero $\lambda\in\Lambda$ and a cochain $f_u^+\colon C_q\to\Lambda$ such that $\lambda f_u=\delta(f_u^+)$. Letting $\rho'(u)(c)=f_u^+(c)/\lambda\in Q(\Lambda)/\Lambda$ for $c\in Z_q(C)$, we obtain a well-defined $\Lambda$-homomorphism $\rho'\colon TH^{q+1}(C; \Lambda)\to E^1H_q(C)$.

**Lemma 5.1.** *For the map $\rho\colon TH^{q+1}(C;\Lambda)\to E^1H_q(C)$ appearing in UCES, there is a natural $\Lambda$-isomorphism $\rho''\colon E^1H_q(C)\cong E^1H_q(C)$ such that $\rho=\rho''\rho'$.*

Proof. Recall that $\rho$ is the composite

$$TH^{q+1}(C;\Lambda) = K^{q+1}(C;\Lambda) \overset{\rho_1}{\cong} \mathrm{Coker}\, i_1^q \xrightarrow{\alpha_5} \mathrm{Coker}\, i^q \overset{\rho_2}{\cong} E^1H_q(C)\,.$$

For $u=\{f_u\}\in TH^{q+1}(C;\Lambda)$, we have $f_u(Z_{q+1}(C))=0$ and hence a map $f_u^B\colon B_q(C) \overset{\partial^{-1}}{\cong} C_{q+1}/Z_{q+1}(C)\xrightarrow{f_u}\Lambda$. Then note that $\rho_1(u)=\{f_u^B\}\in\mathrm{Coker}\, i_1^q$. The map $\alpha_5$ is an obvious surjection. We shall construct a natural $\Lambda$-isomorphism $\rho_2'$: Coker $i^q\cong E^1H_q(C)$. For $f^B\in B^q(C)$ we have a non-zero $\lambda\in\Lambda$ and $f^Z\in Z^q(C)$ such that $f^Z|B_q(C)=\lambda f^B$[Note that Coker $i^q\cong E^1H_q(C)$ is $\Lambda$-torsional]. Letting $\rho_2'(f^B)(c) =f^Z(c)/\lambda\in Q(\Lambda)/\Lambda$ for $c\in Z_q(C)$, we obtain a well-defined $\Lambda$-homomorphism $\rho_2'$: Coker $i^q\to E^1H_q(C)$. The naturality of $\rho_2'$ is clear. Given a $\Lambda$-homomorphism

$f\colon H_q(C) \to Q(\Lambda)/\Lambda$, we have a $\Lambda$-homomorphism $\tilde{f}\colon Z_q(C) \to Q(\Lambda)$ inducing $f$, because $Z_q(C)$ is $\Lambda$-projective. Then $\tilde{f}(B_q(C)) \subset \Lambda$ and we can see that the correspondence $\{f\} \in E^1H_q(C) \to \{\tilde{f} \mid B_q(C)\} \in \text{Coker } i^q$ is the well-defined inverse of $\rho_2'$. So, $\rho_2'$ is a natural $\Lambda$-isomorphism. The identity $\rho_2'\alpha_5\rho_1 = \rho'$ is easily checked. Letting $\rho'' = \rho_2\rho_2'^{-1}$, we obtain the identity $\rho = \rho''\rho'$, completing the proof.

5.2 Proof of the First Duality Theorem. By UCES and Lemma 5.1, $\rho'$ induces a $\Lambda$-isomorphism $TH_\Lambda^{r+1}(\tilde{M}, \tilde{A}')/DH_\Lambda^{r+1}(\tilde{M}, \tilde{A}') \cong E^1H_r(\tilde{M}, \tilde{A}')/DE^1H_r(\tilde{M}, \tilde{A}')$, also denoted by $\rho'$. By Lemma 3.5, the latter is identical with $E^1(TH_r(\tilde{M}, \tilde{A}')/DH_r(\tilde{M}, \tilde{A}')) = \text{Hom}_\Lambda(TH_r(\tilde{M}, \tilde{A}')/DH_r(\tilde{M}, \tilde{A}'), Q(\Lambda)/\Lambda) = \text{Hom}_\Lambda(E^1E^1H_r(\tilde{M}, \tilde{A}'), Q(\Lambda)/\Lambda)$. By the Reidemeister duality, we have a $t$-anti $\Lambda$-isomorphism $\tilde{D}$: $TH_\Lambda^{r+1}(\tilde{M}, \tilde{A}')/DH_\Lambda^{r+1}(\tilde{M}, \tilde{A}') \cong TH_p(\tilde{M}, \tilde{A})/DH_p(\tilde{M}, \tilde{A}) = E^1E^1H_p(\tilde{M}, \tilde{A})$. Then we define a pairing

$$
\begin{aligned}
L\colon & E^1E^1H_p(\tilde{M}, \tilde{A}) \times E^1E^1H_r(\tilde{M}, \tilde{A}') \\
&= TH_p(\tilde{M}, \tilde{A})/DH_p(\tilde{M}, \tilde{A}) \times TH_r(\tilde{M}, \tilde{A}')/DH_r(\tilde{M}, \tilde{A}') \to Q(\Lambda)/\Lambda
\end{aligned}
$$

by $L(x, y) = \varepsilon(p+1)\rho'\tilde{D}^{-1}(x)(y)$. By construction, (2) and (4) are satisfied. To see (1), let $f\colon (M_1; A_1, A_1') \to (M_2; A_2, A_2')$ be a proper oriented homotopy equivalence wiht $f^*(\gamma_2) = \gamma_1$. The lift $\tilde{f}$ induces the following commutative diagram (Use $\tilde{f}_*[\tilde{M}_1] = [\tilde{M}_2]$ for the left square):

