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Intersection-based architectures for decentralized diagnosis of discrete event systems[☆]

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ABSTRACT

In this paper, two intersection-based architectures, named the normal-state-estimator-intersection-based architecture (N-SEI architecture) and the failure-state-estimator-intersection-based architecture (F-SEI architecture), are examined for decentralized diagnosis of discrete event systems. For each of these architectures, the corresponding notion of codiagnosability is defined. These defined notions of codiagnosability are incomparable with inference diagnosability for the inference-based architecture. In addition, codiagnosability for the N-SEI architecture is weaker than the existing notion of intersection-based codiagnosability, while codiagnosability for the F-SEI architecture is incomparable with it. For each of the N-SEI and F-SEI architectures, a method for verifying the corresponding notion of codiagnosability is developed.

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1. Introduction

For discrete event systems (DESs), the failure diagnosis framework introduced by Sampath et al. (1995) has been extended to the decentralized setting, where multiple local diagnosers jointly diagnose the system so that the occurrence of any failure is correctly detected within a certain number of steps. Several architectures have been developed for decentralized diagnosis. In the disjunctive architecture (Debouk et al., 2000; Qiu & Kumar, 2006), the global failure decision is issued if at least one local diagnoser issues a local failure decision. On the other hand, in the conjunctive architecture (Wang et al., 2007), if all local diagnosers issue local failure decisions, then the global failure decision is issued. The notions of disjunctive codiagnosability (Qiu & Kumar, 2006) and conjunctive codiagnosability (Wang et al., 2007) were defined as the diagnosability properties for the disjunctive and conjunctive architectures, respectively. Later, inference-based approaches have been developed (Kumar & Takai, 2009; Takai & Kumar, 2017; Wang et al., 2007). In the inference-based approaches, each local diagnoser issues a local diagnosis decision by inferring diagnosis decisions of other local diagnosers. The notion of N -inference diagnosability, where N is a nonnegative integer that represents

the level of inference, characterizes the class of diagnosable systems in the inference-based architecture of Takai and Kumar (2017), which is more general than the conditional disjunctive and conjunctive architectures of Wang et al. (2007). However, for a system that is not N -inference diagnosable, another architecture has to be employed to diagnose it.

In the decentralized architectures mentioned above, each local diagnoser issues a local diagnosis decision. In Debouk et al. (2000), a decentralized architecture was proposed, where each local diagnoser sends a certain local state estimate to the coordinator and the coordinator makes the global diagnosis decision based on the intersection of the received local state estimates, as shown in Fig. 1. More precisely, each local diagnoser computes the set of all possible states regardless whether they are reached after the occurrence of a failure, as its local state estimate, and if these intersection only includes states reached after the occurrence of a failure, then the coordinator decides that a failure has occurred. For this intersection-based architecture, the codiagnosability property, which we call intersection-based codiagnosability in this paper, was defined and a verification method was developed for it (Panteli & Hadjicostis, 2013). As shown in this paper, however, intersection-based codiagnosability of Panteli and Hadjicostis (2013) should be weakened to characterize diagnosable systems in this intersection-based architecture. The intersection-based architecture is also employed for distributed diagnosis by Keroglou and Hadjicostis (2018). Besides, the intersection-based architecture was developed for decentralized supervisory control and the corresponding notion of intersection-based coobservability was introduced by Yin and Lafortune (2016). Interestingly, intersection-based coobservability is incomparable with N -inference observability

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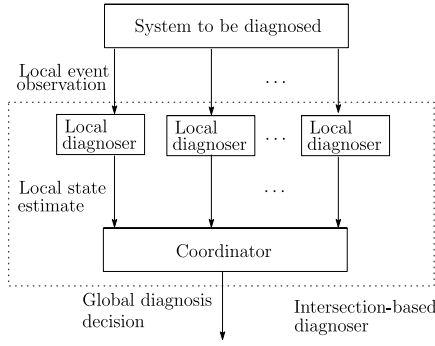


Fig. 1. Intersection-based architecture.

for the inference-based decentralized supervisory control architecture of Kumar and Takai (2007).

In this paper, we examine two intersection-based architectures, named the normal-state-estimator-intersection-based architecture (N-SEI architecture) and the failure-state-estimator-intersection-based architecture (F-SEI architecture). The main purpose of examining these two architectures is identifying new notions of codiagnosability that are different from the existing ones. In the N-SEI architecture (respectively, F-SEI architecture), each local diagnoser computes the set of possible states that are reached by executing normal strings (respectively, executing at least m events, where m is a given nonnegative integer, after the occurrence of a failure), as its local state estimate, and if the intersection of these local state estimates is empty, then the coordinator decides that a failure has occurred (respectively, a normal string has occurred or, after the occurrence of a failure, at most $m - 1$ events have occurred). The N-SEI architecture is equivalent to the existing intersection-based architecture in the sense that the occurrence of a failure is detected in the N-SEI architecture if and only if it is detected in the existing intersection-based architecture. However, the state set explored by local diagnosers to compute their local state estimates in the N-SEI architecture is smaller than the state set explored by them in the existing intersection-based architecture. For each of these architectures, we define the corresponding notion of codiagnosability to characterize diagnosable systems. Codiagnosability for the N-SEI architecture and that for the F-SEI architecture are incomparable, which implies that the N-SEI and F-SEI architectures complement each other. In addition, they are compared with the existing notions of codiagnosability. Codiagnosability for the N-SEI architecture is weaker than intersection-based codiagnosability of Panteli and Hadjicostis (2013), while codiagnosability for the F-SEI architecture is incomparable with it. An example of a system that is codiagnosable in the N-SEI architecture but not characterized by intersection-based codiagnosability of Panteli and Hadjicostis (2013) is presented. Moreover, codiagnosability for the N-SEI architecture and that for the F-SEI architecture are weaker than disjunctive codiagnosability and conjunctive codiagnosability, respectively, and incomparable with N -inference diagnosability for any $N \geq 1$. This means that there exists a system that is diagnosable in the N-SEI or F-SEI architecture but not diagnosable in the inference-based architecture of Takai and Kumar (2017). To diagnose such a system, the N-SEI or F-SEI architecture should be employed. These comparisons justify developing the N-SEI and F-SEI architectures for decentralized diagnosis. Then, for each of the N-SEI and F-SEI architectures, we present a method for verifying the corresponding notion of codiagnosability.

The present paper is an extended version of the authors' conference papers (Takai & Yamamoto, 2023, 2024). It contains

the proofs of the technical results, which were omitted in Takai and Yamamoto (2023) and Takai and Yamamoto (2024), two motivating examples, and additional explanations on the presented results.

2. Preliminaries

A DES to be diagnosed is modeled as a finite automaton $G = (Q, \Sigma, \delta, q_0)$, where Q is the finite set of states, Σ is the finite set of events, a partial function $\delta : Q \times \Sigma \rightarrow Q$ is the transition function, and $q_0 \in Q$ is the initial state. A sequence $q^{(0)} \xrightarrow{\sigma^{(0)}} q^{(1)} \xrightarrow{\sigma^{(1)}} \dots q^{(l-1)} \xrightarrow{\sigma^{(l-1)}} q^{(l)}$ ($l \geq 1$) of transitions such that $\delta(q^{(h)}, \sigma^{(h)}) = q^{(h+1)}$ for each $h \in \{0, 1, \dots, l-1\}$ is called a path of G , and it is also called a cycle if $q^{(0)} = q^{(l)}$. A finite automaton $G' = (Q', \Sigma, \delta', q'_0)$ is called a subautomaton of G if $Q' \subseteq Q$, $q'_0 = q_0$, and, for any $q \in Q'$ and any $\sigma \in \Sigma$, $\delta'(q, \sigma) \Rightarrow \delta(q, \sigma) = \delta(q, \sigma)$. Let Σ^* be the set of all finite strings of elements of Σ , including the empty string ε . The transition function δ can be generalized to $\delta : Q \times \Sigma^* \rightarrow Q$ in the usual manner. For any $q \in Q$ and any $s \in \Sigma^*$, $\delta(q, s)!$ means that $\delta(q, s)$ is defined, that is, $\delta(q, s) \in Q$ and $\neg\delta(q, s)!$ denotes the negation of $\delta(q, s)!$. Let $\mathbb{N}$ be the set of all nonnegative integers. For any $s \in \Sigma^*$, $|s| \in \mathbb{N}$ denotes its length, and s^m represents m concatenations of s , where $m \in \mathbb{N}$. Besides, for any $m \in \mathbb{N}$, we let $\Sigma^{\geq m} = \{s \in \Sigma^* \mid |s| \geq m\}$. A subset of Σ^* is called a language. The generated language of G , denoted by $L(G)$, is defined as $L(G) = \{s \in \Sigma^* \mid \delta(q_0, s)!\}$. Let $\text{Reach}(G)$ be the set of all reachable states of G , that is, $\text{Reach}(G) = \{q \in Q \mid (\exists s \in L(G)) q = \delta(q_0, s)\}$. For a language $K \subseteq \Sigma^*$, the set of all prefixes of strings in K is denoted by $\text{pr}(K)$, that is, $\text{pr}(K) = \{s \in \Sigma^* \mid (\exists t \in \Sigma^*) st \in K\}$. If $K = \text{pr}(K)$, then K is said to be (prefix-)closed. Clearly, the generated language $L(G)$ of G is nonempty and closed.

In the setting of decentralized diagnosis, n local diagnosers locally observe the occurrence of an event in G . Let $I = \{1, 2, \dots, n\}$ be the index set of local diagnosers. For the i th local diagnoser ($i \in I$), $\Sigma_{o,i} \subseteq \Sigma$ is the set of locally observable events, and $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$ is the natural projection from Σ^* to $\Sigma_{o,i}^*$, which is inductively defined as $P_i(\varepsilon) = \varepsilon$ and, for any $s \in \Sigma^*$ and any $\sigma \in \Sigma$, if $\sigma \in \Sigma_{o,i}$ then $P_i(s\sigma) = P_i(s)\sigma$; otherwise $P_i(s\sigma) = P_i(s)$. If a string $s \in L(G)$ is executed in G , then the locally observable event string $P_i(s) \in \Sigma_{o,i}^*$ is observed by the i th local diagnoser. Two strings $s, s' \in L(G)$ are said to be indistinguishable (under P_i) if $P_i(s) = P_i(s')$. The inverse projection $P_i^{-1} : \Sigma_{o,i}^* \rightarrow 2^{\Sigma^*}$ is defined by $P_i^{-1}(t_i) = \{s \in \Sigma^* \mid P_i(s) = t_i\}$ for any $t_i \in \Sigma_{o,i}^*$. That is, $P_i^{-1}(t_i)$ is the set of strings that are observed as t_i by the i th local diagnoser. For any languages $K \subseteq \Sigma^*$ and $K_i \subseteq \Sigma_{o,i}^*$, let $P_i(K) = \{P_i(s) \in \Sigma_{o,i}^* \mid s \in K\}$ and $P_i^{-1}(K_i) = \{s \in \Sigma^* \mid P_i(s) \in K_i\}$. For any $s \in L(G)$, the estimate of the current state of G with respect to P_i is obtained as $\mathcal{E}_{G,i}(P_i(s)) = \{q \in Q \mid (\exists s_i \in L(G) \cap P_i^{-1}(P_i(s))) \delta(q_0, s_i) = q\}$.

For each $i \in I$, the unobservable reach is defined as a function $UR_{G,i} : 2^Q \rightarrow 2^Q$ such that $UR_{G,i}(Q') = \{q \in Q \mid (\exists q' \in Q')(\exists s \in (\Sigma - \Sigma_{o,i})^*) \delta(q', s) = q\}$ for each $Q' \subseteq 2^Q$. Then, the observer automaton $\text{Obs}_i(G) = (Q_{\text{obs}_i}, \Sigma_{o,i}, \delta_{\text{obs}_i}, q_{\text{obs}_i,0})$ of G is defined, where the state set is $Q_{\text{obs}_i} = 2^Q$, the initial state is $q_{\text{obs}_i,0} = UR_{G,i}(\{q_0\})$, and the transition function $\delta_{\text{obs}_i} : Q_{\text{obs}_i} \times \Sigma_{o,i} \rightarrow Q_{\text{obs}_i}$ is given as

$$\delta_{\text{obs}_i}(q_{\text{obs}_i}, \sigma_{o,i}) = \begin{cases} q'_{\text{obs}_i}, & \text{if } (\exists q \in q_{\text{obs}_i}) \delta(q, \sigma_{o,i})! \\ \text{undefined}, & \text{otherwise,} \end{cases}$$

where $q'_{\text{obs}_i} = UR_{G,i}(\{q' \in Q \mid (\exists q \in q_{\text{obs}_i}) \delta(q, \sigma_{o,i}) = q'\})$, for each $q_{\text{obs}_i} \in Q_{\text{obs}_i}$ and each $\sigma_{o,i} \in \Sigma_{o,i}$. It follows that $L(\text{Obs}_i(G)) = P_i(L(G))$ and, for any $t_i \in P_i(L(G))$, $\delta_{\text{obs}_i}(q_{\text{obs}_i,0}, t_i) = \mathcal{E}_{G,i}(t_i)$ (Cassandras & Lafortune, 2021).

