

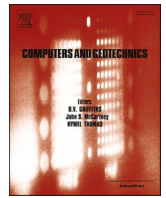


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Research Paper

Modeling of hydraulic fracturing process in crystalline rock reflecting actual mineral distribution using FDEM based on extrinsic cohesive zone model

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ABSTRACT

A grain-based model (GBM) replicating the exact mineral distribution of the experimental specimen was constructed, and hydraulic fracturing simulations were conducted using the finite-discrete element method (FDEM), enabling direct comparison with laboratory observations. To leverage FDEM's ability to represent fracture surfaces explicitly, the GBM incorporated the mineral boundary geometries directly into the mesh. An extrinsic cohesive zone model (ECZM) was adopted to improve stress representation, and verification demonstrated that ECZM avoided the artificial stress oscillations characteristic of the Intrinsic CZM (ICZM). Simulations of Brazilian tensile strength tests and hydraulic fracturing tests exhibited both intergranular and transgranular cracking, producing tortuous fracture paths, and the resulting fracture metrics agreed with experiments. The analyses further revealed that fracture propagation pathways are controlled by the interplay between the stress concentration at the fracture tip and the local mineral strength. This framework provides a robust tool for investigating fracture processes in heterogeneous crystalline rock.

1. Introduction

Rock fracturing is a critically important phenomenon in various deep underground applications, including geothermal energy development, the geological disposal of high-level radioactive waste, carbon capture and storage (CCS), and deep tunneling projects. From a mechanical perspective, the discontinuities generated by rock fracturing have led to a reduction in the bulk strength of rock masses, thereby compromising the structural stability of underground infrastructures. From the viewpoint of fluid flow, the fractures formed through rock fracturing serve as preferential pathways for fluid flow, resulting in an increase in rock permeability. In next-generation geothermal power systems, such as engineered/enhanced geothermal systems (EGSs), a fracture network is

created by injecting high-pressure fluid into the rock mass through hydraulic fracturing, thereby artificially enhancing and controlling the rock permeability within the reservoir. To enhance the power generation performance of EGSs, it is essential to form, through hydraulic fracturing, circulation pathways (i.e., fracture network) that enable the injected water to efficiently recover heat from the surrounding high-temperature rock and be effectively extracted. Accordingly, characterizing the permeability of the fracture network formed through hydraulic fracturing is indispensable for evaluating the power generation performance of EGSs. The permeability characteristics of fractures depend on their propagation paths, which can be quantitatively described by parameters, such as linearity and tortuosity, as well as the fracture aperture at each location. However, the processes of fracture initiation and propagation within a rock mass are influenced by numerous

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Nomenclature		RS-GB-FDEM	real-specimen grain-based FDEM
<i>Abbreviations</i>		GPGPU	general-purpose graphics processing unit
CCS	carbon capture and storage	CZM	cohesive zone model
EGS	engineered/enhanced geothermal system	ICZM	intrinsic CZM
IG crack	intergranular crack	ECZM	extrinsic CZM
TG crack	transgranular/intragranular crack	DFNM-FEM	FEM with discrete fracture network model
GBM	grain-based model	ECZM-FDEM	ECZM-based FDEM
RS-GBM	real-specimen GBM	BTS	Brazilian tensile strength
FEM	finite element method	UCS	uniaxial compressive strength
FDM	finite difference method	DIC	digital image correlation
DEM	discrete element method	DIP	digital image processing
FDEM	combined finite-discrete element method	CT	computed tomography
GB-FDEM	grain-based FDEM	XRF	X-ray fluorescence
		LEFM	linear elastic fracture mechanics

factors—including, in the case of hydraulic fracturing, coupled hydraulic–mechanical interactions—making it challenging to predict and characterize the properties of the induced fractures. Therefore, further research is required to improve the understanding of the complex processes of rock fracturing.

Among the types of crystalline rock widely distributed at EGS candidate sites, granite exhibits macroscopic fracturing behaviors, such as fracture linearity, tortuosity, aperture, and the associated stress–strain responses. These behaviors are strongly influenced by the microstructures at the grain scale, including grain morphology, grain size and spatial distribution, crystallographic anisotropy and orientation, the elastic responses of grains, and the properties of the grain interfaces (Benedetti and Aliabadi, 2013; Fritzen, 2011). According to previous studies (Li et al., 2021; Tham et al., 2005), the heterogeneities in the grain morphology, grain size, and spatial distribution of grains induce stress concentration and distortion. When the local stress exceeds the local strength, microcracks are generated. And many microcracks, namely, those with configurations favorable for coalescence, selectively merge to form macroscopic fractures (Kranz, 1983). The locations where microcracks form significantly influence the fracture propagation paths. Microcracks occurring along grain boundaries are classified as intergranular cracks (referred to as IG cracks in this study), whereas those within grains are categorized as transgranular or intragranular cracks (referred to as TG cracks in this study) (Aboyanah et al., 2024; Kranz, 1983; Li et al., 2019). Mo et al. (2023) conducted Mode I fracture tests on granite, while performing real-time observations by scanning electron microscopy, and reported that TG cracks tend to occur preferentially within biotite among the constituent minerals of granite. Jacobsson and Brander (2024) analyzed digital image correlation (DIC) measurements from wedge-splitting tests and observed that coarse-grained quartz tends to exhibit TG cracks, whereas fine-grained quartz typically exhibits IG cracks. These findings suggest that fracture propagation paths—whether IG or TG—are governed by grain-scale factors and, in turn, affect the linearity and tortuosity of macroscopic fractures.

Numerous experimental studies have reported on evaluations of rock microstructures (Eberhardt et al., 1999; Erarslan, 2016). However, experiments aimed at assessing deformation and fracturing induced by microstructural features involve highly complex procedures, and the results are governed by the inherent characteristics of each rock specimen, such as the grain morphology and orientation. Consequently, it is nearly impossible to fully reproduce such conditions in experimental settings (Abdelaziz et al., 2018). To overcome the limitations of these experimental approaches, a wide range of numerical methods has been developed. The numerical methods often employ grain-based models (GBMs), which incorporate microstructural features—particularly grain morphology, size, and spatial distribution—into the modeling process (See Zhang and Wong (2018)). These numerical simulations are constructed contingent on either continuum-based or discontinuum-based

approaches, depending on how the fractures are represented. Continuum-based methods model the entire computational domain as a continuous medium, where each continuum element represents an average volume that may include fractures, and damage evolution is described through the stress softening of the elements. In contrast, discontinuum-based methods model the entire domain as an assembly of discrete bodies (i.e., rock grains), with rigid particles assigned to individual elements, and fractures explicitly represented at the interfaces between these elements.

In continuum-based methods, finite element methods (FEMs) and finite difference methods (FDMs) are widely employed. Examples of microstructure-informed analyses include FEM-based studies (Li et al., 2025; Li et al., 2024; Yu et al., 2015) and FDM-based work (Chen et al., 2004). In these approaches, mechanical properties corresponding to specific minerals are assigned to each element; elastic moduli are used to compute stress and deformation, while strength parameters govern the initiation and evolution of damage. Continuum-based methods offer advantages such as numerical stability and the ability to capture the overall deformation and failure mechanisms based on internal stress and deformation. However, fractures are not represented explicitly in conventional continuum models; and thus, it is difficult to account for the grain boundaries, making it impossible to simulate IG cracking. Furthermore, such methods cannot capture key discontinuity-related phenomena, including displacements (e.g., aperture and slip) along fracture surfaces or contact behavior at newly formed cracks. As such, continuum-based methods are not well-suited for resolving localized fracture initiation and propagation driven by microstructural heterogeneity.

In contrast, discontinuum-based methods, particularly the discrete element method (DEM) (Cundall and Strack, 1979), are widely used for simulating grain-scale fracture phenomena. Numerous DEM studies have incorporated grain distributions into their models (Chen et al., 2018; Kong et al., 2021, 2022; Li et al., 2018; Liu et al., 2024; Qi et al., 2022; Saadat and Taheri, 2020; Wang et al., 2021; Zhang et al., 2020; Zhang et al., 2019; Zhu et al., 2024). While DEM does not generally resolve a continuous stress–strain field as continuum methods do, it allows for the explicit representation of fractures, making it well-suited to capturing phenomena along the fracture surfaces, including IG cracks. In addition, DEM allows for flexible modeling by assigning different mechanical properties, depending on whether the fractured zone is TG or IG. Recent advances have demonstrated the potential of grain-based DEM (GB-DEM) for simulating hydraulic fracturing under realistic grain-scale heterogeneity. For instance, Kong et al. (2024) highlighted the ability of GB-DEM to reproduce grain-scale microcrack nucleation and propagation under fluid loading. In GB-DEM, individual mineral grains and their contacts are explicitly represented, allowing non-uniform stress development, bond breakage, and crack coalescence that reflect actual grain-scale behavior. Furthermore, GB-DEM

simulations of hydraulic fracturing (e.g., Li et al., 2022, Wang et al., 2024b) have shown that fluid injection can trigger complex fracture growth patterns in heterogeneous rock. Nonetheless, GB-DEM has several limitations. As reviewed by Kong et al. (2024), commonly used contact models (e.g., parallel-bond bonded contacts) may fail to reproduce realistic mixed-mode (tensile–shear) fracture propagation and may not reliably capture the transition from intact to fully fractured rock. Moreover, GB-DEM requires a large number of micro-parameters (grain stiffnesses, bond strengths, friction coefficients, fluid–grain coupling parameters, etc.), and their calibration is highly non-unique. High-resolution hydraulic fracturing simulations also involve substantial computational costs.

Recently, attention has turned to hybrid methods, such as the finite-discrete element method (FDEM) (Munjiza, 2004), which integrates the advantages of both continuum and discontinuum approaches. Examples of studies on FDEM that incorporate grain distributions include Abdelaziz et al. (2018), Yahaghi et al. (2023), Zhang et al. (2021), and Zhou et al. (2020). FDEM solves the stress and deformation within each continuum element, and allows for the explicit modeling of fractures. It can also simulate the transition from intact continuum behavior to discontinuous fracturing, making it effective for capturing the progressive evolution from continuous deformation to discrete failure. Moreover, FDEM provides both a continuous stress–strain field and explicit fracture representation, offering a distinct advantage over DEM for problems where accurate stress resolution is essential. In addition, FDEM has been implemented with parallel computation using general-purpose graphics processing unit (GPGPU) technology, achieving computational speeds several hundred times faster than those of conventional serial computation (Fukuda et al., 2019; Fukuda et al., 2020a; Lisjak et al., 2018). This capability makes FDEM especially attractive for high-resolution simulations that incorporate detailed grain distributions. Given these characteristics, many previous studies, focusing on grain-based modeling, have adopted hybrid approaches. They are better suited for simulating localized fracture initiation and propagation induced by microstructural heterogeneity.

Various modeling approaches have been investigated for constructing simulation models, particularly within the context of GBMs. Recent studies incorporating grain distributions have sought to reproduce experimentally observed fracture propagation by developing numerical models that replicate the actual mineralogical structure of rock specimens, including both those used in experiments and others with similar characteristics. Zhang et al. (2019) demonstrated that Voronoi-based GBMs with randomly distributed grains tend to overestimate the role of IG cracking when compared to models in which the rock structure is directly mapped from real rock specimens. They concluded that such random models are inadequate for accurately interpreting fracture mechanisms. As one of the direct mapping techniques, digital image processing (DIP) has been utilized to create GBMs by processing digital images with computer algorithms (Gao-Feng et al., 2025; Zhang et al., 2019). For example, Zhang et al. (2019) classified the surface of a Kowloon granite specimen into four minerals, namely, quartz, alkali feldspar, plagioclase, and biotite, based on color characteristics extracted from digital images, and then constructed a corresponding numerical model. Zhou et al. (2025) successfully employed X-ray computed tomography (CT) scanning to generate a GBM of granite composed of three primary minerals, namely, quartz, feldspar, and mica. Furthermore, Mahabadi et al. (2014) developed a GBM from the CT images of a specimen used in Brazilian tensile strength (BTS) tests and conducted numerical simulations using the hybrid FDEM method. Their model, which retained the same grain distribution as the very specimen tested in the experiment (hereafter referred to as the real-specimen grain-based model, RS-GBM), enabled a direct comparison with the experimental results. Such analyses employing RS-GBMs are of critical importance for elucidating the complex processes of fracture initiation and propagation governed by microstructural heterogeneity and other interacting factors. For example, Li et al. (2024) employed a continuum method to

simulate uniaxial compressive strength (UCS) tests, while Mahabadi et al. (2014) used a hybrid method to simulate BTS tests. Both studies used RS-GBMs and compared their fracture patterns. However, such analyses remain extremely rare and, to the best of the authors' knowledge, no similar attempts have been made within the context of hydraulic fracturing. Chen et al. (2018) performed DEM-based hydraulic fracturing simulations using a GBM generated from a single rock surface image via DIP technology. However, the rock captured in the image was not one used in any hydraulic fracturing experiments. Similarly, other studies involving DEM-based hydraulic fracturing simulations that consider grain distributions (Kong et al., 2021, 2024; Li et al., 2022; Wang et al., 2023, 2024a) employed models with randomly assigned grain phases rather than experimentally matched distributions. Furthermore, to the best of the authors' knowledge, no hydraulic fracturing simulations have been conducted using the grain-based FDEM (GB-FDEM) that incorporates mineralogical heterogeneity.

Based on the above discussion, in order to accurately capture hydraulic fracturing behavior influenced by grain distributions, it is desirable to employ hybrid methods, such as FDEM, as the simulation approach, and to use RS-GBMs. This integrated framework is hereafter referred to as RS-GB-FDEM (real-specimen grain-based FDEM). As noted above, however, such integration has not yet been realized. Moreover, even in simulations focusing solely on the mechanical behavior, excluding the hydraulic effects, two major limitations remain in the current GB-FDEM analyses. The first limitation lies in their inability to capture the orientation of the grain boundaries, even though the overall grain distribution is reasonably represented. For instance, Mahabadi et al. (2012, 2014) developed numerical models by first generating a mesh and then mapping mineral information extracted from CT images onto each mesh element. While this procedure enables a satisfactory representation of the spatial arrangement of the grains, it does not preserve the morphology of the grain boundaries, particularly when coarse meshes are used; the resulting grain shapes are constrained by the pre-defined mesh geometry. Since FDEM represents fractures explicitly along the element boundaries, capturing realistic fracture propagation requires that the mesh boundaries coincide with the potential fracture paths, such as the grain interfaces. However, due to the increased computational cost associated with such modeling, previous GB-FDEM studies have paid little attention to aligning the mesh boundaries with actual grain interfaces.

The second limitation in the current GB-FDEM analyses that incorporate grain distributions is the accuracy of the stress computation, which remains an area with room for improvement. In most FDEM implementations, fracture initiation and propagation are modeled using the cohesive zone model (CZM) (Barenblatt, 1962; Dugdale, 1960), wherein cohesive elements are inserted along the continuum element boundaries to explicitly simulate crack evolution, while the stress–strain response of the bulk material is concurrently resolved. CZM can be broadly classified into two types: the intrinsic cohesive zone model (ICZM) and the extrinsic cohesive zone model (ECZM) (Papoulia et al., 2003; Zhang et al., 2007). Originally, CZM was designed to model the behavior following crack initiation and does not account for the intact (uncracked) state. ECZM adopts a natural approach by inserting cohesive elements only at the moment a crack is initiated. In contrast, ICZM pre-embeds cohesive elements across the entire model from the beginning of the simulation, and assigns them finite stiffness (a cohesive penalty) to reproduce pseudo-elastic behavior in the intact state. Although ICZM is widely used due to its ease of implementation and compatibility with GPGPU-based parallel computation, it introduces artificial compliance into the bulk material because the pre-inserted cohesive elements deform even before crack initiation—despite their physical absence in the intact state. This artificial compliance leads to degraded stress computation accuracy, as noted in previous studies (Fukuda et al., 2020b; Maeda et al., 2024a). To date, all GB-FDEM simulations have been based on ICZM, raising concerns about the degradation of stress computation accuracy in such analyses. In rock

materials, heterogeneity in stress fields arises due to differences in the deformation properties of individual minerals, often leading to the nucleation of microcracks under locally elevated stresses. Moreover, in hydraulic fracturing, pore pressure induces deformation in the bulk rock matrix. Therefore, accurate stress computation is critically important in fracture simulations—particularly hydraulic fracturing—that account for mineralogical heterogeneity.

In this study, a hydraulic fracturing simulation is conducted using RS-GB-FDEM. In particular, by constructing a model that explicitly traces the actual grain boundaries, the simulation can more realistically capture the fracture propagation in FDEM, where the cracks are represented along the mesh boundaries. Furthermore, for the fracture model in FDEM, the GPGPU-parallelized ECZM (Maeda et al., 2024b) is employed, which solves the problem of the degradation of the stress computation accuracy associated with the conventional ICZM. This approach allows for the numerically accurate resolution of stress–strain contrasts arising from spatially heterogeneous grain distributions and enables the simulation of the entire fracturing process—from microcrack initiation to macroscopic specimen failure—based on mineral-scale heterogeneity.

The remainder of this paper is organized as follows. Section 2 describes the methodology of the numerical simulation, provides experimental overviews of the BTS and hydraulic fracturing tests, and explains the modeling strategy for constructing RS-GBMs. Section 3 verifies the proposed simulator through benchmark analyses of the coupled hydraulic–mechanical behavior, and examines the differences in stress field representation between ICZM and ECZM under intact conditions. Section 4 presents the results of both the BTS and hydraulic fracturing tests, comparing the experimental observations with numerical simulations. Section 5 provides a detailed discussion on fracture propagation path selection. Section 6 summarizes the conclusions of this study.

2. Method

2.1. Numerical method

A hydraulic fracturing simulator, based on RS-GB-FDEM, was developed in this study. The simulator integrates FDEM with a GBM reflecting the actual mineral distribution of the experimental specimen, enabling an accurate and quantitative reproduction of the fracture initiation and propagation governed by mineralogical heterogeneity. The developed simulator comprises two main components: a mechanical solver that handles stress–strain behavior along with fracture initiation and propagation, and a hydraulic solver that models fluid flow within both the rock matrix and the fractures. For the fluid analysis, a finite element method coupled with a discrete fracture network model (Herbert, 1996), referred to as DFN-FEM, was adopted. In particular, the DFN-FEM used in this study was improved so that the fluid pressures in the rock matrix and in the fractures would be treated as separate variables. As a result, the developed simulator can accurately capture dominant fracture-flow behavior and the associated complex fracture propagation phenomena. For the mechanical analysis, an FDEM framework incorporating a GPGPU-parallelized ECZM (Maeda et al., 2024b), referred to as ECZM-FDEM, was employed. The simulator proposed in this study utilized the same hydraulic–mechanical coupling scheme and hydraulic calculation as the hydraulic fracturing simulator developed by Takuma et al. (2024); however, the mechanical calculation was different. In the study by Takuma et al. (2024), the mechanical behavior was analyzed using FEM rather than FDEM, which can explicitly represent local contact phenomena along discontinuous fracture surfaces. Moreover, their simulator did not implement a GPGPU-based parallel computation and was not designed to account for mineralogical heterogeneity. In contrast, the present study adopted a GPGPU-parallelized FDEM framework capable of handling analyses that incorporate mineral distributions. The proposed simulator enables high-resolution grain-based modeling, including grain boundaries, allowing

for an accurate simulation of the stress–strain responses and fracture evolution arising from mineralogical heterogeneity. Moreover, in simulations that incorporate grain distributions, the simulator can flexibly account for differences in the deformation properties across mineral types, as well as the strength characteristics of individual minerals and their interfaces. As described above, both ECZM-FDEM and DFN-FEM can explicitly distinguish between the rock matrix and the fractures. This capability facilitates the realistic modeling of complex physical phenomena.

All the numerical simulations in this study were conducted under plane strain conditions. Tensile stress and compressive stress were defined as positive and negative, respectively. Hydraulic fracturing scenarios were conducted assuming fully saturated rock conditions.

The computational flow for the calculation loop of the developed simulator is illustrated in Fig. 1. The fluid analysis follows an implicit time integration scheme, whereas the mechanical analysis follows an explicit time integration scheme. In explicit schemes, the time step must be set to an extremely small value to ensure numerical stability, making it difficult to simulate durations in the order of several seconds, which is the actual time scale of hydraulic fracturing. To overcome this limitation, the present study adopted a quasi-static approach, similar to those used in many existing hydraulic–mechanical coupled simulators (Viré et al., 2012; Yang et al., 2020), where the mechanical response is tracked until a quasi-static equilibrium state has been achieved. The fluid solver operates with a global time variable t and its increment Δt , both of which correspond to actual physical times. Each fluid computation calculates fluid flow over the time interval from t_{start} to t_{end} , as indicated in the

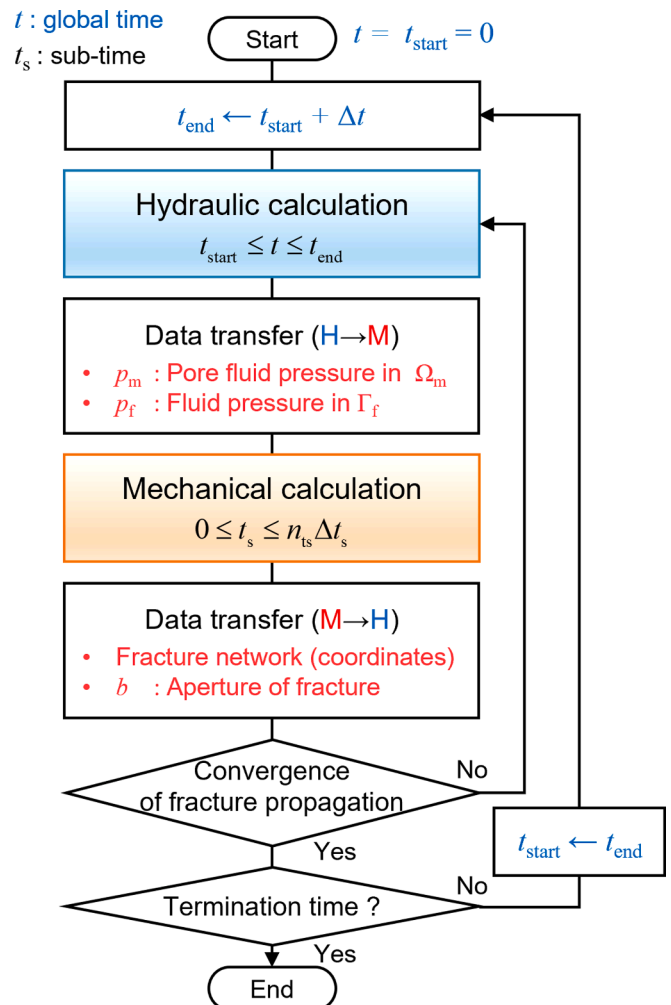


Fig. 1. Computational flow for calculation loop of developed simulator.

figure. Based on the pore pressure field at time $t = t_{\text{end}}$, the mechanical solver performs iterative calculations to simulate the instantaneous processes of fracture initiation and propagation. The time variable used for these iterative calculations is referred to as sub-time t_s , with its increment denoted as Δt_s , and the iterations were performed up to the prescribed maximum number of steps, denoted as n_{ts} . In this study, n_{ts} was set to 100,000 steps, based on prior evaluations confirming that this setting ensures a sufficiently quasi-static condition with respect to the ratio of kinetic energy to strain energy. At the beginning of each mechanical calculation, sub-time t_s was reset to zero. Furthermore, during fracture propagation, it was assumed that the interaction between fracture growth and fluid flow within the newly formed fractures would occur instantaneously and repeatedly. To incorporate this instantaneous fluid–mechanical interaction within the newly formed fractures, fluid and mechanical computations were alternately performed without advancing the global time until fracture initiation and propagation had converged. It should be noted that, in hydraulic fracturing analyses, there may be a moment when an unstable and rapid fracture growth occurs, causing the fracture to reach the model boundary before convergence has been achieved. To more accurately reflect the instantaneous fluid–mechanical interactions that would occur in reality at such a moment, the maximum number of steps per mechanical calculation, n_{ts} , was reduced to 10,000 steps.