$$
\begin{array}{ccccc}
E^1E^1H_p(\tilde{M}_1, \tilde{A}_1) & \underset{\cong}{\overset{\tilde{D}}{\longleftarrow}} & TH_\Lambda^{u+1}(\tilde{M}_1, \tilde{A}_1')/DH_\Lambda^{r+1}(\tilde{M}_1, \tilde{A}_1') & \underset{\cong}{\overset{\rho'}{\longrightarrow}} & \text{Hom}_\Lambda(E^1E^1H_r(\tilde{M}_1, \tilde{A}_1'), Q(\Lambda)/\Lambda) \\
\cong \downarrow \tilde{f}_* & & \cong \uparrow \tilde{f}^* & & \cong \uparrow \tilde{f}^* \\
E^1E^1H_p(\tilde{M}_2, \tilde{A}_2) & \underset{\cong}{\overset{\tilde{D}}{\longleftarrow}} & TH_\Lambda^{r+1}(\tilde{M}_2, \tilde{A}_2')/DH_\Lambda^{r+1}(\tilde{M}_2, \tilde{A}_2') & \underset{\cong}{\overset{\rho'}{\longrightarrow}} & \text{Hom}_\Lambda(E^1E^1H_r(\tilde{M}_2, \tilde{A}_2'), Q(\Lambda)/\Lambda).
\end{array}
$$

This means $L(\tilde{f}_*(x), \tilde{f}_*(y)) = L(x, y)$, showing (1). To see (3), let $x = \{c_x\} \in TH_p(\tilde{M}, \tilde{A})$, $y = \{c_y\} \in TH_r(\tilde{M}, \tilde{A}')$, $u_x = \{f_x\} \in TH_c^{r+1}(\tilde{M}, \tilde{A}')$ and $u_y = \{f_y\} \in TH_c^{p+1}(\tilde{M}, \tilde{A})$ with $u_x \cap [\tilde{M}] = x$ and $u_y \cap [\tilde{M}] = y$. Then there are non-zero $\lambda_x$, $\lambda_y \in \Lambda$ and $c_x^+ \in \Delta_{p+1}(\tilde{M}, \tilde{A})$ and $c_y^+ \in \Delta_{r+1}(\tilde{M}, \tilde{A}')$ such that $\partial c_x^+ = \lambda_x c_x$ and $\partial c_y^+ = \lambda_y c_y$. Since $\bar{\lambda}_x u_x = \bar{\lambda}_y u_y = 0$, there are $f_x^+ \in \Delta_c^r(\tilde{M}, \tilde{A}')$ and $f_y^+ \in \Delta_c^p(\tilde{M}, \tilde{A})$ such that $\delta(f_x^+) = \bar{\lambda}_x f_x$ and $\delta(f_y^+) = \bar{\lambda}_y f_y$. By definition,

$$L(x, y) = \varepsilon(p+1) \textstyle\sum_{i=-\infty}^{+\infty} f_x^+(t^i c_y) t^{-i}/\bar{\lambda}_x \pmod{\Lambda}\,.$$

**Assertion 5.3.** *$L(x, y) = \varepsilon((p+1)r) \sum_{i=-\infty}^{+\infty} f_y^+(t^{-i}c_x)t^{-i}/\lambda_y$ (mod $\Lambda$).*

From this, we have $L(x, y) = \varepsilon(pr+1)\overline{L(y, x)}$, showing (3), since $L(y, x) = \varepsilon(r+1) \sum_{i=-\infty}^{+\infty} f_y^+(t^i c_x)t^{-i}/\bar{\lambda}_y$ (mod $\Lambda$). This completes the proof of the First Duality Theorem, except for the proof of Assertion 5.3.

5.4. Proof of Assertion 5.3. By Lemma 2.4, we have a $t$-invariant cycle $z \in \Delta_n^c(\tilde{M}; \tilde{A}, \tilde{A}')$ representing $[\tilde{M}]$. The map $\cap z\colon \Delta_c^k(\tilde{M}, \tilde{A}) \to \Delta_{n-k}(\tilde{M}, \tilde{A}')$, $\cap z$:

$\Delta_c^k(\tilde{M},\tilde{A}')\to\Delta_{n-k}(\tilde{M},\tilde{A})$ or $\cap\tilde{z}:\Delta_c^k(\tilde{M};\tilde{A},\tilde{A}')\to\Lambda_{n-k}(\tilde{M})$ is defined to be the limit (on $K$) of the cap product map $\cap\tilde{z}_K$: $\Delta^k(\tilde{M},(\tilde{M}-K)\cup\tilde{A})\to\Delta_{n-k}(\tilde{M},\tilde{A}')$, $\cap\tilde{z}_K$: $\Delta^k(\tilde{M},(\tilde{M}-K)\cup\tilde{A}')\to\Delta_{n-k}(\tilde{M},\tilde{A})$ or $\cap\tilde{z}_K$: $\mathrm{Hom}_Z(\Lambda_k(\tilde{M})/(\Delta_k(\tilde{M}-K)+\Delta_K(\tilde{A})+\Delta_k(\tilde{A}')), Z)\to\Delta_{n-k}(\tilde{M})$ with respect to the Alexander/Whitney diagonal approximation, respectively. Assume that $f_x\cap\tilde{z}=c_x$ and $f_y\cap\tilde{z}=c_y$. Let $T$: $\tilde{M}\times\tilde{M}\to\tilde{M}\times\tilde{M}$ be the map changing the factors and $T'$: $\Delta_\sharp(\tilde{M})\otimes\Delta_\sharp(\tilde{M})\to\Delta_\sharp(\tilde{M})\otimes\Delta_\sharp(\tilde{M})$ be the chain map defined by $T'(c^p\otimes c^q)=\varepsilon(pq)c^q\otimes c^p$. Let $\tau$: $\Delta_\sharp(\tilde{M}\times\tilde{M})\to\Delta_\sharp(\tilde{M})\otimes\Delta_\sharp(\tilde{M})$ be a natural chain equivalence so that $\tau d_\sharp$ is the Alexander/Whitney diagonal approximation, where $d$: $\tilde{M}\to\tilde{M}\times\tilde{M}$ is the diagonal map. Since there is a natural chain homotopy $D_1$: $\tau T_\sharp\simeq T'\tau$ (cf. [Sp, 5.3.8]), we have $\delta d^\sharp D_1^\sharp+d^\sharp D_1^\sharp\delta=d^\sharp T^\sharp\tau^\sharp-d^\sharp\tau^\sharp T'^\sharp$, where each summand is regarded as a homomorphism $\Delta_c^\sharp(\tilde{M},\tilde{A}')\otimes\Delta_c^\sharp(M,A)\to\Delta_c^\sharp(\tilde{M};\tilde{A},\tilde{A}')$ of degree 0. Using that $\tau d_\sharp$ and $\tau T_\sharp d_\sharp$ are the Alexander/Whitney diagonal approximations, we obtain $\delta d^\sharp D_1^\sharp(t^if_x^+\otimes f_y)+d^\sharp D_1^\sharp\delta(t^if_x^+\otimes f_x)=t^if_x^+\cup f_y-\varepsilon((p+1)r)f_y\cup t^if_x^+$. But, $\delta d^\sharp D_1^\sharp(t^if_x^+\otimes f_y)\cap\tilde{z}=0$ and $\sum_{i=-\infty}^{+\infty}\varepsilon_{\tilde{M}}(d^\sharp D_1^\sharp\delta(t^if_x^+\otimes f_y)\cap\tilde{z})t^{-i}=\lambda_x\sum_{i=-\infty}^{+\infty}\varepsilon_{\tilde{M}}(d^\sharp D_1^\sharp(t^if_x\otimes f_y)\cap\tilde{z})t^{-i}$. Hence

