The semantics of differential *no*

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Abstract This short squib explores the semantics of *no* in *no more than* + numeral constructions. Three types of analyses have been proposed in the literature: *no* as zero, constituent negation, and a degree quantifier. I argue that the degree quantifier analysis is the most successful of the three. The zero analysis is too strong, as it predicts only an exactly + numeral reading in non-modal contexts and upper-/lower-bound readings in modal contexts. The constituent negation analysis accounts for the correct range of interpretations but fails to explain why explicit constituent negation does not exhibit the same behavior as *no*. The degree quantifier analysis makes a correct prediction without any known complications.

Keywords *no*, *no more than*, differentials, degrees, degree quantifiers, constituent negation

1. Introduction

This short squib examines the semantics of *no* in *no more/less than* in English. At least three approaches have been proposed in the literature: *no* as zero, constituent negation, and a degree quantifier. I argue that it should be analyzed as a degree quantifier of differentials rather than as a degree of differentials or as constituent negation. The analysis itself is neither new nor original; rather, my aim here is to clarify the range of predictions these approaches make and to determine which one best explains and predicts the facts.

This paper focuses on *no more/less than* + numeral, as exemplified in (1a)-(1b). It is evident that these expressions originate from comparatives, and one intriguing aspect of them is that they are often interpreted as *exactly* + numeral. Nouwen (2008) notes that in both (1a) and (1b), the number of people who came is understood as exactly 10.

(1) a. No more than ten students attended the lecture.

b. No less than ten students attended the lecture.

Jespersen (1917/1966:83) also states that *no less than 30* means exactly 30, implying surprise or wonder at the high number. *He has no less than ten children*he has ten, and isnt that a large family? This is reflected in the fact that *no more than* + numeral is often paraphrased as *only* + numeral in descriptive grammars for Japanese learners of English.[1]

In spite of this, it is also true that *no more/less than* + numeral can have below/above + numeral interpretations. Consider the following examples:

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[1] As Jespersen (1917/1966) suggested, *no more/less than* is often associated with a connotation of unexpectedness. This can be described as negative evaluativity, which indicates that *no* + *many-er* implies that the number is small, while *no* + *little-er* implies that the number is large. I will not explore how this connotation arises.

(2) a. My students really fight the thumbnail sketch idea. I keep telling them that once they have the composition and darks and lights planned out, the painting will just fall off their brushes. **The thumbnail should be no more than two inches and take no more than two minutes.** COCA, emphasis mine

b. The hamburgers should contain no less than 50 % meat.

In (2a)(2b), *no more/less than* + numeral is most naturally interpreted as below/above + numeral.

One would not want to claim that *no more than* + numeral is ambiguous in a way that simply covers these cases; rather, one would seek to derive both readings from a single lexical semantics of *no*. In the next section, I introduce three types of approaches to the semantics of *no* in *no more/less than* + numeral. Then, in Section 3, I examine the predictions these approaches make and identify the most promising one.

2. The semantics of *no*: Introducing three approaches

Some dictionaries deal with *no more/less than* as a kind of idiom, but I take it as an ordinary comparative with *many/much/little* as its base adjectives.

Some dictionaries treat *no more/less than* as an idiom, but I take it to be an ordinary comparative with *many/much/little* as its base adjectives.

All the descriptive grammars I consulted agree that *no* in *no more/less than* is not a sentential negation, but there are at least two ways to characterize what it is. One approach is to analyze it as a constituent negation, in which *no* negates more/less. This seems to be the path taken by Yagi (2015), who notes that *no* in (3) is an adverb that negates *more* (cf. Quirk et al. (1985), Nouwen (2008)).

(3) She is no more hard-working than me.

Another approach is to regard *no* as a differential, functioning in the same way as *2 inches/much* in (4). This is the position taken by Huddleston & Pullum (2002), who classify *no* among differential expressions. Hirasawa (2012) and Honda (2017) also adopt this view, describing *no* in *no more/less than* as denoting zero difference.[2]

(4) John is { 2 inches/much/no } taller than Bill.

This section presents the theoretical implementations of these analyses before exploring their predictions.

2.1. *No* as differentials

Let us first assume the following semantics for the comparative morpheme. I will represent both the inflectional comparative morpheme *-er* and the analytic one *more* as MORE.[3]

[2] Hirasawa (2012) and Honda (2017) focus on the so-called whale construction, where the comparison concerns the unlikeliness of propositions, as exemplified in (i): (i) A whale is no more a fish than a horse is. I do not include this type of construction, as it exhibits several idiosyncratic properties that may set it apart from ordinary comparative constructions. See Sawada (2005) for details.

