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# Theoretical Calculation on Shape of Fusion Boundary and Temperature Distribution around Moving Heat Source (Report I)<sup>†</sup>

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and Yoshiaki ARATA\*\*\*\*

## Abstract

*Shape of steady state fusion boundary and temperature distribution around a point heat source moving a constant velocity on a thick plate are calculated theoretically by solving the free boundary problem of heat conduction where the emission and absorption of the latent heat at the boundary and the difference between the thermal diffusivity in solid phase and that in liquid phase are taken into account.*

*The fusion boundary is determined so that the heat balance equation on the fusion boundary including the latent heat is satisfied by successive approximation.*

*In a typical case, the fusion boundary is displaced forward and its length is 10% shorter, when the latent heat and the difference of the thermal diffusivity are ignored.*

*Obtained results are compared with experimental data quoted from Christensen's work. The theoretical pool widths agree with the experimental ones, but the ripple lag and pool lengths obtained by experiments are longer than those obtained by the present theory.*

## 1. Introduction

Since the spatial and temporal temperature distribution around a moving heat source on a plate under welding influences on the mechanical and metallurgical quality of the welded zone of the plate, the theoretical studies to determine the temperature distribution has been developed by many authors.

D. Rosenthal applied the mathematical theory on the Helmholtz equation to the problem and calculated the temperature field around a moving point source and that around a moving line source<sup>1)</sup>. He remarked that this theory applies only the region far from the source of heat. Despite his remark, the temperature field in the neighborhood of the fused zone has been calculated based on this theory. Almost all of these studies are simple extensions of this theory<sup>2)</sup>. In this theory, the following assumptions are necessary to obtain the temperature field as the solution of the Helmholtz equation.

- (1) Latent heats of melting and solidification are negligible.
- (2) Thermal conductivity, density and specific heat are constant.
- (3) There are no heat losses from the plate surface.

Despite the above assumption, fairly good agreements of the theoretical results with the experimental ones could be possible under certain conditions<sup>3)</sup>. But, in actual, material parameters showing thermal properties discontinuously change at the melting point where the phase of the material changes, and the latent heat is emitted or absorbed at the weld pool boundary. Therefore, in order to calculate the shape of the fusion boundary and the temperature distribution rigorously, we have to use some other mathematical methods than that based on the Rosenthal theory.

One of the methods has been developed by Y. Arata and M. Kanayama<sup>4)</sup>, and Z. Paley and P. D. Hibbert<sup>5)</sup>. In these papers, the temperature field outside the isothermal fusion boundary is calculated by solving the boundary value problem of equation of heat conduction, where the fusion boundary is fixed and given beforehand.

Recently, T. Ishimura, who is one of the authors, has developed the method which deal with the problem to solve the shape of fusion boundary as the free boundary problem of heat conduction, where the latent heat of melting and solidification as well as the difference between the thermal diffusivity of the material

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in solid state and that in liquid state are taken into account<sup>6)</sup>.

The aim of this paper is to apply this method under the condition that the heat source is a point source and to compare the obtained results with Rosenthal's solutions and experimental data and to discuss the effect of the two facts, that is, the emission and absorption of latent heat and the difference between the thermal diffusivity in solid phase and that in liquid phase, on the shape of fusion boundary. Furthermore we describe the temperature distribution in the neighborhood of the fusion boundary compared with Rosenthal's solutions.

## 2. Problem Formulation and Analytical Method

### 2.1 Mathematical formulation of the problem\*

Consider that a constant heat input  $Q$  is supplied at a point moving with a constant speed  $v$  along a straight line on a thick plate which is regarded as a semi-infinite plate, then a steady state of temperature distribution is established soon after the beginning of heating. In this steady state, an observer moving with the heat source will see a molten pool as if it is rest and will notice no temporal change in the temperature distribution around the heat source. So, we use the moving coordinate system shown in Fig. 1, hereafter, then the problem is reduced to that independent of time.

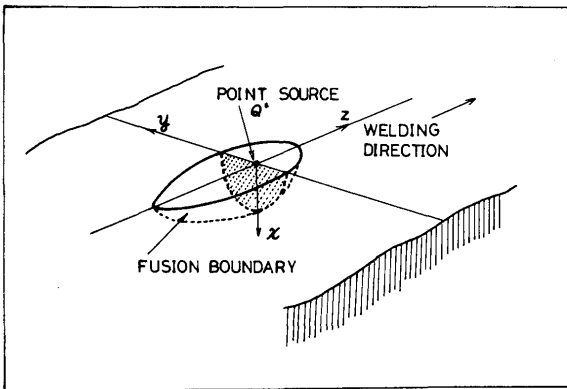


Fig. 1 Physical situation for steady state fusion boundary problem around moving point source and coordinate system.