$\sum_{i=-\infty}^{+\infty}f_x^+(t^ic_y)t^{-i}/\bar{\lambda}_x=\sum_{i=-\infty}^{+\infty}\varepsilon_{\tilde{M}}((t^if_x^+\cup f_y)\cap\tilde{z})t^{-i}/\bar{\lambda}_x\equiv\varepsilon((p+1)r)\sum_{i=-\infty}^{+\infty}\varepsilon_{\tilde{M}}((f_y\cup t^if_x^+)\cap\tilde{z})t^{-i}/\bar{\lambda}_x \pmod{\Lambda}=\varepsilon((p+1)r)\sum_{i=-\infty}^{+\infty}\varepsilon_{\tilde{M}}((\delta(f_y^+)\cup t^if_x^+)\cap\tilde{z})t^{-i}/\bar{\lambda}_x\lambda_y=\varepsilon((p+1)r)\varepsilon(p+1)\sum_{i=-\infty}^{+\infty}\varepsilon_{\tilde{M}}(f_y^+\cup t^i\delta(f_x^+))\cap\tilde{z})t^{-i}/\bar{\lambda}_x\lambda_y=\varepsilon((p+1)r)\varepsilon(p+1)\sum_{i=-\infty}^{+\infty}f_y^+(t^{-i}c_x)t^{-i}/\lambda_y$.

The result follows.

REMARK 5.5. Assume that $x, y$ are represented by $c_x, c_y$ with $|c_x|\cap|t^ic_y|=\emptyset$ for all $i$. For example, if $M$ is triangulable, then this assumption is satisfied. Then the intersection numbers $\mathrm{Int}(c_x^+, t^ic_y)$ are defined (cf. [$\mathrm{Ka}_3$, Appendix A]) and we have

$$L(x,y)=\sum_{i=-\infty}^{+\infty}\mathrm{Int}(c_x^+,\ t^ic_y)t^{-i}/\bar{\lambda}_x\,.$$

In fact, $L(x,y)=\varepsilon(p+1)\sum_{i=-\infty}^{+\infty}f_x^+(t^i\partial c_y^+)t^{-i}/\bar{\lambda}_x\lambda_y=\varepsilon(p+1)\sum_{i=-\infty}^{+\infty}f_x(t^i\bar{\lambda}_xc_y^+)t^{-i}/\bar{\lambda}_x\lambda_y=\varepsilon(p+1)\sum_{i=-\infty}^{+\infty}\mathrm{Int}(c_x,t^i\bar{\lambda}_xc_y^+)t^{-i}/\bar{\lambda}_x\lambda_y$ (cf. [$\mathrm{Ka}_3$, A. 4]) $=\varepsilon(p+1)\sum_{i=-\infty}^{+\infty}\mathrm{Int}(\partial c_x^+, t^ic_y^+)t^{-i}/\bar{\lambda}_x\lambda_y=\sum_{i=-\infty}^{+\infty}\mathrm{Int}(c_x^+,t^i\partial c_y^+)t^{-i}/\bar{\lambda}_x\lambda_y=\sum_{i=-\infty}^{+\infty}\mathrm{Int}(c_x^+,t^ic_y)t^{-i}/\bar{\lambda}_x$.

## 6. Proof of the Second Duality Theorem

Since the infinite cyclic covering $\tilde{M}\to M$ is the pullback of the exponential covering exp: $R\to S^1$ by a map $f_\gamma$: $M\to S^1$ representing $\gamma$, the lift $\tilde{f}_\gamma$: $\tilde{M}\to R$ of $f_\gamma$ is a proper map. Let $M_i^+=\tilde{f}_\gamma^{-1}(i,+\infty)$ and $M_i^-=\tilde{f}_\gamma^{-1}(-\infty,-i)$. Let $H^*(\tilde{M},\varepsilon(\pm)\cup\tilde{A})=\lim_{i\to+\infty}H^*(\tilde{M},M_i^\mp\cup A)$. Taking the limit $i\to+\infty$ of the Mayer/Vietoris sequence for $(M;\ M_i^+\cup\tilde{A}, M_i^-\cup\tilde{A})$, we obtain an exact sequence

$$\to H_c^q(\tilde{M},\tilde{A})\to H^q(\tilde{M},\varepsilon(+)\cup\tilde{A})\oplus H^q(\tilde{M},\varepsilon(-)\cup\tilde{A})\to H^q(\tilde{M},\tilde{A})\xrightarrow{\delta_c}H_c^{q+1}(\tilde{M},\tilde{A})\to.$$

**Lemma 6.1.** *There is one and only one element* $\mu$ *of* $H_{n-1}(\tilde{M}, \partial\tilde{M})$ *such that*

(1) $(t-1)\mu=0$,

(2) *The map* $p_*\colon H_{n-1}(\tilde{M}, \partial\tilde{M}) \to H_{n-1}(M, \partial M)$ *sends* $\mu$ *to* $\gamma \cap [M]$, *where* $p$ *denotes the covering projection.*

*Further,* $\mu$ *is given by* $\delta_c(1) \cap [\tilde{M}]$ *for* $\delta_c\colon H^0(\tilde{M}) \to H^1_c(\tilde{M})$.