3. Existing intersection-based architecture

In this section, we review an existing intersection-based architecture proposed by [Debouk et al. \(2000\)](#) for decentralized diagnosis and the corresponding notion of intersection-based codiagnosability introduced by [Panteli and Hadjicostis \(2013\)](#).

Let the normal behavior of the system $G = (Q, \Sigma, \delta, q_0)$ be described by a nonempty closed regular sublanguage $K \subseteq L(G)$. The occurrence of an event $\sigma \in \Sigma$ such that $s\sigma \in L(G) - K$ for some $s \in K$ models a failure. A string in K (respectively, $L(G) - K$) is called a normal (respectively, failure) string. For the sake of simplicity, we assume in the remainder of the paper that the system G is deadlock free, that is, for any $s \in L(G)$, there exists $\sigma \in \Sigma$ such that $s\sigma \in L(G)$. In addition, we assume without loss of generality that a nonempty closed regular sublanguage $K \subseteq L(G)$ that models the normal behavior of the system G is generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G . Then, it holds that $L(G_N) = K$. To characterize failure strings in $L(G) - K$, G_N is augmented by adding a dump state $q_d \notin Q_N$. The augmented automaton is defined as $\tilde{G}_N = (Q_N, \Sigma, \tilde{\delta}_N, q_0)$, where the state set is $\tilde{Q}_N = Q_N \cup \{q_d\}$, and the transition function $\tilde{\delta}_N : \tilde{Q}_N \times \Sigma \rightarrow \tilde{Q}_N$ is given as

$$\tilde{\delta}_N(\tilde{q}_N, \sigma) = \begin{cases} \delta_N(\tilde{q}_N, \sigma), & \text{if } \tilde{q}_N \in Q_N \wedge \delta_N(\tilde{q}_N, \sigma)! \\ q_d, & \text{otherwise} \end{cases}$$

for each $\tilde{q}_N \in \tilde{Q}_N$ and each $\sigma \in \Sigma$ ([Qiu & Kumar, 2006](#)). It follows from the definition of the transition function $\tilde{\delta}_N$ that $L(\tilde{G}_N) = \Sigma^*$. Then, we construct the synchronous composition $G \parallel \tilde{G}_N = (Q \times \tilde{Q}_N, \Sigma, \alpha, (q_0, q_0))$ of G and $\tilde{G}_N$, where the transition function $\alpha : (Q \times \tilde{Q}_N) \times \Sigma \rightarrow (Q \times \tilde{Q}_N)$ is given as

$$\alpha((q, \tilde{q}_N), \sigma) = \begin{cases} (\delta(q, \sigma), \tilde{\delta}_N(\tilde{q}_N, \sigma)), & \text{if } \delta(q, \sigma)! \\ \text{undefined}, & \text{otherwise} \end{cases}$$

for each $(q, \tilde{q}_N) \in Q \times \tilde{Q}_N$ and each $\sigma \in \Sigma$. Since $L(G \parallel \tilde{G}_N) = L(G) \cap L(\tilde{G}_N) = L(G) \cap \Sigma^* = L(G)$, $G \parallel \tilde{G}_N$ can be considered as a refined model of the system. In the refined model $G \parallel \tilde{G}_N$, a string $s \in L(G)$ is a failure one with $s \in L(G) - K$ if and only if the second element of the state reached by the execution of $s \in L(G)$ is the dump state q_d .

A certain intersection-based architecture shown in [Fig. 1](#) was proposed by [Debouk et al. \(2000\)](#) for decentralized diagnosis¹. When $s \in L(G)$ is executed in the system G , the i th local diagnoser observes $P_i(s)$, computes the set $\mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s))$ of possible states in the refined model $G \parallel \tilde{G}_N$, as its local state estimate, and sends it to the coordinator. Then, the global diagnosis decision is issued by the coordinator based on the intersection $\bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s))$. If $\bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s)) \subseteq Q \times \{q_d\}$, then the intersection-based diagnoser decides that a failure has occurred. Note that, since G_N is a subautomaton of G , a reachable state of $G \parallel \tilde{G}_N$ is of the form (q, q) or (q, q_d) , where $q \in Q$. For this intersection-based architecture, the notion of intersection-based codiagnosability introduced by [Panteli and Hadjicostis \(2013\)](#) can be described by the following condition:

$$\begin{aligned} (\exists m \in \mathbb{N}) \\ (\forall s \in L(G) \cap (L(G) - K) \Sigma^{\geq m}) (\forall q, q' \in Q) \\ \{(q, q), (q', q_d)\} \not\subseteq \bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s)). \end{aligned} \quad (1)$$

The condition (1) is weaker than disjunctive codiagnosability ([Panteli & Hadjicostis, 2013](#)).

¹ In [Debouk et al. \(2000\)](#), a failure is modeled by the occurrence of an event in the failure event set Σ_f . In this paper, the intersection-based architecture proposed by [Debouk et al. \(2000\)](#) is slightly modified to adapt to the case where a failure is modeled by the occurrence of an event $\sigma \in \Sigma$ such that $s\sigma \in L(G) - K$ for some $s \in K$.

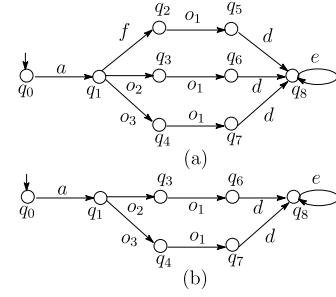


Fig. 2. (a) Finite automaton G that models plant and (b) its subautomaton G_N .

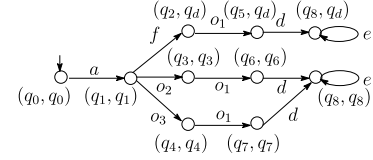


Fig. 3. Refined model $G \parallel \tilde{G}_N$.

4. Motivating examples

In this section, we first show that intersection-based codiagnosability of [Panteli and Hadjicostis \(2013\)](#) should be weakened to characterize diagnosable systems in the intersection-based architecture presented in Section 3.

Example 1. We consider the finite automaton G shown in [Fig. 2\(a\)](#), which models a simple plant that processes a job. The event set Σ of G is $\Sigma = \{a, d, o_1, o_2, o_3, f, e\}$, where a and d denote arrival and departure of a job, respectively, o_j ($j = 1, 2, 3$) are three kinds of operations, f denotes the occurrence of a failure, and e is the fictitious event added to satisfy the deadlock-free assumption. A departure after solely o_1 (without o_2 or o_3) is considered as a failure. The finite automaton G_N that describes the normal behavior of G is shown in [Fig. 2\(b\)](#).

Let $I = \{1, 2\}$, $\Sigma_{0,1} = \{a, o_1, o_2, d\}$, and $\Sigma_{0,2} = \{a, o_1, o_3, d\}$. In the intersection-based architecture presented in Section 3, each local diagnoser estimates the current state of the refined model $G \parallel \tilde{G}_N$ shown in [Fig. 3](#). We consider a failure string $afo_1 \in L(G) - K$. Since $\mathcal{E}_{G \parallel \tilde{G}_N, 1}(P_1(afo_1)) = \mathcal{E}_{G \parallel \tilde{G}_N, 1}(ao_1) = \{(q_7, q_7), (q_5, q_d)\}$ and $\mathcal{E}_{G \parallel \tilde{G}_N, 2}(P_2(afo_1)) = \mathcal{E}_{G \parallel \tilde{G}_N, 2}(ao_1) = \{(q_6, q_6), (q_5, q_d)\}$, we have $\bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(afo_1)) = \{(q_5, q_d)\}$. Thus, the occurrence of the failure event f is detected. However, the condition (1) of intersection-base codiagnosability of [Panteli and Hadjicostis \(2013\)](#) is not satisfied. For any $m \in \mathbb{N}$, we consider a failure string $afo_1de^m \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$. Since $\mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(afo_1de^m)) = \{(q_8, q_8), (q_8, q_d)\}$ for $i = 1, 2$, we have $\bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(afo_1de^m)) = \{(q_8, q_8), (q_8, q_d)\}$. Therefore, intersection-based codiagnosability of [Panteli and Hadjicostis \(2013\)](#) should be weakened to characterize diagnosable systems in the intersection-based architecture.

Next, we show an example of a system that is not diagnosable in the inference-based architecture of [Takai and Kumar \(2017\)](#). To diagnose such a system, another architecture must be employed. G is said to be N -inference diagnosable with respect to a nonempty closed regular sublanguage $K \subseteq L(G)$ if there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$ ([Takai & Kumar, 2017](#)), where a monotonically decreasing sequence $\{(F_k(m), H_k(m))\}_{k \geq 0}$ of language pairs is defined as follows:

- Base step:

$$F_0(m) = L(G) \cap (L(G) - K) \Sigma^{\geq m},$$

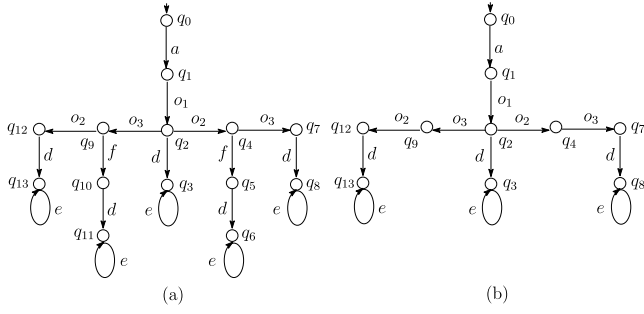


Fig. 4. (a) Finite automaton G that models plant and (b) its subautomaton G_N .

$$H_0(m) = K.$$

• Induction step:

$$F_{k+1}(m) = F_k(m) \cap \left(\bigcap_{i \in I} P_i^{-1} P_i(H_k(m)) \right),$$

$$H_{k+1}(m) = H_k(m) \cap \left(\bigcap_{i \in I} P_i^{-1} P_i(F_k(m)) \right).$$

Remark 2. Disjunctive (respectively, conjunctive) codiagnosability is equivalent to the existence of $m \in \mathbb{N}$ such that $F_1(m) = \emptyset$ (respectively, $H_1(m) = \emptyset$) (Takai & Kumar, 2017).

The following example shows a system that is not N -inference diagnosable for any $N \in \mathbb{N}$.

Example 3. We consider the finite automaton G shown in Fig. 4(a), which models another simple plant that processes a job. The event set Σ of G is $\Sigma = \{a, d, o_1, o_2, o_3, f, e\}$, where the meaning of each event is the same as that of Example 1. Normally, a departure of a job occurs following one of the operation sequences $o_1, o_1 o_2 o_3$, and $o_1 o_3 o_2$. A departure following $o_1 o_2$ or $o_1 o_3$ is considered as a failure. The finite automaton G_N that describes the normal behavior of G is shown in Fig. 4(b).

Let $I = \{1, 2\}$, $\Sigma_{0,1} = \{a, o_2, d\}$, and $\Sigma_{0,2} = \{a, o_3, d\}$. Then, for any $m \in \mathbb{N}$ with $m \geq 1$, we have $F_0(m) = \{ao_1 o_2 f d e^h, ao_1 o_3 f d e^h \mid h \geq m - 1\}$ and $H_0(m) = pr(\{ao_1 d e^h, ao_1 o_2 o_3 d e^h, ao_1 o_3 o_2 d e^h \mid h \geq 0\})$. In addition, for any $k \in \mathbb{N}$ with $k \geq 1$, we have $F_k(m) = \{ao_1 o_2 f d e^h, ao_1 o_3 f d e^h \mid h \geq m - 1\} \neq \emptyset$ and $H_k(m) = \{ao_1 d e^h, ao_1 o_2 o_3 d e^h, ao_1 o_3 o_2 d e^h \mid h \geq 0\} \neq \emptyset$. It follows that, for any $N \in \mathbb{N}$, G is not N -inference diagnosable with respect to K .

5. N-SEI and F-SEI architectures

In this paper, we examine two intersection-based architectures, which we call the normal-state-estimator-intersection-based architecture (N-SEI architecture) and failure-state-estimator-intersection-based architecture (F-SEI architecture), and introduce the corresponding notions of codiagnosability.