The weld piece is semi-infinite, or the media is bounded by the plane  $x=0$  and extends to infinity in the direction of  $x$  positive. But, we assume no loss of

heat from surface, therefore we can suppose the media continued on the negative-side of the plane  $x=0$  symmetrically. Since the source is a point source, the heat flux through the surface of the hemisphere drawn around the source must be the value of the total heat  $Q$  delivered to the thick plate or semi-infinite media. So the value of the heat of source in infinite media must be double of the one in semi-infinite media. And then, mathematically we shall consider the infinite media where a point source, the value of heat of which is  $2Q$ , exists at the origin of the coordinates and the fusion boundary is symmetrical with respect to the plane  $x=0$ .

The equation of heat conduction inside of the fusion boundary is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -\frac{v}{\alpha_L} \frac{\partial T}{\partial z}, \quad (1)$$

and that outside of the fusion boundary is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -\frac{v}{\alpha_S} \frac{\partial T}{\partial z}, \quad (2)$$

except for the point through which the heat is supplied. Here  $\alpha_L$  and  $\alpha_S$  are thermal diffusivity in liquid phase and that in solid one, respectively. It should be remarked here that the value of thermal diffusivity  $\alpha_S$  in solid phase differs from that in liquid one  $\alpha_L$ .

Of course,  $T$  must satisfy the following boundary conditions at the origin and at the point at infinity,

$$\lim_{r \rightarrow 0} 4\pi r^2 \kappa_L \frac{\partial T}{\partial r} = 2Q, \quad (3)$$

$$\lim_{r \rightarrow \infty} T = T_0, \quad (4)$$

where  $r = \sqrt{x^2 + y^2 + z^2}$  and  $T_0$  is room temperature\*\*.

Furthermore,  $T$  must satisfy the boundary conditions

$$T = T_f, \quad (5)$$

$$P = -L\rho v(\vec{\nu} \cdot \vec{Z}) = \kappa_L \left( \frac{\partial T}{\partial \nu} \right)_L - \kappa_S \left( \frac{\partial T}{\partial \nu} \right)_S, \quad (6)$$

on the fusion boundary

$$f(x, y, z) = 0, \quad (7)$$

where,  $\vec{\nu}$  = unit vector in the direction of the outward normal to the boundary,

$\vec{z}$  = unit vector in the direction of positive  $z$ ,

$\kappa_S$  = thermal conductivity in solid,

$\kappa_L$  = thermal conductivity in liquid,

$P$  = rate of emission of latent heat of solidification per unit area of the boundary.

After all, the problem is resolved into the mathe-

\* In this paper, only an analysis of the temperature field produced by a moving point source is presented but it is easy to extend the analysis to the case of other kinds of heat source.

\*\* Usually,  $T_0$  is chosen as 0.

mathematical one to obtain the temperature distribution and the fusion boundary satisfying these equations.

**2.2 Outline of the method for solution**

We shall treat that the free boundary problem which is given at the end of the preceding subsection. We choose a suitable function  $f_1(x, y, z)$  so that the surface given by the following equation

$$f_1(x, y, z) = 0 \tag{8}$$

is an approximation of the fusion boundary. Then, we consider a problem to obtain the temperature distribution  $T_1(x, y, z)$  which satisfies equation (1)~(4) and equation (5) on the surface given by equation (8). If this problem is solved, we can find the right side of equation (6), where  $T$  is replaced by  $T_1$ , however, it may not agree with the left side of the same equation. Next, we calculate the variations of both side of equation (6) corresponding to the variations of  $f_1(x, y, z)$ , and the choose linear combination of variations of  $f_1$  so that the corresponding variations of both side of equation (6) cancels out the difference of the both side of equation (6). Now, we find the next approximation of fusion boundary given by

$$f_2(x, y, z) = 0 \tag{9}$$

where  $f_2(x, y, z)$  is the sum of  $f_1(x, y, z)$  and the linear combination of variations of  $f_1$ . This procedure is repeated and a series of functions  $f_1, f_2, f_3, \dots$  is established. Finally, we can find  $f(x, y, z)$  from the equation

$$f(x, y, z) = \lim_{n \rightarrow \infty} f_n(x, y, z). \tag{10}$$

Usually, after several repetitions, the correct fusion boundary is obtained. The temperature distribution is calculated by solving equation (1) and (2) with the conditions on the obtained fusion boundary where temperature and its differential coefficient are given. Then, this procedure is similar to initial value problem of partial differential equation, though the partial equation is an elliptic type.

