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Macroscopic Constraint-Based Modeling and Dynamic Simulation for Manipulation with a Belt-Attached Hand

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Abstract

This study aims to develop a novel modeling and analysis method based on macroscopic constraints for manipulation with a belt-attached hand. In a mesh-based model where a flexible body is divided into smaller parts and analysis is based on local constraints, the complexity of the model depends on the number of elements and scale of the belt. In contrast, the proposed model has the advantage of allowing dynamic analysis with a small number of unknown variables and constraints because it formulates the adaptive deformation of the belt into the object shape as macroscopic constraints. First, a generalized complementarity condition concerning tautness and adaptive deformation of a cloth-like flexible body and tension is introduced, and the belt-object system is modeled. Subsequently, a dynamic analysis combining algorithms to update the contact state and a mixed nonlinear complementarity problem to satisfy constraints is introduced. Finally, translation involving rolling and slipping, continuous rotation by fingertip vibration, and fixation by grasping the object on the belt are simulated based on the proposed method, and the results are compared with those of a real experiment. We confirmed the efficacy of the proposed method in reproducing the various dynamic behaviors of the belt-object system, including rolling, slipping, and collisions.

Keywords

Dynamics, grippers, flexible robots, manipulation, cloth simulation.

1 Introduction

Soft hands equipped with flexible elements are attracting attention as novel robot manipulation methods. Soft hands exhibit the capability to deform to fit the object better and are, therefore, expected to improve grasping and manipulation stability (Hughes et al., 2016). Flexible elements with moderate elasticity in the stretch and bending direction are primarily used in soft hands. For instance, hands that move by expanding and contracting elastic materials such as silicone by applying air pressure (Zhou et al., 2018; Pagoli et al., 2021; Zhou et al., 2019) and hands that move by deforming thinly printed resin fingers using the tension of tendons or belts (Dragusanu et al., 2022; Cai and Yuan, 2023) have been proposed. In these methods, the viscoelastic deformation is actively controlled to produce the hand movements required for grasping and in-hand manipulation. Conversely, we focus on flexible elements such as a string, belt, and cloth, which have high stiffness in the stretch direction and low stiffness in the bending direction (hereafter called a cloth-like flexible body, or CFB for short). Several studies have been performed on grasping and manipulation by surrounding the object with a rope or a

belt (Matl and Bajcsy, 2021; Govindan et al., 2022; Donald et al., 2000; Maneewarn and Detudom, 2005; Iwamasa and Hirai, 2015). Actively controlling the deformation of CFBs can be challenging, but they have excellent bending deformability and can contact objects over a wide area. Thus, a hand with a CFB can stably grasp and manipulate objects with various shapes. In previous studies (Ichikura and Higashimori, 2022, 2024), the authors developed hands with a belt or cloth between the fingertips and proposed nonprehensile manipulation methods that rotate an object on the CFB using vibrational inputs to the fingertips, as illustrated in Fig. 1. In these previous studies, the shape of the object was limited to isotropic shapes such as cylinders or spheres, which maintained a nearly constant CFB shape

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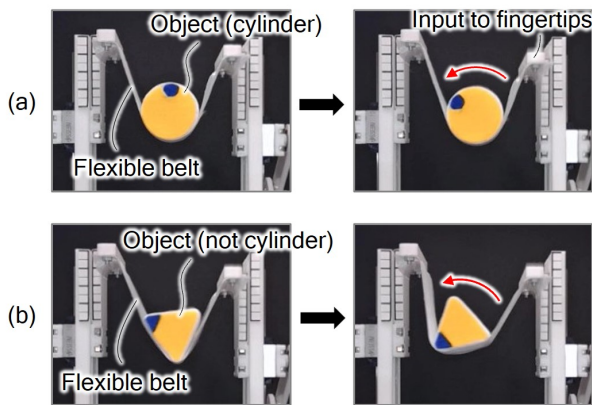


Figure 1. Manipulation with belt-attached hand. (a) For a cylindrical object. The belt shape is approximately constant regardless of the object orientation. (b) For a non-cylindrical object. The belt shape changes significantly due to the change in the object orientation.

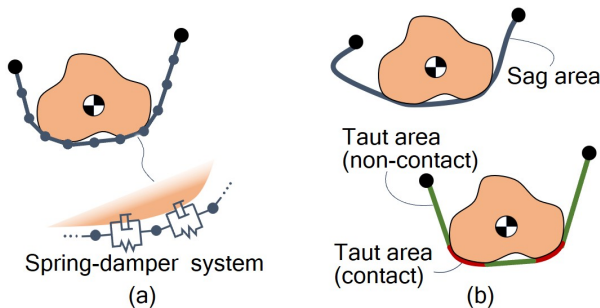


Figure 2. Modeling approaches of a CFB and an object system. (a) Mesh-based model. (b) Proposed model based on macroscopic geometric constraints.

regardless of the object orientation (Fig. 1(a)).

For practical use of this manipulation method, we consider generalizing the object shape to non-cylindrical and non-spherical. In this case, the shape of the CFB changes depending on the object orientation, making the geometric relationship between the CFB and the object complex (Fig. 1(b)). Modeling and dynamic simulation of the CFB and object system should be performed to understand their interaction. However, modeling a deformable flexible body is challenging as it has infinite degrees of freedom. The main modeling method to this end is the mesh-based method shown in Fig. 2(a) (Long et al., 2011; Bridson et al., 2002; Selle et al., 2009; Sifakis et al., 2007). In this method, the flexible body is represented as a finely divided mesh, and the force distribution and displacement on each mesh node are calculated using a finite element method or position-based dynamics. For the distances and angles between adjacent mesh nodes, local constraints using viscoelastic parameters are imposed (Selle et al., 2009; Sifakis et al., 2007). This modeling method

tends to increase the number of unknown variables and constraints significantly depending on the mesh resolution, and the connection parameters (spring, damper, penetration, etc.) between the nodes require trial-and-error adjustments. For a non-stretchable CFB, the calculation tends to fail to converge due to the stiff system caused by high stretch stiffness and low bending stiffness of the CFB. These are inherent challenges in designing systems leveraging a flexible body (Lipson, 2014; Bridson et al., 2002).

Conversely, we focus on the “macroscopic” features of the system comprising the CFB and object, as illustrated in Fig. 2(b). At the area where no tension occurs, the CFB deforms to sag, and at the area where tension occurs, the CFB in contact with the object adaptively deforms to the object shape (Fig. 2(b) red lines). The area not in contact with the object deforms along a straight line (Fig. 2(b) green lines). This study formulates the deformation of the CFB and the presence or absence of tension as a complementarity constraint and models the system of the CFB and object. Dynamic analysis can be performed without a model complexity dependent on the resolution of the CFB by representing the shape of the CFB macroscopically without a mesh. To represent cloth without a mesh, Han and Zhu (2005) proposed a method to learn real cloth folds and reproduce the cloth shapes in two and three dimensions. The stretch and slippage of a belt have been studied extensively by geometrically formulating belt-pulley drive systems (Bechtel et al., 2000; Rubin, 2000). The authors’ previous research statically analyzed the system of the CFB and cylindrical or spherical objects with a geometric formulation (Ichikura and Higashimori, 2022, 2024). This study limits the system to one where the CFB is in contact with the object and is taut, and a versatile modeling and dynamic analysis method is constructed for any shape of the object. Various dynamic behaviors of the system are represented with a small number of unknown variables and mathematical approaches.

In this paper, we propose a novel modeling and dynamic analysis method based on macroscopic constraints on a system comprising a belt-attached hand and an object, involving manipulation by input to the fingertips of the hand. By formulating the geometric features without a mesh, the problem complexity is made independent of the belt resolution (i.e., the spatial discretization interval of the belt nodes), and dynamic analysis can be performed with fewer unknown variables and constraints. First, a two-dimensional analysis model comprising a belt and an object on the belt is introduced. The complementary generalized relationship between the geometric properties of a CFB to deform linearly or adaptively to an object shape and the presence or absence of tension is formulated. The contacts with slipping and equations of motion are formulated. Next, a dynamic analysis flow is introduced.

Combining algorithms for updating the contact state that can handle rolling and tumbling and a mixed nonlinear complementarity problem (mNCP) to satisfy constraints, an analysis flow to solve the behavior of the belt-object system sequentially is constructed. Finally, experimental validations are performed. Basic movements with simple displacements to the fingertips, rotational manipulation, and grasp of the object on the belt are studied in the simulation based on the proposed method. The results are compared with those of a real experiment, and the effectiveness of the proposed method is verified.

2 Modeling

2.1 Basic idea: generalized constraints on CFB

First, in a system comprising a CFB and an object, an absolute coordinate system Σ_{abs} and a coordinate system fixed to the CFB (a cloth coordinate system) Σ_{cloth} are defined. Σ_{abs} is in two or three dimensions. Σ_{cloth} is a coordinate system in which the CFB is unfolded, and its dimension is one smaller than that of Σ_{abs} . Hereafter, at any point p , the position vectors of p in Σ_{abs} and Σ_{cloth} are denoted by $\mathbf{x}$ and $\tilde{\mathbf{x}}$, respectively. We consider the case where two points, p_1 and p_2 , are on a CFB, as illustrated in Fig. 3(a). There are two possible correspondences between Σ_{abs} and Σ_{cloth} for the positions of the two points p_1 and p_2 . When the distance between the two points in Σ_{cloth} is shorter than the distance in Σ_{abs} (Fig. 3(b)), the line connecting the two points in Σ_{cloth} is not straight in Σ_{abs} . Thus, the CFB sags and tension is not generated in the area. When the distance between the two points in Σ_{abs} is equal to the distance in Σ_{cloth} (Fig. 3(c)), the line connecting the two points in Σ_{cloth} is straight in Σ_{abs} . In this area, the CFB is taut. Conversely, there is no case where the distance between the two points in Σ_{abs} is longer than the distance in Σ_{cloth} . Denoting the tension between the two points as f_t , f_t and the distance in Σ_{cloth} minus the distance in Σ_{abs} are both non-negative, and the product of the two is zero. This constraint is called a complementarity constraint and is expressed using the symbol $\perp$ as follows:

$$0 \leq |\tilde{\mathbf{x}}_1 - \tilde{\mathbf{x}}_2| - |\mathbf{x}_1 - \mathbf{x}_2| \perp f_t \geq 0 \quad (1)$$

This holds for any two points on a CFB. Hereafter, a model based on this constraint is developed.

2.2 Analytical model of the belt-object system

Consider a two-dimensional model comprising a flexible belt and rigid body with a constant cross-sectional shape (prismatic object in 3D), as illustrated in Fig. 4. The belt is anchored at both endpoints (hereafter called belt anchor points), and their positions are controlled. The object is

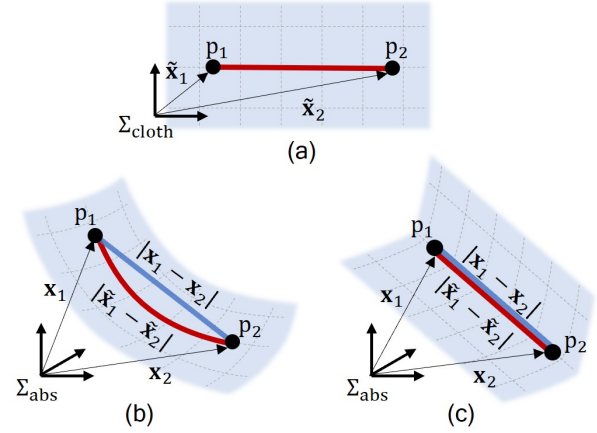


Figure 3. Correspondence of two points on the CFB between the Σ_{abs} and Σ_{cloth} . (a) Cloth coordinate system. (b) Case with $|\tilde{\mathbf{x}}_1 - \tilde{\mathbf{x}}_2| > |\mathbf{x}_1 - \mathbf{x}_2|$. (c) Case with $|\tilde{\mathbf{x}}_1 - \tilde{\mathbf{x}}_2| = |\mathbf{x}_1 - \mathbf{x}_2|$.

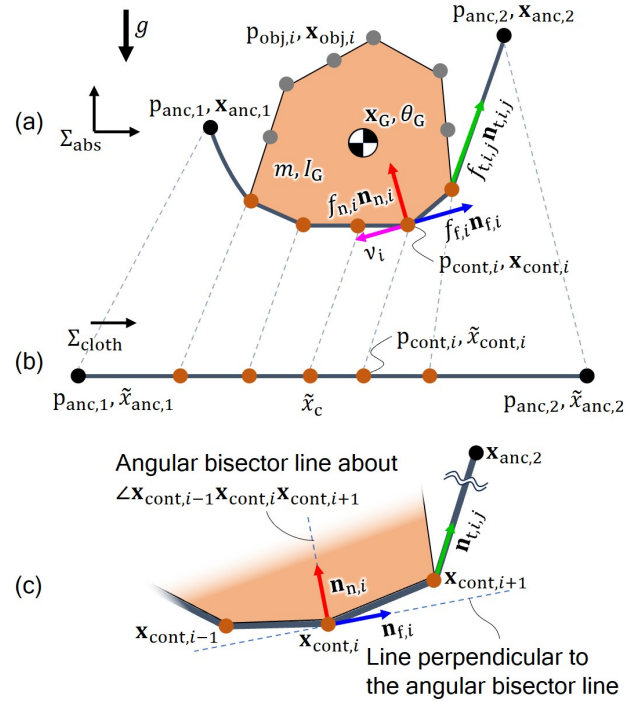


Figure 4. Two-dimensional analysis model. (a) Absolute coordinate system. (b) Cloth coordinate system. (c) Contact force and tension direction.

represented by points and lines connecting the points. Forces are represented as the product of a scalar f and a direction vector $\mathbf{n}$ ($|\mathbf{n}| = 1$). The symbols in Fig. 4 are defined as follows.

