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# Robust predictions and hard information in the market for lemons

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ABSTRACT

The literature on informationally robust predictions has focused mostly on *soft* information. In a stylized adverse selection model, we show that hard information enables trade, even when the unique equilibrium outcome without it is no-trade.

## 1. Introduction

Informationally robust predictions aim to identify the set of outcomes that can arise in (Bayesian) equilibrium under *some* admissible information structure. The literature on such predictions has focused mostly on *soft* information; that is, information that cannot be credibly verified or disclosed to others. Roesler and Szentes (2017) and Bergemann et al. (2015) study stylized trading environments between a monopoly seller and buyer(s). Roesler and Szentes (2017) show that the optimal information structure for the buyer is not (necessarily) the one in which he knows everything; it may be better for him not to know his precise valuation. Bergemann et al. (2015) characterize the entire set of possible equilibrium payoffs, assuming that buyers know their precise valuations while the seller may have imprecise information about them. Kartik and Zhong (2025) further extend this approach to settings with interdependent values, maintaining the assumption that information is soft.

In practice, however, information is often *hard*; that is, it is accompanied by *evidence* and can be credibly disclosed to others. For example, a car manufacturer obtains a third-party certification for the quality of its car. As is well understood in the literature on hard information/evidence, such information may be elicitable without satisfying the same sort of incentive compatibility as soft information.[1] Therefore, the set of possible trading outcomes could be different if we allow for soft *and* hard information.

In this paper, we show in a stylized setup of adverse selection (Lemon) market à la Akerlof (1970) that such hard information is indeed critical in informationally robust predictions. The buyer is assumed to have no information (as the seller in Roesler and Szentes, 2017). The seller may have any (possibly noisy) information about the quality. First, we consider an arbitrary soft information structure; that is, the seller may obtain an arbitrary signal about the quality of the good, but no evidence is attached to it. We show that, under certain conditions, the unique equilibrium outcome is no-trade regardless of how rich the seller's (soft) information structure is. Next, we demonstrate by example that, with hard information, non-trivial trade outcomes could be supported. Moreover, the difference is economically significant because the players' payoffs and total surplus are strictly improving over the no-trade outcome. These results highlight the importance of allowing for both soft *and* hard information when studying informationally robust predictions.

Our model differs from Ali et al. (2024) and Dasgupta et al. (2022), two recent papers that study hard information design in standard monopoly pricing settings, by assuming an initially uninformed sender (seller), interdependent values, and no commitment on the part of the receiver (buyer). In our model, both players are initially uninformed, and the seller learns her type through either soft or hard information. By contrast, Ali et al. (2024) consider a setting in which the sender knows his type and strategically discloses the hard information he acquires to influence the receiver's behavior. Dasgupta et al. (2022) is closer to our model in that the sender is initially uninformed and learns his type through hard information. They show that hard information can expand the set of implementable outcomes. However, they assume

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[1] See Green and Laffont (1986) and Bull and Watson (2007).

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private values and the receiver's commitment to a pricing scheme, whereas we assume interdependent values and no commitment on the part of the receiver.

In the adverse selection literature, the importance of hard information has long been recognized (e.g., see Viscusi, 1978), though much of the work assumes some fixed information structure. Our contribution is to highlight its importance even for informationally robust predictions, and in the environment beyond those previously studied in the literature. We hope this short note can be used as a small but first step toward a fuller understanding of this subject.

## 2. Model

There is a seller (she) who has a good to sell to a buyer (he). There are $n$ possible quality levels (types) that the good can take, denoted by $q \in \{1, 2, \dots, n\}$. The common prior over types is given by the CDF $F$. If the good is of type $q$, the seller's valuation is $r_q \in \mathbb{R}_+$ and the buyer's valuation is $v_q \in \mathbb{R}_+$. We order the types such that $r_1 < r_2 < \cdots < r_n$ and $v_1 < v_2 < \cdots < v_n$. We assume that the valuations are linearly related and that trade is ex post efficient for the highest type but inefficient for the lowest one; specifically, $v_1 < r_1$ and $r_n < v_n$. The seller observes a signal $s$ about $q$. The buyer then makes a take-it-or-leave-it price offer of $p$ to the seller.[2] Trade occurs if and only if the seller's expected valuation conditional on the realized signal weakly exceeds $p$.[3] If a type $q$ good is traded at a price $p$, the seller gets $p - r_q$ and the buyer gets $v_q - p$; if no trade occurs, both get 0.