Proof. For uniqueness, let $\mu$, $\mu'$ have (1) and (2). By the Wang exact sequence $H_{n-1}(\tilde{M}, \partial\tilde{M}) \xrightarrow{t-1} H_{n-1}(\tilde{M}, \partial\tilde{M}) \xrightarrow{p_*} H_{n-1}(M, \partial M)$ (cf. [$Mi_2$]), we have $\mu-\mu'=(t-1)x$ for an $x \in H_{n-1}(\tilde{M}, \partial\tilde{M})$. By (1), $(t-1)^2x=0$. By the Reidemeister duality and UCES, $TH_{n-1}(\tilde{M}, \partial\tilde{M}) \cong TH^1_\Lambda(\tilde{M}) \cong E^1H_0(\tilde{M})$ and the last is easily seen to be a direct sum of modules of type $\Lambda/(t^q-1)\,(q\neq 0)$ (cf. [$Ka_1$, Lemma 1]). Hence $(t-1)^2x=0$ means $(t-1)x=0$ and $\mu=\mu'$. Next, let $\mu''=\delta_c(1) \cap [\tilde{M}]$. Since $t1=1$ and $t[\tilde{M}]=[\tilde{M}]$, $\mu''$ has (1). To see that it has (2), first assume that $f_\gamma$ has a leaf $V$ in $M$ (cf. [$Ka_3$]). Regard $V \subset \tilde{M}$ and thicken $V \times I \subset \tilde{M}$ so that $\tilde{f}_\gamma^{-1}(0)=V$ and $\tilde{f}_\gamma^{-1}I=V\times I$ and $\tilde{f}_\gamma | V\times I\colon V\times I \to I$ is the projection, where $I=[0, \varepsilon]$ for a small $\varepsilon>0$. The following commutative diagram is obtained ($\mathring{I}=I-\partial I$):

$$
\begin{array}{ccccc}
H^1(I, \partial I) & \xrightarrow{(\tilde{f}_\gamma | V\times I)^*} & H^1(V\times I, V\times\partial I) & \xrightarrow{\cap [V\times I]} & H_{n-1}(V\times I, (\partial V)\times I) \\
\cong\downarrow & & \downarrow & & \downarrow \cap [\tilde{M}]_{V\times \mathring{I}} \\
H^1(R, R-\mathring{I}) & \xrightarrow{\tilde{f}_\gamma^*} & H^1(\tilde{M}, \tilde{M}-V\times\mathring{I}) & \searrow & \\
\cong\downarrow & & \downarrow & & \\
H^1_c(R) & \xrightarrow{\tilde{f}_\gamma^*} & H^1_c(\tilde{M}) & \xrightarrow{\cap [\tilde{M}]} & H_{n-1}(\tilde{M}, \partial\tilde{M})
\end{array}
$$

Since $\delta_c(1)=\tilde{f}_\gamma^*[R]$ (cf. [$Ka_1$, p. 98]), we see that $\mu''=[V] \in H_{n-1}(\tilde{M}, \partial\tilde{M})$. So, $p_*(\mu'')=[V] \in H_{n-1}(M, \partial M)$, which equals $\gamma \cap [M]$. Hence $\mu''$ has (2). If $\gamma$ has no leaf, then we take $M_P=M\times CP^2$ and $\gamma_P=\gamma\times 1 \in H^1(M_P)$. Then by [$K/S$] $\gamma_P$ has a leaf. By the identity $(\delta_c(1)\times 1)\cap([\tilde{M}]\times[CP^2])=(\delta_c(1)\cap[\tilde{M}])\times[CP^2]$, $\mu''$ has also (2). This completes the proof.

We call $\mu$ of Lemma 6.1 the *fundamental class* of the covering $\tilde{M} \to M$. By Lemmas 3.4 and 3.5, the epimorphism $\rho\colon TH^{q+1}_\Lambda(\tilde{M}, \tilde{A}) \to E^1H_q(\tilde{M}, \tilde{A})$ in UCES induces an epimorphism $DH^{q+1}_\Lambda(\tilde{M}, \tilde{A}) \to E^1BH_q(\tilde{M}, \tilde{A})$, also denoted by $\rho$. We define a $t$-anti epimorphism

$$\theta\colon DH_p(\tilde{M}, \tilde{A}) \to E^1BH_{s+1}(\tilde{M}, \tilde{A}')$$

by the composite $DH_p(\tilde{M}, \tilde{A}) \overset{\tilde{D}^{-1}}{\cong} DH^{s+2}_\Lambda(\tilde{M}, \tilde{A}') \xrightarrow{\rho} E^1BH_{s+1}(\tilde{M}, \tilde{A}')$. Clearly, any proper oriented homotopy equivalence $f\colon (M_1; A_1, A_1') \to (M_2; A_2, A_2')$ with $f^*(\gamma_2)=\gamma_1$ induces the following commutative square:

$$\begin{array}{ccc} DH_p(\tilde{M}_1, \tilde{A}_1) & \xrightarrow{\theta} & E^1BH_{s+1}(\tilde{M}_1, \tilde{A}_1') \\ \cong \downarrow \tilde{f}_* & & \cong \uparrow \tilde{f}^* \\ DH_p(\tilde{M}_2, \tilde{A}_2) & \xrightarrow{\theta} & E^1BH_{s+1}(\tilde{M}_2, \tilde{A}_2') . \end{array}$$

