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# Studies on Inorganic Electrochemical Reactions <br> by the Derivative Voltammetry 

## by

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List of symbols and abbreviations

| A | electrode area |
| :---: | :---: |
| $\mathrm{a}_{\mathrm{i}}$ | activity of species i |
| C | differential capacity of the electrical double |
|  | layer |
| $C_{f}$ | equivalent-series-capacitive component of |
|  | faradaic impedance |
| $C_{i}$ | concentration of species i |
| $C^{*}$ | initial (bulk) concentration |
| $E^{\circ}$ | standard redox potential |
| $\mathrm{E}_{1 / 2}^{\mathrm{r}}$ | reversible half-wave potential |
| $E(t)$ | instantaneous applied potential |
| $E_{d c} \text { or } E$ | dc component of potential |
| $\Delta E_{a c} \text { or } \Delta E$ | amplitude of applied alternating potential |
| $\left[E_{\mathrm{dc}}\right]_{\max }$ | dc potential of maximum cot $\phi$ |
| $\left[E_{d c}\right]_{\text {peak }}$ | dc potential of maximum alternating |
| or E ${ }_{p}$ | current |
| F | Faraday's constant |
| f | frequency |
| $\mathrm{f}_{\mathrm{i}}$ | activity coefficient of species i |
| $\mathrm{D}_{\mathrm{i}}$ | diffusion coefficient of species i |
| $\mathrm{I}_{\mathrm{d}}$ or $\mathrm{i}_{\mathrm{d}}$ | dc current or dc diffusion current |
| $I_{p}$ or $i_{p}$ | peak faradaic alternating current |


| $I(\omega t)$ | fundamental harmonic faradaic alternating current |
| :---: | :---: |
| $I(n \omega t)$ | nth-harmonic faradaic alternating current |
| i ( t ) | total faradaic current |
| k | (first-order) chemical reaction rate constant |
| $\mathrm{k}_{\mathrm{s}}$ | heterogeneous rate constant for charge transfer at $E^{\circ}$ |
| n | number of electrons transferred in heterogeneous <br> charge transfer step |
| R | ideal-gas constant |
| $\mathrm{R}_{\mathrm{f}}$ | equivalent-series-resistance component of |
|  | faradaic impedance |
| $\mathrm{R}_{\Omega}$ or $\mathrm{R}_{S}$ | uncompensated ohmic resistance or solution |
|  | resistance |
| T | absolute temparature |
| t | time |
| u | auxiliary variable of integration |
| v | dc potential scan rate |
| x | distance from electrode surface |
| $\mathrm{Z}_{\mathrm{f}}$ | faradaic impedance(absolute value) |
| $z_{i}$ | charge on species i |
| $\alpha$ | charge-transfer coefficient for reduction |
| B | charge-transfer coefficient for oxidation |
|  |  |
| $\Gamma$ | surface concentration |


| $\psi_{i}$ | current relative to applied alternating |
| :---: | :---: |
|  | potential |
| $\phi_{i}$ or | phase angle of alternating current relative to |
| $\phi$ | applied alternating potential |
| $\phi_{2}$ | potential at outer Helmholtz plane relative to |
|  | potential in bulk solution |
| $\omega$ | angular frequency $(\omega=2 \pi f)$ |
| C.E. | counter electrode |
| CPE | cotrolled potential electrolysis |
| CV | cyclic voltammetry |
| DME | dropping mercury electrode |
| HMDE | hanging mercury drop electrode |
| R.E. | reference electrode |
| W.E. | working electrode |

Chapter 1. General introduction

Oxidation and reduction of chemical species are utilized very often in the course of the study of the method for the chemical analysis of inorganic materials and the analysis of electron transfer mechanism in the redox systems is one of the major areas in analytical chemistry. Although many brilliant works have been performed to elucidate those mechanism of inorganic species in aqueous media and succeeded, most of multiple steps of oxidation or reduction of a single ion still remained under the controversy. $\mathrm{CrO}_{4}^{2-}$ is one of those ions whose kinetic character in reduction process have not been cleared yet. Various techniques have been introduced to observe kinetically the redox reaction of inorganic ions in aqueous solution.

Voltammetry is the most widely used electrochemical technique for the chemical analysis of inorganic ions in aqueous medium. And the technique is very useful for the determination of micro-amounts of chemical species. One can also get by this method various informations related to the electron transfer between chemical species and the electrode.

Direct current(dc) polarography is one of the oldest and most popular method in voltammetry. One of its major assets is its ability to the simultaneous determinations of various ions at low concentration in liquid medium, while the major limitation of this technique is located to the difficulty in the separation of non-faradaic current, that is a charging current to an electrochemical double layer, from the faradaic current developed by the electrode reaction of chemical species to be interested. This makes rather inadequate to apply dc polarography to the kinetic analysis of multistep electron transfer systems.

Differential pulse(DP) polarography is the modified method of dc polarography and can reduce the charging current in the observed signal. But it is not applicable to the kinetic study of the fast reaction, unfortunately.

Although cyclic voltammetry can control the scanning rate of applied voltage to the electrolytic circuit and is useful to observe the electrode reaction with a large rate constant, it can not distinguish the phase shift of the faradaic and non-faradaic components of current at the onset of the applied voltage.

The differential capacity for investigating on the vicinity of electrode cannot be isolated by those methods above described.

In the mathematical expression, the faradaic
alternating current through the electrochemical cell is developed nonlinearly in the power series[1]. In other words, the equivalent circuit of the cell cannot be expressed by the linear combination of the electric parameters such as a resistance and capacitance. The fundamental harmonic(lf) mode(the current component of the same frequency as the input ac voltage, that is to say, the first developed term of the power series of output ac signal) is, conventionally, called as the ac polarography. In this sence, the ac polarography can be called one of the nonlinear analyses[2-15]. The ac polarography[16-20] is available to the analysis of the comparatively fast electrochemical reaction. The use of the phase selective mode could reduce the charging current, too[16-33]. Unless the reaction is reversible in process, however, the polarogram in the $1 f$ mode is not a real first derivative form of the dc polarogram in the second harmonic(2f) mode (the current component which have twice as high frequency as the input, e.g., the second term of the output power series) is again not the derivative form of lf mode. However, these modes are the extreme function of the kinetic character of the electrochemical reactions.

Though the phase selective mode may be useful for the electrochemistry, only $\operatorname{lf}\left(0^{\circ}\right)\left(0^{\circ}\right.$ means the same phase angle as the input ac voltage) and $2 f\left(90^{\circ}\right)$ have been mainly
utilized for analytical purposes so far, e.g., the quantitative analyses[34-41], studies on kinetics[42-44] and determination of reversible half-wave potentials for organic compounds[45-47]. And the phase angle component other than $1 \mathrm{f}\left(0^{\circ}\right)$ and $2 \mathrm{f}\left(90^{\circ}\right)$ modes have never been employed. The differential capacity can be also obtained by the use of the phase selective mode. Therefore, the ac polarography could be widely applied to the studies on reaction kinetics and to the chemical analyses.

In the present work, therefore, special instruments designed for the derivative voltmmetry were constructed, and phase angle components of $1 f\left(45^{\circ}\right), 1 f\left(90^{\circ}\right), 2 f\left(0^{\circ}\right)$ and $2 f\left(45^{\circ}\right)$ in addition to the $1 f\left(0^{\circ}\right)$ and $2 f\left(90^{\circ}\right)$ components to elucidate the kinetic character of $\mathrm{CrO}_{4}^{2-}$, catalytic reduction of $\mathrm{NO}_{3}^{-}$to establish the kinetic electrochemical analysis by means of non-linear methods. And some new applications developed by the phase selective ac polarographic technique were performed.

1-1. Properties of some phase angle components

According to the theoretical analysis, the reversible process is carried out at the phase angle of $45^{\circ}$ in $1 f$ mode and in a quasi-reversible or an irreversible process, it changes from $0^{\circ}$ to $45^{\circ}$ depending on the dc potential and other kinetic parameters such as $\mathrm{k}_{\mathrm{s}}$ (heterogeneous rate
constant for charge transfer step) and $\alpha$ (charge transfer coefficient for reduction). When the reaction process is reversible, the current amplitude of $45^{\circ}$ component is highest in all phase angle components. Therefore, the $1 f\left(45^{\circ}\right)$ is the highest sensitive component in order to elucidate the rate determining step for the electrochemical reaction including successive multi-step charge transfers.

Since Cakenberghe[53] has tried basically to the second harmonic mode, the $2 f$ mode has come into use for kinetic and analytical applications[25,37-44,47-49,54] and determination of reversible half-wave pootentials for many organic compounds[50-52].

Though all phase angle components in $2 f$ mode are reduced in the charging current as compared with those in $1 f$ mode, only the $2 f\left(90^{\circ}\right)$ (compared with $1 f\left(0^{\circ}\right)$ ) has been generally utilized for the analytical purpose. The phase angle for the reversible process, in the 2 f mode, is $-45^{\circ}$ or $135^{\circ}$ and that for the quasi-reversible or irreversible process changes from $0^{\circ}$ to $360^{\circ}$ depending on the dc potential and other kinetic parameters. For example, if the reaction process is reversible, $45^{\circ}$ component in 2 f mode gives zero in its current amplitude. If the process is quasi-reversible or irreversible, the current amplitude of this component is not zero. From this principle, $2 f\left(45^{\circ}\right)$ can be used for the separation analyses of two species
having similar redox potentials and having different reaction rates with each other.

Except for the faradaic current, the charging current to the electrochemical double layer flows through the electrochemical cell. The differential capacity of the adsorption of chemical species on electrode and for the structure of the vicinity of electrode[55-59]. An ac bridge method can be employed in these purposes, but the technical treatment is troublesome. Therefore, the differential capacity has not been obtained in complicated systems so far. The phase selective ac polarography is a very useful technique for these purposes. The differential capacity is calculated easily from the observed current values of $1 f\left(0^{\circ}\right)$ and $1 f\left(90^{\circ}\right)$ modes. The techniques are applied in the present work, to the analysis for the vicinity of the electrode of zirconium(IV)-nitrate system.

1-2. Utilization of $\operatorname{lf}\left(45^{\circ}\right)$ component for chromium(VI) system

The oxidation and reduction of chemical species have been one of the interesting subjects to the chemist. As the reaction of such a multivalent species involves a multi-step electron transfer process, the understanding of such reaction mechanism is one of the purposes of this research. A chromium(VI) has been known as one of these elements
[60-68]. The existence of chromium(V)[60,61] and chromium(IV)[62-68] were certified as the reduction products and as the intermediates, respectively. The kinetic mechanism for the reduction of chromium(VI) also changed with the property of reductant employed.

The electrode reduction of chromium(VI) in alkaline solution has also been investigated by the dc polarography and by other electrochemical techniques[69-75], but the rate determining step for total reaction of chromium(VI) to chromium(III) has never been determined and the kinetics has not been investigated quantitatively by those studies. The dc polarogram of this system shows only single reduction wave. However, from the ac polarographic data in the lf( $45^{\circ}$ ) mode observed in this work, it was clearly shown that the reaction is two step mechanism.

1-3. Application of differential capacity measurement to zirconium(IV)-nitrate ion system

Though the oxidation capacity of nitric acid has been well-known [76,77], the nitrate anion is hardly reduced electrochemically in univalent supporting electrolytes [78,82]. The nitrate ion is, however, reduced at a more positive reduction potential when a multi-valent cation is present than when it is absent[78-82].

The reduction potential of nitrate ion shifts 1.3 V to
the more positive potential in the presence of zirconium( IV) than that in its absence(in the presence of a univalent supporting electrolyte)[83,84]. The kinetic mechanism has not been fully clarified, especially because of the lackin of the data concerning the structure in the vicinity of the electrode. In this work, a role of zirconium(IV) in this system was discussed from the measurement of the differential capacities by using $1 f\left(0^{\circ}\right)$ and $1 f\left(90^{\circ}\right)$ components and from some other electrochemical and spectral

1-4. Application of $2 f\left(45^{\circ}\right)$ and $2 f\left(0^{\circ}\right)$ to chemical analyses

In the chemical analysis, the problem of elimination of interference should be settled in any case.

In electroanalytical methods, it is well known that cadmium(II) usually interferes in the analysis of indium (III) [the half-wave potential ( $E_{1 / 2}$ ) difference is 45 $m V]$, and that nickel(II) also interferes in the analysis of zinc(II)(the $E_{1 / 2}$ difference is 60 mV ). In the present work, the usefullness of the phase selective ac polarogram even in the quantitative analysis was shown again. The quantitative analises of indium(III) and zinc(II) in large excess of cadmium(II) and of nickel(II) solutions respectively, can be performed even without any pretreatments by the phase selective ac polarographic technique.

Consequently, the purpose of this work is to utilize the validity of various phase angle components in the ac polarography: (1) $1 f\left(45^{\circ}\right)$, for analyzing the kinetic mechanism of the $\operatorname{Cr}(V I) / C r(I I I)$ in alkaline solution; $1 f\left(0^{\circ}\right)$ and $1 f\left(90^{\circ}\right)$, for investigating on the role of zirconium(IV) in the reduction of nitrate ion from the data of the differential capacity obtained from these components and by other electrochemical and spectral techniques; (3) $2 f\left(45^{\circ}\right)$ and $2 f\left(0^{\circ}\right)$, for applying to the chemical analysis of one of two metal ions having similar reduction potentials by considering the difference between reduction mechaniss of these ions.

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Chapter 2. Experimental and theory

2-1. Experimental device

A block diagram of the experimental device for the dc and phase-selective ac polarographies used in this work, is given in Fig. 2-1. A hand-made function generator produces a dc lamp wave(sawtooth or triangular). An oscillator (Kikusui Electronics Corp., RC Oscillator Model 4045) was used as a generator of sine wave ac voltage. The dc lamp wave(typically $1 \mathrm{mVs}^{-1}$ ) and the sine wave are fed to a hand-made potentiostat. This potentiostat has a positive feedback for iR compensation circuit(Fig. 2-2)[1,2]. All amplifiers used for its construction were of Burr-Brown 3507 J (fast slewing OP AMP). The fundamental and second harmonic alternating currents were detected by employing a lock-in amplifier(NF Circuit Design Block Co., Model LI 507). The ac polarogram was recorded by using an $X-Y$ recorder (Yokogawa Type 3086).

The iR compensation at the potential where the faradaic alternating current was not detected, was attained by adjusting the variable resistance of the compensator(R3


Fig. 2-1. Block diagram of experimental device for the phase-selective ac polarography.


Fig. 2-2. The circuit diagram of the potentiostat with positive feedback iR compensation.
voltage(E $\mathrm{ac}^{\mathrm{c}}$ ) is given by the equation: $\mathrm{E}_{\mathrm{ac}}=\Delta \mathrm{E}_{\mathrm{ac}} \cos \omega t$, and $i_{a c}$ can be expressed as:

$$
i_{a c}=a_{1} \Delta E_{a c} \cos \omega t+a_{2} \Delta E_{a c}^{2} \cos ^{2} \omega t+a_{3} \Delta E_{a c}^{3} \cos ^{3} \omega t+
$$

And using the relations: $\cos ^{2} x=(1+\cos 2 x) / 2, \cos ^{3} x=$ $(3 \cos x+\cos 3 x) / 4, \cos ^{4} x=(3+4 \cos 2 x+\cos 4 x) / 8$, -----, Eq. (2-2) can be rewritten:

$$
\begin{align*}
\mathrm{i}_{\mathrm{ac}}= & {\left[\left(\mathrm{a}_{2} / 2\right) \Delta \mathrm{E}_{\mathrm{ac}}^{2}+\left(3 \mathrm{a}_{4} / 8\right) \Delta \mathrm{E}_{\mathrm{ac}}^{4}+---\right]+\left[\mathrm{a}_{1} \Delta \mathrm{E}_{\mathrm{ac}}+\right.} \\
& \left.\left(3 \mathrm{a}_{3} / 4\right) \Delta \mathrm{E}_{\mathrm{ac}}^{3}+---\right] \cos \omega t+\left[\left(\mathrm{a}_{2} / 2\right) \Delta \mathrm{E}_{\mathrm{ac}}^{2}+\right. \\
& \left.\left(\mathrm{a}_{4} / 2\right) \Delta \mathrm{E}_{\mathrm{ac}}^{4}+---\right] \cos 2 \omega t+\left[\left(\mathrm{a}_{3} / 4\right) \Delta \mathrm{E}_{\mathrm{ac}}^{3}+\ldots--\right] \times \\
& \cos 3 \omega t+\left[\left(\mathrm{a}_{4} / 8\right) \Delta \mathrm{E}_{\mathrm{ac}}^{4}+---\right] \cos 4 \omega t+, \tag{2-3}
\end{align*}
$$

where the phase difference of the output current replied to the input voltage is not included in this equation so that the relationship between them is simply expressed.

And when the input amplitude $\Delta E$ ac ${ }^{\text {is }}$ small, the leading term in each square bracket is significant. Namely, the amplitude of cos wt (fundamental harmonic) is proportinal to $\Delta E_{a c}$, that of $\cos 2 \omega t$ (second harmonic) to $\Delta E_{a c}^{2}$, that of $\cos 3 \omega t$ (third harmonic) to $\Delta E_{a c}^{3}$ and so on. In other words, the amplitude of cos nwt (nth harmonic) varies in proportion to the nth power term developed the total alternating current. This procedure will be explained in detail later (cf. Appendix $I, I I)$.