The interaction between the mechanical field (M) and the hydraulic field (H), considered in the developed simulator, is explained below, following the actual computational sequence. In the model, the intact rock domain is defined as Ω_m , the fracture boundary as Γ_f , the external boundary as Γ_o , and the injection borehole boundary as Γ_{in} . The influence from the hydraulic field to the mechanical field (H $\rightarrow$ M) is represented by transferring injection pressure p_{in} on Γ_{in} , pore pressure p_m on Ω_m and, if fractures have already occurred, fracture fluid pressure p_f on Γ_f to the mechanical solver. In the mechanical computation, the stress and deformation resulting from each of these types of pressure are explicitly considered. Conversely, the influence of the mechanical field on the hydraulic field (M $\rightarrow$ H) involves passing the coordinates of the newly generated fractures and the aperture of all the fractures from a mechanical computation to a hydraulic computation. The hydraulic solver updates the fracture network geometry accordingly and performs the calculation using fracture permeability determined based on the given apertures.

2.1.1. Mechanical calculation in ECZM-FDEM

In ECZM-FDEM, the deformation behavior of the rock matrix, the processes of fracture initiation and propagation, and the contact mechanics of discontinuous surfaces, including generated fractures, are governed by continuum mechanics, ECZM (Papoulia et al., 2003; Zhang et al., 2007), and contact mechanics, respectively. After computing the nodal forces resulting from deformation, external loads, and fluid pressure, ECZM-FDEM solves the equation of motion (Eq. (1)).

$$\mathbf{M} \frac{\partial^2 \mathbf{u}}{\partial t_s^2} = -\mathbf{f}_{\text{int}} + \mathbf{f}_{\text{ext}} + \mathbf{f}_{\text{coh}} + \mathbf{f}_{\text{con}} + \mathbf{f}_{\text{pre}} \quad (1)$$

where $\mathbf{M}$ [kg] is the nodal lumped mass matrix, $\mathbf{u}$ [m] is the nodal displacement vector, and t_s [s] is the sub-time. $\mathbf{f}_{\text{int}}$ [N] denotes the nodal force arising from internal stress, and it is computed by transforming the Cauchy stress $\boldsymbol{\sigma}$ [Pa] obtained from the constitutive model of a hyper elastic solid based on continuum mechanics (Eqs. (2) and (3) in Fukuda et al. (2019)). In simulations considering grain distributions, the elastic moduli of the continuum elements are assigned according to the mechanical properties of each mineral, enabling the representation of deformation heterogeneity due to the mineralogical variation. $\mathbf{f}_{\text{ext}}$ [N] represents the nodal force due to external loads, which includes confining pressure in the present study. $\mathbf{f}_{\text{coh}}$ [N] is the cohesive traction acting on the cohesive elements representing fractures and is calculated based on ECZM. $\mathbf{f}_{\text{con}}$ [N] corresponds to the contact force that arises

during contact between discontinuous surfaces, including generated fracture surfaces, and is evaluated using contact mechanics. $\mathbf{f}_{\text{pre}}$ [N] denotes the nodal force due to hydraulic loads induced by fluid pressure and constitutes the H $\rightarrow$ M coupling term (See Section 2.1.2).

Fracture initiation is determined based on normal stress σ_n and shear stress τ_n along the boundaries of the continuum elements (i.e., mesh interfaces), as formulated in Eq. (2) of Maeda et al. (2024b). Equations (2) and (3) define the tensile and shear failure indicators, denoted as S_t [-] and S_s [-], respectively, which are evaluated at the mesh boundaries based on the local stress state. Unlike ICZM, where failure is typically assessed using strain- or displacement-based criteria because cohesive elements are embedded prior to failure, ECZM does not contain cohesive elements before fracture initiation; thus a stress-based criterion is employed. When either failure indicator reaches a value of 1, a cohesive element is activated at the corresponding mesh boundary to represent the fracture. The activated cohesive elements are subjected to normal and tangential cohesive tractions, σ^{coh} [Pa] and τ^{coh} [Pa], respectively.

$$S_t = \frac{\sigma_n}{f_t} \quad (2)$$

$$S_s = \frac{|\tau_n|}{\max(\varepsilon, c - \sigma_n \tan \phi)} \quad (3)$$

where f_t [Pa], c [Pa], and ϕ [degree] are the tensile strength, cohesion, and internal friction angle of the mesh boundaries, respectively. ε [-] is a small positive constant introduced to prevent division by zero; it was set to 1×10^{-10} in this study. In simulations that consider grain distributions, the strength properties of the mesh interfaces are assigned based on the specific minerals or mineral interfaces they intersect. This allows for flexible modeling, such as assigning lower strength values to the mineral interfaces, which are considered to be mechanical weak planes.

After the activation of a cohesive element, effective opening o [m] and sliding displacement $|s|$ [m] are calculated using Eqs. (19) and (20) from Maeda et al. (2024b). Because ECZM inherently suffers from a time-discontinuity issue—namely, a mismatch between the internal stresses and newly assigned cohesive tractions immediately before and after element insertion—the nominal displacement derived from the pre-insertion stresses is incorporated into the effective displacement to ensure a smoother transition, following an approach conceptually similar to that of Sam et al. (2005) and Woo and Cairns (2019). While this treatment mitigates the discontinuity to some extent, fully eliminating it would require a node-based traction equilibration framework such as that of Lei et al. (2024), which remains a subject for future development due to its limited compatibility with large-scale GPGPU parallel computations.

Once a cohesive element is activated, the procedures in ICZM and ECZM are identical: the increase in the effective displacements leads to the softening of the cohesive element, thereby representing fracture propagation. In conventional ICZM and ECZM, the effective displacements are dependent on the element size, whereas the critical displacements for CE4 softening are constant parameters, i.e., independent of the element size. Consequently, the resulting fracture propagation becomes dependent on the element size. To address this issue, recent studies have introduced approaches in which the critical displacements are explicitly linked to the element size (Deng et al., 2021), or both the effective and critical displacements are reformulated in terms of element-independent strain (Maeda et al., 2024a). In this study, following the model proposed by Maeda et al. (2024a), effective normal strain ε_o [-], effective shear strain ε_s [-], critical opening strain $\varepsilon_{o,c}$ [-], and critical shear strain $\varepsilon_{s,c}$ [-] are computed using element size h [m], as defined in Eqs. (4) to (7). As the effective strain approaches the critical strain, damage coefficient D [-] increases from 0 to 1, reducing the cohesive traction accordingly and thereby modeling fracture propagation.

$$\varepsilon_o = \frac{o - o_p}{h} \quad (4)$$

$$\varepsilon_s = \frac{|s| - s_p}{h} \quad (5)$$

$$\varepsilon_{o,c} = \lambda_o \frac{f_t}{E} \quad (6)$$

$$\varepsilon_{s,c} = \lambda_s \frac{c}{E} \quad (7)$$

where o_p [m] and s_p [m] are the artificial elastic limits of o and $|s|$, respectively, λ_o [-] and λ_s [-] are coefficients, and the values of λ_o and λ_s in this study were set to 10 and 7 based on a preliminary study. Damage variable D [-] was calculated from tension-induced damage D_o [-] and shear-induced damage D_s [-], as follows:

$$D = \min\left(1, \sqrt{D_o^2 + D_s^2}\right) \quad (8)$$

$$D_o = \max\left(0, \frac{\varepsilon_o}{\varepsilon_{o,c}}\right) \quad (9)$$

$$D_s = \max\left(0, \frac{\varepsilon_s}{\varepsilon_{s,c}}\right) \quad (10)$$

2.1.2. Hydraulic calculation in DFNM-FEM and coupled interactions

In the DFNM-FEM framework, the pore fluid pressure in rock matrix p_m on Ω_m and, if fractures have been generated, the fluid pressure within fracture p_f on Γ_f , are computed based on the mass conservation law and Darcy's law, using injection pressure p_{in} on Γ_{in} at the borehole as the boundary condition (Eqs. (14) to (18) in Takuma et al. (2024)). Prior to fracture initiation, the fluid analysis involves the computation of only p_m on Ω_m . Once a fracture has been generated in the mechanical simulation, the interaction from mechanical field to hydraulic field ($M \rightarrow H$) involves incorporating the newly created fractures into the model

geometry of DFNM-FEM, upon which fracture fluid pressure p_f on Γ_f is subsequently solved. After the fracture initiation, equivalent fracture aperture b [m], which is calculated from the opening displacement of cohesive elements (Eqs. (19) to (21) in Takuma et al. (2024)), is transferred from a mechanical analysis to a fluid flow analysis after each mechanical step. Fracture permeability k_f [m²] is then computed from the equivalent aperture and applied in the fluid analysis. To ensure a clear distinction between fractured and intact regions, the minimum value of the equivalent aperture is set to be ten times the aperture corresponding to the intact permeability, thereby guaranteeing that the permeability of the newly formed fractures is always greater than that of the intact matrix.

After the fluid computation has been completed, injection pressure p_{in} on Γ_{in} , pore fluid pressure p_m on Ω_m , and fracture fluid pressure p_f on Γ_f at the end of the fluid step are transferred to the mechanical calculation. Based on these pressures, nodal force $\mathbf{f}_{pre}$, arising from the hydraulic loads, is evaluated (Eqs. (22) to (27) in Takuma et al. (2024)). This force $\mathbf{f}_{pre}$ is expressed as the sum of the contributions from p_{in} , p_m , and p_f , denoted as $\mathbf{f}_{pre,in}$ [N], $\mathbf{f}_{pre,m}$ [N], and $\mathbf{f}_{pre,f}$ [N], respectively (Fig. 2).

$$\mathbf{f}_{pre} = \mathbf{f}_{pre,in} + \mathbf{f}_{pre,m} + \mathbf{f}_{pre,f} \quad (11)$$

The influence of the injection pressure p_{in} on Γ_{in} is evaluated by assuming a distributed load acting on each edge $i-j$ along Γ_{in} , as illustrated in Fig. 2 (1). The nodal forces $\mathbf{f}_{pre,in}$ acting on nodes i and j are calculated based on the following equation. This computation is performed for all edges along Γ_{in} .

$$\mathbf{f}_{pre,in} = \frac{p_{in}l}{2} \mathbf{n} \quad \text{in } \Gamma_i \quad (12)$$

where l [m] and $\mathbf{n}$ are the lengths and the outward unit normal vectors of the edge.

The influence of the pore pressure p_m on Ω_m is modeled as a rock expansion effect based on Biot's theory. As illustrated in Fig. 2 (2), for each edge $i-j$, the nodal forces $\mathbf{f}_{pre,m}$ acting on nodes i and j are computed

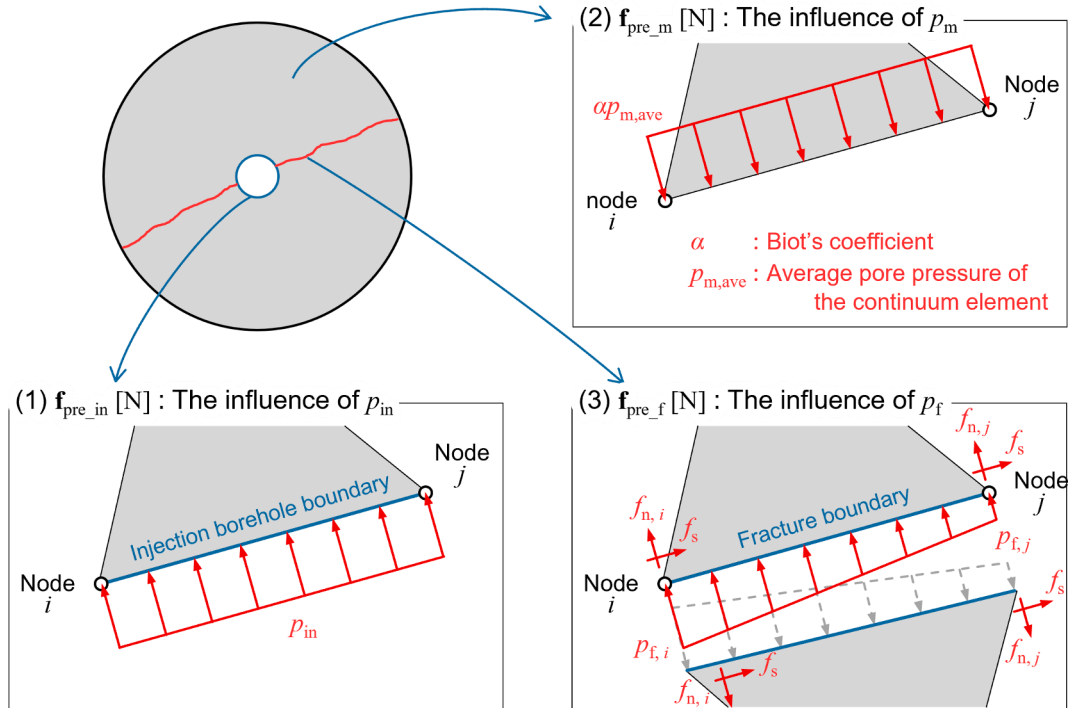


Fig. 2. Coupled Interactions from the hydraulic field (H) to the mechanical field (M): (1) Effect of injection pressure, (2) Effect of pore fluid pressure, (3) Effect of fracture fluid pressure.

using Eq. (13). This procedure is applied to all edges of all continuum elements.

$$\mathbf{f}_{\text{pre},m} = -\frac{\alpha p_{m,\text{ave}} l}{2} \mathbf{n} \quad \text{in } \Omega_m \quad (13)$$

where α [-] is Biot's coefficient; $p_{m,\text{ave}}$ [Pa] is calculated by the average of p_m in nodes comprising the continuum element.

The influence of the fracture pressure p_f on Γ_f is illustrated in Fig. 2 (3). Focusing on an edge i – j , a linear distribution of fluid pressure $p_{f,i}$ and $p_{f,j}$ is assumed. Based on this distribution, the nodal forces in the normal and tangential directions, f_n [N] and f_s [N], acting on the fracture are computed and then converted into the nodal force $\mathbf{f}_{\text{pre},f}$ (Liu et al., 2018; Munjiza et al., 1999).

$$f_{n,i} = \left(\frac{p_{f,i} l}{3} + \frac{p_{f,j} l}{6} \right) \quad \text{in } \Gamma_f \quad (14)$$

$$f_{n,j} = \left(\frac{p_{f,i} l}{6} + \frac{p_{f,j} l}{3} \right) \quad \text{in } \Gamma_f \quad (15)$$

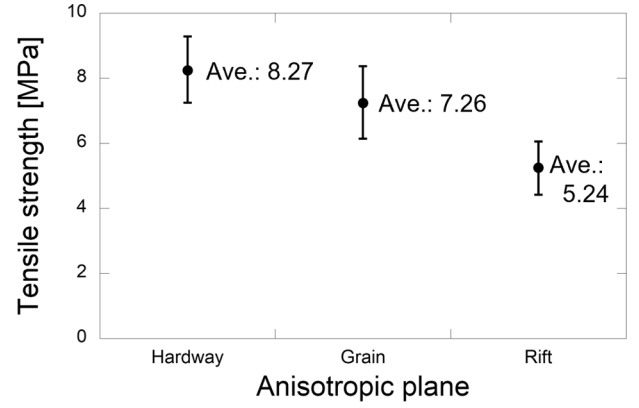
$$f_s = \frac{(p_{f,i} - p_{f,j}) b}{4} \quad \text{in } \Gamma_f \quad (16)$$

2.2. Experimental method

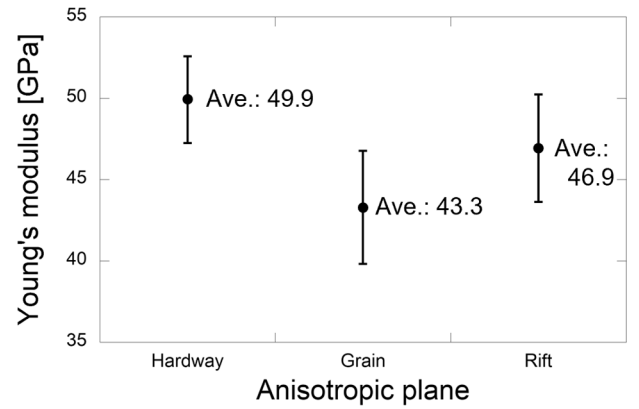
In this study, to perform simulations that incorporate mineralogical heterogeneity, reference was made to the results of the BTS tests and hydraulic fracturing tests conducted on specimens for which grain distributions had been characterized in advance. Prior to both experiments, the elemental composition on the specimen surfaces was quantified using an X-ray fluorescence (XRF) analyzer. Additionally, high-resolution surface images of the specimens were captured using a digital microscope before and after the tests, which were subsequently used for a crack pattern analysis. This section provides an overview of the BTS and hydraulic fracturing experiments, as well as the associated measurement and imaging procedures.

Before explaining the two experiments, the anisotropic mechanical properties of granite, which significantly affect the test results, will be described. Although granite is generally regarded as weakly anisotropic compared with strongly foliated rocks such as shale and slate, certain granites are known to exhibit pronounced mechanical anisotropy depending on their geological history, mineral fabric, and the preferential orientation of pre-existing microcracks (Bouchez, 1997). One such example is the Inada granite used in the series of experiments presented in this study, quarried from Kasama City, Ibaraki Prefecture, Japan, in which a well-developed set of orthotropic planes—Hardway, Grain, and Rift—has been reported. In this study, specimens were prepared with their orientations aligned to these three anisotropic planes to quantitatively evaluate their directional mechanical properties (Lin et al., 2008). In this study, specimens were prepared with their orientations aligned to these three anisotropic planes to quantitatively evaluate their directional mechanical properties, and BTS tests and UCS tests were conducted. The resulting tensile strength and Young's modulus are shown in Fig. 3 (a) and (b), respectively, while Table 1 summarizes the directional variations in tensile strength, Young's modulus, cohesion, and internal friction angle. These results confirm that Inada granite exhibits significant anisotropy in both strength and deformation characteristics.

Here, the tensile strength for each orientation (e.g., the Hardway plane) corresponds to the average value obtained from the BTS tests in which the fracture plane coincided with that orientation (e.g., the Hardway plane). Similarly, the Young's modulus for each orientation corresponds to the average value obtained from the UCS tests in which the loading axis was aligned with that orientation. The cohesion and internal friction angle values were determined by applying the following equations, derived from Mohr's stress circles based on the results of the



(a) Anisotropic tensile strength



(b) Anisotropic Young's modulus

Fig. 3. Experimental data on anisotropic mechanical properties: (a) tensile strength and (b) Young's modulus.

Table 1

Experimental data on anisotropic mechanical properties.

Property	Hardway plane	Grain plane	Rift plane
Tensile strength (MPa)	8.27	7.26	5.24
Young's modulus (GPa)	49.9	43.3	46.9
Cohesion (MPa)	20.0	17.5	14.8
Internal friction angle (degree)	62.6	62.4	67.6

BTS and UCS tests:

$$c = \frac{f_c f_t}{2\sqrt{f_t(f_c - 3f_t)}} \quad (17)$$

$$\phi = \tan^{-1} \left(\frac{f_c - 4f_t}{2\sqrt{f_t(f_c - 3f_t)}} \right) \quad (18)$$

where f_c [Pa] is the uniaxial compressive strength for each anisotropic orientation, obtained as the average value from the UCS tests in which the compression loading axis was aligned with that orientation, consistent with the definition of Young's modulus.

2.2.1. Overview of BTS tests

Cylindrical specimens with a diameter of 30 mm and a height of 30 mm were prepared for the BTS tests. In this study, the simulations targeted two experimental cases with different anisotropic conditions. Fig. 4 (a) and (b) illustrate the anisotropic configurations of these two

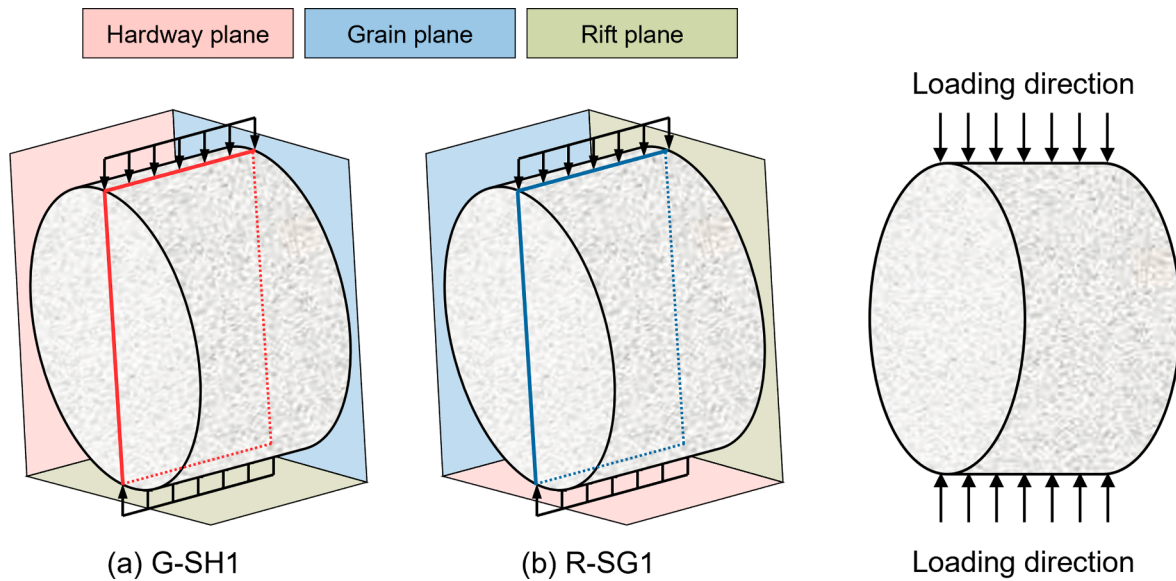


Fig. 4. Anisotropic configurations of BTS tests conducted and modeled in this study.

cases, where the specimen exhibiting a fracture surface aligned with the Hardway plane is referred to as G-SH1, and the one aligned with the Grain plane is referred to as R-SG1. Basic information on G-SH1 and R-SG1 is summarized in Table 2. The detailed experimental procedure is provided in Appendix A.