**3. Calculation**

Symbols and normalization are shown in **Table 1**. The values of these normalized parameters are depend on the heat input condition and material used.

For the numerical calculation we used the NEAC-2200-500 computer system of Osaka University.

**Table 1** Symbols and normalizations

$Q$ :	Heat Input	(cal/sec)
$T$ :	Temperature	(°C)
$T_f$ :	Melting Point	(°C)
$L$ :	Latent Heat of Solidification	(cal/g)
$v$ :	Moving Speed of Source	(cm/sec)
$\kappa$ :	Thermal Conductivity	(cal/sec·cm·°C)
$\alpha$ :	Thermal Diffusivity	(cm <sup>2</sup> /sec)
$C$ :	Specific Heat	(cal/g °C)
suffix	S: solid	
	L: liquid	

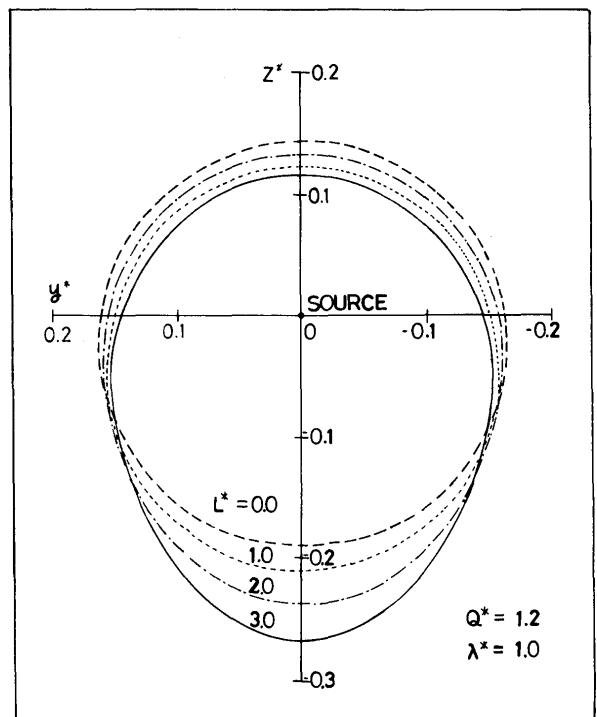
$$Q^* = \frac{Qv}{2\kappa_s\alpha_s T_f}, \quad L^* = \frac{L}{C_s T_f}, \quad \lambda^* = \frac{\alpha_s}{\alpha L}, \quad \kappa^* = \frac{\kappa L}{\kappa_s},$$

$$z^* = \frac{zv}{2\alpha_s}$$

**4. Results and Discussions**

**4.1 Effect of latent heat and difference of thermal diffusivity**

In **Fig. 2**, the shapes of fusion boundary calculated in the case of the latent heat  $L^* = 0.0, 1.0, 2.0$  and  $3.0$  are shown, where the heat input  $Q^* = 1.2$  and  $\lambda^* = 1.0$ . In **Fig. 3**, the shapes of the fusion boundary obtained in the cases of the heat input  $Q^* = 2.0, 4.0, 6.0$  and  $8.0$  are shown in the respective case of  $L^* = 0.0$  and  $\lambda^* = 1.0$  and  $L^* = 0.3$  and  $\lambda^* = 1.0$ . As seen from these figures, the backward displacement of the molten pool and the pool length along the  $z$ -axis become larger with increase of relative latent heat  $L^*$ .