For the simple electrode reaction, $0 x+n e^{-} \rightleftharpoons$ Red,
in Fig. 2-2), until the alternating current amplitude of the $1 f\left(0^{\circ}\right)$ component vanished or reached minimum amount. At the potential, the chargeing current of the electric double layer should be detected only in the $1 \mathrm{f}\left(90^{\circ}\right)$ component, provided the resistance of the solution is 0 ohm. Figure $2-3$ shows the effect on the $i R$ compensation. If there is no compensation, the $1 f\left(90^{\circ}\right)$ suffers a significant loss of amplitude.

The iR compensation circuit was tested with a $\mathrm{Zn}(\mathrm{II}) / \mathrm{Zn}(\mathrm{Hg})$ system in 1 M potassium chloride solution. The potentiostat gave theoretical phase angle in the frequency range $9-500 \mathrm{~Hz}[3]$.

When the dc polarogram is recorded, the oscillator and lock-in amplifier are disconnected from the circuit, while the potentiostat output signal of the direct current was directly fed to the $X-Y$ recorder.

The polarograms were obtained with a conventional three-electrode system. The working electrode was a dropping mercury electrode(DME). A hanging mercury drop electrode(HMDE) (Metrohm AG., EA 290) was also used for a cyclic voltammetry(CV): drop diameter was 0.84 mm , drop surface area was $2.20 \pm 0.05 \mathrm{~mm}^{2}$. The counter electrode was a platinum spiral wire. The reference electrode was a saturated silver silver chloride electrode(SSE) (Ag/AgCl[sat. KCl]).

 ac polarogram vs. $\omega^{1 / 2}$. (O) $1 f\left(0^{\circ}\right)$, ( ( ) $1 f\left(90^{\circ}\right)$ without iR compensation, and ( $\left.{ }^{( }\right) 1 f\left(0^{\circ}\right)$, (X) $1 f\left(90^{\circ}\right)$ with iR compensation. 0.2 mM cadmium chloride in 1 M potassium chroride(pH2). $\Delta E_{a c}=2 \mathrm{mV}(r . m . s$.$) .$

2-2. Reagents

All chemicals were of analytical-reagent grade and used without further purification. Deionized-distilled water was used for the preparation of all solutions. A stock solution of cadmium(II) was prepared from $\mathrm{CdCl}_{2} \cdot 21 / 2 \mathrm{H}_{2} \mathrm{O}$. Other reagents used for studies in later chapters will be shown in the experimental sections of these chapters. Temperture of the solutions were regurated at $25 \pm 0.5{ }^{\circ} \mathrm{C}$. Dissolved oxygen was removed from the sample solution for $15-20$ min prior to recording of polarograms or voltamograms.
$2-3$. Phase angle adjustment

The phase angle of the lock-in amplifier was calibrated by the use of a dummy cell[4]. A resistance was used as a dummy cell for the fundamental alternating current. The phase shifter of the lock-in amplifier are adjusted so that the lock-in amplifier output is maximized (in-phase ( $0^{\circ}$ ) current), and then a quadrature (90 $)$ current amplitude vanished.

For the second-harmonic alternating current, an output signal of the potentiostat is fed to a full wave rectifier (the resistance was used as the dummy cell). The phase angle of the output current is determined by the following relationship,

```
input signal: y = A sin x,
output signal: y = (2A/\pi)[1-(2cos 2x)/3-
                        (2cos 4x)/3\cdot5-(2\operatorname{cos}6x)/5\cdot7 \cdots].
```

The circuit diagram of the full wave rectifier is given in Fig. 2-4. As is found from Eq. (2-1), the phase angle in the $2 f$ mode becomes $-90^{\circ}$ when the maximum signal is detected in the lock-in amplifier.

If one wants to obtain the $45^{\circ}$ component of the fundamental harmonic alternating current[1f(45*)], the phase shifter of the lock-in amplifier should be ajusted to the point so that the output current is $I_{l f\left(0^{\circ}\right)} \cos 45^{\circ}$ $=I_{1 f\left(0^{\circ}\right)} / \sqrt{2}$.

2-4. Theoretical background

The total alternating current(inch through a nonlinear circuit element such as an electrochemical cell is developed nonlinearly the power series. When the input


Fig. 2-4. The circuit diagram of the full wave rectifier used as the phase adjuster of 2 f mode in lock-in amplifier.
where the rate is controlled by diffusion and/or heterogeneous charge transfer kinetics, the fundamental and second harmonic alternating currents are given by the following equations[5,6](Appendix I, II):

$$
\begin{align*}
& I(\omega t)=I(\omega t)_{\operatorname{rev}} \mathrm{F}_{1}(t) G_{1}(\omega) \sin \left(\omega t+\phi_{1}\right)  \tag{2-4}\\
& I(2 \omega t)=I(2 \omega t)_{\operatorname{rev}_{2}} F_{2}(t) G_{2}(\omega) \sin \left(2 \omega t+\phi_{2}\right), \tag{2-5}
\end{align*}
$$

where

$$
\begin{equation*}
I(\omega t)_{r e v}=(n F)^{2} A C_{0}^{*}\left(\omega D_{0}\right)^{1 / 2} \Delta E_{a c}[4 R T \cosh 2(j / 2)]^{-1} \tag{2-6}
\end{equation*}
$$

$I(2 \omega t)_{r e v}=(n F)^{3} A C_{0}^{*}\left(2 \omega D_{0}\right)^{I / 2} \Delta E_{a c}^{2}\left[16 R^{2} T^{2} \cosh ^{3}(j / 2)\right]^{-1}$
$\phi_{1}=\cot ^{-1}\left[1+(2 \omega)^{1 / 2 / \lambda],}\right.$
$\phi_{2}=\cot ^{-1}\left\{\left[\mathrm{~L}\left(2 \omega^{1 / 2} / \lambda+1\right)+\mathrm{P}\right] /\left[\mathrm{L}-\mathrm{P}\left(2 \omega^{1 / 2} / \lambda+1\right)\right]\right\}$,
where
$j=n F(R T)^{-1}\left(E_{d c}-E_{1 / 2}^{r}\right) ; \quad \lambda=\left(k_{s} f_{a} / R T\right)\left(e^{-\alpha j}+e^{B j}\right)$. (2-10)

The current component $I(\theta)$ at a phase angle, $\theta$ is given by the following equation,
$I(\theta)=I_{\text {total }} \cos (\phi-\theta)$,
where the $I_{\text {total }}$ is the amplitude of Eq. (2-4) or (2-5).

Appendix $I$. The reversible ac polarographic wave[5]

For the system presented by the reaction scheme $0 x+$ $n e^{-} \rightleftharpoons$ Red, the partial-differential equations, initial and boudary conditions for diffusion to a stationary planar electrode are:

$$
\begin{align*}
& d C_{0} / d t=D_{0} d^{2} C_{0} / d x^{2}  \tag{I-1}\\
& d C_{R} / d t=D_{R} d^{2} C_{R} / d x^{2} . \tag{I-2}
\end{align*}
$$

For $t=0$, any $x$,

$$
\begin{align*}
& C_{0}=C_{0}^{*}  \tag{I-3}\\
& C_{R}=C_{R}^{*} \tag{I-4}
\end{align*}
$$

For $t>0, x \rightarrow \infty$,

$$
\begin{align*}
& c_{0} \longrightarrow c_{0}^{*},  \tag{I-5}\\
& c_{R} \longrightarrow c_{R}^{*} . \tag{I-6}
\end{align*}
$$

For $t>0, x=0$,
$D_{0} d C_{0} / d x=-D_{R} d C_{R} / d x=i(t) / n F A$,
$C_{0}=C_{R}\left(D_{R} / D_{0}\right)^{1 / 2} \exp \left[(n F / R T)\left[E(t)-E_{1 / 2}^{r}\right]\right]$.
The assumptions that Fick's law is applicable to each species independently, that coupled chemical reactions exert to no influence, and that electrode curvature and motion relative to the solution have a negative effect are incorporated in Eqs. (I-1) and (I-2). Equation (I-7) assumes that each reacting species is soluble either in the solution or electrode phase, that is, there is no accumulation of electroactive material at the interface.

Equation ( $I-8$ ) states the Nernst equation in terms of the reversible dc polarographic half-wave potential, $E_{1 / 2}^{r}$, where

$$
\begin{equation*}
E_{1 / 2}^{r}=E^{0}-(R T / n F) \ln \left(f_{R} / f_{0}\right)\left(D_{0} / D_{R}\right)^{1 / 2} \tag{I-9}
\end{equation*}
$$

The assumption of nernstian behavior is obeyed if heterogeneous charge transfer kinetics are very rapid.

Application of the method of Laplace transformation to
Eqs. ( $I-1$ ) through ( $I-7$ ) yields, for surface concetrations,

$$
\begin{align*}
& C_{0 x=0}=C_{0}^{*}-\int_{0}^{t} i(t-u) d u / n F A\left(D_{0} \pi u\right)^{1 / 2}  \tag{I-10}\\
& C_{R x=0}=C_{R}^{*}+\int_{0}^{t} i(t-u) d u / n F A\left(D_{R} \pi u\right)^{1 / 2}
\end{align*}
$$

Derivation of Eqs. $(I-10)$ and $(I-11)$ is well known and contains no features specific to ac polarography. For simplicity, we shall assume at this stage that the reduced form is initially absent from the solution; that is, $C_{R}^{*}=$ 0. Substituting Eq. $(I-10)$ and $(I-11)$ in $E q .(I-8)$ and rearranging yields the integral expression(given in a form most convenient for subsequent steps):

$$
\begin{align*}
e^{-j(t)}- & e^{-j(t)} \int_{0}^{t} \psi(t-u) d u /(\pi u)^{1 / 2}= \\
& \int_{0}^{t} \psi(t-u) d u /(\pi u)^{1 / 2} \tag{I-12}
\end{align*}
$$

where

$$
\begin{align*}
& \psi(t)=i(t) / n F A C{ }_{0}^{*} 1 / 2  \tag{I-13}\\
& j(t)=(n F / R T)\left[E(t)-E_{1 / 2}^{r}\right] \tag{I-14}
\end{align*}
$$

In ac polarography the applied potential is given by

$$
\begin{equation*}
E(t)=E_{d c}-\Delta E_{a c} \sin \omega t \tag{I-15}
\end{equation*}
$$

The dc potential term is considered constant. This assumes
the rate of scan is slow relative to the rate of change of alternating potential and that the dc potential does not change significantly over the life of a single mercury drop. The latter restriction applies also to de polarography. Substitution of Eq. (I-15) in the exponential term gives as:

$$
\begin{equation*}
e^{-j(t)}=e^{-j} \exp \left[\left(n F \Delta E_{a c} / R T\right) \sin \omega t\right], \tag{I-16}
\end{equation*}
$$

where

$$
\begin{equation*}
j=(n F / R T)\left(E_{d c}-E_{1 / 2}^{r}\right) \tag{I-17}
\end{equation*}
$$

One then develops the power series:

$$
\begin{array}{ll}
\exp \left[\left(n F \Delta E_{a c} / R T\right) \sin \omega t\right]=\left(n F \Delta E_{a c} / R T\right)^{p}(\sin \omega t)^{p} / p!, \\
\psi(t)=\sum_{p=0}^{\infty} \psi_{p}(t)\left(n F \Delta E_{a c} / R T\right)^{p}, & (I-1)  \tag{I-18}\\
p=0,1,2,3 \ldots . & (I-2)
\end{array}
$$

Substituting Eqs. ( $I-18$ ) and (I-19) in Eq. (I-12) and equating coefficients of equal powers of ( $\left.n=\Delta E_{a c} / R T\right)^{p}$, one obtains a system of integral equations:

$$
\begin{gather*}
e^{-j}(\sin \omega t)^{p} / p!-\sum_{r=0}^{p}\left\{\left[e^{-j}(\sin \omega t)^{r} / r!\right] x\right. \\
\left.\left.\int_{0}^{t} \psi_{p-r}(t-u) d u /(\pi u)^{1 / 2}\right]\right\}=\int_{0}^{t} \psi_{p}(t-u) d u /(\pi u)^{1 / 2} \tag{I-21}
\end{gather*}
$$

Solutions of Eq. (I-21) for different values of pepresent the various faradaic current components in the following manner. If $k$ is the order of the current harmonic $(k=0$ for $d c, k=1$ for fundamental-harmonic ac, etc.), contributions to a particular harmonic $k$ are obtained from values of $p$ given by

$$
\begin{equation*}
p=2 q+k \tag{I-22}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{q}=0,1,2,3 \ldots \tag{I-23}
\end{equation*}
$$

Thus, de components are obtained for $p=2 q=0,2,4---$ and fundamental harmonic ac terms for $p=2 q+1=1,3,5,7 \ldots-$. The contribution to a given harmonic decreases with increasing $p$ for $\Delta E_{a c}<0.1 V$, a condition always obeyed in ac polarography. At sufficiently small amplitudes( $\Delta E_{a c}<$ $8 / \mathrm{nmV}$ ), only the term corresponding to the lowest value of p need be considered. For $p=0$, one obtains:

$$
\begin{equation*}
\int_{0}^{t} \psi_{0}(t-u) d u /(\pi u)^{1 / 2}=1 /\left(1+e^{j}\right) \tag{I-24}
\end{equation*}
$$

The solution for $\psi_{0}(t)$ is obtained most conveniently by application of the method of Laplace transformation to Eq. ( $\mathrm{I}-24$ ). Applying Eqs. ( $\mathrm{I}-13$ ) and $(\mathrm{I}-19)$ to the solution for $\psi_{0}(t)$, we get:

$$
\begin{equation*}
i_{d c}(t)=n F A C_{0}^{*} D_{0}^{1 / 2} /\left(1+e^{j}\right)(\pi t)^{1 / 2} \tag{I-25}
\end{equation*}
$$

This is well-known expression for the potentiostatic direct current at a planar electrode with a reversible process[7].

$$
\begin{align*}
& \text { For } p=1, \text { one has: } \\
& \quad e^{-j}\left[1-\int_{0}^{t} \psi_{0}(t-u) d u /(\pi u)^{1 / 2}\right] \sin \omega t= \\
& \left(1+e^{-j}\right) \int_{0}^{t} \psi_{1}(t-u) d u /(\pi u)^{1 / 2} \tag{I-26}
\end{align*}
$$

Note that the integral equation for the small-amplitude fundamental harmonic contains a term in the dc component $\psi_{0}(t)$. This represents a general feature of the
theoretical method. As will be seen, the integral equation for the second-harmonic contains terms $\psi_{1}(t)$ and $\psi_{0}(t)$. In general, the integral equation for any higher-order term contains all the lower-order terms. For the reversible case, explicit solution of the lower-order terms $\left(\psi_{p}(t)\right)$ is not required, since the integral equations provide the required relationships. Thus, substitution of Eq. ( $\mathrm{I}-24$ ) in Eq. (I-26) eliminates $\psi_{0}(t)$, yielding:

$$
\int_{0}^{t} \psi_{1}(t-u) d u /(\pi u)^{1 / 2}=\sin \omega t / 4 \cosh ^{2}(j / 2) . \quad(I-27)
$$

However, for most electrode-reaction mechanisms, explicit solution for $\psi_{0}(t)$ is required to obtain a general solution for $\psi_{1}(t)$, solution of $\psi_{0}(t)$ and $\psi_{1}(t)$ is required for $\psi_{2}(t)$, etc. The need to obtain an explicit solution for the dc component presents the most serious obstacle to obtaining completely general ac polarographic wave equations for a particular kinetic scheme.

The form of Eq. (I-27) is such that $\psi_{1}(t)$ can contain only fundamental harmonic terms. Thus, we may write:

$$
\begin{equation*}
\psi_{1}(t)=A \operatorname{Ain} \omega t+B \cos \omega t . \tag{I-28}
\end{equation*}
$$

Substituting this relation in Eq. (I-27), employing the trigonometric identities:

$$
\begin{align*}
& \sin \omega(t-u)=\sin \omega t \cos \omega u-\cos \omega t \sin \omega u,  \tag{I-29}\\
& \cos \omega(t-u)=\cos \omega t \cos \omega u+\sin \omega t \sin \omega u, \tag{I-30}
\end{align*}
$$

applying the steady-state approximation $\left(\int_{0}^{t}=\int_{0}^{\infty}\right.$ ) [8] and the relations[9]:

$$
\begin{align*}
& \int_{0}^{\infty} \cos \omega u d u /(\pi u)^{1 / 2}=\int_{0}^{\infty} \sin \omega u d u /(\pi u)^{1 / 2} \\
& \quad=1 /(2 \omega)^{1 / 2} \tag{I-31}
\end{align*}
$$

we obtain the result:

$$
\begin{align*}
& (A+B) \sin \omega t-(A-B) \cos \omega t= \\
& {\left[(2 \omega)^{1 / 2} / 4 \cosh ^{2}(j / 2)\right] \sin \omega t} \tag{I-32}
\end{align*}
$$

The steady-state approximation neglects transient behaviour in the ac concentration profile. As shown by Berzins and Delahay[10], transients are negligible when ( $\omega t)^{1 / 2} \gg 1$, where $t$ is time elapsed after application of the alternating potential.