5.1. N-SEI architecture

In the N-SEI architecture, each local diagnoser computes the set of possible states of the normal behavior model G_N , as its local state estimate, and the coordinator makes the global diagnosis decision based on the intersection of these local state estimates. More precisely, when $s \in L(G)$ is executed in the system G , the i th local diagnoser observes $P_i(s) \in P_i(L(G))$, computes the estimate of possible states in the normal behavior model G_N as

$$\mathcal{E}_i^{N-SEI}(P_i(s)) = \begin{cases} \mathcal{E}_{G_N, i}(P_i(s)), & \text{if } P_i(s) \in P_i(L(G_N)) \\ \emptyset, & \text{otherwise} \end{cases}$$

and sends it to the coordinator. Based on the intersection of $\mathcal{E}_i^{N-SEI}(P_i(s))$ over I , the coordinator issues the global diagnosis decision.

The following lemma shows that if the intersection of the local state estimates is empty, we can conclude that the executed string is a failure string.

Lemma 4. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , we consider any $s \in L(G)$. If $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) = \emptyset$ then $s \in L(G) - K$.

Proof. We prove the contrapositive. If $s \in K = L(G_N)$ then $P_i(s) \in P_i(L(G_N))$ for any $i \in I$. Since $\delta_N(q_0, s) \in \mathcal{E}_{G_N, i}(P_i(s)) = \mathcal{E}_i^{N-SEI}(P_i(s))$ for any $i \in I$, we have $\delta_N(q_0, s) \in \bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) \neq \emptyset$. $\square$

The condition $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) = \emptyset$ means that the local state estimates $\mathcal{E}_i^{N-SEI}(P_i(s))$ ($1 = 1, 2, \dots, n$) do not have a common element. It does not necessarily imply that there exists $i \in I$ such that $\mathcal{E}_i^{N-SEI}(P_i(s)) = \emptyset$. It is worth mentioning that disjunctive codiagnosability requires the existence of $i \in I$ such that $\mathcal{E}_i^{N-SEI}(P_i(s)) = \emptyset$. By Lemma 4, the intersection-based diagnoser is defined as a function $D_N : L(G) \rightarrow \{0, 1\}$ such that

$$D_N(s) = \begin{cases} 1, & \text{if } \bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) = \emptyset, \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

for each $s \in L(G)$.

The following proposition shows that the N-SEI architecture is equivalent to the existing intersection-based architecture presented in Section 3 in the sense that the occurrence of a failure is detected in the N-SEI architecture if and only if it is detected in the existing intersection-based architecture.

Proposition 5. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , we consider any $s \in L(G)$. Then, $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) = \emptyset$ if and only if $\bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s)) \subseteq Q \times \{q_d\}$.

Proof. First, we prove the contrapositive of the sufficiency part. We suppose that $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) \neq \emptyset$. We consider any $q_N \in \bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) \neq \emptyset$. For any $i \in I$, since $q_N \in \mathcal{E}_{G_N, i}(P_i(s))$, there exists $s_i \in L(G_N)$ such that $P_i(s_i) = P_i(s)$ and $\delta_N(q_0, s_i) = q_N$. Then, we have $\alpha((q_0, q_0), s_i) = (q_N, q_N) \in \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s))$. It follows that $(q_N, q_N) \in \bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s)) \not\subseteq Q \times \{q_d\}$.

Next, we prove the contrapositive of the necessity part. We suppose that $\bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s)) \not\subseteq Q \times \{q_d\}$. Then, there exists $q_N \in Q$ such that $(q_N, q_N) \in \bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s))$. For any $i \in I$, there exists $s_i \in L(G)$ such that $P_i(s_i) = P_i(s)$ and $\alpha((q_0, q_0), s_i) = (q_N, q_N)$. Since $(q_N, q_N) \notin Q \times \{q_d\}$, we have $s_i \in L(G_N)$, which implies $\delta_N(q_0, s_i) = q_N$ and $P_i(s) = P_i(s_i) \in P_i(L(G_N))$. It follows that $q_N \in \mathcal{E}_{G_N, i}(P_i(s)) = \mathcal{E}_i^{N-SEI}(P_i(s))$. Thus, we have $q_N \in \bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) \neq \emptyset$. $\square$

Remark 6. Although the N-SEI architecture is equivalent to the existing intersection-based architecture, the state set Q_N explored by local diagnosers to compute their local state estimates in the N-SEI architecture is smaller than the state set $Q \times \tilde{Q}_N$ explored by them in the existing intersection-based architecture.

To guarantee that the occurrence of any failure is correctly detected within a certain number of steps, we impose the following two conditions on D_N :

$$\begin{aligned} & (\exists m \in \mathbb{N}) \\ & (\forall s \in L(G) \cap (L(G) - K) \Sigma^{\geq m}) (\exists t \in pr(\{s\})) \\ & D_N(t) = 1 \end{aligned} \quad (3)$$

and

$$(\forall s \in K) D_N(s) = 0. \quad (4)$$

Remark 7. The condition (3) requires the existence of $m \in \mathbb{N}$ such that, for any $s \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$, there exists a prefix $t \in pr(\{s\})$ such that $D_N(t) = 1$. Even if D_N satisfies (3),

$$(\exists m \in \mathbb{N})(\forall s \in L(G) \cap (L(G) - K) \Sigma^{\geq m}) D_N(s) = 1 \quad (5)$$

does not necessarily hold. As an example, we consider the system G of Example 1. For any $m \in \mathbb{N}$, we consider a failure string $afo_1de^m \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$. For its prefix $afo_1 \in pr(\{afo_1de^m\})$, we have $\mathcal{E}_1^{N-SEI}(P_1(afo_1)) \cap \mathcal{E}_2^{N-SEI}(P_2(afo_1)) = \{q_7\} \cap \{q_6\} = \emptyset$, which implies together with (2) that $D_N(afo_1) = 1$. However, since $\mathcal{E}_1^{N-SEI}(P_1(afo_1de^m)) \cap \mathcal{E}_2^{N-SEI}(P_2(afo_1de^m)) = \{q_8\} \cap \{q_8\} = \{q_8\} \neq \emptyset$, we have $D_N(afo_1de^m) = 0$. Therefore, D_N only satisfies (3).

By Lemma 4, D_N always satisfies (4). As a condition under which D_N satisfies (3), normal-state-estimator-intersection-based codiagnosability (N-SEI codiagnosability) is defined as follows:

Definition 8. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , G is said to be N-SEI codiagnosable with respect to K if

$$(\exists m \in \mathbb{N})$$

$$(\forall s \in L(G) \cap (L(G) - K) \Sigma^{\geq m})(\exists t \in pr(\{s\}))$$

$$\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(t)) = \emptyset.$$

N-SEI codiagnosability is a necessary and sufficient condition for D_N to satisfy (3) and (4), as shown in the following proposition.

Proposition 9. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , the intersection-based diagnoser $D_N : L(G) \rightarrow \{0, 1\}$ given by (2) satisfies (3) and (4) if and only if G is N-SEI codiagnosable with respect to K .

Proof. First, we prove the sufficiency part. Since G is N-SEI codiagnosable with respect to K , there exists $m \in \mathbb{N}$ such that

$$(\forall s \in L(G) \cap (L(G) - K) \Sigma^{\geq m})(\exists t \in pr(\{s\}))$$

$$\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(t)) = \emptyset.$$

By (2), D_N satisfies (3). In addition, by Lemma 4, D_N satisfies (4).

Next, we prove the necessity part. Since D_N satisfies (3), by (2), G is N-SEI codiagnosable with respect to K . $\square$

Introducing N-SEI codiagnosability resolves the issue raised in Example 1. N-SEI codiagnosability is weaker than intersection-based codiagnosability of Panteli and Hadjicostis (2013), as shown in the following proposition.

Proposition 10. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , if the condition (1) holds, then G is N-SEI codiagnosable with respect to K .

Proof. We prove the contrapositive. If G is not N-SEI codiagnosable with respect to K , then it holds that

$$(\forall m \in \mathbb{N})$$

$$(\exists s \in L(G) \cap (L(G) - K) \Sigma^{\geq m})(\forall t \in pr(\{s\}))$$

$$\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(t)) \neq \emptyset.$$

Since $s \in pr(\{s\})$, we have $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) \neq \emptyset$, which implies together with Proposition 5 that $\bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s)) \not\subseteq Q \times \{q_d\}$. Then, there exists $q_N \in Q$ such that $(q_N, q_N) \in \bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s))$. Moreover, since $s \in L(G) \cap (L(G) - K) \Sigma^{\geq m} \subseteq L(G) - K$, we have $\alpha((q_0, q_0), s) = (q, q_d) \in \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s))$, where $q = \delta(q_0, s) \in Q$, for each $i \in I$. Thus, we have $\{(q_N, q_N), (q, q_d)\} \subseteq \bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_N, i}(P_i(s))$, which implies that (1) does not hold. $\square$

As shown in the following example, the reverse implication of Proposition 10 does not hold in general.

Example 11. Again, we consider the system G of Example 1, which does not satisfy the condition (1). We consider any $s \in L(G) \cap (L(G) - K) \Sigma^{\geq 1}$. For its prefix $afo_1 \in pr(\{s\})$, we have $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(afo_1)) = \emptyset$, as shown in Remark 7. Thus, G is N-SEI codiagnosable with respect to K .

In addition, the issue raised in Example 3 can be resolved by employing the N-SEI architecture.

Example 12. We consider the system G of Example 3, which is not N -inference diagnosable with respect to K for any $N \in \mathbb{N}$. Any $s \in L(G) \cap (L(G) - K) \Sigma^{\geq 1}$ can be written as $s = ao_1o_2fde^h$ or $s = ao_1o_3fde^h$, where $h \geq 0$. In the case of $s = ao_1o_2fde^h$, we have $ao_1o_2fd \in pr(\{s\})$ and $\mathcal{E}_1^{N-SEI}(P_1(ao_1o_2fd)) \cap \mathcal{E}_2^{N-SEI}(P_2(ao_1o_2fd)) = \mathcal{E}_1^{N-SEI}(ao_2d) \cap \mathcal{E}_2^{N-SEI}(ad) = \{q_8, q_{13}\} \cap \{q_3\} = \emptyset$, which implies that the occurrence of the failure event f is detected after ao_1o_2fd is executed. Also, in the case of $s = ao_1o_3fde^h$, we have $ao_1o_3fd \in pr(\{s\})$ and $\mathcal{E}_1^{N-SEI}(P_1(ao_1o_3fd)) \cap \mathcal{E}_2^{N-SEI}(P_2(ao_1o_3fd)) = \mathcal{E}_1^{N-SEI}(ad) \cap \mathcal{E}_2^{N-SEI}(ao_3d) = \{q_3\} \cap \{q_8, q_{13}\} = \emptyset$. Thus, G is N-SEI codiagnosable with respect to K , so the occurrence of the failure event f can be correctly detected in the N-SEI architecture.

5.2. F-SEI architecture

In the F-SEI architecture, for a given nonnegative integer $m \in \mathbb{N}$, each local diagnoser computes, as its local state estimate, the set of possible states that are reached by executing at least m events after the occurrence of a failure. A nonnegative integer $m \in \mathbb{N}$ is a parameter of the F-SEI architecture, which represents an allowable detection delay.

For a given nonnegative integer $m \in \mathbb{N}$, $L(G) \cap (L(G) - K) \Sigma^{\geq m}$ is the set of failure strings such that at least m events occur after the occurrence of a failure. To characterize such failure strings in $L(G) \cap (L(G) - K) \Sigma^{\geq m}$, we augment G_N by adding $m + 1$ dump states $q_{d0}, q_{d1}, \dots, q_{dm} \notin Q_N$. The augmented automaton is defined as $\tilde{Q}_{N_m} = (\tilde{Q}_{N_m}, \Sigma, \tilde{\delta}_{N_m}, q_0)$, where the state set is $\tilde{Q}_{N_m} = Q_N \cup \{q_{d0}, q_{d1}, \dots, q_{dm}\}$, and the transition function $\tilde{\delta}_{N_m} : \tilde{Q}_{N_m} \times \Sigma \rightarrow \tilde{Q}_{N_m}$ is given as

$$\tilde{\delta}_{N_m}(\tilde{q}_{N_m}, \sigma) = \begin{cases} \delta_N(\tilde{q}_{N_m}, \sigma), & \text{if } \tilde{q}_{N_m} \in Q_N \wedge \delta_N(\tilde{q}_{N_m}, \sigma)! \\ q_{d0}, & \text{if } \tilde{q}_{N_m} \in Q_N \wedge \neg \delta_N(\tilde{q}_{N_m}, \sigma)! \\ q_{d(h+1)}, & \text{if } m \geq 1 \wedge \tilde{q}_{N_m} = q_{dh} \\ & (h \in \{0, 1, \dots, m-1\}) \\ q_{dm}, & \text{otherwise} \end{cases}$$

for each $\tilde{q}_{N_m} \in \tilde{Q}_{N_m}$ and each $\sigma \in \Sigma$ (Yamamoto & Takai, 2015). The augmented automaton $\tilde{G}_N$ defined in Section 3 can be considered as a special case of $\tilde{G}_{N_m}$ with $m = 0$. It follows from the definition of the transition function $\tilde{\delta}_{N_m}$ that $L(\tilde{G}_{N_m}) = \Sigma^*$. Then, we construct the synchronous composition $G \parallel \tilde{G}_{N_m} = (Q \times \tilde{Q}_{N_m}, \Sigma, \alpha_m, (q_0, q_0))$ of G and $\tilde{G}_{N_m}$, where the transition function $\alpha_m : (Q \times \tilde{Q}_{N_m}) \times \Sigma \rightarrow (Q \times \tilde{Q}_{N_m})$ is defined in a similar way to the transition function $\alpha : (Q \times \tilde{Q}_N) \times \Sigma \rightarrow (Q \times \tilde{Q}_N)$ of $G \parallel \tilde{G}_N$. Since $L(G \parallel \tilde{G}_{N_m}) = L(G)$, $G \parallel \tilde{G}_{N_m}$ can also be regarded

as a refined model of the system. In the refined model $G \parallel \tilde{G}_{N_m}$, a string $s \in L(G)$ is a failure one with $s \in L(G) \cap (L(G) - K)\Sigma^{\geq m}$ if and only if the second element of the state reached by the execution of $s \in L(G)$ is the final dump state q_{dm} .