2.2.2. Overview of hydraulic fracturing tests

A cylindrical specimen with a diameter of 30 mm and a height of 60 mm was prepared for the hydraulic fracturing tests, and a blind borehole with a diameter of 4 mm and a depth of 40 mm was drilled at the center of the cylinder (Fig. 5 (a)). Fig. 5 (b) presents information on the anisotropy of the specimen targeted in the numerical simulation of this study. Fig. 5 (c) shows a photograph of the specimen. The specimen was cored such that the circular end face of the cylinder would align with the Grain plane and, within the end face, the Hardway and Rift planes would intersect. Basic information on the specimen dimensions is summarized in Table 3. The detailed experimental procedure is provided in Appendix B.

2.2.3. XRF-based elemental mapping, microscale surface observation, and quantitative fracture roughness analysis

To obtain the mineral distribution to be used in the numerical simulations, elemental mapping was conducted on the end surfaces of the cylindrical specimens prior to the BTS and hydraulic fracturing tests. This mapping was performed using the micro-XRF spectrometer (M4 TORNADO PLUS, manufactured by BRUKER) shown in Fig. 6 (a). The instrument irradiates the target surface with X-rays having a spot diameter of 20 μm and measures the emitted fluorescent X-rays to determine the elemental composition at each spot. The spatial resolution, defined by the pixel size of the measurement, was set to 45 μm . The measured elements were aluminum (Al), silicon (Si), potassium (K), iron (Fe), sodium (Na), and calcium (Ca), for a total of six elements. An example of the elemental mapping results is shown in Fig. 6 (b). In the figure, the brightness indicates the relative elemental concentration, with brighter regions corresponding to higher elemental contents. Based

on the obtained elemental concentrations, an RS-GBM was constructed for use in the simulations (See in Section 2.3).

To observe the generated fractures, high-resolution surface images were taken both before and after the BTS and hydraulic fracturing tests. These images were captured using a digital microscope (DVM6, manufactured by Leica). The captured images were used to extract information on fracture propagation paths, including fine fractures that were difficult to identify visually. Furthermore, the tortuosity of the fractures was quantitatively evaluated using the method described below, enabling a comparison between the experimental and numerical results. In this study, a quantitative index for fracture tortuosity Z_2 that is widely used in the field of rock mechanics was calculated (Barton, 1973; Barton and Choubey, 1977). It is calculated by placing measurement points at regular intervals along a fracture trace, and normalizing the coordinate differences between adjacent points by their spacing.

$$Z_2 = \left[\frac{1}{n-1} \sum \left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right)^2 \right]^{1/2} \quad (19)$$

where x [m] and y [m] are the coordinates of an observed point, and n is the total number of observed points. In this study, the sampling interval was set to 1.0 mm.

2.3. Modeling procedure for GBM

This section describes the procedure for constructing an RS-GBM, in which grain boundaries coincide with mesh boundaries. Among the six elemental distributions obtained by the XRF analysis, four elements—Al, Si, K, and Fe—were used to generate image data representing the mineral distribution. Based on this image data, the geometry and mesh of the numerical model were constructed.

2.3.1. Creation of mineral distribution image

In this study, a mineral distribution map was created for granite based on the elemental distributions of Al, Si, K, and Fe, following the method proposed by Ueki and Niwa (2017). This method classifies mineral phases by utilizing the differences in their elemental compositions. The mineral phases considered here are quartz, K-feldspar, plagioclase, and biotite. Their typical chemical compositions are as follows: quartz – SiO_2 ; K-feldspar – KAlSi_3O_8 ; plagioclase – $\text{NaAlSi}_3\text{O}_8$ or $\text{CaAl}_2\text{Si}_2\text{O}_8$; and biotite – $\text{K}(\text{Mg,Fe})_3(\text{AlSi}_3\text{O}_{10})(\text{OH})_2$. While Si is present in all these minerals, regions with high Si contents are identified as

Table 2

Basic dimensions of specimen used in Brazilian tensile tests.

Case	Diameter [mm]	Height [mm]	Density [g/cm^3]
G-SH1	29.76	29.67	2.62
R-SG1	29.76	30.23	2.62

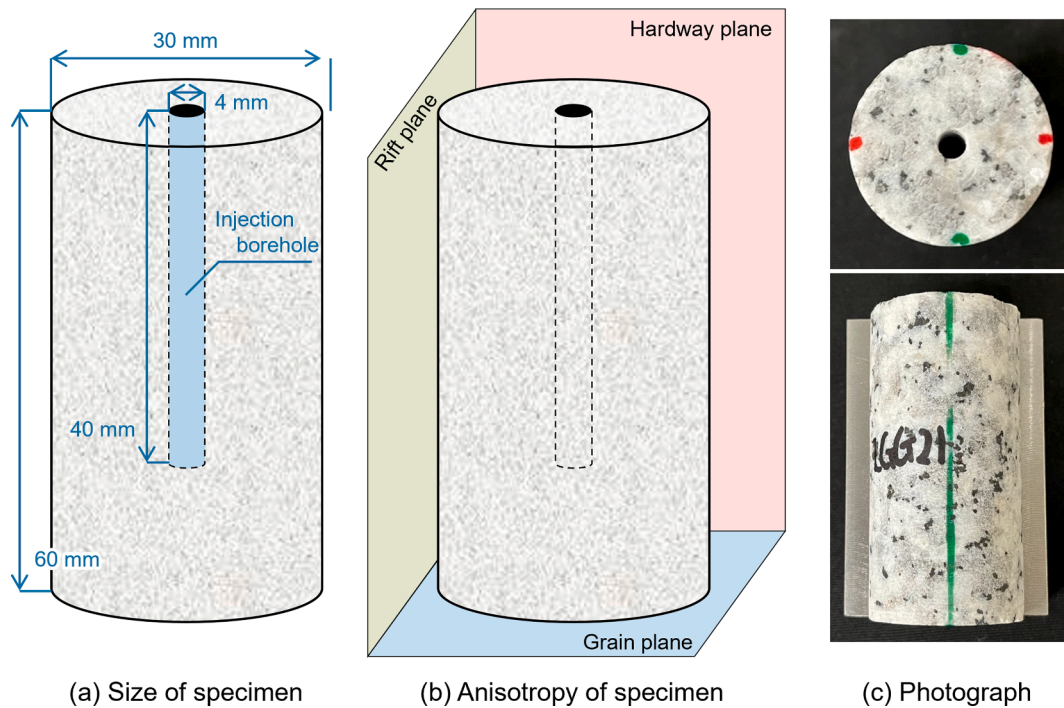


Fig. 5. Overview of specimen used in hydraulic fracturing tests.

Table 3

Basic dimensions of specimen used in hydraulic fracturing tests.

Diameter [mm]	Height [mm]	Borehole diameter [mm]	Borehole depth [mm]
29.78	60.33	4.02	39.56

quartz. Al is contained in K-feldspar, plagioclase, and biotite; thus, areas with Al are classified as one of these three minerals. Among them, regions containing Fe are identified as biotite. If Fe is absent, but K is present, the mineral is classified as K-feldspar. If both Fe and K are absent, the mineral is classified as plagioclase. Therefore, by superimposing the elemental distribution maps in order of Al, K, and Fe, the three minerals excluding quartz can be identified. Since regions with high Si contents rarely overlap those containing Al, K, or Fe, the order in which the Si map is overlaid does not significantly affect the classification results. In this study, following the order described in the reference, the mineral distribution map was generated by superimposing the elemental maps of Al, Si, K, and Fe in sequence (Fig. 7). The elemental distribution maps used in the above procedure were obtained by XRF analysis, and correspond to the concentration distributions of Al, Si, K, and Fe. Since the raw data may include signals from pixels where the target element is not actually present, each elemental map was binarized using Otsu's method (Otsu, 1979) to distinguish regions that contain the

target element from those that do not. It should be noted that Otsu's method is an automatic thresholding algorithm that determines an optimal threshold value by minimizing the intra-class variance of the grayscale histogram. In the case of Si, the threshold was manually adjusted to distinguish regions with significantly high concentrations from others. Fig. 7 shows the binarized images of each elemental distribution in the upper row, and the sequential process of superposition from left to right in the lower row, with the bottom-right image representing the final mineral distribution map. In these images, red indicates quartz, green indicates K-feldspar, blue indicates plagioclase, and black indicates biotite.

The final mineral distribution map may contain unwanted features (e.g., stray dots) arising from various factors, such as noise during the XRF analysis and variations in mineral colors (Zhang et al., 2019). Such artifacts are typically corrected through the application of computational image-processing algorithms. In this study, a median filter method (Gonzalez and Woods, 2012) was employed, in which the value of each pixel was replaced by the median of its neighboring pixels, thereby effectively reducing noise while preserving the essential boundaries between mineral grains (Zhang et al., 2019).

2.3.2. Numerical model construction: boundary extraction and vectorization from pixel-based data

When creating numerical models, it is common to firstly define the

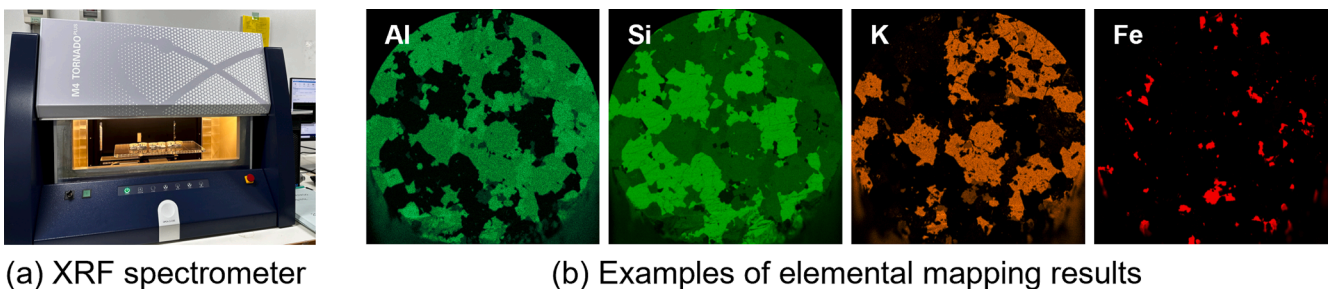


Fig. 6. Overview of XRF spectrometer: (a) photograph of XRF spectrometer and (b) examples of elemental analysis results obtained by XRF analysis.

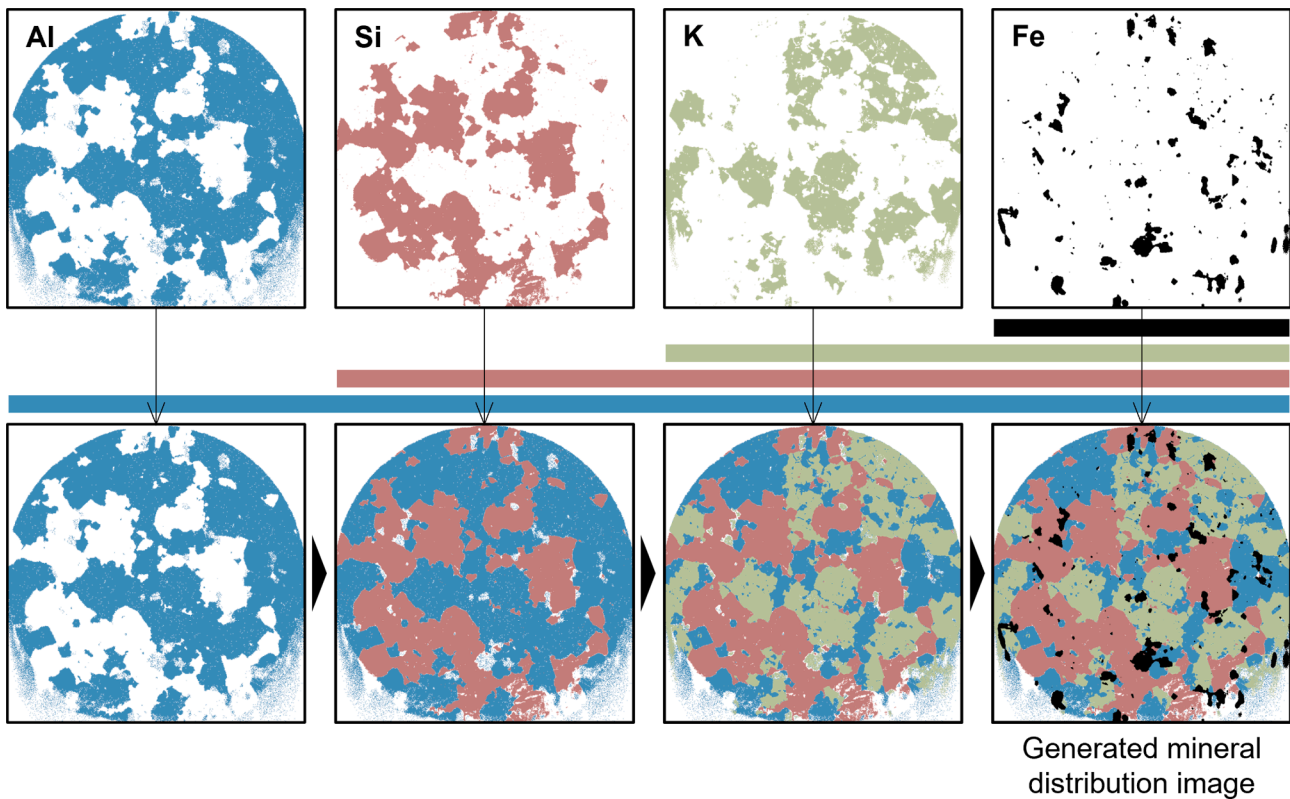


Fig. 7. Procedure for generating mineral distribution. The upper row shows the binarized images of each elemental distribution. The lower row illustrates the process of superposition from left to right, with the bottom-right image representing the final mineral distribution. In the mineral image, red indicates quartz, green indicates K-feldspar, blue indicates plagioclase, and black indicates biotite.

target components, such as the rock specimen and loading plates, as geometries. Then, a mesh generation is performed for each geometry. To explicitly incorporate grain boundaries into the simulation model, those boundaries must be defined on the geometry in advance. The mineral distribution image is a bitmap consisting of pixels. If grain boundaries are directly mapped onto the geometry from the pixel-based image, those boundaries that run diagonally will appear as stair-step patterns. This leads to unnatural stress concentrations in mechanical simulations. These stress concentrations are not due to the actual grain shapes. Rather, they result from the fact that the elemental distribution data used to construct the mineral map have been structured on a regular grid. To mitigate this effect, smoothing of the boundaries is necessary. As shown in Fig. 8 (a), a smoothing approach was introduced in this study in which the configuration of the surrounding minerals is evaluated at each central grid point shared by four adjacent pixels. Among the eight candidate grain boundaries (indicated as red lines in the figure), appropriate segments were selected to reconstruct smooth boundaries. Examples of mineral patterns formed by the four pixels and the applied boundary segments for each pattern are shown in Fig. 8 (b). It is noted that, for mineral patterns that are equivalent by rotation or reflection, only one representative pattern is illustrated. In Examples 3 and 4, the assignment of minerals to areas with increased or decreased coverage is determined based on a pre-defined priority order of minerals. The mineral with the highest priority is assigned to the expanding region. In this study, the priority order was set as quartz, K-feldspar, plagioclase, and biotite. By applying this procedure to all combinations of four pixels, a mineral distribution that is both suitable for numerical simulation and consistent with actual grain morphology could be reasonably reconstructed.

In this study, to incorporate the anisotropy of granite into the numerical analysis, a grid-based layer structure simulating anisotropic planes was additionally constructed on the geometry that includes

smoothed grain boundaries (Fig. 8 (c)). The layer spacing was set to three pixels, so that as many layers as possible could be included without overly densifying the mesh. In the analysis, anisotropy was introduced using this layered structure, where the direction along the Hardway plane was set as the reference direction. In contrast, the tensile strength, Young's modulus, and cohesion were reduced along the Grain and Rift planes. To incorporate anisotropy for each mineral, the assigned values were treated as the reference material properties in the direction of the Hardway plane. Anisotropic behavior was represented by applying reduction ratios, derived from Table 1, to the properties in the directions of the Grain and Rift planes. For example, the tensile strength in the Rift direction was reduced by multiplying it by 5.24/8.27. As for Young's modulus, the components along the Grain and Rift directions were reduced. For tensile strength and cohesion, the reduction was applied to the mesh boundaries oriented within $\pm 5^\circ$ of the Grain or Rift directions.

2.4. Numerical setup

This section describes the simulation conditions, including the numerical model incorporating the generated grain distribution, the mesh and material properties, and the loading conditions for the BTS tests as well as the injection conditions for the hydraulic fracturing tests.

2.4.1. Simulation conditions for the BTS tests

Fig. 9 (a) and (b) show the RS-GBMs for the BTS test specimens, (a) G-SH1 and (b) R-SG1, which were constructed following the procedure described in Section 2.3. From left to right, the figures display microscopic images of the rock specimen, corresponding mineral distribution, constructed numerical model, and generated finite element mesh. As shown in the numerical model, loading plates were placed at the top and bottom of the rock specimen. The mesh was composed of unstructured triangular elements, with the mesh size determined based on the

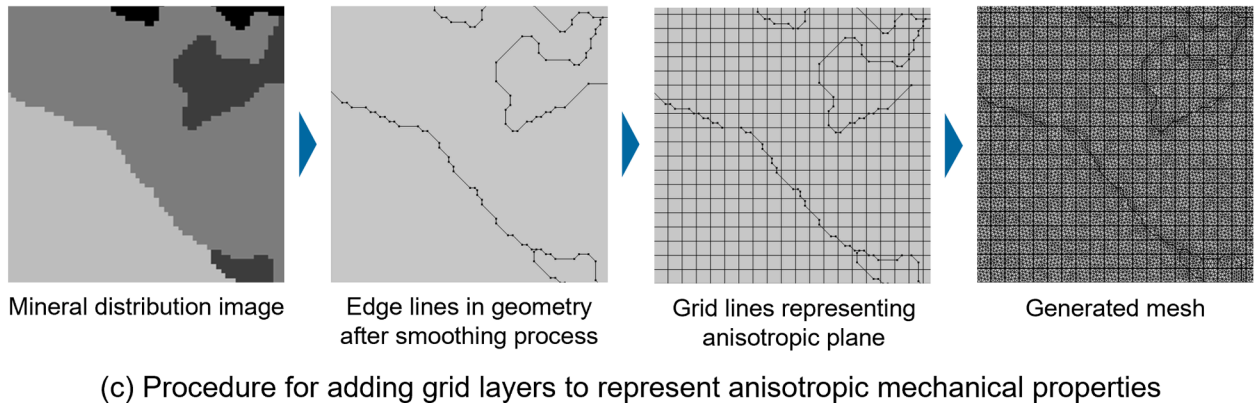
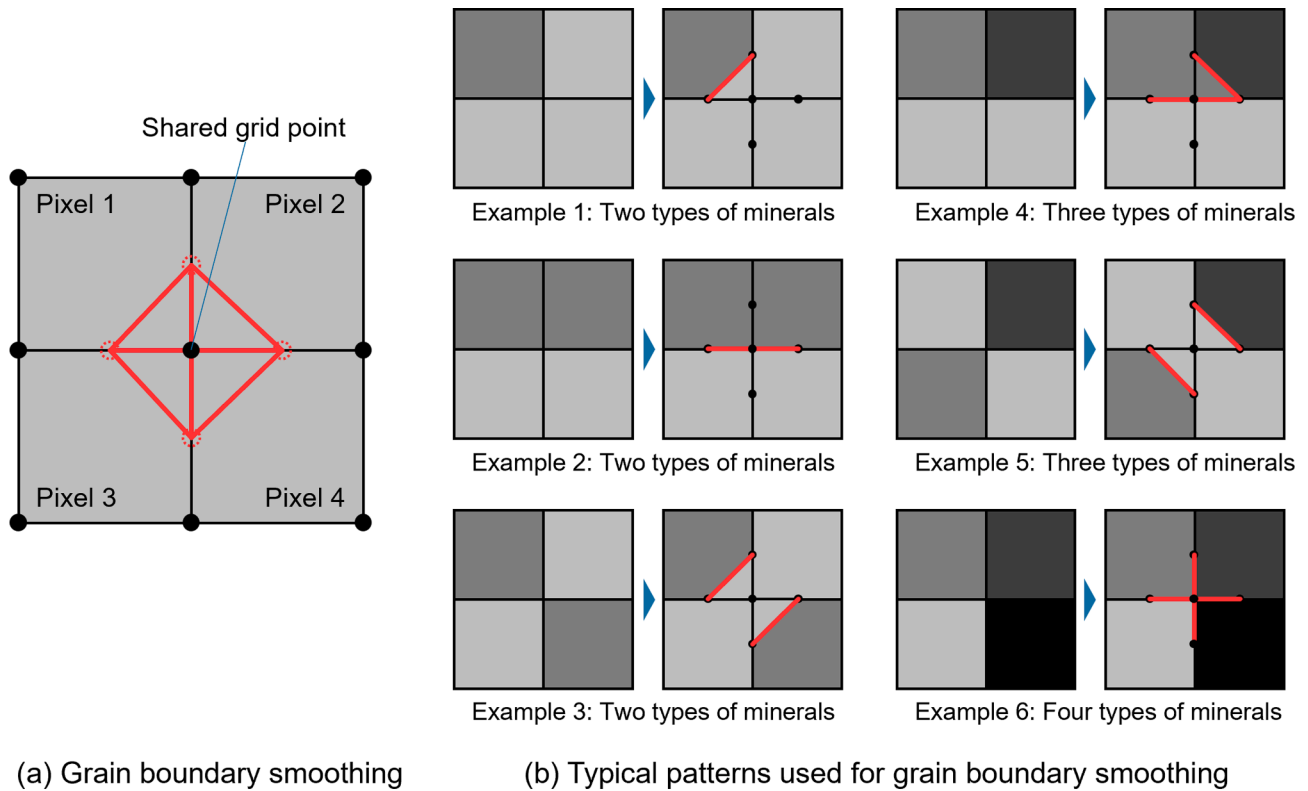


Fig. 8. Approach for boundary extraction and vectorization from pixel-based data: (a) grain boundary smoothing, (b) typical patterns used for grain boundary smoothing, and (c) procedure for adding grid layers to represent anisotropic mechanical properties.

