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Selecting Nodes to Protect in Interdependent Networks Using Shapley Value Analysis

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Abstract—Interdependent networks consist of entities that mutually rely on each other. These networks are particularly vulnerable to cascading failures due to their complex interdependencies. This study aims to mitigate the cascading effects of node failures caused by disastrous events. Specifically, it addresses the problem of selecting a fixed number of nodes for protection to minimize the impact of initial failures in interdependent networks. A key challenge in this problem, referred to as the hardening problem, is that the significance of individual nodes cannot be assessed in isolation due to mutual dependencies. To address this, we propose a method that quantifies node significance using Shapley values, a concept from cooperative game theory, to account for the combined effects of multiple node failures. Once all nodes are evaluated for their significance, the most significant ones are selected for protection in order. We conducted simulation experiments to compare the proposed method with other approaches. The results show that our method achieves the best performance.

Keywords—interdependent networks, hardening problem, Shapley value, cascading failure

1. INTRODUCTION

Social systems often mutually depend on each other for their operation. A power system is a typical example, where its power lines and control networks rely on each other interdependently. Such an interdependent system sometimes experiences cascading failures, where a failure in one of its constituent systems could cause another failure in the other constituent system, which could in turn lead to yet another failure.

This study aims to protect interdependent networks against cascading failures. We use a mathematical model of interdependent networks, where an interdependent network consists of many nodes that depend on each other. The specific problem we address is selecting nodes to protect in order to mitigate the impact of disastrous events, given that a fixed number of nodes are allowed to be protected or *hardened* in advance.

A reasonable approach to this problem could be to select nodes whose failures have the most impact on the entire network. However, the nature of interdependent networks makes this approach more difficult to apply than it appears. In an interdependent network, the failures of nodes are related to each other in a complex manner; thus each node cannot be evaluated in isolation with respect to its failure's impact.

To overcome this obstacle, we propose using the Shapley value to quantify the impact of each node. The Shapley value [1], a concept arising from cooperative game theory, is used to distribute the total gains among a group of cooperating players by quantifying each player's contribution. In this study, we considered the situation impacted by a disastrous event as a game. In the game, the initially failed nodes are regarded as players who collaborate to impact the entire network by causing cascading failures. The Shapley value measures the contribution of each of the initially failed nodes in the game. The weighted sum of Shapley values for all games provides the node's significance. Once the significance values are calculated for all nodes in the network, the nodes with the highest significance are chosen for protection because they have the greatest impact on the network's operation.

The performance of the proposed method was evaluated experimentally, comparing several different methods. The results of the experiments show that the proposed method exhibited the best performance in terms of the number of failed nodes induced by the initial failures.

Contributions In this paper, we make the following key contributions:

- 1) We present a novel approach to quantifying node significance using the Shapley value.
- 2) We propose a failure mitigation strategy based on node significance quantified using the Shapley value, demonstrating its effectiveness in reducing failure propagation more efficiently than existing methods.

The remainder of this paper is organized as follows. Section 2 describes the model of interdependent networks and the problem of hardening nodes. Section 3 describes the concept of Shapley values. Section 4 presents the proposed method that uses the Shapley value to select nodes to be hardened. Section 5 presents the experiments and their results. Section 6 briefly summarizes related works. Section 7 concludes the paper.

2. MODEL AND PROBLEM DEFINITION

This section first describes the model of interdependent networks and then formally defines the problem to address.

2.1 Implicative Interdependency Model

We adopt the *Implicative Interdependency Model (IIM)* as the formal model of interdependent networks [2], as it generalizes other common models.

Definition 1 (IIM). An IIM represents an interdependent network and is defined as a tuple $(N, \mathcal{B})$, where N is a finite set of nodes and $\mathcal{B}$ is a finite set of Boolean formulas. We identify the nodes in N with the Boolean variables that represent these nodes, meaning that N is a set of nodes as well as a set of Boolean variables, each representing a node in the network. A Boolean formula in $\mathcal{B}$ is of form $i \rightarrow b$ where $i \in N$, $\rightarrow$ is the implication operator, and b is a sub-formula of the form $term_1 + term_2 + \dots +$ such that $+$ is the OR operator and $term_i$ is a Boolean product of variables in N .