Σ_{abs} Two-dimensional absolute coordinate system. The x -axis is horizontal, and the y -axis is vertical.

Σ_{cloth} One-dimensional cloth coordinate system.

- $p_{\text{anc},i}$ Belt anchor point. $i = 1, 2$. The positions of $p_{\text{anc},i}$ in Σ_{abs} and Σ_{cloth} are denoted as $\mathbf{x}_{\text{anc},i}$ and $\tilde{\mathbf{x}}_{\text{anc},i}$, respectively.
- $\mathbf{x}_G$ Center position of the object in Σ_{abs} . $\mathbf{x}_G = [x_G, y_G]^T$.
- θ_G Orientation of the object in Σ_{abs} .
- n_{obj} Number of points comprising the object.
- $p_{\text{obj},i}$ Points comprising the object. $i = 1, \dots, n_{\text{obj}}$. The positions of $p_{\text{obj},i}$ in Σ_{abs} and Σ_{cloth} are denoted as $\mathbf{x}_{\text{obj},i}$ and $\tilde{\mathbf{x}}_{\text{obj},i}$, respectively.
- $\mathbf{x}_{\text{obj},i}^*$ Position vector of $p_{\text{obj},i}$ in Σ_{abs} in the reference state ($\mathbf{x}_G = \mathbf{0}$, $\theta_G = 0$). $\mathbf{x}_{\text{obj}}^* = [\mathbf{x}_{\text{obj},1}^*, \dots, \mathbf{x}_{\text{obj},n_{\text{obj}}}^*]^T$.
- n_{cont} Number of contact points between the object and the belt.
- $p_{\text{cont},i}$ Contact point between the object and the belt. $i = 1, \dots, n_{\text{cont}}$. $p_{\text{cont},i} \subset \{p_{\text{obj},1}, \dots, p_{\text{obj},n_{\text{obj}}}\}$. The positions of $p_{\text{cont},i}$ in Σ_{abs} and Σ_{cloth} are denoted as $\mathbf{x}_{\text{cont},i}$ and $\tilde{\mathbf{x}}_{\text{cont},i}$, respectively.
- $\tilde{\mathbf{x}}_c$ Position of representative point of contact area between the belt and the object in Σ_{cloth} .
- $\tilde{\mathbf{x}}_{\text{obj},i}^*$ Relative position of $p_{\text{obj},i}$ with respect to $\tilde{\mathbf{x}}_c$ in Σ_{cloth} . $\tilde{\mathbf{x}}_{\text{obj}}^* = [\tilde{\mathbf{x}}_{\text{obj},1}^*, \dots, \tilde{\mathbf{x}}_{\text{obj},n_{\text{obj}}}^*]^T$.
- m Mass of the object.
- I_G Moment of inertia of the object about an axis through $\mathbf{x}_G$ and perpendicular to the xy -plane.
- g Magnitude of gravitational acceleration.
- $f_{n,i}$ Scalar multiplier of the normal force on $p_{\text{cont},i}$.
- $\mathbf{n}_{n,i}$ Direction vector of the normal force on $p_{\text{cont},i}$.
- $f_{f,i}$ Scalar multiplier of the frictional force on $p_{\text{cont},i}$.
- $\mathbf{n}_{f,i}$ Direction vector of the frictional force on $p_{\text{cont},i}$.
- $f_{t,i,j}$ Scalar multiplier of the belt tension generated between $p_{\text{cont},i}$ and $p_{\text{anc},j}$.
- $\mathbf{n}_{t,i,j}$ Direction vector of the belt tension generated between $p_{\text{cont},i}$ and $p_{\text{anc},j}$.
- ν_i Scalar multiplier of the relative velocity between the belt and the object at $p_{\text{cont},i}$. The direction of the relative velocity is opposite to that of $\mathbf{n}_{f,i}$. $\boldsymbol{\nu} = [\nu_1, \dots, \nu_{n_{\text{cont}}}]^T$.

Note that Σ_{cloth} is one-dimensional, and thus the position in Σ_{cloth} is denoted as a scalar. The sets $\mathbf{x}_{\text{anc},i}$ and $\tilde{\mathbf{x}}_{\text{anc},i}$ ($i = 1, 2$), $\mathbf{x}_{\text{obj},i}$ and $\tilde{\mathbf{x}}_{\text{obj},i}$ ($i = 1, \dots, n_{\text{obj}}$), and $\mathbf{x}_{\text{cont},i}$ and $\tilde{\mathbf{x}}_{\text{cont},i}$ ($i = 1, \dots, n_{\text{cont}}$) are denoted as $\mathbf{x}_{\text{anc}}$, $\tilde{\mathbf{x}}_{\text{anc}}$, $\mathbf{x}_{\text{obj}}$, $\tilde{\mathbf{x}}_{\text{obj}}$, $\mathbf{x}_{\text{cont}}$, and $\tilde{\mathbf{x}}_{\text{cont}}$, respectively.

To simplify the shape of the belt, we employ the following assumptions:

Assumption 1 The mass of the belt, viscoelasticity in the bending direction, and deformation in the stretching direction are negligible. The belt is freely deformable only in the bending direction.

Assumption 2 No wrinkles occur at the contact area of the belt.

Assumption 3 From assumptions 1 and 2, the contact area of the belt adapts to the object and deforms to the convex hull of the object.

In this paper, we discuss the model under ideal conditions that satisfy these assumptions. $\mathbf{x}_{\text{obj},i}$ and $\tilde{\mathbf{x}}_{\text{obj},i}$, the positions of $p_{\text{obj},i}$ in Σ_{abs} and Σ_{cloth} , are expressed using $\mathbf{x}_G$, θ_G , $\tilde{\mathbf{x}}_c$, $\mathbf{x}_{\text{obj},i}^*$ and $\tilde{\mathbf{x}}_{\text{obj},i}^*$ as follows:

$$\mathbf{x}_{\text{obj},i} = \mathbf{x}_G + \mathbf{R}(\theta_G) \mathbf{x}_{\text{obj},i}^* \quad (2)$$

$$\tilde{\mathbf{x}}_{\text{obj},i} = \tilde{\mathbf{x}}_c + \tilde{\mathbf{x}}_{\text{obj},i}^*, \quad (3)$$

where, $\mathbf{R}(\theta_G)$ is the rotation matrix with respect to the orientation θ_G . As depicted in Fig. 4(c), let $\mathbf{n}_{n,i}$ be the direction of the angular bisectors of the two sides connected to $p_{\text{cont},i}$. $\mathbf{n}_{f,i}$ and $\mathbf{n}_{t,i,j}$ are expressed as follows:

$$\mathbf{n}_{f,i} = \mathbf{R}\left(-\frac{\pi}{2}\right) \mathbf{n}_{n,i}, \quad (4)$$

$$\mathbf{n}_{t,i,j} = \frac{\mathbf{x}_{\text{anc},j} - \mathbf{x}_{\text{cont},i}}{|\mathbf{x}_{\text{anc},j} - \mathbf{x}_{\text{cont},i}|}. \quad (5)$$

Note that the object point $p_{\text{obj},i}$ can be placed not only on the apex but also on the side of the object. When $p_{\text{obj},i}$ is placed segmentally on the side of the object, we can consider the contact forces on the side.

2.3 Formulation of basic constraints

In this subsection, the equation of motion and other constraints are formulated based on a typical modeling method for rigid multibody dynamics (Bender et al., 2014).

2.3.1 Equations of motion The equation of motion based on contact forces applied to the object from the belt is expressed as follows:

$$\mathbf{M} \begin{bmatrix} \ddot{x}_G \\ \ddot{y}_G \\ \ddot{\theta}_G \end{bmatrix} - \mathbf{J}_n^T \mathbf{f}_n - \mathbf{J}_f^T \mathbf{f}_f - \mathbf{f}_{\text{ext}} = \mathbf{0}, \quad (6)$$

where, $\mathbf{M}$ is the mass matrix of the object and $\mathbf{f}_{\text{ext}} = [0, -mg, 0]^T$ is the external force vector. $\mathbf{M}$ is expressed as follows:

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_G \end{bmatrix}. \quad (7)$$

$\mathbf{f}_n$ and $\mathbf{f}_f$ are the vectors arranging $f_{n,i}$ and $f_{f,i}$ at each contact point $\mathbf{p}_{\text{cont},i}$, and $\mathbf{J}_n$ and $\mathbf{J}_f$ are the Jacobi matrices that transform $\mathbf{f}_n$ and $\mathbf{f}_f$ into a combined force-moment, respectively. The matrix components $\mathbf{J}_{n,i}$ and $\mathbf{J}_{f,i}$ of $\mathbf{J}_n$ and $\mathbf{J}_f$, corresponding to $f_{n,i}$ and $f_{f,i}$ are expressed as follows:

$$\mathbf{J}_{n,i} = \begin{bmatrix} \mathbf{n}_{n,i} \\ (\mathbf{x}_{\text{cont},i} - \mathbf{x}_G) \times \mathbf{n}_{n,i} \end{bmatrix}^T \quad (8)$$

$$\mathbf{J}_{f,i} = \begin{bmatrix} \mathbf{n}_{f,i} \\ (\mathbf{x}_{\text{cont},i} - \mathbf{x}_G) \times \mathbf{n}_{f,i} \end{bmatrix}^T. \quad (9)$$

As for the system comprising the object and the belt around the object, the contact forces cancel between the object and the belt. Thus, this system is subjected to tensions and external force, with combined force-moment equal to (6). The equation of motion of the system comprising the object and the belt is expressed as follows:

$$\mathbf{M} \begin{bmatrix} \ddot{x}_G \\ \ddot{y}_G \\ \ddot{\theta}_G \end{bmatrix} - \mathbf{J}_t^T \mathbf{f}_t - \mathbf{f}_{\text{ext}} = \mathbf{0}. \quad (10)$$

$\mathbf{f}_t$ is the vector arranging $f_{t,i,j}$, and $\mathbf{J}_t$ is the Jacobi matrix that transforms $\mathbf{f}_t$ into a combined force-moment. The matrix component $\mathbf{J}_{t,i,j}$ of $\mathbf{J}_t$ corresponding to $f_{t,i,j}$, is expressed as follows:

$$\mathbf{J}_{t,i,j} = \begin{bmatrix} \mathbf{n}_{t,i,j} \\ (\mathbf{x}_{\text{cont},i} - \mathbf{x}_G) \times \mathbf{n}_{t,i,j} \end{bmatrix}^T. \quad (11)$$

2.3.2 Constraints of belt-object geometry From (1), the geometric constraint for the contact point $\mathbf{x}_{\text{cont},i}$, the belt anchor point $\mathbf{x}_{\text{anc},j}$, and the tension between them $f_{t,i,j}$ is expressed as follows:

$$C_{\text{ten},i,j} = |\tilde{x}_{\text{anc},j} - \tilde{x}_{\text{cont},i}|^2 - |\mathbf{x}_{\text{anc},j} - \mathbf{x}_{\text{cont},i}|^2 \geq 0 \quad (12)$$

$$0 \leq C_{\text{ten},i,j} \perp f_{t,i,j} \geq 0. \quad (13)$$

This constraint prevents the object from penetrating the belt. Let $\mathbf{p}_{\text{cont,edge},i}$ be the point on the $\mathbf{x}_{\text{anc},i}$ -side edge of the contact area. The minimum value of $C_{\text{ten},i,j}$ is always obtained at $\mathbf{p}_{\text{cont,edge},i}$ and the case where $C_{\text{ten},i,j} = 0$ and $f_{t,i,j} > 0$ can only occur at $\mathbf{p}_{\text{cont,edge},i}$. Thus, the tension in (10) only needs to be considered at $\mathbf{p}_{\text{cont,edge},i}$, ($i = 1, 2$). At other points, $f_{t,i,j} = 0$ and the constraint (13) does not need to be considered. If $C_{\text{ten},i,j} > 0$ and $f_{t,i,j} = 0$ at $\mathbf{p}_{\text{cont,edge},i}$, the belt sags, and the object is close to a jump.

