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Optimal capital structure with earnings above a floor☆

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ABSTRACT

This paper derives the optimal capital structure of a firm whose earnings follow a geometric Brownian motion with a lower reflecting barrier. The barrier can be interpreted as a market intervention threshold (e.g., a price floor) by the government or an exit threshold of weak competitors in the market. Unlike in the standard model with no barrier, the firm is able to issue riskless debt to a certain capacity determined by the barrier. The higher the barrier, the larger the riskless debt capacity, and the firm prefers riskless capital structure rather than risky capital structure. Notably, with intermediate barrier levels, the firm can choose riskless capital structure with lower leverage than the level with no barrier. This mechanism can help explain debt conservatism observed in practice. The paper also entails several implications of public intervention by examining the lowest barrier (i.e., the weakest intervention) to achieve riskless capital structure.

1. Introduction

This paper analyzes an optimal capital structure model of a firm that receives stochastic flows of earnings above a floor. The floor can be interpreted as the government's intervention to protect particular companies or industries (e.g., utility, agricultural, or financial industries) against downside risks. Apart from regulated markets, the floor may represent an exit threshold of weak competitors. Then, the model can approximate a firm with a certain competitive advantage or protection against downside risks. In the model, we reveal that such a firm can determine capital structure through a mechanism that diverges from standard trade-off theory; indeed, a firm can maximize debt level within the riskless debt capacity generated by the floor. This paper sheds new light on capital structure of firms with competitive advantage or in regulated markets and suggests potential policies to prevent corporate bankruptcy efficiently.

The baseline model builds on the standard real options models of optimal capital structure (e.g., Goldstein, Ju, & Leland, 2001; Leland, 1994; Shibata & Nishihara, 2012). As in the standard literature, we assume that a firm has an option to issue consol debt at an initial time and that shareholders of the firm have an option to default debt in place. The firm's earnings are modeled by a geometric Brownian motion (GBM) with a lower reflecting barrier (i.e., a floor), which is a difference from the standard models. We also extend the baseline model to a model with a debt financing constraint. In the models with a barrier, we analytically derive the equity, debt, firm values, leverage, and credit spreads, as well as their sensitivities to barrier levels. The results are explained below.

A most notable difference from the standard results with no barrier (e.g., Goldstein et al., 2001; Leland, 1994; Shibata & Nishihara, 2012) is that a barrier generates a capacity of riskless debt financing. Naturally, the higher the barrier, the larger the riskless debt capacity. Compared to risky debt, riskless debt has an advantage of no bankruptcy cost but a disadvantage of the debt level being limited by the capacity. If the barrier is lower than a critical level (i.e., the riskless debt capacity is insufficient), then the firm prefers risky capital structure. In this case, the presence of a barrier hardly affects the equity, debt, firm values, leverage, and credit spreads because the firm chooses leverage by the standard trade-off between the tax benefits and bankruptcy costs of debt.

If the barrier is higher than the critical level (i.e., the riskless debt capacity is sufficient), then the firm prefers riskless capital structure. In the no-default case, barrier levels greatly affect all the values because the riskless debt capacity (depending on barrier levels) rather than the standard trade-off is a key determinant of capital structure. Notably,

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[1] As we will explain in Section 3.2, riskless debt in our theory can be identified as debt with very low default risk in empirics.

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the barrier close to the critical level leads to lower debt and leverage levels than the optimal levels with no barrier. That is, in contrast to the straightforward intuition that the firm increases debt with a floor, the firm voluntarily reduces debt to take advantage of having no bankruptcy risk. This result can help explain empirical observations of debt conservatism (e.g., Ghoul, El, Guedhami, Kwok, & Zheng, 2018; Graham, 2000; Strebulaev & Yang, 2013); some firms have quite low leverage and bankruptcy risk compared to the optimal levels predicted by standard trade-off theory. Indeed, our model suggests that firms do not choose risky capital structure based on the standard trade-off but optimally choose riskless capital structure with low leverage if they have certain degrees of competitive advantage or protection against downside risks.[1] Our theory can also help understand intra-industry variation in leverage (e.g., Graham & Leary, 2011; MacKay & Phillips, 2005). In fact, our result is consistent with empirical evidence that a firm that occupies a stronger position within its industry tends to have lower leverage (e.g., MacKay & Phillips, 2005; Mitani, 2014).

We also examine the comparative statics with respect to key parameters. With a given barrier level, a higher volatility, bankruptcy cost, lower growth rate, corporate tax rate, and stronger debt issuance constraint tend to lead to the no-default case. The switch to the no-default case can cause the comparative static results to differ from those of the standard trade-off models. For instance, with a constant barrier level, a higher volatility increases leverage and firm value in the no-default case because the barrier becomes more effective. Although this result is contrasted to the standard result, it can explain empirical findings of Ovtchinnikov (2010). Indeed, he found a strong positive relation between leverage and volatility in regulated markets, in contrast to the strong negative relation observed after deregulation. We argue that public protection against downside risks in regulated markets can lead to the positive relation.

This paper focuses on the lowest barrier to attain the no-default case because it can be interpreted as the weakest intervention by the government that prevents the firm from bankruptcy. We show that the critical level is lower than the level necessary to save the firm from bankruptcy ex post. This result emphasizes the importance of the ex ante information disclosure of the intervention policy. The appropriate commitment by the government leads a firm to adopt riskless capital structure with low leverage rather than leading the firm to take moral hazard behavior of increasing debt. A higher volatility and lower growth rate decrease the critical barrier level but increase the frequency of hitting the barrier. A higher bankruptcy cost, lower corporate tax rate, and stronger debt issuance constraint decrease advantages of risky debt, decreasing the critical barrier level and the frequency of hitting the barrier. These results suggest that the government can weaken and reduce market interventions with a more stringent bankruptcy law (i.e., a higher bankruptcy cost), lower corporate tax rate, and stronger leverage regulation.

Finally, we will briefly explain technically related literature. Leahy (1993) solves a competitive market model, where a price ceiling and floor arise as entry and exit thresholds in equilibrium. Dixit and Pindyck (1994) solve the entry and exit timing models with a price ceiling and floor and entail many implications of market competition and regulation. By extending the models, Dobbs (2004) shows that the optimal price ceiling delays investment in a monopoly, whereas Roques and Savva (2009) show that it accelerates investment in an oligopoly. Evans and Guthrie (2012) study a firm's production capacity adjustments under a price ceiling and quantity floor and show that with economies of scale, the firm invests in smaller, more frequent, increments than the social planner. Adkins, Paxson, Pereira, and Rodrigues (2019) examine the optimal duration of regulation in the investment timing model with a finite/retractable price ceiling and floor. Unlike this paper, the above papers assume all-equity firms and do not examine any capital structure problem.

Sarkar (2016) develops a Leland-type capital structure model with a price ceiling and shows that the price ceiling significantly increases leverage. He also shows that the price ceiling can counterintuitively decrease consumer welfare. In contrast to Sarkar (2016), we show that leverage can either increase or decrease (i.e., it can be nonmonotonic) with floor levels. Rodrigues (2025) is closest to our paper. He investigates the investment timing and capital structure model with a revenue ceiling and floor, which is more generalized than our baseline model. However, our paper has four advantages. First, we derive the explicit solutions and sensitivities, while he relies on numerical analysis. Second, we investigate the impact of a debt issuance constraint, which he does not address. Third, we show the positive relation between leverage and volatility in the no-default region, which is not observed in his paper. Finally, our reflecting barrier model, unlike the shadow process models in Rodrigues (2025) and Sarkar (2016), can capture not only public protection but also competitive advantage against downside risks.

The paper is organized as follows. Section 2 explains the model setup. In Section 3.1, we explain the solutions in the benchmark model with no barrier, and in Section 3.2, we derive the explicit solutions in the baseline model with a barrier. We also analytically derive the sensitivities to barrier levels. In Section 3.3, we derive the explicit solutions in the extended model with a debt issuance constraint. Section 4 examines the sensitivities to the key parameters numerically, and Section 5 concludes.

2. Model setup

The baseline model builds on the standard capital structure model based on trade-off theory (e.g., Goldstein et al., 2001; Leland, 1994). Tables 1 and 2 summarize the notations used throughout this paper. Consider a firm that receives continuous streams of earnings before interest and taxes (EBIT) $X(t)$ until bankruptcy. Under the risk-neutral measure, EBIT $X(t)$ follows a GBM[2]

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t)\ (t > 0), \quad X(0) = x \tag{1}$$

with lower reflecting barrier $x_L(> 0)$, where $B(t)$ denotes the standard Brownian motion defined in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ and growth rate μ, volatility $\sigma(> 0)$, and initial value $X(0) = x(\geq x_L)$ are constants. For convergence, $r > \mu$ is assumed, where a positive constant r denotes the risk-free interest rate, and $X(0) = x$ is assumed to be sufficiently high level so that the firm is not bankrupt at time 0.

At time 0, the firm issues consol debt to maximize the firm value, where the tax benefits and bankruptcy costs of debt will be clarified in the next section. For debt in place, shareholders can stop coupon payments (i.e., declare default) to maximize the equity value. In the default case, shareholders receive nothing, and debt holders receive the post-bankruptcy firm value, which is equal to the unlevered firm value multiplied by $(1-\alpha)$. This means that a fraction $\alpha \in (0,1)$ of the unlevered firm value is lost to the deadweight costs of bankruptcy. Equity, debt, and firm values are fairly priced based on the rational expectation of ex post shareholders' default behavior.[3]

The presence of lower reflecting barrier x_L is a difference from the standard capital structure model. Intuitively, lower reflecting barrier x_L means that $X(t)$ is pulled back to x_L and moves again from x_L immediately after $X(t)$ falls below x_L. It is different from the assumption

[2] Debt financing models (e.g., Charalambides & Koussis, 2018; Eisdorfer, Morellec, & Zhdanov, 2024; Elkamhi, Kim, & Salerno, 2024; Goldstein et al., 2001; Hackbarth, Mathews, & Robinson, 2014; Jeon, 2021) often assume EBIT following a GBM, although Luo, Wang, and Yang (2016) assume EBIT with jump risk.

[3] For simplicity, this paper assumes market debt that is nonrenegotiable. Such papers as Morellec, Valta, and Zhdanov (2015) and Shibata and Nishihara (2015) also consider bank debt that is negotiable. This paper's main results (i.e., the no-default case) remain unchanged even if bank debt is considered. The results may change when lenders have market power as modeled in Huberts, Wen, Dawid, Huisman, and Kort (2025).

that $X(t)$ equals a GBM $S(t)$ (i.e., the shadow process) for $S(t) \geq x_L$ but remains at x_L for $S(t) < x_L$. Although the shadow process model requires the value function in regions $S(t) < x_L$ and $S(t) \geq x_L$, the reflecting barrier model requires only one region, $X(t) \geq x_L$.[4] By this technical simplicity, we can solve the model explicitly in the next section. In addition, as we will explain below, the reflecting barrier model, unlike the shadow process model, can approximate a firm that occupies a strong position within its industry.

The model with barrier x_L can approximate the following two situations. First, governments might try to intervene in markets to protect specific firms or industries (e.g., utility, agricultural, or financial industries) against downside risks. For instance, European Union countries' governments purchase particular agricultural products to prevent their prices from dropping to unsustainably low levels. Although such market interventions require direct and indirect costs, governments can adopt the market measures if the bankruptcy costs of these firms, including indirect costs, such as threats to national security, are higher than the intervention costs. In this case, x_L is interpreted as the intervention threshold.[5] Chapter 9 of Adkins et al. (2019), Dixit and Pindyck (1994), and Rodrigues (2025) also examine real options models with floors (and ceilings) in terms of public intervention. Sections 3.3 and 4.6 study the effects of leverage regulation in addition to the public intervention by incorporating a debt issuance constraint into the baseline model.

Second, the baseline model may capture firms with strong competitive advantage against downside risks. For instance, consider oil prices. Relatively weak shale oil producers tend to exit the markets when oil prices fall to unsustainably low levels for them. After the exit of shale oil producers, oil prices are likely to rebound. Thus, the biggest oil companies, which have sufficient competitive advantage to survive downturns, could receive cash flows above certain levels. More generally, the cash flow dynamics of resilient firms may have such a trend. Our model can formally represent a market with one strong firm and an infinite number of weaker firms, where x_L arises as the weak firms' exit threshold in equilibrium, as in Leahy (1993). Leahy (1993) examines a competitive market with homogeneous firms with no leverage. Baldursson (1998) and Chapter 8 of Dixit and Pindyck (1994) further explore competitive and oligopoly markets using reflecting barriers, providing theoretical foundations for our model structure.

Finally, we clarify model limitations. While incorporating an upper reflecting barrier to represent a price cap in regulated markets or the entry threshold for small firms is logical, the presence of both upper and lower reflecting barriers prevents the explicit derivation of model solutions. To analytically prove the intriguing effects of a revenue floor on valuations and capital structure, we have structured the baseline model to include only a floor. Appendix F numerically verifies that the baseline results are unchanged when extending the model to include both a floor and ceiling. Another potential extension involves examining the firm's dynamic leverage adjustments and the government's dynamic intervention policy to prevent bankruptcy. However, we acknowledge that this would significantly complicate the model, and we currently lack a solution framework for it.