**Fig. 2** Dependence of fusion boundary on latent heat  $L^*$  in the case of  $Q^* = 1.2$  and  $\lambda^* = 1.0$ .

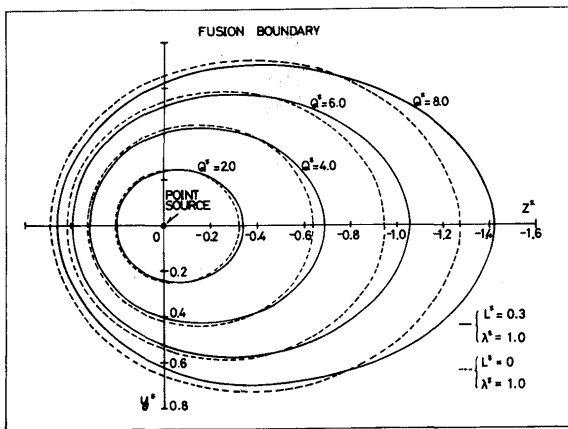


Fig. 3 Comparison between the fusion boundary in the case of  $L^*=0.3$  and  $\lambda^*=1.0$  and that in the case of  $L^*=0.0$  and  $\lambda^*=1.0$ .

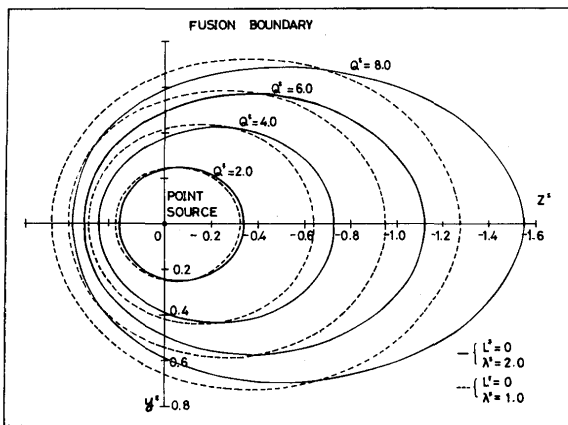


Fig. 4 Comparison between the fusion boundary in the case of  $L^*=0.0$  and  $\lambda^*=2.0$  and that in the case of  $L^*=0.0$  and  $\lambda^*=1.0$ , with respect to  $Q^*=2.0, 4.0, 6.0$  and  $8.0$ .

Figure 4 shows the shapes of fusion boundary in cases where the ratio  $\lambda^*$  of thermal diffusivity in solid state to that in liquid state is 2.0 and 1.0 and the heat input  $Q^*$  are 2.0, 4.0, 6.0 and 8.0. From Fig. 4 we can conclude that the backward displacement of molten pool becomes larger with increase of  $\lambda^*$ . Therefore, if the emission and absorption of latent heat on the fusion boundary and the difference between the thermal diffusivity of the material in solid phase and that in liquid phase are ignored, then the calculated shape of fusion boundary may differ considerably from the correct one, in some conditions. An example showing this fact is given in Fig. 5. In this connection, the value of  $L^*$  is not larger than 1.0 and  $\lambda^*$  takes the value from 1.0 to 2.0 in metals in common use.

#### 4.2 Temperature distribution and heat flux

We examine the results of calculation applied to a practical case where an heat source of  $5 \times 10^3$  watts

moves on an aluminum thick plate with velocity 1 cm/sec.. In this case, the normalized quantities are  $L^*=0.6$ ,  $Q^*=3.0$ ,  $\lambda^*=2.0$  and  $\kappa^*=0.5$ . The calculated fusion boundary and isothermal surfaces are shown by dashed line and solid ones, respectively, on the left side of the z-axis in Fig. 5, where the parameters represent normalized temperature  $T=T/T_f$ . In this calculation, the difference between the thermal conductivity of the material in solid state and that in liquid state is also taken into account to calculate the shape of the isothermal surface, as well as that of the thermal diffusivity. The fusion boundary and isothermal surfaces shown on the right side of the z-axis in Fig. 5 are those calculated under the same condition as before except that the latent heat is assumed to be zero and the thermal constants is to be equal to those in solid state all over the medium. As seen from Fig. 5, the latent heat and the difference between the thermal diffusivity in solid phase and that in liquid phase are not able to ignore in a practical case in order to discuss the details of the fusion boundary and the temperature distribution.