2.3.3 Constraints of contact state The normal forces are non-negative, and the frictional forces and relative velocities are in opposite directions. Thus, the following equations hold:

$$f_{n,i} \geq 0 \quad (14)$$

$$\nu_i - \dot{\tilde{x}}_{\text{cont},i} = 0 \quad (15)$$

$$\nu_i f_{f,i} \leq 0. \quad (16)$$

Slipping is modeled based on Coulomb's friction law. Assuming that the static friction coefficient μ_s and the dynamic friction coefficient μ_d are equal between the belt and the object ($\mu_s = \mu_d = \mu$), the following complementarity constraint holds:

$$0 \leq \mu^2 f_{n,i}^2 - f_{f,i}^2 \perp \nu_i^2 \geq 0. \quad (17)$$

Specifically, slipping does not occur when the contact force is within the friction cone ($\nu_i = 0$ when the left of (17) is positive), and slipping occurs when the contact force is on the side of the friction cone ($|\nu_i| \geq 0$ when the left of (17) is zero).

2.4 Optional constraints

2.4.1 Hypothetical constraints of normal force distribution One of the issues with the proposed constraint-based modeling method is the redundancy of the contact forces. There are as many contact forces as the number of contact points, but there are only three equations, as shown in (6). Thus, the contact forces are not uniquely determined. Therefore, we consider a hypothetical constraint on the distribution of the contact forces, as shown in Fig. 5(a). Assuming that there are two adjacent points $\mathbf{p}_{\text{cont},i}$ and $\mathbf{p}_{\text{cont},i+1}$ in the contact area and that $\mathbf{p}_{\text{cont},i+1}$ is inside the contact area (not $\mathbf{p}_{\text{cont,edge},j}$, ($j = 1, 2$)). The hypothetical constraints are expressed as follows:

$$(1 + \alpha)f_{n,i+1} - f_{n,i} \geq 0 \quad (18)$$

$$\alpha = \xi_{\text{dist}} \frac{|\mathbf{x}_{\text{cont},i} - \mathbf{x}_{\text{cont},i+1}|}{\sum_{i=1}^{n_{\text{obj}}} |\mathbf{x}_{\text{cont},i} - \mathbf{x}_{\text{cont},i+1}|}, \quad (19)$$

where ξ_{dist} is an adjustable parameter. These constraints are defined considering the fact that the belt-object system has a surface contact area and the normal force distribution varies continuously. Moreover, α is determined such that the shorter the distance of the adjacent points, the smaller the change in $f_{n,i}$. The larger ξ allows for variation in $f_{n,i}$, and the smaller ξ becomes more rigid about variation in $f_{n,i}$. Furthermore, (18) is applied bi-directionally to the adjacent points, that is, $f_{n,i}$ has upper and lower bounds when $\mathbf{p}_{\text{cont},i}$ is inside the contact area, and $f_{n,i}$ has an upper bound when $\mathbf{p}_{\text{cont},i}$ is the edge point of the contact area. Note that this constraint is only hypothetical, and there is room for improvement.

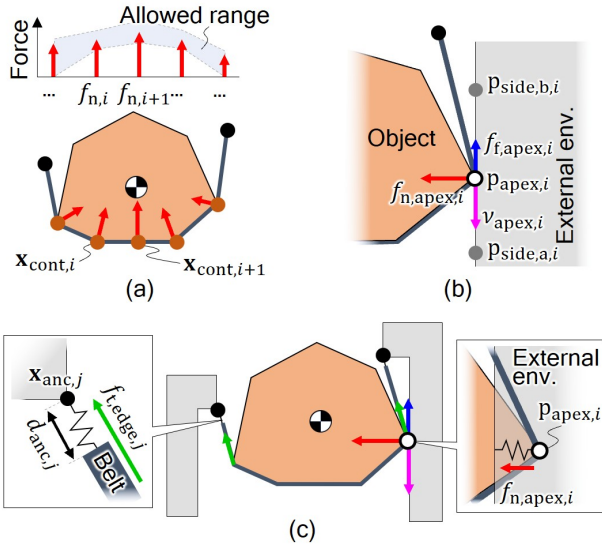


Figure 5. Optional constraints. (a) Constraints for normal force distribution. (b) Constraints for external environments. (c) Constraints for relaxing constraint violations.

2.4.2 External environmental constraints To analyze the belt-attached hand and object system, it is necessary to consider the contact between the object and external surfaces such as the finger pads and fingertips. For this analysis, we can apply the typical rigid multibody dynamics modeling method (Bender et al., 2014). Assuming there is an apex $p_{apex,i}$ and a side $p_{side,a,i}$ - $p_{side,b,i}$ between the external environment and object that may be in contact, as illustrated in Fig. 5(b), the following constraints are satisfied:

$$C_{env,i} = \xi_{env} \frac{(\mathbf{x}_{apex,i} - \mathbf{x}_{side,a,i}) \times (\mathbf{x}_{side,b,i} - \mathbf{x}_{side,a,i})}{|\mathbf{x}_{side,b,i} - \mathbf{x}_{side,a,i}|} \geq 0 \quad (20)$$

$$0 \leq C_{env,i} \perp f_{n,apex,i} \geq 0, \quad (21)$$

where $C_{env,i}$ is the signed distance between $p_{apex,i}$ and $p_{side,a,i}$ - $p_{side,b,i}$ and $f_{n,apex,i}$ is the normal force at $p_{apex,i}$. ξ_{env} is a signed parameter for no penetration ($\xi_{env} = -1$ or $\xi_{env} = 1$). Considering the collision between the object and the external environment, the normal force $f_{n,apex,i}$, friction force $f_{f,apex,i}$ and relative velocity $v_{apex,i}$ are newly defined as unknown variables at the apex points $p_{apex,i}$ ($i = 1, \dots, n_{env}$, n_{env} is the number of possible collision apexes), and the external force $\mathbf{f}_{ext}$ is replaced as follows:

$$\mathbf{f}_{ext} = \mathbf{J}_{n,apex}^T \mathbf{f}_{n,apex} + \mathbf{J}_{f,apex}^T \mathbf{f}_{f,apex} + \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix}, \quad (22)$$

where, $\mathbf{J}_{n,apex}$ and $\mathbf{J}_{f,apex}$ are matrices replacing (8) and (9) with respect to p_{apex} , respectively. The direction of the normal force is perpendicular to $p_{side,a,i}$ - $p_{side,b,i}$ and directed to the object. The direction of the friction force is the direction of the normal force multiplied by $\mathbf{R}(-\pi/2)$. By substituting (22) in the equations of motion (6) and (10) and imposing contact constraints (15), (16), (17), and (21) on $p_{apex,i}$, the analysis can be performed taking the external environment into account. Note that these constraints are applicable irrespective of whether the object point corresponds to the apex $p_{apex,i}$ or sides $p_{side,a,i}$ and $p_{side,b,i}$. Moreover, interactions between the belt and external environment are ignored.

2.4.3 Relaxing constraint violations with elastic elements When a displacement-based input is provided to the belt to grasp the object with the belt or external environment, the geometric constraints can be violated. Thus, we consider relaxing the violation by introducing an elastic element in the constraints (12) and (20), as shown in Fig. 5(c). For the constraint between the belt and the object $C_{ten,i,j}$ shown in (12), the position of the belt anchor point $\tilde{x}_{anc,j}$ is transformed as $\tilde{x}'_{anc,j}$ as follows:

$$\tilde{x}'_{anc,j} = \tilde{x}_{anc,j} + d_{anc,j} \xi_{anc,j}, \quad (23)$$

where, $\xi_{anc,j}$ is a signed parameter so that the belt extends as $d_{anc,j}$ increases ($\xi_{anc,j} = 1$ or $\xi_{anc,j} = -1$). $d_{anc,j}$ is the displacement of the position of the belt anchor point $\tilde{x}_{anc,j}$ (unknown variable). For $d_{anc,j}$, the following constraint is imposed using the tension and a sufficiently large elastic modulus k_{anc} :

$$f_{t,edge,j} - k_{anc} d_{anc,j} = 0 \quad (24)$$

The constraint between the object and the external environment $C_{env,i}$ shown in (20) is added to the terms with the normal force as follows:

$$C'_{env,i} = C_{env,i} + \frac{1}{k_{env}} f_{n,apex,i}, \quad (25)$$

where k_{env} is a sufficiently large elastic modulus. These constraints allow the constraints $C_{ten,i,j}$ and $C'_{env,i}$ to be relaxed according to $f_{t,i,j}$ and $f_{n,apex,i}$, respectively.

3 Analysis of dynamics of the belt-object system

We discuss a problem of solving for the object position $\mathbf{x}_G$, object orientation θ_G , position of the contact representative point $\tilde{x}_c$, normal forces $\mathbf{f}_n$, friction forces $\mathbf{f}_f$, tensions $\mathbf{f}_t$, and belt-object relative velocities $\mathbf{v}$ as unknown variables with the positions of belt anchor points $\tilde{x}_{anc}$ and $\mathbf{x}_{anc}(t)$, standard positions of the object points $\mathbf{x}_{obj}^*$ and $\tilde{x}_{obj}^*$, mass m , and inertia moment I_G as known variables. Hereafter, let $\mathbf{x}_G$, θ_G , $\tilde{x}_c$ are grouped together as the state vector $\mathbf{q} = [\mathbf{x}_G^T, \theta_G, \tilde{x}_c]^T$.

3.1 Solving flow of the problem

Fig. 6 shows the flow of solving the dynamics problem. The problem is solved discretely for each time interval Δt . At the beginning of each time step, the algorithm to update the contact candidate points and contact positions with the position and orientation of one previous step is executed. Next, the mNCP problem is solved to find a solution that satisfies the aforementioned constraints. As the unknown variables, the velocity of the state vector $\dot{\mathbf{q}}$; forces $\mathbf{f}_n$, $\mathbf{f}_f$, and $\mathbf{f}_t$; relative velocity $\mathbf{v}$ are determined. If the optional constraints shown in sections 2.4.2 and 2.4.3 are applied, the external forces/velocities $\mathbf{f}_{n,\text{apex}}$, $\mathbf{f}_{f,\text{apex}}$, $\mathbf{v}_{\text{apex}}$, and velocity of the belt anchor point $\dot{\mathbf{d}}_{\text{anc},i}$ are additionally determined, respectively. Next, penetration between the belt and object is compensated, and the state vector is updated from $\mathbf{q}$. Finally, the time is updated.

3.2 Contact algorithm

Algorithm 1 shows the algorithm for detecting the contact candidate points, assuming the belt anchor points are $x_{\text{anc},1} < x_{\text{anc},2}$ and $\mathbf{x}_{\text{obj}}$ are arranged in a counterclockwise direction. Fig. 7 shows the overview of the algorithm. The first step is to find points contained in the desired direction for $\mathbf{x}_{\text{anc}}$ (Algorithm 1 line 4). Here, “the desired direction” is set vertically downward and $\xi_{\text{BI}} = 1$. The points are defined as belt internal points. The next step is to create a convex hull from $\mathbf{x}_{\text{anc}}$ and the belt internal points. The points in $\mathbf{x}_{\text{obj}}$ that are contained in the convex hull are defined as the contact candidate points. By setting these contact candidate points as $\mathbf{p}_{\text{cont},i}$ in Phase (ii) shown in Fig. 6, the constraints (6) and (14)–(17) can be considered. In the contact candidate points, the point connected with the belt anchor point $\mathbf{p}_{\text{anc},i}$ is defined as the edge point of the contact area $\mathbf{p}_{\text{cont,edge},i}$. The belt area between $\mathbf{p}_{\text{anc},i}$ and $\mathbf{p}_{\text{cont,edge},i}$ is the non-contact taut area shown in Fig. 2(b) green area. The points to be considered in the constraint (13) and the equation of motion (10) are only the edge points of the contact area. This algorithm allows the contact state to be changed when the object rolls. Note that these candidate contact points do not impose the generation of contact force. When the belt is sagging, the contact force or tension at $\mathbf{p}_{\text{cont},i}$ is calculated as zero. Furthermore, note that the areas which are not on the interior side of the belt but are in contact with the belt can exist (for example, if the object is significantly shifted toward one of the fingers), but are excluded from the contact area at this stage. In this paper, this case is assumed not to occur. For Algorithm 2, the highest and lowest points in the vertical direction are initially determined. $\tilde{\mathbf{x}}_{\text{obj}}^*$ is placed so that $\tilde{x}_{\text{obj},i_{\text{top}}}^*$ is minimum, increasing counterclockwise, and $\tilde{x}_{\text{obj},i_{\text{low}}}^* = 0$. When i_{low} differs between time steps t and $t - \Delta t$, $\tilde{x}_c(t - \Delta t)$ is added to $\tilde{x}_{\text{obj},i_{\text{low}}}^*(t) - \tilde{x}_{\text{obj},i_{\text{low}}}^*(t - \Delta t)$. This algorithm allows the