3. Model solutions

3.1. EBIT with no barrier

This subsection explains the benchmark model with no barrier (i.e., $x_L = 0$). The following results are well known in previous literature (e.g., Goldstein et al., 2001; Shibata & Nishihara, 2012; Sundaresan, Wang, & Yang, 2015), and hence, the details of derivation are omitted. First, suppose that the firm issues consol debt with coupon C. For given coupon C, the equity, debt, and firm values are expressed as

$$E_0(x;C) = \pi x - \frac{(1-\tau)C}{r} + \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\frac{(1-\tau)C}{r} - \pi x_0(C)\right) \tag{2}$$

$$D_0(x;C) = \frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\frac{C}{r} - (1-\alpha)\pi x_0(C)\right) \tag{3}$$

$$F_0(x;C) = \pi x + \frac{\tau C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\alpha\pi x_0(C) + \frac{\tau C}{r}\right) \tag{4}$$

for $x \geq x_0(C)$, where τ denotes a corporate tax rate, $x_0(C)$ denotes the default threshold, and

$$\pi = \frac{1-\tau}{r-\mu}, \tag{5}$$

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{6}$$

denote the unlevered firm value's coefficient (i.e., πx is the unlevered firm value) and a negative characteristic root, respectively. Subscript 0 stands for the benchmark model with no barrier. The first, second, and last terms in equity value (2) correspond to the unlevered firm value, perpetual coupon payments, and the value of the default option, respectively. The first and second terms in debt value (3) are the perpetual coupon receipts (i.e., the riskless debt value) and loss due to default risk, respectively. The first, second, and last terms in firm value (4) are the unlevered firm value, perpetual tax benefits of debt, and bankruptcy costs, respectively. Note that shareholders determine $x_0(C)$ to maximize its own value $E_0(x;C)$ for debt in place. By solving $\arg\max_{x_0(C)\geq 0} E_0(x;C)$, we obtain default threshold

$$x_0(C) = C/\delta, \tag{7}$$

where δ is a constant given by

$$\delta = \frac{(\gamma-1)r}{\gamma(r-\mu)} \ (>1). \tag{8}$$

Now, consider the optimal capital structure. The firm chooses coupon C to maximize firm value $F_0(x;C)$ based on the trade-off between the tax benefits and bankruptcy costs of debt. By solving $\arg\max_{C\geq 0} F_0(x;C)$, we obtain optimal coupon

$$C_0(x) = \delta x/h, \tag{9}$$

where h is a constant given by

$$h = \left[1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tau}\right)\right]^{-\frac{1}{\gamma}} \ (>1). \tag{10}$$

The optimally levered firm value $F_0(x)$ is

$$F_0(x) = F_0(x;C_0(x)) = \psi\pi x, \tag{11}$$

where ψ is a constant given by

$$\psi = 1 + \frac{\tau}{(1-\tau)h} \ (>1) \tag{12}$$

and is interpreted as the leverage effect. Indeed, the levered firm value $F_0(x)$ is the unleverd firm value πx multiplied by $\psi(>1)$. From (2), (3), and (9), the optimally levered equity and debt values, $E_0(x) = E_0(x;C_0(x))$ and $D_0(x) = D_0(x;C_0(x))$, respectively, also become linear functions of initial EBIT x, and the firm's leverage $LV_0(x) = D_0(x)/F_0(x)$ and credit spreads $CS_0(x) = C_0(x)/D_0(x) - r$ are independent of x, although the detailed expressions are omitted for space limitations.

3.2. EBIT with a lower reflecting barrier

This subsection solves the baseline model with lower reflecting barrier $x_L = k_L x$, where we use the notation $k_L \in (0,1)$ to simplify

[4] We do not think that the technical difference greatly affects the results. Indeed, Rodrigues (2025), who studies a more complicated model based on the shadow process, find similar results to our results. For instance, a higher floor also leads to riskless capital structure in Rodrigues (2025).

[5] When the product price follows a GBM with a floor, EBIT also follows a GBM with a floor in the standard setups (e.g., Dixit & Pindyck, 1994). Then, for simplicity, this paper directly assumes EBIT with a floor.

Table 1
Notations. The table also includes the baseline parameter values used in Section 4.

Notation	Description	Baseline value
Parameters:		
$x = X(0)$	Initial value of EBIT $X(t)$ following GBM (1).	1
μ, σ	Growth rate and Volatility of EBIT $X(t)$.	0.01, 0.2
r	Risk-free interest rate.	0.05
τ	Corporate tax rate.	0.15
α	Fraction of bankruptcy costs to the unlevered firm value.	0.4
$x_L = k_L x$	Lower reflecting barrier.	0.2
$\bar{C} = k_C x$	Upper limit of coupon.	
Constants:		
π	Unlevered firm value's coefficient (5).	21.25
γ	Negative constant (6).	−1.351
δ	Positive constant (8).	2.175
h	Positive constant (10).	3.491
ψ	Positive constant (12).	1.051

Table 2
Notations. The table also presents the baseline values in Section 4 , computed based on the parameter values in Table 1.

Notation	Description	Baseline value
No-barrier model:		
$E_0(x;C)$	Equity value (2) for coupon C.	
$D_0(x;C)$	Debt value (3) for coupon C.	
$F_0(x;C)$	Firm value (4) for coupon C.	
$x_0(C)$	Default threshold (7) for coupon C.	
$C_0(x)$	Optimal coupon (9).	0.623
$E_0(x), D_0(x), F_0(x)$	Equity, Debt, and Firm values for optimal coupon $C_0(x)$.	11.49, 10.84, 22.32
$LV_0(x)$	Leverage.	0.485
$CS_0(x)$	Credit spreads.	0.00751
Baseline model:		
$E_d(x;C)$	Equity value (14) for coupon C in the default-possible case.	
$D_d(x;C)$	Debt value (15) for coupon C in the default-possible case.	
$F_d(x;C)$	Firm value (16) for coupon C in the default-possible case.	
$E_n(x;C)$	Equity value (17) for coupon C in the no-default case.	
$D_n(x;C)$	Debt value (18) for coupon C in the no-default case.	
$F_n(x;C)$	Firm value (19) for coupon C in the no-default case.	
$E(x;C)$	Equity value for coupon C.	
$D(x;C)$	Debt value for coupon C.	
$F(x;C)$	Firm value for coupon C.	
$C(x)$	Optimal coupon.	0.435
$E(x), D(x), F(x)$	Equity, Debt, and Firm values for optimal coupon $C(x)$.	14.21, 8.7, 22.91
$LV(x)$	Leverage.	0.38
$CS(x)$	Credit spreads.	0
Constrained model:		
$\bar{C}(x)$	Optimal coupon.	
$\bar{E}(x), \bar{D}(x), \bar{F}(x)$	Equity, Debt, and Firm values for optimal coupon $\bar{C}(x)$.	
$\bar{LV}(x)$	Leverage.	
$\bar{CS}(x)$	Credit spreads.	

the equations derived in this section. First, suppose that the firm issues debt with coupon C. For given C, shareholders choose whether they default. Then, equity value $E(x;C)$ is expressed as

$$E(x;C) = \max\{E_d(x;C), E_n(x;C)\}, \tag{13}$$

where $E_d(x;C)$ and $E_n(x;C)$ represent the equity values in the default-possible and no-default cases, which will be defined later. The next proposition shows the equity, debt, and firm values, denoted by $E(x;C)$, $D(x;C)$, and $F(x;C)$, respectively, for given coupon C. For proof, see Appendix A.

Proposition 1. *For $C > \delta k_L x$, the firm goes bankrupt at default threshold $x_0(C) = C/\delta$ (i.e., the default-possible case). The equity, debt, and firm values are given by*

$$E(x;C) = E_d(x;C) = E_0(x;C), \tag{14}$$

$$D(x;C) = D_d(x;C) = \underbrace{\frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\frac{C}{r} - (1-\alpha)\pi x_0(C)\right)}_{=D_0(x;C)} - \frac{k_L^{1-\gamma}(1-\alpha)\pi x}{\gamma}, \tag{15}$$

$$F(x;C) = F_d(x;C) = \underbrace{\pi x + \frac{\tau C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma}\left(\alpha\pi x_0(C) + \frac{\tau C}{r}\right)}_{=F_0(x;C)} - \frac{k_L^{1-\gamma}(1-\alpha)\pi x}{\gamma}. \tag{16}$$

Otherwise, the firm never goes bankrupt (i.e., the no-default case). The equity, debt, and firm values are given by

$$E(x;C) = E_n(x;C) = \pi x - \frac{(1-\tau)C}{r} - \frac{k_L^{1-\gamma}\pi x}{\gamma}, \tag{17}$$

$$D(x;C) = D_n(x;C) = \frac{C}{r}, \tag{18}$$

$$F(x;C) = F_n(x;C) = \pi x + \frac{\tau C}{r} - \frac{k_L^{1-\gamma}\pi x}{\gamma}. \tag{19}$$

It follows from (7) that $C > \delta k_L x$ is equivalent to $x_0(C) > k_L x$. First, we explain the default-possible case, in which $x_0(C)$ is higher than $k_L x$. Equity value $E_d(x;C)$ does not depend on k_L because $X(t)$ does not hit the barrier before bankruptcy. Then, $E_d(x;C)$ is the same as the benchmark value $E_0(x;C)$ (see (14)). However, debt and firm values, $D_d(x;C)$ and $F_d(x;C)$, respectively, benefit by the barrier. The first and second terms in (15) coincide with $D_0(x;C)$, and the last term is the value added by the barrier (note that $\gamma < 0$). The additional value arises from the fact that the post-default value, which debt holders obtain, increases with higher barrier level k_L.[6] Indeed, in (28) in Appendix A, the post-default value $(1-\alpha)F_n(x_0(C);0)$ is higher than $(1-\alpha)\pi x_0(C)$ (i.e., the post-default value in the model with no barrier). Firm value $F_d(x;C)$ has the same benefit from the barrier (see the last terms in (15) and (16)) because of $F_d(x;C) = E_0(x;C) + D_d(x;C)$.

Now, we explain the no-default case. By $C \le \delta k_L x$, $E_n(k_L x;C) \ge 0$ holds in (17). Then, for any $X(t) \ge k_L x$, shareholders are better off continuing operation with coupon payments rather than declaring default. The first, second, and third terms in (17) represent the unlevered firm value, perpetual coupon payments, and value created by the barrier (note that $\gamma < 0$). Shareholders benefit by the barrier. Debt holders also benefit by the barrier because it removes the default risk. Then, $D_n(x;C)$ agrees with the riskless debt value in (18). In (19), firm value $F_n(x;C)$ consists of the unlevered firm value, perpetual tax benefits, and additional value by the barrier. Unlike $F_0(x;C)$, $F_n(x;C)$ does not include any term representing bankruptcy costs.

Proposition 1 implies that $D_n(x;\delta k_L x) = \delta k_L x/r$ is the capacity of riskless debt. Of course, for $C \le k_L x$, the firm always receives nonnegative cash flows $X(t) - C$, and hence, shareholders continue operation perpetually. Note that $\delta > 1$. Considering the possibility that $X(t)$ goes beyond $k_L x$ due to volatility σ, shareholders prefer to operate perpetually for $C \le \delta k_L x$. Indeed, the expected cash flows of perpetual operation are nonnegative (i.e., $E_n(x;C) \ge 0$) for $C \le \delta k_L x$. This is how the presence of barrier $x_L = k_L x$ creates the riskless debt capacity $\delta k_L x/r$. Proposition 1 nests the benchmark case with no barrier as the limiting case of $k_L \to 0$. Indeed, $\lim_{k_L\to 0} E(x;C) = E_0(x;C), \lim_{k_L\to 0} D(x;C) = D_0(x;C)$, and $\lim_{k_L\to 0} F(x;C) = F_0(x;C)$ hold.

Next, consider the optimal capital structure. We need to solve $\max_{C\ge 0} F(x;C)$. By (9) and (16), we have $\arg\max_{C\ge 0} F_d(x;C) = \arg\max_{C\ge 0} F_0(x;C) = C_0(x)$, which reflects the standard trade-off between the tax benefits and bankruptcy costs of debt. By (19), $F_n(x;C)$ increases linearly in C, implying that $\arg\max_{C\in[0,\delta k_L x]} F_n(x;C) = \delta k_L x$. This reflects the fact that a higher debt level increases firm value via greater tax benefits in the no-default case. Comparing (16) and (19), we have $F_d(x;C) < F_n(x;C)$ for any (x,C) because $F_d(x;C)$, unlike $F_n(x;C)$, includes the term of bankruptcy costs. Therefore, $\max_{C\ge 0} F(x;C) = \max\{F_d(x;C_0(x)), F_n(x;\delta k_L x)\}$ holds. For graphical images of $F(x;C)$, see Fig. 1. By (11) and (16), we obtain

$$F_d(x;C_0(x)) = \left(\psi - \frac{k_L^{1-\gamma}(1-\alpha)}{\gamma}\right)\pi x, \tag{20}$$

and by (19), we obtain

$$F_n(x;\delta k_L x) = \left(1 + \frac{\tau\delta k_L}{r\pi} - \frac{k_L^{1-\gamma}}{\gamma}\right)\pi x. \tag{21}$$

From (12), (20), and (21), we have

$$F_n(x;\delta x_L) - F_d(x;C_0(x)) = g(k_L)\pi x,$$

where

$$g(k_L) = \frac{\tau\delta k_L}{r\pi} - \frac{\tau}{(1-\tau)h} - \frac{k_L^{1-\gamma}\alpha}{\gamma}. \tag{22}$$

The next proposition shows the equity, debt, firm values, coupon, leverage, and credit spreads, denoted by $E(x), D(x), F(x), C(x), LV(x)$, and $CS(x)$ respectively, under optimal capital structure. For proof, see Appendix B.

Proposition 2. *There exists a unique solution $k_L^* \in (0, \gamma/((\gamma-1)h))$ to $g(k_L^*) = 0$, and $k_L < k_L^*$ is equivalent to $g(k_L) < 0$.*

For $k_L < k_L^*$, the firm issues risky debt with coupon $C_0(x)$ and goes bankrupt at default threshold $x_0(C_0(x)) = x/h$ (i.e., the default-possible case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $E(x) = E_0(x;C_0(x)), D(x) = D_d(x;C_0(x)), F(x) = F_d(x;C_0(x)), C(x) = C_0(x), LV(x) = D_d(x;C_0(x))/F_d(x;C_0(x))$, and $CS(x) = C_0(x)/D_d(x;C_0(x)) - r$, respectively.

Otherwise, the firm issues riskless debt with coupon $\delta k_L x$ and never goes bankrupt (i.e., the no-default case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $E(x) = E_n(x;\delta k_L x), D(x) = \delta k_L x/r, F(x) = F_n(x;\delta k_L x), C(x) = \delta k_L x, LV(x) = \delta k_L x/(rF_n(x;\delta k_L x))$, and $CS(x) = 0$, respectively.

In Proposition 2, firm value $F(x) = \max\{(20),(21)\}$ is a linear function of initial EBIT $x = X(0)$. As in (11), $F(x)$ is the unlevered firm value (i.e., πx) multiplied by the constant that represents the leverage and barrier effects. Similarly, $E(x), D(x)$, and $C(x)$ are linear with respect to x, and $LV(x)$ and $CS(x)$ are independent of x.

First, we explain the default-possible case (i.e., $k_L < k_L^*$). The firm prefers to issue risky debt due to the insufficient riskless debt capacity. In this case, the optimal coupon and default timing are the same as those of the benchmark model with no barrier. The other values are obtained by substituting coupon $C_0(x)$ into the default-possible case of Proposition 1. Note that $C_0(x) > \delta k_L x$ holds by $F_n(x;\delta k_L x) < F_d(x;C_0(x)) < F_n(x;C_0(x))$.

Next, we focus on the no-default case (i.e., $k_L \ge k_L^*$). The firm is better off issuing riskless debt $\delta k_L x/r$ because of the sufficient riskless debt capacity. Notably, $\delta k_L x$ is not necessarily higher than $C_0(x)$. Both $\delta k_L x < C_0(x)$ and $F_n(x;\delta k_L x) \ge F_d(x;C_0(x))$ can be satisfied in (20) and (21) because $F_n(x;\delta k_L x)$, unlike $F_d(x;C_0(x))$, includes no term of bankruptcy cost α (e.g., see the top panel of Fig. 1). In other words, the firm chooses riskless capital structure if the gain from having no bankruptcy risk dominates the inefficiency from the upper limit of riskless debt.

