Transportation Research Interdisciplinary Perspectives 34 (2025) 101732

Contents lists available at ScienceDirect

Transportation Research Interdisciplinary Perspectives

journal homepage: www.elsevier.com/locate/trip

Continuous multi-dimensional deterioration process model for highway pavement management planning

Kotaro Sasai *, Felix Obunguta, Manish Man Shakya, Kiyoyuki Kaito

Department of Civil Engineering, Division of Global Architecture, Graduate School of Engineering, The University of Osaka, Japan

ARTICLE INFO

Keywords:
Deterioration prediction model
Copula
Pavement management
Infrastructure planning

ABSTRACT

For maintenance, repair, and rehabilitation of highway pavements, implementing efficient repairs to extend pavement service life can be achieved through proper actions on various types of deterioration process. In this study, a multi-dimensional deterioration process model using continuous quantities is developed considering the interaction between pavement surface and load-bearing capacity deterioration. The proposed model contributes to formulating preventive maintenance repair plans cognizant of this interaction. Specifically, progression of pavement deterioration is modeled by the continuous deterioration hazard model to estimate the heterogeneity that represents the deterioration rate specific to each pavement deterioration process; i.e., surface and sub-surface deterioration. Furthermore, the developed multi-dimensional deterioration process model expresses the correlation of the heterogeneity parameters of the interacting deterioration processes using an Archimedian copula and is used to estimate the progression of each deterioration process. The characteristics of each deteriorating process are discussed from the viewpoint of timely repair planning. Finally, the proposed model is verified through an application case study using actual highways inspection data.

1. Introduction

Efficient pavement repair is advocated to extend the service life of pavements for which the introduction of preventive maintenance strategies is important. In the maintenance and management of highway pavements, it is necessary to consider the performance of pavements in terms of safety and driving comfort. Adey et al. (2020) lists important considerations for road service that include comfort and safety; so road managers should develop new and improve existing strategies to better pavement condition. The road surface condition indices including cracking rate, rutting depth, and International Roughness Index (IRI), may be regularly obtained through road condition surveys in countries with the economic and human resources to do so. Pavement condition inspection data can be used to estimate deterioration rates and develop plans for maintenance, repair and rehabilitation of highway pavements. In addition, pavement sub-surface conditions may be known through estimating the deflection rate using the Falling Weight Deflectometer (FWD). The FWD survey can be used to estimate pavement load-bearing capacity by measuring its deflection rate at specific locations. Pavement surface conditions such as cracking, rutting, and roughness, and sub-surface conditions like load-bearing capacity derived from FWD tests, are inherently interrelated (e.g., Nakamura et al. (2022)). Understanding and quantifying these interrelationships is essential for comprehensive pavement deterioration modeling.

In road asset management, various statistical approaches have been developed to predict infrastructure deterioration using inspection data. While a range of deterioration modeling frameworks exist, the methodology proposed in this study is rooted in the Markov model, which has served as a foundational approach in the field. Among these, the Markov deterioration hazard model developed by Tsuda et al. (2006) has been widely applied due to its intuitive formulation and compatibility with condition-state-based inspection data. This family of models has been further extended—for example, into the mixed Markov deterioration hazard model that enables quantification of heterogeneity for each evaluation unit (Obama et al., 2008), and the hidden Markov model that handles incomplete monitoring data for latent deterioration processes (Lethanh and Adey, 2012). While the Markov framework is commonly used and forms a historical foundation for deterioration modeling, it has limitations in capturing continuous deterioration processes and the interdependence among multiple deterioration indicators. To overcome these limitations, the present study adopts a modeling approach based on continuous quantities and explicitly incorporates dependency structures using copula functions.

* Corresponding author.
E-mail addresses: k.sasai@civil.eng.osaka-u.ac.jp (K. Sasai), f.obunguta@civil.eng.osaka-u.ac.jp (F. Obunguta), shakya.m@civil.eng.osaka-u.ac.jp (M.M. Shakya), kaito@ga.eng.osaka-u.ac.jp (K. Kaito).

https://doi.org/10.1016/j.trip.2025.101732
Received 19 January 2025; Received in revised form 21 September 2025; Accepted 4 November 2025
Available online 27 November 2025

Conventional Markov models often focus on a single deterioration indicator and are limited in their ability to represent the statistical dependency among multiple deterioration processes. Such models do not adequately capture the joint progression of various types of pavement degradation – such as surface cracking, rutting, and sub-surface structural weakening – which may evolve simultaneously and influence one another. To overcome these limitations, this study proposes a modeling framework that describes the entire deterioration process using multiple continuous condition indices.

In this framework, the deterioration of each indicator is modeled using a continuous quantity hazard model that accounts for heterogeneity across pavement sections. To represent the dependency structure among these indicators, copula functions are introduced. Copulas are particularly suitable for this application because they allow for flexible modeling of the joint distribution of multiple random variables, while preserving the marginal distributions of each deterioration process. This enables the characterization of interdependence among indicators, even when the data are only partially observed or exhibit asymmetric relationships. By combining continuous hazard models with copula-based dependency modeling, the proposed method facilitates multi-dimensional assessment of pavement condition and supports improved planning for maintenance, repair, and rehabilitation strategies.

The rest of the paper is organized as follows. The next section describes the research methodology followed by a description of the basic concept of the study. The subsequent section details the proposed multidimensional deterioration process model using continuous quantities and expression of the correlation among deterioration events using copulas. Lastly, the applicability of the multidimensional deterioration process model using continuous quantities is demonstrated through a case study on highway pavements and the conclusion and future research work are stated.

2. Basic concept of this study

2.1. Multi-dimensional evaluation of structural deterioration rate

It is common practice in infrastructure management to express deterioration processes using a single structural health monitoring index. In pavement management specifically, two approaches are typically employed: (1) aggregating various distress indicators such as cracking, rutting, and roughness into a composite index and modeling its temporal progression, and (2) modeling the progression of each distress indicator individually over time. Both approaches have their own strengths and are widely recognized in practice. However, since infrastructure deterioration often involves multiple interacting mechanisms, a modeling framework capable of jointly considering multiple deterioration indicators may provide additional insights.

Inspection data for a single deterioration process may not be sufficient to model the entire infrastructure deterioration progression. Moreover, the inspection data observed in reality contains a mixture of such multiple cases. For this reason, when using data synthesized as a single indicator, it is difficult to determine which indicator best represents the deterioration process. In addition, the deterioration process model using a single deterioration event does not provide information on the heterogeneity of deterioration processes for each infrastructure facility and the correlation between deterioration events.

The term "entire infrastructure" refers to the overall condition of a facility as assessed by considering multiple deterioration indicators and members jointly, providing a comprehensive view of its performance. In contrast, when referring to "each infrastructure facility" in the single-indicator context, it denotes the same physical unit (e.g., a homogeneous pavement section) but evaluated separately for each indicator.

The multi-dimensional deterioration process model proposed in this study is applicable to the case where the deterioration management index of a structure is expressed as a continuous quantity and takes into account the heterogeneity of the deterioration processes in question. The continuous deterioration hazard model uses a baseline model as a benchmark case. The heterogeneity of the category is represented by the heterogeneity parameter ε that is distributed according to a probability density function $p(\varepsilon)$ which makes it possible to evaluate the relative deterioration rates of individual structure categories. $\varepsilon = 0$ corresponds to the benchmark, while $\varepsilon > 0$ is a structure that deteriorates relatively faster than the benchmark, and $\varepsilon < 0$ is a structure that deteriorates relatively slower than the benchmark. Please note that the deterioration speed refers to the overall deterioration tendency of a facility category, aggregated from the deterioration rates of its constituent sections.

The heterogeneity parameter introduced in this study captures the variability observed in pavement deterioration across different pavement sections. This variability naturally arises from differences in factors such as construction quality, material characteristics, environmental exposure, traffic loads, and maintenance practices. Explicitly modeling this heterogeneity is crucial because pavement sections do not deteriorate uniformly, and neglecting this variability could result in less accurate deterioration predictions.

2.2. Modeling the correlation structure using copulas

When evaluating the deterioration rate of infrastructure facilities using multiple deterioration management indices, the heterogeneity parameter, which represents the heterogeneity of the deterioration rate for each degradation process may show the correlation between the processes. In this study, multivariate copulas are used to express the dependency structure among event multivariate marginal distributions (see e.g., Nelsen (1999) for the use of copulas). The copulas are used to merge the marginal distribution functions of multiple random variables with their joint distribution functions. This study is unique in that joint distribution functions can be estimated while maintaining the probability structure of the multivariate marginal distributions.