[3] It is well known that the analytic form has broader functions than the inflectional one. For instance, it can co-occur with non-gradable adjectives, often resulting in the so-called metalinguistic comparison: (i) The problem is more political than financial. Metalinguistic *more* can co-occur with a wide range of syntactic categories, including NPs, VPs, and PPs. I do not include these cases in the semantics of MORE.

The first argument of (5a) is intended for a differential phrase.

(5) a. $[\![\text{MORE}]\!] = \lambda d_1.\ \lambda d_2.\ \lambda D_{\langle d,t \rangle}.[\max_d(D) = d_1 + d_2]$, where max is the maximality operator.

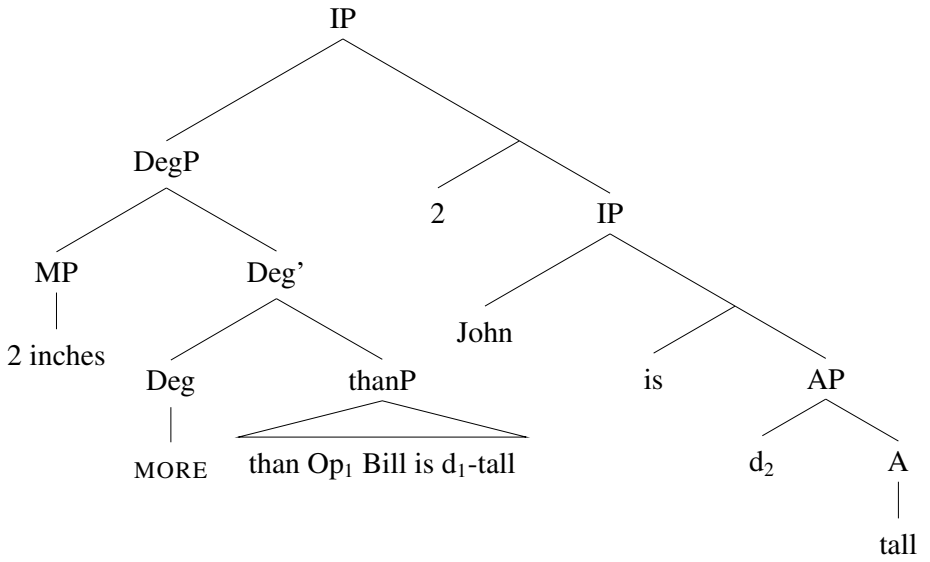
b. Let D be a set of degrees:
max(D) = the unique d such that $\forall$d' $\in$ D. d' $\leq$ d.

There may be two ways to analyze measure phrases that function as differentials. The first option is straightforward: a differential measure phrase (MP) denotes a degree on the relevant scale. In other words, differential MPs are of type d.

(6) $[\![2 \text{ inches}]\!]$ = 2-inches type d

Given the standard assumption about null operator movement in the comparative standard clause, as in (7a), along with QR of DegP in the matrix clause, this yields the truth conditions for *John is two inches taller than Bill (is)*, as shown in (7c). Under this truth condition, if Bill is 6' tall, then John should be 6'2" tall, as required.

(7) a.



b. $[\![\text{than}]\!] = \lambda D_{\langle d,t \rangle}.\max(D)$

c. $[\![$ John is 2 inches taller than Bill (is) $]\!]$
= [$\max_d$(john is d-tall) = $\max_d$(bill is d-tall) + 2-inches]

d. In prose: John's height equals Bill's height + 2 inches

Taking this at face value, we can assume the semantics of *no* as zero, as in (8a). This yields the correct truth conditions for *John is no taller than Bill (is)*, where John and Bill are of the same height.

(8) a. $[\![\text{no}_d]\!] = 0$ type d

b. $[\![$ John is no taller than Bill (is) $]\!]$ = [$\max_d$(john is d-tall) = $\max_d$(bill is d-tall) + 0]

Let us extend this analysis to *no more than* + numeral in the attributive position. I assume the LF in (9a), which would be interpreted as in (9c).