Given the linear relationship between valuations, we can, without loss of generality, assume that the seller observes an *unbiased signal*; that is, $\mathbb{E}[r \mid s] = s$. Under this assumption, the buyer's posterior mean is a linear function of $s$:

$$v(s) \equiv \mathbb{E}[v \mid s] = \frac{v_n - v_1}{r_n - r_1} s - \frac{r_1 v_n - v_1 r_n}{r_n - r_1}.$$

Let $r_F \equiv \int_{r_1}^{r_n} r \mathrm{d}F(r)$ denote the seller's expected valuation under the prior. We assume that no trade occurs under the prior:

$$v(r_F) < r_F \iff r_F < \frac{r_1 v_n - v_1 r_n}{v_n - r_n + r_1 - v_1} \equiv \underline{p}.$$

As in Roesler and Szentes (2017), a CDF $G$ is the distribution of some unbiased signal if and only if $F$ is a mean-preserving spread of $G$. Hence, a signal distribution $G$ is feasible if it satisfies

$$\int_{r_1}^{t} F(r)\mathrm{d}r \geq \int_{r_1}^{t} G(s)\mathrm{d}s \text{ for all } t \in \left[r_1, r_n\right] \quad \text{and} \quad \int_{r_1}^{r_n} s\mathrm{d}G(s) = r_F.$$

## 3. Only soft information

Suppose that the seller can acquire only soft information. Proposition 1 demonstrates that such information acquisition does not mitigate the equilibrium inefficiency.

**Proposition 1.** *There exists no signal distribution under which trade occurs in equilibrium.*

**Proof.** Consider an arbitrary feasible signal distribution $G$. If the buyer offers a price $p$, the seller accepts it if $s \leq p$, which yields the buyer's expected payoff of

$$\int_{r_1}^{p} [v(s) - p] \, \mathrm{d}G(s).$$

Since $v(\cdot)$ is increasing and $v(p) - p \leq 0$ for all $p \leq \underline{p}$, the buyer's expected payoff is negative if he offers $p \in \left[r_1, \underline{p}\right]$. If he offers $p \in \left(\underline{p}, r_n\right]$ satisfying $G(p) > 0$, his expected payoff is

$$\begin{aligned}
&\int_{r_1}^{p} \left[\frac{v_n - v_1}{r_n - r_1} s - \frac{r_1 v_n - v_1 r_n}{r_n - r_1} - p\right] \mathrm{d}G(s) \\
&= \frac{v_n - v_1}{r_n - r_1} \int_{r_1}^{p} s\mathrm{d}G(s) - \left[\frac{r_1 v_n - v_1 r_n}{r_n - r_1} + p\right] \int_{r_1}^{p} \mathrm{d}G(s) \\
&= G(p) \left\{ \frac{v_n - v_1}{r_n - r_1} \mathbb{E}\left[s \mid r_1 \leq s \leq p\right] - \frac{r_1 v_n - v_1 r_n}{r_n - r_1} - p \right\}.
\end{aligned}$$

This is negative if $r_F < \frac{r_1 v_n - v_1 r_n}{v_n - v_1} + \frac{r_n - r_1}{v_n - v_1} p$. Hence, it suffices to have

$$r_F \leq \frac{r_1 v_n - v_1 r_n}{v_n - v_1} + \frac{r_n - r_1}{v_n - v_1} \underline{p} = \underline{p}.$$

Since $r_F < \underline{p}$ by assumption, the unique equilibrium outcome is no-trade, whatever signal distribution we consider.

## 4. Soft and hard information

Suppose now that the seller can obtain and disclose hard information about her type. Specifically, upon observing a signal $s$, the seller obtains a set of evidence $M(s)$. She then discloses some $m \in M(s)$ to the buyer, after which the buyer makes an offer. We continue to assume that the seller accepts the offer if and only if her expected valuation conditional on the realized signal (and the set of evidence) weakly exceeds the offer. We demonstrate by example that, in equilibrium, trade can occur and the seller can obtain a positive expected payoff.