Consider a case in which two types of deterioration events, A and B, are used to evaluate the deterioration state of multiple infrastructure facilities. The relationship between copulas, heterogeneity, marginal and joint distributions of parameters is shown schematically in the three-dimensional space (Fig. 1). The vertical planes in Fig. 1 show the marginal distribution functions of each heterogeneity parameter, allowing for relative evaluation of the deterioration rate of each event. The upper horizontal plane in Fig. 1 shows the joint distribution of the heterogeneity parameter pairs (ε_A, ε_B) plotted in a two-dimensional space.

The information on the spatial distribution state of the heterogeneity parameters provides transition trends based on correlations among the heterogeneity parameters, as well as deterioration rates for individual deterioration events. For example, if two types of deterioration events are considered, the total deterioration rate for the two types of events can be expressed in terms of the rate of deterioration of each event. Additionally, in the two-dimensional space for two types of deterioration events, the deterioration characteristics of a structure can be classified into four categories in each quadrant. In the first quadrant ($\varepsilon_A > 0$ and $\varepsilon_B > 0$), the two deterioration events are both relatively faster than in the benchmark case. The second ($\varepsilon_A > 0$ and $\varepsilon_B < 0$) and fourth ($\varepsilon_A < 0$ and $\varepsilon_B > 0$) quadrants correspond to the progression of either deterioration events A or B is superior to that of the benchmarking case, while the other is inferior, respectively. The third ($\varepsilon_A < 0$ and $\varepsilon_B < 0$) quadrant can be evaluated as the progression of both events being slower than that of the benchmarking case. Moreover, the conditional probability density of a heterogeneity parameter, given that the other heterogeneity parameter is known, can be calculated using a copula and marginal distributions of the heterogeneity parameters as shown in the lower horizontal plane in Fig. 1.

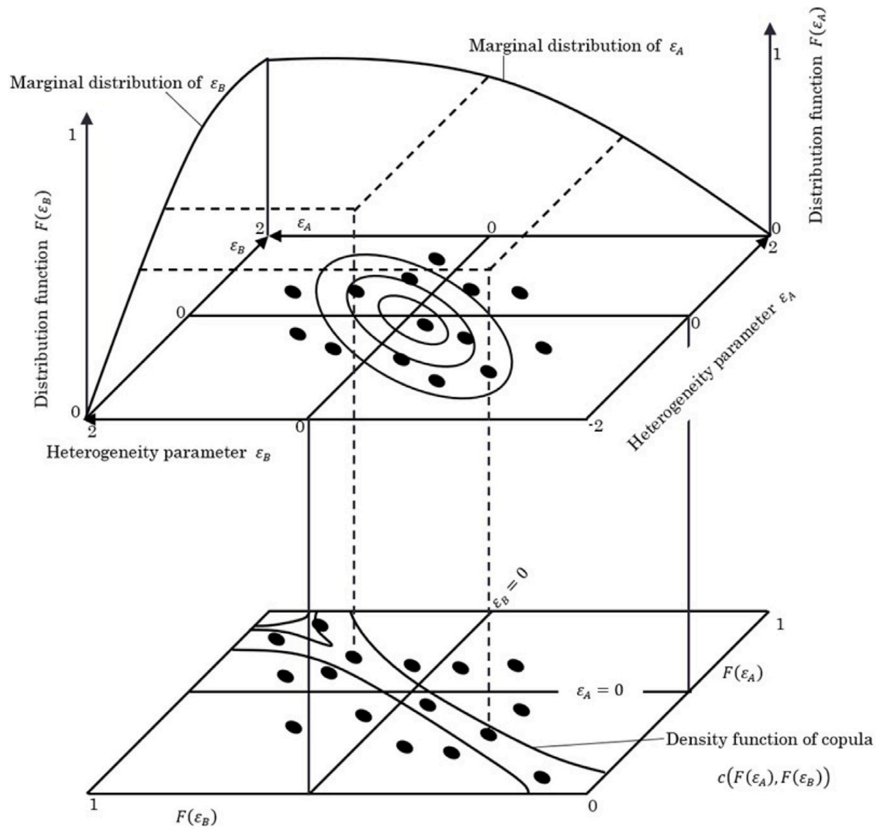


Fig. 1. Joint distribution of heterogeneity parameters and copula.

3. Multi-dimensional deterioration process model with continuous quantities

3.1. Continuous quantity deterioration hazard model

For each multidimensional deterioration event d ($d = 1, \dots, D$), the deterioration management index of facility i ($i = 1, \dots, I$) is denoted as x^i_d and the elapsed time since the most recent construction (renewal) as t^i. The more the deterioration progresses, the larger the value of the deterioration indicator becomes. The progression of the deterioration process is formulated as

$$x^i_d = \exp(-B^i_d) f_d(t^i, \boldsymbol{\beta}_d) \tag{1a}$$

$$B^i_d = \mathbf{z}^i \boldsymbol{\theta}'_d - \varepsilon^k_d + \sigma_d w^i_d \tag{1b}$$

where B^i_d is the deterioration characteristic coefficient of facility i, which can be expressed using the characteristic variable term $\mathbf{z}^i \boldsymbol{\theta}'_d$, the heterogeneity term ε^k_d and the error term $\sigma_d w^i_d$ as shown in Eq. (1b). In Eq. (1b), $\mathbf{z}^i = (z^{i,1}, \dots, z^{i,M})$ is the characteristic variable vector affecting the deterioration of facility i, $\boldsymbol{\theta}_d = (\theta^1_d, \dots, \theta^M_d)$ is the characteristic parameter vector, ε^k_d is the heterogeneity parameter expressing the unique deterioration rate in the group k ($k = 1, \dots, K$) to which facility i belongs, w^i_d is the random error term expressing the deterioration factor unique to facility i, and σ_d is the deviation parameter. The heterogeneity parameter ε^k_d is assumed to follow a normal distribution with mean 0 and variance ϕ^2_d. to reflect symmetric variability around the mean deterioration rate, without imposing skewness toward faster or slower deterioration. Also, $f_d(t^i, \boldsymbol{\beta}_d)$ is a deterioration model that represents the baseline deterioration process (hereafter, the baseline model) and is a monotonically increasing function with respect to t^i. Also, $\boldsymbol{\beta}_d = (\beta^1_d, \dots, \beta^N_d)$ is the unknown parameter vector that characterizes the baseline model. If $\exp(-B^i_d) = 1$ holds, the deterioration curve is identical to the baseline model. If the theoretical deterioration curve can be derived from a kinematic model, then it can be used as the baseline model. If no theoretical model exists, it is necessary to approximate the baseline model using, for example, a flexible function. By taking the logarithm on both sides of Eq. (1a),

$$\begin{aligned} y^i_d &= \ln f_d(t_i, \boldsymbol{\beta}_d) \\ &= \ln x^i_d + \mathbf{z}^i_d \theta'_d - \varepsilon^i_d + \sigma_d w^i_d \end{aligned} \tag{2}$$

is obtained where $y^i_d = \ln f_d(t^i, \boldsymbol{\beta}_d)$ is the non-linearized lifetime index. The stochastic error term w^i_d is assumed to follow the probability density function of the standard Gumbel distribution expressed as

$$g_w(w^i_d) = \exp\{-w^i_d - \exp(-w^i_d)\} \tag{3}$$

where $E(w^i_d) = \gamma = 0.57722\cdots$ is an Euler constant.

Rewrite Eq. (2) as

$$w^i_d = \frac{y^i_d - \ln x^i_d - \mathbf{z}^i \boldsymbol{\theta}'_d + \varepsilon^k_d}{\sigma_d} \tag{4}$$

and perform a variable transformation of the probability density function Eq. (3). As a result, the probability density function representing the conditional distribution of the lifetime index y^i_d until the deterioration management index value x^i_d is reached for facility i with deterioration characteristic $\mathbf{z}^i$ is expressed as

$$\begin{aligned} q_y\left(y^i_d \mid x^i_d, \boldsymbol{z}^i_d\right) &= \frac{1}{\sigma_d} g_w\left(\frac{y^i_d - \ln x^i_d - \boldsymbol{z}^i_d \boldsymbol{\theta}'_d + \varepsilon^i_d}{\sigma_d}\right) \\ &= \frac{1}{\sigma_d} \exp\left\{ - \exp\left(-\frac{y^i_d - \ln x^i_d - \boldsymbol{z}^i_d \boldsymbol{\theta}'_d + \varepsilon^i_d}{\sigma_d}\right) \right. \\ &\quad \left. - \frac{y^i_d - \ln x^i_d - \boldsymbol{z}^i_d \boldsymbol{\theta}'_d + \varepsilon^i_d}{\sigma_d} \right\} \end{aligned} \tag{5}$$

Table 1
Selected Archimedian copulas.