(9) a. LF: [$_{IP}$ [$_{DegP}$ [no [MORE than 10]]] [1 [$_{IP}$ [t_1-many students] attended the lecture]]]

b. $[\![$ no more than 10 $]\!] = \lambda D_{\langle d,t\rangle}.\ [\max_d(D) = 10 + 0]$

c. $[\![$ No more than 10 students attended the lecture $]\!]$ = [$\max_d$(d-many students attended the lecture) = 10 + 0]

Another approach is to assume that differentials are degree quantifiers (cf. Schwarzschild (2005)). I adopt the implementation proposed by Alrenga & Kennedy (2014), in which *no* is defined as in (10a). This denotation suggests that there is no degree d such that the number of students who attended the lecture reaches d and d exceeds 10. In other words, the number of students is 10 or fewer, per (11).

(10) a. $[\![\text{no}_{\langle dt,t\rangle}]\!] = \lambda D_{\langle d,t\rangle}.\ \{\ d \mid D(d)\ \} = \emptyset$

b. $[\![$ No more than 10 students attended the lecture $]\!]$ = $\{\ d \mid$ d-many students attended the lecture $\wedge\ d > 10\ \} = \emptyset$

(11) a. Exactly 11 students attended. FALSE

b. Exactly 10 students attended. TRUE

c. Exactly 9 students attended. TRUE

This truth condition clearly differs from that of the *no*-as-zero analysis in that it allows the below 10 interpretation and does not yield the exactly' + numeral reading.

2.2. *No* as a constituent negation

Nouwen (2008) proposes an analysis of *no* as constituent negation. This analysis is partially motivated by data from Dutch and German, where regular negation is used in contexts where *no* combines with a comparative in English.

(12) **Dutch**
Sjeng heeft niet meer dan tien knikkers gevonden.
Sjeng has not more than ten marbles found
'Sjeng found no more than ten marbles.' Nouwen (2008:276)

Nouwen (2008) implements the constituent negation analysis as shown in (13c). (13a)(13b) present reconstructed denotations based on his analysis of *no more than 10*.

(13) a. $[\![\text{MORE}]\!] = \lambda d.\ \lambda D_{\langle d,t\rangle}.\ \max_d(D) > d$

b. $[\![$ no $]\!] = \lambda P_{\langle\langle d,t,t\rangle\rangle}.\ \lambda D'_{\langle d,t\rangle}.\ \neg\ P(D')$

c. $[\![$ no MORE than 10 $]\!] = \lambda D'_{\langle d,t\rangle}.\ \neg\ [\max_d(D') > 10]$
Nouwen (2008:276, slightly adapted)

Note that this analysis again does not yield the 'exactly' + numeral reading. The interpretation in (13c) allows for a situation in which exactly 9 students attended the lecture.

3. Predictions of the three approaches

The two approaches other than the zero analysis do not derive the exactly + numeral interpretation, as described in (1a)(1b). In the next section, we first examine how this issue is resolved through scalar implicature. In Section 3.2, I show that interpretations in modal contexts help us distinguish the predictions of these three approaches.

3.1. Deriving the 'exactly' + numeral reading via scalar implicatures

The most straightforward way to give rise to the 'exactly'+ numeral reading in the constituent negation and differentials as degree quantifier approaches is to invoke scalar implicature, as proposed in Nouwen (2008).

Scalar implicatures arise when a weaker proposition is asserted to implicate the negation of its stronger alternatives. These alternative propositions are assumed to be generated based on the same level of structural complexity. In the case at hand, propositions of the form no more than n students attended the lecture, where $n \neq 10$, serve as alternatives to no more than 10 students attended the lecture.

Among these alternatives, propositions where $n < 10$ are stronger in that they asymmetrically entail *no more than 10 students attended the lecture*.

Let us illustrate how the exactly + numeral interpretation can be derived under the differential-as-degree quantifier approach. Given the truth condition in (14a), (14a) is asymmetrically entailed by (14b): if there is no degree such that the number of students who attended the lecture reaches d and $d > 9$, then it is necessarily true that there is no degree such that the number of students who attended the lecture reaches d and $d > 10$. This entailment pattern applies to all alternatives where $d < 10$.