resolution of the XRF analysis, approximately 45 μm . The minimum, maximum, and average element sizes were 13, 59, and 35 μm for G-SH1, and 13, 60, and 35 μm for R-SG1, respectively. The total number of nodes was 690,006 for G-SH1 and 690,028 for R-SG1, while the total number of triangular elements was 1,377,221 for G-SH1 and 1,377,944 for R-SG1. The RS-GBMs were composed of four minerals—quartz, K-feldspar, plagioclase, and biotite—based on granite. The volume fractions of each mineral were 38 %, 29 %, 27 %, and 6 % for G-SH1, and 30 %, 30 %, 34 %, and 6 % for R-SG1, respectively. The mechanical properties assigned to each mineral and mineral interface (i.e., the boundary between different mineral phases) are listed in Table 4, based on Xu et al. (2023). Although mineral interface properties may differ depending on the mineral pairing, experimentally determining such interface-specific parameters is extremely difficult, and the associated calibration problem is highly non-unique, as also noted by Aboyanah et al. (2022, 2024). Therefore, this study adopts a simplified assumption in which all interfaces share the same parameters, allowing us to focus

on reproducing the macroscopic mechanical behavior rather than resolving the full variability of grain-scale interfaces. In Xu et al. (2023), feldspar properties were presented as a combined value for both K-feldspar and plagioclase; hence, the same property values were applied to both minerals in this study. While the mechanical behavior of the two feldspars was thus treated identically, the model explicitly represented the boundary between K-feldspar and plagioclase. The cohesion and internal friction angle were adopted from the experimentally obtained values reported by Xu et al. (2023). Furthermore, the Young's modulus and tensile strength values were calibrated through numerical fitting using the BTS test simulation of specimen G-SH1 (described in Section 4.1). In this calibration, a uniform scaling factor was applied to all mineral and interface properties without altering the relative ratios between them, as reported in the literature. It should be noted that these microscopic parameters do not necessarily coincide with the bulk-scale properties because the macroscopic mechanical response of the rock is strongly influenced by heterogeneous mineral distributions, grain-

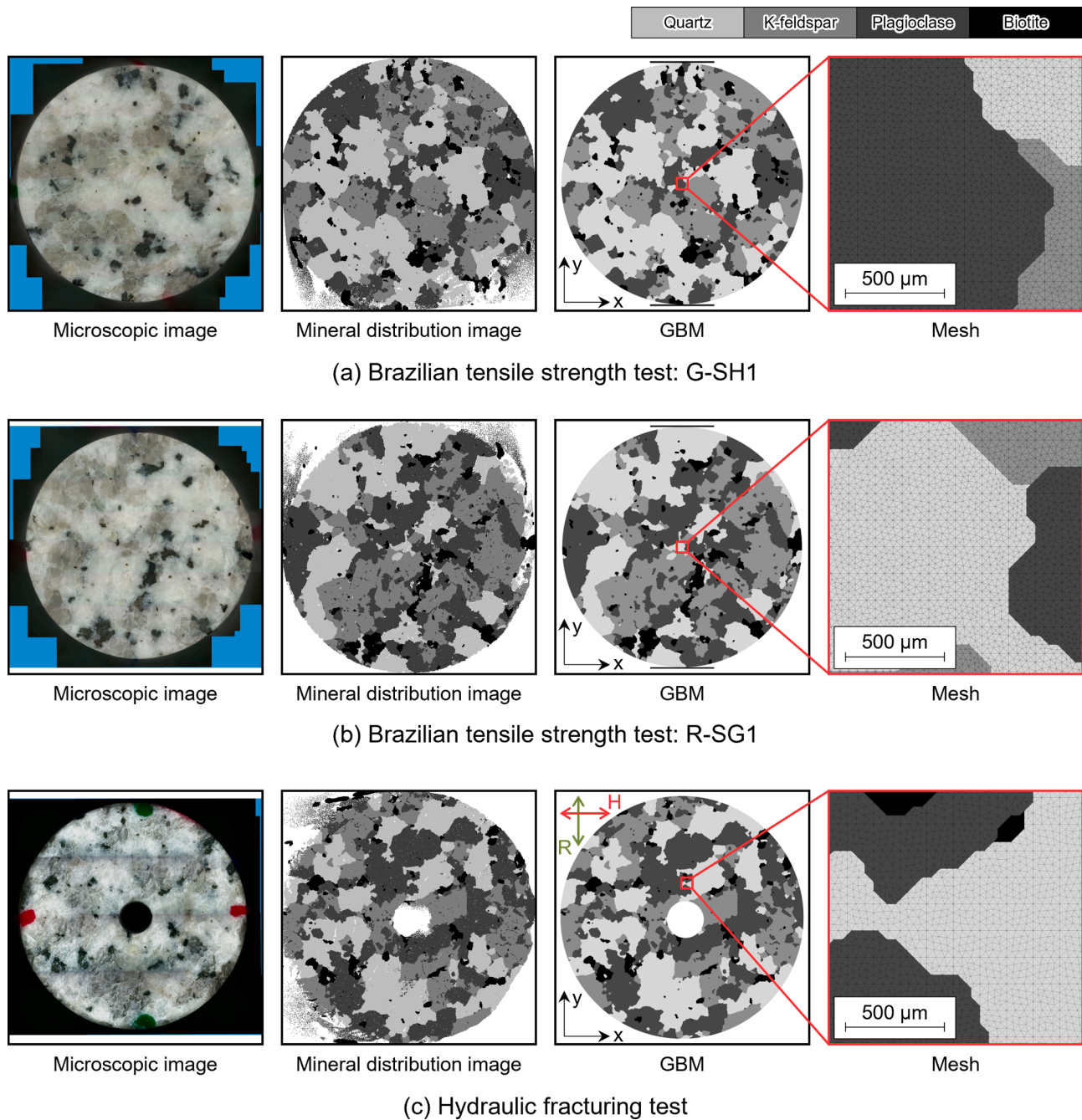


Fig. 9. Illustration of grain-based model: (a) BTS test specimen G-SH1, (b) BTS test specimen R-SG1, and (c) hydraulic fracturing test specimen. From left to right: microscopic images of the rock specimen, corresponding mineral distribution, constructed numerical model, and generated finite element mesh.

Table 4

Input parameters assigned to each mineral grain.

Property	Quartz	K-feldspar	Plagioclase	Biotite	Interface
Density (kg/m^3)	2650	2600	2600	2850	
Young's modulus (GPa)	12.1	8.69	8.69	5.80	
Poisson's ratio (—)	0.08	0.27	0.27	0.25	
Tensile strength (MPa)	56.4	39.6	39.6	30.0	12.0
Cohesion (MPa)	1891	606	606	16	40
Internal friction angle (degree)	9	21	21	45	50

boundary compliance, and microdefects, which cannot be represented by the intrinsic properties of individual mineral phases alone.

In this analysis, the loading conditions replicated those of the experiment: the lower loading plate was fixed in displacement, while the upper loading plate was displaced downward. The loading plates were modeled as rigid bodies, and thus, did not deform. The downward velocity of the upper plate was set to 0.05 m/s, which corresponds to a strain rate of approximately 1.7 [1/s]. Although this loading rate is higher than that used in the experiment (0.0005 mm/s), viscous damping was introduced to simulate the quasi-static BTS testing conditions in Mahabadi et al. (2014).

The loading rate was selected based on preliminary studies that confirmed it to be sufficiently low. It was also lower than the strain rates recommended in previous studies, such as 6.7 [1/s] (loading velocity:

0.25 m/s and specimen diameter: 37.37 mm) in 2D BTS simulations by Mahabadi et al. (2014), and 5.0 [1/s] (loading velocity: 0.2 m/s and specimen diameter: 40 mm) in 3D BTS simulations by Mohammadnejad et al. (2020). The time step for the simulation was set to 4.0×10^{-10} [s/step], which was also determined through preliminary tests to ensure sufficient resolution. The simulation results were visualized every 10,000 steps.

2.4.2. Simulation conditions for hydraulic fracturing tests

Fig. 9 (c) shows the RS-GBM for the hydraulic fracturing tests, created following the procedure described in Section 2.3. From left to right, the figure presents the microscopic image, mineral distribution image, numerical model, and mesh. Regarding anisotropy, the Hardway plane is aligned with the x-axis, while the Rift plane is aligned with the y-axis in the figure. The mesh was generated with a target element size of 45 μm , matching the resolution of the XRF analysis. The minimum, maximum, and average element sizes were 15, 65, and 39 μm , respectively. The total number of nodes was 522,029, and the number of triangular elements was 1,041,524. The RS-GBM consisted of four mineral types commonly found in granite—quartz, K-feldspar, plagioclase, and biotite—with area fractions of 33 %, 37 %, 25 %, and 5 %, respectively. The material properties assigned to each mineral and mineral interface were identical to those used in the BTS tests and are listed in Table 4. These values were determined through calibration based on the BTS simulation of specimen G-SH1.

A confining pressure of 7 MPa was applied to the outer boundaries, consistent with the experimental conditions. The confining pressure was imposed as a constant normal stress and maintained throughout the entire simulation period to reproduce the continuous confinement used in the laboratory tests. The outer boundary condition was set as undrained, while drainage was permitted at locations where the propagating fracture intersected the boundary. At the injection borehole, fluid was injected according to the experimentally observed pressure ramp rate, with the injection pressure defined as $p_{\text{in}} = 0.5 \text{ [MPa/s]} \times t \text{ [s]}$. Global time step Δt , for alternating fluid and mechanical calculations, was set to 1.0 s. For the mechanical calculations, the internal time step was set to 1.0×10^{-10} [s/step] to ensure numerical stability.

3. Verification of numerical simulator

In this study, three types of benchmark analyses—steady-state and time-dependent, and fracture-aperture validation—were conducted to verify the fundamental performance of the developed simulator for hydraulic-mechanical coupling, assuming hydraulic fracturing conditions. The numerical solutions obtained from the simulations were compared with the analytical solutions, and are summarized in Appendix C and Appendix D. Through these three benchmark analyses, it was confirmed that the simulator could accurately capture the hydraulic loading effects on both the stress and deformation responses of the rock matrix.

Furthermore, through practical hydraulic fracturing simulations, the rock matrix stresses obtained through the use of ECZM and ICZM were compared to demonstrate the superiority of ECZM. In all the analyses presented in this section, the strength parameters were intentionally set to high values to prevent fracture initiation, thereby isolating the influence of the fracture process on the surrounding stress field.

The stress distribution in the intact rock during hydraulic fracturing simulations with RS-GBMs was compared between ICZM and ECZM. Unlike ECZM, in which cohesive elements are activated only at the mesh interfaces for which the failure indicator reaches a value of 1 during the simulation, ICZM embeds and activates cohesive elements throughout the entire domain from the start of the analysis. To represent a pseudo-elastic response, the stiffness of the cohesive elements (cohesive penalty) was set to be 100 times the Young's modulus, following the approach of Mohammadnejad et al. (2020). A previous study by Maeda et al. (2024b) demonstrated that, in UCS simulations of homogeneous rock, ICZM

produces artificially heterogeneous stress fields even in the intact state, whereas ECZM yields smooth and continuous stress distributions. As of yet, however, no studies have examined these effects under conditions which incorporate mineralogical heterogeneity or in the context of hydraulic fracturing. In this study, the extent to which ICZM-induced stress heterogeneity affects the stress distribution in GBMs for hydraulic fracturing was investigated. It is shown that ECZM appropriately captured the intrinsic heterogeneity of the stress field arising from mineralogical variations.

The simulation conditions were essentially identical to those described in Section 2.4, which outlined the hydraulic fracturing analysis conducted in this study. The only difference was that, to isolate the influence of mineral-induced stress heterogeneity from that caused by fracture propagation, the material strength was set to an extremely high level to ensure that no fractures would occur. Fig. 10 presents the distributions of the maximum principal tensile stress at 30 s (corresponding to $p_{\text{in}} = 15 \text{ MPa}$), along with the mineral map, for both ICZM and ECZM. At this time point, no cohesive elements have been activated in ECZM, all cohesive elements remain undamaged ($D < 0$), and the system continues in an intact state with no fracture initiation. The results show that ICZM yields a highly heterogeneous stress distribution, whereas ECZM produces a relatively smooth and continuous field. In the following discussion, these stress distributions are compared with the underlying mineral map to examine the differences between ICZM and ECZM in representing the intact stress state.

Fig. 11 shows plots of the maximum principal tensile stress and principal strain along the observation lines defined in Fig. 10. Two observation lines were set in the vertical and horizontal directions. In the graphs, the results from ICZM and ECZM are represented by black and red lines, respectively, with the background color indicating the mineral type at each position. The results of both ICZM and ECZM reveal stress contrasts among the different minerals. In particular, higher tensile stresses are observed within quartz and biotite compared to feldspars. Quartz exhibits the highest Young's modulus among the minerals, and thus, develops the greatest stress under comparable strain conditions. Although biotite has the lowest Young's modulus, it is surrounded by stiffer minerals, leading to constraint-induced deformation. This effect is also evident in the principal strain distribution, where biotite shows significantly more compressive (contractive) behavior than the surrounding regions. Such constrained deformation is considered to be the primary cause of the tensile stress concentration within the biotite grains. In the ECZM results, the stress distribution within each individual mineral is continuous. It is noted that K-feldspar and plagioclase are assigned to identical material properties in the model; and thus, they behave as a single mineral phase with continuous stress and strain responses. In contrast, the ICZM results exhibit oscillations in the distributions of both stress and strain even within the same mineral phase in the intact (no fracture) state. These artifacts arise because, in ICZM, cohesive elements are embedded throughout the domain from the beginning of the analysis, acting like elastic springs and connecting adjacent finite elements. Their finite stiffness causes them to bear part of the load and deform even prior to fracture initiation and subsequent damage. Notably, in some regions, these artificial fluctuations within a single mineral phase are even larger than the actual stress contrasts between different minerals. These observations demonstrate that ECZM is more suitable for simulations involving GBMs, where capturing stress-strain behavior and fracture initiation based on material heterogeneity is essential. It should be noted that the heterogeneous stress distribution within a single mineral observed in the ICZM model results from the introduction of cohesive interfaces in the intact state. Such heterogeneity is not physically intended. If the microstructural weaknesses within minerals—such as microcracks, sub-grain boundaries, or other latent weak surfaces—are to be represented, it should be modeled explicitly rather than imposed uniformly across the entire domain. Moreover, while ICZM can implicitly express these latent weak surfaces, assigning cohesive interfaces to all potential planes may introduce

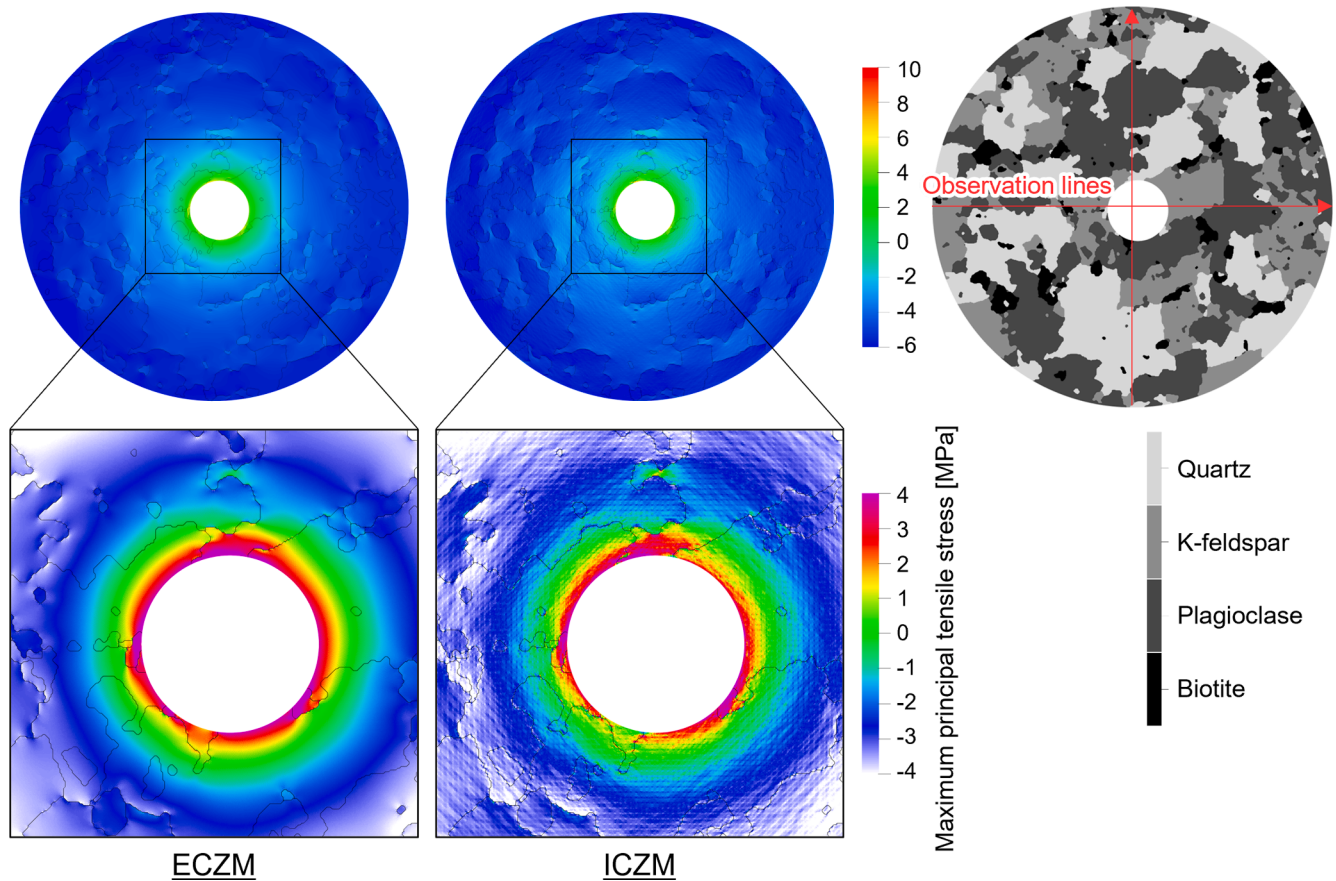


Fig. 10. Comparison of maximum principal tensile stress distributions obtained using ECZM (left) and ICZM (center) under the same mineralogical structure. The grain distribution used in the simulations is shown on the right, with observation lines indicated.

artificial stress perturbations and does not reflect the actual spatial distribution of microdefects in rocks. A more appropriate strategy would be an ECZM-based framework in which latent weak surfaces are selectively introduced only where necessary. Although this hybrid approach is not implemented in the present study, we consider it a promising direction for future development.

In addition to comparing stress distributions, the computational performances of ICZM and ECZM were also evaluated. In the aforementioned analyses, the elapsed computation times from the start of the simulation to the output of the results, shown in Figs. 10 and 11, were 12,073 s for ICZM and 9,391 s for ECZM, indicating that ECZM is approximately 1.29 times faster than ICZM. This difference arises because, in ICZM, cohesive elements are activated across the entire domain from the beginning of the simulation, necessitating additional computations throughout the whole region, and thus, increasing the total computation time. Furthermore, Maeda et al. (2024b) reported that, in the UCS tests under intact conditions prior to fracture initiation, ECZM allows for stable time steps up to three times larger than those used in ICZM. Similarly, in the present analysis, further computational efficiency could be achieved with ECZM by employing larger time steps. These findings collectively demonstrate that, from the perspective of computational cost, ECZM is more advantageous than ICZM.

4. Results of experiments and numerical simulation

4.1. Results of Brazilian tensile strength tests

In this section, the experimental and numerical results of the BTS tests are presented. Two experimental cases are considered: one in which the fracture plane coincides with the Hardway plane (G-SH1) and

the other in which it coincides with the Grain plane (R-SG1). To incorporate anisotropy, the material properties along the Hardway plane were treated as the reference values, whereas those along the other planes (the Grain plane and the Rift plane) were reduced in terms of Young's modulus and strength. Accordingly, the calibration of the mineral and mineral interface properties in this study was performed using the first case, namely, the case with the fracture plane aligned with the Hardway plane (G-SH1). The calibration was conducted with respect to two parameters obtained from the stress–strain curve of the BTS tests: the tensile strength and the slope. For the slope, the comparison was made at the stress corresponding to 50 % of the tensile strength. In the experiments, stress and strain were calculated from the applied load and displacement recorded by the servo-controlled testing machine. Here, stress refers to the indirectly obtained tensile stress, and strain denotes the axial strain in the loading direction. Fig. 12 presents the stress–strain curves obtained from both the experiments and the simulations. It should be noted that the strain measured in the experiments included not only the deformation of the rock specimen, but also the strain of the servo-controlled testing machine and the slack of the loading plates prior to loading. In contrast, the simulations provided the strain of the rock specimen alone. In the stress–strain plots, the strain axis is displayed with two scales so that the strains corresponding to 50 % of the tensile strength in the experimental results and those in the numerical results are equal, thereby facilitating a direct comparison. The tensile strength and slope obtained from the stress–strain curves of the BTS tests were 8.97 MPa and 13.44 MPa in the experiment, and 8.83 MPa and 13.20 MPa in the simulation, respectively. The relative errors with respect to the experimental results were -1.6 % for the tensile strength and -1.8 % for the slope, confirming that the numerical results accurately reproduced the experimental observations.

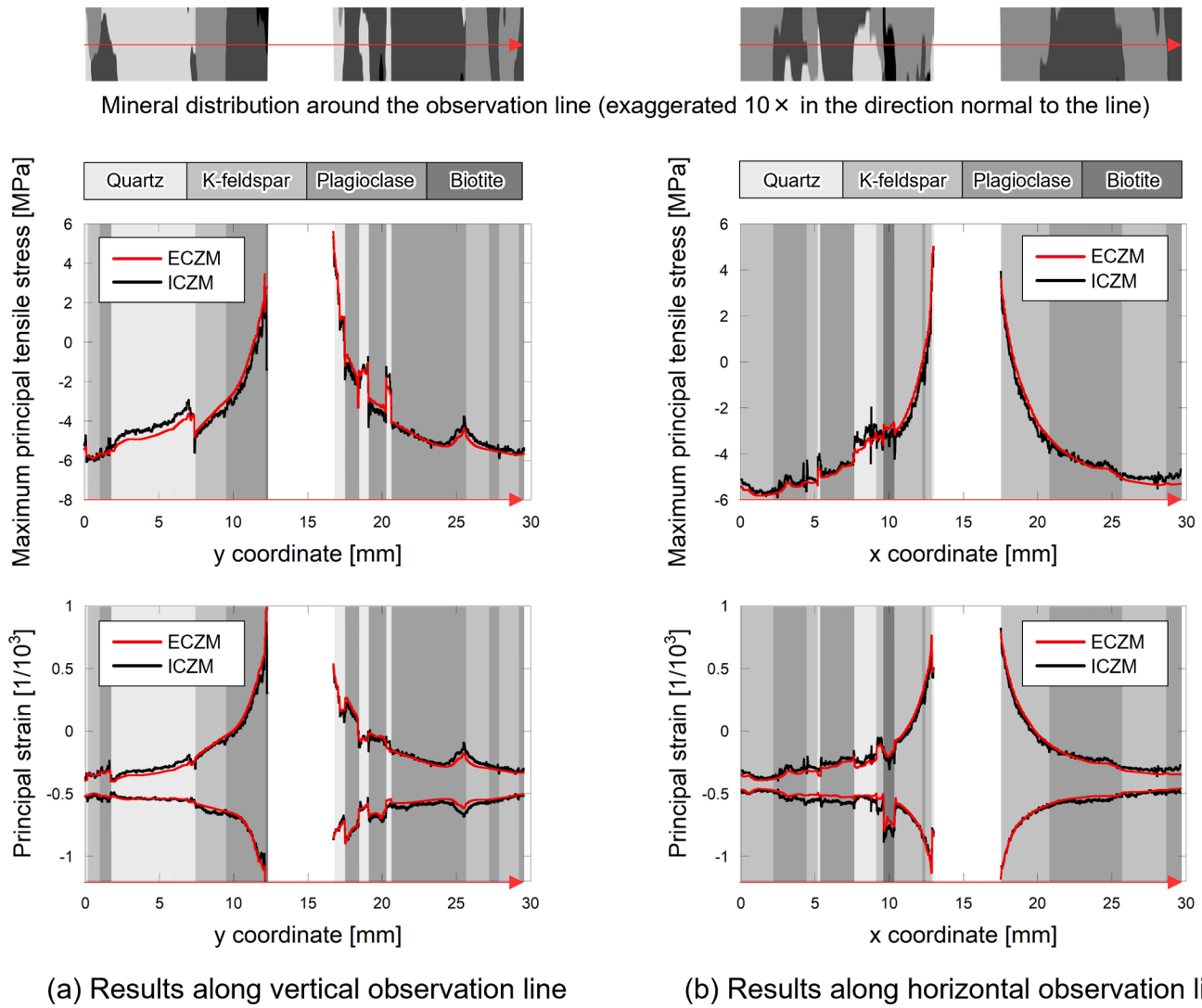


Fig. 11. Profiles of maximum principal tensile stress and principal strain along observation lines shown in previous figure: (a) vertical observation line and (b) horizontal observation line. The stress and strain distributions obtained by ECZM and ICZM are compared, with background shading indicating mineral phases.