Equating coefficients of sin $\omega t$ and cos $\omega t$ on both sides of Eq. ( $I-32$ ), and solving for $A$ and $B$, we obtain:
$A=B=(2 \omega)^{1 / 2} / 8 \cosh ^{2}(j / 2)$.
( $I-33$ )
Application of the trigonometric identity:
asin $\omega t+b \cos \omega t=\left(a^{2}+b^{2}\right)^{1 / 2} \sin \left[\omega t+\cot ^{-1}(a / b)\right]$,
yields:

$$
\begin{equation*}
\psi_{1}(t)=\left[\omega^{1 / 2} / 4 \cosh ^{2}(j / 2)\right] \sin (\omega t+\pi / 4) \tag{I-35}
\end{equation*}
$$

Applying Eqs (I-13) and (I-19), one obtains as the solution for the small-amplitude fundamental harmonic current:

$$
\begin{align*}
I(\omega t)= & {\left[n^{2} F^{2} A C_{0}^{*}\left(\omega D_{0}\right)^{1 / 2} \Delta E_{a c} / 4 R T \cosh \right.} \\
& \sin (\omega t+\pi / 4) \tag{I-36}
\end{align*}
$$

One may proceed in a similar manner to obtain
higher-order current components. Equation (I-21) has the form, for $p=2$,

$$
\begin{align*}
& \left(e^{-j} / 2\right)\left[1-\int_{0}^{t} \psi_{0}(t-u) d u /(\pi u)^{1 / 2}\right] \sin ^{2} \omega t- \\
& {\left[e^{-j} \int_{0}^{t} \psi_{1}(t-u) d u /(\pi u)^{1 / 2}\right] \sin \omega t=} \\
& \left(1+e^{-j}\right) \int_{0}^{t} \psi_{2}(t-u) d u /(\pi u)^{1 / 2} . \tag{I-37}
\end{align*}
$$

Substituting Eqs. (I-24) and (I-27) in Eq. (I-37), rearranging, and applying the trigonometric identity:

$$
\begin{equation*}
\sin ^{2} \omega t=(1-\cos 2 \omega t) / 2 \tag{I-38}
\end{equation*}
$$

we get

$$
\begin{align*}
\int_{0}^{t} \psi_{2}(t-u) d u /(\pi u)^{1 / 2}= & {\left[\sinh (j / 2) / 16 \cosh ^{3}(j / 2)\right] \times } \\
& (1-\cos 2 \omega t) . \tag{I-39}
\end{align*}
$$

The form of Eq. (I-39) indicates that $\psi_{2}(t)$ is made up of dc and second-harmonic components, so that one can write:

$$
\begin{equation*}
\psi_{2}(t)=A_{0}(t)+A_{2} \sin 2 \omega t+B_{2} \cos 2 \omega t . \tag{I-40}
\end{equation*}
$$

Substituting Eq. (I-40) in Eq. (I-39) generates expressions for the dc and ac coponentspapplying Eqs (I-29), (I-30), and (I-31) to the ac portion] given by the equation:

$$
\begin{equation*}
\int_{0}^{t} A_{0}(t-u) d u /(\pi u)^{1 / 2}=\sinh (j / 2) / 16 \cosh ^{3}(j / 2) \tag{I-41}
\end{equation*}
$$

and

$$
\begin{align*}
& \left(A_{2}-B_{2}\right) \cos 2 \omega t-\left(A_{2}+B_{2}\right) \sin 2 \omega t= \\
& {\left[\omega^{1 / 2} \sinh (j / 2) / 8 \cosh ^{3}(j / 2)\right] \cos 2 \omega t,}  \tag{I-42}\\
& \text { Solution of Eq. (I-41) is accomplished readily by the }
\end{align*}
$$ method of Laplace transformation and, after application of Eqs. (I-13) and (I-19), the dc component corresponding to $A_{0}(t)$ is shown to be:

$$
I_{2}(d c)=n^{3} F^{3} A C_{0}^{*}{\underset{o}{1 / 2} \Delta E_{a c}^{2} \sinh (j / 2) /}^{1 / 2}
$$

$$
\begin{equation*}
16 R^{2} T^{2}(\pi t)^{1 / 2} \cosh ^{3}(j / 2) \tag{I-43}
\end{equation*}
$$

This represents the small-amplitude faradaic-rectification dc component flowing under the experimental conditions described. Addition of this term to the normal direct current[Eq. (I-25)] flowing in absence of alternating current yields an expression for the Fournier polarogram[11-13] within the framework of the planar-diffusion model. As will be pointed out below, the planar-diffusion model sometimes appears accurate ac components observed with a reversible process at a DME, but yields rather inaccurate expressions for the dc components. Influence of electrode growth and geometry must be considered in deriving the dc terms.

Proceeding as in the derivation of the fundamental harmonic component, one obtains solution for $A_{2}$ and $B_{2}$ in Eq. (I-42), corresponding to a second-harmonic component given by the equation:

$$
\begin{align*}
I(2 \omega t)= & {\left[2^{1 / 2} n^{3} F^{3} A C_{O}^{*}\left(\omega D_{O}\right)^{1 / 2} \Delta E_{a c}^{2} \sinh (j / 2) /\right.} \\
& \left.16 R^{2} T^{2} \cosh ^{3}(j / 2)\right] \sin (2 \omega t-\pi / 4) . \tag{I-44}
\end{align*}
$$

One may solve the integral equation for large values of $p$ to obtain expressions for third, fourth, etc., harmonics and larger amplitude contributions to all current components.[11,14].

The small amplitude expression given by Eq. (I-36) corresponds to a faradaic impedance(magnitude):

$$
\begin{equation*}
Z_{f}=4 R T \cosh { }^{2}(j / 2) / n^{2} F^{2} A C_{0}^{*}\left(\omega D_{0}\right)^{1 / 2} \tag{I-45}
\end{equation*}
$$

which is equivalent to a series RC circuit with :

$$
\begin{equation*}
R_{f}=1 / \omega C_{f}=4 R T \cosh { }^{2}(j / 2) / n^{2} F^{2} A C_{0}^{*}\left(2 \omega D_{0}\right)^{1 / 2} \tag{I-46}
\end{equation*}
$$

Appendix II. The quasi-reversible ac polarographic

$$
\text { wave }[5,6]
$$

Theoretical treatment of the ac polarographic wave kinetically controlled by both charge transfer and diffusion(the "quasi-reversible" case) requires minor modification in the mathematical formation of the reversible case. One deletes the assumption of nernstian behavior [Eq. (I-8)] and replaces it by the absolute rate expression[15]:

$$
\begin{aligned}
i(t) / n F A k_{s}= & C_{0 x=0} \exp \left[(-\alpha n F / R T)\left(E(t)-E^{\circ}\right)\right]- \\
& C_{R x=0} \exp [(1-\alpha) n F / R T]\left(E(t)-E^{\circ}\right) .(I I-1)
\end{aligned}
$$

One substitutes the expression for surface concentrations[Eqs. (I-10) and (I-11)] in Eq. (II-1) and, proceeding as for the reversible case, one obtains Eqs. (2-2) and (2-3), where

$$
\begin{align*}
& F_{1}(t)=1+\left(\alpha e^{-j}-\beta\right) D^{1 / 2} \psi_{0}(t) / k_{s} e^{-\alpha j},  \tag{II-2}\\
& G_{1}(\omega)=\left\{2 /\left[1+\left[1+\left[(2 \omega)^{\left.1 / 2 / \lambda]]^{2}\right\}^{1 / 2},}\right.\right.\right.\right.  \tag{II-3}\\
& F_{2}(t)=\left[P^{2}+L^{2}\right]^{1 / 2}, \tag{II-4}
\end{align*}
$$

$$
\begin{equation*}
G_{2}(\omega)=\left\{2 /\left[1+\left[1+\left(2 \omega^{1 / 2} / \lambda\right)\right]^{2}\right]\right\}^{1 / 2}, \tag{II-5}
\end{equation*}
$$

where

$$
\begin{align*}
& P=\left(\alpha^{2} / 2\right)\left(e^{j / 2}+2 e^{-j / 2}+e^{-3 j / 2}\right)-\left(\alpha e^{-j / 2}-\beta e^{j / 2}\right) x \\
& {\left[2+(2 \omega)^{1 / 2} / \lambda\right]\left[1+\left(\alpha e^{-j}-\beta\right) D^{1 / 2} \psi_{0}(t) / k_{s} e^{-\alpha j}\right] /} \\
& {\left[1+\left[1+\left[(2 \omega)^{\left.1 / 2 / \lambda]]^{2}\right],}\right.\right.\right.} \\
& L=\left(\alpha e^{-j / 2}-\beta e^{j / 2}\right)\left[(2 \omega)^{1 / 2 / \lambda]\left[1+\left(\alpha e^{-j}-\beta\right) D^{1 / 2} \times\right.}\right. \\
& \left.\psi_{0}(t) / k_{s} e^{-\alpha j}\right] /\left[1+\left[1+\left[(2 \omega)^{\left.1 / 2 / \lambda]]^{2}\right],} \quad(I I-7)\right.\right.\right.
\end{align*}
$$

where

$$
\begin{equation*}
\psi_{0}(t)=\left(k_{s} e^{-\alpha j} / D^{1 / 2}\right) e^{\lambda^{2} t} \operatorname{erfc}\left(\lambda t^{1 / 2}\right), \tag{II-8}
\end{equation*}
$$

where

$$
\begin{aligned}
& \operatorname{erfc}(x)=1-\operatorname{erf}(x)=\left(2 / \pi^{1 / 2}\right) \int_{x}^{\infty} \exp \left(-u^{2}\right) d u= \\
& {\left[\exp \left(-x^{2}\right) / \pi^{1 / 2} x\right]\left[1-1 / 2 x^{2}+1 \times 3 /\left(2 x^{2}\right)^{2}-\right.}
\end{aligned}
$$

$$
\begin{equation*}
\left.1 \times 3 \times 5 /\left(2 x^{2}\right)^{2}+\cdots \cdot \cdot\right] \tag{II-9}
\end{equation*}
$$

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Chapter 3. Kinetic analysis for the electrochemical reduction of chromium(VI) in alkaline solution
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3-1. Introduction

As mentioned in general introduction, the analysis of the oxidation and reduction mechanisms of multivalent chemical species is a continuing problem. In paticular, many studies[1-7] have been published to date on the polarographic reduction of the chromium(VI) species. The polarographic study of chromium(VI) in neutral and alkaline solutions was first performed by Kolthoff and Lingane[1]. Tondeur et al.[2] studied double layer effects on this system, and Miller[3] investigated the dc polarography of this system in the presence of complexing agents.

Delahay et al. $[4,5]$ also studied this system by employing dc polarography and chronopotentiometry. They suggested that chromium(IV) is an intermediate product of the reduction of chromium(VI) to chromium(III).

There have, however, been few reports[6] on its ac polarography and no report on the kinetics of this system.

In this chapter, the precise analysis of the
electrochemical reduction of chromium(VI) to chromium(II) in alkaline solution, already in use for the quantitative chemical analysis of chromium(VI)[1,7], was attempted by employing fundamental harmonic(lf) phase-selective ac polarography.

In general, $\operatorname{lf}\left(0^{\circ}\right)$ component of alternating current has been almost used for the quantitative analysis and other purpose(cf. Chap. l). The ac polarogram of this chromium(VI) system at high frequency, especially in lf(45 ) component, was not a first derivative form of the dc polarogram. The non-linear phenomenon was investigated by a curve fitting calculation. The curve fitting analysis was performed by using the theoretical equations of Hung and Smith[8] for a stepwise reaction controlled by ac polarography.

3-2. Experimental

A stock solution of chromium(VI) was prepared from sodium chromate(VI), $\mathrm{Na}_{2} \mathrm{CrO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$, and its concentration was determined gravimetrically with barium chloride[9].

The working electrode was a dropping mercury electrode; the capillary had a drop time of 4.25 s at -1.0 V in 1 M sodium hydroxide and the mercury flow was 1.769
mgs ${ }^{-1}$ at a height of 70 cm .
The typical compensation resistance of the iR compensator[l0,11](cf. Chap. 2) was $75 \Omega$ for the $\mathrm{Zn}(\mathrm{II}) / \mathrm{Zn}(\mathrm{Hg})$ and $60 \Omega$ for the $\mathrm{Cr}(\mathrm{VI})$ system.

The theoretical and curve fitting calculations were performed by using an $A C O S$ NEAC 1000 at the Computation Center, Osaka University.

3-3. Results and discussion

Figure 3-1 shows the ac polarograms, $1 f\left(0^{\circ}\right)$ and lf( $45^{\circ}$ ), of chromium(VI) in 1 M sodium hydroxide solution taken at various frequencies. $1 f\left(0^{\circ}\right)$ is the fundamental harmonic current at zero phase-angle. The dc polarogram is also included in the figure. It has been reported that in a neutral solution, the dc polarogram displays a small pre-wave and an irreversible main-wave[1-3]. The dc polarogram in 1 M sodium hydroxide in Fig. 3-l(A) does not show any pre-wave, but the ac polarogram at 15 Hz presents a small shoulder(Sl at -0.5 V ) which precedes the main peak(M at -0.8 V) as shown in Fig. 3-1(B).

The involvement of the specific adsorption in the present system is not probable because; (l) both the diffusion of the dc polarogram and the peak current of $M$


Fig. 3-1. Dc and ac polarograms for $5 \mathrm{mM} \mathrm{Na} \mathrm{NrO}_{4}$ in 1 M NaOH . (A) dc polarogram, (B) ac polarogram. Solid lines: lf( $0^{\circ}$ ); broken lines: $1 \mathrm{f}\left(45^{\circ}\right)$; (1) 15 Hz , (2) 50 Hz , (3) $500 \mathrm{~Hz}, \Delta \mathrm{E}_{\mathrm{ac}}=5 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}) . \mathrm{M},. \mathrm{S} 1, \mathrm{~S} 2:$ see text for caption.
for $1 f\left(0^{\circ}\right)$ are proportional to the concentration of chromium(VI) in the range 0.2-5.0 mM(Fig. 3-2), (2) the shapes of polarograms of both $1 f\left(0^{\circ}\right)$ and $1 f\left(90^{\circ}\right)$ components do not change with the concentration, (3) the phase angle is less than $45^{\circ}$.

As the frequency increases, another shoulder(S2) appears in the $1 f\left(0^{\circ}\right)$ polarogram between -0.9 and -1.0 V . This shoulder(S2) is more clearly seen in $1 f\left(45^{\circ}\right)$ than in $1 f\left(0^{\circ}\right)$.

The comparison between the experimental polarograms and the theoretical curves for one-step reversible, irreversible and quasi-reversible process[12-14](cf. Chaps. 2 and 5) leads to the following conclusions. (1) The theory for reversible process (ks $\gg \mathrm{cms}^{-1}$ ) predicts that the half-width of the ac polarogram is $90 / n \mathrm{mV}[12](\mathrm{n}$; the number of electrons transferred in the charge transfer step) for all frequencies, whereas that of the experimental polarograms is larger than 200 mV at low frequency. (2) The theory for an irreversible process[13](ks $<10^{-5} \mathrm{cms}^{-1}$ ) predicts polarograms similar in shape to that observed at low frequency, whereas the theoretical one has neither the shoulder $S 1$ nor $S 2$. (3) The theoretical polarogram for a quasi-reversible process[12,l4] presents the shoulder Sl when $\alpha<0.5$ and $k_{s} \simeq 10^{-4} \mathrm{cms}^{-1}$, whereas the shoulder $\mathrm{c}^{2}$ does not appear in any case of the theory.


Fig. 3-2. Calibration curves for chromium(VI) in 1 M sodium hydroxide. (O) dc diffusion current(id); ac peak current $\left(I_{\mathrm{p}}\right):(\bigcirc) 50 \mathrm{~Hz},(\mathrm{X}) 500 \mathrm{~Hz}, \Delta \mathrm{E}_{\mathrm{ac}}=5 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}$.$) .$

- For the quasi-reversible two step(EE) mechanism, Hung and Smith have developed a theory based on the stationary plane diffusion model on the assumption of the Butler type equation for i-E relation[cf. Eq. (II-1) in Appendix II]. They have shown several theoretical polarograms in their paper which, however, are not similar to the experimental ones. Since their calculation was limited to the case of $\mathrm{k}_{\mathrm{s}}>10^{-3} \mathrm{cms}^{-1}$, it is extended to the case of a slower rate. For the stepwise electrode reactions:

$$
\begin{align*}
& 0+\mathrm{n}_{1} \mathrm{e}^{-} \rightleftharpoons \mathrm{Y},  \tag{3-1}\\
& \mathrm{Y}+\mathrm{n}_{2} \mathrm{e}^{-} \rightleftharpoons \mathrm{R}, \tag{3-2}
\end{align*}
$$

the fundamental harmonic alternating current is written as follows[8](cf. Appendix III):

$$
\begin{equation*}
I(\omega t)=I_{1 f} \sin (\omega t+\phi), \tag{3-3}
\end{equation*}
$$

where

$$
I_{1 f}=\left[2 \mathrm{~F}^{2} A C_{0}^{*} \Delta \mathrm{E}_{\mathrm{ac}}\left(2 \omega \mathrm{D}_{\mathrm{O}}\right)^{1 / 2 / R T]\left[\left(\mathrm{Y}^{2}+\mathrm{z}^{2}\right)^{1 / 2} / \mathrm{Q}\right], \quad(3-4), ~}\right.
$$

where (see also Appendix III for $Q, Y$ and $Z$ )
$Q=g_{1}\left(\omega, \lambda_{i}, j_{i}\right)$,
$Y=g_{2}\left(\omega, \lambda_{i}, j_{i}, n_{i}\right)$,
$z=g_{3}\left(\omega, \lambda_{i}, j_{i}, n_{i}\right)$,
$\phi=\cot ^{-1}(Y / Z)$,
where
$\lambda_{i}=\left[\left(k_{s, i} f_{i}\right) / D_{i}^{l / 2}\right]\left[\exp \left(-\alpha_{i} j_{i}\right)+\exp \left(\beta_{i} j_{i}\right)\right]$,
$j_{i}=\left(n_{i} F / R T\right)\left(E_{d c}-E_{1 / 2, i}^{r}\right)$.
$Y / Q$ and $Z / Q$ in Eqs. (3-4)-(3-7) are the quantities


Fig. 3-3. Theoretical ac polarograms for two-step
mechanism. Solid lines: $1 \mathrm{f}\left(0^{\circ}\right)$; broken lines: $1 \mathrm{f}\left(45^{\circ}\right)$;
15 Hz , (2) $500 \mathrm{~Hz}, \mathrm{D}_{0}=9.61 \times 10^{-6} \mathrm{~cm}^{2} \mathrm{~s}^{-1}$ (calculated from
diffusion current for dc polarogram), $D_{Y}=D_{R}=1 \times 10^{-5}$ $\mathrm{cm}^{2} \mathrm{~s}^{-1}, \mathrm{~A}=0.0327 \mathrm{~cm}^{2}, \mathrm{t}=4.25 \mathrm{~s}, \Delta \mathrm{E} \mathrm{ac}=0.005 \mathrm{v}, \mathrm{C}_{\mathrm{O}}^{*}=$ $5.0 \times 10^{-3} \mathrm{M}, \mathrm{T}=298 \mathrm{~K}$. Values of $\mathrm{n}_{1}, \mathrm{n}_{2}, \alpha_{1}, \alpha_{2}, \mathrm{k}_{\mathrm{s}, 1}$, $k_{s, 2}$ and $\left(E_{1 / 2,1}^{r}-E_{1 / 2,2}^{r}\right)$ are given in Table 3-1.