(14) Differential-as-degree quantifier approach

a. $[\![$ No more than 10 students attended the lecture. $]\!] = \{ d \mid$ d-many students attended the lecture $\wedge\ d > 10 \} = \emptyset$

b. $[\![$ No more than 9 students attended the lecture. $]\!] = \{ d \mid$ d-many students attended the lecture $\wedge\ d > 9 \} = \emptyset$

This asymmetric entailment relation gives rise to a scalar implicature: the negation of all these stronger alternatives is assumed to be true. Combining this with the assertion and the implicatures, we obtain the 'exactly 10' interpretation.

(15) a. Implicatures:
$\neg[\{ d \mid$ d-many students attended the lecture $\wedge\ d > 9 \} = \emptyset]$
$= \{ d \mid$ d-many students attended the lecture $\wedge\ d > 9 \} \neq \emptyset$
In prose: there is some d such that the number of students who attended the lecture reaches d and $d > 9$

b. Assertion (14a) + Implicatures
$9 <$ the number of students who attended the lecture ≤ 10

The same is true for the constituent negation analysis.

This means that the absense of the *exactly*-n interpretation of these analyses is no longer a problem, as it can be derived as an implicature.

3.2. Modal contexts: where the predictions diverge

Let us now examine contexts in which *no more than* + numeral is embedded under modals, as in (16)(17)(=(2)), where the three analyses yield different predictions.

(16) Your paper is required to be no more than 10 pages.

(17) a. The thumbnail should be no more than two inches and take no more than two minutes. COCA

b. The hamburgers should contain no less than 50 % meat.

Let us first consider the zero analysis. Since the maximality operator takes scope, there should be scopal ambiguity with a modal. The predicted interpretations for (16) are as follows (Acc_{w0} is shorthand for accessible worlds from the evaluation world w_0):

(18) a. $\forall w \in Acc_{w0} \rightarrow [max_d(\text{your paper is d-many pages at w}) = 10 + 0]$

b. $max_d(\forall w \in Acc_{w0} \rightarrow \text{your paper is d-many pages at w}) = 10 + 0$

What (18a) indicates is that in every accessible world w, the maximum length of your paper in w is 10. This sets the upper bound on your paper, excluding any paper of length 11 or more; we call this the upper-bound reading. On the other hand, (18b) is a requirement about the minimum length of your paper: the maximum degree d such that the length of your paper reaches d in every accessible world is 10. This means that your paper must have at least 10 pages. A similar interpretation holds for the attributive *no more than* + numeral:

(19) Cody is not eligible for tenure at MIT. He didn't publish enough articles. He can try to get tenure at Acme University, however. There, one is required to have published no more than three articles in order to get tenure. Nouwen (2008:280)

Nouwen (2008) observes that (19) states that the minimum requirement for tenure at Acme University is three articles. I will refer to this interpretation as the lower-bound reading.

In addition to these readings, there is another, much weaker reading than the former two. Consider the following attested examples (20):

(20) a. Hello all, Ive been playing around with a new concept for an advanced book club. I just wanted to think of something more engaging and fun while reading at a quicker pace. So the foundation of this concept is to add some gamification, increase participation and read 1 book per month. In order to do that I was thinking of putting page limit at 500 pages, meaning **we'd have to read no more than 17 pages a day for each book that's 500 pages or less.** [4]

b. This course includes a substantial reading load but not an onerous one. **During most weeks you will have to read no more than 200 pages of text, usually fewer.**[5]

[4] https://www.librarygirl.net/post/level-up-book-club-gamification-for-an-epic-win

[5] https://english.ufl.edu/wp-content/uploads/sites/45/ENL2930_17A7_Harpold.pdf

What (20a) means can be paraphrased as "we don't have to read more than 17 pages a day." Similarly, the natural interpretation of (20b) would be "you will not have to read more than 200 pages of text." These interpretations are much weaker: they allow situations where you read 15 or 190 pages, but do not falsify the sentences if you read more than 17 or 200 pages.

Neither (18a) nor (18b) predicts this weaker interpretation. Therefore, the 'zero' analysis fails in the context of modals.

The constituent negation approach proposed by Nouwen (2008) fares better than the zero analysis. It predicts weaker truth conditions but can be strengthened by scalar implicatures to derive the upper- and lower-bound interpretations. Let us first examine an existential modal, such as *be allowed to*, with *no more than* + numeral, as discussed in Nouwen (2008).