In Proposition 2, k_L^* stands for the lowest level to achieve the no-default case. The critical barrier $k_L^* x$ is lower than the benchmark default threshold $x_0(C_0(x)) = x/h$ by $k_L^* < \gamma/((\gamma-1)h) < 1/h$. This result has the following implication for public intervention. Suppose that the government attempts to prevent the firm from bankruptcy. Without commitment to an intervention threshold, the firm issues debt with coupon $C_0(x)$ (as in the benchmark case with no barrier). In this scenario, by Proposition 1 with $C = C_0(x)$, the government needs bailout threshold $x_L = C_0(x)/\delta = x/h$ to prevent the firm from bankruptcy. In contrast, the government can prevent the firm from bankruptcy by committing to bailout threshold $k_L^* x (< x/h)$. This is because by considering the committed barrier, the firm strategically reduces debt (i.e., $\delta k_L^* x < C_0(x)$) and chooses riskless capital structure. Although in both cases, the bailout happens precisely when bankruptcy would occur without intervention, our model shows that government commitment can effectively lower the firm's leverage and the frequency of bailouts. As we will check numerically in Section 4, the ex ante required level $k_L^* x$ is much lower than the ex post required level x/h, which highlights the importance of the credible commitment to the public bailout policy.[7]

We can analytically prove the comparative statics with respect to barrier level k_L because Proposition 2 derives all the values explicitly. For proof, see Appendix C.

[6] As in Goldstein et al. (2001) and Leland (1994), our model does not specify either liquidation or reorganization bankruptcy but assumes that the post-default firm value is the unlevered value discounted by bankruptcy costs. The unlevered value increases in barrier level k_L.

[7] This paper focuses on the public intervention policy to prevent the firm from bankruptcy. An alternative policy of annual fixed subsidies does not necessarily prevent default (see Appendix G), while subsidies are effective in the context of investment (e.g., Lukas & Thiergart, 2019; Zhang, Chronopoulos, Kyriakou, & Dimitrova, 2024).

Proposition 3. *For $k_L < k_L^*$ (i.e., the default-possible case), $D(x), F(x)$, and $LV(x)$ increase in x_L, $CS(x)$ decreases in k_L, and $C(x) = C_0(x)$, $x_0(C_0(x)) = x/h$, and $E(x) = E_0(x; C_0(x))$ are constant.*

At $k_L = k_L^*$ (i.e., the switching point), $E(x)$ jumps upward, $D(x), C(x)$, $LV(x)$, and $CS(x)$ jump downward, and $F(x)$ is continuous.

For $k_L \geq k_L^*$ (i.e., the no-default case), $D(x), F(x), C(x) = \delta k_L x$, and $LV(x)$ increase in k_L, $E(x)$ decreases in k_L, and $CS(x)$ is 0.

Note that the values approach the benchmark values with no barrier for $k_L \to 0$. At $k_L = k_L^*$, the values, except firm value $F(x)$, jump because the firm switches coupon $C(x)$ from $C_0(x)$ (i.e., the default-possible case) to $\delta k_L^* x$ (i.e., the no-default case). The switch results from maximization of $F(x)$, and hence, the switch does not cause a jump in $F(x)$. Interestingly, $E(x), D(x), C(x)$, and $LV(x)$ are nonmonotonic with respect to k_L because of the switch. Section 4.1 will show the quantitative effects of k_L on the results in numerical examples.

The no-default case with k_L close to k_L^* is most intriguing. In this region, $D(x)$ and $LV(x)$ are lower than $D_0(x)$ and $LV_0(x)$ because of $\delta k_L^* x < C_0(x)$. As explained previously, this result implies that by the credible commitment to the market intervention threshold, the government can decrease the firm's debt and remove its bankruptcy risk.

Furthermore, this result can help explain debt conservatism. It is well known as debt conservatism that many firms have low leverage and bankruptcy risk compared to the optimal level predicted by trade-off theory (e.g., Ghoul et al., 2018; Graham, 2000; Strebulaev & Yang, 2013). For instance, Ghoul et al. (2018) observe that about 40% of firms have nonpositive net debt, which implies very low default risk. Myers (2001) criticize trade-off theory, stating, "if theory is right, a value-maximizing firm should never pass up interest tax shields when the probability of financial distress is remotely low. Yet there are many established, profitable companies with superior credit ratings operating for years at low debt ratios, including Microsoft and the major pharmaceutical companies".

Debt conservatism is often explained by theories of dynamic (and infrequent) leverage adjustment and financial slack for future investments and downside risks, but our model adds an alternative mechanism. Indeed, firms can optimally choose low leverage with remotely low default risk if they have a certain degree of competitive advantage or protection against downside risks. Here, as in Leary and Roberts (2010), riskless debt in our model should be understood as debt with very low default risk rather than debt entirely without risk. In our model, due to the assumption that the government will intervene at the barrier or that the firm's strong market position prevents EBIT from falling below the threshold. However, in practice, a bailout may fail, or a firm may lose its market position due to unforeseen events (e.g., corporate scandals), resulting in potential bankruptcies. Empirical findings by Ovtchinnikov (2010) and Sanyal and Bulan (2011) indicate significantly lower bankruptcy probabilities for regulated firms, aligning with our model's outcome of riskless debt.

Our result can also help explain the empirical relation between competitive advantage and leverage. It is widely observed that leverage ratios vary across firms within an industry (e.g., Graham & Leary, 2011; MacKay & Phillips, 2005). In the oil industry, relatively weak shale oil producers tend to have much higher leverage and bankruptcy risk than those of biggest oil companies. More broadly, as Myers (2001) criticized, strong firms appear to forgo tax benefits despite their very low default probabilities. MacKay and Phillips (2005) show that incumbents tend to have lower leverage and higher profitability than entrants and exiters, whereas Mitani (2014) shows that firms with higher market share tend to have lower leverage. These empirical observations are frequently explained through theories related to the difficulties weak firms face in executing debt buybacks, debt overhang, and dynamic (and infrequent) leverage adjustments. Our model, however, provides an alternative explanation for these phenomena.

Following the standard models (e.g., Goldstein et al., 2001; Leland, 1994), our model assumes the post-bankruptcy value as the discounted value of the unlevered value. However, some papers, including Lambrecht and Myers (2008), Mella-Barral and Perraudin (1997), Nishihara and Shibata (2021), and Shibata and Nishihara (2018) assume a constant component of the post-bankruptcy value (e.g., constant scrap value). The presence of constant liquidation value generates the possibility of riskless debt financing, but its mechanism is different from that of this paper. In these models, shareholders can retire the principal of debt by a part of the constant liquidation value and obtain the residual value. In such a situation, the firm exits the market, but debt becomes riskless.[8] In contrast, in our model, protection against downside risks can lead the firm to operate perpetually in the market, which makes debt riskless.

Finally, it should be noted that our model does not help resolve the credit spread puzzle. The credit spread puzzle means that observed credit spreads are much higher than those implied by structural models (especially those based on a GBM). High observed spreads require unrealistically high default probabilities in structural models. Specifically, historical default probabilities of investment grade bonds are much lower than the model-implied default probabilities. Many studies, such as Huang and Huang (2001), argue that observed spreads incorporate components beyond default probabilities (e.g., liquidity and asset risk premiums), framing the puzzle as a pricing issue. As in the standard structural models (e.g., Goldstein et al., 2001; Leland, 1994), our model does not account for liquidity and asset risk premiums. Therefore, while our model explains why firms with very low default probabilities may adopt conservative leverage, it does not address why such firms' credit spreads remain elevated despite these low default probabilities. This pricing question lies outside our paper's scope.

3.3. Debt issuance constraint

This subsection interprets $x_L = k_L x$ as the public intervention threshold. In such regulated markets (e.g., utility, agricultural, and financial industries), governments might not only save firms from financial distress but also regulate excessive uses of debt to remove bankruptcy risk. To explore the effects of such a leverage regulation on the outcome, this subsection extends the baseline model to a model with an upper limit of debt issuance. The extended model assumes that coupon C must satisfy $C \leq \bar{C} = k_C x$ for a given upper limit $\bar{C} = k_C x$, where we use the notation $k_C (> 0)$ to simplify the equations derived in this section. The constraint is interpreted as a constraint on the book value of debt (i.e., $C/r \leq k_C x/r$). However, as we will see in Section 4.5, $D(x)$ and $LV(x)$ monotonically increases in k_C. Hence, the results will remain unchanged even if we assume a constraint on the market value of debt or leverage. Assume that $\bar{C} < C_0(x)$, i.e., $k_C < \delta/h$ because the firm is unconstrained otherwise.

For $k_C \leq \delta k_L$, the firm optimally chooses the maximum coupon $k_C x$ and obtain firm value $F_n(x; k_C x)$ because there is no possibility of bankruptcy (see Proposition 1). For $k_C \in (\delta k_L, \delta/h)$, the firm solves $\max\{F_d(x; k_C x), F_n(x; \delta k_L x)\}$ because $F_d(x; C)$ increases in $C \leq C_0(x) = \delta x/h$. It follows from (16) that

$$F_d(x; k_C x) = \left(1 + \frac{\tau k_C}{r\pi} - \left(\frac{k_C}{\delta}\right)^{1-\gamma}\left(\alpha + \frac{\tau k_C \delta}{r\pi}\right) - \frac{k_L^{1-\gamma}(1-\alpha)}{\gamma}\right)\pi x. \tag{23}$$

By (21) and (23), we have

$$F_n(x; \delta k_L x) - F_d(x; k_C x) = \bar{g}(k_L)\pi x,$$

[8] Luo et al. (2016) and Wang, Yang, and Zhang (2015) examine joint investment and financing decisions with guarantee contracts. With such guarantee contracts, debt is also riskless for lenders because insurers will compensate lenders for loss due to bankruptcy.

where

$$\bar{g}(k_L) = \frac{\tau(\delta k_L - k_C)}{r\pi} + \left(\frac{k_C}{\delta}\right)^{1-\gamma}\left(\alpha + \frac{\tau k_C \delta}{r\pi}\right) - \frac{k_L^{1-\gamma}\alpha}{\gamma}. \tag{24}$$

The next proposition shows the equity, debt, firm values, coupon, leverage, and credit spreads, denoted by $\bar{E}(x), \bar{D}(x), \bar{F}(x), \bar{C}(x), \bar{LV}(x)$, and $\bar{CS}(x)$, respectively, under the debt issuance constraint. For proof, see Appendix D.

Proposition 4. *There exists a unique solution $\bar{k}_L \in (0, \min\{k_L^*, k_C/\delta\})$ to $\bar{g}(\bar{k}_L) = 0$. The solution $\bar{k}_L$ increases in k_C.*

For $k_L < \bar{k}_L$, the firm issues risky debt with coupon $k_C x$ and goes bankrupt at default threshold $x_0(k_C x)$ (i.e., the default-possible case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $\bar{E}(x) = E_0(x; k_C x), \bar{D}(x) = D_d(x; k_C x), \bar{F}(x) = F_d(x; k_C x), \bar{C}(x) = k_C x, \bar{LV}(x) = D_d(x; k_C x)/F_d(x; k_C x)$, and $\bar{CS}(x) = k_C x/D_d(x; k_C x) - r$, respectively.

For $k_L \in [\bar{k}_L, k_C/\delta]$, the firm riskless debt with coupon $\delta k_L x$ and never goes bankrupt (i.e., the no-default case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $\bar{E}(x) = E_n(x; \delta k_L x), \bar{D}(x) = \delta k_L x/r, \bar{F}(x) = F_n(x; \delta k_L x), \bar{C}(x) = \delta k_L x, \bar{LV}(x) = \delta k_L x/(rF_n(x; \delta k_L x))$, and $\bar{CS}(x) = 0$, respectively.

For $k_L > k_C/\delta$, the firm riskless debt with coupon $k_C x$ and never goes bankrupt (i.e., the no-default case). The equity, debt, firm values, coupon, leverage, and credit spreads are given by $\bar{E}(x) = E_n(x; k_C x), \bar{D}(x) = k_C x/r, \bar{F}(x) = F_n(x; k_C x), \bar{C}(x) = k_C x, \bar{LV}(x) = k_C x/(rF_n(x; k_C x))$, and $\bar{CS}(x) = 0$, respectively.

As in Proposition 2, in each case of Proposition 4, $\bar{F}(x), \bar{E}(x), \bar{D}(x)$, and $\bar{C}(x)$ are linear with respect to initial EBIT $x = X(0)$, and $\bar{LV}(x)$ and $\bar{CS}(x)$ are independent of x. We can interpret $\bar{k}_L$ as the lowest level to achieve the no-default case. For $k_L < \bar{k}_L$ (i.e., the default-possible case), riskless debt capacity $\delta k_L x/r$ is not sufficient, and hence, the firm issues risky debt with the maximum coupon $k_C x$. For $k_L \in [\bar{k}_L, k_C x/\delta]$ (i.e., the no-default case), riskless debt capacity $\delta k_L x/r$ is large enough to lead the firm to choose riskless debt. For $k_L > k_C/\delta$ (i.e., the no-default case), debt with any $C \le k_C x$ becomes riskless, and hence, the firm issues riskless debt with the maximum coupon $k_C x$.[9] Proposition 4 nests Proposition 2 as the limiting case of $k_C \to \delta/h$ because $\bar{g}(k_L)$ and $\bar{k}_L$ agree with $g(k_L)$ and k_L^* in the limiting case.

Proposition 4 shows that lower k_C decreases $\bar{k}_L$ and firm value $F_n(x; \delta\bar{k}_L x)$. That is, with a stronger leverage regulation, the government can weaken market intervention, but the firm value lowers. In reality, the government's regulation and intervention require direct and indirect costs, causing spillover effects on other firms and industries. The combination of strong regulation (i.e., low k_C) and weak intervention (i.e, low $\bar{k}_L$) will decrease the intervention cost but increase the regulation cost. The government chooses one from the set $\{(k_C, \bar{k}_L) \mid 0 \le k_C \le \delta/h\}$ so that it can minimize the total costs based on the trade-off. It is beyond the scope of this paper to model the total social costs and derive the optimal choice. Note that the critical barrier $\bar{k}_L x$ is lower than $x_0(k_C x) = k_C x/\delta$. As in Proposition Proposition 2, this implies that by the ex ante commitment, the government can improve the efficiency of the market intervention policy.

Although this paper interprets k_C as a leverage regulation, it can be interpreted as a financing constraint imposed by debt holders. In this context, some papers investigate the effects of a borrowing constraint on the investment and financing timing problems with no barrier (e.g., Shibata & Nishihara, 2012, 2015, 2018). In particular, Nishihara et al. (2023) and Shibata and Nishihara (2018) show that under very hard borrowing constraints, the firm tends to issue riskless debt in the models with constant liquidation value.[10] The previous results align with our result that lower k_C decreases $\bar{k}_L$.

As in Proposition 3, we can analytically prove the comparative statics with respect to barrier level k_L. For proof, see Appendix E.