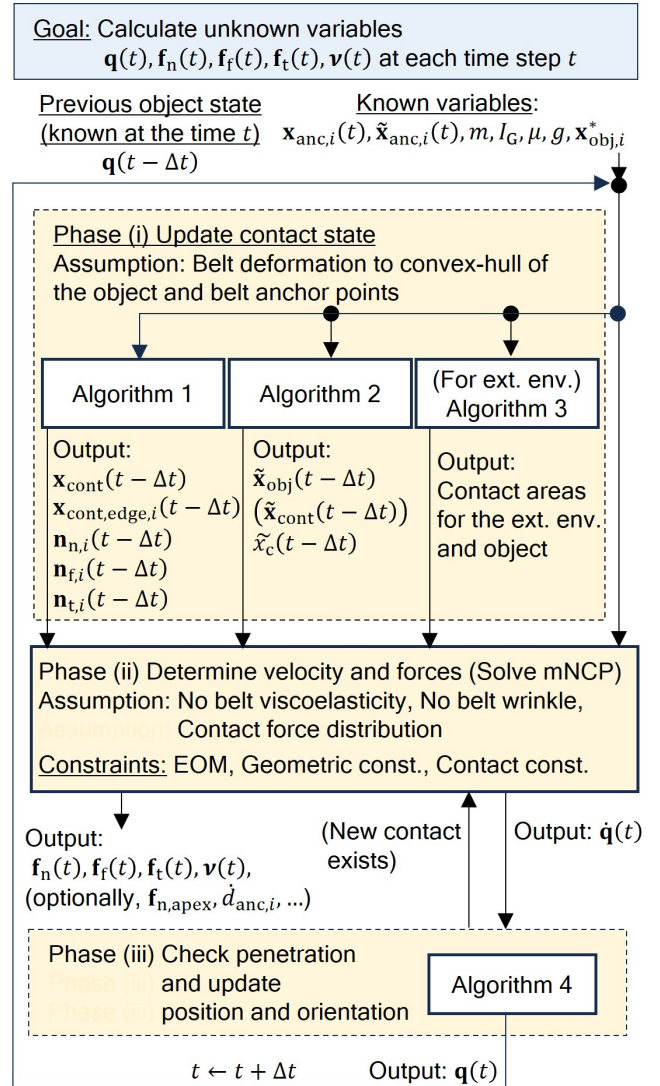


Figure 6. Flow of solving the dynamics.

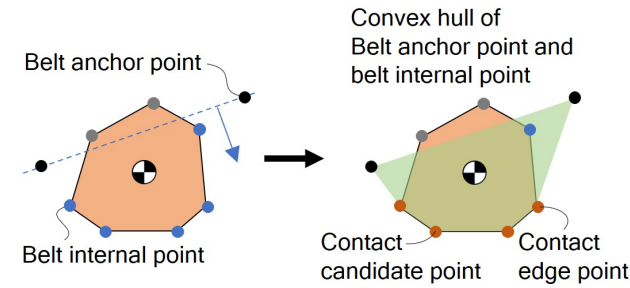
contact position $\tilde{x}_{\text{cont},i}$ to be synchronized when the contact state changes due to rolling, etc. When contacts with external environments are enabled, possible contact points are detected at Phase (i). Algorithm 3 shows the contact detection algorithm between the external environment and the object used in this paper. Note that the algorithm can be applied to nonconvex external environments and objects by decomposing the geometry into convex hulls. Note that the object’s position and orientation used in Algorithms 1, 2, and 3 are the results obtained in Phase (iii) of the previous time step, and that the positions of the external environment and belt anchor points are used as values at the current time, given as known variables.

Algorithm 1 Finding contact candidate points.

```

1:  $CHP \leftarrow$  empty set ▷ Input points for convexhull
2:  $CCP \leftarrow$  empty set ▷ Contact candidate point
3: for  $i = 1$  to  $n_{obj}$  do
4:   if  $\xi_{BI}(\mathbf{x}_{obj,i} - \mathbf{x}_{anc,1}) \times (\mathbf{x}_{anc,2} - \mathbf{x}_{anc,1}) > 0$  then
5:      $IP \leftarrow \mathbf{x}_{obj,i}$  ▷ Belt internal point
6:   end if
7: end for
8:  $CHP \leftarrow \mathbf{x}_{anc}(t)$  and  $IP$ 
9:  $CH =$  convexhull of  $CHP$ 
10: for  $i = 1$  to  $n_{obj}$  do
11:   if  $\mathbf{x}_{obj,i}$  is contained to  $CH$  then
12:      $CCP \leftarrow P_{obj,i}$ 
13:   if  $\mathbf{x}_{obj,i}$  is connected to  $\mathbf{x}_{anc,1}$  or  $\mathbf{x}_{anc,2}$  then
14:      $P_{obj,i}$  is defined to contact edge point
15:   end if
16: end if
17: end for
18: return  $CCP$  as  $P_{cont,1}, \dots, P_{cont,n_{cont}}$ 

```

**Figure 7.** Overview of Algorithm 1.**Algorithm 2** Updating contact position in Σ_{cloth} .

```

1:  $i_{low}(t) \leftarrow$  Index of  $\min y_{obj}(t - \Delta t)$ 
2:  $i_{top}(t) \leftarrow$  Index of  $\max y_{obj}(t - \Delta t)$ 
3: for  $j = 1$  to  $n_{obj}$  do
4:   if  $j = 1$  then
5:      $\tilde{x}_{obj,i_{top}}^* = 0$ 
6:   else
7:      $i' = (i_{top} + j) \bmod n_{obj}$ 
8:      $\tilde{x}_{obj,i'}^* = \tilde{x}_{obj,i'-1}^* + |\mathbf{x}_{obj,i'}^* - \mathbf{x}_{obj,i'-1}^*|$ 
9:   end if
10: end for
11:  $\tilde{\mathbf{x}}_{obj}^* \leftarrow \tilde{\mathbf{x}}_{obj}^* - \tilde{x}_{obj,i_{low}}^*$ 
12:  $\tilde{x}_c(t - \Delta t) \leftarrow \tilde{x}_c(t - \Delta t) + \tilde{x}_{obj,i_{low}}^*(t) - \tilde{x}_{obj,i_{low}}^*(t - \Delta t)$ 

```

3.3 Determination of velocity and force

The problem of determining a solution that satisfies constraints (6)–(17) (and optionally (18)–(25)) is solved using mNCP as a general-purpose solver. Thus, the constraints should be simplified so that unknown variables of the state vector are only velocities. For the equations

Algorithm 3 Finding contact candidate pairs of the external environment and the object.

```

1: for  $i$  to number of the external environment points do
2:    $\mathbf{x}_{apex,i} \leftarrow$  the external environment point positions
3:   for  $j$  to number of the object sides do
4:      $\mathbf{x}_{side,a,j}, \mathbf{x}_{side,b,j} \leftarrow$  the object side
5:      $l_{env,i,j} \leftarrow$  signed distance between
6:      $\mathbf{x}_{apex,i}$  and  $\mathbf{x}_{side,a,j} - \mathbf{x}_{side,b,j}$ 
7:     if  $\angle \mathbf{x}_{apex,i} \mathbf{x}_{side,a,j} \mathbf{x}_{side,b,j} \leq \frac{\pi}{2}$  and
8:        $\angle \mathbf{x}_{apex,i} \mathbf{x}_{side,b,j} \mathbf{x}_{side,a,j} \leq \frac{\pi}{2}$  then
9:        $Internal_{i,j} = \text{True}$ 
10:    end if
11:  end for
12:  if All  $j$  in  $l_{env,i,j} \leq 0$  then ▷ Internal of the object
13:     $ContPair = (i, j)$  in  $Internal_{i,j} = \text{True}$  and  $\max(l_{env,i,j})$ 
14:     $Constraints \leftarrow$  Const. about  $\mathbf{x}_{apex,i}, \mathbf{x}_{side,a,j}, \mathbf{x}_{side,b,j}$ 
15:  else ▷ outside of the object
16:     $ContPair = (i, j)$  in  $Internal_{i,j} = \text{True}$  and  $0 \leq l_{env,i,j} \leq \varepsilon$ 
17:     $Constraints \leftarrow$  Const. about  $\mathbf{x}_{apex,i}, \mathbf{x}_{side,a,j}, \mathbf{x}_{side,b,j}$ 
18:  end if
19: end for
20: Switch side and apex for external environment and object

```

of motion (6) and (10) at time t , the Jacobi matrices $\mathbf{J}_n$, $\mathbf{J}_f$, and $\mathbf{J}_t$ are obtained by substituting the object position and orientation at the previous time step $t - \Delta t$ into the equations (8)(9)(11). The object accelerations are obtained as follows:

$$\begin{bmatrix} \ddot{x}_G(t) \\ \ddot{y}_G(t) \\ \ddot{\theta}_G(t) \end{bmatrix} = \frac{1}{\Delta t} \begin{bmatrix} \dot{x}_G(t) - \dot{x}_G(t - \Delta t) \\ \dot{y}_G(t) - \dot{y}_G(t - \Delta t) \\ \dot{\theta}_G(t) - \dot{\theta}_G(t - \Delta t) \end{bmatrix}. \quad (26)$$

Through Taylor series expansion, the constraints (12)(20) at the time t are linearized as follows:

$$C_{ten,i,j}(t) = C_{ten,i,j}(t - \Delta t) + \frac{\partial C_{ten,i,j}(t - \Delta t)}{\partial \mathbf{q}} \dot{\mathbf{q}}(t) \Delta t + \frac{\partial^2 C_{ten,i,j}(t - \Delta t)}{\partial t^2} \Delta t^2 \quad (27)$$

$$C_{env,i}(t) = C_{env,i}(t - \Delta t) + \frac{\partial C_{env,i}(t - \Delta t)}{\partial \mathbf{q}} \dot{\mathbf{q}}(t) \Delta t + \frac{\partial^2 C_{env,i}(t - \Delta t)}{\partial t^2} \Delta t^2. \quad (28)$$

For the complementarity problem in (13), (17), and (21), the smoothed Fischer-Bermister function (Fukushima et al., 1998) is used as a differentiable continuous function. When $0 \leq a \perp b \geq 0$, the complementarity constraint using the smoothed Fischer-Bermeister function is expressed as follows:

$$\sqrt{a^2 + b^2} + \varepsilon - a - b = 0, \quad (29)$$

where ε is a sufficiently small number. With these processes, the unknown variables satisfying constraints can

be determined by a solver for mNCP.

Table 1 shows the numbers of the unknown variables and constraints in Phase (ii). The mesh-based model is modeled according to (Bender et al., 2014), where the belt and belt-object contact are represented by n_{belt} mass nodes and n_{cont} nodes of the belt, respectively (see Appendix A for details). The unknown variables and constraints need to be defined for each belt node: thus, there is a trade-off between the simplicity of the problem and the reproducibility of the adaptive deformation of the belt. By contrast, in the proposed model, the unknown variables to be determined in Phase (ii) are $\dot{\mathbf{q}}, \mathbf{f}_n, \mathbf{f}_t, \boldsymbol{\nu}, \mathbf{f}_t$, for a total of $6 + 3n_{\text{cont}}$. The constraints are the equations of motion (6)(10), contact constraints (14)–(17), and geometric constraints (13), for a total of $8 + 4n_{\text{cont}}$. When the force distribution (18), external environments (20), and elastic relaxation (23) and (25) are considered, the number of variables increases by 0, $3n_{\text{env}}$, and 2, and the number of constraints increases by $2n_{\text{cont}} - 2$, $4n_{\text{env}}$, and 2, respectively. From Phase (i), the number of the contact points n_{cont} depends on the number of the points of the object. Thus, the numbers of the unknown variables and the constraints depend on n_{obj} . Therefore, the feature that the proposed model is independent of the resolution and the scale of the belt is a significant advantage in the analysis that considers the adaptability of the belt. Note that several linearization methods and fast numerical solution methods have been proposed for the mesh-based model. If the normal force at each belt node is calculated based on geometric penetration, such as in (25), the effect of redundancy of the contact force is reduced. Note that the geometric constraint (12) can be linearly approximated, and this problem can be described as a mixed linear complementarity problem by linearizing (16) and (17), but this is beyond the scope of this paper.

Table 1. Numbers of unknown variables and constraints.

	Mesh-based model (2D)	Proposed model
Unknown variables	$3(n_{\text{belt}} + n'_{\text{cont}}) - 2$	$6 + 3n_{\text{cont}}$
EOM	$2n_{\text{belt}} - 1$	6
Geometric const.	$n_{\text{belt}} - 1$	2
Contact const.	$4n'_{\text{cont}}$	$4n_{\text{cont}}$

3.4 Compensation for penetration

Fig. 8 shows the switching of the contact state by Phases (i) and (ii). When the contact point detaches, the belt is momentarily deformed concave at the edge point of the contact area $p_{\text{cont,edge},i}$ after the optimization of Phase (ii). In Phase (i), at the next time step, $p_{\text{cont,edge},i}$ is out of the contact candidate points. When the object point makes a new contact, the point temporarily penetrates from the belt after Phase (ii) and becomes the new contact candidate

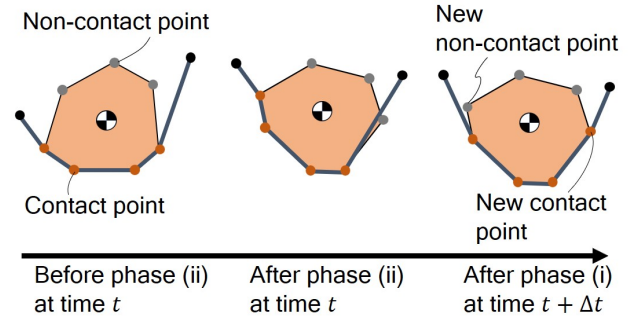


Figure 8. Change of contact state between the belt and object.