Fig. 3-3 (D) and (E).

Table 3-1. Parameters for the calculation.

| Fig. | $n_{1}$ | $n_{2}$ | $\alpha_{1}$ | $\alpha_{2}$ | $k_{s, 1} / \mathrm{cms}^{-1}$ | $\mathrm{k}_{\mathrm{s}, 2} / \mathrm{cms}^{-1}$ | $\mathrm{E}_{1 / 2,1}^{\mathrm{r}}-\mathrm{E}_{1 / 2,2}^{\mathrm{r}} / \mathrm{V}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3-3A | 2 | 1 | 0.25 | 0.5 | $1 \times 10^{-4}$ | $1 \times 10^{-3}$ | 0.15 |
| B | 2 | 1 | 0.25 | 0.5 | $1 \times 10^{-6}$ | $1 \times 10^{-5}$ | 0.15 |
| C | 2 | 1 | 0.25 | 0.5 | $1 \times 10^{-3}$ | $1 \times 10^{-2}$ | 0.20 |
| D | 2 | 1 | 0.25 | 0.5 | $1 \times 10^{-5}$ | $1 \times 10^{-6}$ | 0.10 |
| E | 1 | 2 | 0.5 | 0.25 | $1 \times 10^{-5}$ | $1 \times 10^{-6}$ | 0.10 |

corresponding to the amplitude of $\sin \omega t$ and cos $\omega t$, respectively. The current $I_{1 f}(\theta)$ at the phase-angle, $\theta$ is given by the following equation:

$$
\begin{equation*}
I_{I f}(\theta)=I_{I f} \cos (\phi-\theta) . \tag{3-11}
\end{equation*}
$$

For the dc part of the theoretical calculation, the following approximation was used:

$$
\begin{equation*}
\exp \left(x^{2}\right) \operatorname{erfc}(x) \simeq(2 / \sqrt{\pi}) \int_{x}^{x+z} \exp \left(x^{2}-u^{2}\right) d u \tag{3-12}
\end{equation*}
$$

A value of 10 for $z$ in $E q .(3-12)$ is adequate to obtain a quasi-reversible theoretical ac polarogram[8].

The selected parameters in the calculation are $n_{1}$, $\mathrm{n}_{2}$ (number of electrons transferred in the first and the second heterogeneous charge transfer steps, respectively), $\alpha_{1}, \alpha_{2}$ (transfer coefficients), $k_{s, 1}, k_{s, 2}$ (heterogeneous rate constants) and $\left(E_{1 / 2,1}^{r}-E_{1 / 2,2}^{r}\right)$ (difference between the two reversible half wave potentials). Some of the typical theoretical polarograms obtained are shown in Fig. 3-3. The values of parameters used for the calculation are given in Table 3-1.

The following predictions can be derived from the calculations:
(1) $\alpha_{1}$ and $\alpha_{2}$ determine the width of the peak, as is the case for one step quasi-reversible process[10,13].
(2) The shoulder Sl appears only when $k_{s, 1}$ is of the order of $10^{-4} \mathrm{cms}^{-1}$ and $\alpha_{1}$ is smaller than 0.5 .
(3) The shape of the shoulder $S 2$ changes according to
the ratio $\left(k_{s, 1} / k_{s, 2}\right)$, the potential difference( $E_{1 / 2,1}^{r}-$ $\mathrm{E}_{1 / 2,2}^{\mathrm{r}}$ ) and the frequency.

The polarograms in Fig. 3-3(A) look quite similar to the experimental ones. In order to determine the kinetic parameters, a curve fitting procedure of the ac polarograms was performed by taking the values used for the polarograms in Fig. 3-3(A) as initial values. The flow chart for the procedure is given in Fig. 3-4. The FORTRAN programs have been written in quadruple precision. The curves were fitted by employing the gradient-expansion algorithm of Marquart[15]. Gans concluded[16] that the Marquart approach was a useful general purpose method for nonlinear least squares problems arising in data fitting. This method has already been used in the deconvolution of electrochemical data by Boudreau and Perone[17].

The curve fittings were performed for nineteen polarograms. The results are given in Table 3-2 together with the relevant experimental constants and related parameters. Figure 3-5 shows the best fit for the theoretical and experimental ac polarograms lf( $0^{\circ}$ ) and lf( $45^{\circ}$ ). The agreement is good if one considers that the theoretical calculation is based on a simple model of $E E$ mechanism coupled with stationary plane diffusion. The change of shape of the $45^{\circ}$ component with frequency matches the experimental results very well, probably because the


Fig. 3-4. Flow chart for curve fitting. $I_{i}$ : experimental alternating current; $E_{i}$ : experimental dc potential; $I\left(E_{i}\right)$ : calculated alternating current; $S_{f i t}^{2}=\left[\left[I_{i}-I\left(E_{i}\right)\right]^{2}\right.$.

Table 3-2. Fitting parameters and other constants for curve fitting.

| Parameter | 9 Hz |  | 15 mz |  |  | 30 Hz |  | 50 yz |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $0 \cdot$ | $90^{\circ}$ | $0^{\circ}$ | 45* | $90^{\circ}$ | 0 - | $90^{\circ}$ | 0 - | 45* | $90^{*}$ |
| ${ }^{\circ} 1$ | 0.19 | 0.22 | 0.19 | 0.19 | 0.17 | 0.19 | 0.17 | 0.18 | 0.18 | 0.18 |
| $a_{2}$ | 0.63 | 0.84 | 0.54 | 0.50 | 0.51 | 0.52 | 0.47 | 0.34 | 0.53 | 0.52 |
| $10^{5} \mathrm{k}_{\mathrm{s,1}} / \mathrm{cmas}^{-1}$ | 5.3 | 4.8 | 5.0 | 6.5 | 10.1 | 5.3 | 9.4 | 4.7 | 10.0 | 11.0 |
| $10^{3} \mathrm{k}, 2 / \mathrm{cma}^{-1}$ | 1.8 | 0.3 | 1.9 | 2.2 | 1.2 | 2.3 | 7.2 | 1.9 | 1.4 | 2.3 |
|  | 0.18 | 0.26 | 0.20 | 0.21 | 0.17 | 0.20 | 0.21 | 0.21 | 0.19 | 0.27 |




Fig. 3-5. Theoretical and experimented ac polarograms for 5 $\mathrm{mM} \mathrm{Na} 2_{2} \mathrm{CrO}_{4}$ in 1 M NaOH . (A) 50 Hz , (B) 500 Hz ; (1,3) $\theta=$ $0^{\circ}$, $(2,4) \theta=45^{\circ}$; solid line: experimental; broken lines: theoretical. See Table 3-2 for fitting parameters and other constants.
first step is less reversible than the second, and that $S 2$ is more emphasized in the $45^{\circ}$ than in the $0^{\circ}$ component. In other words, it is the $45^{\circ}$ component which should be used to evaluate the relative rate constants, because its shape is the most sensitive to the ratio (k, $\mathrm{k}_{\mathrm{s}, 1} / \mathrm{k}_{\mathrm{s}, 2}$ ).

One can conclude that the electrochemical reduction of chromium(VI) to chromium(III) is controlled by the following stepwise process:

$$
\begin{aligned}
& \mathrm{Cr}(\mathrm{VI})+2 \mathrm{e}^{-} \rightleftharpoons \operatorname{Cr}(\mathrm{IV}), \\
& \mathrm{Cr}(\mathrm{IV})+\mathrm{e}^{-} \rightleftharpoons \operatorname{Cr}(\mathrm{II}),
\end{aligned}
$$

and that the first reduction step of $\operatorname{Cr}(V I)$ to $\operatorname{Cr}(I V)$ is the rate determining step. The kinetic parameters for the Cr(VI) to $C r(I I I)$ reaction in $l$ sodium hydroxide solution $\operatorname{are} k_{s, 1}=(7.5 \pm 2.9) \times 10^{-5} \mathrm{cms}^{-1}, k_{s, 2}=(1.4 \pm 0.7) \times 10^{-3}$ $\mathrm{cms}^{-1}, \alpha_{1}=0.18 \pm 0.01, \alpha_{2}=0.54 \pm 0.08, \mathrm{E}_{1 / 2,1}^{\mathrm{r}}=-0.32 \mathrm{v} \mathrm{vs}$. NHE and $E_{1 / 2,2}^{r}=-0.52 \quad \mathrm{~V}$.

The redox potential of $\mathrm{E}^{\circ}=-0.39 \mathrm{~V}$ for $\operatorname{Cr}(V I)+3 e^{-}$
$\operatorname{Cr}(I I I)$ calculated from $E_{1 / 2,1}^{r}$ and $E_{1 / 2,2}^{r}$ may well be compared to the following standard redox potentials to estimate the electrogenerated $\operatorname{Cr}(I I I)$ species. When $\left[\mathrm{H}^{+}\right]=$ $10^{-14} \mathrm{M}[18]$,
$\mathrm{CrO}_{4}^{2-}+8 \mathrm{H}^{+}+3 \mathrm{e}^{-} \rightleftharpoons \mathrm{Cr}^{3+}+4 \mathrm{H}_{2} \mathrm{O} \quad(-0.726 \mathrm{~V})$,
$\mathrm{CrO}_{4}^{2-}+7 \mathrm{H}^{+}+3 \mathrm{e}^{-} \rightleftharpoons \mathrm{Cr}(\mathrm{OH})^{2+}+3 \mathrm{H}_{2} \mathrm{O} \quad(-0.526 \mathrm{~V})$,
$\mathrm{CrO}_{4}^{2-}+6 \mathrm{H}^{+}+3 \mathrm{e}^{-} \rightleftharpoons \mathrm{Cr}(\mathrm{OH})_{2}^{+}+2 \mathrm{H}_{2} \mathrm{O} \quad(-0.373 \mathrm{~V})$,
$\mathrm{CrO}_{4}^{2-}+\mathrm{nH}_{2} \mathrm{O}+5 \mathrm{H}^{+}+3 \mathrm{e}^{-} \rightleftharpoons \mathrm{Cr}(\mathrm{OH})_{3} \cdot \mathrm{nH}_{2} \mathrm{O}+\mathrm{H}_{2} \mathrm{O} \quad(-0.133 \mathrm{~V})$.

Therefore, the $\operatorname{Cr}(\mathrm{III})$ could be in the form of $\operatorname{Cr}(\mathrm{OH})_{2}^{+}$.
The so-called pre-wave in the dc polarogram in neutral or weak alkaline solutions has been studied by Tondeur et a1.[2], who reported that its shape and current change with the supporting electrolytes. They have explained it in terms of double layer effects. The experiments indicated that the shoulder(Sl) in $1 f\left(0^{\circ}\right)$ does not correspond to the so-called pre-wave, because the peak current in $2 \mathrm{f}\left(90^{\circ}\right)$ corresponding to the $S 1$ in 1 M sodium hydroxide decreases with frequency, while that of the pre-wave in sodium chloride is independent of frequency.

Appendix III. Alternating current polarogram with multi-step charge transfer(EE mechanism)

Q, $Y$ and $Z$ in Eqs. (3-4)-(3-7) can be written as follows[8]:
$Q=g_{1}\left(\omega, \lambda_{i}, j_{i}\right)=4\left[1-\left(1+e^{-j_{1}}\right)\left(1+e^{j 2}\right)\right]^{2} /\left(1+e^{-j 1}\right)^{2} x$
$\left(1+e^{j 2}\right)^{2}+4(2 \omega)^{1 / 2}\left[\left(1+e^{-j_{1}}\right)\left(1+e^{j_{2}}\right)-1\right]\left(\lambda_{1}+\lambda_{2}\right) /$
$\left(1+e^{-j 1}\right)\left(1+e^{j 2}\right) \lambda_{1} \lambda_{2}+4 \omega\left(\lambda_{1}+\lambda_{2}\right)^{2} / \lambda_{1}^{2} \lambda_{2}^{2}+2(2 \omega)^{3 / 2} x$ $\left(\lambda_{1}+\lambda_{2}\right) / \lambda_{1}^{2} \lambda_{2}^{2}+(2 \omega)^{2} / \lambda_{1}^{2} \lambda_{2}^{2} \quad$,
(III-1)
$Y=g_{2}\left(\omega, \lambda_{i}, j_{i}, n_{i}\right)=n_{1}^{2} G_{1}(t)\left[\left(1+e^{j_{2}}+n_{2} / n_{1}\right) \times\right.$

$$
\begin{align*}
& {\left[\left(1+e^{j 2}\right)\left(1+e^{-j_{1}}\right)-1\right] /\left(1+e^{-j 1}\right)\left(1+e^{j_{2}}\right)^{2}+} \\
& (2 \omega)^{1 / 2}\left[n_{1}\left(1+e^{j_{2}}\right)+n_{2}\right]\left(\lambda_{1}+\lambda_{2}\right) / n_{1}\left(1+e^{j 2}\right) \lambda_{1} \lambda_{2}+ \\
& 2 \omega\left[\lambda_{1}+\lambda_{2}\left[\left(n_{2}+2 n_{1}\left(1+e^{j_{2}}\right)\right] / 2 n_{1}\left(1+e^{j_{2}}\right)\right] / \lambda_{1} \lambda_{2}^{2}+\right. \\
& (2 \omega)^{\left.3 / 2 / 2 \lambda_{1} \lambda_{2}^{2}\right]+n_{2}^{2} G_{2}(t)\left[( 1 + e ^ { - j _ { 1 } } + n _ { 1 } / n _ { 2 } ) \left[\left(1+e^{j_{2}}\right) \times\right.\right.} \\
& \left.\left(1+e^{-j_{1}}\right)-1\right] /\left(1+e^{-j_{1}}\right)^{2}\left(1+e^{j_{2}}\right)+(2 \omega)^{1 / 2}\left[n_{2}\left(1+e^{-j_{1}}\right)\right. \\
& \left.+n_{1}\right]\left(\lambda_{1}+\lambda_{2}\right) / n_{2}\left(1+e^{-j_{1}}\right) \lambda_{1} \lambda_{2}+2 \omega\left[\lambda_{2}+\lambda_{1}\left[n_{1}+\right.\right. \\
& \left.\left.\left.2 n_{2}\left(1+e^{-j_{1}}\right) / 2 n_{2}\left(1+e^{-j_{1}}\right)\right] / \lambda_{1}^{2} \lambda_{2}\right]+(2 \omega)^{3 / 2} / 2 \lambda_{1}^{2} \lambda_{2}\right], \tag{III-2}
\end{align*}
$$