Proposition 5. *For $k_L < \bar{k}_L$ (i.e., the default-possible case), $\bar{D}(x), \bar{F}(x)$, and $\bar{LV}(x)$ increase in k_L, $\bar{CS}(x)$ decreases in k_L, and $\bar{C}(x) = k_C x$, $x_0(\bar{C}) = k_C x/\delta$, and $\bar{E}(x) = E_0(x; k_C x)$ are constant.*

At $k_L = \bar{k}_L$ (i.e., the switching point), $\bar{E}(x)$ jumps upward, $\bar{D}(x), \bar{C}(x)$, $\bar{LV}(x)$, and $\bar{CS}(x)$ jump downward, and $\bar{F}(x)$ is continuous.

For $k_L \in [\bar{k}_L, k_C/\delta]$ (i.e., the no-default case), $\bar{D}(x), \bar{F}(x), \bar{C}(x)$, and $\bar{LV}(x)$ increase in k_L, $\bar{E}(x)$ decreases in k_L, and $\bar{CS}(x)$ is 0.

For $k_L > k_C/\delta$ (i.e., the no-default case), $\bar{E}(x)$ and $\bar{F}(x)$ increase in x_L, $\bar{LV}(x)$ decrease in k_L, and $\bar{D}(x) = k_C x/r, \bar{C}(x) = k_C x$, $\bar{CS}(x) = 0$ are constant.

Proposition 5 shows that the comparative statics with respect to barrier level k_L are mostly unchanged from Proposition 3, even if the model includes the debt issuance constraint. At $k_L = \bar{k}_L$, the values, except firm value $F(x)$, jump because coupon $\bar{C}(x)$ jumps from $k_C x$ (i.e., the default-possible case) to $\delta\bar{k}_L x$ (i.e., the no-default case). All the values are continuous at $k_L = k_C/\delta$ because $\bar{C}(x)$ is continuous. Due to the switching point $k_L = \bar{k}_L$, $\bar{E}(x), \bar{D}(x), \bar{C}(x)$, and $\bar{LV}(x)$ become nonmonotonic with respect to k_L.

4. Numerical analysis and implications

4.1. Baseline results

This section conducts numerical analyses, including comparative statics with respect to barrier level k_L, volatility σ, growth rate μ, bankruptcy cost α, and debt limit level k_C. The baseline parameter values are set as in Table 1, where the values of r, μ, σ, τ, and α are standard in dynamic corporate finance literature and reflect a typical S&P firm (e.g., Arnold, 2014; Morellec, 2001; Nishihara et al., 2023). The initial EBIT value is normalized as $x = X(0) = 1$. For these parameter values, π, γ, δ, h, and ψ are computed as in Table 1, and the lowest level to achieve the no-default case becomes $k_L^* = 0.153$. In the baseline case, we set $k_L = 0.2$ (i.e., the no-default case), which is close to $k_L^* = 0.153$, so that the outcome will switch between the no-default and default-possible cases with varying levels of other parameters (cf. Section 4.3, 4.4, and 4.5). We calculated the expected time for $X(t)$ to hit the barrier, finding it to be $\log(k_L)/(\mu - 0.5\sigma^2) = 160.9$ and 187.7 years for $k_L = 0.2$ and $k_L^* = 0.153$, respectively. However, the expected time may be less informative as it becomes infinite when $\mu - 0.5\sigma^2 > 0$ due to the assumption that $X(t)$ follows a GBM. As an alternative, we consider the probability of $X(t)$ hitting the barrier within five years, yielding probabilities of 11.6% and 6.6% for $k_L = 0.2$ and $k_L^* = 0.153$, respectively.

Although a full model calibration is outside the scope of this paper, we briefly outline the process for estimating k_L. First, we calibrate a GBM $X(t)$ using a firm's EBIT. When the firm's EBIT is primarily determined by a specific commodity price (e.g., oil price for an oil producer or a specific agricultural product price for its producer), we estimate the EBIT function of the commodity price using historical price and EBIT data. The price floor can then be estimated using historical thresholds, such as break-even prices or exit points for shale oil producers, and break-even prices or intervention thresholds for agricultural producers. Substituting this price floor into the EBIT function provides k_L. When

9 Although the debt issuance constraint is imposed even for riskless debt in this paper, it may be imposed only for risky debt to reduce default risk (see Nishihara, Shibata, & Zhang, 2023). In that setup, the firm issues riskless debt $\delta k_L x/r$ for any $k_L \ge \bar{k}_L$, and it does not matter whether $\bar{k}_L$ is higher than k_C/r.

10 In these previous models, a firm optimally chooses risky debt financing in the first-best case with no financing constraint. This differs from this paper's result (cf. Proposition 2). Regarding this issue, Shibata and Nishihara (2023) show that with a high degree of information asymmetry between managers and shareholders, the firm can use riskless debt financing.

EBIT cannot be directly linked to a specific price, estimating k_L may be challenging. An alternative approach is to estimate the probability of critical events (e.g., competitor exits or market interventions) within a certain time horizon (e.g., five years) using historical data. From this probability, k_L can be calibrated indirectly.

Fig. 1 depicts firm value $F(x;C)$ for varying levels of coupon C and barrier level k_L. As shown by (19) in Proposition 1, $F(x;C)$ increases linearly in C up to $C = \delta k_L x = 0.435, 0.333$, and 0.217 in the top, center, and bottom panels, respectively. In each panel, $F(x;C)$ jumps downward after point $C = \delta k_L x$ because $F_n(x;\delta k_L x) > F_d(x;\delta k_L x)$ holds in (16) and (19). All the results are shown by line graphs in Section 4, and lines that look vertical stand for jumps. For the baseline parameter values, we have $C_0(x) = 0.623$. For $C > \delta k_L x$, $F(x;C) = F_d(x;C)$ takes its maximum value at $C = C_0(x) = 0.623$, whereas for $C \le \delta k_L x$, $F(x;C) = F_n(x;C)$ takes its maximum value at $C = \delta k_L x = 0.435, 0.333$, and 0.217 in the panels. In the center panel (i.e., $k_L = k_L^* = 0.153$), $F_n(x;\delta k_L x)$ agrees with $F_d(x;C_0(x))$, and hence, the firm is indifferent to the choice between coupon $\delta k_L x = 0.333$ or $C_0(x) = 0.623$. In the baseline case (i.e., $k_L = 0.2$; see the top panel), $F_n(x;\delta k_L x)$ is higher than $F_d(x;C_0(x))$, and the firm chooses $C(x) = \delta k_L x = 0.435$ (i.e., riskless debt). In the bottom panel (i.e., $k_L = 0.1$), $F_n(x;\delta k_L x)$ is lower than $F_d(x;C_0(x))$, and the firm chooses $C(x) = C_0(x) = 0.623$ (i.e., risky debt).

Table 2 presents the values with no barrier and barrier level $k_L = 0.2$, which are computed for the baseline parameter values in Table 1. In the baseline model, the firm prefers riskless capital structure (i.e., the no-default case). The firm issues debt up to the riskless debt capacity (i.e., $D(x) = \delta x_L / r = 8.7$) to obtain the maximum tax benefits. Leverage becomes $LV(x) = 0.38$, but credit spreads are $CS(x) = 0$ because of riskless debt. In the no-barrier model, the firm chooses coupon $C_0(x) = 0.623$ based on the trade-off between the tax benefits and bankruptcy costs of debt. The firm will go bankrupt when $X(t)$ falls to $x_0(C_0(x)) = 0.287$. Hence, $D_0(x) = 10.84$ is discounted from $C_0(x)/r = 12.46$ due to default risk, and credit spreads are positive (i.e., $CS_0(x) = 0.00751$).

In Table 2, it holds that $C(x) < C_0(x), E(x) > E_0(x), D(x) < D_0(x), F(x) > F_0(x), LV(x) < LV_0(x)$, and $CS(x) < CS_0(x)$. It is straightforward that $F(x) > F_0(x)$ and $CS(x) < CS_0(x)$, and $E(x) > E_0(x)$ readily follows from $C(x) < C_0(x)$. Inequalities $C(x) < C_0(x), D(x) < D_0(x)$, and $LV(x) < LV_0(x)$ are notable. As discussed after Proposition 3, these inequalities imply that barrier level $k_L = 0.2$ leads the firm to strategically reduce debt to take advantage of riskless capital structure rather than to increase debt. In particular, $LV(x) = 0.38$ is much lower than $LV_0(x) = 0.485$. As explained in Section 3.2, the model can help explain firms with very low default probabilities observed in the real world.

4.2. Effects of barrier level k_L

Fig. 2 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x)$, and $CS(x)$ for varying levels of barrier k_L.[11] Region $k_L < k_L^* = 0.153$ is the default-possible case, whereas region $k_L \ge k_L^* = 0.153$ is the no-default case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Fig. 2 also depicts the benchmark results with no barrier by dashed lines. The benchmark results do not depend on x_L.

Although Proposition 3 has already shown the comparative static results analytically, Fig. 2 shows them more closely and quantitatively.[12]

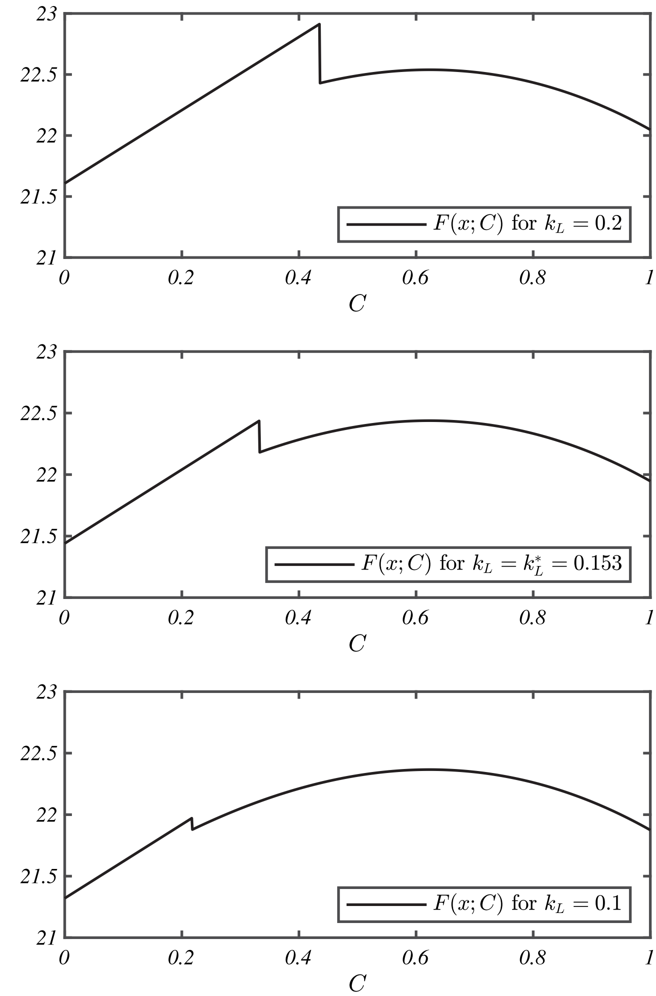

Fig. 1. Firm value $F(x;C)$ for varying levels of coupon C. The top, center, and bottom panels show firm value $F(x;C)$ for barrier level $k_L = 0.2$ (baseline), $k_L = k_L^* = 0.153$, and $k_L = 0.1$, respectively, where $F(x;C)$ agrees with the no-default firm value $F_n(x;C)$ for $C \le \delta k_L x = 0.435, 0.333$, and 0.217 and the default-possible firm value $F_d(x;C)$ for $C > \delta k_L x = 0.435, 0.333$, and 0.217, respectively. The parameter values are set as in Table 1. In all the panels, the default-possible firm value $F_d(x;C)$ takes the maximum at $C = C_0(x) = 0.623$.

For instance, we find that the effects of k_L on $D(x), F(x), LV(x)$, and $CS(x)$ are very weak in the default-possible region (i.e., $k_L < k_L^* = 0.153$). This is because $C(x) = C_0(x)$ and $E(x) = E_0(x)$ do not depend on k_L and the third term in (15) is very small. Thus, k_L does not largely change $D(x), F(x), LV(x)$, and $CS(x)$ from the benchmark values $D_0(x), F_0(x), LV_0(x)$, and $CS_0(x)$. However, in the no-default region (i.e., $k_L \ge k_L^* = 0.153$), the effects of x_L on $E(x), D(x), F(x)$, and $LV(x)$ are strong because $C(x) = \delta k_L x$ increases linearly in k_L. As discussed after Proposition 3, $C(x), D(x)$, and $LV(x)$ are lower than $C_0(x), D_0(x)$, and $LV_0(x)$ for k_L close to $k_L^* = 0.153$, whereas $C(x), D(x)$, and $LV(x)$ are higher than $C_0(x), D_0(x)$, and $LV_0(x)$ for $k_L > 0.3$.

These results entail several implications. First, we interpret k_L as the degree of competitive advantage. Then, the model shows that leverage can be nonmonotonic with respect to the degree of competitive advantage. Notably, firms with intermediate levels of competitive advantage can take low leverage with only riskless debt. As discussed after Proposition 3, this result aligns with empirical evidence that a firm that has competitive advantage in its industry tends to have lower leverage than relatively weaker competitors (e.g., MacKay & Phillips, 2005; Mitani, 2014).

Next, we interpret k_L as the strength of public intervention. The critical intervention threshold $k_L^* x = 0.153$ is not very high. In absence

[11] Rodrigues (2025) also studies the comparative statics with respect to floor levels in numerical examples. Although his model is more complicated than our model, the effects of floor levels on capital structure are qualitatively unchanged from our results. Indeed, Rodrigues (2025) also shows that a higher floor leads to riskless capital structure.

[12] Fig. 7 in Appendix F shows that these results hold robustly in the extended model with both upper and lower reflecting barriers.