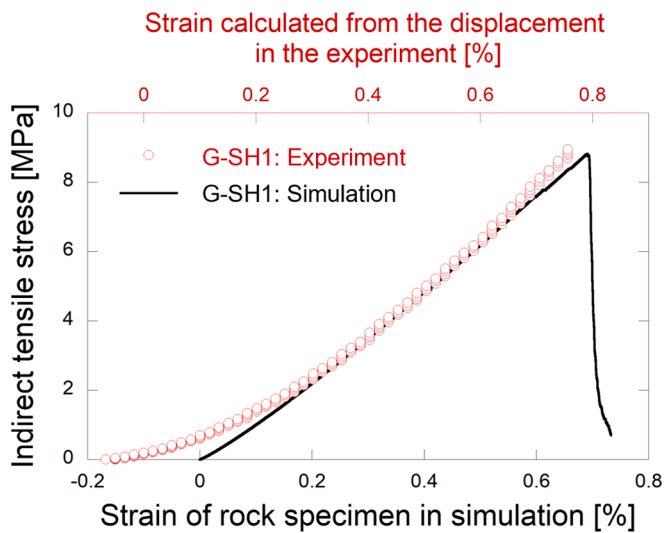


Fig. 12. Stress-strain curves obtained from experiment and numerical simulation for G-SH1 case.

Fig. 13 shows the experimental results for the G-SH1 case. On the left, a post-fracture microscopic image after the test is shown, where the fracture trace identified from the image is highlighted in red. On the right, enlarged microscopic images before and after the test, together with the corresponding mineral distribution map, illustrate representative fracture regions where IG and TG cracks are observed. In the BTS tests, fractures generally propagated along the line connecting the upper and lower loading points. However, **Fig. 13** shows that, in G-SH1, a tortuous fracture path developed. **Fig. 14** displays the fractures observed in the experiment superimposed on the mineral distribution map (**Fig. 14** (a)), as well as the numerical results (**Fig. 14** (b)), which include the spatial distribution of the damage coefficient ($D > 0.1$) and the fracture mode. Fracture modes were classified as tensile (blue) when the tensile failure indicator (Eq. (2)) reached a value of 1, and as shear (red) when the shear failure indicator (Eq. (3)) reached a value of 1. The numerical results demonstrated the occurrence of both TG and IG cracks, resulting in a tortuous fracture path. Upon detailed observation, the fracture path appears particularly tortuous in the case of the TG cracks within the biotite generated by shear failure and the IG cracks. As for the fracture mode, the majority of the fractures were tensile, whereas shear failure was dominant within the biotite. With respect to fracture orientation, all the fractures aligned with the loading direction were tensile, whereas the IG cracks oblique to the loading axis also exhibited

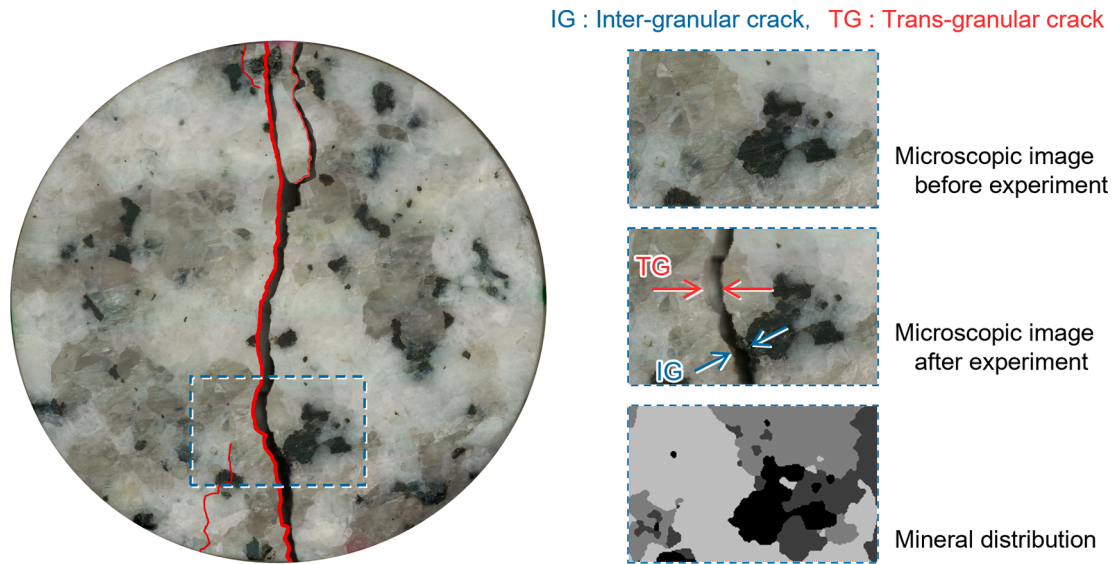
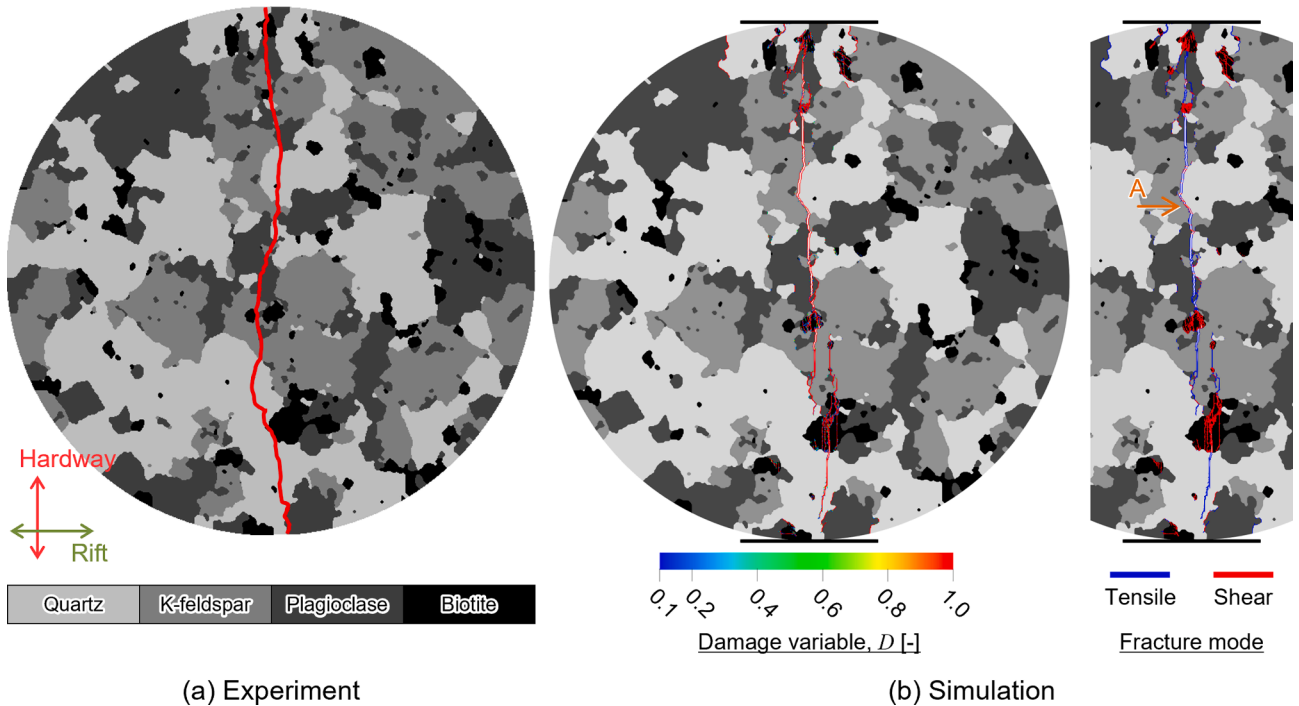


Fig. 13. Experimental observations for G-SH1 specimen.

Fig. 14. Comparison of experimental and numerical fracture paths for G-SH1 specimen: (a) mineral distribution image superimposed with experimentally observed fractures, and (b) spatial distribution of damage variable ($D > 0.1$) and fracture mode obtained from numerical simulation.

localized shear failure (Fig. 14 (b)-A).

Fig. 15 shows the fracture propagation process obtained from the numerical simulation. Up to $\varepsilon = 0.6840$ %, shear failure occurred within the biotite near the loading plate (labeled A in the figure). The initiation of macroscopic fracture propagation leading to specimen failure originated from the occurrence of a tensile fracture at the quartz–K-feldspar boundary at $\varepsilon = 0.6907$ % (labeled B). Following the fracture initiation at B, the fracture propagated upward and downward along the grain boundaries. In the downward direction, the fracture once penetrated through a K-feldspar grain (labeled C) and then extended through multiple weak planes, such as the quartz–plagioclase boundary, the K-feldspar–plagioclase boundary (labeled F), and the biotite grains. When no further grain boundaries existed ahead of the fracture path, the

fracture propagated intragranularly (labeled G). Upon encountering quartz, which possesses relatively high strength, the fracture selectively followed the grain boundary (labeled H). As shown by I, when no favorable boundaries or weak paths were available in the direction of the loading plate, the fracture traversed the quartz grain. In the upward direction from the initiation point B, the fracture passed through a K-feldspar grain (labeled D). The subsequently encountered plagioclase grain was oriented with its boundary perpendicular to the fracture propagation direction, causing the fracture to penetrate intragranularly rather than along the boundary. The following quartz grain was bypassed, and the fracture extended toward the specimen edge along the biotite and grain boundaries.

At $\varepsilon = 0.6967$ %, the evaluation range for fracture roughness is

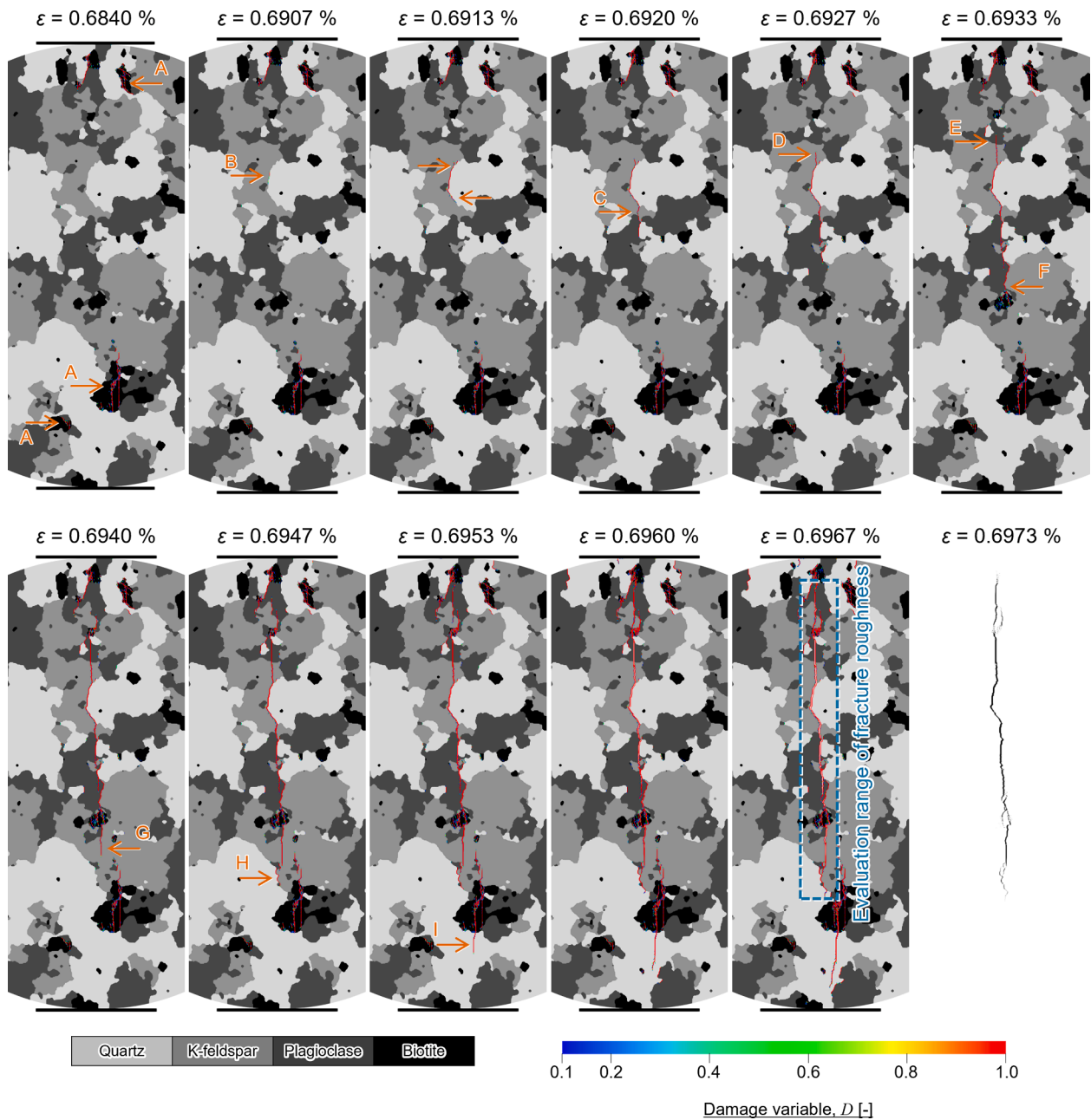


Fig. 15. Fracture propagation process for G-SH1 specimen obtained from numerical simulation.

illustrated, focusing on a continuous fracture within the numerical results. The timing of this evaluation is defined at $\varepsilon = 0.6973$ %, corresponding to the point when the stress has decreased to 80 % of the tensile strength on the stress-strain curve. In the experiment, the fracture within the same range as in the numerical analysis was selected for the quantitative assessment of fracture tortuosity. The calculated tortuosity, Z_2 (Eq. (19)), was 0.230 in the experiment and 0.234 in the simulation. The close agreement between the experimental and numerical results is evident. These findings demonstrate that, in the G-SH1 simulation, both IG and TG cracks induced by mineral heterogeneity were successfully captured. As a result, the quantitatively derived fracture tortuosity closely matched the experimental outcome, confirming the high reproducibility of the simulation.

Fig. 16 shows the experimental results for R-SG1. On the left, a post-fracture microscopic image after the experiment is shown, with the observed cracks delineated in red, while on the right, enlarged microscopic images before and after the experiment, together with the

corresponding mineral distribution image, highlight characteristic regions where IG and TG cracks are observed. R-SG1 is characterized by the presence of the Grain plane aligned with the loading direction and by the wide distribution of biotite along the line connecting the loading points. Within the biotite regions, both TG cracks within the biotite and IG cracks at the biotite-other mineral boundaries are evident. Fig. 17 illustrates a comparison between the experimental fractures (Fig. 17 (a)) and the simulation results (Fig. 17 (b)), where the latter depict the spatial distribution of the damage coefficient ($D > 0.1$) and the associated fracture modes. In Fig. 17 (b)-A, shear fractures within the biotite grains and tensile fractures along the biotite-other mineral boundaries are observed, exhibiting a fracture path highly consistent with the experimental results. Furthermore, in Fig. 17 (b)-B, TG cracks are also observed within quartz—where the tensile strength is the highest—again consistent with the experimental observations.

Fig. 18 illustrates the fracture propagation process obtained from the simulation results. Similar to G-SH1, shear failure is seen to occur firstly

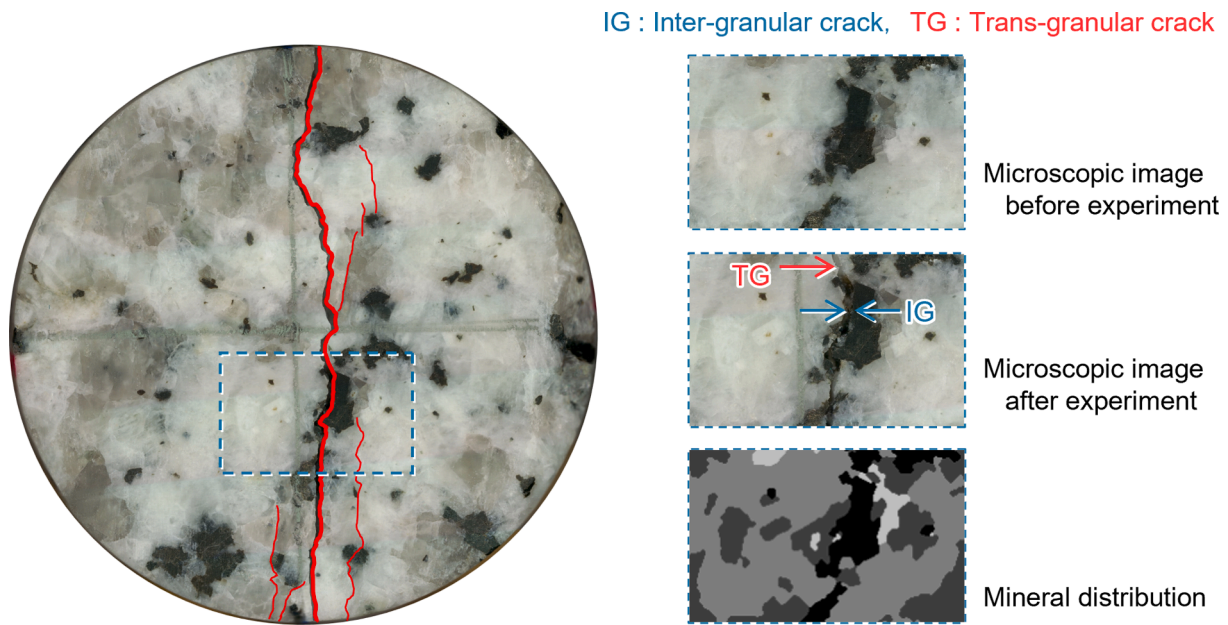
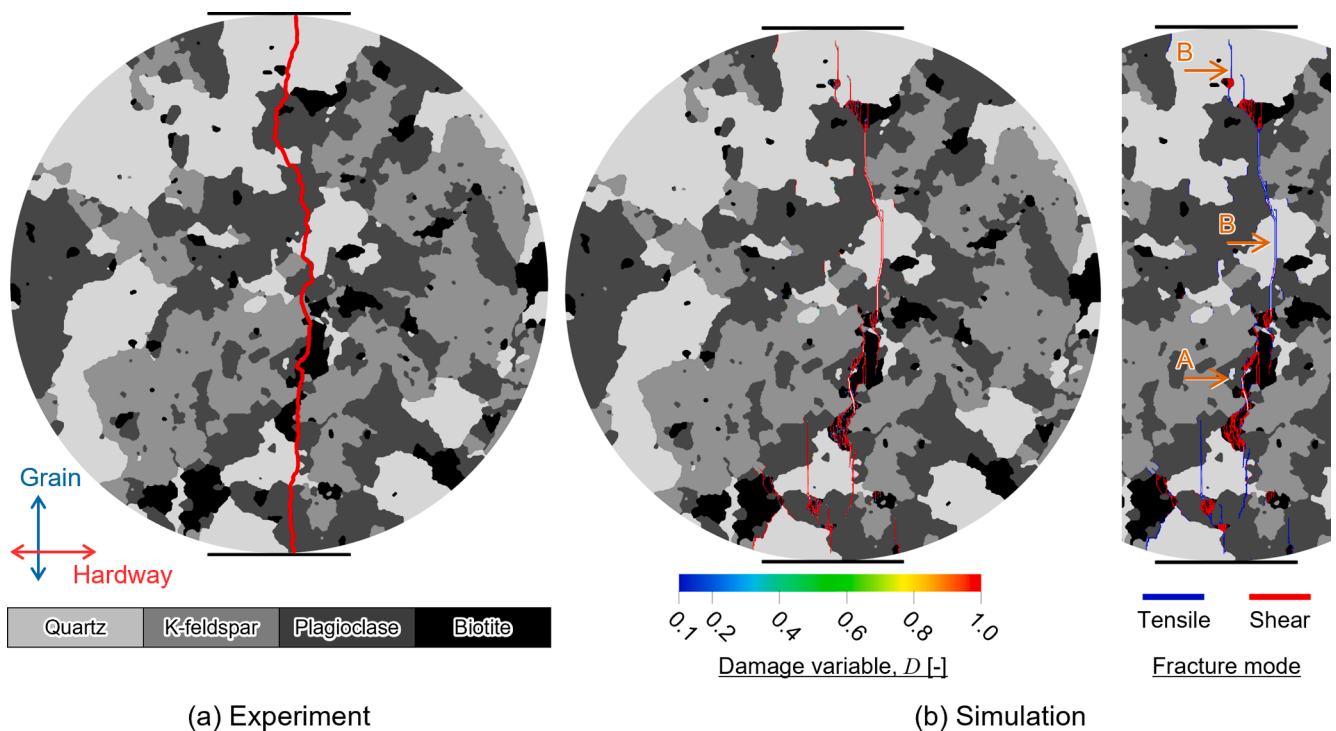


Fig. 16. Experimental observations for R-SG1 specimen.

Fig. 17. Comparison of experimental and numerical fracture paths for R-SG1 specimen: (a) mineral distribution image superimposed with experimentally observed fractures, and (b) spatial distribution of damage variable ($D > 0.1$) and fracture mode obtained from numerical simulation.

within the biotite near the loading plate (labeled A in the figure). The point of initiation of the fracture propagation leading to specimen failure is located at the biotite-plagioclase boundary (labeled B), from which the fracture progresses through the biotite grain (labeled C). As shown in D, E, and F, the fracture path becomes tortuous, with a marked tendency to preferentially follow the biotite boundaries. In G, the fracture alternately penetrates through the quartz grains and quartz-plagioclase boundaries, following the same path as that seen in the experimental results. In F, the fracture extends along the quartz-plagioclase boundary and then penetrates the quartz grain with high

strength (H). Overall, TG cracks predominantly propagate linearly along the Grain plane, whereas in the vicinity of the grain boundaries and the biotite, the fracture path becomes markedly tortuous.