Let $DH_p(\tilde{M}, \tilde{A})^\theta$ be the kernel of $\theta$. By identifying $DH_p(\tilde{M}, \tilde{A})$ with $E^2E^2H_p(\tilde{M}, \tilde{A})$ in a natural way, we also consider $\theta$ as

$$\theta\colon E^2E^2H_p(\tilde{M}, \tilde{A}) \to E^1BH_{s+1}(\tilde{M}, \tilde{A}') .$$

In this case, the kernel of $\theta$ is denoted by $E^2E^2H_p(\tilde{M}, \tilde{A})^\theta$. Note that $DH_s(\tilde{M}, \tilde{A}')^\theta$, $E^2E^2H_s(\tilde{M}, \tilde{A}')^\theta$ are the kernels of $\theta\colon DH_s(\tilde{M}, \tilde{A}') \to E^1BH_{p+1}(\tilde{M}, \tilde{A})$, $\theta\colon E^2E^2H_s(\tilde{M}, \tilde{A}') \to E^1BH_{p+1}(\tilde{M}, \tilde{A})$, respectively. Let $\tau\colon e^1H_p(\tilde{M}, \tilde{A}) \to H^{p+1}(\tilde{M}, \tilde{A})$ be the monomorphism $\rho^{-1}$ in Corollary 1.2 with $\Gamma=F=Z$. Then the following square is commutative:

$$\begin{array}{ccc} e^1H_p(\tilde{M}, \tilde{A}) & \xrightarrow{\tau} & H^{p+1}(\tilde{M}, \tilde{A}) \\ \text{surjection} \uparrow & & \uparrow \delta_{Q/Z} \\ \mathrm{Hom}_Z(H_p(\tilde{M}, \tilde{A}), Q/Z) & \overset{h}{\leftrightarrows} & H^p(\tilde{M}, \tilde{A}; Q/Z) , \end{array}$$

where $\delta_{Q/Z}$ denotes the Bockstein coboundary map. Let $\tau H^{p+1}(\tilde{M}, \tilde{A})=\tau e^1H_p(\tilde{M}, \tilde{A})=\delta_{Q/Z}H^p(\tilde{M}, \tilde{A}; Q/Z)$. By UCES with $\Gamma=F=Z$, $u=\{f\} \in H^{p+1}(\tilde{M}, \tilde{A})$ is in $\tau H^{p+1}(\tilde{M}, \tilde{A})$ iff $f\,|\,Z_{p+1}\Delta_\#(\tilde{M}, \tilde{A})=0$. Let $\tau_\theta H^{p+1}(\tilde{M}, \tilde{A})$ be the $\Lambda$-submodule of $\tau H^{p+1}(\tilde{M}, \tilde{A})$ consisting of all elements $u=\{f\}$ such that $f(c)\equiv 0 \pmod d$ for $\{c_1\} \in DH_p(\tilde{M}, \tilde{A})^\theta$, $c\in \Delta_{p+1}(\tilde{M}, \tilde{A})$ and $d(\neq 0)\in Z$ with $\partial c=dc_1$. Regarding $e^1(H_p(\tilde{M}, \tilde{A})/DH_p(\tilde{M}, \tilde{A})^\theta)\subset e^1H_p(\tilde{M}, \tilde{A})$ in a natural way, we can obtain from an argument similar to [F, §1] the following (whose proof is omitted):

**Lemma 6.2.** $\tau_\theta H^{p+1}(\tilde{M}, \tilde{A})=\tau e^1(H_p(\tilde{M}, \tilde{A})/DH_p(\tilde{M}, \tilde{A})^\theta)$ .

We consider the $t$-anti homomorphism $\cap\,\mu\colon \tau H^{p+1}(\tilde{M}, \tilde{A})\to H_s(\tilde{M}, \tilde{A}')$.

**Lemma 6.3.** $\tau H^{p+1}(\tilde{M}, \tilde{A}) \cap \mu \subset DH_s(\tilde{M}, \tilde{A}')^\theta$.

Proof. By Lemma 6.1, $\tau H^{p+1}(\tilde{M}, \tilde{A}) \cap \mu=((\tau H^{p+1}(\tilde{M}, \tilde{A}) \cup \delta_c(1)) \cap [\tilde{M}]=\delta_c\tau H^{p+1}(\tilde{M}, \tilde{A}) \cap [\tilde{M}]$. For $\{f\} \in \tau H^{p+1}(\tilde{M}, \tilde{A})$, there are $f^\pm \in \Delta^{p+1}(\tilde{M}, M_i^\mp \cup \tilde{A})$ $(i\geq 1)$ such that $f=f^+-f^-$ in $\Delta^{p+1}(\tilde{M},\tilde{A})$. Then $\delta_c\{f\}=\{\delta f^+\}$. Since $f\,|\,Z_{p+1}\Delta_\#(\tilde{M}, \tilde{A})=0$, it follows that $f^+=f^-$ on $Z_{p+1}\Delta_\#(\tilde{M}, \tilde{A})$ and $\phi(f^+)$ is well-defined on it. Let $f_\Lambda=\phi(f^+)\,|\,Z_{p+1}\Delta_\#(\tilde{M}, \tilde{A})\in E^0Z_{p+1}\Delta_\#(\tilde{M}, \tilde{A})$. Noting that some multiple $\lambda f_\Lambda$ is extendable to $\Delta_{p+1}(\tilde{M}, \tilde{A})$ (for $E^1B_p\Delta_\#(\tilde{M}, \tilde{A})\cong E^2H_p(\tilde{M}, \tilde{A})$ is finite) and $\phi(\delta f^+)=\delta f_\Lambda$, we see from Lemma 5.1 that $\{\phi\delta f^+\} \in TH_\Lambda^{p+2}(\tilde{M},\tilde{A})$ and $\rho'\{\phi\delta f^+\}=0=\rho\{\phi\delta f^+\}$. This means that $\tau H^{p+1}(\tilde{M}, \tilde{A}) \cap \mu \subset \mathrm{Ker}[TH_s(\tilde{M}, \tilde{A}') \overset{\tilde{D}^{-1}}{\cong} TH_\Lambda^{p+2}(\tilde{M}, \tilde{A})\overset{\rho}{\to}E^1H_{p+1}(\tilde{M}, \tilde{A})]$, which equals $DH_s(\tilde{M},\tilde{A}')^\theta$ by UCES. This completes the proof.