As an illustrative example, consider a simple interdependent network consisting of four nodes: A , B , C , and D , where these nodes are mutually dependent on each other as follows:

- 1) For A to function properly, both nodes B and C must be operating correctly.
- 2) For B to function properly, either node A or node D must be operating correctly.
- 3) For C to function properly, either node A or node D must be operating correctly.
- 4) For D to function properly, either node B or node C must be operating correctly.

With IIM, this interdependency property is represented as a set of Boolean formulas as follows:

$$A \rightarrow BC \quad (1)$$

$$B \rightarrow A + D \quad (2)$$

$$C \rightarrow A + D \quad (3)$$

$$D \rightarrow B + C \quad (4)$$

Next, we define the behavior of an interdependent network modeled as IIM. The behavior represents the propagation of failures. Each node has a state that is either *operational* or *failed*. The state of a node changes over time depending on the states of other nodes. We consider discrete time starting from time 0 and say that a Boolean formula in $\mathcal{B}$ holds at time t

if it evaluates to true when the variables corresponding to operational nodes are set to true and those corresponding to failed nodes are set to false. For instance, in the above example, suppose that the states of A and B are operational and failed respectively at time 0. Then, $A \rightarrow BC$ does not hold at time 1. (Note that the formula is equivalent to $\neg A + BC$.) The state of a node i at time t is determined depending on the states of the network's nodes at time $t - 1$ as follows. The state of i is operational at time t (≥ 1) if and only if i is operational and the Boolean formula for i in $\mathcal{B}$ holds at time $t - 1$. Equivalently, the state of i is failed at time t if and only if i 's state is failed or the Boolean formula for i in $\mathcal{B}$ does not hold at time $t - 1$. We say that the system is in the *steady-state* at time t if no more state changes occur between time t and time $t + 1$. Additionally, we define a node as an *induced failure node* if it had not failed at time 0 but subsequently failed due to failure propagation originating from the initially failed nodes (i.e., those that failed at time 0).

For example, assume that a failure occurs at node B in the

initial state (i.e., time 0). In this case, according to (1), the state of A changes to failed at time 1, with the initial failure propagating to A . By contrast, (3) and (4) hold at time 0. They also hold at time 1. As a result, no state changes occur between times 1 and 2. Table I illustrates the failure propagation for this case. In this table, each row represents the state changes for a node as time elapses. F represents the failed state.

Let us consider two other cases. Table II illustrates the case where D is the only initially failed node. In this case, the failure of D does not propagate, and the system reaches the steady-state immediately.

Finally, let us consider the case where two nodes B and D suffer from the initial failures. In this case, according to (1), the failure propagates to node A at time 1. The failure of two nodes A and D then propagates to node C at time 2, according to (3). All nodes are therefore in the failed state at time 3, with the system reaching a steady-state. Table III shows the propagation of failures in this case.

Comparing these three cases, one may notice that the impact of each node's failure cannot be measured in isolation. The initial failure of D affects no other nodes; however, if it occurs simultaneously with B 's failures, it leads to the total malfunction of the entire network. In short, the measurement of the impact or significance of each node must take into account the interdependent effects with other nodes.

2.2 Hardening Problem

Here, we formally describe the problem we tackle in this study. We consider a catastrophic event, such as an earthquake, which may result in the failure of many nodes. The objective is to select k nodes to harden, thereby minimizing the impact of the initial failures, under the assumption that hardened nodes remain operational at all times. We refer to this problem as the *hardening problem* because it shares fundamental concepts with a problem posed by an earlier study [2], which was also called the hardening problem.

TABLE I
FAILURE PROPAGATION WHEN B HAS FAILED IN THE INITIAL STATE

nodes	time step (t)		
	0	1	2
A		F	F
B	F	F	F
C			
D			

TABLE II
FAILURE PROPAGATION WHEN D HAS FAILED IN THE INITIAL STATE

nodes	time step (t)		
	0	1	2
A			
B			
C			
D	F	F	F

TABLE III
FAILURE PROPAGATION WHEN B AND D HAVE FAILED IN THE INITIAL STATE

nodes	time step (t)		
	0	1	2
A		F	F
B	F	F	F
C			F
D	F	F	F

This problem can be regarded as an optimization problem where the goal is to minimize an objective function representing the failures' impact. We define the objective function as the expected number of induced failure nodes. Also, we assume that each node has failed at time 0 with probability p . Our hardening problem is defined as follows.

Definition 2 (Hardening problem). Given an IIM $(N, \mathcal{B})$, a probability p that the state of a node is failed at time 0, and integer k such that $1 \leq k < |N|$, select k nodes in N to harden such that the expected number of induced failure nodes is minimized.