Algorithm 4 Compensation for penetration.

```

1:  $\mathbf{q}'(t) = \mathbf{q}(t - \Delta t) + \dot{\mathbf{q}}(t)$   $\triangleright$  Input points for convexhull
2:  $CCP_1 = \text{Algorithm 1} (CHP \leftarrow \mathbf{x}_{\text{anc}}(t), \mathbf{x}_{\text{obj}}(\mathbf{q}(t - \Delta t)))$ 
3:  $CCP_2 = \text{Algorithm 1} (CHP \leftarrow \mathbf{x}_{\text{anc}}(t), \mathbf{x}_{\text{obj}}(\mathbf{q}'(t)))$ 
4: if All  $CCP_2$  is exist on  $CCP_1$  then  $\triangleright$  No new contact point
5:    $\mathbf{q}(t) \leftarrow \mathbf{q}'(t)$ 
6: else
7:    $CCP'_2 \leftarrow CCP_1 + \text{newly added points in } CCP_2$ 
8:   Solve mNCP using  $CCP'_2$ 
9:    $\dot{\mathbf{q}}'(t) \leftarrow \text{Solution mNCP using } CCP'_2$ 
10:   $\mathbf{q}(t) \leftarrow \mathbf{q}(t - \Delta t) + \dot{\mathbf{q}}'(t)\Delta t$ 
11: end if

```

point at the next time step Phase (i). This penetration may cause large contact forces and tension at the next time step Phase (ii). Thus, the process where the change of the contact state is checked again, and the penetration of the object is prevented is introduced. Algorithm 4 shows the algorithm for preventing penetration and updating the state vector.

4 Experimental verification

In this section, we compare the simulation based on the proposed method to real-world experiments for object movements when inputs are provided to the belt anchor points and validate the proposed method. Fig. 9(a) shows the actual belt-attached hand for the experiments. The size of the hand is 180 mm in width, 177 mm in height, and 84 mm in depth. The hand has two fingers with vertically movable fingertips and a gripper that can change the interval between fingers. The belt is 7×12 cm in size and made of 100% polyester satin-like fabric; this fabric approximates the ideal conditions described in Assumptions 1, 2, and 3. Figs. 9(b) and (c) show the objects for the experiments. Both objects are prismatic with a height of 4 cm. The cross-sectional shapes of the objects in Figs. 9(b) and (c) are a circle of 3.0-cm in diameter and an equilateral triangle of 2.9-cm on each side, respectively. The object in Fig. 9(c)

has rounded corners with a radius of 2-mm. The masses of the objects in Figs. 9(b) and (c) are 0.016 kg and 0.012 kg. The friction coefficients were estimated as the static friction coefficient $\mu_s = 0.32$ and the dynamic friction coefficient $\mu_d = 0.27$ by sliding the object at speed 500 mm/min and measuring the force response. The experiment was captured by the vision system in front of the hand. The position and orientation of the object relative to the hand were measured by the red marker on the front of the hand and the blue marker on the bottom of the object.

Next, the simulation settings are described. Figs. 9(d) and (e) show the shapes of the objects $\mathbf{x}_{obj}^*$ for the simulation. The shapes of the objects in Figs. 9(d) and (e) reproduce the object in Fig. 9(b) as a regular hexadecagon, and that in Fig. 9(c) as an equilateral triangle whose corners are rounded with four points, respectively. The masses of the objects are the same as that of the experiments, and the internal moments of the objects in Figs. 9(d) and (e) are set to $0.018 \text{ kg} \cdot \text{cm}^2$ and $0.012 \text{ kg} \cdot \text{cm}^2$, respectively. The friction coefficients between the belt and the object and between the external environments and the object are both set to $\mu = 0.27$. The time step is $\Delta t = 0.005 \text{ s}$. The normal force distribution constraint shown in (18) was enabled, and the normal force distribution parameter in (19) was set to $\xi_{\text{dist}} = 3.0$. In Appendix B, the simulation results for the cases where the constraint (18) was not enabled and where ξ_{dist} was varied are presented. Moreover, the relaxation of constraint violations with elastic elements was enabled. The elastic parameters shown in (24) and (25) were set to $k_{\text{anc}} = 10^7 \text{ N/cm}$, $k_{\text{env}} = 10^5 \text{ N/cm}$, respectively, which were determined by trial and error to sure stability. Considering the shape of the actual belt-attached hand, four points $\mathbf{x}_{\text{anc},1}$, $\mathbf{x}_{\text{anc},1} + [0, 6]^T \text{ mm}$, $\mathbf{x}_{\text{anc},2}$, and $\mathbf{x}_{\text{anc},2} + [0, 6]^T \text{ mm}$, and two sides $x \leq x_{\text{anc},1} - 10 \text{ mm}$ and $x \geq x_{\text{anc},2} + 10 \text{ mm}$ are placed as the external environments. The QHull (Barber et al., 1996) method was used to generate the convex hull shown in Fig. 6 Phase (i), and IPOPT (Wächter and Biegler, 2006) was used to solve the mNCP shown in Fig. 6 Phase (ii) (the objective function is not set). IPOPT is a solver for nonlinear optimization problems based on a principal dual interior point method. Assuming ($n_{\text{cont}} = n_{\text{obj}}$, $n_{\text{env}} = 6$), the maximum number of unknown variables in Phase (ii) for the objects in Figs. 9(d) and (e) are 74 and 80, respectively. The maximum number of constraints for the objects in Figs. 9(d) and (e) are 126 and 138, respectively. Extension #1 shows the movies of the real-world experiments and animations of the results of the simulation analysis. Extension #2 provides a set of graphs showing the object's transitions in position and orientation and quantitative comparisons such as RMSE in the simulation and the real-world experiments.

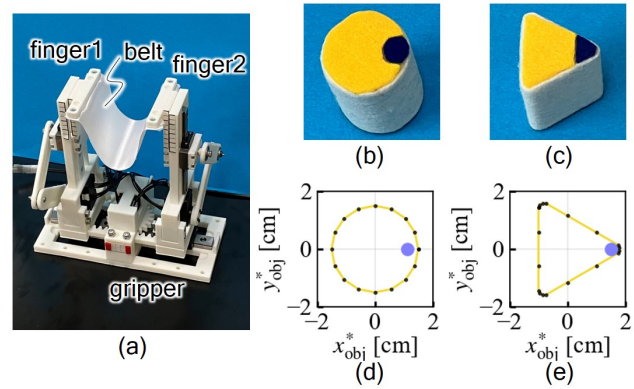


Figure 9. Experimental verification setting. (a) Belt-attached hand for experiments. (b) Cylindrical object for experiments. (c) Triangular prismatic object for experiments. (d) Cylindrical object for simulations. (e) Triangular prismatic object for simulations.

4.1 Behavior of the object with a vertical displacement input

First, we examined the behavior of the object for the vertical displacement inputs. The smoothing step inputs $y_{\text{anc},1} = -y_{\text{step}}$ and $y_{\text{anc},2} = y_{\text{step}}$ (y_{step} is the step input amplitude) were provided to the fingertips for one second. The horizontal interval of fingertips was fixed at a constant value, and this interval was denoted as A . Fig. 10(a) shows the results of the experiments and simulations for the cylindrical object, where the input parameters were for $y_{\text{step}} = 3 \text{ cm}$ and $A = 6 \text{ cm}$. Hereafter, the origin was set to the midpoint of the fingertips (the belt anchor points) in the initial state shown in Fig. 9(a). The experimental results are plotted for each of the three trials. The cylindrical object rolled on the belt for both the experiments and the simulations. The behavior of the experiments and the simulations were in good agreement. In the simulation, the normal forces were smoothly distributed. Figs. 10(b) and (c) show the positions and orientations at $t = 2 \text{ s}$ of the cylindrical object and triangular prismatic object, respectively. The input parameters were set to $A = \{6, 10\} \text{ cm}$ and $y_{\text{step}} = \{2, 3\} \text{ cm}$. As shown in Fig. 10(b), the cylindrical object rolled on the belt, and the object positions and orientations were varied for input parameters A and y_{step} . Under all parameter conditions, the experimental and simulated values were in good agreement. The slight errors in position and orientation are not only caused by inaccuracies of the simulations but also by assembly errors in the belt attachment and measurement errors by the vision system in the experiments. As shown in Fig. 10(c), the triangular prismatic object rolled for $A = 6 \text{ cm}$ in both cases of the experiments and simulations. In the simulation, the object swayed sideways after $t = 1 \text{ s}$ when the belt anchor points stopped, which did not happen

in the experiment. As for $A = 10$ cm, the object was moved on the belt with slipping in both cases of the experiments and simulations, but slipperiness differed between the experiments and simulations. In the experiments, the object did not slip for $y_{\text{step}} = 2$ cm, and slipped without colliding with the fingertip for $y_{\text{step}} = 3$ cm. On the other hand, in the simulations, the object slipped for both $y_{\text{step}} = 2, 3$ cm, and the object slipped and collided with the fingertip for $y_{\text{step}} = 3$ cm. Possible reasons for the oscillatory behavior are that the contact points switched discretely and that the bending viscosity of the belt was neglected. A possible reason why the object slipped more in the simulations is that the friction coefficient in (17) was set to the dynamic friction coefficient estimated in the experiment, and there was no distinction between the dynamic and static friction coefficients. Moreover, it is also possible that inaccurate contact forces from reality were applied to the solution: although the equation of motion (6) and the assumption of contact force distribution (18) guarantees some physical consistency of distributional contact forces, the accuracy of the contact forces is not guaranteed due to the redundancy of the contact forces. The behaviors of the simulations differ in vibration and slipperiness but were qualitatively consistent with the behavior of the experiments. The simulation can reproduce the changes in the experiments in rolling and slipping due to the fingertips interval and the object shapes. Table 2 lists the total simulation computation time ($0 \leq t \leq 2$ s) and the average of iterations required to solve the mNCP (Fig. 6 Phase (ii)) in simulations illustrated in Fig. 10. The average of iterations reflects the difficulty of searching for solutions (i.e., how long it takes to converge). The simulation computation time depends on these iterations, as well as the number of unknown variables and constraints of the model. As depicted in Table 2, for both the cylindrical object and the triangular prismatic object, the simulation computation time and the average of iterations increased for large values of A and y_{step} . This indicates that the search for a solution is more difficult when the contact area changes due to rolling or when collisions with the external environment occur.

Table 2. Calculation costs of the simulations shown in Fig. 10.

A [cm]	y_{step} [cm]	Cyl. obj.		Tri. obj.	
		Sim. time [s]	Ave. iters	Sim. time [s]	Ave. iters
6	2	106	7.0	180	28.4
6	3	131	16.2	197	36.4
10	2	111	14.5	99	7.3
10	3	361	117.2	285	45.8

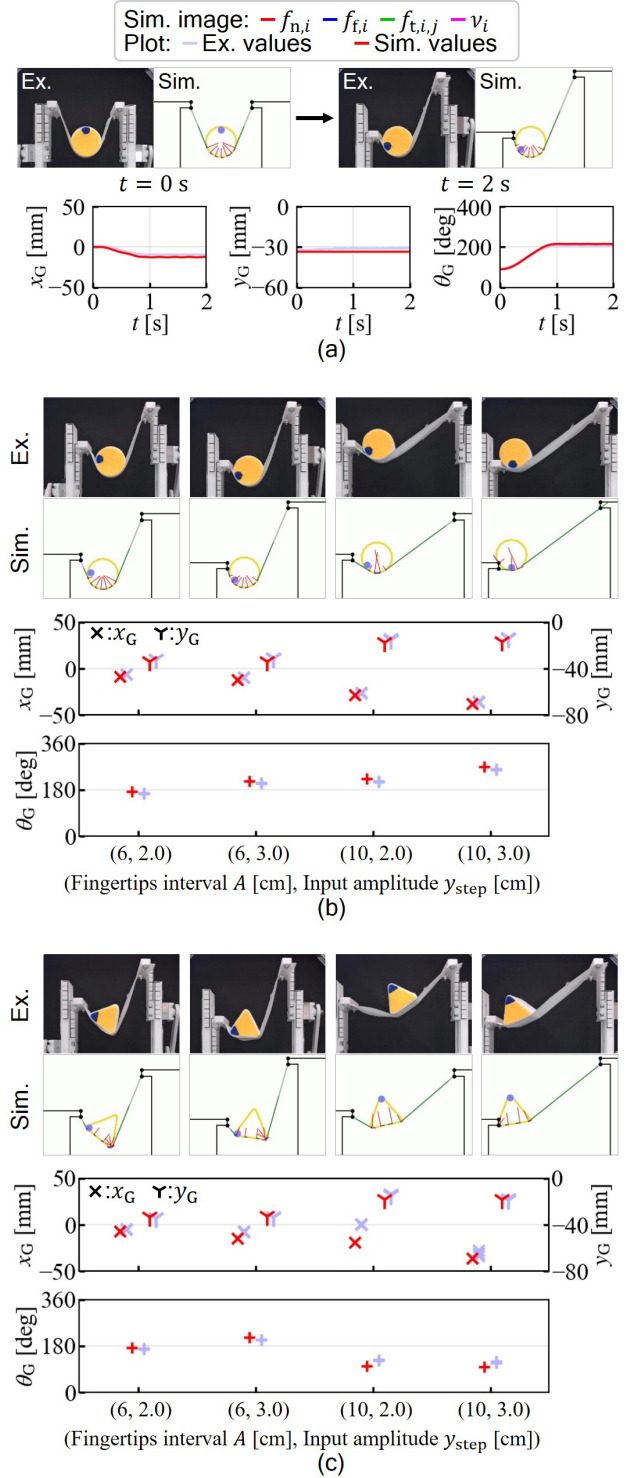


Figure 10. Results for vertical step inputs. (a) Change of the object state by the step input. (b) End position and orientation of the cylindrical object. (c) End position and orientation of the triangular prismatic object.