Copula	Generating function $\zeta(u_d)$	Distribution function $C(u_1,\dots,u_D)$	Probability density function $c(u_1,\dots,u_D)=\frac{\partial^D C(u_1,\dots,u_D)}{\partial u_1 \cdots \partial u_D}$
Gumbel $\alpha \in (1,\infty)$	$(-\ln u_d)^\alpha$	$\exp[-\{\sum_{d=1}^{D}(-\ln u_d)^\alpha\}^{\frac{1}{\alpha}}]$	Partial differentiation of the distribution function at any time since there is no general form
Clayton $\alpha \in (0,\infty)$	$\frac{1}{\alpha}(u_d^{-\alpha}-1)$	$(\sum_{d=1}^{D} u_d^{-\alpha} - D + 1)^{-\frac{1}{\alpha}}$	$\{\prod_{d=1}^{D-1}(1+d\alpha)\}(\prod_{d=1}^{D} u_d^{-\alpha-1})(\sum_{d=1}^{D} u_d^{-\alpha} - D + 1)^{-\frac{1}{\alpha}-D}$
Frank $\alpha \in (0,\infty)$	$\ln\{\exp(-\alpha u_d)-1\} - \ln\{\exp(-\alpha)-1\}$	$-\frac{1}{\alpha}\ln\left[1+\frac{\prod_{d=1}^{D}\{\exp(-\alpha u_d)-1\}}{\{\exp(-\alpha)-1\}^{D-1}}\right]$	Partial differentiation of the distribution function at any time since there is no general form

The lifetime index $y_d^i = \ln f_d(t^i, \boldsymbol{\beta}_d)$ contains an unknown parameter $\boldsymbol{\beta}_d$. If we denote the first derivative of the lifetime index as $\dot{f}_d(t^i, \boldsymbol{\beta}_d) = d f_d(t^i, \boldsymbol{\beta}_d)/dt^i$, then

$$dy_d^i = \frac{\dot{f}_d(t^i, \boldsymbol{\beta}_d)}{f_d(t^i, \boldsymbol{\beta}_d)} dt^i \tag{6}$$

is satisfied. Thus, the probability density function representing the conditional distribution of real life t^i until the control level x_d^i is reached is

$$\tau(t^i | x_d^i, \mathbf{z}^i) = \frac{\dot{f}_d(t^i)}{\sigma_d f_d(t^i)} \cdot \exp\left\{ -\exp\left(-\frac{\ln f_d(t^i) - \ln x_d^i - \mathbf{z}^i \boldsymbol{\theta}'_d + \varepsilon_d^k}{\sigma_d} \right) - \frac{\ln f_d(t^i) - \ln x_d^i - \mathbf{z}^i \boldsymbol{\theta}'_d + \varepsilon_d^k}{\sigma_d} \right\} \tag{7}$$

From Eq. (3), the survival function is expressed as

$$\begin{aligned} S_w(w_d^i) &= 1 - \int_{-\infty}^{w_d^i} g_w(w_d^i) dw \\ &= 1 - \exp\{-\exp(-w_d^i)\} \end{aligned} \tag{8}$$

For a facility i with deterioration characteristic $\mathbf{z}^i$, the probability that the deterioration control index value has not reached x_d^i when the lifetime index y_d^i has passed can be expressed using the survival function

$$\begin{aligned} S_y(y_d^i | x_d^i, \mathbf{z}^i) &= S_w\left(\frac{y_d^i - \ln x_d^i - \mathbf{z}^i \boldsymbol{\theta}'_d + \varepsilon_d^k}{\sigma_d} \right) \\ &= 1 - \exp\left\{ -\exp\left(-\frac{y_d^i - \ln x_d^i - \mathbf{z}^i \boldsymbol{\theta}'_d + \varepsilon_d^k}{\sigma_d} \right) \right\} \end{aligned} \tag{9}$$

Furthermore, the survival function with respect to the real elapsed time t^i is defined as

$$S_t(t^i | x_d^i, \mathbf{z}^i) = 1 - \exp\left\{ -\exp\left(-\frac{\ln f_d(t^i) - \ln x_d^i - \mathbf{z}^i \boldsymbol{\theta}'_d + \varepsilon_d^k}{\sigma_d} \right) \right\} \tag{10}$$

Thus, a survival function Eq. (10) can be derived for the continuous quantity deterioration hazard model.

3.2. Correlation structure of multi-dimensional deterioration events

The joint probability distribution of the heterogeneity parameters ε_d^k for $d(=1,\dots,D)$ types of deterioration events in a certain category $k(=1,\dots,K)$ is represented using a copula C. "Facility categories" refers to groups of infrastructure units that share similar functional or structural characteristics and are therefore expected to exhibit comparable deterioration patterns. An overview of copulas is given below. For a detailed review of copulas, please refer to Nelsen (1999) and Joe (1997, 2014).

Let $P(\varepsilon_1,\dots,\varepsilon_D)$ be the continuous joint distribution function of D random variables $\varepsilon_1,\dots,\varepsilon_D$ with marginal distributions $P_1,\dots,P_D$, then from Sklar's Theorem (Sklar, 1973), there exists a copula C uniquely satisfying

$$P(\varepsilon_1,\dots,\varepsilon_D) = C\left(P_1\left(\varepsilon_1\right),\dots,P_D\left(\varepsilon_D\right)\right) \tag{11}$$

The $C(P_1(\varepsilon_1),\dots,P_D(\varepsilon_D))$ generated by applying the marginal distributions $P_1(\varepsilon_1),\dots,P_D(\varepsilon_D)$ to the copula C is a joint distribution function with the marginal distributions in the interval $[0,1]$.

In addition,

- For any $u_d = P_d(\varepsilon_d) \in [0,1]$ $(d=1,\dots,D)$:

$$C(u_1,\dots,u_{d-1},0,u_{d+1},\dots,u_D) = 0 \tag{12}$$

- For any $u_d = P_d(\varepsilon_d) \in [0,1]$ $(d=1,\dots,D)$:

$$C(1,\dots,1,u_d,1,\dots,1) = u_d \tag{13}$$

- For all $(u_1^1,\dots,u_D^1)$, $(u_1^2,\dots,u_D^2) \in [0,1]^D$ satisfying $u_d^1 \le u_d^2$:

$$\sum_{i_1=1}^{2} \cdots \sum_{i_D=1}^{2} (-1)^{\sum_{s=1}^{D} i_s} C(u_1^{i_1},\dots,u_D^{i_D}) \ge 0 \tag{14}$$

A function C in which all the three properties described above are satisfied is defined as a copula. In this case, the joint probability density function $p(\varepsilon)$ of the heterogeneity parameter vector $\varepsilon_d = (\varepsilon_d^1,\dots,\varepsilon_d^K)$ of individual deterioration events is the copula distribution function $C(P_1(\varepsilon_1),\dots,P_D(\varepsilon_D))$ or probability density function $c(P_1(\varepsilon_1),\dots,P_D(\varepsilon_D))$ can be expressed as

$$\begin{aligned} p(\varepsilon) &= \frac{\partial^D C(P_1(\varepsilon_1),\dots,P_D(\varepsilon_D))}{\partial P_1(\varepsilon_1)\cdots\partial P_D(\varepsilon_D)} \prod_{d=1}^{D} p_d(\varepsilon_d) \\ &= c(P_1(\varepsilon_1),\dots,P_D(\varepsilon_D)) \prod_{d=1}^{D} p_d(\varepsilon_d) \end{aligned} \tag{15}$$

where $\varepsilon = (\varepsilon_1,\dots,\varepsilon_D)$ holds. The probability density function p_d of the marginal distribution P_d follows a normal distribution

$$p_d(\varepsilon_d) = \frac{1}{\sqrt{2\pi\phi_d^2}} \exp\left(-\frac{{\varepsilon_d}^2}{2\phi_d^2} \right) \tag{16}$$

Various copulas have been proposed to represent joint probability distributions using information about the marginal distribution. This study, employs the one-parameter Archimedian copula model (Genest and Rivest, 1993). The model structure has been widely used especially in financial fields due partly to its well-known random number generation method and subsequent practicality. The distribution function $C(u_1,\dots,u_D)$ of the one-parameter Archimedian copula among D variables whose marginal distribution functions are $P_1(x_1) = u_1,\dots,P_D(x_D) = u_D$ can be expressed using the generating function $\zeta(u_d)$ as

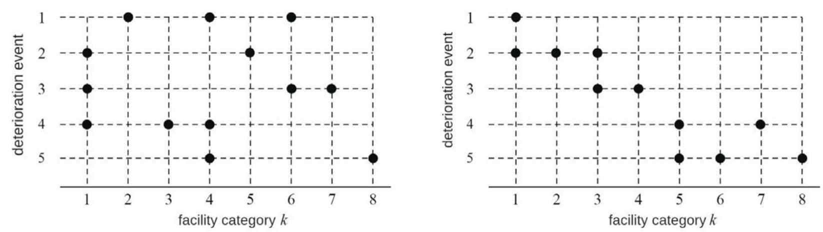
$$C(u_1,\dots,u_D) = \zeta^{-1}\left(\sum_{d=1}^{D} \zeta(u_d) \right) \tag{17}$$

For the Archimedian copula, the following property holds

$$C(1,\dots,1,u_{d_1},1,\dots,1,u_{d_2},1,\dots,1) = C(u_{d_1},u_{d_2}) \tag{18}$$

In this study, three types of Archimedian copulas were considered: the Gumbel copula (Gumbel, 1960), the Clayton copula (Clayton, 1978), and the Frank copula (Frank, 1979). Table 1 shows the generating function, distribution function, and probability density function for the Gumbel, Clayton and Frank copulas. The multivariate joint probability density functions of the Gumbel and Frank copulas are difficult to express in general form and are obtained by partial differentiation of the distribution functions as needed, depending on the number of variables.