The computational complexity of this problem has not yet been evaluated. However, we believe that it is very likely to be NP-hard. This is based on the fact that the hardening problem in a previous study was determined to be NP-hard, even though it assumes much more restrictive conditions [2]. NP-hardness implies that there is no efficient exact solution (unless $P=NP$), meaning that we require an approximate solution.

3. SHAPLEY VALUES

Shapely value is a concept established in the field of game theory, a branch of economics. The main usage of this concept is to fairly distribute the total payoff obtained from a task accomplished by a group of collaborating players. This situation, where players collaborate to achieve the same goal, is mathematically modeled as a *coalitional game*.

Definition 3 (Coalitional game [3]). A coalitional game with transferable payoff (coalitional game for short) is represented as (N, v) , where N is a set of players, and $v : 2^N \rightarrow \mathbb{R}$ is a characteristic function that associates every subset S of N (a coalition) with a real number $v(S)$.

For example, suppose that there are three players, A , B , and C , who cooperate towards a common goal. If all three players cooperate, the goal will be fully accomplished, and they will receive the full reward. Meanwhile, if only some members cooperate, the goal will be partially achieved, and the reward they obtain will be diminished as follows:

- (1) If no one collaborates, no reward is given.
- (2) If only player A collaborates, the reward is 1.
- (3) If only player B collaborates, the reward is 2.
- (4) If only player C collaborates, the reward is 2.
- (5) If two players A and B collaborate, the reward is 3.
- (6) If two players A and C collaborate, the reward is 2.

- (7) If two players B and C collaborate, the reward is 3.
- (8) If all players collaborate, the reward is 4.

This situation is modeled as a coalitional game (N, v) , where $N = \{A, B, C\}$ and v is defined as follows:

- (1) $v(\emptyset) = 0$
- (2) $v(\{A\}) = 1$
- (3) $v(\{B\}) = 2$
- (4) $v(\{C\}) = 2$
- (5) $v(\{A, B\}) = 3$
- (6) $v(\{A, C\}) = 2$
- (7) $v(\{B, C\}) = 3$
- (8) $v(\{A, B, C\}) = 4$

Given a coalitional game (N, v) , the *Shapley value* provides a distribution of the total gains or reward among players based on their individual contributions.

Definition 4 (Shapley value). The Shapley value for player i is defined as follows:

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} (v(S \cup \{i\}) - v(S))$$

Equivalently, the Shapley value is defined in terms of orderings of players as follows.

Definition 5 (Shapley value (ordering-based)). The Shapley value for player i is defined as follows:

$$\phi_i(v) = \frac{1}{|N|!} \sum_{o \in \Pi} v(S_i(o) \cup \{i\}) - v(S_i(o))$$

where Π is the set of all orderings of N , and $S_i(o)$ is the set of players preceding i in the ordering o .

The above formula's $v(S_i(o) \cup \{i\}) - v(S_i(o))$ signifies the *marginal contribution* of player i to the coalition for ordering o .

Let us consider the above example again. Table IV lists all orderings of $N = \{A, B, C\}$ and the marginal contribution of every player for each ordering. For example, for the ordering $(A \ B \ C)$, the marginal contributions are 1, 2, and 1 for A , B , and C , respectively, as $v(\{A\}) - v(\emptyset) = 1$, $v(\{A, B\}) - v(\{A\}) = 2$, and $v(\{A, B, C\}) - v(\{A, B\}) = 1$. The bottom row shows the average of each player's marginal contributions over all orderings, which is equivalent to the Shapley value.

It is well-known that the Shapley value is the only value that satisfies the following properties:

Efficiency $\sum_{i \in N} \phi_i(v) = v(N)$.

Symmetry If two players have the same marginal contribution on each coalition, their Shapley values are the same.

Dummy player $\phi_i(v) = 0$ if i 's marginal contribution is always zero.

Additivity $\phi_i(v + w) = \phi_i(v) + \phi_i(w)$. That is, the Shapley values of the sums of games are the sums of the Shapley values of the individual games.

These properties are often recognized as reasonable indicators of fairness.