At $\varepsilon = 0.6927$ % in the figure, the evaluation range for fracture tortuosity is indicated for both the experimental and numerical results, with the evaluation performed at $\varepsilon = 0.6933$ %, corresponding to the point where the stress decreased to less than 80 % of the tensile strength. The calculated tortuosity Z_2 (Eq. (19)) was 0.290 for the experiment and 0.296 for the simulation. As in the case of G-SH1, the experimental and numerical results demonstrate a good agreement. Furthermore, when

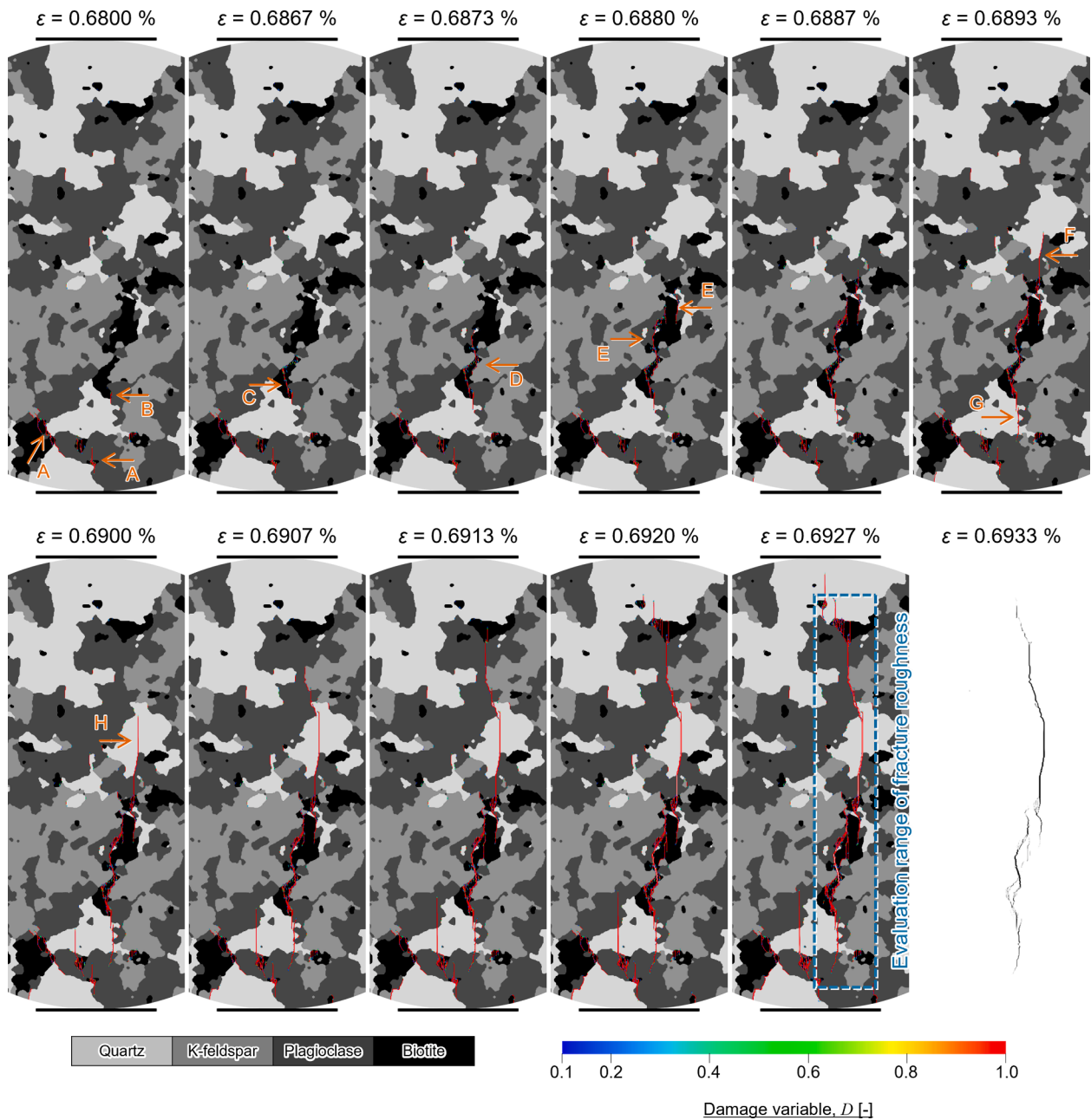


Fig. 18. Fracture propagation process for R-SG1 specimen obtained from numerical simulation.

comparing the tortuosities of G-SH1 and R-SG1, which possess distinct mineral distributions, both the experimental and numerical results indicate larger values for R-SG1 than for G-SH1. These findings confirm that fracture tortuosity is strongly dependent on the mineral distribution, and that the simulation in this study successfully captured the mineral-distribution-dependent variation in tortuosity.

4.2. Results of hydraulic fracturing tests

Fig. 19 presents the experimental results of the hydraulic fracturing tests. Fig. 19 (a) shows the time histories of the injection pressure (water pressure in the figure), confining pressure (lateral pressure in the figure), back pressure, and axial pressure, while Fig. 19 (b) illustrates the post-fracturing specimen. During the hydraulic fracturing tests, the injection pressure increased over time and then abruptly dropped at a certain point; the pressure at this moment is defined as the breakdown pressure, corresponding to the timing of specimen failure. In this

experiment, the breakdown pressure was 19.4 MPa. On the top surface of the specimen after fracturing, fractures propagated from the borehole along the Rift plane in two directions. Observation of the lateral surface indicates that the fractures did not extend to the bottom surface.

Fig. 20 shows a comparison of the experimentally observed fractures superimposed on the mineral distribution map (Fig. 20 (a)) and the simulation results (Fig. 20 (b)), in which the spatial distribution of the damage coefficient ($D > 0.1$) and the fracture mode are displayed. The simulation results are presented at the time when fractures with $D > 0.1$ reached both external boundaries. This corresponds to the 43rd iterative calculation at $t = 37$ s, where the iterative process was continued until fracture initiation and propagation had converged (Section 2.1). The corresponding injection pressure, i.e., the breakdown pressure, was 18.5 MPa, closely matching the experimental value, thereby confirming the reliability of the simulation in reproducing the breakdown pressure.

Next, the fracture propagation paths were examined by comparing the experimental and numerical results. Experimentally, at point A in

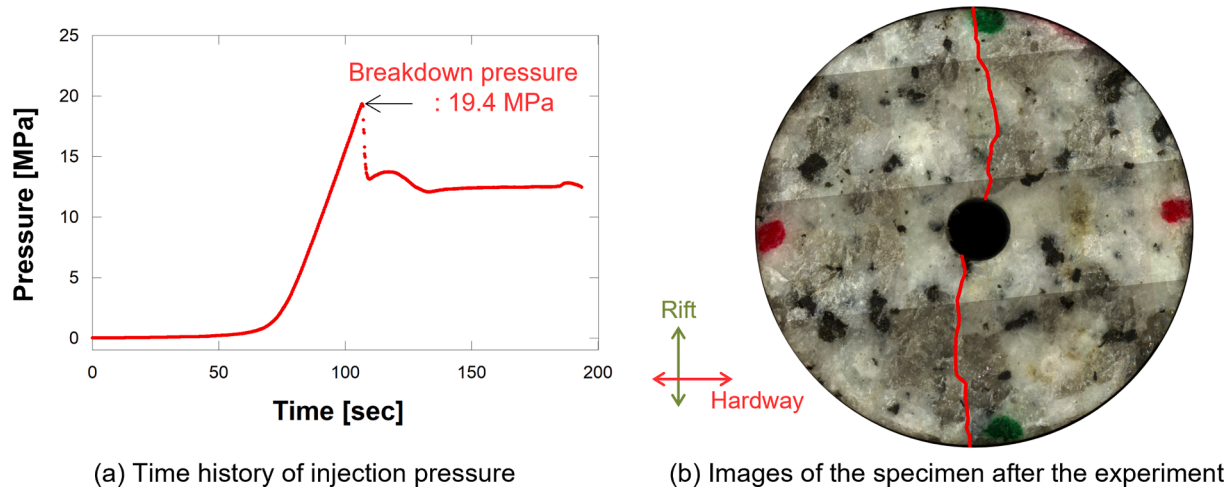


Fig. 19. Experimental results of hydraulic fracturing tests: (a) time history of injection pressure recorded during experiment and (b) post-fracturing image of tested specimen.

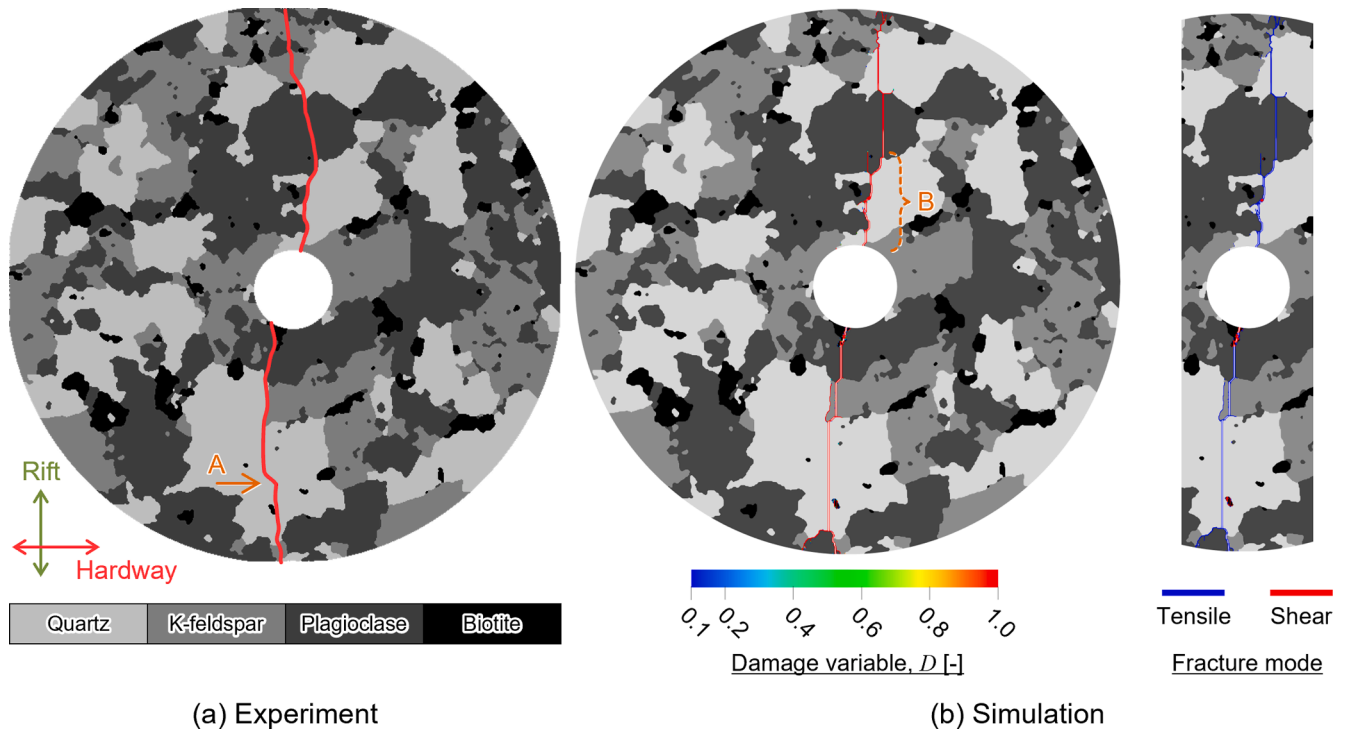


Fig. 20. Comparison of experimental and numerical fracture paths for hydraulic fracturing tests: (a) mineral distribution image superimposed with experimentally observed fractures, and (b) spatial distribution of damage variable ($D > 0.1$) and fracture mode obtained from numerical simulation.

Fig. 20 (b), a sudden change in the propagation direction is seen to occur within a quartz grain, but the reason for this path selection cannot be identified from the surface mineral distribution alone. The numerical results captured the fracture propagation from the borehole in two directions, consistent with the experimental observation, and the propagation paths were also aligned with the Rift plane. Overall, the simulation exhibited high fidelity in reproducing the fracture paths across the specimen, with a particularly strong agreement at point B, where the propagation path successfully connected multiple quartz grains and quartz–plagioclase boundaries. At the initiation point below the borehole, both the experimental and numerical results demonstrated fracture propagation along the boundary of the biotite, further supporting the consistency between the two. In the simulation, TG cracks were observed to propagate preferentially along the Rift plane,

reflecting the inherent weakness of this orientation. However, the tortuous IG crack path observed experimentally at point A, which cannot be explained solely by the mineral distribution at the surface, was not reproduced by the present simulator. Future work must therefore focus on identifying the underlying causes of such behavior through experimental and microstructural analyses, followed by the incorporation of effective modeling approaches into the current simulation framework.

The calculated tortuosity Z_2 (Eq. (19)) of the fractures showed values of 0.248 in the experiment and 0.272 in the simulation for the upper fracture, and 0.245 in the experiment and 0.180 in the simulation for the lower fracture. For the upper fracture, a strong consistency was found between the experimental and numerical results, whereas for the lower fracture, the experimental results exhibited greater tortuosity than the numerical results. It is likely that this discrepancy can be attributed to

the tortuous fracture path observed at location A in Fig. 20 (a) of the experimental results. To examine this effect, tortuosity Z_2 was recalculated for the fracture segment from the borehole to point A, yielding values of 0.186 in the experiment and 0.183 in the simulation, thereby confirming that the tortuosity up to point A has been well captured.

Fig. 21 illustrates the temporal evolution of the spatial distributions of the damage coefficient ($D > 0.1$), fluid pressure, and equivalent fracture aperture, showing the results at each iteration following $t = 37$ s (injection pressure $p_{in} = 18.5$ MPa). The notations “37 s, 1st” in the figure denote the first iteration at $t = 37$ s, where the iterative calculation is performed until fracture initiation and propagation have converged. The figure reveals that fractures propagate rapidly within the iterations at $t = 37$ s. Although the first cohesive element has already been activated at $t = 33$ s ($p_{in} = 16.5$ MPa), the fracture does not propagate immediately. The point of initiation of the fracturing that leads to specimen failure is identified at the quartz–K-feldspar boundary labeled A. However, no upward fracture propagation occurs immediately. Instead, the fracture firstly propagates downward along the biotite boundary. At D, fractures propagate within the K-feldspar grains and at nearby quartz–K-feldspar boundaries, while simultaneously, at C, an TG crack propagates through quartz, marking the onset of upward propagation. At E and F, the fracture propagates through quartz grains with high tensile strength. The figure demonstrates that, as fractures propagate, the aperture within the fractures increases, and the fractures become the dominant flow path, thereby causing fluid pressure inside the fractures to reach levels comparable to the borehole pressure. At G, the fracture firstly reaches the lower external boundary, and drainage from the fracture tip induces a reduction in fluid pressure within the fracture.

Accordingly, using the same set of parameters, three types of analyses were conducted: BTS tests for G-SH1, BTS tests for R-SG1, and hydraulic fracturing tests. In all cases, the simulations successfully captured the tortuosity of the fractures induced by the mineral distribution, yielding quantitative tortuosity indices that closely matched the experimental values. Furthermore, in the hydraulic fracturing tests, the validity of the results was also confirmed from the perspective of the breakdown pressure. These findings demonstrate that the developed simulator is capable of accurately reproducing fracture propagation phenomena governed by mineral distributions, as observed across multiple experimental tests.

5. Discussion

From Section 4, it was confirmed that both experiments and simulations can demonstrate the formation of TG cracks within grains and IG cracks along grain boundaries, leading to the development of tortuous fracture paths through their combination. Even in quartz, which exhibits the highest tensile strength, both experiments and simulations revealed not only IG cracks circumventing the quartz, but also TG cracks penetrating directly through the quartz grains. Therefore, this section discusses why TG cracks formed within the quartz, despite its high tensile strength. Therefore, the aim here is to provide general insights into whether fracture propagation pathways preferentially follow TG or IG cracks and the underlying factors influencing this selection. Specifically, the discussion focuses on regions in the hydraulic fracturing simulations where fractures propagated through quartz grains.

Before discussing fracture propagation within quartz grains, the initiation and propagation sites are firstly examined to clarify the processes leading to failure, and then the characteristics of the propagation sites are summarized. Fig. 22 presents the temporal evolution of the failure indicators (Eqs. (2) and (3)) for five selected mesh boundaries (Targets I–V) given in the figure. When either the tensile failure indicator (Eq. (2), shown in blue) or the shear failure indicator (Eq. (3), shown in red) reaches a value of 1, a cohesive element is activated, indicating the onset of fracturing. It is noted that the tensile failure indicator may take negative values, which correspond to compressive

stresses acting on the mesh boundaries, i.e., continuum element boundaries. Notations such as “37 s, 43rd” in the figure indicate the 43rd iteration at $t = 37$ s, where iterative calculations are performed until fracture initiation and propagation have converged. From the results of the hydraulic fracturing simulations, fracture initiation occurs in the upper region at Target I in area A and in the lower region at Target II in area B, both of which correspond to grain boundaries. Area C in the figure represents a fracture propagation site: Target III reaches a failure indicator value of 1 and undergoes fracturing, whereas Targets IV and V do not experience fracturing. Targets IV and V are located 135 μm and 405 μm away from Target III, respectively. The temporal profiles of the failure indicators show that, at the initiation sites (Targets I and II), the tensile failure indicator gradually increases from the beginning of the calculation and continues until reaching 1. This means that the strength term in the denominator of the failure indicators (Eqs. (2) and (3)) plays a dominant role, as fracturing initiates at the grain boundaries of lower strength. At the propagation site (Target III), the tensile fracture criterion remains at ≤ 0 until just before “37 s, 20th”, but then increases sharply, ultimately reaching 1. This sudden increase is attributed to the stress concentration at the advancing fracture tip from the upper side. At Targets IV and V, similar abrupt increases in the tensile failure indicators are observed. However, the magnitude of the increase diminishes with the distance from Target III, i.e., the fracture tip. These results demonstrate that fracture propagation is strongly governed by the stress concentration at the fracture tip.

In the following, stress concentration at the crack tip, which exerts a significant influence on fracture propagation, is examined to identify the positions and orientations where cracks are most likely to propagate. To this end, the theoretical stress singular field at the fracture tip, derived from linear elastic fracture mechanics (LEFM), is used as a reference to interpret the fracture behavior near the fracture tip. The pre-existing fracture is assumed to be a Mode I (i.e., tensile) fracture. In LEFM, the stress field is expressed as a function in polar coordinates centered at the fracture tip.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \begin{Bmatrix} \cos(\theta/2)[1 - \sin(\theta/2)\sin(3\theta/2)] \\ \cos(\theta/2)[1 + \sin(\theta/2)\sin(3\theta/2)] \\ \sin(\theta/2)\cos(\theta/2)\cos(3\theta/2) \end{Bmatrix} \quad (20)$$

where K_I [Pa $\text{m}^{1/2}$] is a Mode-I stress intensity factor. It should be noted that the representation of fractures and stress in ECZM-FDEM differs in several respects from the assumptions of LEFM. Unlike LEFM, which models fractures as fully separated discontinuities with zero thickness, ECZM-FDEM represents fracture propagation by explicitly capturing the progressive increase in fracture aperture together with damage evolution, expressed through damage coefficient D . Furthermore, in ECZM-FDEM, the stress field is computed as representative values at each element based on a finite-element discretization. Therefore, unless extremely fine meshes are employed, the extremely high stress values expected in the immediate vicinity of the fracture tip in LEFM do not occur. Despite these differences, the stress distributions obtained from both approaches exhibit strong similarities. Accordingly, LEFM solutions are regarded as sufficiently informative for identifying preferential positions and orientations for fracture propagation at the fracture tip. Fig. 23 presents the theoretical stresses near the fracture tip derived from LEFM. The contours show the normalized tensile stresses σ/K_I [$1/\text{m}^{1/2}$] acting on planes oriented (a) horizontally, (b) perpendicularly, and (c) 45° relative to the fracture propagation direction. In the figure, contour maps are plotted only for regions located beyond a certain distance from the fracture tip, which is set to be equal to half the mesh size used in the simulations (i.e., 22.5 μm). In ECZM-FDEM, stress concentration evolves progressively near the fracture tip as the fracture aperture widens and damage accumulates. Therefore, attention should be given not to the absolute stress magnitudes, but rather to the distribution trends in the high-stress regions and the relative stress values across orientations (a) to (c). The results show that the stress values are

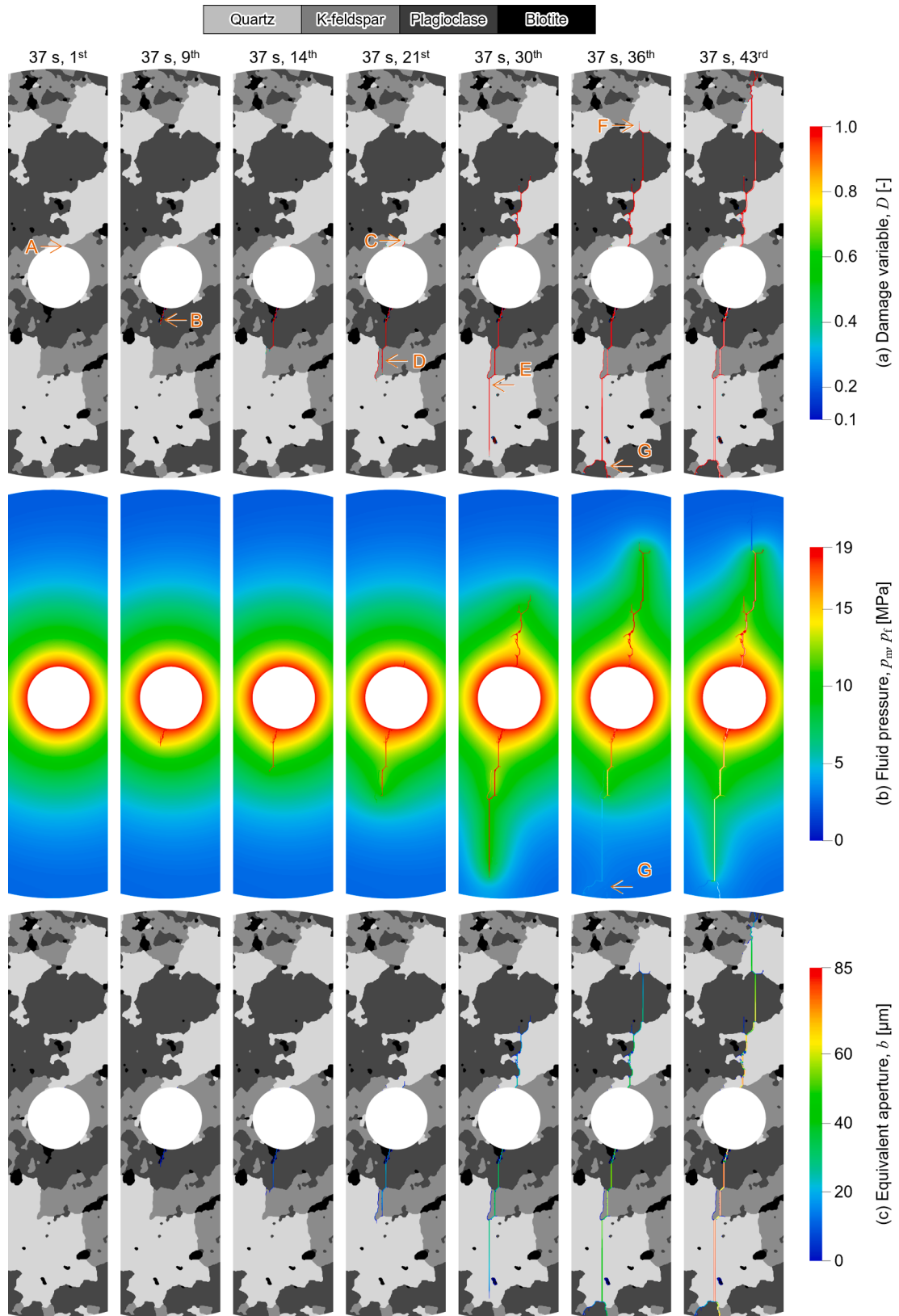


Fig. 21. Temporal evolution of numerical simulation results. Spatial distributions of (a) damage variable ($D > 0.1$), (b) fluid pressure, and (c) equivalent fracture aperture during hydraulic fracturing.