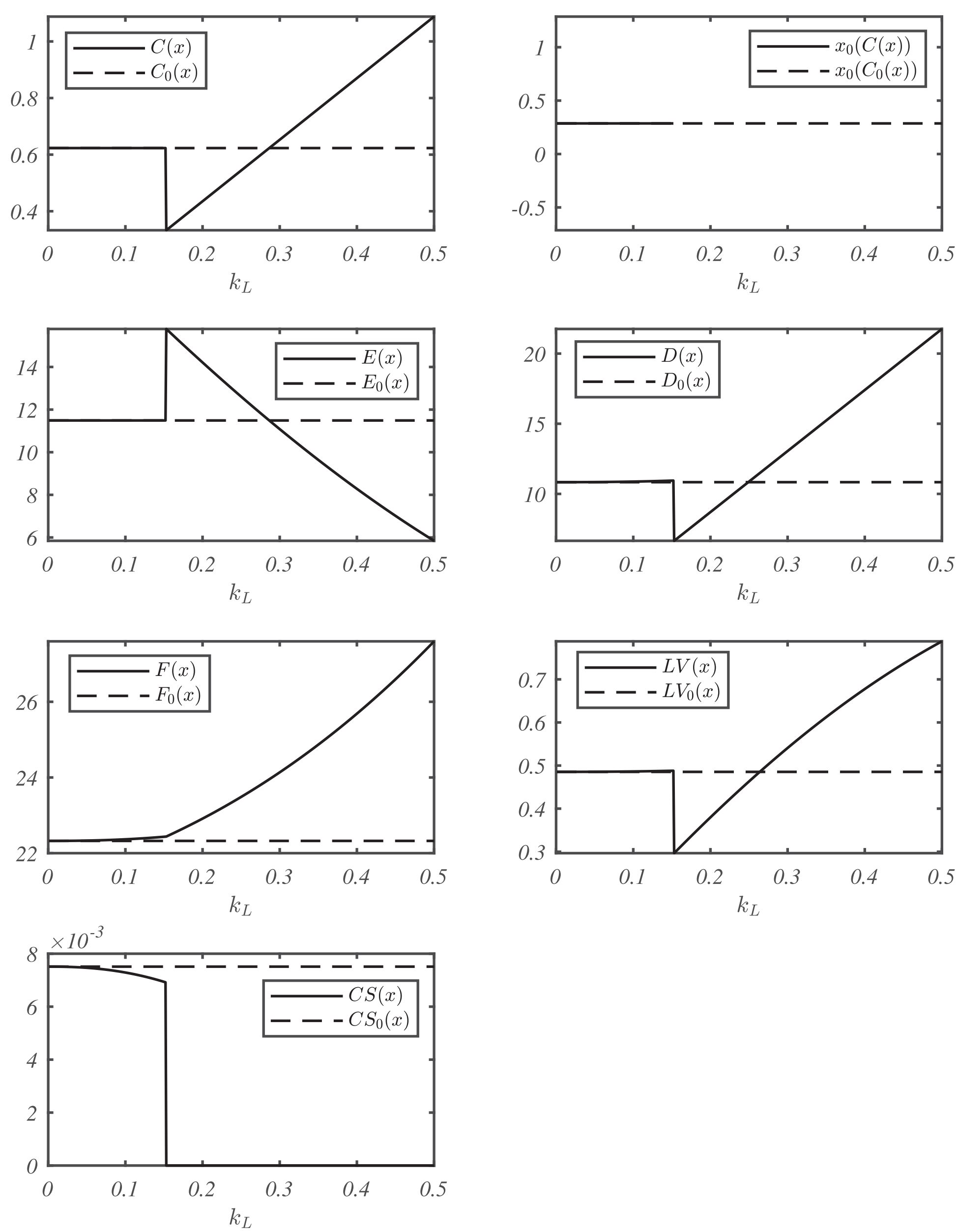

Fig. 2. Comparative statics with respect to barrier level k_L. The other parameter values are set as in Table 1. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, and credit spread $CS(x)$ in the baseline model by solid lines. Region $k_L < k_L^* = 0.153$ is the default-possible case, whereas region $k_L \geq k_L^* = 0.153$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

of the ex ante commitment to the intervention threshold, as explained after Proposition 2, the government would need intervention threshold $x_L = k_L x = x_0(C_0(x)) = 0.287$ to prevent the firm from bankruptcy ex post. With a credible commitment, the government can prevent the firm from bankruptcy with less effort. In fact, the probability of $X(t)$ hitting the barrier within five years becomes 6.6% and 22.5% for $k_L^* = 0.153$ and $k_L = 0.287$, respectively. Of course, in the real world including uncertainty and diversity of firm parameter values, it may be difficult for the government to match $k_L = k_L^*$ perfectly. A low market intervention threshold (i.e., $k_L < k_L^* = 0.153$) hardly influences capital structure, bankruptcy probability, and firm value, whereas a high market intervention threshold (say, $k_L > 0.3$) prevents bankruptcy but leads to the firm's moral hazard (i.e., increasing leverage to gain tax benefits). It is important to set an appropriate intervention threshold (i.e., $k_L \approx k_L^* = 0.153$) to prevent bankruptcy and reduce leverage effectively.

Ovtchinnikov (2010) and Sanyal and Bulan (2011) show that firms in regulated markets tend to have higher leverage than those in unregulated markets, with deregulation reducing leverage by about 25%. This decrease is partly due to changes in bankruptcy risk but also other business factors such as σ and μ. Our model suggests that protection against downside risk in regulated markets might be overly strong (e.g., $k_L = 0.35$), potentially leading to inefficiencies. Bortolotti, Cambini, Rondi, and Spiegel (2011) empirically show that regulated firms strategically increase leverage to obtain better regulatory outcomes (i.e., higher regulated prices). Our model captures this behavior when k_L increases with debt level C. In an extreme scenario where the government's intervention prevents bankruptcy entirely (i.e., $x_L = k_L x = x_d(C)$ for any C), the firm increases leverage and exploits the tax benefits, expecting the guaranteed bailout. This firm behavior is regarded as a moral hazard. We argue that a policy linking increased protection to higher debt level (i.e., $k_L(C)$ as an increasing function) is more problematic than providing protection without commitment (i.e., $k_L =$

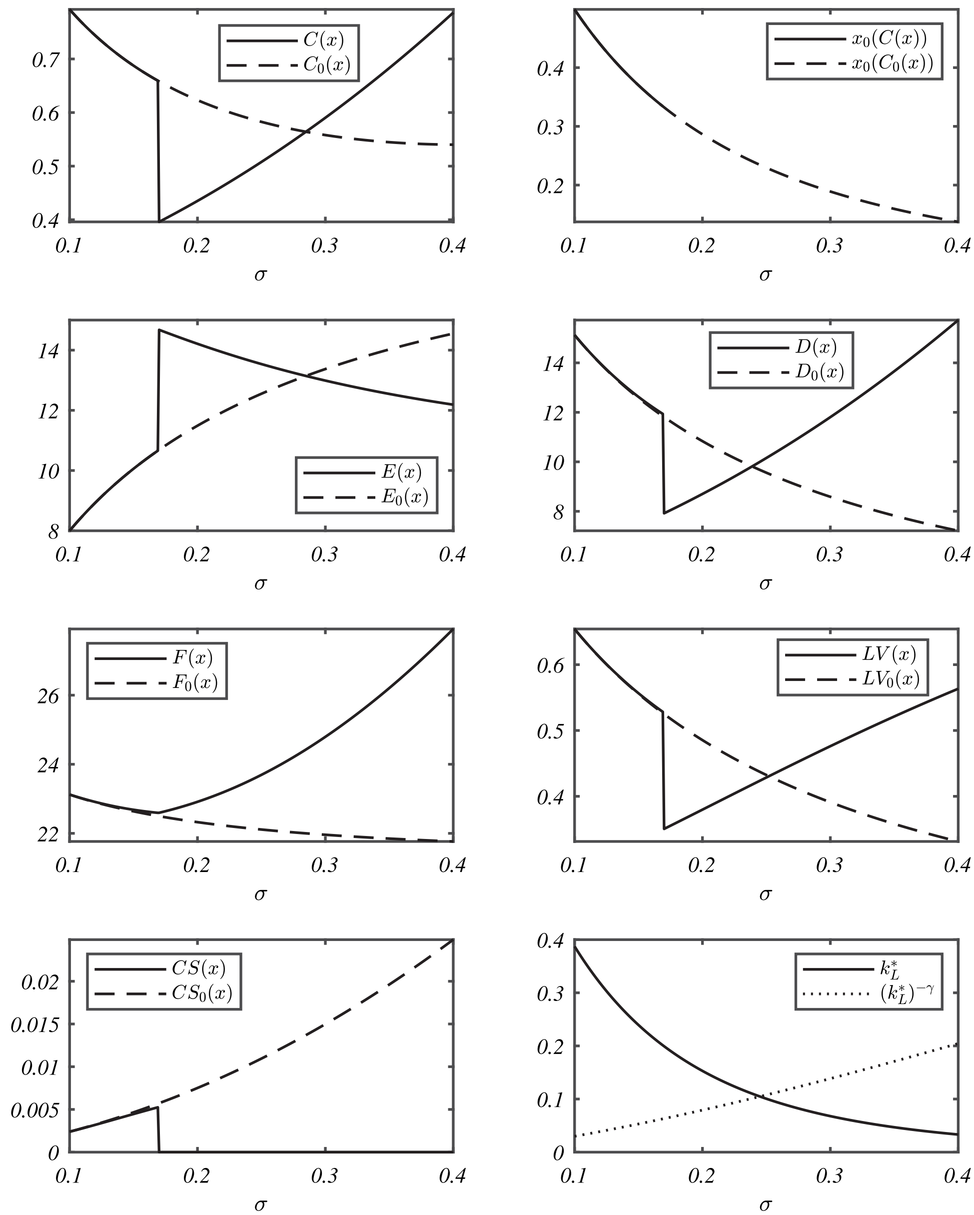

Fig. 3. Comparative statics with respect to volatility σ. The other parameter values are set as in Table 1. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, credit spread $CS(x)$, critical barrier k_L^*, and state price $(k_L^*)^{-\gamma}$ in the baseline model by solid lines. Region $\sigma < 0.17$ is the default-possible case, while region $\sigma \geq 0.17$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

0.287). The commitment to appropriate protection independent of debt level (i.e., $k_L^* = 0.153$) leads to the first-best result (i.e., preventing bankruptcy and reducing leverage).

4.3. Effects of volatility σ

Fig. 3 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x), CS(x), k_L^*$, and $(k_L^*)^{-\gamma}$ for varying levels of volatility σ. Region $\sigma < 0.17$ is the default-possible case, whereas region $\sigma \geq 0.17$ is the no-default case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Fig. 3 also depicts the benchmark results with no barrier by dashed lines.

In the default-possible region (i.e., $\sigma < 0.17$), each value moves in the same way as in the benchmark case with no barrier. In fact, higher σ decreases $C(x), x_0(C(x)), D(x), F(x)$, and $LV(x)$ and increases $E(x)$. These results can be intuitively interpreted as follows. Higher σ increases bankruptcy risk, and the firm reduces leverage to mitigate bankruptcy risk. However, decreased leverage does not fully offset increased bankruptcy risk with higher σ, and hence, credit spreads increase in σ. Firm value decreases in σ due to the lower leverage effect, although equity value increases due to decreased coupon payments. These results align with the standard results in previous literature (e.g., Leland, 1994). As in Fig. 2, Fig. 3 also shows that k_L hardly affects the values in the default-possible region.

At $\sigma = 0.17$, the result switches from the default-possible case to the no-default case. Then, $C(x), E(x), D(x), LV(x)$, and $CS(x)$ jump at this point. In the no-default region (i.e., $\sigma \geq 0.17$), $C(x), E(x), D(x), F(x)$, and $LV(x)$ move contrary to the benchmark case with no barrier. The comparative statics are explained by the sensitivity of riskless debt capacity $\delta k_L x/r$ to σ. Note that $\delta k_L x/r$ increases in σ by $\partial\delta/\partial\sigma > 0$. Then, $C(x) = \delta k_L x, D(x) = \delta k_L x/r, F(x)$, and $LV(x)$ increase in σ, whereas $E(x)$ decreases in σ due to increased $C(x)$. That is, with higher σ, the barrier becomes more effective, allowing the firm to increase leverage and enjoy tax benefits while avoiding default. This result suggests that the effects of volatility on leverage and firm value for firms with sufficient competitive advantage or protection against downside risks can differ from those of standard firms. Ovtchinnikov (2010) found a strong positive relation between leverage and volatility in regulated industries, in contrast to the strong negative relation observed after deregulation. Our model provides a novel mechanism to explain this positive relation in the regulated markets.

Note that the above results are based on the assumption of constant barrier level $k_L = 0.2$. Barrier level $k_L = 0.2$ is more effective with higher σ because the probability of $X(t)$ hitting the barrier increases with higher σ. The bottom-right panel of Fig. 3 shows that the critical level k_L^* and the state price[13] $(k_L^*)^{-\gamma}$ decrease and increase, respectively, in σ. The comparative statics of k_L^* are explained by the decrease in $F_d(x)$ and increase in $F_n(x)$ with higher σ (see $F(x)$ of Fig. 3). By these two effects, k_L^*, which is the unique solution to (22), decreases in σ. Despite the decrease in σ, $(k_L^*)^{-\gamma}$ increases in σ due to $\partial\gamma/\partial\sigma > 0$. In other words, higher σ makes $X(t)$ more volatile and increases the probability of $X(t)$ hitting barrier $k_L^* x$. In terms of public intervention, these results suggest that the government needs a lower market intervention threshold but more frequent interventions to prevent a more volatile firm from bankruptcy.

An acquisition tends to increase EBIT and decrease EBIT volatility through diversification, thus lowering the probability of $X(t)$ hitting the barrier and the firm's benefit from the barrier.[14] This may cause the firm to shift its capital structure from riskless debt to risky debt by increasing leverage after an acquisition. However, the barrier level may also change with acquisition. For example, the government could optimally adjust the intervention threshold so that the firm could maintain its riskless debt structure after an acquisition.

4.4. Effects of growth rate μ

Fig. 4 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x), CS(x), k_L^*$, and $(k_L^*)^{-\gamma}$ for varying levels of growth rate μ. Region $\mu \le 0.0252$ is the no-default case, whereas region $\mu > 0.0252$ is the default-possible case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Fig. 4 also depicts the benchmark results with no barrier by dashed lines.

As in Figs. 2 and 3, Fig. 4 shows that all the values in the baseline case are almost the same as those in the benchmark case in the default-possible region (i.e., $\mu > 0.0252$). One reason is that the firm choose the same coupon $C(x) = C_0(x)$, and the other reason is that the state price contingent on $X(t)$ hitting the barrier (i.e., $k_L^{-\gamma}$) is very low. We omit the details of the comparative statics in the default-possible case because they are the same as those in the standard model with no barrier (e.g., Leland, 1994).

At the switching point $\mu = 0.0252$, $C(x), E(x), D(x), LV(x)$, and $CS(x)$ jump. Even in the no-default region (i.e., $\mu \le 0.0252$), $C(x), E(x), D(x)$, and $F(x)$ change with μ in the same way as in the benchmark values. More notably, $LV(x)$ decreases in μ, contrary to $LV_0(x)$. The reason is as follows. Riskless debt capacity $D(x) = \delta k_L x/r$ increases in μ by $\partial\delta/\partial\mu > 0$, and equity value $E(x) = E(x; \delta k_L x)$ also increases in μ by $\partial\pi/\partial\mu > 0$. The latter effect dominates the former effect, and hence $LV(x)$ decreases in μ. This sensitivity is novel and contrasted with the standard result. In fact, the standard trade-off models (e.g., Leland, 1994) predict a positive relation between leverage and growth rate (see $LV_0(x)$ in Fig. 4), but empirical studies (e.g., Frank & Goyal, 2015; Titman & Wessels, 1988) show a negative relation. This is well known as a deficit of the standard trade-off models (e.g., Demarzo, 2019). While numerous explanations exist for the negative relation between profitability and leverage, our paper provides an additional perspective. Indeed, the model predicts the negative relation for firms with sufficient competitive advantage or protection against downside risks because they set debt level by riskless debt capacity rather than the trade-off between the tax benefits and bankruptcy costs.