TABLE IV
MARGINAL CONTRIBUTION AND SHAPLEY VALUE

order			players		
1st	2nd	3rd	A	B	C
A	B	C	1	2	1
A	C	B	1	2	1
B	A	C	1	2	1
B	C	A	1	2	1
C	A	B	0	2	2
C	B	A	1	1	2
Shapley values			0.83	1.83	1.33

4. PROPOSED METHOD FOR THE HARDENING PROBLEM

This section describes our proposed method for selecting nodes to be hardened. The proposed method consists of two steps:

Step 1. Evaluate the significance of the nodes based on the Shapley value.

Step 2. Selects k nodes to be hardened in descending order of the significance.

Shapley value-based significance evaluation is therefore at the core of the method. In the remainder of this section, we describe how we can represent the network in the face of failures as coalitional games and quantify the significance of each node based on Shapley value.

4.1 Modeling Networks with Failures as Coalitional games

We begin by describing a coalitional game that represents a network in which specific nodes have initially failed. The collective way to represent all possible failure patterns will be presented later.

We consider an interdependent network represented as IIM $(N, \mathcal{B})$. We let $f(S)$ denote the number of the failed nodes at the steady-state when S is the set of initially failed nodes, i.e., the sum of the number of the initially failed nodes (i.e., $|S|$) and the number of induced failure nodes.

For example, consider the example of the four-node network discussed in Section 2. For this network, when B is the only initially failed node, nodes A and B have eventually failed in the steady-state. Thus, $f(B) = |\{A, B\}| = 2$. When B and D have initially failed, all nodes will have failed at the steady-state. Therefore, $f(\{B, D\}) = 4$. Consequently, we have

$$\begin{aligned}
 f(\emptyset) &= 0, \\
 f(\{A\}) &= f(\{D\}) = 1, \\
 f(\{B\}) &= f(\{C\}) = f(\{A, B\}) = f(\{A, C\}) = 2, \text{ and} \\
 f(\{A, D\}) &= f(\{B, C\}) = f(\{B, D\}) = f(\{C, D\}) = \\
 f(\{A, B, C\}) &= f(\{A, B, D\}) = f(\{A, C, D\}) = \\
 f(\{B, C, D\}) &= f(\{A, B, C, D\}) = 4.
 \end{aligned}$$

Now, suppose that a catastrophic event occurs and that the event causes a set of nodes to be failed at time 0. Let I be the set of these initially failed nodes. We model this situation as a coalitional game as follows:

TABLE V
MARGINAL CONTRIBUTIONS AND SHAPLEY VALUES (GAME 1)

(a) Game G_I with $I = \{A, B\}$			
order		nodes	
1st	2nd	A	B
A	B	1	1
B	A	0	2
Shapley values		0.5	1.5

Definition 6 (Coalitional game with initially failed nodes I). Let G_I denote the coalitional game when I is the set of initially failed nodes and define it as follows:

$$G_I = (I, v)$$

where $v(S) = f(S)$ for $S \subseteq I$.

Given G_I , the Shapley value can be obtained for the nodes in I according to its definition. Table V illustrates how the Shapely value is computed for the above example when $I = \{A, B\}$. In the table, each row corresponds to the ordering of I and the marginal contributions of A and B for that ordering. The bottom row shows the Shapley values for these two nodes.

4.2 Node Significance Evaluation Based on Shapley Value

The previous subsection explained how to quantify the impact of an initially failed node as the Shapley value when a set of the initially failed nodes is given. This subsection describes how to holistically evaluate the importance of the nodes in all possible circumstances.

Our idea is that we view the significance of a node as a weighted sum of its Shapley values for all possible games G_I . The weight for a game G_I is the probability that G_I occurs, i.e., the probability that I is the set of the initially failed nodes. Recall that we assume that the state of a node is failed at time 0 with probability p .

Definition 7 (Significance of a node). The significance of node i is defined as follows:

$$s_i = \sum_{I \subseteq N} w_I \psi(G_I, i)$$

where

w_I is a weight and equals the probability that I is the set of the initially failed nodes, i.e., $w_I = p^{|I|}(1-p)^{|N \setminus I|}$.

$\psi(G_I, i)$ is the Shapley value of node i for the game G_I . $\psi(G_I, i) = 0$ if $i \notin I$.