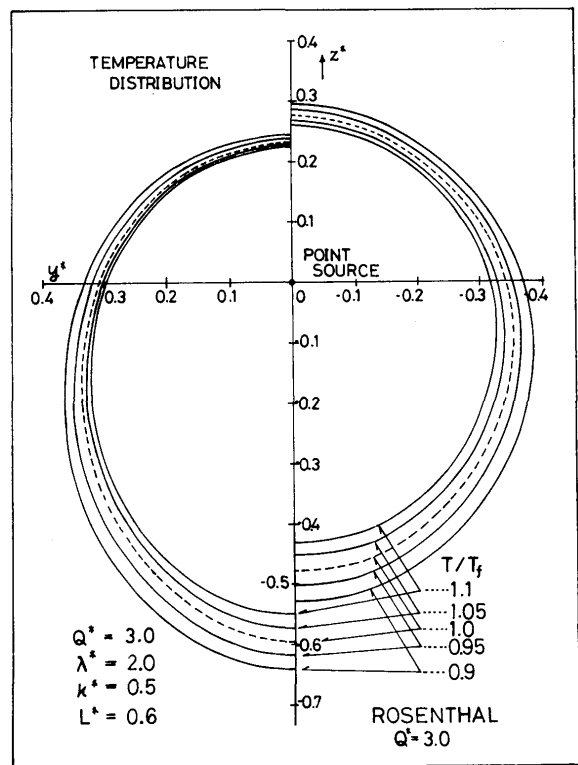


Fig. 5 Comparison between the fusion boundary and isothermal surfaces, which are shown by dashed line and solid ones, respectively, in the case of  $Q^*=3.0$ ,  $L^*=0.6$ ,  $\lambda^*=2.0$  and  $\kappa^*=0.5$  and that in the case of  $Q^*=3.0$ ,  $L^*=0.0$ ,  $\lambda^*=1.0$  and  $\kappa^*=1.0$ .

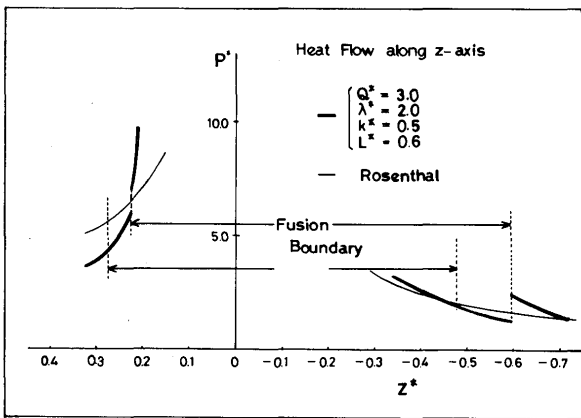


Fig. 6 Heat flux along  $z$ -axis in the neighborhood of the fusion boundary.  $P^*$  represents absolute value of normalized heat flux.

The change of heat flux  $P^*$  along the  $z$ -axis is shown in Fig. 6, where the solid and dashed lines correspond to the temperature distributions shown in left and right half in Fig. 5, respectively. The discontinuous changes of the heat flux appear at the fusion boundary. At  $z^* \approx 0.23$ , that is the front surface, the heat flux toward the fusion boundary is partly absorbed as the latent heat of fusion so that the heat flux from the boundary is smaller than that toward the boundary. On the otherhand, at  $z^* \approx 0.6$ , that is the rear surface, the heat flux from the boundary is larger than that toward the boundary because of the emission of latent heat of solidification. Since the heat flux is proportional to the temperature gradient and the product of the temperature gradient and the velocity of point source is the heating or cooling rate of the media, then we can see from Fig. 9 that the cooling rate just after the solidification is underestimated to some extent when the latent heat is ignored.

### 4.3 Comparison of theoretical results with experimental results

Figure 7, 8 and 9 show the normalized length of molten pool, the normalized width of the molten pool and the normalized length of the weld ripple lag as functions of operating parameter  $n = Q^*/2\pi$  in the case of mild-steel. The points shown in those figures are quoted from the experimental data of TIG welding by Christensen et al.. For comparison, the results of Rosenthal's analysis are also shown. The pool width according to the present theoretical curve and Rosenthal's analysis agrees well and they are represented a single curve. The experimental plots fit well the curve. As to the pool length, the results of the present analysis is closer to the experimental results than those of

Rosenthal's analysis, qualitatively, but appreciable differences between the computed and experimental results are found. Experimental results of weld ripple lag length are longer than the theoretical ones.