$z=g_{3}\left(\omega, \lambda_{i}, j_{i}, n_{i}\right)=n_{1}^{2} G_{1}(t)\left[\left(1+e^{j 2}+n_{2} / n_{1}\right) \times\right.$

$$
\left[\left(1+e^{j 2}\right)\left(1+e^{-j_{1}}\right)-1\right] /\left(1+e^{-j_{1}}\right)\left(1+e^{j 2}\right)^{2}+
$$

$$
(2 \omega)^{1 / 2}\left[\left(1+e^{j_{2}}\right)\left(1+e^{-j_{1}}\right)-1\right] /\left(1+e^{-j 1}\right)\left(1+e^{j 2}\right) \lambda_{2}+
$$

$$
\left.\omega\left[n_{1}\left(1+e^{j_{2}}\right) \lambda_{1}-n_{2} \lambda_{2}\right] / n_{1}\left(1+e^{j 2}\right) \lambda_{1} \lambda_{2}^{2}\right]+n_{2}^{2} G_{2}(t) \times
$$

$$
\left[\left(1+e^{-j_{1}}+n_{1} / n_{2}\right)\left[\left(1+e^{j_{2}}\right)\left(1+e^{-j_{1}}\right)-1\right] /\left(1+e^{-j_{1}}\right)^{2} x\right.
$$

$$
\left(1+e^{j 2}\right)+(2 \omega)^{1 / 2}\left[\left(1+e^{j 2}\right)\left(1+e^{-j_{1}}\right)-1\right] /\left(1+e^{-j 1}\right) \times
$$

$$
\begin{equation*}
\left.\left(1+e^{j 2}\right) \lambda_{1}+\omega\left[n_{2}\left(1+e^{-j 1_{1}}\right) \lambda_{2}-n_{1} \lambda_{1}\right] / n_{2}\left(1+e^{-j 1}\right) \lambda_{1}^{2} \lambda_{2}\right], \tag{III-3}
\end{equation*}
$$

where $G 1(t)$ and $G 2(t)$ are expressed by the equations:
$G_{1}(t)=\left[1 /\left(1+e^{-j_{1}}\right)+\psi_{2,0}(t)\left(1+e^{-j_{2}}\right) / \lambda_{2}\left(1+e^{-j_{1}}\right)+\right.$
$\left.\psi_{1,0}(t)\left(\alpha_{1}+\alpha_{1} e^{-j_{2}}-\beta_{1} e^{j_{1}}\right) / \lambda_{1}\right] /\left(1+e^{-j_{2}}+e^{j_{1}}\right),(I I I-4)$
$G_{2}(t)=\left[1 /\left(1+e^{j_{2}}\right)-\psi_{1,0}(t)\left(1+e^{j_{1}}\right) / \lambda_{1}\left(1+e^{j 2}\right)-\right.$
$\left.\psi_{2,0}(t)\left(\beta_{2}+\beta_{2} e^{j_{1}}-\alpha_{2} e^{-j 2}\right) / \lambda_{2}\right] /\left(1+e^{-j 2}+e^{j 1}\right),(I I I-5)$
where
$\psi_{1,0}(t)=\left[\lambda_{1} /\left(1+e^{j 1}\right)\left(x_{-}-x_{+}\right)\right]\left[\left(x_{-}-\lambda_{2}\right) x\right.$
$\left.\exp \left(X_{-}^{2} t\right) \operatorname{erfc}\left(X_{-} t^{1 / 2}\right)-\left(X_{+}-\lambda_{2}\right) \exp \left(X_{+}^{2} t\right) \operatorname{erfc}\left(X_{+} t^{1 / 2}\right)\right]$,
$\psi_{2,0}(t)=\left[\lambda_{1} \lambda_{2} /\left(X_{-}-X_{+}\right)\left(1+e^{j 1}\right)\left(1+e^{j 2}\right)\right] x$ $\left[\exp \left(X_{+}^{2} t\right) \operatorname{erfc}\left(X_{+} t^{1 / 2}\right)-\exp \left(X_{-}^{2} t\right) \operatorname{erfc}\left(X_{-} t^{1 / 2}\right)\right]$, (III-7) where
$x_{ \pm}=\left[\lambda_{1}+\lambda_{2} \pm\left[\left(\lambda_{1}+\lambda_{2}\right)^{2}-4 \mathrm{~K}\right]^{1 / 2}\right] / 2$,
$K=\lambda_{1} \lambda_{2}\left[e^{j_{2}}+e^{-j_{1}}+e^{\left(j_{2}-j_{1}\right)}\right] /\left(1+e^{j_{2}}\right)\left(1+e^{-j_{1}}\right)$. (III-9)
And D , f and E in Eqs. (3-9) and (3-10) are
obtained from the relations:
$D_{1}=D_{0}^{\beta_{1}} D_{Y}^{\alpha_{1}}, D_{2}=D_{Y}^{\beta_{2}} D_{R}^{\alpha_{2}}, f_{1}=f_{0}^{\beta_{1}} f_{Y}^{\alpha_{1}}, f_{2}=f_{Y}^{\beta_{2}} f_{R}^{\alpha_{2}}$, $\beta_{i}=1-\alpha_{i}$,
$E_{1 / 2,1}^{r}=E_{1}^{o}-\left(R T / n_{1} F\right) \ln \left(f_{Y} / f_{0}\right)\left(D_{0} / D_{Y}\right)^{1 / 2}$,
$E_{1 / 2,2}^{r}=E_{2}^{\circ}-\left(R T / n_{2} F\right) \ln \left(f_{R} / f_{Y}\right)\left(D_{Y} / D_{R}\right)^{1 / 2}$.
On the other hand, the dc polarographic current is
obtained from the following equation:

$$
\begin{equation*}
i_{d c}(t)=\operatorname{FAC}_{0}^{*} D_{0}^{1 / 2}\left[n_{1} \psi_{1,0}(t)+n_{2} \psi_{2,0}(t)\right] \tag{III}
\end{equation*}
$$

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Chapter 4. On the polarographic reduction of nitrate ion in the presence of zirconium(IV)

## 4-1. Introduction

It is well known that nitrate ion produces polarographic waves in certain supporting electrolytes[1-5].

Tokuoka and Ruzicka[l,2] studied the polarography of nitrate ion in the presence of mono-, di- and trivalent cations. They suggested that the nitrate is reduced by splitting into its elementary ions ( $\mathrm{N}^{5+}$ and $\mathrm{O}_{3}^{6-}$ ) in the strong electric field existing in the proximity of the dropping mercury electrode(DME), high-valency cations serving to drag the anion close to the DME interface. Therefore, the DME interface should be investigated. For the effect of high-valence elements upon the polarographic reduction of the nitrate, zirconium(IV) as a tetravalent element has been selected. There is no agreement among previous investigators[6,7] on the mechanism of the reduction of nitrate ion in the presence of this element. Mechelynck and Mechelynck-David[6] concluded that, for the
reduction mechanism, $\mathrm{Zr}^{4+}$ ion acts as a catalyst through the reaction $\mathrm{ZrO}^{2+}+2 \mathrm{H}^{+} \rightleftharpoons \mathrm{Zr}^{4+}+\mathrm{H}_{2} \mathrm{O}$ in the DME interface. Wharton[7] argued for a mechanism, in which the polarographic reduction of the nitrate ion is not catalytic but rather involves the reduction of distinct zirconium(IV)-nitrate complexes.

As mentioned in chapter 1 , the $\operatorname{lf}\left(0^{\circ}\right)$ and $\operatorname{lf}\left(90^{\circ}\right)$ modes of the phase selective ac polarography are valid for a study on the specific adsorption of electrolyte ions on the DME.

In this chapter, therefore, the mechanism of the reduction of the nitrate in the presence of the zirconium(IV) ion was investigated by means of $d c$ and phase-selective ac polarography. As the reduction products in this system have never been eatablished, they were studied by controlled-potential electrolysis(CPE) and by a qualitative and quantitative analysis. In order to investigate the vicinity of the electrode, the differential capacity was measured by ac polarography.

4-2. Experimental

Analytical-reagent grade chemicals were used as received. A stock solution of zirconium(IV) was prepared
from zirconium oxychloride octahydrate(ZrOC1 $\left.2 \cdot 8 H_{2} 0\right)$, and its concentration was determined gravimetrically as zirconium oxide with cupferron (ammonium nitrosophenylhydroxylamine)[8]. The stock solution was allowed to age for at least two weeks prior to use, in order to eliminate the ageing effect of zirconium solution[7]. The stock solution of sodium nitrate was determined gravimetrically with nitron (3,4,6-triphenyl-2, 3,5,6-tetraazabicyclo[2.1.1] hex-1-ene) [9].

The working electrode for $d c$ and ac polarography was the DME; the mercury flow rate was $1.150 \mathrm{mgs}^{-1}$ at a height of 70 cm and the drop time was 5.78 s in water at open circuit.

A hand-made coulomb counter[10] was used in the CPE coulometry. An $H-t y p e c e l l$ was used in the CPE. The working electrode was a mercury pool. The supporting electrolyte solution was degassed and pre-electrolyzed at -0.9 V vs. $\operatorname{SSE}(s a t u r a t e d$ silver chloride electrode[Ag/AgCl, sat.KCl], and the solution was again pre-electrolyzed at -0.3 V after addition of the degassed nitrate solution. The sample solution was electrolyzed at -0.9 V until the current reached the base current of the pre-electrolysis.

The differential capacity was obtained by using phase-selective ac polarography(cf. Appendix IV). The
results of $0.1 \mathrm{M} K C 1$ solution was $\pm 2.5 \%$ of those of Grahame[11]. The electrocapillary maximum(ecm) was measured by a drop time method. The ecm of 0.1 M KCl solution was -0.508 V vs. $\mathrm{NCE}(-0.506 \mathrm{~V}$ according to Grahame[ll]). The surface tention of 0.1 M KCl solution at the ecm was taken $426.7 \mathrm{mNm}^{-1}$ at $25^{\circ} \mathrm{C}[12]$.

Raman spectra were measured with a JASCO $R$ 800, using $514.5 \mathrm{~nm}\left(\mathrm{Ar}^{+}\right)$laser as the excitation source.

Spectrophotometric measurements in the wave length range 200-900 nm were carried out with a HITACHI 228 spectrophotometer.

4-3. Results and discussion

4-3-1. Influence of solution composition on the dc polarographic waves

The dc polarograms of nitrate ion in the presence of zirconium(IV) salts were similar to those of Mechelynck and Mechelynck-David[6] and Wharton[7].

The first wave was observed between -0.7 and -1.1 V vs. SSE and the second was between -1.4 and -1.5 V . The dependence of $i_{d}(d c$ plateau current) for the first and second waves on the concentration of sodium nitrate is
plotted in Fig. 4-1. Log(id $/ \mu A)$ for the first wave varies linearly with $\log \left(\mathrm{C}_{\mathrm{NO}_{3}^{-/ M}}\right)$ for the range $2.0 \times 10^{-6}-4.0 \times 10^{-3} \mathrm{M}$ and the slope is one. In the concentration range > $4.0 \times 10^{-3} \mathrm{M}, \mathrm{i}_{\mathrm{d}}$ is practically constant. The results indicate that the first wave is suitable for the quantitative analysis of nitrate in a wide concentration range.

As the concentration of nitrate ( $\mathrm{pH} 2.93 \pm 0.02$ ) increases, the second wave decreases and the $i_{d}$ of the first wave is limited by the proton concentration. The protons around the DME are consumed and exhausted by the reduction of nitrate ions.

The $i_{d} s$ of the first and second waves change linearly with the square root of the mercury height but the lines does not go through the origin. This suggests that both the polarographic reduction of the nitrate ion and that of the proton are controlled by a diffusion and a chemical reaction.

Figure 4-2(A) shows the change of the dc polarograms with the concentration of zirconium(IV). The dependence of $i_{d}$ (the current at -1.2 V ) on the concentration of zirconium(IV) is given in Fig. 4-2(B). For the first wave, at a low concentration of zirconium(IV), no plateau current is observed. As the concentration of zirconium(IV) increases, it is better defined. At a high concentration


Fig. 4-1. The dependence of $i_{d}$ on $\mathrm{C}_{\mathrm{NO}_{3}^{-}}^{-} \quad \mathrm{x} \mathrm{M} \mathrm{NaNO} 3+y \mathrm{M}$ $\mathrm{NaClO}_{4}+0.2 \mathrm{mM} \mathrm{ZrOCl} 2_{2}+1 \mathrm{mM} \mathrm{HC1O} 4, \mathrm{x}+\mathrm{y}=0.1, \mathrm{pH}=$ $2.93 \pm 0.02$, (O) lst wave; (X) 2nd wave.


Fig. 4-2. (A) Dc polarograms of nitrate in the presence of various concentrations of zirconium(IV). $1 \mathrm{mM} \mathrm{NaNO}_{3}+0.01$ $\mathrm{M} \mathrm{HClO}_{4}+0.09 \mathrm{M} \mathrm{NaClO}_{4} ; \mathrm{C}_{\mathrm{Zr}(\mathrm{IV})}:$ (1) 0 , (2) 0.02 , (3) 0.1 , (4) 0.2 , (5) 0.4 mM . (B) The dependence of $i_{d} u p o n$ $\mathrm{C}_{\mathrm{Zr}(\mathrm{IV})} \cdot(\mathrm{X}) \mathrm{l} \mathrm{mM} \mathrm{NaNO}_{3}+1 \mathrm{mMHClO}_{4}+0.1 \mathrm{M} \mathrm{NaClO}_{4}, \mathrm{pH}=$ $2.95 \pm 0.05$; (O) $1 \mathrm{mM} \mathrm{NaNO} 3+0.01 \mathrm{MHClO}_{4}+0.09 \mathrm{M} \mathrm{NaClO}_{4}$, $\mathrm{pH}=2.02 \pm 0.02$; ( ) $0.05 \mathrm{M} \mathrm{NaNO}_{3}+0.01 \mathrm{M} \mathrm{HClO}_{4}+0.04 \mathrm{M}$ $\mathrm{NaClO}_{4}, \mathrm{pH} 2.02 \pm 0.03$.
of zirconium(IV)(about 0.2 mM ) the $\mathrm{i}_{\mathrm{d}}$ levels off. These results suggest that for the reduction of nitrate, only a certain amount of zirconium(IV) (about 0.2 mM ) is needed.

The dependence of $i_{d}$ and $E_{1 / 2}$ (halfwave potential) for the reduction of nitrate on pH in the bulk solution is given in Figs. 4-3(A) and (B), respectively. When the proton is in excess of the nitrate, both $i_{d}$ and $E_{1 / 2}$ are practically independent of $p H$. When there is a proton deficiency for the reduction of nitrate, $\log \left(i_{d} / A\right)$ changes linearly with the $\mathrm{pH}(\mathrm{slope}$ is -1$)$ and the $E_{1 / 2}$ varies linearly with the $\mathrm{pH}(\mathrm{slope}$ is about 120 mV$)$. The positive slope $\left(\delta E_{1 / 2} / \delta p H\right)$ shows that the greater is the concentration of proton, the less is the nitrate reducible in the presence of zirconium(IV).

4-3-2. Total electrode reaction.

As $n$ (the number of electrons in the charge transfer step) including the total electrode reaction has never been decided for this system, the CPE coulometry was employed. In the solution $\left[\left(0.01 \mathrm{MHClO}_{4}+0.09 \mathrm{M} \mathrm{NaClO} 4+1 \mathrm{mM}\right.\right.$ $\left.\mathrm{ZrOCl}_{2}\right) 25 \mathrm{ml}+\left(25 \mathrm{mM} \mathrm{NaNO} 3 \mathrm{ml} \mathrm{ml}_{\mathrm{M}} \mathrm{n}\right.$ was 5.8士0.4. After the CPE, the sample solution gave positive results with Tollens reagent and for the $\operatorname{TTC}(2,3,5-t r i p h e n y l t e t r a z o l i u m$


```
chloride) spot test[l3]. The solution gave a negative
result with Nessler's reagent. The results shows that
hydroxylamine is in the solution after CPE and the ammonium
ion is not(cf. in LaCl3 and CeCl3, the chief products were
hydroxylamine and ammonia, n was 6.1-7.2[5]). The
hydroxylamine in the solution after CPE was determined
quantitatively by means of dc polarography at pH 6.3(with
K}\mp@subsup{\mp@code{H}}{2}{}\mp@subsup{\textrm{PO}}{4}{}+N\mp@subsup{\textrm{Na}}{2}{}\mp@subsup{\textrm{HPO}}{4}{}\mathrm{ buffer)[14]. 92.4#0.4 % of the available
nitrate ion changed to hydroxylamine in the CPE. Therefore,
it is infered that the total reduction of nitrate in the
presence of zirconium(IV) chiefly follows the reaction:
NO
```

4-3-3. The role of zirconium(IV).

The nitrate anion is reduced at more positive potentials (1.2-1.5 V) in the presence of a small amount of zirconium(IV)[Fig. 4-3(B)) than in only univalent cation supporting electrolyte[2].

Mechelynck and Mechelynck-David[6] asserted that $2 r^{4+}$ ion was a catalyst for the reduction of nitrate. If the $\mathrm{Zr}^{4+}$ is produced by the reaction, $\mathrm{ZrO}^{2+}+2 \mathrm{H}^{+} \rightleftharpoons \mathrm{Zr}^{4+}+$ $\mathrm{H}_{2} \mathrm{O}$, the lower is the pH , the more is the nitrate ion reducible; but Fig. $4-3(B)$ shows the contrary.

Wharton argued[7] that the reduction was not catalytic but rather involved the reduction of distinct zirconium(IV)-nitrate complexes. If such complexes are produced, a band will appear in Raman spectra, at about 750 $\mathrm{cm}^{-1}$ in addition to the band at $715 \mathrm{~cm}^{-1}$ (free solvated nitrate ion) $[15,16]$. In this system, however, the band assigned to the complexes in the bulk solution did not appear in any solution (0-0.01 $\mathrm{MHClO}_{4}+0-0.1 \mathrm{M} \mathrm{ZrOCl}_{2}+$ 0.01-2 $\mathrm{M} \mathrm{NaNO}_{3}$ ). UV spectra were also measured. A band at about 300 nm for nitrate[17,18] did not change in shape or absorption maximum(301.4 $\pm 0.7 \mathrm{~nm})$ in any solution(0-0.01 M $\mathrm{HClO}_{4}+0-0.05 \mathrm{M} \mathrm{ZrOCl}_{2}+1-10 \mathrm{mM} \mathrm{NaNO} 3$ ). Thus, evidence for complexes in the bulk solution in this system were not found from the Raman and UV spectra.

4-3-4. Electrode interface.

Complexes are not produced in the bulk solution. Only 0.2 mM zirconium(IV) is necessary for the reduction of the nitrate ion[Fig. 4-2(B)]. Considering these results, the zirconium(IV) may adsorb on the DME and largely change the double-layer capacity.

Accordingly, in order to obtain the differential capacity and the potential of the outer Helmholtz plane
$\left(\phi_{2}\right)$, the impedance in this system was measured by means of phase-selective ac polarography. The solution components were made up such that the ionic strength was constant and the activity coefficient was also constant. The differential capacity was analyzed by the same method as used by Grahame and Soderberg[19] ( $\Gamma_{\text {salt }}^{\mathrm{ecm}}=0.0 \mu \mathrm{Ccm}{ }^{-2}$, $\left(\partial E^{-} / \partial \mu\right)^{e c m}=0.305, C_{+}^{e c m}=9.59 \mu \mathrm{Fcm}^{-2}$ in the solution, $0.01 \mathrm{M} \mathrm{HCl}+0.09 \mathrm{M} \mathrm{NaCl}+0.2 \mathrm{mM} \mathrm{ZrOC1} 2$ (cf. Appendix IV).