**Lemma 6.4.** $\operatorname{Ker}[\cap \mu: \tau H^{p+1}(\tilde{M}, \tilde{A}) \to H_s(\tilde{M}, \tilde{A}')] \subset \tau_\theta H^{p+1}(\tilde{M}, \tilde{A})$.

Proof. Let $u=\{f\} \in \tau H^{p+1}(\tilde{M}, \tilde{A})$ have $u \cap \mu = 0$. Then $\delta_c u = 0$ and there are $\{f^\pm\} \in H^{p+1}(\tilde{M}, \mathcal{E}(\pm) \cup \tilde{A})$ with $f = f^+ - f^-$ in $\Delta^{p+1}(\tilde{M}, \tilde{A})$. Since $f$ induces the zero map $H_{p+1}(\tilde{M}, \tilde{A}) \to Z$, $f^\pm$ induce the same map $H_{p+1}(\tilde{M}, \tilde{A}) \to Z$. Hence $\phi(f^+) | Z_{p+1}\Delta_\sharp(\tilde{M}, \tilde{A})$ is well-defined and defines an element $f_\Lambda \in E^0 H_{p+1}(\tilde{M}, \tilde{A})$. Take an integer $m>0$ so that $(t^m-1)DH_p(\tilde{M}, \tilde{A})=0$. By UCES and Lemma 3.4, $(1-t^m)f_\Lambda = h\phi^*\{f^c\}$ for some $\{f^c\} \in H_c^{p+1}(\tilde{M}, \tilde{A})$. Then we have $f^+ - f^+ t^m = f^c$ on $Z_{p+1}\Delta_\sharp(\tilde{M}, \tilde{A})$. Define $f_m, f_m^\pm \in \Delta^{p+1}(\tilde{M}, \tilde{A})$ by $f_m(c) = \sum_{k=-\infty}^{+\infty} f^c(t^{km}c)$, $f_m^+(c) = \sum_{k=0}^{+\infty} f^c(t^{km}c)$ and $f_m = f_m^+ - f_m^-$. We have that $\delta(f_m^\pm) = 0 = f_m^\pm | \Delta_{p+1}(M_i^\mp \cup \tilde{A}, \tilde{A})$, taking $i$ so large that $f^c$ represents an element of $H^{p+1}(\tilde{M}, M_i^+ \cup M_i^- \cup \tilde{A})$. Moreovre, for $x \in Z_{p+1}\Delta_\sharp(\tilde{M}, \tilde{A})$, $f_m(x) = \sum_{k=-\infty}^{+\infty} f^c(t^{km}x) = \sum_{k=-\infty}^{+\infty}(f^+(t^{km}x) - f^+(t^{km+m}x)) = 0$ and similarly, $f_m^+(x) = f^+(x)$, so that $f_m^-(x) = f^+(x) = f^-(x)$. Let $f_0 = f - f_m$ and $f_0^\pm = f^\pm - f_m^\pm$. Then $f_0 = f_0^+ - f_0^-$ and $f_0^\pm$ represent elements of $H^{p+1}(\tilde{M}, M_i^\mp \cup \tilde{A})$ for a large $i$ and $f_0^\pm | Z_{p+1}\Delta_\sharp(\tilde{M}, \tilde{A}) = 0$. By construction, $f_m$ and $f_0$ represent elements $u_m$ and $u_0$ of $\tau H^{p+1}(\tilde{M}, \tilde{A})$, respectively. To prove that $u_m, u_0 \in \tau_\theta H^{p+1}(\tilde{M}, \tilde{A})$, let $x = \{c_1\} \in DH_p(\tilde{M}, \tilde{A})^\theta$, $c \in \Delta_{p+1}(\tilde{M}, \tilde{A})$ and $d(\neq 0) \in Z$ such that $\partial c = dc_1$. Since $t^m DH_p(\tilde{M}, \tilde{A})^\theta = DH_p(\tilde{M}, \tilde{A})^\theta$, we can find an element $\{c_1^+\}$ of $H_p(M_i^+ \cup \tilde{A}, \tilde{A})$ of finite order sending to $x$ under the natural map $H_p(M_i^+ \cup \tilde{A}, \tilde{A}) \to H_p(\tilde{M}, \tilde{A})$. That is, there are $h^+ \in \Delta_{p+1}(\tilde{M}, \tilde{A})$, $d_1(\neq 0) \in Z$ and $c^+ \in \Delta_{p+1}(M_i^+ \cup \tilde{A}, \tilde{A})$ such that $c_1 - c_1^+ = \partial h^+$ and $\partial c^+ = d_1 c_1^+$. Let $\bar{c}^+ = d_1 c - dd_1 h^+ - dc^+ \in \Delta_{p+1}(\tilde{M}, \tilde{A})$. Then $\partial \bar{c}^+ = 0$ in $\Delta_p(\tilde{M}, \tilde{A})$ and $f_0^+(\bar{c}^+) = d_1 f_0^+(c) - dd_1 f_0^+(h^+) - df_0^+(c^+) = 0$. But, $f_0^+(c^+) = 0$, so that $f_0^+(c) = df_0^+(h^+) \equiv 0 \pmod d$. Similarly, $f_0^-(c) \equiv 0 \pmod d$. Thus, $f_0(c) \equiv 0 \pmod d$, meaning that $u_0 \in \tau_\theta H^{p+1}(\tilde{M}, \tilde{A})$. To see that $u_m \in \tau_\theta H^{p+1}(\tilde{M}, \tilde{A})$, it suffices to show that $f_m(c_Q) \in Z$ for *some* $Q$-chains $c_Q \in \Delta_{p+1}(\tilde{M}; \tilde{A}, Q)$ with $\partial c_Q = c_1$, where $f_m$ is extended to a map $\Delta_{p+1}(\tilde{M}, \tilde{A}) \otimes Q \to Q$. We may take $c_1$ so that $c_1 = f_1^c \cap \tilde{z}$ for $\{f_1^c\} \in H_c^{s+2}(\tilde{M}, \tilde{A}')$ of finite order and $\tilde{z}$ in Lemma 2.4. Let $f_Q' \in \Delta_c^{s+1}(\tilde{M}, \tilde{A}'; Q)$ have $\delta f_Q' = f_1^c$. We use the same $\phi$ for the $Q$-extension of $\phi: \Delta_c^{s+1}(\tilde{M}, \tilde{A}') \to E^0\Delta_{s+1}(\tilde{M}, \tilde{A}')$. Then $\phi(f_Q'): \Delta_{s+1}(\tilde{M}, \tilde{A}') \to \Lambda_0$ induces a map $\hat{\phi}(f_Q'): H_{s+1}(\tilde{M}, \tilde{A}') \to \Lambda_0/\Lambda$. Since $\{c_1\} \in DH_p(\tilde{M}, \tilde{A})^\theta$, it follows from the definition of $\theta$ that there is a $\Lambda$-homomorphism $f_\Lambda'': H_{s+1}(\tilde{M}, \tilde{A}') \to \Lambda_0$ inducing $\hat{\phi}(f_Q')$. Note that the composite