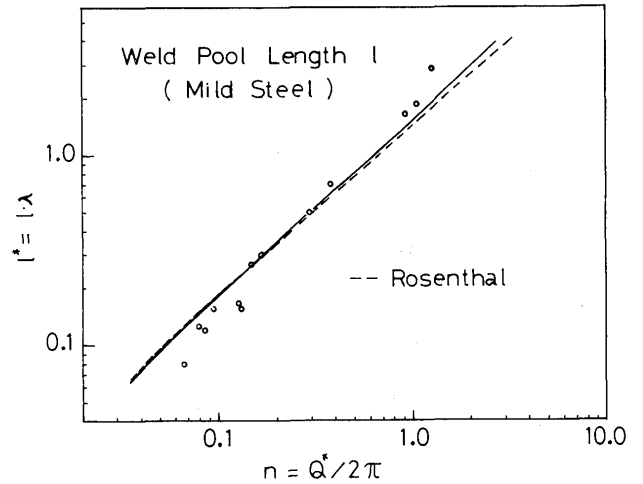


Fig. 7 Comparison between computed and experimental molten pool length.

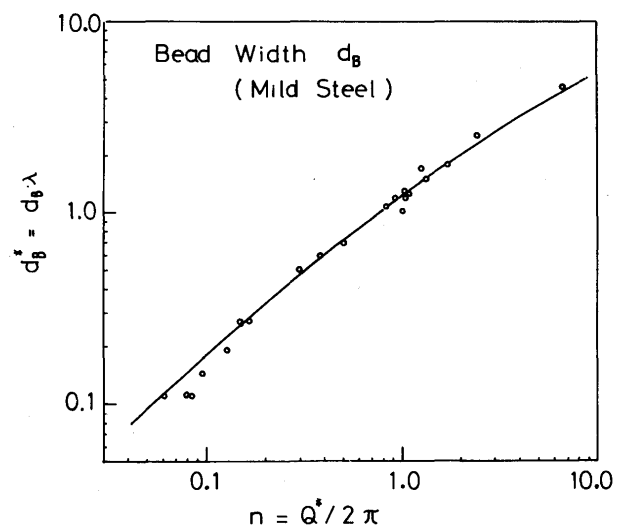


Fig. 8 Comparison between the computed and experimental width of the fusion boundary versus operating parameter  $n = Q^*/2\pi$ . Experimental plots are quoted from Christensen's data.

In the actual process of welding, the fluid dynamical motion of the molten metal in the pool caused by the arc blow, the gravity and the surface tension give rise to the convective heat transfer in the molten pool other than conduction heat transfer. Therefore, we can consider that the difference between the computed and experimental results are ascribed largely to the effect of the convective heat transfer in the molten pool.

In order to take into account the effect of the convective heat transfer within the limit of the present theory, it is necessary to introduce a hypothetical heat

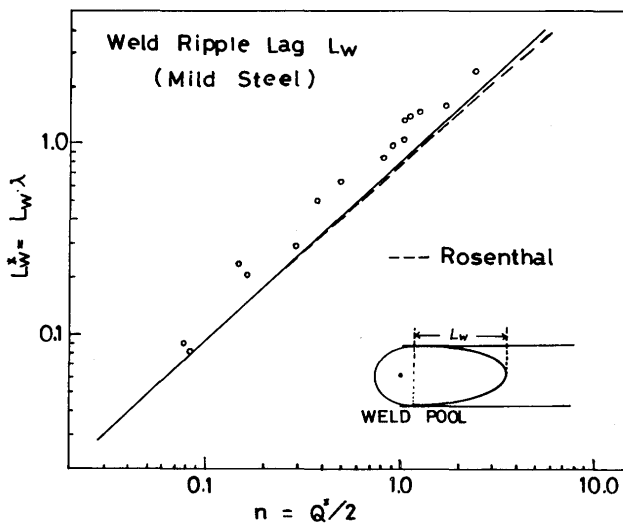


Fig. 9 Comparison between computed and experimental weld ripple lag.

source which is suitably distributed in the molten pool, instead of the real source. Furthermore, we should establish the method to simulate the transfer phenomena of the heat and mass on welding, in order to solve the problem thoroughly.

## 5. Conclusions

Shape of steady state fusion boundary and temperature distribution around a point source moving with a constant speed on a thick plate are calculated theoret-

tically by solving the free boundary problem of heat conduction where the release and absorption of latent heat and the difference between the parameters showing thermal property of material in solid phase and that in liquid phase are taken into account.

In a typical case, when we ignore the latent heat and the difference of the thermal diffusivity, the fusion boundary displaces forward and the pool length becomes about 10% shorter.

In the case of welding of mild steel, the experimental data of pool width agree with the theoretical ones. But the ripple lag length and pool length obtained by experiment are longer than those obtained by our theory. This is considered to be attributed to convective heat transfer in molten pool due to fluid dynamical motion of molten metal.

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