The differential capacity in the presence of zirconium(IV) changed a little from that in its absence (Fig. 4-4). No large capacity difference which might be assigned to strong adsorption was found. The differential capacity in the presence of the nitrate ion at more positve potentials than that at which the nitrate ion is reduced, was the same as that in its absence. The plot of $\phi_{2}$ vs. the dc potential is given in Fig. 4-5. Though there is a difference in the distance of closest approach of the ions[20], $\phi_{2}$ was calculated as a solution of $0.1 \mathrm{M} \mathrm{MeCl}(\mathrm{Me}$ indicates univalent cation), because an aqueous solution of $\mathrm{ZrOCl}_{2} \cdot 8 \mathrm{H}_{2} \mathrm{O}$ exhibits the same pH as a hydrochloric acid solution of the same concentration, $\mathrm{ZrOCl}_{2}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons$ $\mathrm{ZrO}(\mathrm{OH})^{+}+\mathrm{H}^{+}+2 \mathrm{Cl}^{-}$.

From the data of Grahame and Soderberg[19], it follows that the product of the cation charge and its concentration being equal, the higher the valence of the cation is(in


Fig. 4-4. The differential capacity of a solution containing various concentrations of zirconium(IV). $\mathrm{C}_{\mathrm{Zr}(\mathrm{IV})} ;(\longrightarrow) 0,(-\longrightarrow) 2 \times 10^{-5},(---) 4 \times 10^{-5},(---)$ $2 \times 10^{-4} \mathrm{M}$ in $0.01 \mathrm{M} \mathrm{HCl}+0.09 \mathrm{M} \mathrm{NaCl}$.


Fig. 4-5. Plots of $\phi_{2}$ vs. dc potential. (l, $\quad 0.2 \mathrm{mM}$ $2 \mathrm{rOCl}_{2}+0.01 \mathrm{MHCl}+0.09 \mathrm{M} \mathrm{NaCl},(2,----) 0.1 \mathrm{M} \mathrm{KCl}$, $(3,--) 0.1 \mathrm{M} \mathrm{KF},(4,-\cdots) 0.05 \mathrm{M} \mathrm{BaCl}_{2} ;(2,3,4)$ data from ref. 19.
this system, if the species of zirconium(IV) is a polymer [21-25]), the nearer $\phi_{2}$ comes to zero, even if the dc potential is far more negative than the ecm.

Therefore, the difference in $\phi_{2}$ at the reduction potential of nitrate in the absence and presence of zirconium(IV) seems to be at most 0.10-0.15 V. On the other hand, the difference of the reduction potential of nitrate in the absence and presence of zirconium(IV) is 1.2-1.5 V. Consequently, the contribution of $\phi_{2}$ to the difference of the reduction potential of nitrate is at most about $10 \%$. In other words, the mass transfer of nitrate ion to the vicinity of the electrode against the negative charge of electrode is not a very important process for the reduction of nitrate ion. Therefore, a large part of the difference of the reduction potential seems to be in the charge transfer step and/or in a chemical reaction step in the vicinity of the DME.

Figure $4-6$ shows the relationship between the reduction potential of nitrate ion in the presence of various metal ions and the reduction potential of the metal ion to the metal[26]. It is thought that the metal ion acts as a intermediary for the charge transfer of the electrode to the nitrate ion. Therefore, the more the metal ion is reducible(that is to say, an electrochemical depolarizability), the more the nitrate ion in the presence


Fig. 4-6. Relationship between the reduction potential
( $\mathrm{ENO}_{3}^{-}$) of nitrate ion in the presence of metal ion and that ( $E_{M}$ ) of metal ion to metal.
(1) Li(I),
(2) $\mathrm{Sr}(\mathrm{II})$,
$\mathrm{Ca}(\mathrm{II}),(4) \mathrm{Mg}(\mathrm{II}),(5) \mathrm{Ce}(\mathrm{III}),(6) \mathrm{Th}(\mathrm{IV}),(7) \mathrm{Hf}(\mathrm{IV})$,
(8) $\operatorname{Zr}(I V) . E_{\mathrm{NO}_{3}^{-}}$data from: (1)-(5) ref. 2; (6) 0.1 M
$\mathrm{NaNO}_{3}+0.1 \mathrm{mM} \mathrm{Th}\left(\mathrm{ClO}_{4}\right)_{4}+0.005 \%$ gelatin; (7) 0.1 M NaNO 3
$+0.01 \mathrm{M} \mathrm{HClO}_{4}+0.2 \mathrm{mM} \mathrm{HfOCl}{ }_{2}$; (8) $0.05 \mathrm{M} \mathrm{NaNO}_{3}+0.05 \mathrm{M}$ $\mathrm{NaClO}_{4}+0.2 \mathrm{mM} \mathrm{ZrOCl} 2_{2} . \mathrm{E}_{\mathrm{M}}$ data from ref. 26; (4')
$\mathrm{Mg}(\mathrm{II}) / \mathrm{Mg}(\mathrm{I})$.
of the metal ion is also reducible.

Appendix IV. The method for obtaining the differential capacity from phase-selective ac polarographic data; and the relationship between the differential capacity and the potential of the outer Helmholtz plane[19].

The equivalent circuit for the impedance of solution-electrode interface is shown in Fig. IV-1. The impedance $Z$ and admittance $Y(=1 / Z)$ of the circuit are as follows:

$$
\begin{equation*}
Z=R+L /(j \omega C) \tag{IV-1}
\end{equation*}
$$

and

$$
\begin{equation*}
Y=R /\left[R^{2}+(1 / \omega C)^{2}\right]-(1 / \omega C) j /\left[R^{2}+(1 / \omega C)^{2}\right] \tag{IV-2}
\end{equation*}
$$

, respectively. The $0^{\circ}$ and $90^{\circ}$ components of the ac polarographic current at a given dc potential from Ohm's law $(I=Y E)$ relate to $R$ and $C$ by the equations:

$$
\begin{align*}
& I\left(0^{\circ}\right)=\left\{R /\left[R^{2}+(1 / \omega C)^{2}\right]\right\} \Delta E_{a c},  \tag{IV-3}\\
& I\left(90^{\circ}\right)=\left\{(1 / \omega C) /\left[R^{2}+(1 / \omega C)^{2}\right]\right\} \Delta E_{a c}, \tag{IV-4}
\end{align*}
$$

and solving for $R$ and $C$, the relationships can be obtained:

$$
\begin{align*}
& \mathrm{R}=\left\{I\left(0^{\circ}\right) /\left[I\left(0^{\circ}\right)^{2}+I\left(90^{\circ}\right)\right]\right\} \Delta \mathrm{E}_{\mathrm{ac}}  \tag{IV-5}\\
& \mathrm{C}=\left\{\left[I\left(0^{\circ}\right)^{2}+I\left(90^{\circ}\right)^{2}\right] / I\left(90^{\circ}\right)\right\}\left(I / \omega \Delta \mathrm{E}_{\mathrm{ac}}\right) \tag{IV-6}
\end{align*}
$$


W.E. $\mid$ interface $\mid$ solution $\mid$ C.E.

Fig. IV-1. The equivalent circuit of the electrochemical cell, in the case of the supporting electrolyte solution in the absence of a depolarizer.

From the drop time(t/s) at a given dc potential and the mercury flow rate $\left(m / m g s^{-1}\right.$ ), the electrode area(A/cm ${ }^{2}$ ) at the dc potential is given by the equation:

$$
\begin{equation*}
A=(4 \pi)^{1 / 3}(3 m t / \rho)^{2 / 3} \times 10^{-2} \tag{IV-7}
\end{equation*}
$$

where $\rho / \mathrm{gcm}^{-3}$ is the density of mercury. And the differential capacity $\left(\mathrm{C} / \mathrm{Fcm}^{-2}\right.$ ) is obtained from Eqs. (IV-6) and (IV-7).

The surface charge density (q/Ccm ${ }^{-2}$ ) of the DME can be expressed by the equation:

$$
\begin{equation*}
q=\int_{E}^{E} \operatorname{ecm} \operatorname{cdE} \tag{IV-8}
\end{equation*}
$$

where $E^{e c m}$ is the electrocapillary maximum, and the $E^{e c m}$ is obtained by the measurement of the drop time.

On the other hand, $q$ and $C$ are expressed by the equations:

$$
\begin{align*}
& q=q_{+}+q_{-},  \tag{IV-9}\\
& q_{+}=-z_{+} F \Gamma_{+}, q_{-}=-z_{-} F \Gamma_{-},  \tag{IV-10}\\
& C=C_{+}+C_{-}, \tag{IV-11}
\end{align*}
$$

where $z_{+}\left(z_{-}\right)$is the valence of cation(anion), $\Gamma_{+}\left(\Gamma_{-}\right)$is the excess of cation(anion), $q_{+}\left(q_{\sim}\right)$ is the part of the total surface charge density attributable to the approach or departure of cation(anion) and $C_{+}\left(C_{-}\right)$is the part of the total capacity attributable to the approach or departure of cation(anion).

C and $C_{+}$have also relation to $\mu($ the chemical
potential) and $E(t h e d c$ potential) by the following equation:

$$
\begin{equation*}
z_{+} \nu_{+} F(\partial C / \partial \mu)_{E}=-d C_{+} / d E \tag{IV-12}
\end{equation*}
$$

where $\nu_{+}$is the number of cations formed by the dissociation of one molecule of the electrolyte. Therefore, $C_{+}$is obtained by the integration of $E q$. (IV-12) as follows:

$$
\begin{align*}
& C_{+}=\int\left(d C_{+} / d E\right) d E+k \\
& =z_{+} \nu_{+} F \int_{E}^{E} \operatorname{ecm}(\partial C / \partial \mu) d E+C_{+}^{e c m}, \tag{IV-13}
\end{align*}
$$

where

$$
\begin{equation*}
c_{+}^{e c m}=z_{+} \nu_{+} F C^{e c m}(\partial E / \partial \mu)^{\mathrm{ecm}} \tag{IV-14}
\end{equation*}
$$

And $\Gamma_{+}$has relation to $C_{+}$by the following equation:

$$
\begin{equation*}
z_{+} F\left(\partial \Gamma_{+} / \partial E\right)_{\mu}=-C_{+} \tag{IV-15}
\end{equation*}
$$

Using the relation $z_{+} F \Gamma_{+}=-q_{+}$, one can obtain on
integration of Eq. (IV-15):

$$
\begin{equation*}
-q_{+}=z_{+} F \Gamma_{+}=\int_{E}^{E} \mathrm{ecm} C_{+} d E+z_{+} F \Gamma_{+}^{e c m} \tag{IV-16}
\end{equation*}
$$

where the $\Gamma_{+}^{e c m}$ is obtained from the following relationship:

$$
\begin{equation*}
(\partial \sigma / \partial \mu)^{\mathrm{ecm}}=-\Gamma_{\operatorname{salt}}^{\mathrm{ecm}}=-\Gamma_{+}^{\mathrm{ecm}} / \nu_{+}, \tag{IV-17}
\end{equation*}
$$

where $\sigma$ is the interfacial tention of the mercury contact with the inert electrolyte.

For a $1: l$ electrolyte, the potential of the outer Helmholtz plane $\phi_{2}$ has relation to $\Gamma_{+}$by the equation:

$$
\begin{equation*}
\phi_{2}=-\left(2 R T / z_{+}\right) \ln \left[\left(z_{+} F \Gamma_{+} / K\right)+1\right] \tag{IV-18}
\end{equation*}
$$

where

$$
\begin{equation*}
K=(R T \varepsilon c / 2 \pi)^{1 / 2}, \tag{IV-19}
\end{equation*}
$$

where $\varepsilon$ means the absolute value of the electronic charge, and $c$ is the concentration of cation.

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Chapter 5. Studies on the behavior of the second derivative polarograms of $\mathrm{Pb}(I I), ~ T 1(I), ~ C d(I I), ~ I n(I I)$, $\mathrm{Zn}(I I), \mathrm{Ni}(I I), \mathrm{Co}(I I)$ and $\mathrm{Mn}(I I)$ in aqueous solutions

5-1. Introduction

As mentioned in Chapter 1 , the ac polarographic studies focusing to the analytical aspect, has been reported by several authors[1-10]. The total alternating current polarograms for the reductions of various metal ions have been studied[6]. However, the phase selective mode, especially $2 f$ mode, has never been applied to these systems.

In this chapter, the electrode reactions of several inorganic ions were studied by the ac polarographic analysis based on the phase selective second harmonic ac polarograms. The metal ions employed for the study were lead(II), thallium(I), cadmium(II), indium(III), zinc(II), cobalt(II) and manganese(II) in aqueous solution. The analytical results obtained from $2 f$ mode were compared with those from lf mode in several aspects, in order to evaluate reliabilities for the application for the chemical analysis.

The benefits of the potential usage of $2 f$ mode is explored and discussed in the next chapter.

5-2. Experimental

Except for a indium(II) solution, stock solutions used for the polarographic measurements of copper(II), lead(II), cadmium(II), thallium(I), zinc(II), nickel(II), cobalt(II) and manganese(II) were prepared from the salts: $\mathrm{CuCl}_{2} \cdot 2 \mathrm{H}_{2} \mathrm{O}, \mathrm{PbCl}_{2}, \mathrm{CdCl}_{2} \cdot 21 / 2 \mathrm{H}_{2} \mathrm{O}, \mathrm{TlCl}, \mathrm{ZnCl}_{2}, \mathrm{NiCl} 2 \cdot 6 \mathrm{H}_{2} \mathrm{O}$, $\mathrm{CoCl}_{2} \cdot 6 \mathrm{H}_{2} \mathrm{O}$ and $\mathrm{MnCl}_{2} \cdot 4 \mathrm{H}_{2} \mathrm{O}$, respectively. The indium(III) stock solution was prepared by dissolving a relevant amount of pure metal indium (99.99\%) in its equivalent hydrochloric acid solution.

5-3. Results and discussion

The phase selective fundamental and second harmonic ac polarograms of various metal ions in 1 m potassium chloride solution are given in Figs. 5-1 and 5-2, respectively.

The contributions of the charging current (base current) to the faradaic current in $1 f\left(0^{\circ}\right)$ and $2 f\left(90^{\circ}\right)$ components are less than those in $1 f\left(90^{\circ}\right)$ and $2 f\left(0^{\circ}\right)$


Fig. 5-1. Fundamental harmonic ac polarograms of various metal ions in 1 M potassium chloride. $\Delta \mathrm{E}_{\mathrm{ac}}=10$ $m V(r . m . s),. f=50 \mathrm{~Hz}$. Upper: $1 f\left(0^{\circ}\right)$, lower: $1 f\left(90^{\circ}\right)$.


Fig. 5-2. Second-harmonic ac polarograms of various metal ions in 1 M potassium chloride. Upper: $2 f\left(0^{\circ}\right)$, lower: $2 f\left(90^{\circ}\right)$ 。
components, respectively. As compared with lf and 2 f modes, this contribution in $2 f\left(0^{\circ}\right)$ is far less than that in 1f(90 $)$. Therefore, considering the detection limit, when the $2 f$ mode is monitored, any phase angle component can be used for the trace analysis, while in lf mode only $0^{\circ}$ component can be utilized for the purpose. Such a flexibility on the selection of phase angle components monitored in $2 f$ mode will provide a useful method when it is applied to the system in which a kinetics of redox process plays an important roll.

```
5-3-1. Systems of lead(II), cadmium(II) and thallium(I):
    reversible redox process
```

The systems of $\mathrm{Pb}(\mathrm{II}) / \mathrm{Pb}(\mathrm{Hg}), \mathrm{Cd}(\mathrm{II}) / \mathrm{Cd}(\mathrm{Hg})$ and $\mathrm{Tl}(\mathrm{I}) / \mathrm{Tl}(\mathrm{Hg})$ are well-known as the reversible process[11]. The polarogram of 1 f mode has a shape of the first derivative wave of the dc polarogram, and that in $2 f$ mode has a shape of the second derivative wave of the dc polarogram. When the system is reversible, the following relations are reasonably assumed: $(2 \omega)^{1 / 2} \ll 1$ (in $1 f$ mode) and $2 \omega^{1 / 2} \ll 1$ (in $2 f$ mode). And $F_{1}(t) G_{1}(\omega)$ in Eq. $(2-4)$ become unity $\left(F_{1}(t) G_{1}(\omega)=1\right)$ and $F_{2}(t) G_{2}(\omega)$ in Eq. $(2-5)$ is expressed identically by sinh $(j / 2)\left(F_{2}(t) G_{2}(\omega)=\right.$
sinh (j/2)). The polarographic wave forms of 1 f and 2 f mode, therefore, can be expressed mathematically by [cosh ${ }^{2}$ $(j / 2)]^{-1}$ in Eq. (2-6) for lf mode and by [sinh (j/2)][cosh ${ }^{3}$ $(j / 2)]^{-1}$ in Eq. (2-7) for $2 f$ mode, respectively. The half width of peak current in $1 f$ mode is $90 / n m V$, and the difference between the potentials of two peaks in $2 f$ mode is $68 / \mathrm{n} \mathrm{mV}$.

Eqs. (2-4)-(2-7) show that the current amplitude in $1 f$ mode is proportional to $\Delta E$ ac, while that in $2 f$ mode is proportional to $\Delta \mathrm{E}_{\mathrm{ac}}^{2}$. And these relationships for $\mathrm{Cd}(\mathrm{II})$ system are given in Figs. 5-3 and 5-4. When $\Delta E$ ac become very large, these relationships are not approved any more. The results are understood from the fact that in Eq. (2-3) the second or its higher term in each bracket gives significant value when $\Delta E$ ac is large.