$$H_c^{s+1}(\tilde{M}, \tilde{A}'; Q) \cong H_c^{s+1}(\tilde{M}, \tilde{A}') \otimes Q \overset{\phi^* \otimes 1}{\cong} H_\Lambda^{s+1}(\tilde{M}, \tilde{A}') \otimes Q \overset{h \otimes 1}{\to} E^0 H_{s+1}(\tilde{M}, \tilde{A}') \otimes Q$$
$$\cong \operatorname{Hom}_\Lambda(H_{s+1}(\tilde{M}, \tilde{A}'), \Lambda_0)$$

is onto by UCES. Let $\{f_Q''\} \in H_c^{s+1}(\tilde{M}, \tilde{A}'; Q)$ be a preimage of $f_\Lambda''$. Let $f_Q^c = f_Q' - f_Q''$. Then $f_Q^c$ induces the zero map $H_{s+1}(\tilde{M}, \tilde{A}') \to Q/Z$. Let $c_Q = \mathcal{E}(p+1) f_Q^c \cap \tilde{z} \in \Delta_{p+1}(\tilde{M}, \tilde{A}; Q)$. Then $\partial c_Q = \mathcal{E}(p+1)\mathcal{E}(s+1-n)\delta f_Q^c \cap \tilde{z} = c_1$ (cf. [Sp, p. 253]). Regarding $f_Q^c$ as a cocycle $\Delta_{s+1}(\tilde{M}, \tilde{A}') \to Q/Z$, we have in $Q/Z$

$$f_m(c_Q) = \textstyle\sum_{k=-\infty}^{+\infty} f^c(t^{km}c_Q) = \mathcal{E}(p+1) \sum_{k=-\infty}^{+\infty} \varepsilon_{\tilde{M}}((t^{km}f^c \cup f_Q^c) \cap \tilde{z})$$
$$= \mathcal{E}((p+1)s) \textstyle\sum_{k=-\infty}^{+\infty} \varepsilon_{\tilde{M}}((f_Q^c \cup t^{km}f^c) \cap \tilde{z}) = \mathcal{E}((p+1)s) \sum_{k=-\infty}^{+\infty} f_Q^c(t^{km}f^c \cap \tilde{z}) = 0\,,$$

for $t^{km}f^c \cap \tilde{z} \in Z_{s+1}\Delta_{\sharp}(\tilde{M}, \tilde{A}')$. Thus, $u_m \in \tau_\theta H^{p+1}(\tilde{M}, \tilde{A})$ and $u = u_m + u_0 \in \tau_\theta H^{p+1}$ $(\tilde{M}, \tilde{A})$. This completes the proof.

**Theorem 6.5.** *The maps* $e^1H_p(\tilde{M}, \tilde{A}) \xrightarrow{T} \tau H^{p+1}(\tilde{M}, \tilde{A}) \xrightarrow{\cap \mu} H_s(\tilde{M}, \tilde{A}')$ *induce isomorphisms*

$$e^1DH_p(\tilde{M}, \tilde{A})^\theta \overset{T}{\cong} \tau H^{p+1}(\tilde{M}, \tilde{A})/\tau_\theta H^{p+1}(\tilde{M}, \tilde{A}) \overset{\cap \mu}{\cong} DH_s(\tilde{M}, \tilde{A}')^\theta .$$

Proof. Let $\tau_K H^{p+1}$ be the kernel of $\cap \mu$. By Lemmas 6.2, 6.3 and 6.4, we obtain the following diagram:

$$\begin{array}{ccc} & & \tau H^{p+1}(\tilde{M}, \tilde{A})/\tau_K H^{p+1} \xrightarrow[\text{injection}]{\cap \mu} DH_s(\tilde{M}, \tilde{A}')^\theta \\ & & \downarrow \text{surjection} \\ e^1H_p(\tilde{M}, \tilde{A})/e^1(H_p(\tilde{M}, \tilde{A})/DH_p(\tilde{M}, \tilde{A})^\theta) & \overset{T}{\cong} & \tau H^{p+1}(\tilde{M}, \tilde{A})/\tau_\theta H^{p+1}(\tilde{M}, \tilde{A}) \\ \| & & \\ e^1DH_p(\tilde{M}, \tilde{A})^\theta & & \end{array}$$