For the four-node network example, the importance of node A is computed as follows:

$$\begin{aligned}
& s_A \\
& = p(1-p)^3\psi(G_{\{A\}}, A) + p^2(1-p)^2\psi(G_{\{A,B\}}, A) + \\
& \quad p^2(1-p)^2\psi(G_{\{A,C\}}, A) + p^2(1-p)^2\psi(G_{\{A,D\}}, A) + \\
& \quad p^3(1-p)\psi(G_{\{A,B,C\}}, A) + p^3(1-p)\psi(G_{\{A,B,D\}}, A) + \\
& \quad p^3(1-p)\psi(G_{\{A,C,D\}}, A) + p^4\psi(G_{\{A,B,C,D\}}, A) \\
& = p(1-p)^3(1) + p^2(1-p)^2(0.5 + 0.5 + 2) + \\
& \quad p^3(1-p)(0.33 + 0.83 + 0.83) + p^4(0.5)
\end{aligned}$$

The Shapley values for the games involved can be found in Tables V, VI and VII.

4.3 Approximating Shapley Value

Computing the Shapley value is a computationally intractable task, except for some special cases [4], [5]. Our problem does not belong to these special cases. Therefore, we resort to an approximation approach that works efficiently.

We use Monte Carlo simulation to this end. Algorithm 1 displays the proposed approach. Each simulation loop works as follows. First, the algorithm selects a subset of nodes by choosing each node with probability p . Let S be the subset of nodes. Then, the algorithm selects one from $|S|!$ orderings of the nodes in S uniformly randomly. For the selected ordering, the algorithm computes the marginal contribution for each node in S . The marginal contribution is summed for each node at the end of the loop. After the repetition of the loop has completed, the summed marginal contribution for a node is averaged over the total number of loop iterations, yielding an approximation of the node's significance.

Algorithm 1 Approximating the significance values of nodes

```

 $s_i \leftarrow 0$  for  $i \in N$  ▷ significance for node  $i$ 
for 1 to number_of_loops do
   $S \leftarrow$  a subset of nodes, each selected with probability  $p$ 
   $o \leftarrow$  an ordering of  $S$ , selected uniformly randomly from  $|S|!$  orderings
   $\Delta_i \leftarrow$  the marginal contribution for all  $i \in S$  for  $o$ 
   $s_i \leftarrow s_i + \Delta_i$  for all  $i \in S$ 
end for
 $s_i \leftarrow s_i / \text{number\_of\_loops}$ 

```

As a solution to the hardening problem, this approximation method does not require stringent guarantees of error bounds. In our approach, the top- k nodes in terms of significance are selected to be hardened. Therefore, approximation errors have no effects on the selection of nodes as long as the top- k nodes remain unchanged. Even if errors alter the top- k nodes, the difference usually appears only among lower-ranked nodes, in which case the effects on performance (i.e., the number of failed nodes) become marginal.

5. EVALUATION

This section describes the experiments conducted for evaluation purposes. In the experiments, we applied our proposed method to four interdependent network models presented

TABLE VI
MARGINAL CONTRIBUTIONS AND SHAPLEY VALUES (GAMES 2–6)

(a) Game G_I with $I = \{A, C\}$

order		nodes	
1st	2nd	A	C
A	C	1	1
C	A	0	2
Shapley values		0.5	1.5

(b) Game G_I with $I = \{A, D\}$

order		nodes	
1st	2nd	A	D
A	D	1	3
D	A	3	1
Shapley values		2	2

(c) Game G_I with $I = \{A, B, C\}$

order			nodes		
1st	2nd	3rd	A	B	C
A	B	C	1	1	2
A	C	B	1	2	1
B	A	C	0	2	2
B	C	A	0	2	2
C	A	B	0	2	2
C	B	A	0	2	2
Shapley values			0.33	1.83	1.83

(d) Game G_I with $I = \{A, B, D\}$

order			nodes		
1st	2nd	3rd	A	B	D
A	B	D	1	1	2
A	D	B	1	0	3
B	A	D	0	2	2
B	D	A	0	2	2
D	A	B	3	0	1
D	B	A	0	3	1
Shapley values			0.83	1.33	1.83

(e) Game G_I with $I = \{A, C, D\}$

order			nodes		
1st	2nd	3rd	A	C	D
A	C	D	1	1	2
A	D	C	1	0	3
C	A	D	0	2	2
C	D	A	0	2	2
D	A	C	3	0	1
D	C	A	0	3	1
Shapley values			0.83	1.33	1.83

TABLE VII
MARGINAL CONTRIBUTIONS AND SHAPLEY VALUES (GAME 7)