In addition, the parameters of the Gumbel copula satisfy $\alpha \in (1,\infty)$ and those of the Clayton and Frank copulas satisfy $\alpha \in (0,\infty)$. In order



(a) Acquisition pattern 1. (b) Acquisition pattern 2.

Fig. 2. Data acquisition patterns of a certain facility category k.

to select an appropriate copula, Breymann et al. (2003) proposed a copula selection method based on the information criterion considering the selection among copulas with different number of parameters. In this study, the Widely Applicable Information Criteria (WAIC) by Watanabe (2010), which has asymptotically the same expected value and variance as the generalization loss, was adopted as the information criterion for copula selection. The WAIC is suitable as an evaluation index for statistical models with complex structures including irregular models.

3.3. Observability of deterioration events and connectivity conditions

Deterioration state of a facility may be evaluated by different types of management indices for deterioration. However, it is not always possible to obtain data on the indices at the same inspection period. Furthermore, depending on facilities, it may not be possible to acquire inspection data on all deterioration events. Due to the partial observable nature of such multidimensional deterioration events, it is impossible to evaluate multidimensional deterioration events using a single aggregated deterioration management index. It may however be considered that a correlation exists between the different indices derived from survey data. Therefore, in this study, the entire multidimensional deterioration process is estimated using partially observed information obtained from different deterioration processes.

In formulating the multi-dimensional model by estimating the unknown parameters of copulas, the concurrent observability of deterioration event data in relation to categories must be considered. For instance, consider 8 facility categories $k = 1, \dots, 8$ and 5 deterioration events $d = 1, \dots, 5$ as shown in Fig. 2. A black circle in the figure signifies where a deterioration event index has been measured. In Fig. 2(a), deterioration events 2, 3, and 4 are observed for the category 1, and events 1, 4, and 5 are observed for the category 4. In this example, by using the inspection data for the category 1 and 4, the correlation structure among all deterioration events $1, \dots, 5$ can be estimated using copulas. In this case, the deterioration events are said to satisfy the connectivity condition. In Fig. 2(b), there is no category in which the first three events (1, 2, and 3) and the last two events (4 and 5) are observed at the same time. In this case, the connectivity condition is not satisfied. Since copulas represent the distribution of the concurrent occurrences of heterogeneity parameters, it is desirable that the observed data satisfy the connectivity condition in order to minimize the estimation bias of copulas. In this research, the correlations among deterioration events for each category are expressed using copulas assuming that all events are connectable.

Let ω^k denote deterioration events observed for the category $k (k = 1, 2, \dots, K)$. For instance, in Fig. 2(a), deterioration events $2, 3, 4$ are observed in category 1, and therefore $\omega^1 = \{2, 3, 4\}$. For the category 2, only event 1 can be observed, and the deterioration event group can be expressed as $\omega^2 = \{1\}$. For arbitrary deterioration events d, d' $(d, d' = 1, \dots, D)$, the dummy variable $\delta_{d,d'}$ is defined as

$$\delta_{d,d'} = \begin{cases} 1 & \text{if } k \text{ exists which satisfies } d, d' \in \omega^k \\ 0 & \text{Otherwise} \end{cases} \tag{19}$$

For a $D{\times}D$ matrix $\mathbf{H}$ whose dummy variable $\delta_{d,d'}$ is a (d, d') element, the following condition must be satisfied.

$$\times_D \mathbf{H} = \mathbf{1} \tag{20}$$

The symbol $\times_D$ denotes the Boolean operation of multiplying the matrix $\mathbf{H}$ by D times. The $\mathbf{1}$ is a $D \times D$-dimensional matrix with all elements being 1. The condition Eq. (20) is defined as the connectivity condition.

3.4. Partial observation results and joint probability density function

As elaborated above, for each category, correlations between deteriorating events are expressed using copulas. It is assumed that deterioration events can be linked to each other even if inspection data for all deteriorated events is not necessarily available for each category as illustrated in Fig. 2(a). Based on the set of observable deterioration events ω^k for a category k, the dummy variable δ_d^k representing the relationship between the category and each deterioration event is defined as

$$\delta_d^k = \begin{cases} 1 & \text{for } d \in \omega^k \\ 0 & \text{for } d \notin \omega^k \end{cases} \tag{21}$$

The heterogeneity parameter vector corresponding to observable deterioration events in category k is expressed as $\hat{\varepsilon}^k = \{P_1(\varepsilon_1^k)^{\delta_1^k}, \dots, P_D(\varepsilon_D^k)^{\delta_D^k}\}$. In case $\delta_d^k = 0$, $P_d(\varepsilon_d^k)^{\delta_d^k} = 1$ should be satisfied. The copula distribution function $\tilde{C}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k))$ expressing correlations between marginal distributions of partial heterogeneity parameter vectors $\hat{\varepsilon}^k$ is

$$\tilde{C}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k)) = C(P_1(\varepsilon_1^k)^{\delta_1^k}, \dots, P_D(\varepsilon_D^k)^{\delta_D^k}) \tag{22}$$

From the one-parameter Archimedean copula property, Eqs. (18), (22) represents the copula's marginal distribution function for the heterogeneity parameter focusing only on deterioration events in the set ω^k. Let R^k deterioration events belonging to the set ω^k of observable events in the category k be denoted by $\hat{d}_{r^k}^k$ $(r^k = 1, \dots, R^k)$ respectively. The copula probability density function $\tilde{c}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k))$ for each category k is

$$\tilde{c}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k)) = \frac{\partial^{R^k} \tilde{C}(\hat{\mathbf{P}}^k(\varepsilon^k))}{\partial \hat{P}_{\hat{d}_1^k}(\varepsilon_{\hat{d}_1}^k) \cdots \partial \hat{P}_{\hat{d}_{R^k}^k}(\varepsilon_{\hat{d}_{R^k}}^k)} \tag{23}$$

The joint probability density function $\hat{p}^k(\hat{\varepsilon}^k)$ of the heterogeneity parameter vector $\hat{\varepsilon}^k$ for the category k can be expressed, using the copula's probability density function $\tilde{c}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k))$, marginal probability density function $p_d(\varepsilon_d^k)$, and dummy variable δ_d^k, as

$$\hat{p}^k(\hat{\varepsilon}^k) = \tilde{c}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k)) \cdot \prod_{d=1}^{D} \{p_d(\varepsilon_d^k)\}^{\delta_d^k} \tag{24}$$

Using the joint probability density function Eq. (24), even if there is no observed data for a certain deterioration event d' in a certain category k, given that the inspection data for the remaining deterioration events d is acquired, the marginal distribution of the heterogeneity parameter can be estimated.