When $s \in L(G)$ is executed in the system G , the i th local diagnoser observes $P_i(s) \in P_i(L(G))$ and computes $\mathcal{E}_i^{F-SEI}(P_i(s), m) = \mathcal{E}_{G \parallel \tilde{G}_{N_m}, i}(P_i(s)) \cap (Q \times \{q_{dm}\})$, which is the set of states reached by indistinguishable failure strings in $L(G) \cap (L(G) - K)\Sigma^{\geq m}$ in the refined model $G \parallel \tilde{G}_{N_m}$. That is, $\mathcal{E}_i^{F-SEI}(P_i(s), m)$ is the set of possible states that are reached by executing at least m events after the occurrence of a failure. Then, the i th local diagnoser sends $\mathcal{E}_i^{F-SEI}(P_i(s), m)$ to the coordinator, as its local state estimate. Based on the intersection of $\mathcal{E}_i^{F-SEI}(P_i(s), m)$ over I , the coordinator makes the global diagnosis decision.

The following lemma shows that if the intersection of the local state estimates is empty, we can conclude that the executed string is not an element of $(L(G) - K)\Sigma^{\geq m}$.

Lemma 13. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , we consider any $m \in \mathbb{N}$ and any $s \in L(G)$. If $\bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) = \emptyset$ then $s \notin (L(G) - K)\Sigma^{\geq m}$.

Proof. We prove the contrapositive. If $s \in L(G) \cap (L(G) - K)\Sigma^{\geq m}$, then $\alpha_m((q_0, q_0), s) \in \mathcal{E}_{G \parallel \tilde{G}_{N_m}, i}(P_i(s)) \cap (Q \times \{q_{dm}\}) = \mathcal{E}_i^{F-SEI}(P_i(s), m)$ for any $i \in I$. It follows that $\alpha_m((q_0, q_0), s) \in \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) \neq \emptyset$. $\square$

Similar to the N-SEI architecture, the condition $\bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) = \emptyset$ does not necessarily imply that there exists $i \in I$ such that $\mathcal{E}_i^{F-SEI}(P_i(s), m) = \emptyset$. Note that conjunctive codiagnosability requires the existence of such $i \in I$.

By Lemma 13, the intersection-based diagnoser is defined as a function $D_{F_m} : L(G) \rightarrow \{0, 1\}$ such that, for any $s \in L(G)$,

$$D_{F_m}(s) = \begin{cases} 1, & \text{if } \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

To correctly detect the occurrence of any failure within m steps, we impose the following two conditions on D_{F_m} :

$$(\forall s \in L(G) \cap (L(G) - K)\Sigma^{\geq m})(\exists t \in pr(\{s\})) \quad D_{F_m}(t) = 1 \quad (7)$$

and

$$(\forall s \in K) D_{F_m}(s) = 0. \quad (8)$$

Since D_{F_m} always satisfies (7) as shown in Lemma 13, failure-state-estimator-intersection-based codiagnosability (F-SEI codiagnosability) is defined, as a condition for the existence of $m \in \mathbb{N}$ such that D_{F_m} satisfies (8), as follows:

Definition 14. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , G is said to be F-SEI codiagnosable with respect to K if

$$(\exists m \in \mathbb{N})(\forall s \in K) \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) = \emptyset.$$

F-SEI codiagnosability is a necessary and sufficient condition for the existence of a nonnegative integer $m \in \mathbb{N}$ such that the intersection-based diagnoser D_{F_m} satisfies (7) and (8), as shown in the following proposition.

Proposition 15. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , there exists a nonnegative integer $m \in \mathbb{N}$ such that the intersection-based diagnoser $D_{F_m} : L(G) \rightarrow \{0, 1\}$ given by (6) satisfies (7) and (8) if and only if G is F-SEI codiagnosable with respect to K .

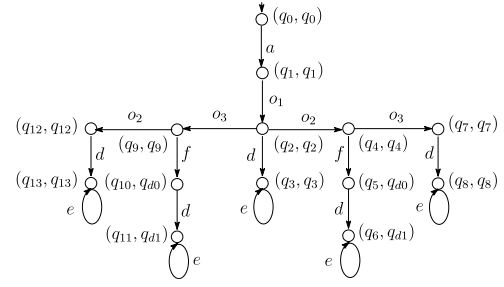


Fig. 5. Refined model $G \parallel \tilde{G}_{N_1}$.

Proof. First, we prove the sufficiency part. Since G is F-SEI codiagnosable with respect to K , there exists $m \in \mathbb{N}$ such that

$$(\forall s \in K) \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) = \emptyset.$$

For such $m \in \mathbb{N}$, by (6), D_{F_m} satisfies (8). In addition, since $s \in pr(\{s\})$, by Lemma 13, D_{F_m} satisfies (7).

Next, we prove the necessity part. Since there exists $m \in \mathbb{N}$ such that D_{F_m} satisfies (8), by (6), G is F-SEI codiagnosable with respect to K . $\square$

Remark 16. By the proof of Proposition 15, we need to find a nonnegative integer $m \in \mathbb{N}$ such that

$$(\forall s \in K) \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) = \emptyset$$

to synthesize the intersection-based diagnoser D_{F_m} that satisfies (7) and (8). Due to the page limit, the issue of computing such $m \in \mathbb{N}$ is considered out of scope for this paper.

Remark 17. Unlike the intersection-based diagnoser D_N in the N-SEI architecture, by Lemma 13, D_{F_m} always satisfies

$$(\forall s \in L(G) \cap (L(G) - K)\Sigma^{\geq m}) D_{F_m}(s) = 1.$$

The issue raised in Example 3 can also be resolved using the F-SEI architecture.

Example 18. Again, we consider the system G of Example 3. Letting $m = 1$, the refined model $G \parallel \tilde{G}_{N_1}$, which is isomorphic to G , is obtained as shown in Fig. 5. For any $s \in K$, we can show that $\bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), 1) = \emptyset$. For example, for $s = ao_1de^h \in K$, where $h \geq 0$, we have $\mathcal{E}_1^{F-SEI}(P_1(s), 1) \cap \mathcal{E}_2^{F-SEI}(P_2(s), 1) = \mathcal{E}_1^{F-SEI}(ad, 1) \cap \mathcal{E}_2^{F-SEI}(ad, 1) = \{(q_{11}, q_{d1})\} \cap \{(q_6, q_{d1})\} = \emptyset$. Also, for $s' = ao_1o_2o_3de^h \in K$ and $s'' = ao_1o_3o_2de^h \in K$, where $h \geq 0$, it follows that $\mathcal{E}_1^{F-SEI}(P_1(s'), 1) \cap \mathcal{E}_2^{F-SEI}(P_2(s'), 1) = \mathcal{E}_1^{F-SEI}(P_1(s''), 1) \cap \mathcal{E}_2^{F-SEI}(P_2(s''), 1) = \mathcal{E}_1^{F-SEI}(ao_2d, 1) \cap \mathcal{E}_2^{F-SEI}(ao_3d, 1) = \{(q_6, q_{d1})\} \cap \{(q_{11}, q_{d1})\} = \emptyset$. Thus, G is F-SEI codiagnosable with respect to K . The occurrence of the failure event f can also be correctly detected after ao_1o_2fd and ao_1o_3fd in $L(G) \cap (L(G) - K)\Sigma^{\geq 1}$ are executed in the F-SEI architecture.

5.3. Relationship between N-SEI and F-SEI architectures

We explore the relationship between the N-SEI and F-SEI architectures. First, we show a certain duality between them. In the N-SEI (respectively, F-SEI) architecture, the intersection-based diagnoser D_N (respectively, D_{F_m} ($m \in \mathbb{N}$)) should satisfy (3) and (4) (respectively, (7) and (8)). The conditions (3) and (7) are imposed to guarantee that there is no missed detection of the occurrence of a failure, while (4) and (8) are required to ensure that there

is no wrong detection. By [Lemmas 4 and 13](#), D_N and D_{F_m} always satisfy (4) and (7). In addition, N-SEI codiagnosability and F-SEI codiagnosability guarantee that D_N and D_{F_m} satisfy (3) and (8), respectively. These facts indicate a certain duality between the N-SEI and F-SEI architectures.

Moreover, the N-SEI and F-SEI architectures complement each other in the sense that, as shown in the following example, N-SEI codiagnosability and F-SEI codiagnosability are incomparable. It is worth mentioning that F-SEI codiagnosability is also incomparable with intersection-based codiagnosability of [Panteli and Hadjicostis \(2013\)](#).

Example 19. We consider the finite automaton G shown in [Fig. 6](#) and let $I = \{1, 2\}$, $\Sigma_{0,1} = \{a, b, c, d, g_1, g_2, o\}$, and $\Sigma_{0,2} = \{a, b, c, e, h_1, h_2, o\}$.

First, we consider the subautomaton G_{N_1} of G that is obtained by removing the transitions labeled by the event f_1 and the states q_7 , q_8 , and q_{10} , as shown in [Fig. 7](#). Let K_1 denote its generated language. Relevant parts of the refined model $G \parallel \tilde{G}_{N_{1m}}$ ($m \in \mathbb{N}$) and the subautomaton G_{N_1} are shown in [Fig. 8\(a\)](#) and (b), respectively. We can verify that

$$(\forall s \in L(G) \cap (L(G) - K_1)\Sigma^{\geq 1})(\exists t \in pr(\{s\}))$$

$$\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(t)) = \emptyset,$$

which implies that G is N-SEI codiagnosable with respect to K_1 . For example, for $s_1 = adf_1o^k \in L(G) \cap (L(G) - K_1)\Sigma^{\geq 1}$, where $k \geq 1$, we have $s_1 \in pr(\{s_1\})$ and $\mathcal{E}_1^{N-SEI}(P_1(s_1)) \cap \mathcal{E}_2^{N-SEI}(P_2(s_1)) = \{q_9\} \cap \{q_6\} = \emptyset$. We can also verify that

$$(\forall s \in L(G) \cap (L(G) - K_1)\Sigma^{\geq 1})(\forall q, q' \in Q)$$

$$\{(q, q), (q', q_d)\} \not\subseteq \bigcap_{i \in I} \mathcal{E}_{G \parallel \tilde{G}_{N,i}}(P_i(s)),$$

which implies that G is intersection-based codiagnosable with respect to K_1 . Again, for example, for $s_1 = adf_1o^k \in L(G) \cap (L(G) - K_1)\Sigma^{\geq 1}$, where $k \geq 1$, we have $\mathcal{E}_{G \parallel \tilde{G}_{N_1,1}}(P_1(s_1)) \cap \mathcal{E}_{G \parallel \tilde{G}_{N_1,2}}(P_2(s_1)) = \{(q_9, q_9), (q_7, q_d), (q_{10}, q_d)\} \cap \{(q_6, q_6), (q_7, q_d)\} = \{(q_7, q_d)\}$. On the other hand, for any $m \in \mathbb{N}$, we have $\mathcal{E}_1^{F-SEI}(P_1(s'_1), m) \cap \mathcal{E}_2^{F-SEI}(P_2(s'_1), m) = \{(q_7, q_{dm}), (q_{10}, q_{dm})\} \cap \{(q_8, q_{dm}), (q_{10}, q_{dm})\} = \{(q_{10}, q_{dm})\}$ for $s'_1 = aedo^m \in K_1$, which implies that G is not F-SEI codiagnosable with respect to K_1 .