(a) Game G_I with $I = \{A, B, C, D\}$

order				nodes			
1st	2nd	3rd	4th	A	B	C	D
A	B	C	D	1	1	2	0
A	B	D	C	1	1	0	2
A	C	B	D	1	2	1	0
A	C	D	B	1	0	1	2
A	D	B	C	1	0	0	3
A	D	C	B	1	0	0	3
B	A	C	D	0	2	2	0
B	A	D	C	0	2	0	2
B	C	A	D	0	2	2	0
B	C	D	A	0	2	2	0
B	D	A	C	0	2	0	2
B	D	C	A	0	2	0	2
C	A	B	D	0	2	2	0
C	A	D	B	0	0	2	2
C	B	A	D	0	2	2	0
C	B	D	A	0	2	2	0
C	D	A	B	0	0	2	2
C	D	B	A	0	0	2	2
D	A	B	C	3	0	0	1
D	A	C	B	3	0	0	1
D	B	A	C	0	3	0	1
D	B	C	A	0	3	0	1
D	C	A	B	0	0	3	1
D	C	B	A	0	0	3	1
Shapley values				0.5	1.16	1.16	1.16

in [2]. These models were constructed based on real-world data of Maricopa County, Arizona, USA. The power system includes power plants and transmission lines, while the communication control system includes buildings and cables. These models represent four disjoint areas of the network where cascading failures could occur. Table VIII summarizes the numbers of nodes composing these models.

We varied the number of nodes to be hardened, k , from one to $n - 1$, where n is the total number of nodes. Also, we tested four different values of p , the probability of a node being initially failed: 0.1, 0.25, 0.5, and 0.75. For comparison purposes, we evaluated three additional methods besides the

TABLE VIII
NETWORK COMPONENTS IN FOUR MODELS

No.	Nodes	Power Plants	Lines	Buildings	Cables
1	48	5	24	15	4
2	46	8	20	12	6
3	48	5	24	15	4
4	53	5	28	16	4

proposed one, which we refer to as Elite, Greedy, and Shapley. The Elite method selects nodes in descending order of the number of failure nodes induced by their individual failure, as follows. First, simulate the failure propagation caused independently by each node's failure to determine the number of induced failures. Then, rank all nodes in descending order by this number and select the top- k nodes accordingly. In other words, this method selects the most influential nodes without considering interactions among failures of other nodes.

The Greedy method selects one node at a time. The selection criterion is that the failure of the selected node results in the greatest number of newly induced failure nodes, assuming the previously selected nodes have already failed. The method works as follows. First, simulate the failure propagation with exactly one initially failed node and select the node that resulted in the greatest number of additional failed nodes. Next, simulate the failure propagation with the previously selected node(s) and one more initially failed node. Select the node that results in the greatest number of additional failures. Repeat this process until k nodes have been selected. This process considers interactions with previously selected nodes, but the scope of consideration is minimal.

When distinct nodes received identical rankings under either method, ties were broken by prioritizing nodes that appeared in a greater number of the IIM's Boolean formulas, excluding appearances as the leftmost variable. Intuitively, such nodes are likely to exert greater influence on other nodes. If nodes remained tied, their relative ranking was determined randomly. The Shapley method was devised in the early stage of the study, around the time when we first conceived the idea of using the Shapley value for node significance evaluation. This method ranks nodes based on their Shapley value in the game G_N , selecting the top- k nodes in descending order. In other words, this method quantifies node importance by evaluating the average marginal contribution across all $n!$ possible orderings of the nodes. As in the proposed method, Monte Carlo simulation was used to approximate the Shapley value in this method.

We set the number of approximation loops to 50,000 in both the Shapley and proposed methods. This ensures that approximation errors have no effects on the selection of nodes, providing a reliable estimate of significance.

Given a fixed k value, a simulation experiment was conducted for each of the node selection methods as follows. First, we selected k nodes to be hardened using the node selection method. The simulation loops were then repeated. In the loop, we first selected a set of initially failed nodes by choosing each node with probability p . Note that these the k hardened nodes were never included in the initially failed nodes. Then, we evaluated the average of the total number of resulting failed nodes by simulating the failure propagation. We repeated 100,000 loop times to ensure numerical stability. The number of iterations was determined based on the requirement that the average number of induced failure nodes converge, indicating that the simulation has reached a reliable estimate of node significance.

All methods, including the proposed approach, as well as the simulation program, were implemented in Python. The experiments were conducted on Google Colab, a cloud-based computing platform, using its default settings and accessed through a Windows 11 laptop.