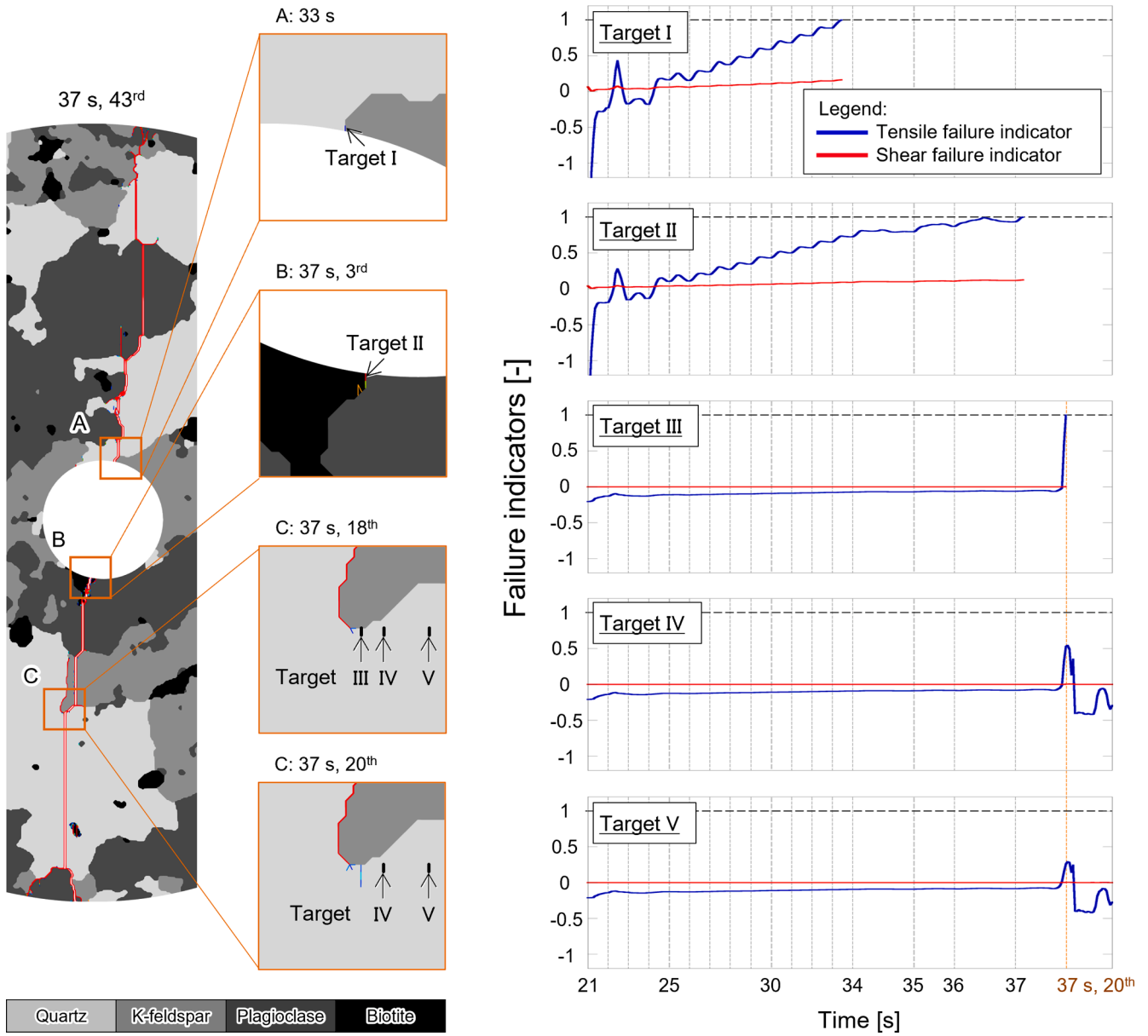


Fig. 22. Temporal evolution of failure indicator for five selected mesh boundaries (Targets I–V).

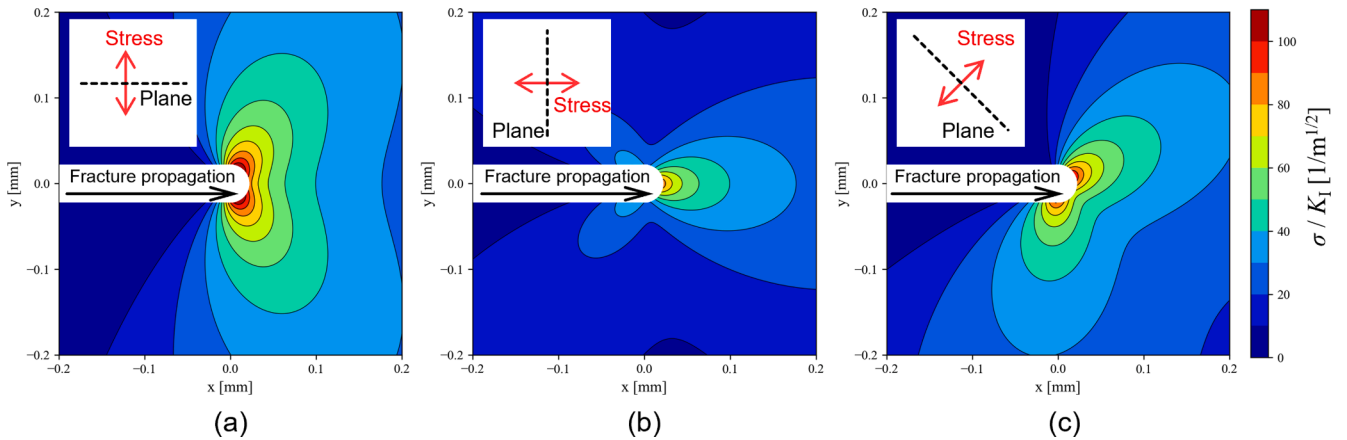


Fig. 23. Theoretical stresses acting on planes oriented (a) horizontally, (b) perpendicularly, and (c) 45° relative to fracture propagation direction, near fracture tip derived from LEFM.

strongly dependent on the orientation. Relative to the crack propagation direction, high-stress regions are distributed laterally along the horizontal plane (a), forward along the perpendicular plane (b), and orthogonally across the 45° plane (c). Among these, the horizontal plane (a) exhibits the highest tensile stress. In addition, the stress consistently increases as the distance to the fracture tip decreases. These findings indicate that the highest tensile stress develops along the horizontal plane (a) near the fracture tip. During fracture propagation, new cracks occur at the locations where the failure indicator—defined as stress divided by strength—reaches its maximum. Thus, the characteristics of stress concentration at the fracture tip suggest that, in homogeneous materials, fractures preferentially follow a straight propagation path. In contrast, in heterogeneous materials, such as granite, if a region of high strength (e.g., quartz) exists ahead of the fracture tip, fractures do not necessarily propagate directly forward. Instead, when weak planes with lower tensile strength are present in the vicinity, the balance between tensile strength and stress concentration may cause cracks to appear along with these weaker planes.

In Fig. 24, the hydraulic fracturing simulation presents contour plots of normal stress for mesh boundaries where the failure indicator has not yet reached a value of 1, but the tensile normal stress exceeds 10 MPa, with already-formed fractures superimposed in black. As explained above, the expression “37 s, 16th” in the figure indicates the 16th iteration at $t = 37$ s, where iterations continue until fracture initiation and propagation have stabilized. In Fig. 24-A, the spatial extent of the regions exceeding 10 MPa is narrow. This is because the fracture propagates along the grain boundaries. As low-strength grain boundaries are located ahead of the advancing fracture, the subsequent cracks occur without excessive stress concentration at the fracture tip. In contrast, when the fracture tip reaches quartz, a high-tensile-strength grain (Fig. 24-B), the stress gradually accumulates with increasing fracture aperture and damage until it attains the tensile strength of quartz, thereby enlarging the region of $\sigma_n > 10$ MPa. At “37 s, 21st”, the fracture propagates within the quartz, and by “37 s, 22nd”, the fracture tip resides within the quartz grain. At this stage, regions of $\sigma_n > 10$ MPa are distributed exclusively within the quartz and, consistent with predictions from LEFM, the distribution of elevated tensile stresses depends on the orientation of the considered plane. For example, on planes oriented (a) horizontally relative to the fracture propagation direction, high-stress regions extend laterally outward from the fracture tip. Thus, the stress distribution predicted by LEFM is consistent with the numerical results. As discussed in the previous section, the locations of new cracks appearing during propagation—whether IG or TG cracks—are

governed by the balance between the tensile strength and the stress concentration imparted by the fracture tip. In addition, the orientation of the grain boundaries strongly influences the stress distribution, thereby exerting a decisive role in fracture path selection. Accordingly, when the fracture tip encounters a grain with high tensile strength (e.g., quartz in granite), TG cracks may form, but the probability of IG cracks along adjacent grain boundaries increases significantly. Indeed, prior studies on hydraulic fracturing in granite, for example, that by Gao et al. (2023), reported that the presence of quartz, with its exceptionally high tensile strength, promotes IG cracking around the quartz grains, leading to increased fracture network complexity. Moreover, the closer a grain boundary is to the fracture tip, the higher the likelihood of IG crack occurrence, and the higher the tensile strength of the grain ahead of the tip, the broader the spatial extent over which IG cracks are likely to develop.

In the following, a numerical experiment is conducted to verify that the higher the tensile strength of the grain located ahead of the fracture tip, the greater the likelihood of IG crack occurrence over a broader region. Specifically, in this numerical experiment, the analysis conditions described in Section 2.4.2 are adopted, except that only the tensile strength of quartz is artificially increased fivefold, from 56.4 MPa to 282 MPa, while all other conditions are left unchanged. To highlight any discernible changes, the tensile strength was intentionally amplified to an extreme value. The results of this newly conducted numerical experiment are presented below. Fig. 25, analogous to Fig. 24, shows contour plots for the normal stress at the mesh boundaries where the failure indicator has not yet reached a value of 1 and the tensile normal stress exceeds 10 MPa, with pre-existing fractures superimposed in black. As shown, until the fracture reaches the quartz with increased tensile strength (37 s, 16th–17th in Fig. 25), the behavior resembles that in Fig. 24. Once the fracture tip encounters the quartz with fivefold tensile strength, stress concentration develops with increasing fracture aperture and damage. However, the stress does not reach the threshold required to break the quartz itself. Instead, the progressive stress concentration elevates the stresses over a broader region. At “37 s, 51st”, cracking occurs not within the quartz grain, but at a distant grain boundary (Fig. 25-C), which differs from the behavior observed in Fig. 24. These numerical results provide a specific example showing that, the higher the tensile strength of the grain ahead of the fracture tip, the wider the spatial extent over which IG cracking is likely to occur. Accordingly, it is suggested that the nucleation sites of new fractures during propagation—whether IG or TG cracks are selected—are governed by a balance between the local tensile strength and the degree of

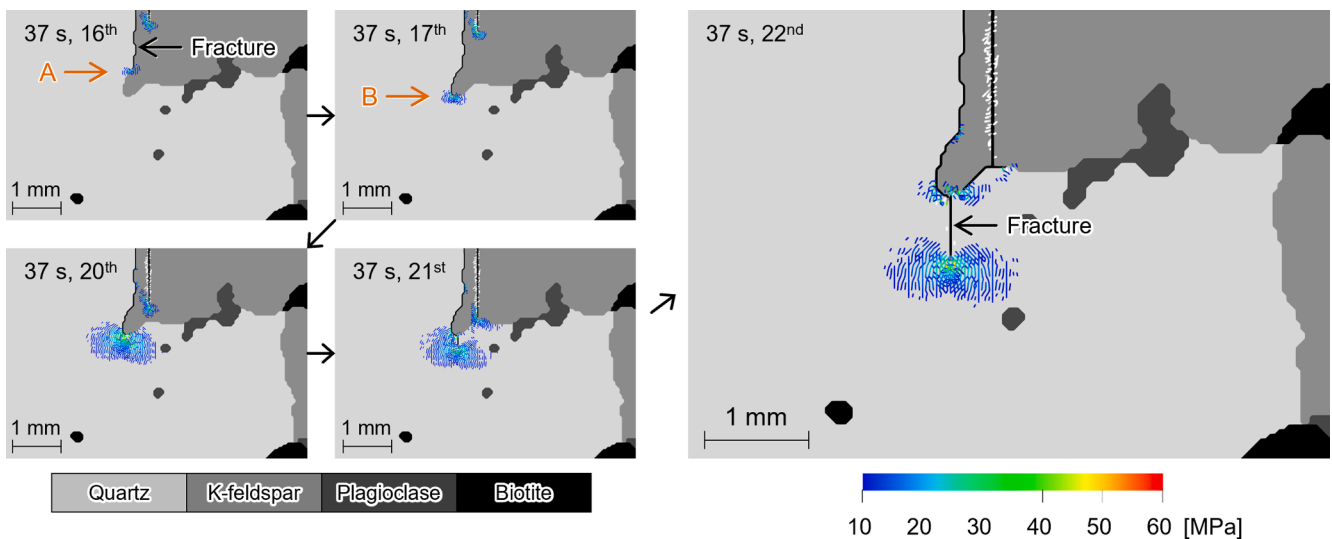


Fig. 24. Contour plots of tensile normal stress (>10 MPa) at mesh boundaries where failure indicator has not yet reached a value of 1 during hydraulic fracturing simulation. Pre-existing fractures are superimposed in black.

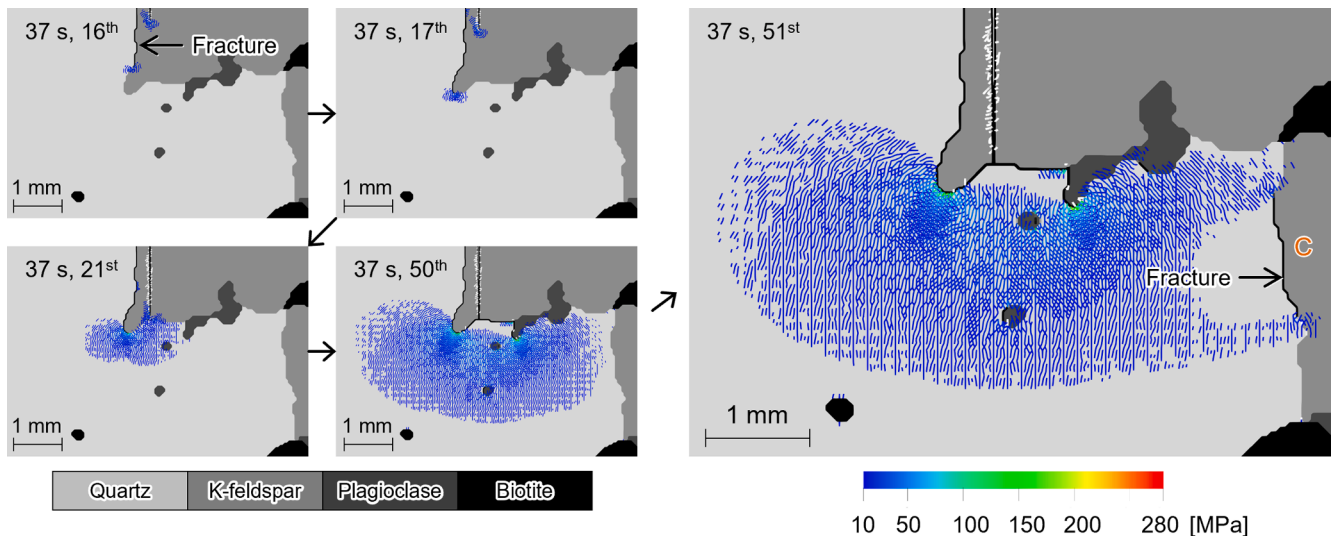


Fig. 25. Contour plots of tensile normal stress (>10 MPa) at mesh boundaries where failure indicator has not yet reached a value of 1, obtained from numerical experiment in which tensile strength of quartz was artificially increased fivefold (from 56.4 MPa to 282 MPa). Pre-existing fractures are superimposed in black.

stress concentration determined by the spatial relationship to the fracture tip.

6. Conclusions

In this study, a real-specimen grain-based model (RS-GBM), reflecting the identical mineral distribution of the experimental specimen, was constructed, and numerical simulations of hydraulic fracturing were performed using the finite-discrete element method (FDEM) for a comparison with the experimental results. By employing FDEM, a hybrid continuum-discontinuum approach, fracture surfaces could be explicitly represented, thereby enabling the realistic reproduction of fracture initiation and propagation along the grain boundaries as well as the failure behavior dependent on grain orientation. Correspondingly, an RS-GBM using a modelling strategy that faithfully mapped the actual grain boundaries onto mesh interfaces was also constructed. In addition, instead of the intrinsic cohesive zone model (ICZM), which has been widely adopted in previous fracture modeling studies, the extrinsic cohesive zone model (ECZM) was implemented to achieve higher accuracy in the stress computation.

Firstly, hydraulic fracturing simulations incorporating the GBM were conducted using the proposed simulator to examine the influence of ICZM and ECZM on the stress distribution. At the evaluation stage, the rock was considered to remain intact without any damage; therefore, the only factor introducing heterogeneity into the stress field was the mineral distribution. The comparison revealed that both methods exhibited stress contrasts among different minerals; however, ECZM provided continuous stress distributions within individual minerals, whereas ICZM produced stress oscillations even within a single mineral, with amplitudes that sometimes exceeded the stress differences between distinct minerals. These results demonstrated that, compared with ICZM, ECZM can represent stress fields arising from mineralogical heterogeneity more robustly and without artificial noise, thereby proving more effective for accurately evaluating fracture initiation and propagation phenomena based on the stress distribution.

Through the subsequent analyses of Brazilian tensile strength tests (BTS) and hydraulic fracturing tests, it was confirmed that both intergranular (IG) and transgranular (TG) cracks occurred depending on the

mineral distribution, and that their combination led to the formation of tortuous fracture paths. Furthermore, quantitative index, such as tortuosity Z_2 , derived from the simulation results, were shown to be consistent with the experimental values. In particular, in the BTS tests conducted on two specimens with distinct mineral distributions, the experimental and numerical values of Z_2 were (0.230, 0.234) for G-SH and (0.290, 0.296) for R-SG, respectively. Both the experiments and simulations indicated that R-SG exhibited greater tortuosity than G-SH. These findings demonstrate that fracture tortuosity is strongly dependent on the mineral distribution and that the proposed simulator is capable of appropriately capturing this behavior.

Based on the results of experiments and simulations using RS-GBMs, the occurrence of both TG and IG cracks was found to depend on the mineral distribution. However, the governing patterns remain complex and unclear, as even in quartz—a mineral with the high tensile strength—both TG and IG cracks were observed. A detailed examination of the simulation results revealed that the selection of fracture propagation pathways is controlled by the balance between the stress concentration at the fracture tip and the local strength. Within the stress field at the fracture tip, the normal stress acting on a plane varies with its orientation, with the highest tensile normal stress occurring on planes oriented horizontally relative to the propagation direction. In homogeneous materials, fractures tend to propagate linearly, whereas in heterogeneous rock, such as granite, the presence of high-strength minerals (e.g., quartz) ahead of the fracture tip does not necessarily result in TG cracks, even under elevated stress concentration. Instead, the likelihood of IG cracks along adjacent weaker grain boundaries increases. Furthermore, in the numerical experiments where the tensile strength of quartz was artificially increased, no TG cracks formed within the quartz where they had previously occurred in the original analysis. Instead, fractures initiated at more distant grain boundaries. These findings demonstrate that the nucleation sites of new cracks during propagation are governed by the balance between local stress concentration and mineral strength, both ahead of and surrounding the fracture tip.

Based on these findings, the simulator developed in this study was demonstrated to accurately and quantitatively reproduce fracture propagation phenomena governed by mineralogical distributions. Furthermore, the comparative assessment of ICZM and ECZM revealed

that ECZM is more suitable for representing stress heterogeneity between different minerals. The discussion on fracture propagation pathways also indicated that stresses at the fracture tip are strongly influenced by orientation, underscoring the effectiveness of constructing models that faithfully incorporate actual grain-boundary orientations. In addition, the results confirmed the applicability of the finite-discrete element method (FDEM), a hybrid continuum-discontinuum approach, in capturing fracture initiation and propagation along grain boundaries. Overall, the proposed methodology provides valuable insights into the understanding of fracture initiation and propagation in heterogeneous crystalline rock, and it is expected that the methodology will contribute to the design and optimization of deep underground applications such as EGS and the geological disposal of high-level radioactive waste.

CRediT authorship contribution statement

Yutaro Maeda: Writing – original draft, Visualization, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Sho Ogata:** Writing – review & editing, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization. **Daisuke Fukuda:** Writing – review & editing, Supervision, Software, Methodology, Conceptualization. **Naoki Kinoshita:** Writing – review & editing, Validation. **Hideaki Yasuhara:** Writing – review & editing, Supervision. **Kaisei Ishii:** Writing – review & editing,

Methodology, Investigation, Formal analysis, Data curation. **Atsushi Okamoto:** Writing – review & editing, Validation. **Shunya Okino:** Writing – review & editing, Methodology, Investigation, Formal analysis, Data curation. **Toru Inui:** Writing – review & editing, Supervision. **Hongyuan Liu:** Writing – review & editing, Software. **Kiyoshi Kishida:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

The BTS tests were conducted using a hydraulically controlled servo testing machine (Model EHF-ED10, Shimadzu Corporation), as shown in Fig. A.1 (a). During testing, the specimen was placed horizontally at the center of the fixed lower loading plate. To ensure the stable placement of the specimen—given that the contact surface is curved—clay was positioned on both sides of the specimen, as shown in Fig. A.1 (b). Throughout the test, the upper loading plate was lowered at a constant displacement rate of 0.0005 mm/s under displacement control. The applied load and displacement were continuously recorded by the servo-controlled testing machine during the experiment.