The current amplitudes in both lf and $2 f$ modes are also proportional to $\omega^{1 / 2}$ (or $f^{1 / 2}$ ) (Fig. 2-3) and to $C_{0}^{*}$.

In the reversible process, the phase angle $\phi_{1}$ of 1 f mode in Eq. $(2-9)$ is $\pi / 4$, and $\phi_{2}$ of 2 f mode in Eq. (2-10) is $-\pi / 4$, which are independent of dc potentials. In both lf and 2 f modes, therefore, the current amplitude of $0^{\circ}$ component is equal to that of $90^{\circ}$ component(Fig. 2-3).

5-3-2. Systems of indium(II) and zinc(II): quasi-


Fig. 5-3. Plots of $i_{p}$ vs. $\Delta E_{a c}$ in $1 f\left(0^{\circ}\right)$. 0.2 mM cadmium chloride in 1 M potassium nitrate(pH2). $\mathrm{f}=50 \mathrm{~Hz}$.


Fig. 5-4. Plots of $i_{p}$ vs $\Delta E_{a c}^{2}$ in $2 f\left(90^{\circ}\right)$. Other conditions are the same as Fig. 5-3.

## reversible process

The systems of $\operatorname{In}(I I I) / I n(H g)$ and $Z n(I I) / Z n(H g)$ in Figs. 5-1 and 5-2 are known as the quasi-reversible case[11]. A lf polarogram does not correspond to a real first derivative form of the dc polarogram, and a $2 f$ polarogram is also not a real second derivative form of the dc polarogram. In the quasi-reversible system, e.g., $\mathrm{Zn}(I I)$ and $\operatorname{In}(I I I)$, the contributions to the current amplitudes of $\mathrm{F}_{1}(\mathrm{t}) \mathrm{G}_{1}(\omega)$ in Eq. $(2-4)$ and $\mathrm{F}_{2}(\mathrm{t}) \mathrm{G}_{2}(\omega)$ in Eq. (2-5) are both less than unity $\left(F_{1}(t) G_{1}(\omega)<1\right)$ and sinh $(j / 2)\left(F_{2}(t) G_{2}(\omega)<\sin (j / 2)\right)$, respectively. Therefore, the current amplitudes in $1 f$ and $2 f$ modes are not proportional to $\omega^{1 / 2}$. On the other hand, in case of the systems having the same n(number of electrons transfered) (e.g., $\mathrm{Zn}(I I), \mathrm{Cd}(I I)$ and $\mathrm{Pb}(I I))$, the current amplitudes between these systems can be compared. And in the quasi-reversible $Z n(I I)$ system, the currents are both more diminished in their amplitudes than those in the reversible systems(Cd(II) and $P b(I I))$.

In the $\operatorname{In}(I I I)$ system, however, the current amplitude in $1 f\left(0^{\circ}\right), 2 f\left(0^{\circ}\right)$ and $2 f\left(90^{\circ}\right)$ components are larger than those of the reversible system(Cd(II) and Pb(II)). This is because the currents in lf and $2 f$ mode are both proportional to $n^{2}$ and $n^{3}$, respectively: in case of

In(III), $n=3$, and in case of $C d(I I)$ and $P b(I I), n=2$.
The half-width in $1 f$ mode is more than $90 / \mathrm{n} \mathrm{mV}$ in the reversible process. The difference between the potentials of two peaks in $2 f$ mode is also more than $68 / n \mathrm{mV}$ in the reversible process.

The phase angle $\phi_{1}$ in lf mode is not $\pi / 4$ (for reversible process) and changes from 0 to $\pi / 4$ with $E_{d c}, k_{s}$ (heterogeneous charge transfer rate constant ), $\alpha$ (transfer coefficient) and f(frequency)[Eq. (2-8)]. The $\phi_{2}$ in $2 f$ mode is not $-\pi / 4$ (for reversible processes) but changes of 0 to $2 \pi$ with $E_{d c}, k_{s}$, and $f[E q .(2-9)]$.

Figure 5-5 shows plots of cot $\phi_{1}$ vs. the dc potential in the system $\mathrm{Zn}(\mathrm{II}) / \mathrm{Zn}(\mathrm{Hg})$. From this plots, the kinetic parameters, $k_{s}$ and a can be obtained by the following equations[5]:

$$
\begin{align*}
& {\left[E_{d c}\right]_{\max }=E_{1 / 2}^{r}+(R T / n F) 1 n(\alpha / \beta)}  \tag{5-1}\\
& {\left[\cot \phi_{1}\right]_{\max }=1+(2 \omega D)^{1 / 2 / k_{s}}\left[(\alpha / \beta)^{-\alpha}+(\alpha / \beta)^{\beta}\right]} \tag{5-2}
\end{align*}
$$

where $\left[E_{d c}\right]_{\text {max }}$ is a maximum at the dc potential of the plot, and $\left[\cot \phi_{1}\right]$ max $i s$ a magnitude of it. The $k_{s}$ in this work was $3.0 \times 10^{-3} \mathrm{cms}^{-1}$ and the was $0.29\left(\mathrm{cf} . \mathrm{k}_{\mathrm{s}}=5.05 \times\right.$ $10^{-3} \mathrm{cms}^{-1}$ and $\left.\alpha=0.30[11]\right)$.

Figure 5-6 shows a unusual shoulder at a less negative dc potential than a main peak in the $\mathrm{Zn}(\mathrm{II}) / \mathrm{Zn}(\mathrm{Hg}) \quad[12,13]$. The unusual shoulder appears kinetically when the $k$ is of the order of $10^{-3} \mathrm{cms}^{-1}$, while $\alpha$ is smaller than 0.5 .


Fig. 5-5. The dependence of $\cot \phi_{1}$ on $E_{d c}$ in the $\mathrm{Zn}(\mathrm{II}) / \mathrm{Zn}(\mathrm{Hg})$ system. $\Delta \mathrm{E}_{\mathrm{ac}}=5 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}),. \mathrm{f}=10 \mathrm{~Hz}, \mathrm{D}_{\mathrm{O}}=$ $0.7 \times 10^{-5} \mathrm{~cm}^{2} \mathrm{~s}^{-1}, \mathrm{D}_{\mathrm{R}}=2.0 \times 10^{-5} \mathrm{~cm}^{2} \mathrm{~s}-1, E_{1 / 2}^{\mathrm{r}}=-0.9685 \mathrm{~V}$, $\left[E_{d c}\right]_{\max }-E_{1 / 2}^{r}=-11.5 \mathrm{mV}$.


Fig. 5-6. Dc, fundamental(lf) and second-harmonic (2f) ac polarograms. 2 mM zinc chloride in 1 M potassium chloride(pH3). $\Delta E_{a c}=5 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}),. \mathrm{f}=100 \mathrm{~Hz}$.

Though the electrochemical charge transfer is one step, the two peaks are observed in lf mode[14] and two maximum peaks and two minimum peaks are in $2 f$ mode[18]. The unusual phenomenon in 2 f mode has been known theoretically [15], but an experimental observation in $2 f$ mode has not been reported.

This phenomenon is not observed in the dc polarography. And in lf mode, this phenomenon is ill-defined, while that in 2 f mode is well-defined(Fig. 5-6). In 2f mode, dependences of the currents for this phenomenon on the applied frequency are given in Fig. 5-7. The curve for the first anodic current shows a peak at a frequency $(f \simeq 15 \mathrm{~Hz})$ and that for the second anodic current increases monotonously. Both first and second cathodic currents decrease monotonously. When in a certain system two-step currents are observed, the data of the dependence of $2 f$ current on the applied frequency will give an indispensable proof for the decision of whether the twostep phenomenon is corresponding to a real two-step process or not.

5-3-3. Systems of nickel(II), cobalt(II) and manganese(II): irreversible process


Fig. 5-7. Plots of $i_{p}$ vs. $\omega^{1 / 2}$ of the $\mathrm{Zn}(\mathrm{II}) / \mathrm{Zn}(\mathrm{Hg})$ system. (o) first anodic peak, (x) first cathodic peak, (o) second anodic peak, (+) second cathodic peak. Other conditions are the same as Fig. 5-6.

In an irreversible system, a dc polarogram does not show a well-defined wave such as the reversible or quasi-reversible process, and the corresponding ac polarogram shows a broad peak[16]. In Figs. 5-1 and 5-2, the systems of $\mathrm{Ni}(\mathrm{II}) / \mathrm{Ni}(\mathrm{Hg})$, $\mathrm{Co}(\mathrm{II}) / \mathrm{Co}(\mathrm{Hg})$ and Mn(II)/Mn(Hg) are known as this category[11].

Theoretical expressions for the irreversible ac polarographic wave induced by very heterogeneous charge transfer[16] can be written as(cf. Appendix $V$ ):
$I(\omega t)=\left[1.644 \alpha n^{2} F^{2} A C_{o}^{*}\left(\omega D_{0}\right)^{1 / 2} \Delta E_{a c} / R_{T e}{ }^{-J}\left(1+e^{1.091 J}\right) \times\right.$ $\left.\left[1+\left(1+Q_{1} e^{J}\right)^{2}\right]^{1 / 2}\right] \sin \left[\omega t+\cot ^{-1}\left(1+Q_{1} e^{J}\right)\right]$,
where

$$
\begin{align*}
J= & (\alpha \mathrm{nF} / \mathrm{RT})\left[\mathrm{E}_{\mathrm{dc}}-\mathrm{E}_{1 / 2}^{\mathrm{r}}\right. \\
& -\left(\mathrm{RT} / \alpha_{\mathrm{nF}}\right) \ln _{\mathrm{n}}\left(1.349 \mathrm{k}_{\mathrm{s}} \mathrm{t}^{\left.\left.1 / 2 / \mathrm{D}^{1 / 2}\right)\right],}\right.  \tag{5-4}\\
\mathrm{Q}_{1}= & 1.907(\omega \mathrm{t})^{1 / 2}, \tag{5-5}
\end{align*}
$$

and

$$
\begin{align*}
& I(2 \omega t)=\left[2.325 \alpha \mathrm{n}^{3} \mathrm{~F}^{3} A C_{0}^{*}\left(\omega \mathrm{D}_{0}\right)^{1 / 2} \Delta \mathrm{E}_{\mathrm{ac}}^{2}\left(\mathrm{X}^{2}+\mathrm{Y}^{2}\right)^{1 / 2} /\right. \\
& \left.4 \mathrm{R}^{2} \mathrm{~T}^{2} \mathrm{e}^{-\mathrm{J}}\left(1+\mathrm{e}^{1.091 \mathrm{~J}}\right)\left[1+\left(1+Q_{2} \mathrm{e}^{\mathrm{J}}\right)^{2}\right]^{1 / 2}\right] \times \\
& \sin \left(2 \omega t+\phi_{2}\right) \tag{5-6}
\end{align*}
$$

where

$$
\begin{aligned}
& Q_{2}=\sqrt{2} Q_{1}=2.697(\omega t)^{1 / 2}, \\
& X=\alpha\left[1-2\left[\left(2+Q_{1} e^{J}\right) /\left[1+\left(1+Q_{1} e^{J}\right)^{2}\right]\right],\right. \\
& Y=2 \alpha\left[Q_{1} e^{J} /\left[1+\left(1+Q_{1} e^{J}\right)^{2}\right]\right], \\
& \phi_{2}=\cot ^{-1}\left[\left[Y\left(Q_{2} e^{J}+1\right)+X\right] /\left[Y-X\left(Q_{2} e^{J}+1\right)\right]\right] .(5-10)
\end{aligned}
$$

The current amplitudes of the irreversible processes (Eqs. (5-3) and (5-6)) are far small and are about $1 / 10$ or $1 / 100$ of that of the reversible process(Eq. (2-6) and (2-7)). As a irreversible process has a very small $k_{s}\left(<10^{-5} \mathrm{cms}^{-1}\right)$, the alternating currents for 1 f and 2 f mode become significant when $E_{d c}$ is far negative from $E_{1 / 2}^{r}(E q .(5-4))$. In the reversible process, $\alpha$ is far from 0.5 for the reversible process. In $2 f$ mode, the polarographic shape is asymmetry for $\alpha$ and $k_{s}$.

Therefore, one can understand the reversibility for a certain system from the data of $i_{p}$ vs. $\omega^{1 / 2}$, of the half width in lf mode or of the difference between the potentials of the two peaks in 2 f mode, and of the phase angle.

The system $\mathrm{Cu}(\mathrm{II}) / \mathrm{Cu}(\mathrm{Hg})$ in chloride solution are known as a two-step reaction $(\mathrm{Cu}(\mathrm{II}) \rightarrow \mathrm{Cu}(\mathrm{I}) \rightarrow \mathrm{Cu}(\mathrm{Hg}))$ [3]. Since the reduction potential for the reaction: $\mathrm{Cu}(\mathrm{II}) \rightarrow \mathrm{Cu}(I)$, is more positive than that for the reaction: $\mathrm{Hg} \rightarrow \mathrm{Hg}_{2} \mathrm{Cl}_{2}$, the alternating current for $\mathrm{Cu}(\mathrm{II}) / \mathrm{Cu}(\mathrm{I})$ is not observed and only the current for $\mathrm{Cu}(\mathrm{I}) / \mathrm{Cu}(0)$ appears. The reaction is also accompanied with the adsorption of $\mathrm{Cu}(\mathrm{I})$ on the electrode. This can be proved from the fact that the phase angle in lf mode is more than $\pi / 4$.

Appendix V. The irreversible ac polarographic wave induced by very slow heterogeneous charge transfer[16]

Under irreversible conditions, Eq. (II-8) in Appendix II is reduced to an analytical expression:

$$
\psi_{0}(t)=(7 / 3 \pi)^{1 / 2} /\left\{\lambda t^{1 / 2}\left[1+\left(1.349 \lambda t^{1 / 2}\right)^{-1.091}\right]\right\}
$$

Substituting Eq. (V-1) in Eq. (II-2) and invoking the relationship:

$$
\begin{align*}
& e^{-\alpha j} \gg 1>e^{\beta j} \gg e>e^{j},  \tag{V-2}\\
& \alpha e^{j} \psi_{0}(t) \gg 1 \tag{v-3}
\end{align*}
$$

and employing the relation:

$$
\begin{equation*}
1.349 \mathrm{k}_{\mathrm{s}} \mathrm{e}^{-\alpha j_{t} 1 / 2_{D}-1 / 2}=\mathrm{e}^{-\mathrm{J}}, \tag{v-4}
\end{equation*}
$$

where

$$
\begin{align*}
J= & (\alpha n F / R T)\left[E_{d c}-E_{1 / 2}^{r}\right. \\
& -(R T / \alpha n F) \ln \left(1.349 k_{s} t^{\left.\left.1 / 2 / D^{1 / 2}\right)\right],}\right. \tag{V-5}
\end{align*}
$$

and algebraic rearrangement yields the results:

$$
\begin{align*}
& I(\omega t)=\left[1.644 \alpha n^{2} F^{2} A C_{0}^{*}\left(\omega D_{0}\right)^{1 / 2} \Delta E_{a c} / R T e^{-J}\left(1+e^{1.091 J}\right) \times\right. \\
& \left.\left[1+\left(1+Q_{1} e^{J}\right)^{2}\right]^{1 / 2}\right] \sin \left[\omega t+\cot ^{-1}\left(1+Q_{1} e^{J}\right)\right], \tag{V-6}
\end{align*}
$$

where

$$
\begin{equation*}
Q_{1}=1.907(\omega t)^{1 / 2} \tag{V-7}
\end{equation*}
$$

and

$$
\begin{align*}
& I(2 \omega t)=\left[2.325 \alpha n^{3} F^{3} A C_{0}^{*}\left(\omega D_{0}\right)^{1 / 2} \Delta E_{a c}^{2}\left(X^{2}+Y^{2}\right)^{1 / 2} /\right. \\
& \left.4 \mathrm{R}^{2} \mathrm{~T}^{2} \mathrm{e}^{-J}\left(1+\mathrm{e}^{1.091 \mathrm{~J}}\right)\left[1+\left(1+Q_{2} \mathrm{e}^{\mathrm{J}}\right)^{2}\right]^{1 / 2}\right] \times \\
& \sin \left(2 \omega t+\phi_{2}\right), \tag{V-8}
\end{align*}
$$

where

$$
\begin{align*}
& Q_{2}=\sqrt{2} Q_{1}=2.697(\omega t)^{1 / 2}  \tag{V-9}\\
& X=\alpha\left[1-2\left[\left(2+Q_{1} e^{J}\right) /\left[1+\left(1+Q_{1} e^{J}\right)^{2}\right]\right]\right.  \tag{V-10}\\
& Y=2 \alpha\left[Q_{1} e^{J} /\left[1+\left(1+Q_{1} e^{J}\right)^{2}\right]\right] \\
& \phi_{2}=\cot ^{-1}\left[\left[Y\left(Q_{2} e^{J}+1\right)+X\right] /\left[Y-X\left(Q_{2} e^{J}+1\right)\right]\right] \tag{V-12}
\end{align*}
$$

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Chapter 6. Analytical application of second harmonic ac polarography

## 6-1. Introduction

As shown in Chap. 2, the nonlinear second expanded term of total alternating current is corresponding to the second harmonic alternating current. And the polarogram of $2 f$ mode has the second derivative shape of dc polarogram. Compared with the current in $l f$ mode, that in $2 f$ mode has little contribution from the charging current over the whole phase angle components(cf. Figs. 5-1 and 5-2). In the chemical analysis, therefore, $2 f$ mode is more advantageous than $1 f$ mode.