For each network model and each probability p , we measured the computation time required to determine the top- k nodes for each method.

The results of our experiments are shown in Figures 1, 2, 3, and 4, each corresponding to one of the four networks. Each figure contains four graphs presenting results for initial failure probabilities $p = 0.1, 0.25, 0.5$, and 0.75 . The horizontal and vertical axes of the graphs indicate the number of hardened nodes k and the average number of induced failure nodes, respectively. Note that induced failure nodes are those that did not initially fail but failed in the steady-state.

Despite minor differences, all cases exhibited common trends, as detailed below. First, as the number of hardened nodes gradually increases from zero, the average number of induced failure nodes decreases sharply and eventually converges to zero. However, this decrease is not always monotonic; in some cases, failure propagation fluctuates before eventually converging to zero.

Among the four methods tested, the proposed one exhibited the best overall performance in nearly all cases. The curves representing the proposed method show the steepest decline as k , the number of nodes that can be hardened, increases from one to larger values. Although the performance difference is less noticeable when k is close to one or very large, the proposed method consistently resulted in the fewest induced failure nodes across a wide range of k values, for nearly all models and values of p .

The Greedy method yielded a small number of induced failure nodes comparable to the proposed approach when the number of hardened nodes was low, for instance, when $k \leq 5$. Because this method selects nodes to harden one at a time, it tends to be effective at identifying the top few most influential nodes. We believe that this characteristic explains its strong performance when k is small. In contrast, as k increases, the reduction in induced failure nodes becomes less significant.

The Shapley method exhibited relatively good performance when k is large. This is likely due to its consideration of all possible orderings of node failures when selecting the top k nodes. However, since the method does not consider failure probabilities, it essentially evaluates node significance under the assumption that all nodes fail. This shortcoming leads to a low performance, especially when k is small.

The Elite method consistently displayed the second-best performance across a wide range of k values in many cases. Unlike the other methods, it selects nodes solely based on their individual influence. This feature explains its strong performance for small k . Interestingly, the relatively strong performance becomes more apparent as k increases, particularly in comparison with the Greedy and Shapley methods. However, as k increases further, the reduction in the number of induced failure nodes slows earlier, leading to a slightly

higher number of failures.

To summarize, the observed performance trend followed the order: proposed $>$ Elite $>$ Greedy $\approx$ Shapley.

For each method, we measured the computation time required to determine the top- k nodes. The results, summarized in Table IX, reveal a consistent trend: Elite $<$ Greedy $\ll$ Shapley $<$ proposed. The Elite and Greedy methods are significantly faster than the Shapley and proposed methods. This is primarily because both the Shapley and proposed methods used Monte Carlo simulation to estimate Shapley values, with a number of simulation loops contributing to their long computation time. The execution time of the proposed methods shows a slight increase as the failure probability p grows. This can be attributed to the fact that higher values of p lead to the selection of more failure nodes in each simulation run, requiring more orderings of them to be checked and thereby increasing the overall computation time.

6. RELATED WORK

The field of network science has attracted considerable attention [6]. This attention is due to the vast amounts of data routinely collected, modeled, and analyzed in various domains, such as social networks [7], [8], economics [9], and biology [10].

In the network science field, interdependent networks have recently gained attentions, as they provide new means of accounting for phenomena that had not been explained by previous studies. In particular, it was shown that cascading failures, which have often been observed in power systems, can be explained using interdependent network models representing the interactions between the power-line and control networks [11], [12].

There have been many studies on the cascading failures in interdependent networks. For example, Parandehgheibi et al. [13] studied the minimum number of initial failure nodes required for a complete power failure, i.e., failure of all nodes in the network. Huang et al. [14] also analyzed a smart grid consisting of interdependent transmission and control networks and proved the existence of a threshold for a fraction of failure nodes at which the system collapses. More recent studies on mathematical nature of failures in interdependent networks include [15], [16].