(21) a. Cody's paper is allowed to have no more than 10 pages.

b. $\exists$w. w$\in$ $\text{Acc}_{\text{w0}} \wedge$ [max_{d} (Cody's paper has_{w} d-many pages) $\leq$ 10pp]

c. max_{d}([$\exists$w. w $\in$ $\text{Acc}_{\text{w0}} \wedge$ Cody's paper has_{w} d-many pages])$\leq$10pp

Nouwen (2008:based on 279)

The reading in (21b) offers a weak interpretation, where a 19-page paper is allowed, but a 21-page paper is also permissible. This occurs because (21c) merely states that the maximum length of Cody's paper is below 20 pages. Crucially, this reading is compatible with a situation where the paper limit is, for instance, 15 pages. (22) indicates that this interpretation is too weak, as it requires the upper-bound reading.

(22) Cody's paper is allowed to have no more than 20 pages. #In fact, the page limit is 15 pages. Nouwen (2008:279)

A stronger interpretation arises through scalar implicature, as before. (21c) has the stronger alternatives given in (23). When combined with scalar implicature, this successfully yields the upper-bound reading, as shown in (24).

(23) a. max_{d}([$\exists$w. w $\in$ $\text{Acc}_{\text{w0}} \wedge$ Cody's paper has d-many pages at w])$\leq$19pp

b. max_{d}([$\exists$w. w $\in$ $\text{Acc}_{\text{w0}} \wedge$ Cody's paper has d-many pages at w])$\leq$18pp

c. ...

(24) max_{d}([$\exists$w. w $\in$ $\text{Acc}_{\text{w0}} \wedge$ Cody's paper has d-many pages at w]) = 20pp

Let us apply the same strategy to the universal modal. The two predicted interpretations for (16) are the following:

(25) a. $\forall$w:w$\in$ $\text{Acc}_{\text{w0}} \rightarrow \neg$ [max_{d}(your paper has_{w} d-many pages) $>$ 10]

b. $\neg$ [max_{d}($\forall$w:w$\in$ $\text{Acc}_{\text{w0}} \rightarrow$ your paper has_{w} d-many pages) $>$ 10]

First, consider the reading in (25b): under this interpretation, the minimum length of the paper is below 10. This allows for scenarios where the paper is 9 pages long, but it is also compatible with a situation where the paper is 11 pages long. Notably, this corresponds to the reading observed in (20a)(20b). This indicates that the constituent negation analysis can account for an interpretation that the 'zero' analysis could not.

As before, the lower-bound readingwhere the minimum length is exactly 10 pagesis derived through scalar implicature.

(26) a. Stronger alternatives:
$\neg$[$\max_d$($\forall$w: w$\in$Acc$_{w0}$ $\rightarrow$ your paper is d-many pages at w) $>$ n], where n $\leq$ 9

b. Scalar implicature:
$\neg$($\neg$ [$\max_d$($\forall$w: w$\in$Acc$_{w0}$ $\rightarrow$ your paper is d-many pages at w) $>$ 9]
$\neg$($\neg$ [$\max_d$($\forall$w: w$\in$Acc$_{w0}$ $\rightarrow$ your paper is d-many pages at w) $>$ 8]
...

c. $\max_d$($\forall$w: w$\in$Acc$_{w0}$ $\rightarrow$ your paper is d-many pages at w) = 10

Next, consider (25a). This states that in every accessible world w, the maximum length of the paper in w is below 10. This interpretation is compatible with scenarios where the paper is 10 pages or fewer. However, crucially, it does not explicitly establish 10 as the upper limit. This is where scalar implicature plays a role.

The stronger alternatives to (25a) should be propositions of the form (27).

(27) $\forall$w: w$\in$Acc$_{w0}$ $\rightarrow$ $\neg$[$\max_d$(your paper is d-many pages at w) $>$ n], where n $\leq$ 9

If these alternatives are negated, we would get (28a). To satisfy both (28a) and (25a), the upper-bound of your paper is 10 pages.

(28) a. $\neg$ [$\forall$w: w$\in$Acc$_{w0}$ $\rightarrow$ $\neg$[$\max_d$(your paper has$_w$ d-many pages) $>$ n]], where n $\leq$ 9

b. = $\exists$w: w $\in$ Acc$_{w0}$ $\wedge$ [$\max_d$(your paper has$_w$ d-many pages) $>$ n], where n $\leq$ 9

One complication of this approach arises from the interpretation of explicit constituent negation in the form of not more than + numeral. As exemplified in (29), not more than + numeral does not undergo scalar implicature to yield the 'exactly' + numeral or the upper-/lower bound interpretations. If *no more than* + numeral were simply a case of constituent negation, this discrepancy would remain unexplained.