3.5. Likelihood function

Consider that several deterioration management indices for each deterioration event are available for a structure i^k ($i^k = 1, \ldots, I^k$) in category k. The inspection data for deterioration event d of structure i^k belonging to category k is defined as $\xi_d^{i^k} = (\bar{\mathbf{x}}_d^{i^k}, \bar{\mathbf{t}}_d^{i^k}, \bar{\mathbf{z}}_d^k)$, and the total visual inspection data collected in $\bar{\Xi}$. The symbol "‾" denotes measured values. Let $\xi^{i^k} = \{\xi_d^{i^k} : d \in \omega^k\}$ specify that the data satisfies the connectivity condition. The likelihood of observing the inspection data can be expressed with the conditional probability density function of the structure's real life and the joint probability density function of the heterogeneity parameters as

$$\mathcal{L}(\lambda|\xi^{i^k}) = \tilde{c}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k)) \prod_{d=1}^{D} \prod_{k=1}^{K} \prod_{i^k=1}^{I^k} \left\{ \tau(\xi^{i^k}|\beta_d, \theta_d, \varepsilon_d^k, \sigma_d) p_d(\varepsilon_d^k) \right\}^{\delta_d^k} \tag{25}$$

where $\lambda = (\boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \boldsymbol{\phi}, \alpha)$ is the parameter vector and

$$\tau(\xi^{i^k}|\beta_d, \theta_d, \varepsilon_d^k, \sigma_d) = \frac{\dot{f}_d(\bar{t}^i)}{\sigma_d f_d(\bar{t}^i)} \cdot \exp\left\{ -\exp\left(-\frac{\ln f_d(\bar{t}^i) - \ln \bar{x}_d^i - \bar{\mathbf{z}}^i \boldsymbol{\theta}_d' + \varepsilon_d^k}{\sigma_d} \right) - \frac{\ln f_d(\bar{t}^i) - \ln \bar{x}_d^i - \bar{\mathbf{z}}^i \boldsymbol{\theta}_d' + \varepsilon_d^k}{\sigma_d} \right\} \tag{26}$$

The likelihood $\mathcal{L}(\lambda|\bar{\Xi})$ of observing the data set $\bar{\Xi}$ is

$$\mathcal{L}(\lambda|\bar{\Xi}) = \prod_{d=1}^{D} \prod_{k=1}^{K} \left[\tilde{c}(\hat{\mathbf{P}}^k(\hat{\varepsilon}^k)) \cdot \prod_{d=1}^{D} \prod_{k=1}^{K} \prod_{i^k=1}^{I^k} \left\{ \tau(\xi^{i^k}|\beta_d, \theta_d, \varepsilon_d^k, \sigma_d) p_d(\varepsilon_d^k) \right\}^{\delta_d^k} \right] \tag{27}$$

The parameters in the defined likelihood function can be estimated using an iterative procedure such as the Markov Chain Monte Carlo (MCMC) method using the Metropolis Hastings (MH) algorithm.

4. Empirical analysis

4.1. Data summary

In the empirical application, the developed multidimensional deterioration process model using continuous quantities was applied to intercity highway pavements in Japan. Intercity highway refers to toll expressways administered by highway companies, which connect major cities and are designed for high-speed, long-distance travel. The road surface condition (cracking rate, rutting area, IRI) and load-bearing capacity data (D_{ind}) obtained from the road surface condition and FWD surveys conducted from 2006 to 2021 were used. The cracking rate is the percentage of the pavement surface area within a section that is visibly cracked. The dataset contains information on each location on the highway pavement, including (1) locational information such as route name, lane classification, and kilometer post, (2) records of repair, i.e, when the repairs were carried out and up to which layers the repairs were undertaken, and (3) pavement composition such as surface layer, base layer, upper roadbed, and lower roadbed type and thickness. Pavement's index for load bearing capacity is defined as

$$D_{ind} = \frac{D_0 - D_{90}}{A} \times 10^6 \tag{28}$$

D_{ind} is an index for evaluating pavement load-bearing capacity based on the estimated deflection in the asphalt layer (hereinafter called As layer), where D_0 is the deflection in mm at the loading point, D_{90} is the deflection in mm at a distance of 90 cm from the loading point, and A is the design thickness in mm of the As layer. The relation among the pavement structure and FWD measurements is illustrated in Fig. 3. D_0 refers to the deflection due to load propagated throughout the entire pavement structure. D_{90} is horizontally 90 cm away from the loading point and shows the deflection from the load propagated through the lower roadbed layer. The rationale behind (28) is that $D_0 - D_{90}$ determines the deflection from the load throughout the As layer. An original form of (28) was proposed by Araki et al. (2018) which was further modified by the authors to account for the effect of layer thickness. The estimated D_{ind} was also used as a measure of the subsurface strength of pavements in the empirical study.

Table 2
Copula candidates and WAIC.

Copula	D_{ind} - Crack rate	D_{ind} - Rutting	D_{ind} - IRI
Gumbel	18,651	16,751	15,993
Clayton	18,568	16,777	16,013
Frank	18,608	16,787	16,004

Pavement deterioration is a complex process consisting of road surface and load-bearing capacity deterioration, and prior research has attempted to examine their relation (e.g., Kaito et al. (2014) and Kamiya (2023)). It is considered that a correlation exists between the road surface and load-bearing capacity deterioration indices. The uncertainty of this correlation was expressed using copulas. In addition, since the FWD survey is a method to diagnose the pavement load-bearing capacity by dropping a weight on the pavement surface and measuring the amount of deflection caused by the weight, it is difficult to obtain load-bearing capacity indices over a wide area and at a high frequency because it may necessitate road traffic restriction which is undesirable for road users. On the other hand, the road surface condition surveys conducted using a moving vehicle do not require traffic restrictions. Thus, even when inspection results are only partially observed in some inspections (deterioration events), the multidimensional deterioration process model can be used to estimate the entire multidimensional deterioration process using consolidated data from multiple deterioration events and structure categories.

The total number of structure categories in the data was 3565 with each pavement section being 10 m long, the minimum unit of the road surface index evaluation length. Of these categories, 312 categories contained both road surface and load-bearing capacity data, thus satisfying the connectivity condition.

The schematic representation of the analytical framework is shown in Fig. 4. The following sections discuss the framework in detail.

4.2. Estimated results

For the estimation of parameters, data samples of drainage pavements, which is a standard type of the surface layer throughout Japan's intercity highway pavement, are used for analysis to avoid potential biases from incorporating different types of surface layer. For the pavement load-bearing capacity, D_{ind} was used, whereas for the road surface condition, three indices were used; i.e., the cracking rate, rutting depth, and IRI. In order to independently evaluate the relationship between the load-bearing capacity index and each road surface index, a multidimensional deterioration process model using individual continuous quantities was applied to analyze a total of three combinations: (1) D_{ind} - cracking rate, (2) D_{ind} - rutting depth, and (3) D_{ind} index - IRI. The characteristic variable groups were defined as granular roadbed and cement stabilized roadbed. The characteristic variable was specified using the following dummy variable.

$$z_i = \begin{cases} 0 & \text{When point } i \text{ is a granular roadbed} \\ 1 & \text{When point } i \text{ is a cement stabilized roadbed} \end{cases} \tag{29}$$

By estimating the parameter θ based on the characteristic variable z_i, the effect of using granular or cement stabilized roadbeds as the subgrade can be evaluated.

Table 3 shows the estimated parameters of the load-bearing capacity indices $\beta^{D_{ind}}, \theta^{D_{ind}}, \sigma^{D_{ind}}$, the 95% confidence intervals of the

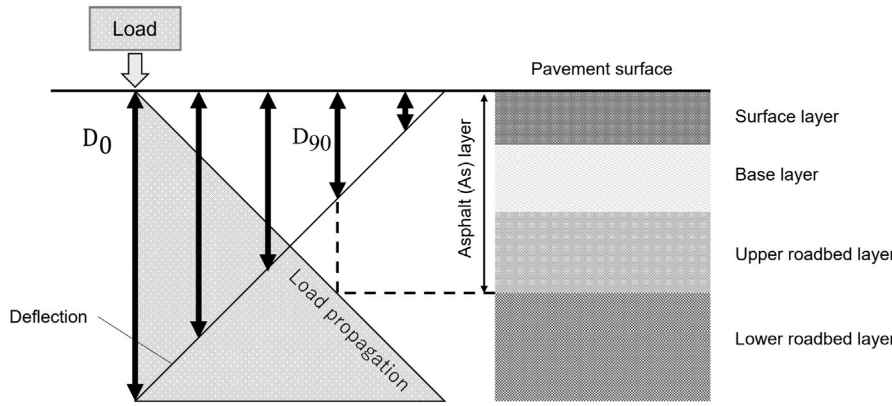


Fig. 3. Pavement structure and FWD measurements.

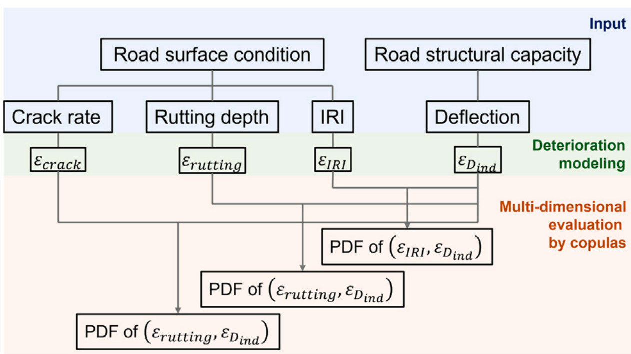


Fig. 4. Overview.