The bottom-right panel of Fig. 4 shows that k_L^* and $(k_L^*)^{-\gamma}$ increase and decrease, respectively, in μ. The former result is caused by $F_d(x)$ increasing in μ more than $F_n(x)$ does. Despite the increase in σ, $(k_L^*)^{-\gamma}$ decreases in μ due to $\partial\gamma/\partial\mu < 0$ (i.e., higher μ decreases the probability of $X(t)$ hitting $k_L^* x$). These results entail a policy implication that the government needs a higher market intervention threshold but less frequent market interventions to prevent a high-growth firm from bankruptcy.

4.5. Effects of bankruptcy cost α

Fig. 5 depicts $C(x), x_0(C(x)), E(x), D(x), F(x), LV(x), CS(x), k_L^*$, and $(k_L^*)^{-\gamma}$ for varying levels of bankruptcy cost α. Region $\alpha < 0.199$ is the no-default case, whereas region $\alpha \ge 0.199$ is the default-possible case. Default threshold $x_0(C(x))$ is depicted only in the default-possible case. For comparison, Fig. 5 also depicts the benchmark results with no barrier by dashed lines.

In the no-default region (i.e., $\alpha \ge 0.199$), neither value depends on α because the firm will never go bankrupt. In the default-possible region (i.e., $\alpha < 0.199$), all the values change with α in the same way as in the benchmark values with no barrier. In this region, higher α increases the disadvantages of debt and hence decreases $C(x)$, $D(x)$, and $LV(x)$. Firm value $F(x)$ and $CS(x)$ also decrease in α due to the decreased leverage effect, whereas $E(x)$ increases in α due to decreased coupon payments.

By (22) and $\partial h/\partial\alpha > 0$, we can easily prove that k_L^* decreases in α. The bottom-right panel of Fig. 3 numerically verifies the sensitivity of k_L^* to α. Note that state price $(k_L^*)^{-\gamma}$ changes in the same way as k_L^* because γ does not depend on α. This result is intuitively explained as follows. Higher α increases the disadvantages of risky debt and decreases the leverage effect. Then, the firm is more likely to be better off using riskless debt rather than relying on risky debt. For the same reason, lower corporate tax rate τ decreases k_L^* and $(k_L^*)^{-\gamma}$, although we omit a figure illustrating the comparative statics with respect to τ. Indeed, lower τ decreases the tax advantages of debt, which decreases the firm's motive to use risky debt. The comparative static results have the following implications of public intervention. The government can prevent the firm from bankruptcy by weaker and fewer market interventions if it imposes a lower corporate tax rate and a more stringent bankruptcy law with higher bankruptcy penalty. This is because with such public policies, the firm has fewer advantages from issuing risky debt and is more likely to choose riskless capital structure.

4.6. Effects of debt limit level k_C

So far, we have examined the effects of the key parameters on the results in the baseline model. This subsection studies the effects of upper limit $\bar{C} = k_C x$ in the constrained model of Section 3.3. For the baseline parameter values (i.e., Table 1), the no-default case holds by $k_L^* = 0.1529 < k_L = 0.2$ in absence of debt issuance constraint. By Proposition 4, we have $\bar{k}_L < k_L^* = 0.153 < k_L = 0.2$, and hence, the no-default case holds for any k_C. We reset $k_L = 0.1$ to depict both the no-default and default-possible cases. The other parameter values are set as in Table 1. Fig. 6 depicts $\bar{C}(x), x_0(\bar{C}(x)), \bar{E}(x), \bar{D}(x), \bar{F}(x), \bar{LV}(x), \bar{CS}(x), \bar{k}_L$, and $(\bar{k}_L)^{-\gamma}$ for varying levels of $k_C (\le \delta/h = 0.623)$. Note that k_C does not bind the firm for $k_C \ge \delta/h = 0.623$. Region $k_C \le 0.261$ is the no-default region, whereas region $k_C > 0.261$ is the default-possible region. Default threshold $x_0(\bar{C}(x))$ is depicted only in the default-possible case. The no-default region is classified into region $k_C \in [0.218, 0.261]$, where $\bar{C}(x) = \delta k_L x = 0.218$ does not depend on k_C, and region $k_C < 0.218$, where $\bar{C}(x) = k_C x$ (see Proposition 4). For comparison, Fig. 5 also depicts the benchmark results with no barrier under upper limit $\bar{C} = k_C x$ by dashed lines. In this benchmark case, the firm chooses coupon

[13] The state price denotes the present values of \$1 contingent on $X(t)$ hitting barrier $k_L^* x$.

[14] In reality, the target's EBIT may not be perfectly correlated with the acquirer's EBIT. However, a model where EBIT follows the sum of GBMs is not analytically tractable.

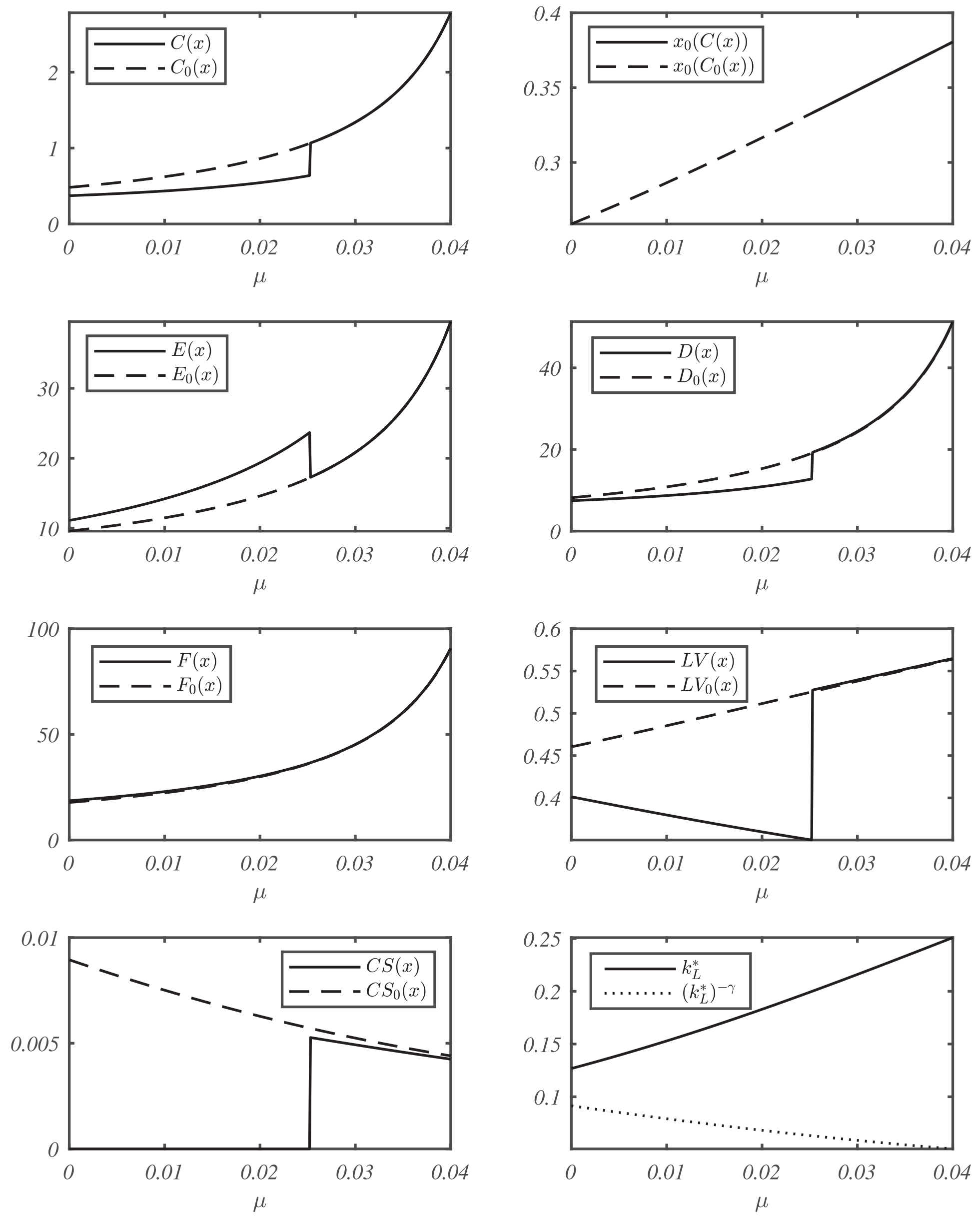

Fig. 4. Comparative statics with respect to growth rate μ. The other parameter values are set as in Table 1. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, credit spread $CS(x)$, critical barrier k_L^*, and state price $(k_L^*)^{-\gamma}$ in the baseline model by solid lines. Region $\mu \leq 0.0252$ is the default-possible case, whereas region $\mu > 0.0252$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

$k_C x$ because firm value $F_0(x; C)$ (see (4)) monotonically increases in C up to $C = C_0(x) = 0.623$.

In the default-possible region (i.e., $k_C > 0.261$) of Fig. 6, the presence of k_L hardly affects each value. The main reason is that the firm chooses the maximum coupon $k_C x$ regardless of k_L. All the comparative static results are straightforward and the same as the benchmark results with no barrier. Higher k_C increases $\bar{D}(x)$ and $\bar{LV}(x)$. The increased leverage effects increase $\bar{F}(x)$, although the increased coupon payments decrease $\bar{E}(x)$ and increase $\bar{CS}(x)$. Note that each value agrees with that of the unconstrained baseline model for $k_C = \delta/h = 0.623$ (i.e., the right end of each panel of Fig. 6).

The no-default region $k_C \in [0.218, 0.261]$ is most intriguing. In this region, the firm chooses riskless capital structure because $\bar{k}_L \leq k_L = 0.1$ (see the bottom-right panel of Fig. 6). Riskless debt capacity $\delta k_L x = 0.218$ rather than debt issuance limit $k_C x$ binds the firm due to $\delta k_L x = 0.218 \leq k_C x$. Then, coupon $\bar{C}(x) = \delta k_L x = 0.218$ is constant in this region. This also implies that $\bar{E}(x)$, $\bar{D}(x)$, $\bar{F}(x)$, and $\bar{LV}(x)$ are constant in this region. These results are contrasted with the benchmark results with no barrier.

Lastly, we turn to the no-default region $k_C < 0.218$. In this region, debt issuance limit $k_C x$ rather than riskless debt capacity $\delta k_L x = 0.218$ binds the firm due to $k_C x < \delta k_L x = 0.218$. Then, the firm chooses the maximum coupon $k_C x$ as in the benchmark case with no barrier. The comparative static results other than $\bar{CS}(x) = 0$ are the same with the standard results with no barrier. Note that each value converges to that of the all-equity firm for $k_C \to 0$ (i.e., the left end of each panel of Fig. 6).

As shown by Proposition 4, the bottom-right panel of Fig. 6 shows that the critical level $\bar{k}_L$ increases in k_C. State price $(\bar{k}_L)^{-\gamma}$ similarly increases in k_C because γ does not depend on k_C. These results show that by regulating leverage, the government can reduce the market intervention threshold and frequency to prevent the firm from bankruptcy. As discussed after Proposition 4, the optimal policy would lie in $\{(k_C, \bar{k}_L) \mid 0 \leq k_C \leq \delta/h\}$, but it may be difficult for the government to find a perfectly optimal pair $(k_C, \bar{k}_L)$. In fact, the government tends to

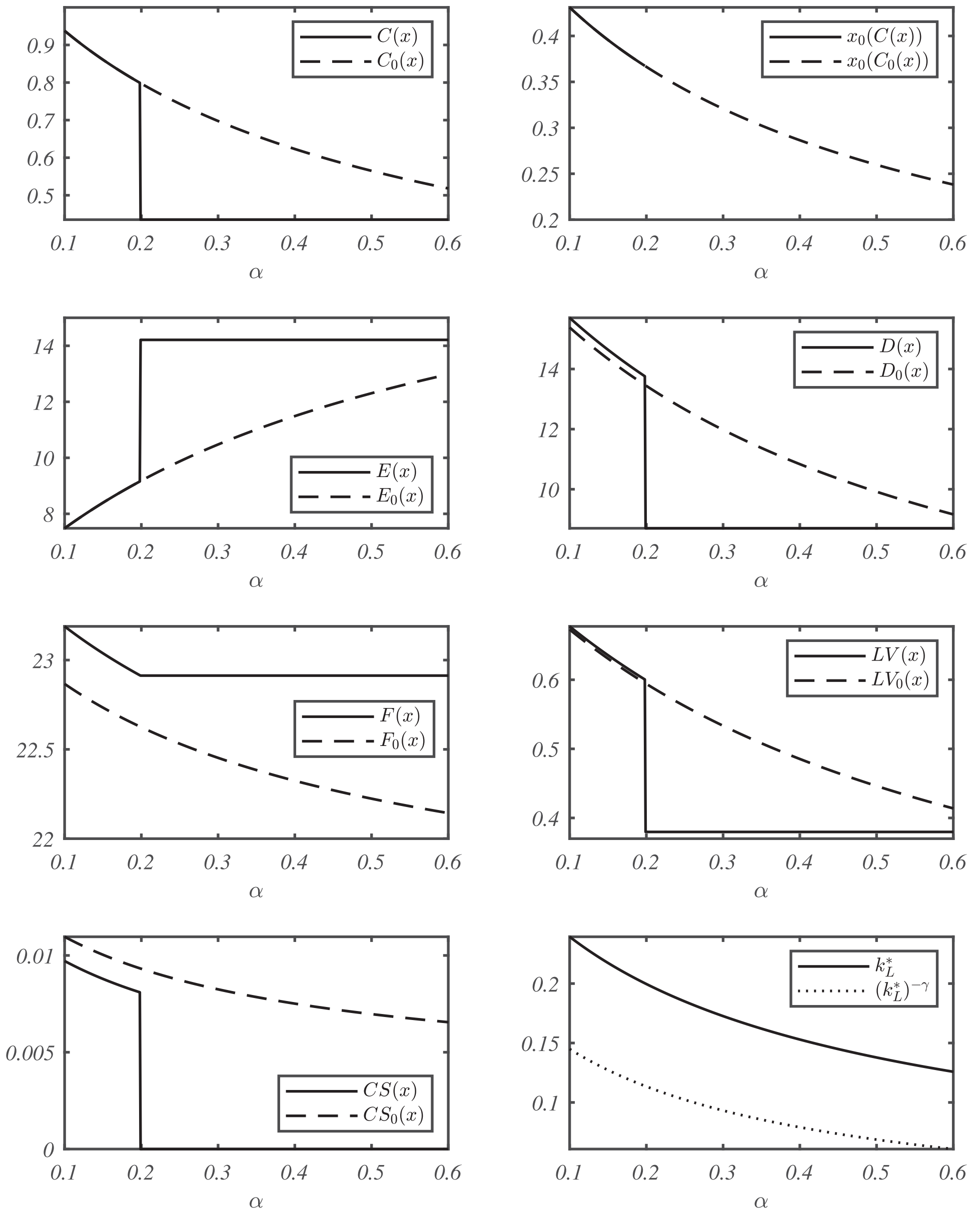

Fig. 5. Comparative statics with respect to bankruptcy cost α. The other parameter values are set as in Table 1. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, credit spread $CS(x)$, critical barrier k_L^*, and state price $(k_L^*)^{-\gamma}$ in the baseline model by solid lines. Region $\alpha < 0.199$ is the default-possible case, whereas region $\alpha \geq 0.199$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

impose a uniform regulation and protection policy over firms within the same industry, although cash flows are affected by firm-specific factors and risks. That is, $(k_C, \bar{k_L})$ differs over firms in the industry, but the government must choose one policy for all the firms. Regulation that is too weak cannot prevent bankruptcy (i.e., the region $k_C > 0.261$), whereas regulation that is too strong decreases firm value inefficiently (i.e., the region $k_C < 0.218$). Even if the government cannot find a perfect solution for all the firms, it can choose a policy within the plausible region (i.e., the region $k_C \in [0.218, 0.261]$).