(29) a. Not more than ten students attended the lecture.

b. Your paper is not required to be more than 10 pages.

c. Your paper is required to not more than 10 pages.

Let us next consider the degree quantifier analysis. This analysis predicts the following two interpretations:

(30) a. $\forall$w$\in$ Acc$_{w0}$ $\rightarrow$ { d | your paper has$_w$ d-many pages $\wedge$ d $>$ 10 } = $\emptyset$

b. { d | $\forall$w$\in$ Acc$_{w0}$ $\rightarrow$ your paper has$_w$ d-many pages$\wedge$ d $>$ 10 } = $\emptyset$

The interpretation in (30a) states that in all accessible worlds w, there are no degrees d such that the length of your paper reaches d and $>$10. This corresponds to a situation where the upper bound of your paper is 10 pages. In contrast, the interpretation in (30b) is considerably weaker, allowing for the possibility that your paper may or may not be exactly 10 pages longconsistent with the readings observed in (20a)-(20b).

This analysis does not require scalar implicature to derive the upper-bound reading. However, obtaining the lower-bound reading does rely on scalar implicature. Since the derivation process is identical to that used in the constituent negation approach, I will not illustrate it here.

Both the constituent negation and degree quantifier analyses account for all three attested interpretations, though through slightly different mechanisms. In the degree quantifier approach, the upper-bound reading follows directly from the truth conditions, whereas in the constituent negation approach, it arises via scalar implicature.

As noted above, the constituent negation approach does not account for why *not more than* remains insensitive to strengthening via scalar implicature. In contrast, the degree quantifier approach does not face this issue.

The following is a summary:

	Degree 'zero'	Degree Quantifier	Constituent negation
exactly n (non-modal)	truth condition	SI	SI
upper-bound ($\forall$-modal)	truth condition	truth condition	SI
lower-bound ($\forall$-modal)	truth condition	SI	SI
weak reading ($\forall$-modal)	not available	NO $> \forall$	$\neg > \forall$

Table 1. Summary of the predications of three approaches

4. Conclusion

This squib has explored the semantics of *no* in *no more/less than* + numeral, and explicates that the degree quantifier approach is the most promising one. I have exhibited that the data point for the current analyses would be whether they can predict the weak reading as well as the upper/lower-bound interpretations in modal contexts.

Expressions containing *no more/less than* are notoriously challenging for Japanese learners of English, partly due to the absence of a direct lexical equivalent to *no* in Japanese. I hope that the theoretical insights presented here can be extended to inform pedagogical strategies.

References

Alrenga, Peter & Christopher Kennedy. 2014. No more shall we part: Quantifiers in english comparatives. *Natural language semantics* 22. 1–53.

Hirasawa, Shinya. 2012. "kujira-no koobun" no "koobun" tositeno imi-wa dokoni arunoka. *Journal of English Grammar and Usage* 19. 50–65.

Honda, Akira. 2017. Kujira-no koosiki-no nazo-o toku (solving mysteries of the whale construction). In *Kobe gaidai ronso*, vol. 67 3, 59–88. Kobe City University of Foreign Studies.

Huddleston, Rodney & Geoffrey K. Pullum. 2002. *The cambridge grammar of the english language*. Cambridge University Press.

Jespersen, Otto. 1917/1966. *Negation in English and other languages*. Kobenhavn.

Nouwen, Rick. 2008. Upper-bounded no more: The exhaustive interpretation of non-strict comparison. *Natural Language Semantics* 16. 271–295.

Quirk, Randolph, Sydney Greenbaum, Geoffrey Leech & Jan Svartvik. 1985. *A comprehensive grammar of the english language*. Longman.

Sawada, Osamu. 2005. The cognitive characteristics of the idiomatic comparative constructions: the case of the no more/less...than constructions. *Proceedings of the 9th Conference of the Pan-Pacific Assocation of Applied Linguistics* 273–280.

Schwarzschild, Roger. 2005. Measure phrases as modifiers of adjectives. *Recherches Linguistiques de Vincennes* 34. 207–28.

Yagi, Katsumasa. 2015. Hikaku koobun to dootei idiomu: no more ...than koobun-no honsitu. (comparative constructions and identity idioms: The essence of no more ...than). *Journal of English Grammar and Usage* 22. 167–182.