4.2 Rotation and frequency response of the object with a vibration input

Based on the previous research (Ichikura and Higashimori, 2022), rotational manipulations for the object on the belt using slipping were performed. The fingertips interval and the vibration inputs to the fingertips were set to $A = 6$ cm and $y_{\text{anc},1} = 0$ cm and $y_{\text{anc},2} = 1.0 \sin(2\pi ft)$ cm. Fig. 11(a) shows the results of the cylindrical object when $f = 6.0$ Hz. As shown in Fig. 11(a), the object rotated continuously with slipping and rolling in both the experiments and the simulations. Figs. 11(b) and (c) show the positions at times $3 \leq t \leq 5$ s for the cylindrical object and triangular prismatic object, respectively. The averages of positions are plotted by markers and standard deviations by error bars. The standard deviation is considered to be an indicator of the amplitude of the object's position due to the vibration input. The input frequency was set to $4 \leq f \leq 7$ Hz ($\Delta f = 0.5$ Hz) to ensure the actual hand could follow the motion. As shown in Fig. 11(b), in the experiment, the cylindrical object swayed more vertically than horizontally in most frequencies, but at $f \approx 5$ Hz, the object resonated and swayed more horizontally. For $f \approx 7$ Hz, the average of x_G was positively increased. In the simulation, the object resonated at $f = 4.5$ Hz. In the case of $f \approx 7$ Hz, the average of x_G did not increase significantly. Moreover, the change in the average of y_G observed in the experiment was not observed in the simulation. As shown in Fig. 11(c), in the experiment, the triangular prismatic object resonated at $f \approx 4.5$ Hz. At $f \geq 5.5$ Hz, where the object began to rotate, the object swayed more horizontally than the cylindrical object due to the larger change in contact area during rotation. The average of x_G was positively increased for $f = 7$ Hz as in the cylindrical object. In the simulation, The object resonated at $f \approx 4$ Hz. For $f \geq 5.5$ Hz, the object rotated and swayed more than in the experiments. For the high input frequencies, the changes in the averages of x_G and y_G were not consistent between experiment and simulation. The reason why the average of x_G became positive when $f = 7$ Hz in the experiment is because the belt sagged more on the right finger (Fig. 9(a) finger 2) side than the left finger side due to the jumping of the object. Even in a sagging state, it was considered that small contact forces were generated due to the bending viscosity and thickness of the belt. Conversely, in the simulation, when the belt sagged, the object was treated as moving completely independently of the belt, with no contact occurring; this is because the combined force of tension and contact force is assumed to be equal, as shown in (6) and (10). The reason why the lateral sway of the triangular prismatic object on rotation in the simulation was larger than that in the experiment is that the damping by the bending viscosity of the belt was not applied. The bending viscoelasticity of the belt may explain

Table 3. Calculation costs of the simulations shown in Fig. 11.

f [Hz]	Cyl. obj.		Tri. obj.	
	Sim. time [s]	Ave. iters	Sim. time [s]	Ave. iters
4.0	1091	52.2	1124	46.0
5.0	1614	79.5	1156	50.6
6.0	1256	62.1	1350	60.6
7.0	1351	72.2	1701	77.9

why the resonance frequency in the experiment was larger than in the simulation: it is because the smaller bending of the belt around the fingertips shortens the length of the displaced belt at resonance. Fig. 12 shows the averages of the angular velocities $\bar{\omega}_G$ of the objects for the input frequencies ($\Delta f = 0.25$ Hz). The average of the angular velocities is calculated from the slope of the first-order approximation of the change in the orientation of the object between times $3 \leq t \leq 5$ s. When f was small, both the cylindrical object and triangular prismatic object only rolled with oscillatory motion, and their position and orientation varied around constant values. As f was increased, the object rotated continuously with slipping. As f was further increased, the angular velocity was increased. For the triangular prismatic object, the change in the orientation was more non-uniform than that for the cylindrical object, and the average of the angular velocity decreased overall. As for the simulations, the average of the angular velocity was smaller overall than in the experiment (approximately 70% at $f = 7$ Hz); this indicates that the amount of slip in the simulation deviates from reality. Although differences were observed in resonance frequency, behavior during jumping, and angular velocity of rotation due to slipping, the simulations succeeded in qualitatively reproducing the transition from the object state of no rotation to the occurrence of lateral oscillation and slippage rotation as f increased. This confirmed the effectiveness of the proposed method in rotational manipulation. Table 3 shows the total simulation computation time ($0 \leq t \leq 10$ s) and the average of iterations in simulations shown in Fig. 11. The simulation time and average number of iterations increase as the input frequency increases or resonance occurs; this indicates that it was difficult to find a solution when the object's acceleration or contact force changes violently due to jumping or collision with the fingertips.

4.3 Validation of applicability

4.3.1 Experiments for the objects with various shapes

The validation shown in sections 4.1 and 4.2 was performed on objects with various shapes. Fig. 13 shows the objects for the validation. The cross-sectional shape of the object shown in Fig. 13(a) has a high aspect ratio and is a rectangle of 1.5 cm $\times$ 3.0 cm. The shape has rounded corners with a radius of 2 mm. The cross-sectional shape of the object

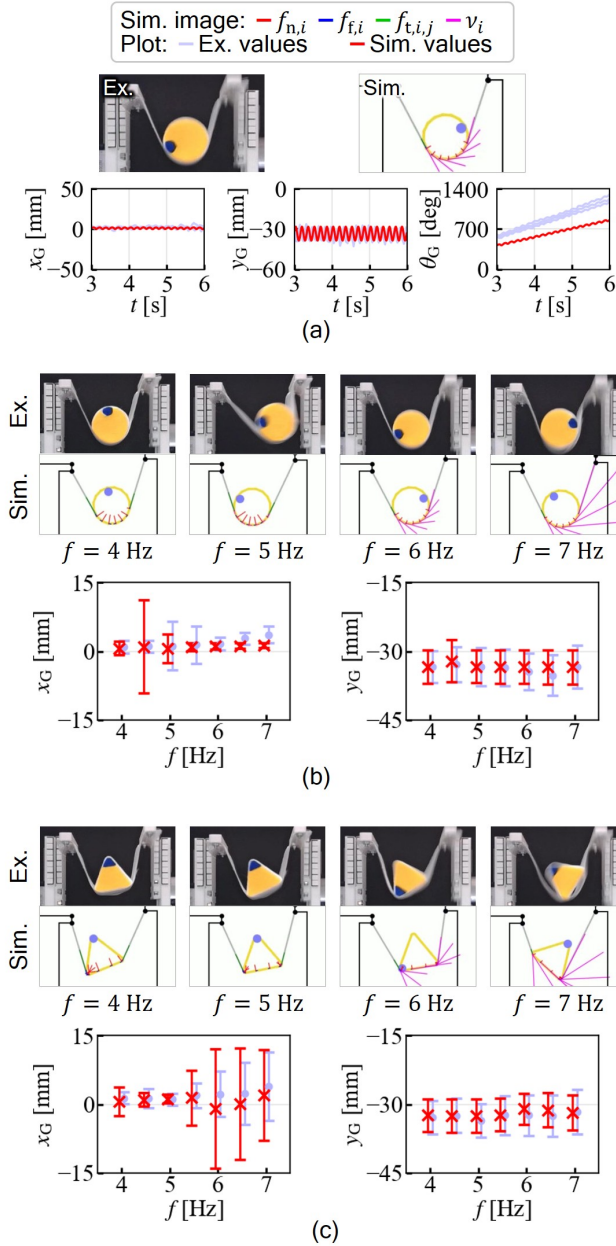


Figure 11. Results for rotational manipulation with the vibrational input. (a) Change of the object state. (b) Positions and displacements of the cylindrical object. (c) Positions and displacements the triangular prismatic object.

shown in Fig. 13(b) is non-convex: the shape consists of two overlapping ellipses, each with a minor axis of 0.9 cm and a major axis of 1.5 cm. The cross-sectional shapes of the object shown in Figs. 13(c) and (d) are a circle of 5.0 cm in diameter and an equilateral triangle of 6.0 cm on each side (rounded corners with a radius of 2 mm), respectively. The material of the object is the same as in Figs. 9(b) and (c). Fig. 14(a) shows the results, where the

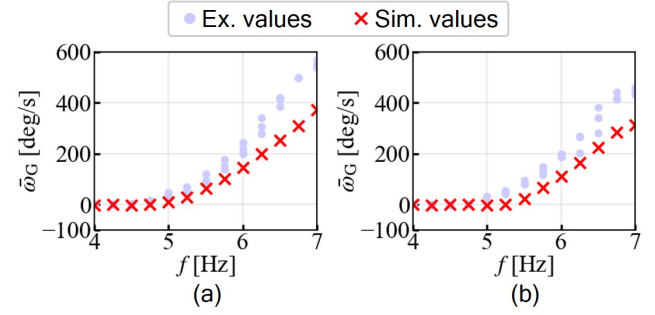


Figure 12. Average of the angular velocity for rotational manipulation. (a) For the cylindrical object. (b) For the triangular prismatic object.

input parameters were $y_{anc,1} = -3.0$ cm, $y_{anc,2} = 3.0$ cm, and $A = 10.0$ cm. The positions and orientations of the objects were plotted at $t = 2$ s. The object shown in Fig. 13(a) (Rec. obj.) slipped and stopped before colliding with the fingertips in the experiment, but in the simulation, the object slipped and collided with the fingertip. After the collision, the object swayed sideways. The object shown in Fig. 13(b) (W-ellip. obj.) rolled and collided with the fingertip in both the experiment and simulation. No contact force was generated in the concave areas of the object; contact force was generated only in the areas excluding the concave areas. The position and orientation of the object in the experiment and simulation were in good agreement. The object shown in Fig. 13(c) (L-cyl. obj.) rolled and collided with the fingertips in both the experiment and simulation, but in the simulation, the object lifted onto the fingertip more than in the experiment; this indicates that the slip displacement after the collision was different from that in the experiment. In contrast, for the object shown in Fig. 13(d) (L-tri. obj.), the simulation failed after collision with the fingertip. This is because the large size of the object and the wide width of the fingertips make it easier to tolerate solutions in which the object moves outside the belt. Fig. 14(b) shows the results, where the input parameters were $y_{anc,1} = 0.0$ cm, $y_{anc,2} = 1.0 \sin(14\pi t)$ cm, and $A = 6.0$ cm. The averages and standard deviation of the object position and the average angular velocity were plotted at $3 \leq t \leq 5$ s. The object shown in Fig. 13(a) (Rec. obj.) rotated, swayed sideways, and collided with the fingertips in both the experiment and simulation. The object shown in Fig. 13(b) (W-ellip. obj.) rotated and swayed unsteadily in both the experiment and simulation, but the amplitude of the lateral motion in the simulation was smaller than that in the experiment; this may be due to the difference in resonance frequencies seen in Fig. 11. Although this object is non-convex, the belt deformed along its convex hull, resulting in behavior similar to that of the cylindrical object in Fig. 9(b). The object shown in Fig. 13(c) (L-cyl. obj.) rotated and collided with the fingertips in both

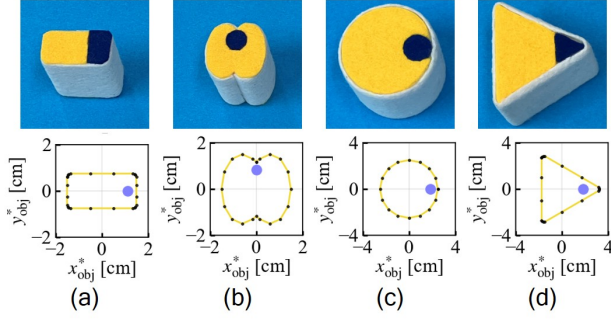


Figure 13. Shapes of the objects. (a) For the rectangular object. (b) For the double-elliptical prismatic object. (c) For the large cylindrical object. (d) For the large triangular prismatic object.

the experiment and simulation. While the average angular velocities of the objects shown in Figs. 13(a) and (b) in the simulations were smaller than that in the experiments, and the average angular velocity of the objects shown in Fig. 13(c) in the simulation was larger than that in the experiment. This is because in the simulation, there were periodic periods when the objects did not collide with the fingertips, but in the experiment, they always collided with the fingertips. Conversely, for the object shown in Fig. 13(d) (L-tri. obj.), the simulation failed at $t \approx 3$ s. This is because the collision between the object and the fingertip/finger pad frequently occurred, and a large contact force was determined as the solution. Consequently, the simulation can be applied to objects with high aspect ratios, non-convex shapes, and various sizes, although the simulation is likely to fail when large contact forces are generated due to collisions with the external environment. Simulation can qualitatively reproduce the geometric state in which the belt tautens while deforming to the object's convex hull shape, as well as the object's slipping and rolling behavior, suggesting that the proposed method is highly applicable to object shape.