Table 3
Estimated parameters for load-bearing capacity β, θ, σ.

Parameter	Mean	Upper 5%	Lower 5%	Geweke statistics
$\beta_1^{D_{ind}}$	74.81	96.58	44.57	−0.156
$\beta_2^{D_{ind}}$	1.126	1.137	1.109	0.086
$\beta_3^{D_{ind}}$	151.5	162.5	142.2	0.677
$\theta^{D_{ind}}$	1.602	2.271	0.353	1.697
$\sigma^{D_{ind}}$	1.322	1.422	1.171	−0.257

Bayesian estimation, and the Geweke statistics. Similarly, Table 4 shows results for road indices (cracking: $\beta^{crack}, \theta^{crack}, \sigma^{crack}, \alpha^{crack}$, rutting: $\beta^{rut}, \theta^{rut}, \sigma^{rut}, \alpha^{rut}$, IRI: $\beta^{IRI}, \theta^{IRI}, \sigma^{IRI}, \alpha^{IRI}$). For cracking, only β_1^{crack} and β_2^{crack} are shown assuming that $\beta_3^{crack} = 0$ for a newly constructed or repaired pavement. The parameters showed high convergence with the Geweke diagnostics falling within the [−1.96, 1.96] limits with 0 specifying perfect convergence. These parameters can be used to generate illustrative performance curves shown in Figs. 5–8. The results showed that cement stabilized pavements have longer life compared to granular roadbed pavements for all performance curves as cement stabilization increases subbase strength.

The WAIC of the entire model including copulas is shown in Table 2. Three types of copulas were used: Gumbel, Clayton, and Frank copulas. From the same table, the Clayton copula was selected as the best copula for the D_{ind} - cracking rate, and the Gumbel copula for the D_{ind} - rutting and D_{ind} - IRI as they achieved minimum WAIC scores. The flexible function $f_d(t, \beta)$ (polynomial function, power function, exponential function) representing the baseline model for each indicator was also evaluated using WAIC, from which the exponential function $f_d(t, \beta) = \beta_1(\beta_2{}^t - 1) + \beta_3$ was employed for each pair of indicators.

Table 4
Estimated parameters for road surface indices β, θ, σ.

	Parameter	Mean	Upper 5%	Lower 5%	Geweke statistics
Cracking	β_1^{crack}	0.395	0.405	0.375	0.071
	β_2^{crack}	1.311	1.315	1.304	0.838
	θ^{crack}	1.410	1.443	1.356	0.883
	σ^{crack}	1.120	1.129	1.103	−1.491
	α^{crack}	0.866	1.330	0.210	0.273
Rutting	β_1^{rut}	5.007	5.154	4.784	−1.629
	β_2^{rut}	1.086	1.087	1.083	1.531
	β_3^{rut}	2.164	2.191	2.117	0.655
	θ^{rut}	0.302	0.312	0.284	0.924
	σ^{rut}	0.451	0.457	0.438	−0.981
	α^{rut}	1.085	1.182	1.002	0.938
IRI	β_1^{IRI}	0.771	0.793	0.727	0.816
	β_2^{IRI}	1.143	1.145	1.139	−0.548
	β_3^{IRI}	0.041	0.047	0.034	−0.143
	θ^{IRI}	0.270	0.286	0.243	0.403
	σ^{IRI}	0.606	0.613	0.593	−1.439
	α^{IRI}	1.084	1.178	1.003	−1.571

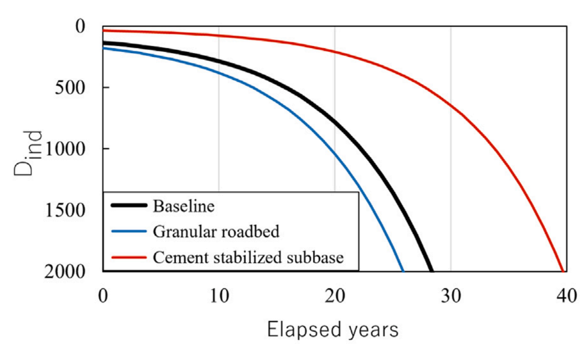


Fig. 5. Performance curve based on D_{ind}.

4.3. Joint probability density functions for heterogeneity parameters

In the multidimensional deterioration process model with continuous quantities, the copula probability density function and the marginal probability density function can be used to represent the joint probability density functions of the heterogeneity parameters for multiple deterioration events. The joint probability density functions of the heterogeneity parameters of the D_{ind}-crack rate, D_{ind}-rutting and D_{ind}-IRI are shown in Figs. 9–11. From the joint probability plots, although the variation of the heterogeneity parameter of the D_{ind}-crack rate of Fig. 9 was the largest among the three road surface indices, its correlation coefficient with the heterogeneity parameter of the D_{ind}

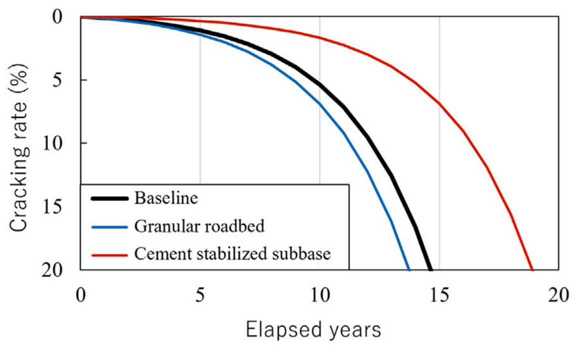


Fig. 6. Performance curve based on cracking.

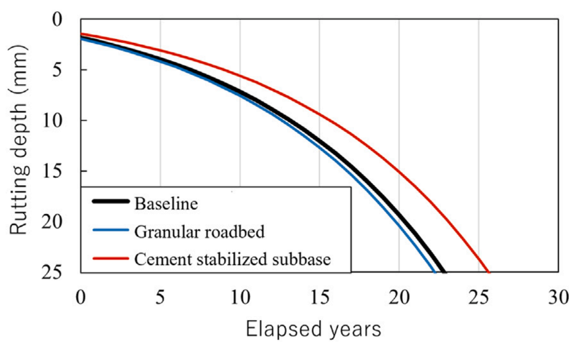


Fig. 7. Performance curve based on rutting.

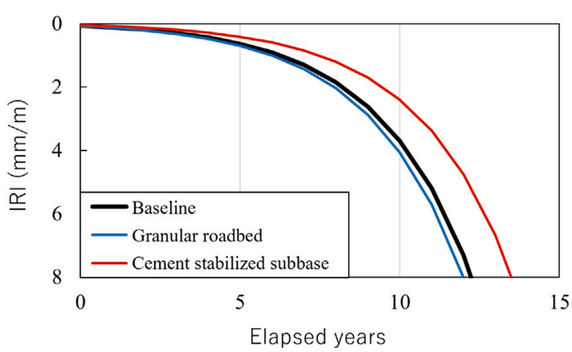


Fig. 8. Performance curve based on IRI.

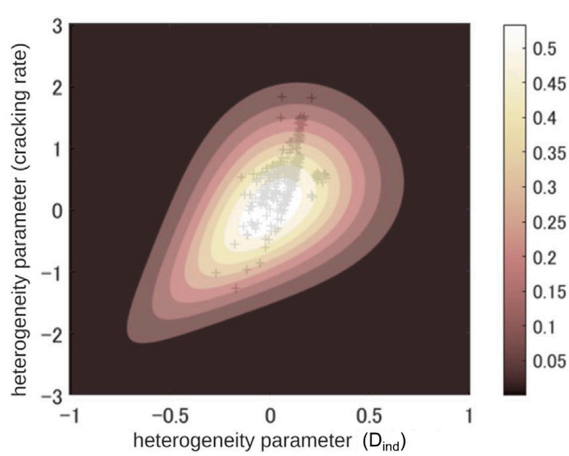


Fig. 9. Joint probability density for $(\varepsilon_{D_{ind}}, \varepsilon_{crack})$.

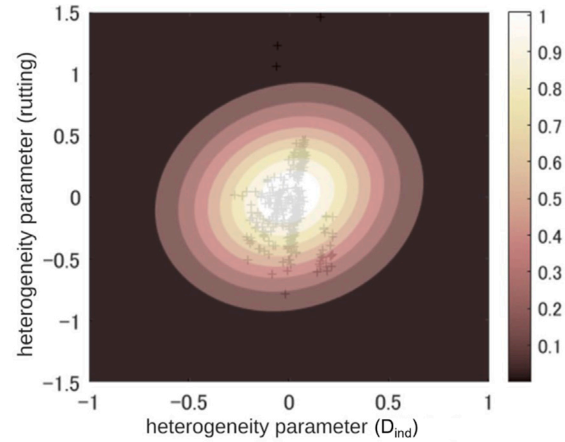


Fig. 10. Joint probability density for $(\varepsilon_{D_{ind}}, \varepsilon_{rut})$.