Next, we consider the subautomaton G_{N_2} of G that is obtained by removing the transitions labeled by the event f_2 and the states q_{16} and q_{19} , as shown in [Fig. 9](#). Let K_2 denote its generated language. Relevant parts of the refined model $G \parallel \tilde{G}_{N_2,1}$ and the subautomaton G_{N_2} are shown in [Fig. 10\(a\)](#) and (b), respectively. It can be verified that

$$(\forall s \in K_2) \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), 1) = \emptyset,$$

which implies that G is F-SEI codiagnosable with respect to K_2 . For example, for $s_2 = bdo \in K_2$, we have $\mathcal{E}_1^{F-SEI}(P_1(s_2), 1) \cap \mathcal{E}_2^{F-SEI}(P_2(s_2), 1) = \{(q_{19}, q_{d1})\} \cap \{(q_{16}, q_{d1})\} = \emptyset$. On the other hand, G is not N-SEI codiagnosable with respect to K_2 . For any $m \in \mathbb{N}$, we consider $s'_2 = bedf_2o^m \in L(G) \cap (L(G) - K_2)\Sigma^{\geq m}$ and any prefix $t'_2 \in pr(\{s'_2\})$. If $t'_2 \in K_2$, then $\delta_{N_2}(q_0, t'_2) \in \mathcal{E}_1^{N-SEI}(P_1(t'_2)) \cap \mathcal{E}_2^{N-SEI}(P_2(t'_2))$. In the case of $t'_2 = bedf_2$, we have $\mathcal{E}_1^{N-SEI}(P_1(t'_2)) \cap \mathcal{E}_2^{N-SEI}(P_2(t'_2)) = \{q_{12}, q_{14}, q_{15}\} \cap \{q_{13}, q_{14}, q_{15}\} = \{q_{14}, q_{15}\}$. Moreover, in the case of $t'_2 = bedf_2o^k$, where $1 \leq k \leq m$, we have $\mathcal{E}_1^{N-SEI}(P_1(t'_2)) \cap \mathcal{E}_2^{N-SEI}(P_2(t'_2)) = \{q_{17}, q_{20}\} \cap \{q_{18}, q_{20}\} = \{q_{20}\}$. In addition, by [Proposition 10](#), G is not intersection-based codiagnosable with respect to K_2 .

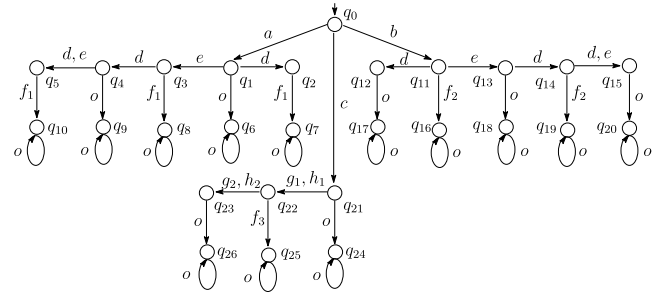


Fig. 6. Finite automaton G .

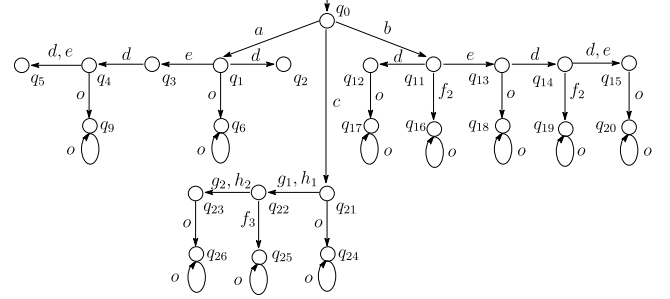


Fig. 7. Subautomaton G_{N_1} .

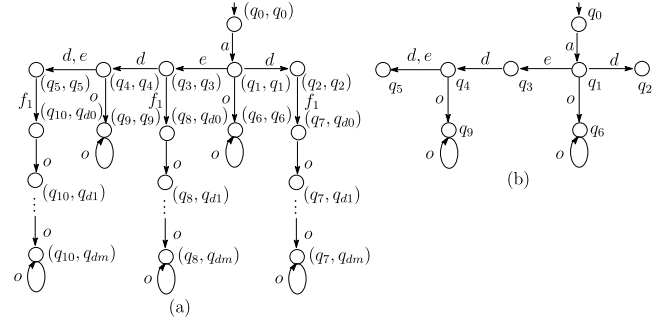


Fig. 8. Relevant parts of (a) refined model $G \parallel \tilde{G}_{N_{1m}}$ and (b) subautomaton G_{N_1} .

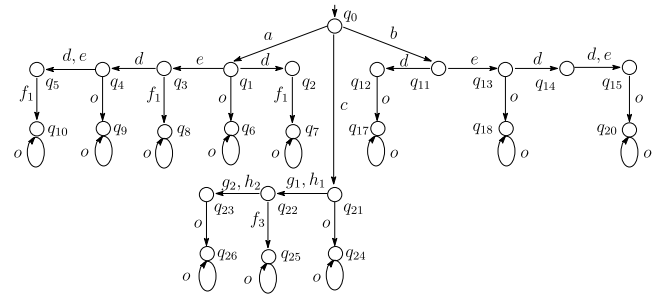


Fig. 9. Subautomaton G_{N_2} .

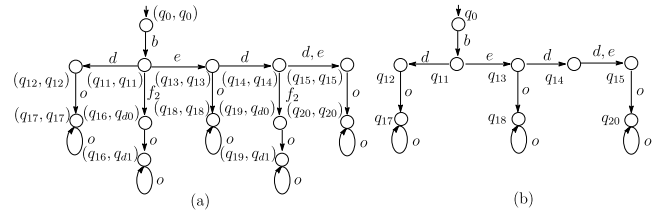
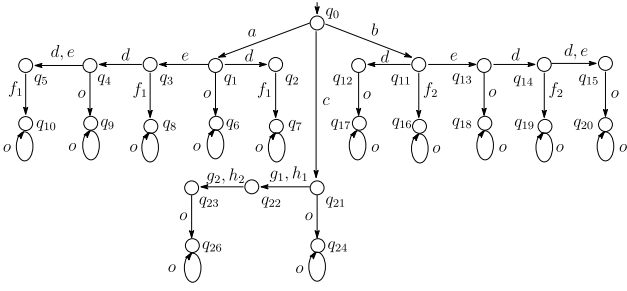
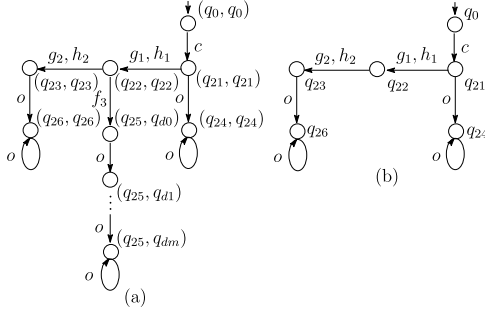


Fig. 10. Relevant parts of (a) refined model $G \parallel \tilde{G}_{N_{2,1}}$ and (b) subautomaton G_{N_2} .

Fig. 11. Subautomaton G_{N_3} .Fig. 12. Relevant parts of (a) refined model $G \parallel \tilde{G}_{N_3m}$ and (b) subautomaton G_{N_3} .

5.4. Comparison with N -inference diagnosability

We compare N -SEI codiagnosability and F -SEI codiagnosability with N -inference diagnosability. We obtain the following proposition on 0-inference diagnosability.

Proposition 20. *For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , if G is 0-inference diagnosable with respect to K , then it is N -SEI codiagnosable or F -SEI codiagnosable with respect to K .*

Proof. There exists $m \in \mathbb{N}$ such that $F_1(m) = \emptyset$ or $H_1(m) = \emptyset$. First, we consider the case where $F_1(m) = \emptyset$. Then, for any $s \in L(G) \cap (L(G) - K)\Sigma^{\geq m} = F_0(m)$, there exists $i \in I$ such that $s \notin P_i^{-1}P_i(H_0(m)) = P_i^{-1}P_i(K)$. For such $i \in I$, we have $P_i(s) \notin P_i(L(G_N))$, which implies $\mathcal{E}_i^{N-SEI}(P_i(s)) = \emptyset$. Thus, G is N -SEI codiagnosable. Next, we consider the case where $H_1(m) = \emptyset$. Then, for any $s \in K = H_0(m)$, there exists $i \in I$ such that $s \notin P_i^{-1}P_i(F_0(m)) = P_i^{-1}P_i(L(G) \cap (L(G) - K)\Sigma^{\geq m})$. For such $i \in I$, we have $P_i(s) \notin P_i(L(G) \cap (L(G) - K)\Sigma^{\geq m})$, which implies $\mathcal{E}_i^{F-SEI}(P_i(s), m) = \emptyset$. Thus, G is F -SEI codiagnosable. $\square$

As shown in the following example, the reverse implication of Proposition 20 does not necessarily hold, and, for any $N \geq 1$, N -inference diagnosability is incomparable with N -SEI codiagnosability and F -SEI codiagnosability.

Example 21. We consider the setting of Example 19. As shown in Example 19, G is N -SEI codiagnosable and F -SEI codiagnosable with respect to the languages K_1 and K_2 , respectively. However, for K_1 , we have $adf_1o^m, aef_1o^m \in F_k(m)$ and $ao^m, aedo^m \in H_k(m)$ for any $m \in \mathbb{N}$ and any $k \in \mathbb{N}$. It follows that, for any $N \in \mathbb{N}$, G is not N -inference diagnosable with respect to K_1 . For K_2 , we have $bf_2o^m, bedf_2o^m \in F_k(m)$ and $bdo^m, beo^m \in H_k(m)$ for any $m \in \mathbb{N}$ and any $k \in \mathbb{N}$. Thus, for any $N \in \mathbb{N}$, G is not N -inference diagnosable with respect to K_2 . To detect the occurrence of f_1 (respectively, f_2) correctly, the N -SEI (respectively, F -SEI) architecture should be used.

We consider the subautomaton G_{N_3} of G that is obtained by removing the transition labeled by the event f_3 and the state q_{25} , as shown in Fig. 11. Let K_3 denote its generated language. Relevant parts of the refined model $G \parallel \tilde{G}_{N_3m}$ ($m \in \mathbb{N}$) and the subautomaton G_{N_3} are shown in Fig. 12(a) and (b), respectively. In the case of $m = 1$, we have $F_2(1) = \emptyset$ and $H_2(1) = \{co^k \mid k \geq 1\}$ for K_3 . Since $F_2(1) = \emptyset$, G is N -inference diagnosable with respect to K_3 for any $N \geq 1$. However, G is not N -SEI codiagnosable with respect to K_3 . For any $m \in \mathbb{N}$, we consider $s_3 = cg_1f_3o^m \in L(G) \cap (L(G) - K_3)\Sigma^{\geq m}$ and any prefix $t_3 \in pr(\{s_3\})$. If $t_3 \in K_3$, then $\delta_{N_3}(q_0, t_3) \in \mathcal{E}_1^{N-SEI}(P_1(t_3)) \cap \mathcal{E}_2^{N-SEI}(P_2(t_3))$. In the case of $t_3 = cg_1f_3$, we have $\mathcal{E}_1^{N-SEI}(P_1(t_3)) \cap \mathcal{E}_2^{N-SEI}(P_2(t_3)) = \{q_{22}, q_{23}\} \cap \{q_{21}, q_{22}, q_{23}\} = \{q_{22}, q_{23}\}$. Moreover, in the case of $t_3 = cg_1f_3o^k$, where $1 \leq k \leq m$, we have $\mathcal{E}_1^{N-SEI}(P_1(t_3)) \cap \mathcal{E}_2^{N-SEI}(P_2(t_3)) = \{q_{26}\} \cap \{q_{24}, q_{26}\} = \{q_{26}\}$. Thus, G is not N -SEI codiagnosable with respect to K_3 . Furthermore, for any $m \in \mathbb{N}$, we have $\mathcal{E}_1^{F-SEI}(P_1(co^m), m) \cap \mathcal{E}_2^{F-SEI}(P_2(co^m), m) = \{(q_{25}, q_{dm})\} \cap \{(q_{25}, q_{dm})\} = \{(q_{25}, q_{dm})\}$ for $co^m \in K_3$, which implies that G is not F -SEI codiagnosable with respect to K_3 . Therefore, to detect the occurrence of f_3 correctly, the inference-based architecture of Takai and Kumar (2017) should be employed.

Example 21 shows that the N -SEI and F -SEI architectures and the inference-based architecture complement each other.

6. Verification of N -SEI codiagnosability and F -SEI codiagnosability

In this section, we present methods for effectively verifying N -SEI codiagnosability and F -SEI codiagnosability.