4.3.2 Effect of contact configuration Fig. 15 shows the results of the simulation without the fillet of the triangular prismatic object used in Fig. 9(e). The inputs in Figs. 15(a) and (b) were the same as that in Figs. 14(a) and (b), respectively. As shown in Fig. 15, when the fillets at the corners of the object were invalid, unrealistic solutions were determined, and the simulation failed in both cases shown in Figs. 15(a) and (b). This is because the contact point positions and contact force directions change discontinuously, even though the contact force distribution constraint is imposed by the equation (18). This indicates that it is important for the stability of the calculation that the contact points of the object be placed so that the contact point position and contact force direction vary smoothly. Next, the effect of the simulation results on the number of

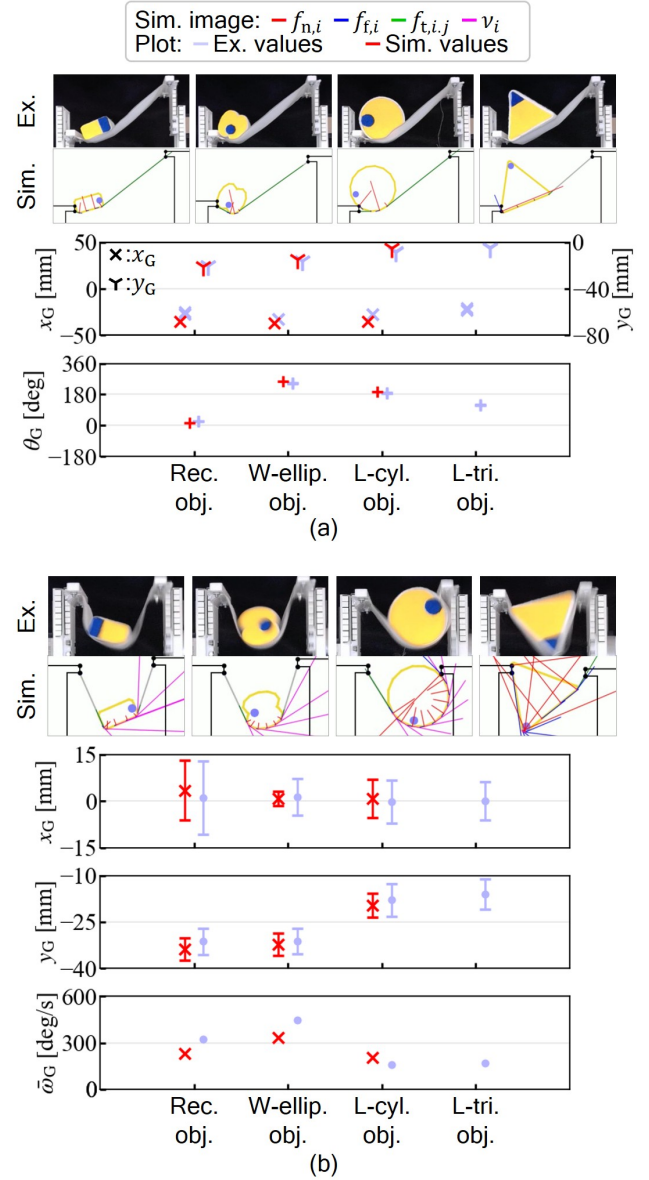


Figure 14. Results for the objects with various shapes. (a) For the vertical step input. (b) For the vertical vibrational input.

vertices of a regular polygon on the circle shown in Fig. 9(d) is investigated. The numbers of the regular polygon vertices n_{obj} were set to $4 \leq n_{obj} \leq 24$ ($\Delta n_{obj} = 4$). Fig. 16(a) shows the results where the inputs were the same as that in Fig. 14(a). The positions and orientations of the objects were plotted at $t = 2$ s. The total simulation computation time and the average of iterations are in the range of $0 \leq t \leq 2$ s. As for $n_{obj} = 4, 8$, the simulation result deviated from the experiment; this is because the object's circumference deviates significantly from the experiment due to the small number of vertices. As for $n_{obj} \geq 12$, the positions and angular velocities were almost the same for

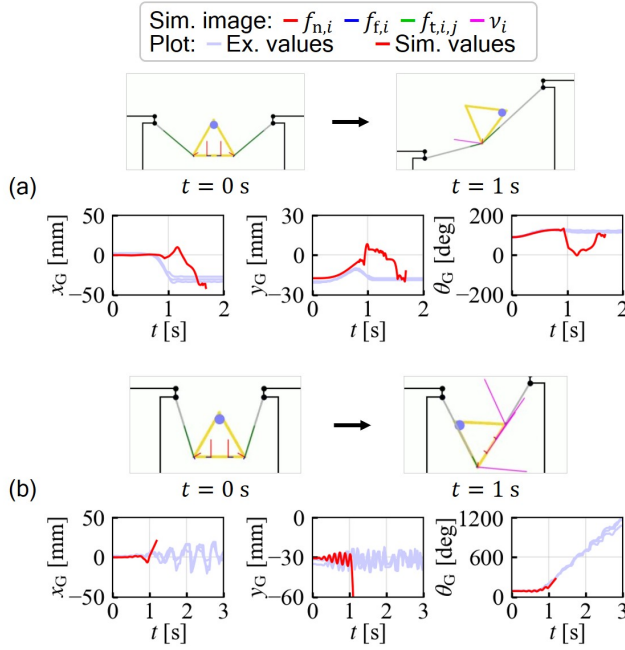


Figure 15. Results for the triangular prismatic object with no fillet in the simulation. (a) For the vertical step input. (b) For the vertical vibrational input.

n_{obj} . As for the calculation costs, both the computation time and iterations were significantly larger at $n_{obj} = 16$. This may be because there existed a point of slight contact, and the calculation of the contact force at that point took a long time. Fig. 16(b) shows the results where the inputs were the same as that in Fig. 14(b). The averages and standard deviation of the object position and the average angular velocity were plotted at $3 \leq t \leq 5$ s. The total simulation computation time and the average of iterations are in the range of $0 \leq t \leq 10$ s. As for $n_{obj} = 4$, the simulation result deviated from the experiment, and the simulation failed at $t \approx 2$ s; this is because the rolling of the cylindrical object changed to a tumbling due to the small n_{obj} and unrealistic solutions were determined as in Fig. 15 due to the non-continuity of the contact point. As for $n_{obj} \geq 8$, the simulations successfully reproduced the slip rotation, and the positions and angular velocities were almost the same for n_{obj} , but for $n_{obj} = 12$, the object swayed sideways, unlike the other n_{obj} . As for the calculation costs, as n_{obj} increased, the computation time increased at an accelerating rate, and the average of iterations increased linearly; this indicates that an increase in n_{obj} not only increases the unknown variables and constraints but also complicates the determination of the solution. According to Fig. 15 and Fig. 16, the positions of the object points $\mathbf{x}_{obj}$ should be placed sparsely to reduce the computational cost, while in regions with high

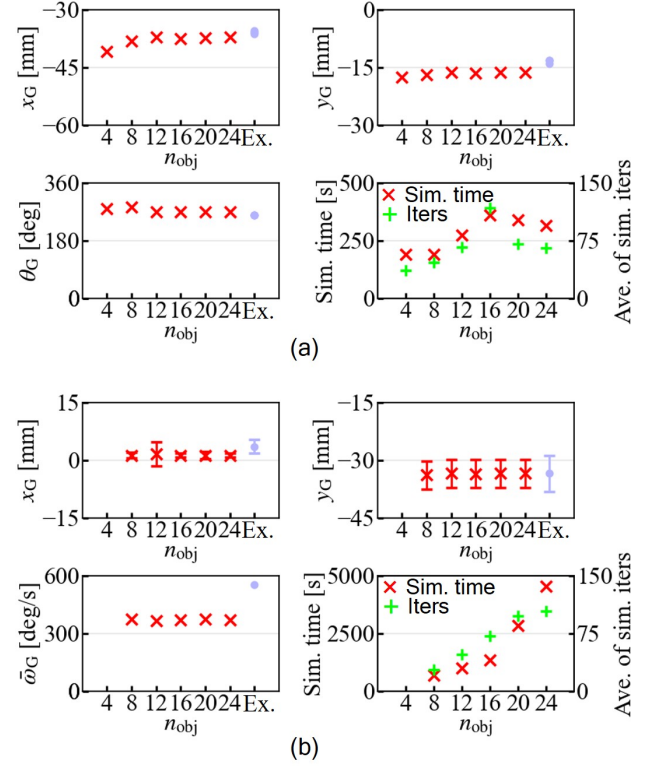


Figure 16. Effect of n_{obj} of the cylindrical object. (a) For the vertical step input. (b) For the vertical vibrational input.

curvature, $\mathbf{x}_{obj}$ should be placed densely for computational stability.

4.4 Grasping of the object on the belt with a horizontal displacement input

Grasping of the object on the belt was performed. Fig. 17 shows the results. The objects in Figs. 17(a) and (b) are the same as that shown in Figs. 13(c) and (d), respectively. As the inputs to the fingertips, $y_{anc,1} = y_{anc,2} = 0$ cm, and $A = 3.3, 3.4$ cm were provided in Figs. 17(a) and (b), respectively. The inputs were provided over for one second. Note that the scales of the force plots in Fig. 17 are smaller than those in Fig. 10(a) and Fig. 11(a) because the contact forces and tensions were significantly large after $t = 1$ s. In the real experiments, both the cylindrical object and the triangular prismatic object were locked with contact forces from the fingertips, finger pads, and belts. The triangular prismatic object collided with the fingertip, slipped, changed its orientation, and became locked. In the simulation, the behavior was qualitatively consistent with this experimental behavior. Particularly, the contact forces from the belt were significant for the cylindrical object, and the contact forces from the external environments were significant for the triangular prismatic object. The computation took considerable time because of the large

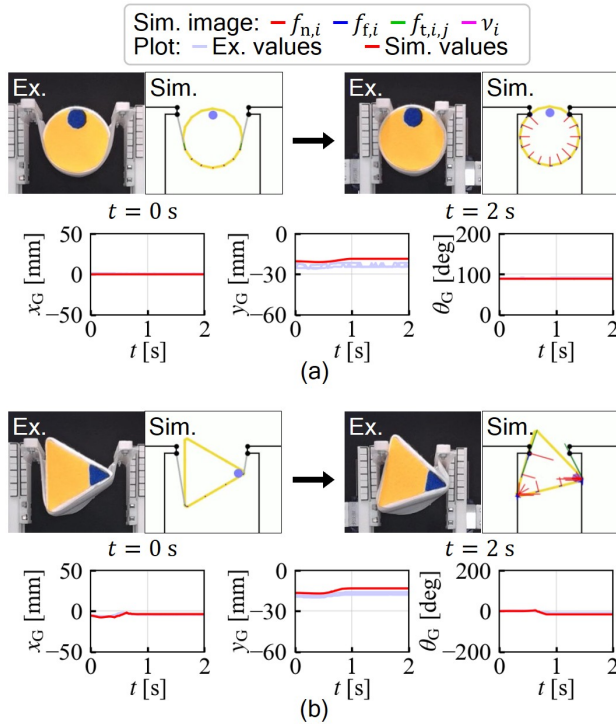


Figure 17. Results of the grasp for the object on the belt. (a) For the cylindrical object. (b) For the triangular prismatic object.

contact force. The simulation in Fig. 17(a) took 158s due to the small amount of geometric constraint violation, whereas the simulation in Fig. 17(b) took 6825s due to the large amount of violation. The belt-attached hand can lock (grasp) an object on the belt by narrowing the interval between the fingertips. However, the capability to grasp an object depends on the size of the object and the length of the belt, and the input to the fingertips needs to be adjusted. The proposed method is an effective tool for the consideration of these parameters.