**Example 1.** Assume $n = 2$ and $0 \leq v_1 < r_1 < r_2 < v_2$. We maintain the assumption that no trade occurs under the prior: $v_F \equiv v(r_F) < r_F$. Consider a signal distribution $G$ such that, for some $g \in \left(\frac{(r_F - r_1)(r_2 - v_F)}{(v_2 - r_2)(r_F - r_1) + (r_2 - r_1)(r_2 - v_F)}, \frac{r_F - r_1}{r_2 - r_1}\right)$, the signal $s$ is distributed as

$$s = \begin{cases} r_1 & \text{with probability } \frac{r_2 - r_F}{r_F - r_1} g \\ r_F & \text{with probability } 1 - \frac{r_2 - r_1}{r_F - r_1} g \\ r_2 & \text{with probability } g. \end{cases}$$

It is straightforward to verify that the common prior $F$ is a mean-preserving spread of $G$.[4] Suppose that the set of evidence is given by $M(r_1) = \{x\}$ and $M(r_F) = M(r_2) = \{x, y\}$; that is, if the seller observes $s = r_1$, she can only disclose $m = x$, whereas if she observes $s = r_F$ or $s = r_2$, she can disclose either $m = x$ or $m = y$. This evidence structure can be interpreted as a certification system; only when the seller receives the signal $r_F$ or $r_2$ does she obtain the certification $y$, which she can then disclose to the buyer. We show that the following strategy profile, together with the belief system derived by the profile and $G$ through Bayes' rule, constitutes a weak perfect Bayesian equilibrium:

- Seller's strategy:
    1. Discloses $m = x$ if she observes $s = r_1$;
    2. Discloses $m = y$ if she observes $s = r_F$ or $s = r_2$.
- Buyer's strategy:
    1. Offers some $p \in [0, r_1)$ if he receives $m = x$;
    2. Offers $p = r_2$ if he receives $m = y$.

Given the buyer's strategy, the sequential rationality of the seller's strategy is obvious. The first part of the buyer's strategy is also optimal

[2] The assumption that the (uninformed) buyer is an offerer makes the problem simpler, by avoiding the informed-principal problem. Though not innocuous, it seems a reasonable assumption in some contexts, such as when the (uninformed) buyer is a potential acquirer of a firm who often has a stronger bargaining power than the (informed) seller.

[3] We take this seller's acceptance criterion for granted to simplify the analysis, but it is not crucial for the results.

[4] Indeed, $\mathbb{E}_G[s] = r_F$, and the area between $F$ and $G$ on the interval $[r_1, r_F]$ is equal to that on $[r_F, r_2]$, which establishes the required inequality.

since, upon receiving $m = x$, he believes $s = r_1$ with probability 1, and no gain from trade exists in this case. It remains to verify the optimality of the second part of the buyer's strategy. Suppose that the buyer receives $m = y$. If he offers $p \in [r_F, r_2)$, the seller accepts only when $s = r_F$, yielding a negative expected payoff to the buyer because $v_F < r_F$ by assumption. If he offers $p \geq r_2$, the seller accepts regardless of whether $s = r_F$ or $s = r_2$. Hence, the buyer's optimal offer is $p = r_2$, which yields an expected payoff of

$$\frac{1}{1 - \frac{r_2 - r_F}{r_F - r_1} g} \left[ g(v_2 - r_2) + \left(1 - \frac{r_2 - r_1}{r_F - r_1} g\right)(v_F - r_2) \right],$$

which is positive if

$$\begin{aligned} & g(v_2 - r_2) + \left(1 - \frac{r_2 - r_1}{r_F - r_1} g\right)(v_F - r_2) > 0 \\ \iff\ & g > \frac{(r_F - r_1)(r_2 - v_F)}{(v_2 - r_2)(r_F - r_1) + (r_2 - r_1)(r_2 - v_F)}. \end{aligned}$$

Therefore, offering $p = r_2$ is optimal for the buyer. Note that trade occurs with positive probability and that the seller obtains a positive expected payoff. Thus, the availability of hard information mitigates the equilibrium inefficiency.

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## Data availability

No data was used for the research described in the article.

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