6.1. Verification of N -SEI codiagnosability

For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of $G = (Q, \Sigma, \delta, q_0)$, we consider the synchronous composition $(G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N)) = (X_N, \Sigma, \delta_{T_N}, x_{N0})$ of the refined model $G \parallel \tilde{G}_N = (Q \times Q_N, \Sigma, \alpha, (q_0, q_0))$ and the n observers $Obs_i(G_N) = (Q_{Nobs_i}, \Sigma_{o,i}, \delta_{Nobs_i}, q_{Nobs_i,0})$ ($i = 1, 2, \dots, n$) of G_N , where the state set is

$$X_N = (Q \times \tilde{Q}_N) \times \prod_{i \in I} Q_{Nobs_i},$$

the initial state is

$$x_{N0} = ((q_0, q_0), q_{Nobs_1,0}, q_{Nobs_2,0}, \dots, q_{Nobs_n,0}),$$

and the transition function $\delta_{T_N} : X_N \times \Sigma \rightarrow X_N$ is given as

$$\delta_{T_N}(x_N, \sigma) = \begin{cases} x'_N, & \text{if } \delta(q, \sigma)! \wedge \\ & [(\forall i \in I) \sigma \in \Sigma_{o,i} \Rightarrow \delta_{Nobs_i}(q_{Nobs_i}, \sigma)!] \\ \text{undefined,} & \text{otherwise} \end{cases}$$

for each $x_N = ((q, \tilde{q}_N), q_{Nobs_1}, q_{Nobs_2}, \dots, q_{Nobs_n}) \in X_N$ and each $\sigma \in \Sigma$, where

$$x'_N = (\alpha((q, \tilde{q}_N), \sigma), q'_{Nobs_1}, q'_{Nobs_2}, \dots, q'_{Nobs_n})$$

and

$$q'_{Nobs_i} = \begin{cases} \delta_{Nobs_i}(q_{Nobs_i}, \sigma), & \text{if } \sigma \in \Sigma_{o,i} \\ q_{Nobs_i}, & \text{otherwise} \end{cases}$$

for each $i \in I$. It holds that $L((G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N))) = L(G) \cap (\bigcap_{i \in I} P_i^{-1}(L(Obs_i(G_N)))) = L(G) \cap (\bigcap_{i \in I} P_i^{-1}P_i(L(G_N)))$.

To verify N -SEI codiagnosability, we construct a subautomaton, denoted by T_N , of $(G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N))$ as follows:

$$T_N = (\hat{X}_N, \Sigma, \hat{\delta}_{T_N}, x_{N0}), \quad (9)$$

where the state set is

$$\hat{X}_N = \{((q, \tilde{q}_N), q_{Nobs_1}, q_{Nobs_2}, \dots, q_{Nobs_n}) \in X_N \mid \bigcap_{i \in I} q_{Nobs_i} \neq \emptyset\}$$

and the transition function $\hat{\delta}_{T_N} : \hat{X}_N \times \Sigma \rightarrow \hat{X}_N$ is given as

$$\hat{\delta}_{T_N}(\hat{x}_N, \sigma) = \begin{cases} \delta_{T_N}(\hat{x}_N, \sigma), & \text{if } \delta_{T_N}(\hat{x}_N, \sigma) \in \hat{X}_N \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

for each $\hat{x}_N \in \hat{X}_N$ and each $\sigma \in \Sigma$. Since $q_0 \in \bigcap_{i \in I} q_{Nobs_i,0} \neq \emptyset$, we have $x_{N0} \in \hat{X}_N$, which implies that the subautomaton T_N is well-defined.

The following lemma shows that T_N generates a string $s \in L(G)$ such that $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) \neq \emptyset$, and will be used to prove the correctness of the method developed later in [Theorem 23](#) to verify N-SEI codiagnosability.

Lemma 22. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , we consider the subautomaton T_N of $(G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N))$ given by (9). For any $s \in L(T_N)$, it holds that $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) \neq \emptyset$.

Proof. For any $s \in L(T_N) \subseteq L((G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N)))$, we have $P_i(s) \in L(Obs_i(G_N)) = P_i(L(G_N))$ for any $i \in I$ and

$$\begin{aligned} \hat{\delta}_{T_N}(x_{N0}, s) \\ = (\alpha((q_0, q_0), s), q_{Nobs_1}, q_{Nobs_2}, \dots, q_{Nobs_n}), \end{aligned}$$

where $q_{Nobs_i} = \delta_{Nobs_i}(q_{Nobs_i,0}, P_i(s)) = \mathcal{E}_{G_N,i}(P_i(s))$ for each $i \in I$. For any $i \in I$, since $P_i(s) \in P_i(L(G_N))$, we have $q_{Nobs_i} = \mathcal{E}_i^{N-SEI}(P_i(s))$. It follows from $\hat{\delta}_{T_N}(x_{N0}, s) \in \hat{X}_N$ that $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(s)) = \bigcap_{i \in I} q_{Nobs_i} \neq \emptyset$. $\square$

By [Lemma 22](#), the violation of N-SEI codiagnosability can be characterized by the existence of an arbitrarily long failure string $s \in L(G) - K$ that is generated by T_N . Based on this observation, the following theorem is obtained.

Theorem 23. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , G is not N-SEI codiagnosable with respect to K if and only if there exists a reachable cycle $\hat{x}_N^{(0)} \xrightarrow{\sigma^{(0)}} \hat{x}_N^{(1)} \xrightarrow{\sigma^{(1)}} \dots \hat{x}_N^{(l-1)} \xrightarrow{\sigma^{(l-1)}} \hat{x}_N^{(0)}$ ($l \geq 1$) such that $\tilde{q}_N^{(h)} = q_d$ for any $h \in \{0, 1, \dots, l-1\}$, where $\hat{x}_N^{(h)} = ((q^{(h)}, \tilde{q}_N^{(h)}), q_{Nobs_1}^{(h)}, q_{Nobs_2}^{(h)}, \dots, q_{Nobs_n}^{(h)})$ for each $h \in \{0, 1, \dots, l-1\}$, in the subautomaton T_N of $(G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N))$ given by (9).

Proof. First, we prove the sufficiency part. We consider any reachable cycle $\hat{x}_N^{(0)} \xrightarrow{\sigma^{(0)}} \hat{x}_N^{(1)} \xrightarrow{\sigma^{(1)}} \dots \hat{x}_N^{(l-1)} \xrightarrow{\sigma^{(l-1)}} \hat{x}_N^{(0)}$ ($l \geq 1$) such that $\tilde{q}_N^{(h)} = q_d$ for any $h \in \{0, 1, \dots, l-1\}$ in T_N . Then, there exists $s \in L(T_N)$ such that $\hat{\delta}_{T_N}(x_{N0}, s) = \hat{x}_N^{(0)}$. Since $\tilde{q}_N^{(0)} = q_d$, we have $s \in L(G) - K$. Letting $t = \sigma^{(0)}\sigma^{(1)} \dots \sigma^{(l-1)}$, we have $st^m \in L(T_N) \cap (L(G) \cap (L(G) - K)\Sigma^{\geq m})$ for any $m \in \mathbb{N}$. We consider any $u \in pr(\{st^m\})$. Since $u \in L(T_N)$, by [Lemma 22](#), we have $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(u)) \neq \emptyset$. Thus, G is not N-SEI codiagnosable.

Next, we prove the necessity part. We consider any $m \in \mathbb{N}$ such that $m \geq |\hat{X}_N|$. Since G is not N-SEI codiagnosable, there exists $s \in L(G) \cap (L(G) - K)\Sigma^{\geq m}$ such that, for any $t \in pr(s)$, $\bigcap_{i \in I} \mathcal{E}_i^{N-SEI}(P_i(t)) \neq \emptyset$. For each $i \in I$, since $\mathcal{E}_i^{N-SEI}(P_i(t)) \neq \emptyset$, we have $P_i(t) \in P_i(L(G_N))$ and $\mathcal{E}_i^{N-SEI}(P_i(t)) = \mathcal{E}_{G_N,i}(P_i(t)) = \delta_{Nobs_i}(q_{Nobs_i,0}, P_i(t))$. It holds that $t \in L(G) \cap (\bigcap_{i \in I} P_i^{-1}P_i(L(G_N))) = L((G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N)))$ and $\bigcap_{i \in I} \delta_{Nobs_i}(q_{Nobs_i,0}, P_i(t)) \neq \emptyset$, which implies $s \in L(\hat{T}_N)$. Since $s \in L(G) \cap (L(G) - K)\Sigma^{\geq m}$, s can be written as $s = uv$, where $u \in L(G) - K$ and $|v| \geq m \geq |\hat{X}_N|$. Letting

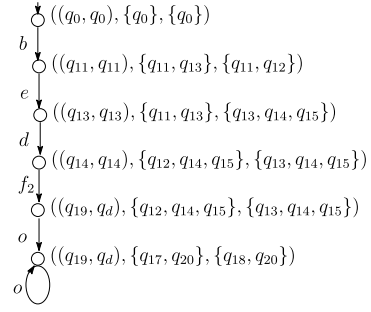


Fig. 13. A part of the subautomaton T_N of $(G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N))$.

$\hat{\delta}_{T_N}(x_{N0}, u) = ((q, \tilde{q}_N), q_{Nobs_1}, q_{Nobs_2}, \dots, q_{Nobs_n})$, since $u \in L(G) - K$, we have $\tilde{q}_N = q_d$. By $|v| \geq |\hat{X}_N|$, there exists a reachable cycle $\hat{x}_N^{(0)} \xrightarrow{\sigma^{(0)}} \hat{x}_N^{(1)} \xrightarrow{\sigma^{(1)}} \dots \hat{x}_N^{(l-1)} \xrightarrow{\sigma^{(l-1)}} \hat{x}_N^{(0)}$ ($l \geq 1$) such that $\tilde{q}_N^{(h)} = q_d$ for any $h \in \{0, 1, \dots, l-1\}$ in T_N . $\square$

Remark 24. Since the number of reachable states of the subautomaton T_N of $(G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G_N))$ is at most $2|Q| \times 2^{n|Q|}$, the complexity of constructing the reachable part of T_N is $O(|Q| \times 2^{n|Q|} \times |\Sigma|)$. [Theorem 23](#) shows that N-SEI codiagnosability can be verified by examining all maximal strongly connected components of T_N . The computational complexity of computing all maximal strongly connected components of T_N is $O(|Q| \times 2^{n|Q|} \times |\Sigma|)$. Thus, the computational complexity of verifying N-SEI codiagnosability by [Theorem 23](#) is $O(|Q| \times 2^{n|Q|} \times |\Sigma|)$.

Remark 25. In [Viana and Basilio \(2019\)](#), the synchronous composition $(G \parallel \tilde{G}_N) \parallel (\|_{i \in I} Obs_i(G \parallel \tilde{G}_N))$ of the refined model $G \parallel \tilde{G}_N$ and its observers $Obs_i(G \parallel \tilde{G}_N)$ ($i = 1, 2, \dots, n$) is used to verify disjunctive codiagnosability. In this paper, the observers $Obs_i(G_N)$ ($i = 1, 2, \dots, n$) of the normal behavior model G_N , instead of $Obs_i(G \parallel \tilde{G}_N)$, are used for verifying N-SEI codiagnosability.

Example 26. We consider the setting of [Example 19](#). For the regular sublanguage $K_2 \subseteq L(G)$ generated by the subautomaton G_{N_2} of G , a part of the subautomaton T_N of $(G \parallel \tilde{G}_{N_2}) \parallel (\|_{i \in I} Obs_i(G_{N_2}))$ is shown in [Fig. 13](#). This part shows that there exists a reachable cycle $((q_{19}, q_d), \{q_{17}, q_{20}\}, \{q_{18}, q_{20}\}) \xrightarrow{o} ((q_{19}, q_d), \{q_{17}, q_{20}\}, \{q_{18}, q_{20}\})$ in T_N . By [Theorem 23](#), we can conclude that G is not N-SEI codiagnosable with respect to K_2 , as shown in [Example 19](#).