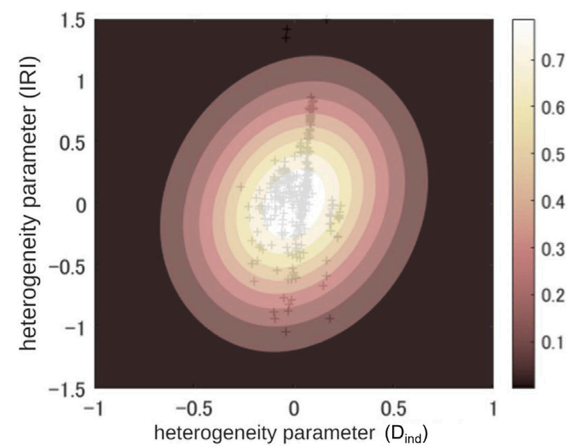


Fig. 11. Joint probability density for $(\varepsilon_{D_{ind}}, \varepsilon_{IRI})$.

was 0.597, confirming a positive correlation between pavement surface cracking and subsurface deterioration. The D_{ind}-rutting in Fig. 10 had the smallest correlation coefficient at 0.160, with both heterogeneity parameters concentrated around 0. The D_{ind}-IRI of Fig. 11 also showed the same trend as the D_{ind}-rutting, but the variability of the IRI heterogeneity parameter was relatively larger, with a correlation coefficient of 0.219.

The joint probability density functions of the heterogeneity parameters presented above showed that in this empirical study, the D_{ind}-crack rate combination has interactive effects for both deterioration processes (surface and subsurface). In the D_{ind}-crack rate combination, the joint probability density function has a distorted shape in the third quadrant because the Clayton copula with the lowest WAIC score was used for estimation. For the Clayton copula, the degree of dependence among random variables is relatively strong in the lower left and weak in the upper right of the plot. This means that the correlation is particularly strong in the third quadrant, where both heterogeneities take small values, and that when either the D_{ind} or the crack rate takes small values.

Three types of Archimedian copulas; i.e., the Gumbel, Clayton, and Frank copulas were used in the estimation of the correlation between the surface and subsurface deterioration events. However, it is desirable to estimate using a variety of copulas as candidates so as to select models that further reduce the WAIC.

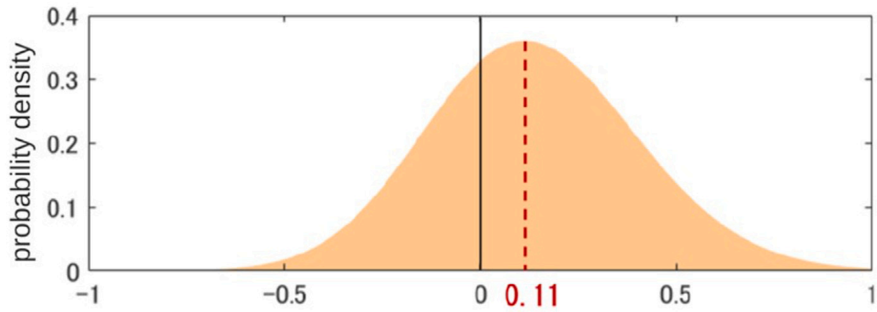


Fig. 12. Probability density function for $\varepsilon_{D_{ind}}$ (when $\varepsilon_{crack} = 1$).

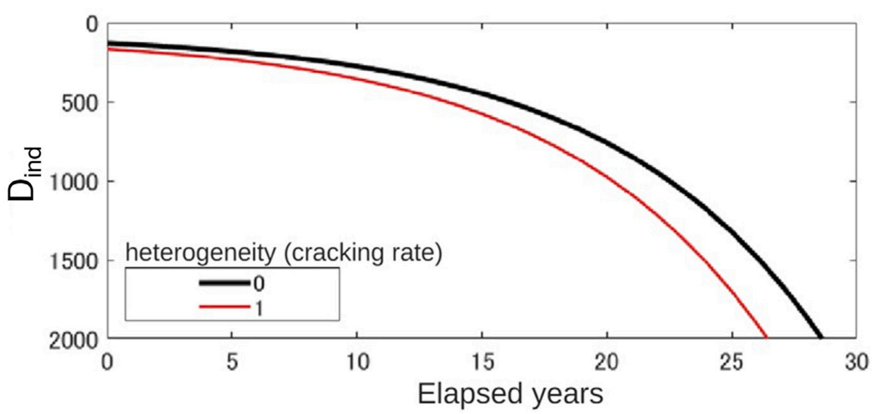


Fig. 13. Variation of load bearing capacity index due to differences in road surface index.

4.4. Multidimensional deterioration process

As discussed in detail earlier, the FWD test used to estimate pavement load-bearing capacity requires traffic restrictions, whereas road surface condition surveys do not. Hence, cases where the load-bearing capacity index is unknown and the road surface index is known (measured multiple times) are prevalent in the data set. Therefore, the unknown load-bearing capacity index can be quantitatively predicted using the joint probability density functions of the load-bearing capacity index and the heterogeneity of each road surface index. In this application case, a positive correlation was observed between the cracking rate and the load bearing capacity index, and there was a weak correlation for the combination of the load bearing capacity index - rutting area and the load bearing capacity index - IRI, so a joint probability density function for the heterogeneity parameters ($\varepsilon_{D_{ind}}, \varepsilon_{crack}$) shown in Fig. 9 was used to predict the unknown load bearing capacity index.

If the heterogeneity $\varepsilon_{crack} = 1$ of the crack rate at a point i is known, then through application of the joint probability density function, the probability density distribution of the heterogeneity $\varepsilon_{D_{ind}}$ of the load bearing capacity index as shown in Fig. 12 can be estimated. The expected value of $\varepsilon_{D_{ind}}$ was 0.11, suggesting that the deterioration rate was comparatively larger. It is also possible to use the probability density function to assess hazard risk where heterogeneity is greater than the expected value. The expected deterioration performance curves of the load-bearing capacity index are shown in Fig. 13 assuming only crack rate and $\varepsilon_{crack} = 0$ and $\varepsilon_{crack} = 1$ predicted using the results in Fig. 12. From the same figure, it is possible to quantitatively evaluate load-bearing capacity deterioration rates when the crack rate heterogeneity is higher.

4.5. Practical implications

In this research, the correlation among heterogeneity parameters in a continuous quantity deterioration hazard model that takes heterogeneity into account was quantified using copulas. Through the evaluation of the deterioration rate of infrastructural facilities from multiple perspectives, the following pavement intervention considerations can be made. Focusing on Fig. 9, the group located in the first quadrant, where the heterogeneity parameter is greater than 0 for both the road surface and load bearing capacity indices, shows faster deterioration progress for both the road surface and load bearing capacity. For this group, intensive interventions should be performed, taking into account the need for not only repairs of the shallow layers such as the surface and base layers, but also large-scale maintenance and renewal involving deeper layers including the upper and lower roadbeds. Next, for groups in the fourth quadrant, where the heterogeneity parameter of the road surface index is greater than 0 and the heterogeneity parameter of the load-bearing capacity index is less than 0, replacement of the surface and base layers should be carried out as pavement damage is likely confined only to shallow layers. For groups in the second quadrant, where the heterogeneity parameter of the road surface index is less than 0 and the heterogeneity parameter of the load-bearing capacity index is greater than 0, there is a possibility that deterioration is developing in the deeper pavement layers and therefore intensive intervention should be conducted and a rehabilitation plan that takes into account large-scale rehabilitation and renewal when damage appears on the road surface be established. No intervention may be required in the quadrant with both heterogeneity parameters less than 0, however, regular inspection can be encouraged. Furthermore, the joint probability that a group is located in each quadrant of Fig. 9 can be calculated as 0.33 for the first quadrant, 0.17 for the second quadrant, 0.33 for the third quadrant and 0.17 for the fourth quadrant using the joint probability density function for the heterogeneity parameters. These probabilities represent the proportion of the total road sections under observation located in each quadrant, and may provide useful information for budget planning and major repair projects.

Additionally, when only road surface indices are obtained, it is possible to quantitatively evaluate whether the group in question is located in the first or second quadrant, or the third or fourth quadrant, using fragmentary information in the form of heterogeneity parameters of the load bearing capacity index. The ability to represent multidimensional deterioration processes probabilistically from partial observation information is an important feature of this research with practical applications in the case of incomplete infrastructure monitoring data.