In this chapter, phase-selective second harmonic ac polarography is used to determine one of two species having similar reduction potentials without any preliminary separation, selective complexation[l,2], masking of certain electrode process[3], or using computer techniques[4-10].

The present technique is different from the methods used by Devay et al.[ll] and Fajioli et al.[12] in that it utilizes the difference in the dependence of phase angle on
dc potential for two species, which results from the kinetics of two-electrode processes. As examples, two systems are chosen, i.e., in 1 M potassium chloride solution, indium(III) in the presence of excess of cadmium(II), and zinc(II) in the presence of excess of nickel(II).

6-2. Experimental

The working electrode was a dropping mercury electrode; the capillary had a drop time of 3.96 s at -1.0 $V$ vs. Ag/AgCl in 1 M potassium chloride solution and the mercury flow was $2.025 \mathrm{mgs}^{-1}$ at a height of 70 cm .

The phase angle of faradaic alternating current was determined by recording in-phase( $0^{\circ}$ ) and quadrature (90 $)$ components. The phase angle of the lock-in amplifier was calibrated by use of a dummy cell[13](cf. Chap. 2). Applied ac voltage ( $\Delta E$ ac $)$ and frequency $(f)$ were $5-10$ mV(r.m.s.) and $15-200 \mathrm{~Hz}$, respectively.

6-3. Results and discussion

For the simple electrode reaction, $0 x+n e^{-} \rightleftharpoons$ Red,
where rate is controlled by diffusion and/or heterogeneous charge-transfer kinetics, the fundamental and second harmonic alternating currents are given by the well-known equations[14,15](see also Chap. 2):
$I(\omega t)=I(\omega t)_{r e v} F_{1}(t) G_{1}(\omega) \sin \left(\omega t+\phi_{1}\right)$,
$I(2 \omega t)=I(2 \omega t)_{\operatorname{rev}^{F}}{ }_{2}(t) G_{2}(\omega) \sin \left(2 \omega t+\phi_{2}\right)$,
where
$I(\omega t)_{r e v}=(n F)^{2} A C_{0}^{*}\left(\omega D_{0}\right)^{1 / 2} \Delta E_{a c}\left[4 R T \cosh { }^{2}(j / 2)\right]^{-1},(6-3)$
$I(2 \omega t)_{r e v}=(n F)^{3} A C_{0}^{*}\left(2 \omega D_{0}\right)^{1 / 2} \Delta E_{a c}^{2}\left[16 R^{2} T^{2} \cosh ^{3}(j / 2)\right]^{-1}$,
$\phi_{1}=\cot ^{-1}\left[1+(2 \omega)^{1 / 2} / \lambda\right]$,
$\phi_{2}=\cot ^{-1}\left[\left[L\left(2 \omega^{1 / 2} / \lambda+1\right)+P\right] /\left[L-P\left(2 \omega^{1 / 2} / \lambda+1\right)\right]\right]$,
where
$j=n F(R T)^{-1}\left(E_{d c}-E_{1 / 2}^{r}\right) ; \lambda=\left(k_{s} f_{a} / R T\right)\left(e^{-\alpha j}+e^{\beta j}\right) \cdot(6-7)$
In Eqs. (6-3) and (6-4), if n(number of electrons transfered in heterogeneous charge transfer step) for a system is 2 and that for another system is 3 (and the both systems are reversible processes), the current amplitudes in $1 f$ mode are in the ratio $4: 9$ and those in $2 f$ mode are in the ratio 8:27, respectively. Therefore, when a chemical species( $n=3$ ) is analyzed in the presence of excess of another species(n $=2$ ), the $2 f$ mode is more advantageous than the $1 f$ mode.

Equations (6-5) and (6-6) show that the fundamental
and second harmonic current phase angles ( $\phi_{1}$ and $\phi_{2}$ ) change with dc potential. But for a process which displays a reversible wave in the ac or dc polarographic sence $\left(2 \omega^{1 / 2} / \lambda \ll 1\right.$ or $\left.\lambda t^{1 / 2} \gg 50\right)$, the $\phi_{1}$ and $\phi_{2}$ values are $\pi / 4$ and $-\pi / 4$, respectively, and these are independent of dc potential. These phase angle relationships prove useful for separation of two species with similar redox potentials.

A mixed solution of cadmium(II) and indium(III) in 1 M potassium chloride(pH 3) was investigated as one such example. As the $C d(I I) / C d(H g)$ system in 1 M potassium chloride behaves as a reversible process $\mathrm{k}_{\mathrm{s}}=2.5 \pm 0.3$ $\left.\mathrm{cms}^{-1}, \alpha=0.65 \pm 0.03\right)[16]$, the phase angle of the system is $45^{\circ}$ for $1 f$ over the peak potential region and that for $2 f$ is $-45^{\circ}$ at the more positive potential than the zero crossing potential and $135^{\circ}$ at the more negative potential. In contrast, as the In(III)/In(Hg) system in 1 M potassium chloride behaves as a quasi-reversible process $\mathrm{k}_{\mathrm{s}}=(6 \pm 2) \times$ $\left.10^{-2} \mathrm{cms}^{-1}, \beta=0.9 \pm 0.1\right)[16]$, the phase angles of the system for both lf and $2 f$ are not constant but change with dc potential. Therefore, as expected from the above theoretical relation, when the phase angle of the lock-in amplifier is adjusted to $135^{\circ}$ for 1 f or to $45^{\circ}$ for 2 f , the alternating current only for indium(II) is detected without interference from cadmium(II).

The principle of the separation in $1 f$ mode is explained by a vector illustration in Fig. 6-1. Namely, $\phi_{1}$ for the reversible process(Cd(II)/Cd(Hg) system) is $\pi / 4$ independent of dc potential, while that in the quasi-reversible process(In(III)/In(Hg) system) changes from 0 to $\pi / 4$ with dc potential(Eq. (6-5)). Therefore, in the $135^{\circ}$ component, the alternating current for the revesible process vanishes over the whole range of dc potential, while that for the quasi-reversible process remains.

Curves 1 and 2 in Fig. 6-2 were obtained individually for each of cadmium(II) and indium(III) at 0.2 mM concentrations in 1 M potassium chloride acidified to pH 3 with hydrochloric acid. The peak potential difference between them for lf was 45 mV . Curve 3 is for the mixed solution of 0.2 mM cadmium(II) and 0.2 mM indium(III). The polarograms of the mixture (curve 3) for the $0^{\circ}$ component of 1 f and for the $90^{\circ}$ component of 2 f show overlapping of the components. In contrast, the $135^{\circ}$ component of $1 f$ and the $45^{\circ}$ component of $2 f$ for the mixture display simple peaks corresponding to a single electrode reaction, and peaks for only indium(III) are detected; those for cadmium(II) are eliminated. As the contribution of the charging current is less in the $45^{\circ}$ component of $2 f$ than in the $135^{\circ}$ component of $1 f(F / C>50$ for $2 f$ and $F / C \simeq 1$


Fig. 6-1. Phase angle relationship between fundamental harmonic ac polarographic total currents and the current components at some phase angles.


Fig. 6-2. Fundamental (lf) and second(2f) harmonic ac polarograms at various phase angles for cadmium(II), indium(III) and their mixture. (1) $0.2 \mathrm{mM} \mathrm{Cd}(\mathrm{II}) ;(2) 0.2$ $\mathrm{mM} \operatorname{In}(\mathrm{III}) ;(3) 0.2 \mathrm{mM} C d(I I)+0.2 \mathrm{mM} \operatorname{In}(I I I)$ in $\mathrm{I} M$ $\operatorname{KCl}(\mathrm{pH} 3) \cdot \Delta \mathrm{E}_{\mathrm{ac}}=10 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}),. \mathrm{f}=25 \mathrm{~Hz}$.
for $1 f$, where $F$ is the faradaic current and $C$ is the charging current), the $2 f$ mode is superior to the $1 f$ mode in this case.

The calibration curves of indium(II) in the presence and in the absence of 0.2 mM cadmium(II) for $2 f$ are given in Fig. 6-3. The upper curve is for the $45^{\circ}$ component and the lower is for the $90^{\circ}$ component in the absence of cadmium(II). Peak current ( $I_{p}$ ) for the $90^{\circ}$ component was measured at -0.538 V for indium(III), because the peak at -0.570 V for indium(II) overlaps with the peak at - 0.572 V for cadmium(II) when the concentration of indium(III) is low. The $I_{p}$ for the $45^{\circ}$ component is the sum of positive and negative peak currents. The $I_{p}$ value for the $90^{\circ}$ component in the presence of cadmium(II) deviates from the calibration curve even at the concentration ratio of $\mathrm{C}_{\mathrm{In}(\mathrm{III})} / \mathrm{C}_{\mathrm{Cd}(I I)}=1 / 5$, but that for the $45^{\circ}$ component in the presence of cadmium(II) lies on the calibration curve up to a ratio of $1 / 50$ when $C_{\text {In (III) }}$ is $4 \times 10^{-6} \mathrm{M}$. A mixed solution of $z i n c(I I)$ and nickel(II) in 1 M potassium chloride was also investigated. Figure 6-4(A) shows the dependence of the phase angle of the alternating current upon de potential for $z i n c(I I)$ in 1 M potassium chloride, and Fig. 6-4(B) shows that for nickel(II). As the $\mathrm{Zn}(\mathrm{II}) / \mathrm{Zn}(\mathrm{Hg})$ system in chloride solution behaves as a quasi-reversible process in kinetics(ks $=(5.05 \pm 0.05) \times$


Fig. 6-3. Calibration curves for indium(III) in l M KCl(pH 3): (X) with 0.2 mM cadmium(II), and (O) without cadmium(II). The curves are for $45^{\circ}$ (upper) and $90^{\circ}$ (lower) components in the 2 f mode. $\quad \Delta \mathrm{E}_{\mathrm{ac}}=10 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}),. \mathrm{f}=25$ Hz.


Fig. 6-4. The phase angle of fundamental(lf) and second(2f) harmonic currents for $z i n c(I I)$ and nickel(II) in 1 MKCl .
(A) $0.2 \mathrm{mM} \mathrm{Zn}(I I) ;$ (B) $0.8 \mathrm{mM} \mathrm{Ni}(I I) . \Delta \mathrm{E}_{\mathrm{ac}}=5 \mathrm{mV}$ (r.m.s.), $f=15 \mathrm{~Hz}$.
$\left.10^{-3} \mathrm{cms}^{-1}, \alpha=0.3 \pm 0.02\right)[16-18]$, the phase angle changes with dc potential. In contrast, the phase angle for nickel(II) in If is zero(ks $=1.2 \times 10^{-2} \mathrm{cmlmol} \mathrm{s}^{-1} \mathrm{~s}^{-1}, \mathrm{NiOH}^{+}+$ $2 \mathrm{e}^{-}+\mathrm{Hg} \longrightarrow \mathrm{Ni}(\mathrm{Hg})+\mathrm{OH}^{-}$in 0.1 M KC 1$)[16]$. The phase angle in 2 f changes from $-90^{\circ}$ to $+90^{\circ}$ at the zero crosing potential. It has been suggested that the irreversibility of the $\mathrm{Ni}(\mathrm{II}) / \mathrm{Ni}(\mathrm{Hg})$ system in chloride solution is induced not by a very slow heterogeneous charge transfer reaction but other mechanisms[19-21].

The phase angle-potential relationships indicate that the current of lf for nickel(II) vanishes when the phase angle of the lock-in amplifier is adjusted to $90^{\circ}$, and that the current of $2 f$ vanishes when the phase angle is adjusted to $0^{\circ}$, while the currents of both modes for zinc(II) should be detected at these phase angles. Figure 6-5 shows ac polarograms of the $0^{\circ}$ and $90^{\circ}$ components for $1 f$ and 2 f . Curves 1 and 2 were obtained individually for each of 2 mM nickel(II) and 0.1 mM zinc(II). The peak potential difference between them for $1 f$ was 65 mV . Curve 3 is for a mixed solution of 2 mM nickel(II) and 0.1 mM zinc(II).

Practically, the polarograms of the mixture(curve 3) for the $0^{\circ}$ component of $1 f$ and the $90^{\circ}$ component of $2 f$ show overlapping of the components and one cannot separate them well from each other. The $90^{\circ}$ component in $1 f$ and the $0^{\circ}$ component in $2 f$ for the mixture are almost the same as


Fig. 6-5. Fundamental(lf) and second(2f) harmonic ac polarograms for zinc(II), nickel(II) and their mixture. (1) $2 \mathrm{mM} \mathrm{Ni}(I I) ;(2) 0.1 \mathrm{mM} \mathrm{Zn}(I I) ;(3) 2 \mathrm{mM} \mathrm{Ni}(I I)+0.1 \mathrm{mM}$ $\mathrm{Zn}(\mathrm{II})$ in $1 \mathrm{MKCl} . \Delta \mathrm{E}_{\mathrm{ac}}=10 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}),. \mathrm{f}=100 \mathrm{~Hz}$.
those for zinc(II). The contribution of charging current to the whole current for the former is about $98 \%$ and that for the latter is about $10 \%$. Therefore, the $2 f$ mode is superior to the lf mode for quantitative work.

Figure $6-6$ shows the calibration curves obtained by the use of the $2 f$ mode for $z i n c(I I)$ in the presence and in the absence of 2 mM nickel(II) in 1 M potassium chloride. The upper curve is for the $0^{\circ}$ component and the lower is for the $90^{\circ}$ component. The $I_{p}$ is measured by the same method as in the case of indium(II) and cadmium(II). When the ratio of $C_{Z_{n}(I I)} / C_{N i(I I)}$ is less than $1 / 5$, the $I_{p}$ for the $90^{\circ}$ component in the presence of nickel(II) deviates from the calibration curve obtained in the absence of nickel(II). Yet the $I_{p}$ values for the $0^{\circ}$ component in the presence and in the absence of nickel(II) are equal even when the $C_{Z_{n(I I)}} / C_{N i(I I)}$ ratio is $1 / 100$ at $C_{Z n(I I)}=2 \times 10^{-5}$ M.

The phase-selective second harmonic ac polarographic technique can be used to distinguish two species having similar peak potentials. The method presented, which utilizes phase angle detection, can be used even when the peak potentials are the same, provided that each electrochemical system has a different phase angle depending on its behavior as reversible, quasi-reversible or irreversible. When a given species reacts with the


Fig. 6-6. Calibration curves for zinc(II) in 1 M KCl: (X) with 2 mM nickel(II), and (O) without nickel(II). The curves are for $0^{\circ}$ (upper) and $90^{\circ}$ (lower) components in the 2 f mode. $\Delta \mathrm{E}_{\mathrm{ac}}=10 \mathrm{mV}(\mathrm{r} . \mathrm{m} . \mathrm{s}),. \mathrm{f}=100 \mathrm{~Hz}$.
electrode through two different electrode processes simultaneously, this method could be also applied to analyze the kinetics for each electrochemical reaction, provided that they give different phase angles.

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Chapter 7. Concluding remarks

The properties of the various phase angle components were precisely analyzed for the purpose of the analyses of electron transfer reaction kinetics and for the chemical analysis of inorganic ions in aqueous medium.

In the ac polarography, the total alternating current through the electrochemical cell can be expressed by power series. In the reversible process, the first term of the power series, lf mode, is a real first derivative form of dc polarogram and the second term, $2 f$ mode, is also that of lf mode. In the quasi-reversible and irreversible process, however, $1 f$ and $2 f$ modes do not show the real derivative forms of dc and lf, respectively. Therefore, this method gives useful informations in assessing the reaction kinetics of electrochemical process.

In the electrochemical reduction of chromium(VI) in alkaline solution, a characteristic electrochemical spectrum was observed by $1 f\left(45^{\circ}\right)$ component of the first derivative polarography. The spectrum and rate constant for the electron transfer of various steps of oxidation state was determined by the curve fitting method. From the results of
the analysis, it was proved that the rate determining step was the reduction of chromium(VI) to chromium(IV) in the total reaction of chromium(VI) to chromium(III) and that the final electrode reaction products would be $\operatorname{Cr}(\mathrm{OH})_{2}^{+}$.

In the zirconium(IV)-nitrate ion system, the dc polarographic data for the reduction of nitrate ion suggested the contribution of adsorbed zirconium(IV). The differential capacity measurement by using $1 f\left(0^{\circ}\right)$ and lf(90 $)$, however, showed no evidence of its adsorption on the electrode. The role of zirconium(IV) in the reduction of nitrate was discussed. It was reasonably suggested that on the vicinity of the electrode, the zirconium(IV) might act as an intermediary role for charge transfer from the electrode to the nitrate ion.

The $2 f$ mode in the phase selective ac polarography was investigted throughly for the purpose of the establishment of non-linear electrochemical method for chemical analysis. A new technique was proposed in analyzing two species having similar redox potentials by utilizing the difference in the electron transfer rates between them. The theoretical interpretation of the principle of this technique was attempted and applied in this analysis. Actually, it was successfully proved that the $2 f\left(45^{\circ}\right)$ and $2 f\left(0^{\circ}\right)$ were valid for the analyses of indium(III) in the presence of excess of cadmium(II) and of zinc(II) in the presence of nickel(II),
respectively.

Kinetic mechanisms of the reduction of multivalent ions, $\mathrm{CrO}_{4}^{2-}$ and $\mathrm{NO}_{3}^{-}$were studied by means of phase selective voltammetric techniques. These electrochemical methods were also applied to quantitative analyses.

The information involved in various phase angle components will be useful for studies on reaction mechanisms, for understanding of properties of chemical species, and even for chemical analyses.

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