6.2. Verification of F-SEI codiagnosability

For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of $G = (Q, \Sigma, \delta, q_0)$, we use the verifier ([Wang et al., 2007; Yamamoto & Takai, 2014](#)) that was constructed to verify conjunctive codiagnosability. It is a finite automaton

$$V_F = (X_F, \Sigma_F, \delta_{V_F}, x_{F0}) \quad (10)$$

that consists of G_N and n copies of $G \parallel \tilde{G}_N$, which are used to trace a normal string in K and n indistinguishable failure strings in $L(G) - K$, respectively. Each element of V_F is given as follows:

- The state set is

$$\begin{aligned} X_F = Q_N \\ \times \underbrace{(Q \times \tilde{Q}_N) \times (Q \times \tilde{Q}_N) \times \dots \times (Q \times \tilde{Q}_N)}_{n \text{ times}}. \end{aligned}$$

- The initial state is

$$x_{F0} = (q_0, (q_0, q_0), (q_0, q_0), \dots, (q_0, q_0)).$$

- The event set is

$$\begin{aligned} \Sigma_F = \{ & (\sigma, \tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_n) \in \tilde{\Sigma}^{n+1} \mid \\ & \sigma \in \Sigma \wedge [(\forall i \in I) \tilde{\sigma}_i = P_i(\sigma)] \\ & \cup \{(\varepsilon, \tilde{\sigma}_1, \tilde{\sigma}_2, \dots, \tilde{\sigma}_n) \in \tilde{\Sigma}^{n+1} \mid \\ & (\exists i \in I) \\ & \tilde{\sigma}_i \in \Sigma - \Sigma_{0,i} \wedge [(\forall j \in I - \{i\}) \tilde{\sigma}_j = \varepsilon]\}, \end{aligned}$$

where

$$\tilde{\Sigma}^{n+1} = \underbrace{(\Sigma \cup \{\varepsilon\}) \times (\Sigma \cup \{\varepsilon\}) \times \dots \times (\Sigma \cup \{\varepsilon\})}_{n+1 \text{ times}}.$$

- For any

$$x_F = (q_N, (q_1, \tilde{q}_{N1}), (q_2, \tilde{q}_{N2}), \dots, (q_n, \tilde{q}_{Nn})) \in X_F$$

and any

$$\sigma_F = (\tilde{\sigma}, \tilde{\sigma}_1, \dots, \tilde{\sigma}_n) \in \Sigma_F,$$

$\delta_{V_F}(x_F, \sigma_F)!$ if the following two conditions hold:

- $\tilde{\sigma} \neq \varepsilon \Rightarrow \delta_N(q_N, \tilde{\sigma})!$, and
- $(\forall i \in I) \tilde{\sigma}_i \neq \varepsilon \Rightarrow \delta(q_i, \tilde{\sigma}_i)!$.

If $\delta_{V_F}(x_F, \sigma_F)!$, then

$$\begin{aligned} \delta_{V_F}(x_F, \sigma_F) \\ = (q'_N, (q'_1, \tilde{q}'_{N1}), (q'_2, \tilde{q}'_{N2}), \dots, (q'_n, \tilde{q}'_{Nn})), \end{aligned}$$

where

$$q'_N = \begin{cases} \delta_N(q_N, \tilde{\sigma}), & \text{if } \tilde{\sigma} \neq \varepsilon \\ q_N, & \text{otherwise} \end{cases}$$

and, for any $i \in I$,

$$(q'_i, \tilde{q}'_{Ni}) = \begin{cases} \alpha((q_i, \tilde{q}_{Ni}), \tilde{\sigma}_i), & \text{if } \tilde{\sigma}_i \neq \varepsilon \\ (q_i, \tilde{q}_{Ni}), & \text{otherwise.} \end{cases}$$

For any $x_F = (q_N, (q_1, \tilde{q}_{N1}), (q_2, \tilde{q}_{N2}), \dots, (q_n, \tilde{q}_{Nn})) \in X_F$, we let $\pi(x_F) = q_N$ and $\pi_i(x_F) = (q_i, \tilde{q}_{Ni})$ for each $i \in I$. Similarly, for any $\sigma_F = (\tilde{\sigma}, \tilde{\sigma}_1, \dots, \tilde{\sigma}_n) \in \Sigma_F$, we let $\pi(\sigma_F) = \tilde{\sigma}$ and $\pi_i(\sigma_F) = \tilde{\sigma}_i$ for each $i \in I$. In addition, for each $s_F \in \Sigma_F^*$, $\pi(s_F)$ and $\pi_i(s_F)$ for any $i \in I$ are defined as

$$\pi(s_F) = \begin{cases} \varepsilon, & \text{if } s_F = \varepsilon \\ \pi(\sigma_{F1})\pi(\sigma_{F2}) \dots \pi(\sigma_{F|s_F|}), & \text{otherwise,} \end{cases}$$

and

$$\pi_i(s_F) = \begin{cases} \varepsilon, & \text{if } s_F = \varepsilon \\ \pi_i(\sigma_{F1})\pi_i(\sigma_{F2}) \dots \pi_i(\sigma_{F|s_F|}), & \text{otherwise,} \end{cases}$$

where $s_F = \sigma_{F1}\sigma_{F2} \dots \sigma_{F|s_F|}$ if $s_F \neq \varepsilon$. By the construction of V_F , the following lemma is straightforwardly obtained (Yamamoto & Takai, 2014), so its proof is omitted.

Lemma 27. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , we consider the finite automaton V_F given by (10). Then, for any $s_F \in L(V_F)$, it holds that $\pi(s_F) \in K$, $\pi_i(s_F) \in L(G)$, $P_i(\pi(s_F)) = P_i(\pi_i(s_F))$ for any $i \in I$, and

$$\delta_{V_F}(x_{F0}, s_F) = (q_N, (q_1, \tilde{q}_{N1}), (q_2, \tilde{q}_{N2}), \dots, (q_n, \tilde{q}_{Nn})),$$

where $q_N = \delta_N(q_0, \pi(s_F))$ and, for any $i \in I$, $(q_i, \tilde{q}_{Ni}) = \alpha((q_0, q_0), \pi_i(s_F))$. In addition, for any $s \in K$ and any $s_1, s_2, \dots, s_n \in L(G)$, if $P_i(s) = P_i(s_i)$ for any $i \in I$, then there exists $s_F \in L(V_F)$ such that $\pi(s_F) = s$ and $\pi_i(s_F) = s_i$ for any $i \in I$.

For any $m \in \mathbb{N}$ and any $s_F \in L(V_F)$, the following lemma shows a condition under which $\bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(\pi(s_F)), m) \neq \emptyset$ holds.

Lemma 28. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , we consider the finite automaton V_F given by (10). For any $m \in \mathbb{N}$ and any $s_F \in L(V_F)$, if $\pi_i(s_F) \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$ for any $i \in I$ and $\pi_i(\delta_{V_F}(x_{F0}, s_F)) = \pi_j(\delta_{V_F}(x_{F0}, s_F))$ for any $i, j \in I$, then $\bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(\pi(s_F)), m) \neq \emptyset$.

Proof. For any $m \in \mathbb{N}$ and any $s_F \in L(V_F)$, we suppose that $\pi_i(s_F) \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$ for any $i \in I$ and $\pi_i(\delta_{V_F}(x_{F0}, s_F)) = \pi_j(\delta_{V_F}(x_{F0}, s_F))$ for any $i, j \in I$. For any $i \in I$, by Lemma 27, we have $P_i(\pi(s_F)) = P_i(\pi_i(s_F))$ and $\pi_i(\delta_{V_F}(x_{F0}, s_F)) = \alpha((q_0, q_0), \pi_i(s_F))$. Let $\alpha((q_0, q_0), \pi_i(s_F)) = (q_i, \tilde{q}_{Ni})$ for any $i \in I$. Since $P_i(\pi(s_F)) = P_i(\pi_i(s_F))$ and $\pi_i(s_F) \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$, we have $\alpha_m((q_0, q_0), \pi_i(s_F)) = (q_i, q_{dm}) \in \mathcal{E}_{G \parallel \tilde{G}_{Nm}, i}(P_i(\pi(s_F))) \cap (Q \times \{q_{dm}\}) = \mathcal{E}_i^{F-SEI}(P_i(\pi(s_F)), m)$. Since $\pi_i(x_F) = \pi_j(x_F)$, where $x_F = \delta_{V_F}(x_{F0}, s_F)$, for any $i, j \in I$, we can let $\hat{q} = q_i$ for any $i \in I$. It follows that $(\hat{q}, q_{dm}) \in \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(\pi(s_F)), m) \neq \emptyset$. $\square$

By Lemma 28, the violation of F-SEI codiagnosability can be characterized by the condition that, for any $m \in \mathbb{N}$, there exists $s_F \in L(V_F)$ that satisfies $\pi_i(s_F) \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$ for any $i \in I$ and $\pi_i(\delta_{V_F}(x_{F0}, s_F)) = \pi_j(\delta_{V_F}(x_{F0}, s_F))$ for any $i, j \in I$. Such $s_F \in L(V_F)$ must visit cycles so that, for each $i \in I$, $\pi_i(s_F)$ can be extended to an arbitrarily long failure string. In general, multiple cycles need to be executed sequentially to elongate $\pi_i(s_F)$ for all $i \in I$. Based on this observation, we construct a nondeterministic acyclic automaton

$$\mathcal{V}_F = (Y_F, \Sigma_F, \xi_{\mathcal{V}_F}, y_{F0}) \quad (11)$$

from V_F , which has the same event set Σ_F as V_F (Yamamoto & Takai, 2014). Except for Σ_F , each element of $\mathcal{V}_F$ is given as follows:

- The state set Y_F is the set of all maximal strongly connected components of V_F .
- The initial state $y_{F0} \in Y_F$ is a maximal strongly connected component of V_F such that $x_{F0} \in y_{F0}$.
- The nondeterministic transition function $\xi_{\mathcal{V}_F} : Y_F \times \Sigma_F \rightarrow 2^{Y_F}$ is given as, for any $y_F \in Y_F$ and any $\sigma_F \in \Sigma_F$,

$$\begin{aligned} \xi_{\mathcal{V}_F}(y_F, \sigma_F) \\ = \{y'_F \in Y_F \mid y_F \neq y'_F \\ \wedge [(\exists x_F \in y_F)(\exists x'_F \in y'_F) \delta_{V_F}(x_F, \sigma_F) = x'_F]\}. \end{aligned}$$

A labeling function $J_F : Y_F \rightarrow 2^I$ is defined as

$$\begin{aligned} J_F(y_F) = \{i \in I \mid [(\exists x_F \in y_F) \pi_i(x_F) \in Q \times \{q_d\}] \\ \wedge [(\exists x'_F \in y'_F)(\exists \sigma_F \in \Sigma_F) \\ \delta_{V_F}(x_F, \sigma_F) = x'_F \wedge \pi_i(\sigma_F) \neq \varepsilon]\} \end{aligned} \quad (12)$$

for each $y_F \in Y_F$ (Yamamoto & Takai, 2014). For any $i \in I$, if $i \in J_F(y_F)$, then $\pi_i(s_F)$ can be extended to an arbitrarily long failure string by executing a cycle that includes $\sigma_F \in \Sigma_F$ with $\pi_i(\sigma_F) \neq \varepsilon$ an arbitrary number of times. We consider any path $p_{\mathcal{V}_F} : y_{F0} \xrightarrow{\sigma_{F0}} y_{F1} \xrightarrow{\sigma_{F1}} \dots y_{F(l-1)} \xrightarrow{\sigma_{F(l-1)}} y_{Fl} \ (l \geq 1)$ that begins at the initial state y_{F0} in $\mathcal{V}_F$. Its label $J_F(p_{\mathcal{V}_F}) \in 2^I$ is given as

$$J_F(p_{\mathcal{V}_F}) = \bigcup_{h \in \{1, 2, \dots, l\}} J_F(y_{Fh}).$$

Then, the condition $J_F(p_{\mathcal{V}_F}) = I$ ensures that, by visiting these maximal strongly connected components $y_{Fh} \ (h = 1, 2, \dots, l)$ and executing appropriate cycles an arbitrary number of times, $\pi_i(s_F)$ can be extended to an arbitrarily long failure string for any $i \in I$.

The following theorem presents how to verify F-SEI-codiagnosability using $\mathcal{V}_F$.

Theorem 29. For a regular sublanguage $K \subseteq L(G)$ generated by a subautomaton $G_N = (Q_N, \Sigma, \delta_N, q_0)$ of G , G is not F-SEI codiagnosable with respect to K if and only if there exists a path $p_{V_F} : y_{F0} \xrightarrow{\sigma_{F0}} y_{F1} \xrightarrow{\sigma_{F1}} \dots y_{F(l-1)} \xrightarrow{\sigma_{F(l-1)}} y_{Fl}$ ($l \geq 1$) that begins at the initial state y_{F0} in V_F given by (11) such that

$$J_F(p_{V_F}) = I \quad (13)$$

and

$$(\exists x_F \in y_{Fl})(\forall i, j \in I) \pi_i(x_F) = \pi_j(x_F). \quad (14)$$

Proof. First, we prove the sufficiency part. We consider any path $p_{V_F} : y_{F0} \xrightarrow{\sigma_{F0}} y_{F1} \xrightarrow{\sigma_{F1}} \dots y_{F(l-1)} \xrightarrow{\sigma_{F(l-1)}} y_{Fl}$ ($l \geq 1$) in V_F such that (13) and (14) hold. Then, there exists a path $x_{F0} \xrightarrow{\sigma_F^{(0)}} x_F^{(1)} \xrightarrow{\sigma_F^{(1)}} \dots x_F^{(l'-1)} \xrightarrow{\sigma_F^{(l'-1)}} x_F^{(l')}$ ($l' \geq 1$) in V_F such that the following two conditions hold:

- $(\forall i \in I)(\exists h_{i1}, h_{i2}, h_{i3} \in \mathbb{N})[0 < h_{i1} \leq h_{i2} < h_{i3} \leq l'] \wedge [x_F^{(h_{i1})} = x_F^{(h_{i3})}] \wedge [\pi_i(x_F^{(h_{i2})}) \in Q \times \{q_d\}] \wedge [\pi_i(x_F^{(h_{i2})}) \neq \varepsilon]$, and
- $(\forall i, j \in I) q_i^{(l')} = q_j^{(l')}$,

where $\pi_i(x_F^{(l')}) = (q_i^{(l')}, \tilde{q}_{Ni}^{(l')})$ for each $i \in I$.