5. Conclusion

This paper investigates the capital structure model with earnings above a reflecting barrier. The model can approximate a firm with competitive advantage or public protection against downside risks. In the former, the barrier represents an exit threshold of competitors, whereas in the latter, it represents a public intervention threshold. This paper explicitly derives the equity, debt, firm values, leverage, and credit spreads and shows their comparative statics with respect to barrier levels. The main results are summarized below.

First, and most notably, the barrier generates the riskless debt capacity, and the firm chooses either riskless or risky capital structure by comparing the values with the maximum riskless debt and with risky debt. The higher the barrier, the larger the riskless debt capacity, and the firm tends to prefer riskless capital structure. With intermediate barrier levels, the firm chooses lower leverage than the level with no barrier to take advantage of riskless debt.

This result can help explain debt conservatism observed in the real world. Indeed, the model predicts that firms with certain degrees of competitive advantage or public protection can issue lower levels of riskless debt rather than adjusting risky debt levels based on the trade-off between the tax benefits and bankruptcy costs of debt. The result can also help explain why strong firms may forgo tax benefits despite their low default probabilities. In the no-default case, leverage increases

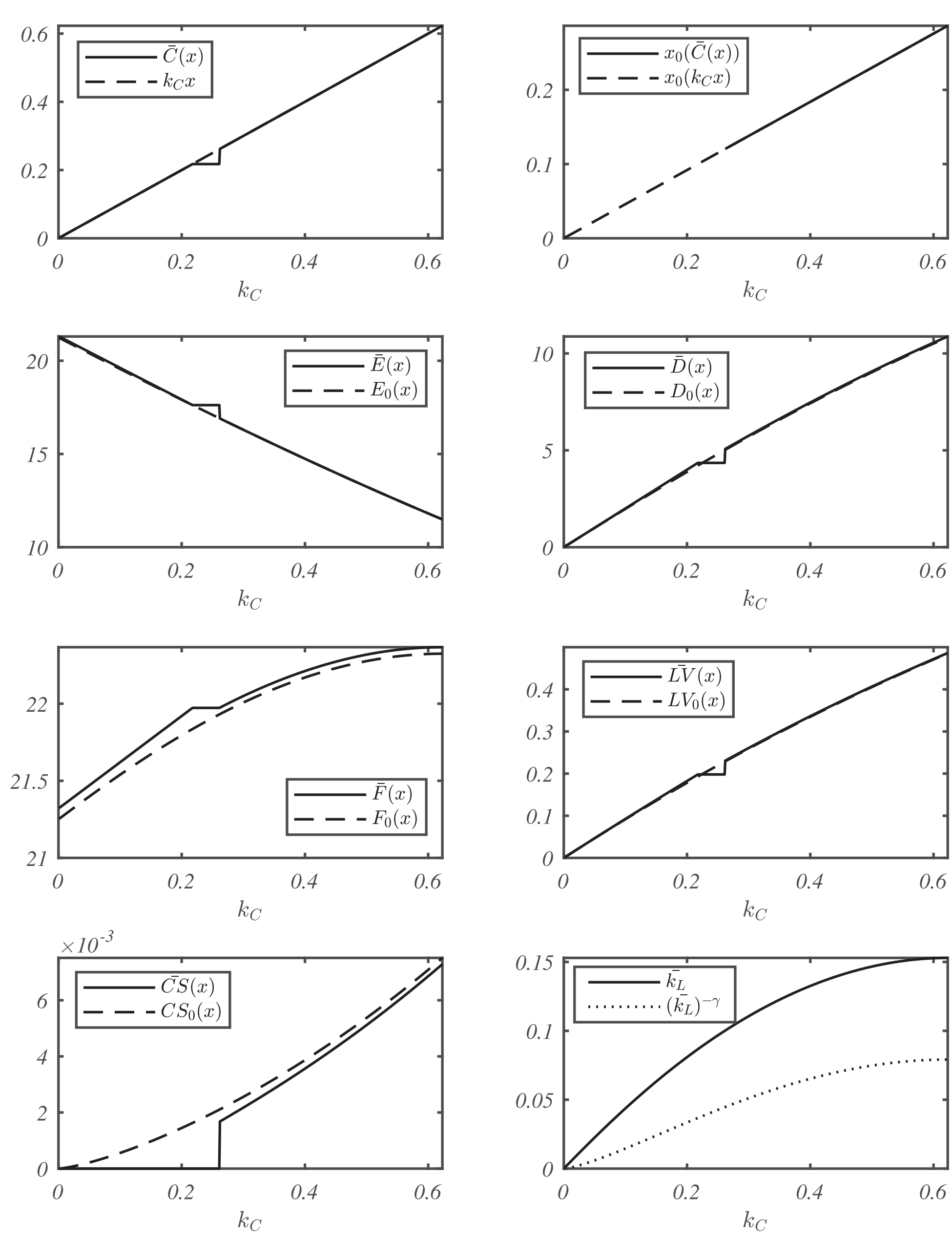

Fig. 6. Comparative statics with respect to coupon limit level k_C. Barrier level k_L is set at 0.1. The other parameter values are set as in Table 1. The figure depicts coupon $\bar{C}(x)$, default threshold $x_0(\bar{C}(x))$, equity value $\bar{E}(x)$, debt value $\bar{D}(x)$, firm value $\bar{F}(x)$, leverage $\bar{LV}(x)$, credit spread $\bar{CS}(x)$, critical barrier $\bar{k_L}$, and state price $(\bar{k_L})^{-\gamma}$ in the model with upper limit $\bar{C} = k_C x$ by solid lines. Region $k_C \leq 0.261$ is the no-default case, whereas region $k_C > 0.261$ is the default-possible case. The dashed lines represent the benchmark results with no barrier under upper limit $\bar{C} = k_C x$.

with higher volatility and lower growth rates, contrasting with standard trade-off theory. The former result aligns with empirical observations of a positive relation between leverage and volatility in regulated markets, while the latter supports empirical evidence of a negative relation between leverage and profitability.

The model also entails several implications of public intervention to protect specific firms or industries from financial distress. High leverage levels observed in regulated markets imply that public protection against downside risks may be excessively strong, potentially leading to inefficiencies. Using the ex ante commitment to an appropriate intervention threshold, the government can efficiently lead firms to adopt riskless capital structure with low leverage. With a more stringent bankruptcy law (i.e., higher bankruptcy cost), lower corporate tax rate, and stronger leverage regulation, the government needs weaker and fewer market interventions to prevent the firms from bankruptcy.

CRediT authorship contribution statement

Michi Nishihara: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Takashi Shibata:** Writing – review & editing, Validation, Supervision, Funding acquisition, Conceptualization.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGpt in order to improve language and readability. After using ChatGpt, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

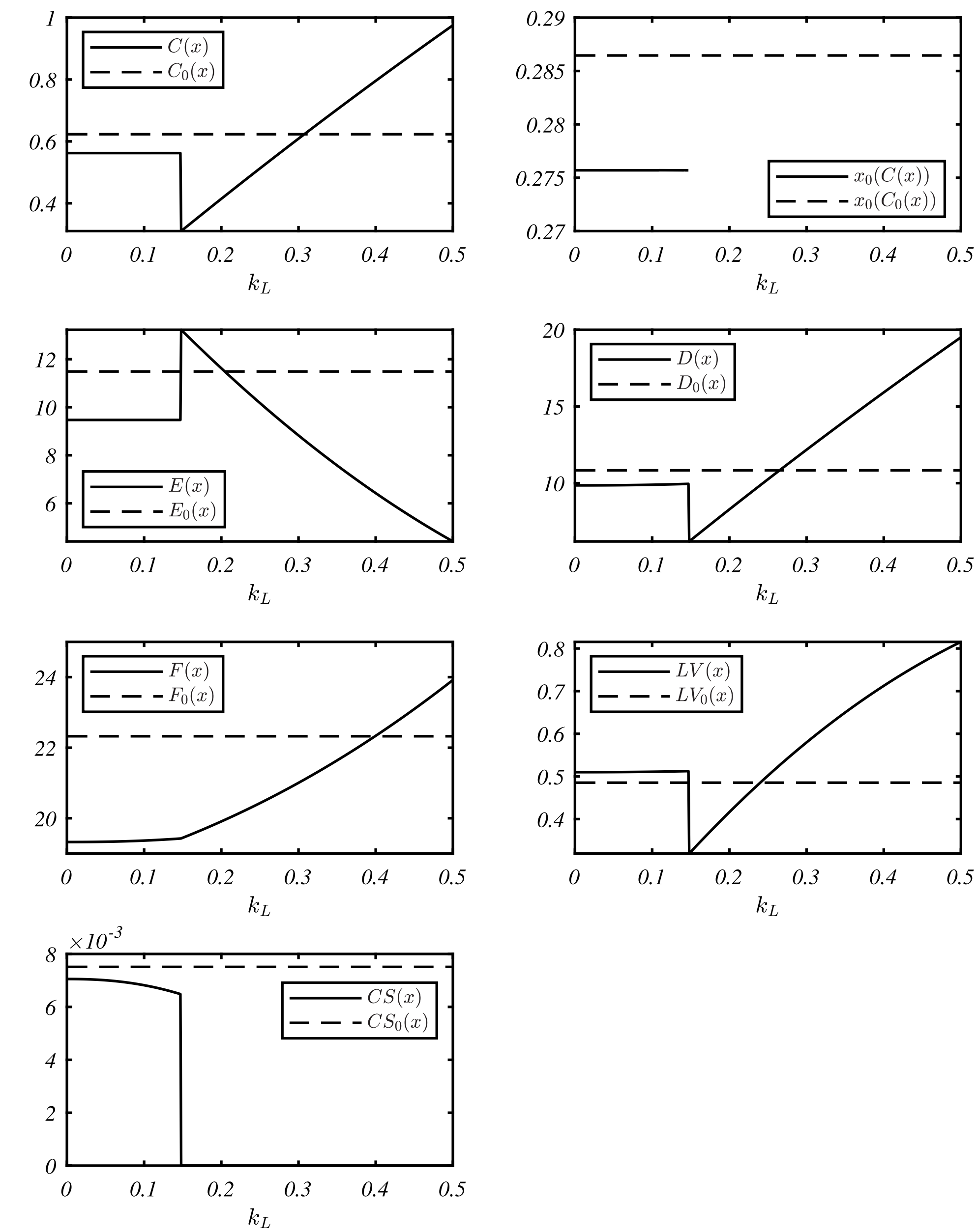

Fig. 7. Comparative statics with respect to lower reflecting barrier level k_L in the extended model, where the upper reflecting barrier is set at 5, and the other parameter values are set as in Table 1. The figure depicts coupon $C(x)$, default threshold $x_0(C(x))$, equity value $E(x)$, debt value $D(x)$, firm value $F(x)$, leverage $LV(x)$, and credit spread $CS(x)$ in the model with upper and lower reflecting barriers by solid lines. Region $k_L < 0.148$ is the default-possible case, whereas region $k_L \geq 0.148$ is the no-default case. The dashed lines represent the benchmark results with no barrier.

Appendix A. Proof of Proposition 1

First, derive the equity value of the firm that operates perpetually, i.e., $E_n(x; C)$. The derivation process is the same as in the reflecting barrier models in Chapter 8 of Dixit and Pindyck (1994). Equity value $E_n(x; C)$ satisfies the differential equation

$$\mu x \frac{\partial E_n(x;C)}{\partial x} + \frac{\sigma^2 x^2}{2} \frac{\partial^2 E_n(x;C)}{\partial^2 x} + (1-\tau)(x - C) = r E_n(x;C) \tag{25}$$

for $x > x_L$ with the boundary conditions

$$\frac{\partial E_n(x_L;C)}{\partial x} = 0, \tag{26}$$

$$\lim_{x\to\infty} \frac{E_n(x;C)}{\pi x} < \infty. \tag{27}$$

Note that (26) means that the derivative of $E_n(x; C)$ must be 0 at reflecting barrier x_L because $X(t)$ surely increases from $X(0) = x_L$, while (27) stems from the fact the probability of $X(t)$ hitting x_L approaches 0 for $X(0) \to \infty$. By (25) and (27), $E_n(x; C)$ is expressed as

$$E_n(x;C) = \pi x - \frac{(1-\tau)C}{r} + A x^{\gamma},$$

where A is a constant. By (26), we can derive

$$A = -\frac{(1-\alpha)\pi x_L^{1-\gamma}}{\gamma}.$$

Then, $E_n(x; C)$ is expressed as (17), where x_L is replaced with $k_L x$. Note that $E_n(k_L x; C) \geq 0$ holds if and only if $C \leq \delta k_L x$. Accordingly, for $C \leq \delta k_L x$, $E_n(x; C) \geq 0$ holds for all $x \geq k_L x$, which implies that shareholders do not prefer to receive default value 0 by declaring default. Then, equity value $E(x; C)$ becomes $E_n(x; C)$ in this case. Debt is riskless, and hence $D_d(x; C) = C/r$ holds. By summing this and $E_n(x; C)$, we have $F_n(x; C)$ as (19).