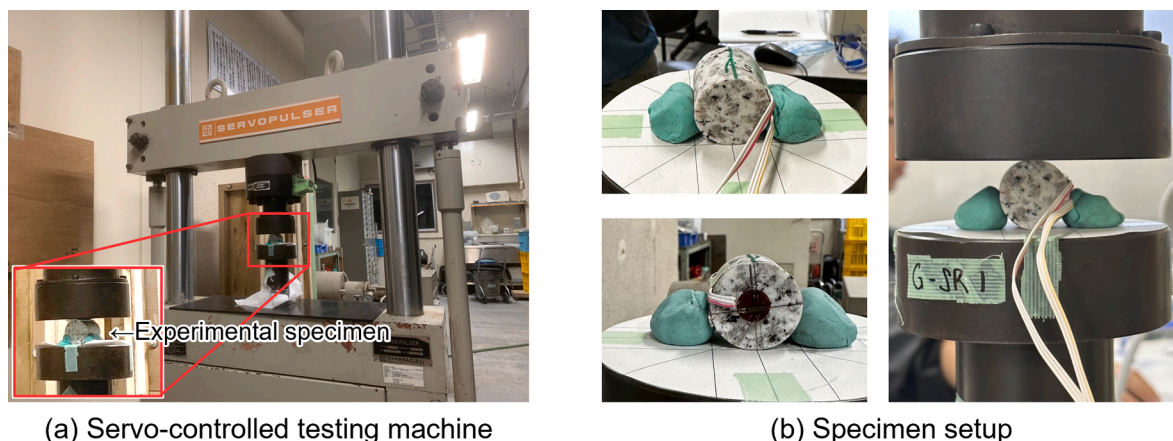


Fig. A.1. Experimental system for BTS tests: (a) photograph of servo-controlled testing machine and (b) specimen setup.

Appendix B

A schematic diagram of the laboratory hydraulic fracturing test apparatus is presented in Fig. B.1 (a). As shown in Fig. B.1 (b), when installing the specimen into the apparatus, the upper end face of the specimen was covered with a Teflon sheet (manufactured by Nikola Co., Ltd.) followed by a urethane rubber sheet to allow the concentration of permeation at the borehole section. The Teflon sheet had a thickness of 0.3 mm, while the urethane rubber sheet had a thickness of 3 mm. Both sheets had a diameter of 30 mm and were shaped with a central hole 4 mm in diameter. To

improve adhesion among the specimen, Teflon sheet, urethane rubber sheet, and pedestal, vacuum grease (high-vacuum grease manufactured by Toray Dow Corning Co., Ltd.) was applied to the interfaces. When placing the specimen into the apparatus, a neoprene rubber sleeve (manufactured by Marui Co., Ltd.) and a heat-shrink tube were used to cover the specimen, as shown in Fig. B.1 (b). The purpose of this covering was to prevent the intrusion of silicone oil into the specimen surface and to suppress the scattering of fractured fragments after failure. After the specimen was installed in the cell, a triaxial permeameter for high-temperature and high-pressure conditions (Model: MIS-235–1–55–30, manufactured by Marui Co., Ltd.) was used to fill the cell with silicone oil (Barrel Silicone Fluids M–2, manufactured by Matsumura Oil Research Corp.). An axial stress of 50 MPa and a confining pressure of 7 MPa were applied. Following the application of confining pressure, deionized water was injected into the cell using a syringe pump (manufactured by TELEDYNE ISCO) at a constant rate of 8 mL/min to increase the pore pressure. During the entire test duration, data on the axial stress, confining pressure, injection pressure, and back pressure were recorded at 0.1-second intervals using a data logger (Model: midiLOGGER GL820, manufactured by Graphtec Corp.).

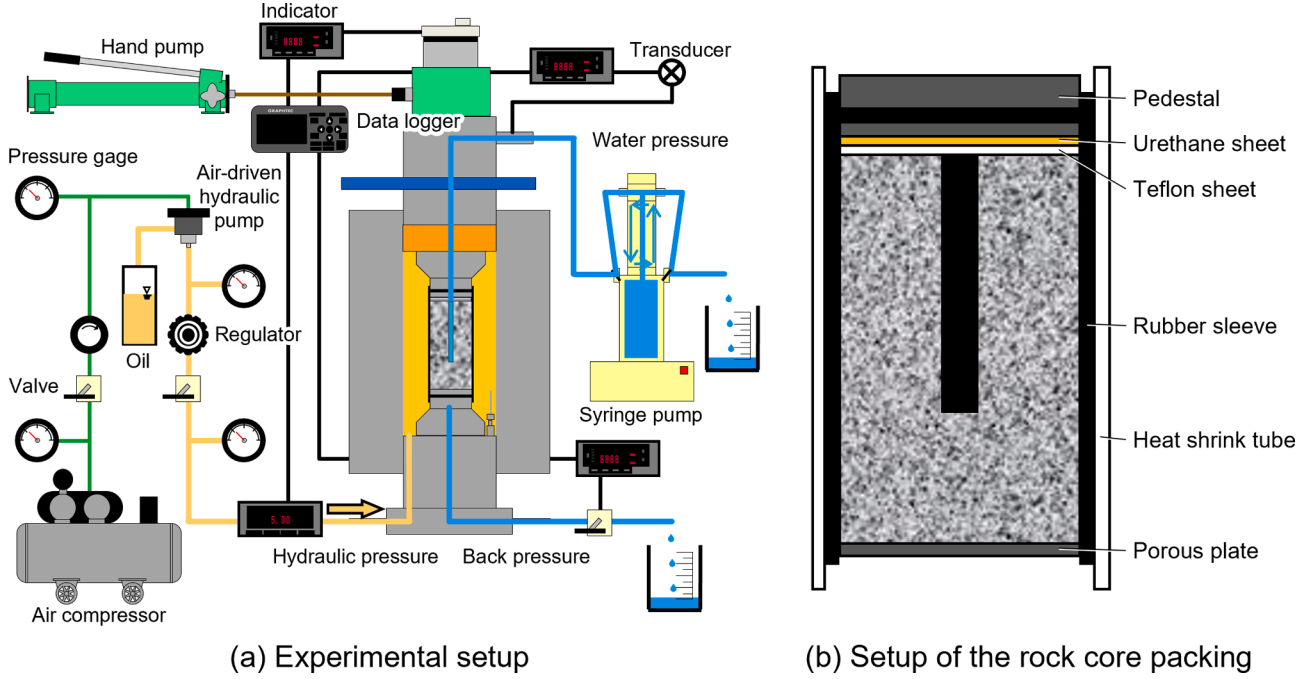


Fig. B.1. Experimental system for hydraulic fracturing tests: (a) experimental setup and (b) setup of rock core packing.

Appendix C

To verify the fundamental performance of the developed simulator, two types of benchmark analyses were conducted to examine the effects of the fluid pressure on the solid-phase stress in the rock: steady-state and time-dependent. The same geometry and mesh were used for both analyses. The geometry consisted of a two-dimensional horizontal cross section representing a hydraulic fracturing test, modeled as a circular specimen with a radius of $r_b = 30$ mm and a central borehole with a radius of $r_a = 4$ mm. The mesh was configured such that the maximum element size would be $45 \mu\text{m}$, consistent with the hydraulic fracturing simulation. In both cases, the spatial distributions of the pore fluid pressure and solid-phase stress were computed either in the steady-state or at a specified time, under constant fluid pressure applied at the borehole. The results are compared with the analytical solutions in the following.

For the benchmark analysis under the steady-state condition, the analytical solution proposed by Zhan et al. (2009) was referenced (Eqs. (C.1) to (C.4)). The outer boundary was assumed to be a free surface, with no confining pressure applied. At the injection borehole, a constant pressure of 0.5 MPa was imposed, consistent with the increment used in the numerical simulation for the hydraulic fracturing, where the pressure was increased by 0.5 MPa per loop. The material properties of granite, listed in Table 5, were used in the simulation. Fig. C.1 (a) presents a comparison of the numerical and analytical solutions for the spatial distribution of pore pressure p_m and the solid-phase stresses (σ_{rr} : radial and $\sigma_{\theta\theta}$: tangential). The sampling line used in the numerical analysis is also shown in the figure. The results are organized with respect to the radial coordinate r of the polar coordinates, centered at the borehole. The numerical results for p_m , σ_{rr} , and $\sigma_{\theta\theta}$ are seen to be in good agreement with the analytical solutions. These findings confirm that the calculation of the effective stresses induced by injection pressure p_{in} and resulting pore pressure p_m has been correctly implemented by the simulator.

$$p_m = p_{in} + S_k \ln \frac{r}{r_a} \quad (\text{C.1})$$

$$\sigma_{rr} = \frac{(1+\nu)(r_a r_b)^2}{2(r_b^2 - r_a^2)} S_k \left[\frac{\ln(r_b/r_a)}{r^2} + \frac{\ln(r/r_b)}{r_a^2} - \frac{\ln(r/r_a)}{r_b^2} \right] \quad (C.2)$$

$$\sigma_{\theta\theta} = \frac{(1+\nu)(r_a r_b)^2}{2(r_b^2 - r_a^2)} S_k \left[-\frac{\ln(r_b/r_a)}{r^2} + \frac{\ln(r/r_b)}{r_a^2} - \frac{\ln(r/r_a)}{r_b^2} \right] - \frac{1-\nu}{2} S_k \quad (C.3)$$

$$S_k = \frac{p_{in}}{\ln(r_b/r_a)} \quad (C.4)$$

where r [m] is the radial coordinate, r_a [m] is the radius of the injection borehole, r_b [m] is the radius of the specimen, and ν [-] is Poisson's ratio.

Table 5
Input parameters employed in steady-state benchmark analysis.

Property	Value
Density (kg/m ³)	2600
Young's modulus (GPa)	50
Poisson's ratio (-)	0.25
Biot's coefficient (-)	1.0

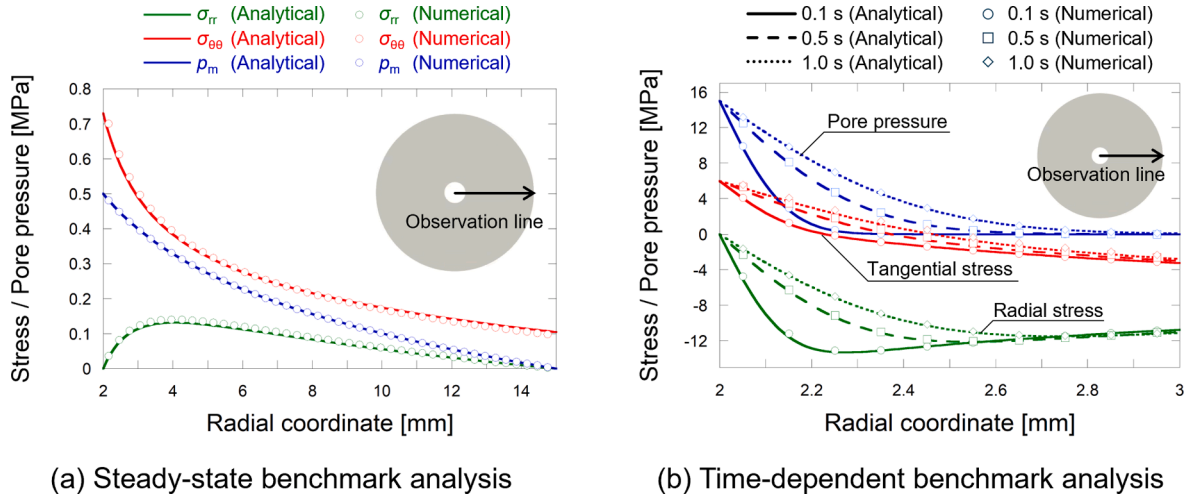


Fig. C.1. Comparison of analytical and numerical solutions for stress and pore pressure: (a) steady-state benchmark analysis and (b) time-dependent benchmark analysis.

For the benchmark analysis under the time-dependent condition, the analytical solution presented by [Detournay and Cheng \(1988\)](#) was referenced. Their solution considers a drained response of rock containing a borehole, under an initial pore pressure and subjected to confining stress. In contrast, the hydraulic fracturing simulations conducted in this study assumed zero initial pore pressure, applied confining stress to the rock, and injected fluid through the borehole. Accordingly, the original analytical expressions were reformulated to match the physical conditions of hydraulic fracturing, leading to modified solutions for pore pressure p_m and the solid-phase stresses (σ_{rr} : radial and $\sigma_{\theta\theta}$: tangential), as shown in Eqs. (C.5) to (C.21). Total solid-phase stress components σ_{rr} and $\sigma_{\theta\theta}$ are expressed as the sum of the stress components due to injection pressure p_{in} ($\sigma_{rr,p_{in}}$ and $\sigma_{\theta\theta,p_{in}}$), the isotropic components of confining pressure P_0 (σ_{rr,P_0} and $\sigma_{\theta\theta,P_0}$), the anisotropic components of confining pressure S_0 (σ_{rr,S_0} and $\sigma_{\theta\theta,S_0}$), and pore pressure ap . An isotropic confining pressure of 7 MPa was applied to the external boundaries, and a constant injection pressure of 15 MPa was assumed at the injection borehole. The rock properties were set to represent granite, using the values listed in [Table 6](#). The global time increment was set to $\Delta t = 0.1$ s, and fluid and mechanical calculations were executed alternately at each time step. The simulation was continued for a total of 10 steps, finally reaching 1.0 s. Fig. C.1 (b) presents a comparison of the numerical and analytical solutions for the spatial distributions of pore pressure p_m , and solid stresses σ_{rr} and $\sigma_{\theta\theta}$ at 0.1, 0.5, and 1.0 s after the start of the injection. The observation line used in the numerical simulation is also illustrated in the figure, and the results are organized as a function of radial distance r from the center of the borehole in a polar coordinate system. Angle θ was measured with respect to the direction of the minimum confining stress. The spatial distributions of p_m , σ_{rr} , and $\sigma_{\theta\theta}$ obtained from the simulation agree well with the analytical solutions. These results confirm that the developed simulator can accurately capture the time-dependent fluid infiltration behavior and fluid–mechanical interactions, thereby demonstrating its fundamental capability for hydraulic fracturing analyses.

$$p_m = p_{in} \times \left\{ \sqrt{\frac{r_a}{r}} \operatorname{erfc}\left(\frac{r-r_a}{\sqrt{4ct}}\right) + \frac{1}{8} \sqrt{\frac{r_a}{r}} \left(1 - \frac{r_a}{r}\right) \left[\sqrt{\frac{4ct}{r_a^2 \pi}} \times \exp\left(-\frac{(r-r_a)^2}{4ct}\right) - \left(\frac{r}{r_a} - 1\right) \operatorname{erfc}\left(\frac{r-r_a}{\sqrt{4ct}}\right) \right] \right\} \quad (C.5)$$

$$\sigma_{rr} = \sigma_{rr,p_{in}} + \sigma_{rr,p_0} + \sigma_{rr,S_0} + \alpha p_m \quad (C.6)$$

$$\sigma_{rr,p_{in}} = -p_{in} \frac{r_a^2}{r^2} - p_{in} \times 2\eta \left\{ \left(\frac{r_a}{r}\right)^{\frac{3}{2}} \left[\sqrt{\frac{4ct}{r_a^2 \pi}} \exp\left(-\frac{(r-r_a)^2}{4ct}\right) - \left(\frac{r}{r_a} - 1\right) \operatorname{erfc}\left(\frac{r-r_a}{\sqrt{4ct}}\right) \right] - \frac{r_a^2}{r^2} \sqrt{\frac{4ct}{r_a^2 \pi}} \right\} \quad (C.7)$$

$$\sigma_{rr,p_0} = P_0 \times \left(-1 + \frac{r_a^2}{r^2} \right) \quad (C.8)$$

$$\sigma_{rr,S_0} = S_0 \times \left[1 - 4 \frac{r_a^2}{r^2} + 3 \frac{r_a^4}{r^4} \right] \cos 2\theta \quad (C.9)$$

$$\sigma_{\theta\theta} = \sigma_{\theta\theta,p_{in}} + \sigma_{\theta\theta,p_0} + \sigma_{\theta\theta,S_0} + \alpha p_m \quad (C.10)$$

$$\begin{aligned} \sigma_{\theta\theta,p_{in}} = & p_{in} \frac{r_a^2}{r^2} - p_{in} \times 2\eta \left\{ \left(\frac{r_a}{r}\right)^{\frac{3}{2}} \left[\sqrt{\frac{4ct}{r_a^2 \pi}} \exp\left(-\frac{(r-r_a)^2}{4ct}\right) + \operatorname{erfc}\left(\frac{r-r_a}{\sqrt{4ct}}\right) \right] \right. \\ & \left. - \frac{r_a^2}{r^2} \sqrt{\frac{4ct}{r_a^2 \pi}} + \frac{1}{8} \sqrt{\frac{r_a}{r}} \left(1 - \frac{r_a}{r}\right) \left[\sqrt{\frac{4ct}{r_a^2 \pi}} \exp\left(-\frac{(r-r_a)^2}{4ct}\right) - \left(\frac{r}{r_a} - 1\right) \operatorname{erfc}\left(\frac{r-r_a}{\sqrt{4ct}}\right) \right] \right\} \end{aligned} \quad (C.11)$$

$$\sigma_{\theta\theta,p_0} = P_0 \times \left(-1 - \frac{r_a^2}{r^2} \right) \quad (C.12)$$

$$\sigma_{\theta\theta,S_0} = S_0 \times \left[-1 + 4 \frac{\nu_u - \nu}{1 - \nu} \sqrt{\frac{r_a}{r}} \operatorname{erfc}\left(\frac{r-r_a}{\sqrt{4ct}}\right) - 3 \frac{r_a^4}{r^4} \right] \cos 2\theta \quad (C.13)$$

$$P_0 = -\frac{\sigma_1 + \sigma_3}{2} \quad (C.14)$$

$$S_0 = -\frac{\sigma_1 - \sigma_3}{2} \quad (C.15)$$

$$c = \frac{2\kappa B^2 G(1 - \nu)(1 + \nu_u)^2}{9(\nu_u - \nu)(1 - \nu_u)} \quad (C.16)$$

$$\kappa = \frac{k}{\mu} \quad (C.17)$$

$$B = \frac{3(\nu_u - \nu)}{\alpha(1 - 2\nu)(1 + \nu)} \quad (C.18)$$

$$G = \frac{E}{2(1 + \nu)} \quad (C.19)$$

$$\eta = \alpha \frac{1 - 2\nu}{2(1 - \nu)} \quad (C.20)$$

$$S = \frac{9(\nu_u - \nu)(1 - 2\nu_u)}{2GB^2(1 - 2\nu)} \quad (C.21)$$

where ν_u [-] is the undrained Poisson's ratio, k [m²] is the rock permeability, μ [Pa · s] is the fluid viscosity, and α [-] is Biot's coefficient.

Table 6

Input parameters employed in time-dependent benchmark analysis.

Property	Value
Density (kg/m ³)	2600
Young's modulus (GPa)	50
Poisson's ratio (-)	0.25
Undrained Poisson's ratio (-)	0.26
Rock permeability (m ²)	4.0×10^{-20}
Viscosity (Pa · s)	0.001
Biot's coefficient (-)	1.0
Storage coefficient (1/Pa)	6.0×10^{-10} (Eq. (32) in Takuma et al. (2024))

Appendix D

To verify the performance of the developed simulator, a benchmark analysis was conducted to evaluate the effect of fluid pressure within a fracture on the resulting fracture opening. The model geometry and boundary conditions used in the calculation are shown in Fig. D.1. The computational domain is a 100 m × 100 m square containing a 20 m-long horizontal fracture embedded at its center. The rock matrix is assumed to be elastic, isotropic, and impermeable. The mesh was generated such that the maximum element size was 1 m. In this benchmark study, the numerical fracture aperture induced by a constant fluid pressure was compared with the analytical solution (Parker (1981), Eq. (D.1)), enabling verification of the fluid–mechanical coupling implemented in the simulator.

A horizontal stress of $\sigma_h = 16$ MPa and a vertical stress of $\sigma_v = 11.5$ MPa were applied as confining stresses, while the internal fluid pressure within the fracture was set to $p_{in} = 20$ MPa. The Young's modulus and Poisson's ratio of the rock matrix were set to $E = 40$ GPa and $\nu = 0.22$, respectively. Because this benchmark focuses solely on the elastic response of the rock, unrealistically high strength parameters were assigned so that no new fractures would be generated.

Fig. D.1 presents a comparison between the numerical and analytical solutions for the spatial distribution of the fracture aperture. The solid line denotes the analytical solution, while the plotted points represent the numerical results. The two results show a good agreement. These findings confirm that the effect of fluid pressure on fracture aperture is correctly captured by the simulator, demonstrating that the fluid–mechanical coupling—an essential mechanism in hydraulic fracturing—is accurately represented.

$$b = \frac{4(\sigma_v - p_{in})(1 - \nu^2)}{E} \sqrt{L^2 - x^2} \quad (D.1)$$

where x [m] is the distance from the fracture center and L [m] is the half-length of the fracture.

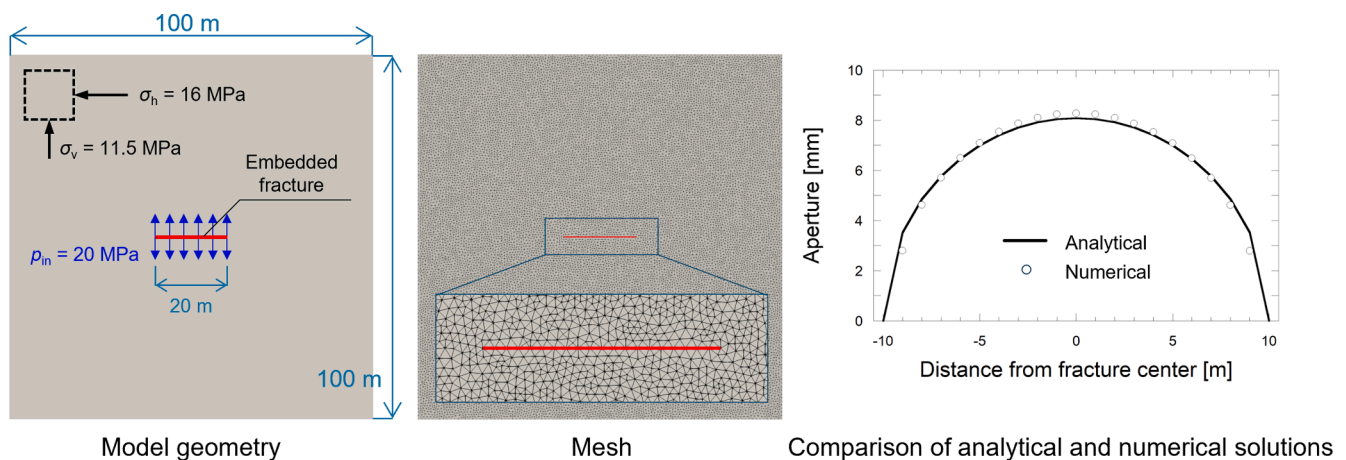


Fig. D.1. Model geometry, computational mesh, and comparison between analytical and numerical solutions for the fracture aperture under uniform fluid pressure.

Data availability

Data will be made available on request.

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