We consider any $m \in \mathbb{N}$. By Algorithm 1², we construct $s_F \in \Sigma_F^*$ and $t_{F,i} \in \Sigma_F^*$ for each $i \in I$. Then, we have $s_F, t_{F,i} \in L(V_F)$ and $t_{F,i} \in pr(\{s_F\})$ for each $i \in I$. Let $s = \pi(s_F)$. By Lemma 27, we have $s \in K$. For each $i \in I$, since $t_{F,i} \in pr(\{s_F\})$, there exists $u_{F,i} \in \Sigma_F^*$ such that $s_F = t_{F,i} u_{F,i}$. Let $t_i = \pi_i(t_{F,i})$ and $u_i = \pi_i(u_{F,i})$. By the construction of $t_{F,i}$ in Algorithm 1 and Lemma 27, we have $\delta_{V_F}(x_{F0}, t_{F,i}) = x_F^{(h_{i1})}$ and $\alpha((q_0, q_0), t_i) = \pi_i(x_F^{(h_{i1})}) = \pi_i(x_F^{(h_{i3})})$. Since $\pi_i(x_F^{(h_{i2})}) \in Q \times \{q_d\}$, we have $\alpha((q_0, q_0), t_i) \in Q \times \{q_d\}$, which implies $t_i \in L(G) - K$. In addition, since $|u_i| \geq m$ by $\pi_i(\sigma_F^{(h_{i2})}) \neq \varepsilon$, we have $\pi_i(s_F) = t_i u_i \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$. Furthermore, by Lemma 27, we have $\pi_i(\delta_{V_F}(x_{F0}, s_F)) = \alpha((q_0, q_0), t_i u_i) = (q_i^{(l')}, q_d)$. For any $i, j \in I$, since $q_i^{(l')} = q_j^{(l')}$, we have $\pi_i(\delta_{V_F}(x_{F0}, s_F)) = \pi_j(\delta_{V_F}(x_{F0}, s_F))$. It follows from Lemma 28 that $\bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) \neq \emptyset$. Therefore, G is not F-SEI codiagnosable.

We next prove the necessity part. Since G is not F-SEI codiagnosable, for any $m \in \mathbb{N}$, there exists $s \in K$ such that $\bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) \neq \emptyset$. We consider any $(\hat{q}, q_{dm}) \in \bigcap_{i \in I} \mathcal{E}_i^{F-SEI}(P_i(s), m) = \bigcap_{i \in I} \mathcal{E}_{G||\tilde{G}_{Nm},i}(P_i(s)) \cap (Q \times \{q_{dm}\})$. For each $i \in I$, there exists $s_i \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$ such that $P_i(s) = P_i(s_i)$ and $\alpha_m((q_0, q_0), s_i) = (\hat{q}, q_{dm})$. Then, by Lemma 27, there exists $s_F \in L(V_F)$ such that $\pi(s_F) = s$ and $\pi_i(s_F) = s_i$ for each $i \in I$. For any $i \in I$, by $s_i \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$, we have $s_i \neq \varepsilon$, which implies $s_F \neq \varepsilon$. We let $s_F = \sigma_F^{(0)} \sigma_F^{(1)} \dots \sigma_F^{(l-1)}$ ($l \geq 1$).

In the case of $m > |X_F|$, we consider the path $p_{V_F} : x_{F0}^{(0)} \xrightarrow{\sigma_F^{(0)}} x_F^{(1)} \xrightarrow{\sigma_F^{(1)}} \dots x_F^{(l-1)} \xrightarrow{\sigma_F^{(l-1)}} x_F^{(l)}$, where $x_{F0}^{(0)} = x_{F0}$, obtained by executing s_F in V_F . For each $i \in I$, since $\pi_i(s_F) = s_i$, by Lemma 27, we have $\pi_i(x_F^{(l)}) = \alpha((q_0, q_0), s_i) = (\hat{q}, q_d)$. For the path p_{V_F} , there exists the path $p_{V_F} : y_{F0} \xrightarrow{\sigma_{F0}} y_{F1} \xrightarrow{\sigma_{F1}} \dots y_{F(l'-1)} \xrightarrow{\sigma_{F(l'-1)}} y_{Fl'}$ ($l' \geq 1$) in V_F such that

$$\{x_F^{(0)}, \dots, x_F^{(h_0)}\} = y_{F0},$$

$$\{x_F^{(h_0+1)}, \dots, x_F^{(h_1)}\} = y_{F1},$$

:

:

² Algorithm 1 is presented to show a procedure for constructing the strings $s_F \in \Sigma_F^*$ and $t_{F,i} \in \Sigma_F^*$ for each $i \in I$.

Algorithm 1 Constructions of $s_F, t_{F,i} \in \Sigma_F^*$ ($\forall i \in I$)

Require: $\sigma_F^{(0)} \sigma_F^{(1)} \dots \sigma_F^{(l'-1)}, h_{i1}, h_{i3}$ ($\forall i \in I$), m

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1:  $s_F \leftarrow \varepsilon, t_{F,i} \leftarrow \varepsilon$  ( $\forall i \in I$ )
2:  $h \leftarrow 0$ 
3: while  $h \leq l' - 1$  do
4:    $\Phi(h) \leftarrow \{i \in I \mid h = h_{i1}\}$ 
5:   while  $\Phi(h) \neq \emptyset$  do
6:     Pick any  $i \in \Phi(h)$ 
7:      $t_{F,i} \leftarrow s_F$ 
8:      $s_F \leftarrow s_F(\sigma_F^{(h_{i1})} \sigma_F^{(h_{i1}+1)} \dots \sigma_F^{(h_{i3}-1)})^m$ 
9:      $\Phi(h) \leftarrow \Phi(h) - \{i\}$ 
10:  end while
11:   $s_F \leftarrow s_F \sigma_F^{(h)}$ 
12:   $h \leftarrow h + 1$ 
13: end while
```

$$\{x_F^{(h_{l'-1}+1)}, \dots, x_F^{(l)}\} = y_{Fl'}$$

for some $h_0, h_1, \dots, h_{l'-1} \in \{0, 1, \dots, l-1\}$ such that $0 \leq h_0 < h_1 < \dots < h_{l'-1} < l$.

We show that the path p_{V_F} satisfies (13). For each $i \in I$, since $s_i \in L(G) \cap (L(G) - K) \Sigma^{\geq m}$, it can be written as $s_i = t_i u_i$, where $t_i \in L(G) - K$ and $|u_i| \geq m > |X_F|$. By $|u_i| \geq m > |X_F|$, there exist $h_{i1}, h_{i2}, h_{i3} \in \mathbb{N}$ that satisfy $0 < h_{i1} \leq h_{i2} < h_{i3} \leq l$, $x_F^{(h_{i1})} = x_F^{(h_{i3})}$, $\pi_i(\sigma_F^{(h_{i2})}) \neq \varepsilon$, and $t_i \in pr(\pi_i(\sigma_F^{(0)} \sigma_F^{(1)} \dots \sigma_F^{(h_{i1}-1)}))$. In addition, since $t_i \in L(G) - K$ and $t_i \in pr(\pi_i(\sigma_F^{(0)} \sigma_F^{(1)} \dots \sigma_F^{(h_{i1}-1)})) \subseteq pr(\pi_i(\sigma_F^{(0)} \sigma_F^{(1)} \dots \sigma_F^{(h_{i2}-1)}))$, we have $\pi_i(x_F^{(h_{i2})}) = \alpha((q_0, q_0), \pi_i(\sigma_F^{(0)} \sigma_F^{(1)} \dots \sigma_F^{(h_{i2}-1)})) \in Q \times \{q_d\}$. It follows that $i \in J_F(y_{Fl'})$ for some $h \in \{1, 2, \dots, l'\}$. Thus, we have p_{V_F} satisfies (13). In addition, since $\pi_i(x_F^{(l)}) = (\hat{q}, q_d)$ for each $i \in I$, p_{V_F} satisfies (14). $\square$

Remark 30. Conjunctive codiagnosability (Wang et al., 2007) is verified by testing the existence of a path p_{V_F} in the non-deterministic acyclic automaton V_F such that the condition (13) holds (Yamamoto & Takai, 2014). Theorem 29 shows that, by additionally imposing (14), F-SEI codiagnosability can be verified.

Remark 31. The number of reachable states of the finite automaton V_F is at most $2^n |Q|^{n+1}$, and the number of events of V_F is at most $(n+1)|\Sigma|$. Therefore, the complexity of constructing V_F is $O(2^n |Q|^{n+1} \times n |\Sigma|)$. To construct the nondeterministic acyclic finite automaton V_F , we need to compute all maximal strongly connected components of V_F . Its computational complexity is $O(2^n |Q|^{n+1} \times n |\Sigma|)$.

Similar to verification of conjunctive codiagnosability (Wang et al., 2015; Yamamoto & Takai, 2014) and N -inference diagnosability (Takai & Kumar, 2017) that involves multiple arbitrarily long failure strings, by Theorem 29, F-SEI codiagnosability can be verified by exploring paths $p_{V_F} : y_{F0} \xrightarrow{\sigma_{F0}} y_{F1} \xrightarrow{\sigma_{F1}} \dots y_{Fl}$ ($l \geq 1$) of V_F which end with deadlocking states. Since the events σ_{Fh} ($h = 0, 1, \dots, l-1$) are not relevant to the conditions (13) and (14), the number of such paths that have to be explored is at most $(|Y_F| - 1)!$, where Y_F is the state set of V_F and $|Y_F|$ is at most $2^n |Q|^{n+1}$. To verify whether there exists a path p_{V_F} that satisfies (13) and (14), we have to construct the labeling function $J_F : Y_F \rightarrow 2^I$ defined by (12) and identify all states $y_F \in Y_F$ that satisfy

$$(\exists x_F \in y_F)(\forall i, j \in I) \pi_i(x_F) = \pi_j(x_F). \quad (15)$$

Its computational complexity is $O(|Y_F| \times n \times |X_F| \times |\Sigma_F|)$. Once the labeling function $J_F : Y_F \rightarrow 2^I$ is constructed and all states $y_F \in Y_F$ that satisfy (15) are identified, the computational complexity of verifying the existence of a path p_{V_F} that satisfies (13) and (14) is $O((|Y_F| - 1)!)$.

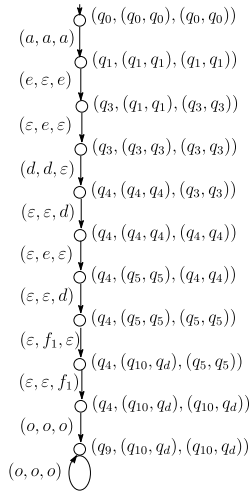


Fig. 14. A part of the finite automaton V_F .

Example 32. We consider the setting of Example 19. For the regular language $K_1 \subseteq L(G)$ generated by the subautomaton G_{N_1} of G , a part of the finite automaton V_F is shown in Fig. 14. Letting $x_F = (q_9, (q_{10}, q_d), (q_{10}, q_d))$, by Fig. 14, a singleton state $\{x_F\}$ is reachable in the nondeterministic acyclic automaton V_F . We consider any path p_{V_F} that ends with $\{x_F\}$. Then, we have $\pi_1(x_F) = \pi_2(x_F) = (q_{10}, q_d) \in Q \times \{q_d\}$. In addition, we have $\delta_{V_F}(x_F, (o, o, o)) = x_F$ for $(o, o, o) \in \Sigma_F$ with $\pi_1((o, o, o)) = \pi_2((o, o, o)) = o \neq \varepsilon$, as shown in Fig. 14. It follows that $J_F(p_{V_F}) = I$ and $\pi_1(x_F) \neq \pi_2(x_F)$. By Theorem 29, we can conclude that G is not F-SEI codiagnosable with respect to K_1 , as shown in Example 19.

7. Conclusion

The two intersection-based decentralized diagnosis architectures, named the N-SEI architecture and the F-SEI architecture, were considered. The notions of N-SEI codiagnosability and F-SEI codiagnosability were defined for the N-SEI architecture and the F-SEI architecture, respectively, and compared with the existing notions of codiagnosability. Then, methods for verifying N-SEI codiagnosability and F-SEI codiagnosability were developed. To verify F-SEI codiagnosability, paths of the acyclic automaton constructed for verifying F-SEI codiagnosability are explored. Developing an efficient exploration method for this purpose is important future work.

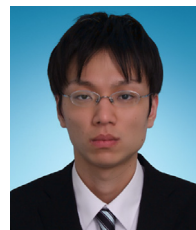
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