On the other hand, for $C > \delta k_L x$, $E_n(k_L x; C) < 0$ holds, which implies that shareholders prefer to declare default at a sufficiently low threshold $x_d (\geq k_L x)$. Note that $C > \delta k_L x$ is equivalent to $x_0(C) \geq k_L x$. As in the standard literature (e.g., Goldstein et al., 2001; Shibata & Nishihara, 2012; Sundaresan et al., 2015), the equity value of the firm that defaults at the optimal timing, i.e., $E_d(x; C)$, is expressed as

$$E_d(x;C) = \sup_{x_d \geq x_L} \left(\pi x - \frac{(1-\tau)C}{r} + \left(\frac{x}{x_d}\right)^{\gamma} \left(\frac{(1-\tau)C}{r} - \pi x_d \right) \right)$$
$$= \pi x - \frac{(1-\tau)C}{r} + \left(\frac{x}{x_0(C)}\right)^{\gamma} \left(\frac{(1-\tau)C}{r} - \pi x_0(C) \right)$$
$$= E_0(x;C).$$

Hence, equity value $E(x;C)$ becomes $E_d(x;C) = E_0(x;C)$ in this case. It should be noted that $E_0(x;C) > E_n(x;C)$ holds for $x \geq \max\{k_L x, x_0(C)\}$ if and only if $C > \delta k_L x$.

Debt value is derived as

$$D_d(x;C) = \frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma} \left(\frac{C}{r} - (1-\alpha) F_n(x_0(C); 0) \right) \tag{28}$$
$$= \frac{C}{r} - \left(\frac{x}{x_0(C)}\right)^{\gamma} \left(\frac{C}{r} - (1-\alpha)\pi x_0(C) \right) - \frac{k_L^{1-\gamma}(1-\alpha)\pi x}{\gamma}.$$

By summing this and $E_0(x;C)$, we also obtain $F_d(x;C)$ as (16).

Appendix B. Proof of Proposition 2

By (22) and $\gamma < 0$, $g(k_L)$ is continuously increases in $k_L \in [0,1]$. By (8) and (22), we also have

$$g(0) = -\frac{\tau x}{(r-\mu)h} < 0 \tag{29}$$

$$g(\tilde{k}_L) = -\frac{\tilde{k}_L^{1-\gamma}\alpha}{\gamma} > 0, \tag{30}$$

where $\tilde{k}_L = \gamma/((\gamma-1)h)$. Therefore, a unique solution $k_L^* \in (0, \tilde{k}_L)$ exists to $g(k_L^*) = 0$.

For $k_L < k_L^*$, $g(k_L) < 0$ holds, which leads to $\max_{C\geq 0} F(x;C) = \max\{F_d(x; C_0(x)), F_n(x; \delta k_L x)\} = F_d(x; C_0(x))$. Hence, the firm chooses coupon $C(x) = C_0(x)$ at time 0. Note that $C_0(x) = \delta x/h > \delta k_L x$ follows from $k_L x < k_L^* x < \gamma x/((\gamma-1)h)$. Then, the equity, debt, firm values, coupon, default threshold, leverage, and credit spreads are equal to those of the default-possible case with $C = C_0(x)$ in Proposition 1.

For $k_L \geq k_L^*$, $g(k_L) \geq 0$ holds, which leads to $\max_{C\geq 0} F(x;C) = \max\{F_d(x; C_0(x)), F_n(x; \delta k_L x)\} = F_n(x; \delta k_L x)$. Hence, the firm chooses coupon $C(x) = \delta k_L x$ at time 0. Then, the equity, debt, firm values, coupon, default threshold, leverage, and credit spreads are equal to those of the no-default case with $C = \delta k_L x$ in Proposition 1.

Appendix C. Proof of Proposition 3

By Propositions 1 and 2, for $k_L < k_L^*$, $D(x) = D_d(x; C_0(x))$ increases in k_L, while $C(x) = C_0(x)$, $x_0(C_0(x)) = x/h$, and $E(x) = E_0(x; C_0(x))$ are constant. Then, $F(x)$ and $LV(x)$ increase in k_L, $CS(x)$ decreases in k_L.

At $k_L = k_L^*$, coupon $C(x)$ changes from $C_0(x) = \delta x/h$ to $\delta k_L^* x$. It follows from $k_L^* < \gamma/((\gamma-1)h)$ that $C_0(x) = \delta x/h > \delta k_L^* x$ (i.e., a downward jump). At $k_L = k_L^* x$, $E(x)$ changes from $E_0(x; C_0(x))$ to

$$E_n(x; \delta k_L^* x) = \pi x - \frac{(1-\tau)\delta k_L^* x}{r} - \frac{(k_L^*)^{1-\gamma}\pi x}{\gamma}$$
$$= E_0(x; \delta k_L^* x)$$
$$> E_0(x; C_0(x))$$

(i.e., an upward jump), where we obtained the last inequality by $C_0(x) > \delta k_L^* x$. By definition of k_L^* (i.e., $g(k_L^*) = 0$), $F_d(x; C_0(x))$ continuously changes to $F_n(x; \delta k_L^* x)$ at $k_L = k_L^*$. By the continuity of $F(x)$ and the upward jump of $E(x)$, $D(x)$ must jump downward at $k_L = k_L^*$. Then, $LV(x) = D(x)/F(x)$ jumps downward at $k_L = k_L^*$, and $CS(x)$ also jumps downward to 0 (i.e., riskless debt).

By Propositions 1 and 2, for $k_L \geq k_L^*$, $D(x) = \delta k_L x/r$ and $F(x) = F_n(x; \delta k_L x), C(x) = \delta k_L x$ increase in k_L, while $CS(x)$ is 0. Define

$$H(k_L) = E_n(x; \delta k_L) = \pi x - \frac{(1-\tau)\delta k_L x}{r} - \frac{k_L^{1-\gamma}\pi x}{\gamma}$$

and compute the derivative

$$\frac{\mathrm{d}H(k_L)}{\mathrm{d}k_L} = -\frac{(\gamma-1)(1-k_L^{-\gamma})\pi x}{\gamma} < 0,$$

where the last inequality follows from $k_L < 1$ and $\gamma < 0$. Hence, $E(x) = E_n(x; \delta k_L x)$ decreases in k_L. By the decrease of $E(x)$ and increase of $D(x)$, $LV(x)$ increases in k_L.

Appendix D. Proof of Proposition 4

By (24), and $\gamma < 0$, $\bar{g}(k_L)$ is continuously increases in $k_L \in [0,1]$. By (24), we can show that

$$\bar{g}(0) = -\frac{\tau k_C}{r\pi} + \left(\frac{k_C}{\delta}\right)^{1-\gamma} \left(\alpha + \frac{\tau k_C \delta}{r\pi}\right)$$
$$= \frac{\pi x - F_0(x; k_C x)}{\pi x} < 0, \tag{31}$$
$$\bar{g}(k_L^*) = \frac{F_n(x; \delta k_L^* x) - F_d(x; k_C x)}{\pi x}$$
$$= \frac{F_d(x; C_0(x)) - F_d(x; k_C x)}{\pi x} > 0, \tag{32}$$
$$\bar{g}(k_C/\delta) = \left(\frac{k_C}{\delta}\right)^{1-\gamma} \left(\alpha + \frac{\tau k_C \delta}{r\pi}\right) - \left(\frac{k_C x}{\delta}\right)^{1-\gamma} \frac{\alpha}{\gamma} > 0,$$

where we used $\pi x = F_0(x; 0) < F_0(x; k_C x)$ in (31), and we used $F_n(x; \delta k_L^* x) = F_d(x; C_0(x))$ and the optimality of $C_0(x)$ in (32). Hence, a unique solution $\bar{k}_L \in (0, \min\{k_L^*, k_C/\delta\})$ exists to $\bar{g}(\bar{k}_L) = 0$.

For $k_L < \bar{k}_L$, $\bar{g}(k_L) < 0$ holds, which leads to $\max_{C\in[0,k_C x]} F(x;C) = \max\{F_d(x; k_C x), F_n(x; \delta k_L x)\} = F_d(x; k_C x)$. Then, the firm chooses coupon $C(x) = k_C x$ at time 0. The results follow from the default-possible case with $C = k_C x$ in Proposition 1.

For $k_L \in [\bar{k}_L, k_C/\delta]$, $\bar{g}(k_L) \geq 0$ holds, which leads to $\max_{C\in[0,k_C x]} F(x;C) = \max\{F_d(x; k_C x), F_n(x; \delta k_L x)\} = F_n(x; \delta k_L x)$. Then, the firm chooses coupon $C(x) = \delta k_L x$ at time 0. The results follow from the no-default case with $C = \delta k_L x$ in Proposition 1.

For $k_L > k_C/\delta$, debt with any coupon $C(\leq k_C x)$ becomes riskless, which leads to $\max_{C\in[0,k_C x]} F(x;C) = F_n(x; k_C x)$. Then, the firm chooses coupon $C(x) = k_C x$ at time 0. The results follow from the no-default case with $C = k_C x$ in Proposition 1.

Appendix E. Proof of Proposition 5

By Propositions 1 and 4, for $k_L < \bar{k}_L$, $\bar{D}(x) = D_d(x; k_C x)$ increases in k_L, while $\bar{C}(x) = k_C x$, $x_0(k_C x) = k_C x/\delta$, and $\bar{E}(x) = E_0(x; k_C x)$ are constant. Then, $\bar{F}(x)$ and $L\bar{V}(x)$ increase in k_L, while $C\bar{S}(x)$ decreases in k_L.

At $k_L = \bar{k}_L$, coupon $\bar{C}(x)$ changes from $k_C x$ to $\delta \bar{k}_L x$. By Proposition 4, $k_C > \delta \bar{k}_L$ holds. At $k_L = \bar{k}_L$, $\bar{E}(x)$ changes from $E_0(x; k_C x)$ to

$$E_n(x; \delta \bar{k}_L x) = \pi x - \frac{(1-\tau)\delta \bar{k}_L x}{r} - \frac{(\bar{k}_L)^{1-\gamma}\pi x}{\gamma}$$
$$= E_0(x; \delta \bar{k}_L x)$$
$$> E_0(x; k_C x)$$

(i.e., an upward jump), where we obtained the last inequality by $k_C x > \delta \bar{k}_L x$. By definition of $\bar{k}_L$ (i.e., $\bar{g}(\bar{k}_L) = 0$), $F_d(x; k_C x)$ continuously changes to $F_n(x; \delta \bar{k}_L x)$ at $k_L = \bar{k}_L$. By the continuity of $\bar{F}(x)$ and the upward jump of $\bar{E}(x)$, $\bar{D}(x)$ must jump downward at $k_L = \bar{k}_L$. Then, $L\bar{V}(x)$ jumps downward at $x_L = \bar{x}_L$, and $C\bar{S}(x)$ also jumps downward to 0.

For $k_L \in [\bar{k}_L, k_C/\delta]$, the results follow from proof of Proposition 3 (see the third paragraph of Appendix C).

By Propositions 1 and 4, for $k_L > k_C/\delta$, $\bar{E}(x) = E_n(x; k_C x)$ increases in k_L, while $\bar{D}(x) = k_C x/r, \bar{C}(x) = k_C x$, and $C\bar{S}(x) = 0$ are constant. Then, $\bar{F}(x)$ increases in k_L, and $L\bar{V}(x)$ decreases in k_L.

Appendix F. EBIT with both upper and lower reflecting barriers

With the inclusion of both upper and lower reflecting barriers, we cannot derive the default threshold explicitly in the default-possible case (in contrast to $x_0(C)$ in (7) of Proposition 1). Indeed, for given coupon C, the default threshold must be computed numerically as a solution to a nonlinear equation. Hence, we need to compute the optimal coupon by solving the firm value maximization problem numerically (in contrast to $C_0(x)$ in (9) of Proposition 2). Thus, we have constrained our analysis to the baseline model with only a lower reflecting barrier to establish the analytical results. Nonetheless, we verified numerically that the primary results are unchanged in the extended model incorporating both barriers. Fig. 7 illustrates the comparative statics with respect to lower reflecting barrier level k_L in the extended model, where the upper reflecting barrier is set at 5, and the other parameter values are set as in Table 1. Region $k_L < 0.148$ is the default-possible case, whereas region $k_L \geq 0.148$ is the no-default case. Fig. 7 shows that the baseline results in Proposition 3 and Fig. 2 hold robustly in the extended model. Additionally, we confirmed numerically that the sensitivities to the other parameters are unchanged from the baseline results, although we opted not to present those figures due to space constraints.

Appendix G. Alternative policy of annual subsidies

Assume that the government provides a constant subsidy flow $S > 0$ rather than setting an intervention threshold. Define $S' = S/(1-\tau)$. As in Section 3.1, for given coupon $C > S'$, the equity, debt, and firm values are expressed as

$$\tilde{E}(x;C) = \pi x - \frac{(1-\tau)(C-S')}{r} + \left(\frac{x}{\tilde{x}(C)}\right)^{\gamma}\left(\frac{(1-\tau)(C-S')}{r} - \pi\tilde{x}(C)\right),$$
$$\tilde{D}(x;C) = \frac{C}{r} - \left(\frac{x}{\tilde{x}(C)}\right)^{\gamma}\left(\frac{C}{r} - (1-\alpha)\left(\pi\tilde{x}(C) + \frac{S}{r}\right)\right),$$
$$\tilde{F}(x;C) = \pi x + \frac{\tau C + S}{r} - \left(\frac{x}{\tilde{x}(C)}\right)^{\gamma}\left(\alpha\left(\pi\tilde{x}(C) + \frac{S}{r}\right) + \frac{\tau C}{r}\right)$$

for $x \geq \tilde{x}(C)$, where default threshold $\tilde{x}(C) = (C-S')/\delta$. For $C \leq S'$, shareholders never default on their debt, and equity, debt, and firm values are expressed as

$$\tilde{E}(x;C) = \pi x + \frac{(1-\tau)(S'-C)}{r},$$
$$\tilde{D}(x;C) = \frac{C}{r},$$
$$\tilde{F}(x;C) = \pi x + \frac{\tau C + S}{r}.$$

Firm value $\tilde{F}(x;C)$ increases with $C \leq S'$ and is continuous at $C = S'$ because of $\lim_{C\downarrow S'} \tilde{x}(C)^{-\gamma} = 0$. We can also show that $\lim_{C\downarrow S'} \partial\tilde{F}/\partial C(x;C) > 0$ for $\gamma < -1$. Hence, $\arg\max_{C\geq 0} \tilde{F}(x;C) > S'$ holds for $\gamma < -1$, implying that the firm chooses risky capital structure regardless of subsidy levels S.[15] This scenario arises for the baseline parameter value (i.e., $\gamma = -1.351$ in Table 1).

For $-1 \leq \gamma < 0$ (which tends to hold for higher volatility σ), higher S can lead to $\arg\max_{C\geq 0} \tilde{F}(x;C) = S'$, implying that the firm issues debt up to the riskless debt capacity. In this case, we confirmed numerically that the results resemble those of our baseline model; indeed, intermediate subsidy levels lead to lower leverage and riskless capital structure, though the figure is excluded from the paper due to space constraints.

[15] In the presence of $S > 0$, we cannot derive $\arg\max_{C>S'} \tilde{F}(x;C)$ explicitly.

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