Consequently, the characteristics of the simulations based on the proposed method can be summarized as follows:

- Static rolling can be successfully reproduced, but objects with shapes that have a large change in contact area, such as flats, sway significantly after tumbling.
- Static slipping tends to be greater than actual.
- Rotational manipulation using slippage due to a vibrational input results in a smaller angular velocity than the actual one in most cases. The resonance frequency becomes smaller than the actual one, and the lateral sway at resonance becomes larger than the actual.

- Simulation is more likely to fail or take more time when large contact forces occur, such as collisions with the external environment. For computational stability, the contact points should be placed continuously. Conversely, increasing the number of object points increases the computational cost.
- Under conditions of smooth changes in contact state, simulations can qualitatively reproduce the geometric state of the belt-object system and the slipping, rolling, and resonance of the object with various shapes, including aspect-ratio shapes and non-convex shapes.

5 Conclusion

This paper presented a modeling and simulation method based on the macroscopic constraints for the manipulation of a belt-attached hand. The generalized complementarity condition considering geometric properties of tautness and adaptive deformation of a cloth-like flexible body (CFB) and the presence or absence of tension was introduced, and a two-dimensional model with the object on the belt and displacement inputs provided to the belt anchor points was formulated. The flow for the sequential dynamic analysis updating the contact state was constructed. Translation involving rolling and slipping, continuous rotation by fingertip vibration, and fixation by grasping of the object were performed by the simulations based on the proposed method and compared with the real experiments, and the effectiveness of the proposed method was clarified. Modeling and analysis independent of the belt resolution and with few unknown variables and constraints were achieved.

Future work includes improving the reproducibility of the contact force distribution between the CFB and the object. Due to redundancy in the contact force, the accuracy of the contact force is not guaranteed and does not match the actual solution, especially in the amount of slipping. One possible approach to solve these problems is a physics-based machine learning approach. Extending the modeling approach to account for the bending viscoelasticity and thickness of CFB is important. Thick, inflexible belts are difficult to deform adaptively along the target object's shape, as assumed in the proposed method. To develop more realistic modeling, it would be beneficial to experimentally measure the deformation and curvature characteristics of the belt and improve the constraints. We are also considering the 3D version of the proposed method. Although the modeling method described in this paper can be extended to 3-D, it is necessary to consider a new method for detecting the contact area (See Appendix C). Speeding up and stabilizing the simulation is also a major issue. Although this method does not use a mesh and has few variables and constraints, it is currently not advantageous

in computational cost because of the nonlinear problem. Furthermore, the conditions under which this method can be stably computed are limited: for example, computation tends to fail during collisions or when contact points on objects are sparsely placed; contact points cannot be detected when objects move significantly and parts extend beyond the belt; and computation time increases substantially when large contact forces occur, such as during gripping. We would like to enhance the practicality of this method by linearization, GPU implementation, and improvements to the constraints and algorithms. Then, we would like to compare the quantitative performance of the simulations, such as reproducibility and computational cost, between the mesh-based simulations and the proposed method-based simulations.

Eventually, leveraging the proposed simulation method, we construct an in-hand manipulation method with a cloth-attached hand for control positions and orientations of objects with various shapes. Beyond the cloth-attached hand, we hope that this method will provide clues to the development of new technologies leveraging the geometric features of the CFB. The CFB adaptively deforms but has rigid body-like features. A general stretchable flexible body depends on contact and internal forces for its shape. By contrast, geometric constraints at the tension-generating area of a non-stretchable CFB are independent of the amount of tension and contact forces. In the future, we will attempt to develop new technologies using the CFB with both flexible and rigid features.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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A Estimation of the number of unknown variables and constraints in the mesh-based model

Fig. 18 shows the mesh-based analysis model used in the estimation shown in Table 1. This model subdivides

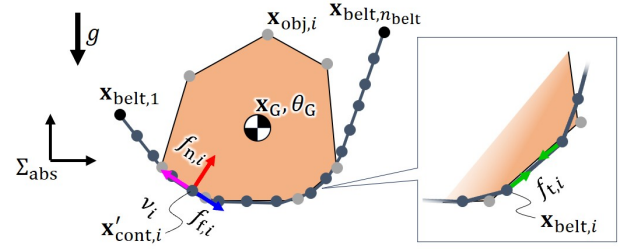


Figure 18. Mesh-based analysis model.

the belt of the proposed model shown in Fig. 18 into n_{belt} nodes. We assume that contact occurs only on the belt nodes and that the number of contact points is n'_{cont} . As unknown variables, the position of the belt node except the edge $\mathbf{x}_{\text{belt},2}, \dots, \mathbf{x}_{\text{belt},n_{\text{belt}}-1}$ and tension between the neighboring belt nodes $f_{t,1}, \dots, f_{t,n_{\text{belt}}-1}$ are defined uniquely. The unknown variables are these plus the object position $\mathbf{x}_G$, object orientation θ_G , contact forces $\mathbf{f}_n$ and $\mathbf{f}_t$, and relative velocities $\mathbf{v}$ at the contact points $\mathbf{p}_{\text{cont},1}, \dots, \mathbf{p}_{\text{cont},n'_{\text{cont}}}$, yielding a total number of $3(n_{\text{belt}} + n'_{\text{cont}}) - 2$. As for constraints, the equation of motion must consider not only the three equations for the object but also $2(n_{\text{belt}} - 2)$ new equations for the belt node position except the belt edges. Instead of (13), the geometric constraints are imposed between the neighboring belt nodes as follows:

$$|\mathbf{x}_{\text{belt},i} - \mathbf{x}_{\text{belt},i+1}| = l_i + \Delta l_i, \quad (30)$$

where, l_i is a natural length between belt nodes and Δl_i is the displacement obtained from the tension $f_{t,i}$ and viscoelasticity parameters. The geometric constraints are a total of $n_{\text{belt}} - 1$ equations. As for contact constraints, the positive normal force constraint (14) is replaced by (21) for the belt node $\mathbf{x}_{\text{belt},i}$ and the object side $\mathbf{x}_{\text{obj},j} - \mathbf{x}_{\text{obj},j+1}$. Adding an elastic term as in (25) allows penetration between the belt node and object. The other contact constraints (15), (16), and (17) are imposed in the same manner as in the proposed model. There are $4n'_{\text{cont}}$ contact constraints in total, where n'_{cont} depends on the number of the belt nodes. The mesh-based model in this appendix is the minimum configuration, and constraints increase considering the interferences of the belt nodes, etc.

B Effects of the normal distributed force parameter in (19)

The effect of the parameter of the normal force distribution ξ_{dist} shown in (19) was examined. Fig. 19 shows the results of the experiments shown in Fig. 12 when the parameter in (19) was set to $\xi_{\text{dist}} = 0.1, 3.0, 100.0$ and (18) was disabled. When ξ_{dist} was small, the normal force distribution varied smoothly, but the search for solutions became more difficult. When the contact force distribution

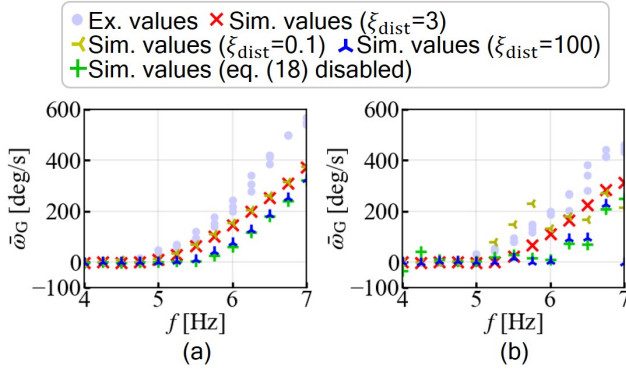


Figure 19. Angular velocity for the parameter of the normal force distribution. (a) Results for the cylindrical object. (b) Results for the triangular prismatic object.

is highly undulating, it is difficult to reproduce the actual contact forces. Moreover, when ξ_{dist} was large or (18) was disabled, the variation of the normal forces was allowed, but unrealistic behaviors such as reverse rotation were often the solutions. Therefore, a smaller ξ_{dist} can better represent the continuity of the contact force when the shape of the object is gentle (for example, the cylinder in Fig. 9(b)). A larger ξ_{dist} can better represent the behavior of the object whose shape is undulating (for example, the triangular prism in Fig. 9(c)).

C Challenges and prospects of extending the theory to three dimensions

The challenges and prospects for 3D modeling and kinetic analysis methods are discussed. The 3D modeling of the CFB and object system can be handled by increasing the dimension of section 2.2 by one dimension. In the three-dimensional model, the positions in Σ_{abs} of the points such as $p_{\text{anc},i}$ and $p_{\text{cont},i}$ are three-dimensional (e.g., $\mathbf{x}_{\text{anc},i} = [x_{\text{anc},i}, y_{\text{anc},i}, z_{\text{anc},i}]^T$), and the positions in Σ_{cloth} of the points such as $p_{\text{anc},i}$ and $p_{\text{cont},i}$ are two-dimensional (e.g., $\tilde{\mathbf{x}}_{\text{anc},i} = [\tilde{x}_{\text{anc},i}, \tilde{y}_{\text{anc},i}]^T$). The constraints of the belt-object geometry (12) extends to three dimensions as follows:

$$C_{\text{ten},i,j} = |\tilde{\mathbf{x}}_{\text{anc},j} - \tilde{\mathbf{x}}_{\text{cont},i}|^2 - |\mathbf{x}_{\text{anc},j} - \mathbf{x}_{\text{cont},i}|^2 \geq 0 \quad (31)$$

For the contact force distribution constraint (18) in 3D, the pairs of adjacent nodes in (19) can be extended to two dimensions, and the constraints are imposed on pairs adjacent at tangent planes. While 3D modeling is possible, the algorithms for detecting contact points (Algorithm 1) and updating contact point changes (Algorithm 2) between the fabric flexible body and the object are not applicable in 3D. Thus, the contact points cannot be represented when the contact geometry is curved or when rolling occurs. 3D

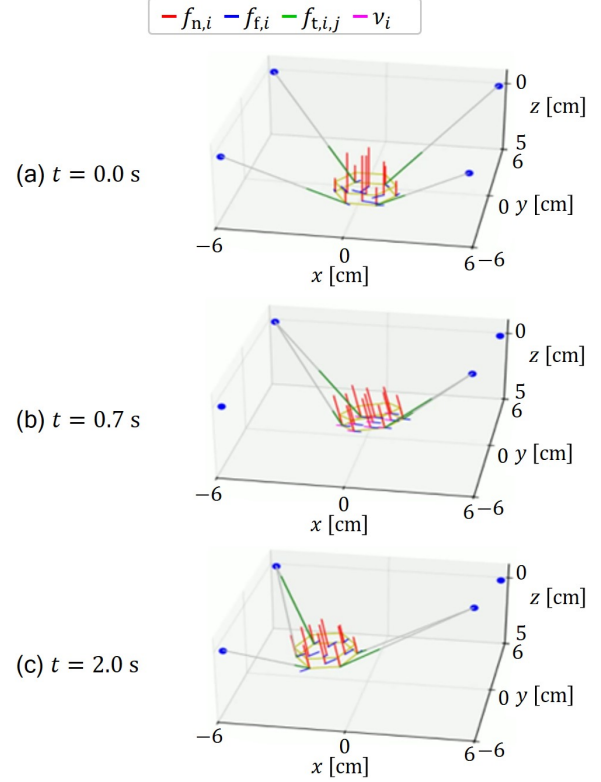


Figure 20. Results of 3D simulation for a plate object. (a) At $t = 0.0$ s. (b) At $t = 0.7$ s. (c) At $t = 2.0$ s.

detection and updating of contact geometry will be the main challenge in three-dimensionalization. On the other hand, it is possible to reproduce limited situations where the configuration of the contact points does not change: for example, a plate sliding on a piece of cloth. Fig. 20 shows the simulation results of a plate sliding on a cloth. The cloth is anchored by 14.1 cm square points on Σ_{cloth} , and the cloth anchor points are placed at 11.3 cm square points in Σ_{abs} . A step input is given to raise one point by 3.0 cm from the same height as all holding points. The cross-sectional shape of the object is a regular hexagon with 1.5 cm per side and a flat plate with 0.38 cm thickness. The contact force distribution constraint (18) is imposed on the contact points on the bottom surface for adjacent edges by Delaunay triangulation. The gray line in Fig. 20 connects the contact point and the cloth anchor point where the tension occurred. As shown in Fig. 20, tension is initially generated for each cloth anchor point, but the plate object slides on the cloth, and the object is finally held at the three cloth anchor points. Thus, we confirm that it is possible to reproduce the sliding behavior of an object in a situation where the contact surface is flat and no change in the contact point occurs.