5. Conclusion

This study introduces a copula-based framework that jointly models multiple continuous pavement deterioration processes with heterogeneity, enabling a more comprehensive representation than conventional single-indicator models. The approach offers practical value by revealing interdependencies among deterioration indicators, supporting optimized inspection schedules, targeted preventive maintenance, and broader application to other transportation infrastructure where multi-indicator data are available.

The proposed model was applied to pavement load-bearing capacity and road surface indices (crack rate, rutting depth, and IRI) estimated through FWD tests and road surface condition surveys, respectively. The correlation structure among deterioration events expressed using an Archimedian copula. This representation enabled the evaluation of the need for large-scale intervention or preventive repair based on the quadrant in which a given pavement section was grouped. Furthermore, when the methodology proposed in this study was applied to actual highway inspection data, a positive correlation was observed between the crack rate and load-bearing capacity index and the correlation was particularly strong in areas where the heterogeneity between the two was small. To our knowledge, no previous study has visualized the interdependent relationships among multiple continuous pavement deterioration processes in this manner.

In this study, all estimated copula correlation parameters were positive, reflecting the empirical finding of predominantly positive dependencies among the deterioration indicators analyzed. While copulas allowing for negative correlations do exist, preliminary analyses indicated that negative correlation structures were not significant for the dataset examined in this study. Nevertheless, future research could explore additional copula structures capable of capturing negative dependencies to verify the robustness of these findings.

By quantifying interdependencies among deterioration indicators, the approach enables maintenance planners to anticipate how changes in one condition (e.g., cracking) may influence others (e.g., rutting or load-bearing capacity), thus supporting targeted preventive strategies such as sealing cracks earlier to slow rutting progression or reinforcing structural layers to prevent rapid surface distress. Although demonstrated using Japanese highway pavement data, the methodology is adaptable to other transportation infrastructure systems – such as bridges, tunnels, or rail track – where multi-indicator deterioration data are available.

As a part of future research, the following two issues need to be addressed. First, applicability of the proposed method must be verified by increasing the number of application cases. The sections analyzed in this application case study are those with particularly advanced deterioration among the highways under a certain jurisdiction of intercity highway managing company, and generalizability of the results is yet to be examined. Therefore, the finding that among the three road surface indices, only cracking rate was strongly correlated with the load-bearing capacity index is valid only for the sections selected for this study application. In the future, it is necessary to apply the proposed method to several sections to clarify the interconnectedness of the road surface indices and the load-bearing capacity index. Second, the different correlations among several deterioration indices need to be expressed in more detail. In this study, one-parameter Archimedian copulas were used for each combination of load-bearing capacity and a road surface index, but it may be preferable to use two-parameter Archimedian copulas, which allow for more flexible expressions of dependence structures. It is also possible to analyze the correlation structure of three or more indicators simultaneously. The correlations between deterioration events can be described more precisely by using the vine copula (Bedford and Cooke, 2002), which defines different types of copulas between each index.

CRediT authorship contribution statement

Kotaro Sasai: Writing – original draft, Visualization, Methodology, Formal analysis, Data curation. **Felix Obunguta:** Validation, Conceptualization. **Manish Man Shakya:** Writing – review & editing, Conceptualization. **Kiyoyuki Kaito:** Writing – review & editing, Supervision, Project administration, Conceptualization.

Notation list

The following symbols are used in this paper:

ε_A = heterogeneity parameter for process A
ε_B = heterogeneity parameter for process B
d = deterioration event
i = facility
k = category
ε_d^k = heterogeneity parameter
x_d^i = deterioration management index of a facility
B_d^i = deterioration characteristic coefficient of a facility
$\boldsymbol{\theta}_d$ = characteristic parameter vector
$\mathbf{z}^i$ = characteristic variable term
$\boldsymbol{\beta}_d$ = unknown parameter vector
y_d^i = lifetime index
w_d^i = error term
S_t = Survival function to time t
t^i = real life until control level
P_d = distribution function for event d
C = copula
ζ = generation function of copula
τ = probability density of real life
D_{ind} = load-bearing capacity index
z_i = dummy variable

Declaration of competing interest

There was no conflict of interest reported by the Authors.

Acknowledgments

This work was supported by Council for Science, Technology and Innovation (CSTI), Cross-ministerial Strategic Innovation Promotion Program (SIP), the 3rd period of SIP "Smart Infrastructure Management System" Grant Number JPJ012187 (Funding agency: Public Works Research Institute). The NEXCO Central Japan is appreciated for providing Japanese roads data used in the study application.

Data availability

The authors do not have permission to share data.

References

Adey, B.T., Burkhalter, M., Martani, C., 2020. Defining road service to facilitate road infrastructure asset management. Infrastruct. Asset Manag. 7 (4), 240–255, URL https://doi.org/10.1680/jinam.18.00045.

Araki, S., Kazato, T., Kaito, K., Kobayashi, K., Tanaka, A., 2018. Analysis for repair effect in each layer of expressway pavements. In: Life Cycle Analysis and Assessment in Civil Engineering: Towards an Integrated Vision: Proceedings of the Sixth International Symposium on Life-Cycle Civil Engineering (IALCCE 2018), 28-31 October 2018, Ghent, Belgium. Vol. 5, CRC Press, p. 315.

Bedford, T., Cooke, R.M., 2002. Vines-a new graphical model for dependent random variables. Ann. Stat. 30 (4), 1031–1068, URL https://www.jstor.org/stable/1558694.

Breymann, W., Dias, A., Embrechts, P., 2003. Dependence structures for multivariate high-frequency data in finance. Quant. Finance 3 (1), 1–14, URL https://doi.org/10.1080/713666155.

Clayton, D.G., 1978. A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. Biometrika 65 (1), 141–151, URL https://doi.org/10.1093/biomet/65.1.141.

Frank, M.J., 1979. On the simultaneous associativity of $F(x, y)$ and $x + y - F(x, y)$. Aequationes Math. 19, 194–226, URL https://doi.org/10.1007/BF02189866.

Genest, C., Rivest, L.-P., 1993. Statistical inference procedures for bivariate Archimedean copulas. J. Amer. Statist. Assoc. 88 (423), 1034–1043.

Gumbel, E.J., 1960. Bivariate exponential distributions. J. Am. Stat. Assoc. 55 (292), 698–707, URL https://doi.org/10.2307/2281591.

Joe, H., 1997. Multivariate Models and Multivariate Dependence Concepts. CRC Press.

Joe, H., 2014. Dependence Modeling with Copulas. CRC Press.

Kaito, K., Kobayashi, K., Obama, K., 2014. Investigating Pavement Structure Deterioration with a Relative Evaluation Model. In: Maintenance and safety of aging infrastructure, Structures and Infrastructures Book Series, vol. 10, CRC Press London, pp. 343–377.

Kamiya, K., 2023. The relationship between surface defects and structural distress on a 50-year-old Japanese toll road. In: Roads and Airports Pavement Surface Characteristics. CRC Press, pp. 403–412.

Lethanh, N., Adey, B.T., 2012. A hidden Markov model for modeling pavement deterioration under incomplete monitoring data. Int. J. Civ. Env. Eng. 6 (1), 7–14.

Nakamura, H., Kameyama, S., Sakuraba, A., Kawabata, S., Tomiyama, K., 2022. The relationship between surface crack features and bearing capacity of the pavement. In: Eleventh International Conference on the Bearing Capacity of Roads, Railways and Airfields. CRC Press, pp. 3–13.

Nelsen, R.B., 1999. An Introduction to Copulas. Springer.

Obama, K., Okada, K., Kaito, K., Kobayashi, K., 2008. Disaggregated hazard rates evaluation and bench-marking. J. Jpn. Soc. Civ. Eng. A 64 (4), 857–874, URL https://doi.org/10.2208/jsceja.64.857.

Sklar, A., 1973. Random variables, joint distribution functions, and copulas. Kybernetika 9 (6), 449–460, URL https://www.kybernetika.cz/content/1973/6/449.

Tsuda, Y., Kaito, K., Aoki, K., Kobayashi, K., 2006. Estimating Markovian transition probabilities for bridge deterioration forecasting. Struct. Eng./Earthq. Eng. 23 (2), 241s–256s, URL https://doi.org/10.2208/jsceseee.23.241s.

Watanabe, S., 2010. Asymptotic equivalence of Bayes cross validation and widely applicable information criterion in singular learning theory. J. Mach. Learn. Res. 11 (DEC), 3571–3591, URL https://doi.org/10.48550/arXiv.1004.2316.