Since $e^1DH_p(\tilde{M}, \tilde{A})^\theta \cong DH_p(\tilde{M}, \tilde{A})^\theta$ as abelian groups, it follows that $|DH_p(\tilde{M}, \tilde{A})^\theta| \leq |DH_s(\tilde{M}, \tilde{A}')^\theta|$. Interchanging the roles of $H_p(\tilde{M}, \tilde{A})$ and $H_s(\tilde{M}, \tilde{A}')$, we have $|DH_s(\tilde{M}, \tilde{A}')^\theta| = |DH_p(\tilde{M}, \tilde{A})^\theta|$. This means that $\tau_K H^{p+1} = \tau_\theta H^{p+1}(\tilde{M}, \tilde{A})$ and $\cap \mu$: $\tau H^{p+1}(\tilde{M}, \tilde{A})/\tau_\theta H^{p+1}(\tilde{M}, \tilde{A}) \cong DH_s(\tilde{M}, \tilde{A}')^\theta$. This completes the proof.

6.6. Proof of the Second Duality Theorem. By Theorem 6.5, we define a pairing

$$l\colon E^2E^2H_p(\tilde{M}, \tilde{A})^\theta \times E^2E^2H_s(\tilde{M}, \tilde{A}')^\theta = DH_p(\tilde{M}, \tilde{A})^\theta \times DH_s(\tilde{M}, \tilde{A}')^\theta \to Q/Z$$

by $l(x, y) = \varepsilon(s+1)f_x(y)$ for $f_x \in e^1DH_s(\tilde{M}, \tilde{A}')^\theta = \mathrm{Hom}_Z(DH_s(\tilde{M}, \tilde{A}')^\theta, Q/Z)$ with $\tau f_x \cap \mu = x \in DH_p(\tilde{M}, \tilde{A})^\theta$ and $y \in DH_s(\tilde{M}, \tilde{A}')^\theta$. By construction, $l$ has (2) and (4). For any $u_x \in H^s(\tilde{M}, \tilde{A}'; Q/Z)$ and $u_y \in H^p(\tilde{M}, \tilde{A}; Q/Z)$ with $\delta_{Q/Z}(u_x) \cap \mu = x$ and $\delta_{Q/Z}(u_y) \cap \mu = y$, we also have in $Q/Z$

$$l(x, y) = \varepsilon(s+1)\varepsilon_{\tilde{M}}((u_x \cup \delta_{Q/Z}(u_y)) \cap \mu) = \varepsilon_{\tilde{M}}((\delta_{Q/Z}(u_x) \cup u_y) \cap \mu)$$

(cf. [F, Lemma 3.8]). We have $l(x, y) = \varepsilon(ps+1)l(y, x)$, showing (3). (1) is obvious, since $\mu$ is invariant under a proper oriented homotopy equivalence $f\colon M_1 \to M_2$ with $f^*(\gamma_2) = \gamma_1$. This completes the proof.

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## References

[B] R.C. Blanchfield: *Intersection theory of manifolds with operators with applications to knot theory,* Ann. of Math. **65** (1957), 340–356.

[C/E] H. Cartan/S. Eilenberg: Homological algebra, Princeton University Press, 1956.

[E] S. Eilenberg: *Homology of spaces with operators,* I, Trans. Amer. Math. Soc. **61**

(1947), 378–417.
[F] M.Š. Farber: *Duality in an infinite cyclic covering and even-dimensional knots,* Math. USSR-Izv. **11** (1977), 749–781.
[J] J.P. Jans: Rings and homology, Athena Series, Holt, Rinehart and Winston, 1964.
[$Ka_1$] A. Kawauchi: *On a 4-manifold homology equivalent to a bouquet of surfaces,* Trans. Amer. Math. Soc. **262** (1980), 95–112.
[$Ka_2$] A. Kawauchi: *A test for the fundamental group of a 3-manifold,* J. Pure Appl. Algebra **28** (1983), 189–196.
[$Ka_3$] A. Kawauchi: *On the signature invariants of infinite cyclic coverings of closed odd dimensional manifolds,* Algebraic and Topological Theories-to the memory of Dr. T. MIYATA, Kinokuniya Co. Ltd. 1985, 52–85.
[$Ka_4$] A. Kawauchi: *On the signature invariants of infinite cyclic coverings of even dimensional manifolds,* Homotopy Theory and Related Topics, Advanced Studies in Pure Math. 9 (1986), 177–188.
[Ke] M.A. Kervaire: *Les noeuds de dimensions suprérieures,* Bull. Soc. Math. France **93** (1965), 225–271.
[K/S] R.C. Kirby/L.C. Siebenmann: Foundational essays on topological manifolds, smoothings, and triangulations, Ann. of Math. Studies, 88, Princeton University Press, 1977.
[L] J. Levine: *Knot modules.* I, Trans. Amer. Math. Soc. **229** (1977), 1–50.
[Ma] S.S. MacLane: Homology, Springer-Verlag OHG, Berlin, 1963.
[$Mi_1$] J.W. Milnor: *A duality theorem for Reidemeister torsion,* Ann. of Math. **76** (1962), 137–147.
[$Mi_2$] J.W. Milnor: *Infinite cyclic coverings,* Conference on the Topology of Manifolds, Prindle, Weber & Schmidt, 1968, 115–133.
[Se] C.S. Seshadri: *Triviality of vector bundles over the affine space $K^2$,* Proc. Nat. Acad. Sci. U.S.A. **44** (1958), 456–458.
[Sp] E.H. Spanier: Algebraic topology, McGraw-Hill, 1966.
[S/S] Y. Shinohara/D.W. Summners: *Homology invariants of cyclic coverings with applications to links,* Trans. Amer. Math. Soc. **163** (1972), 101–121.
[W] C.T.C. Wall: Surgery on compact manifolds, Academic Press, 1970.

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