TABLE IX
COMPUTATION TIME FOR TOP- k NODE SELECTION (S)

Method	Model 1	Model 2	Model 3	Model 4
Elite	0.001	0.001	0.001	0.001
Greedy	0.025	0.016	0.023	0.029
Shapley	404	380	421	566
Proposed Method				
p=0.10	22.7	19.3	23.5	30.8
p=0.25	101	92.8	100	134
p=0.50	248	227	244	321
p=0.75	366	345	373	496

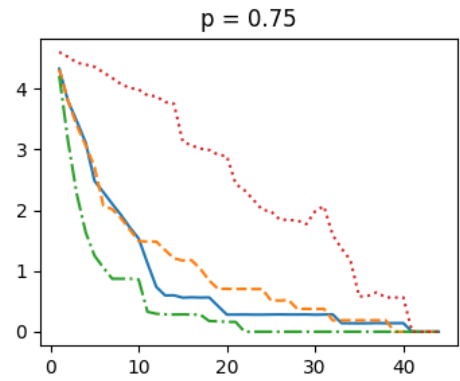
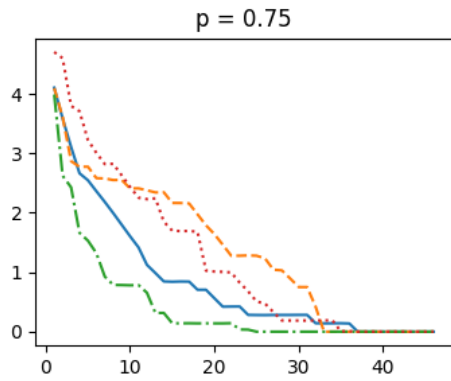
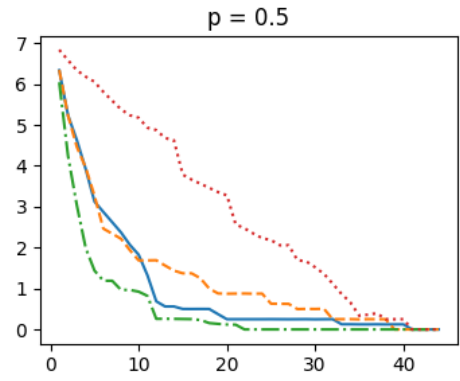
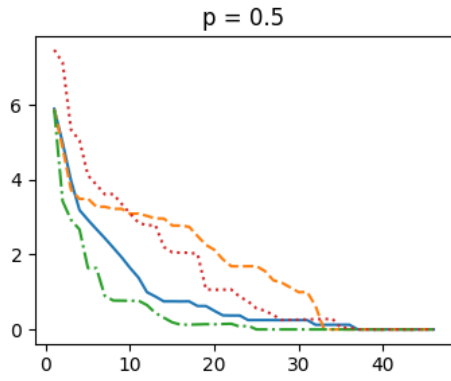
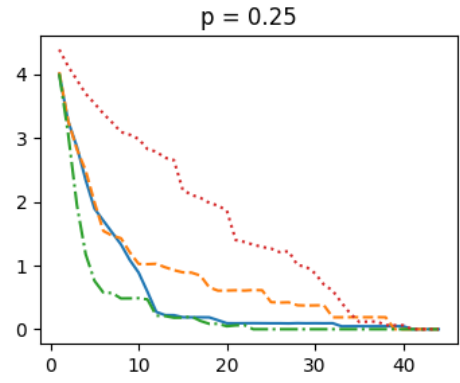
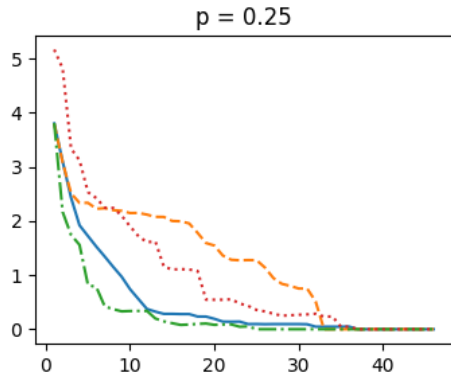
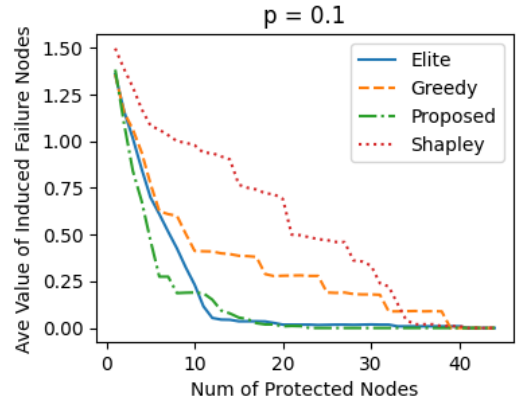
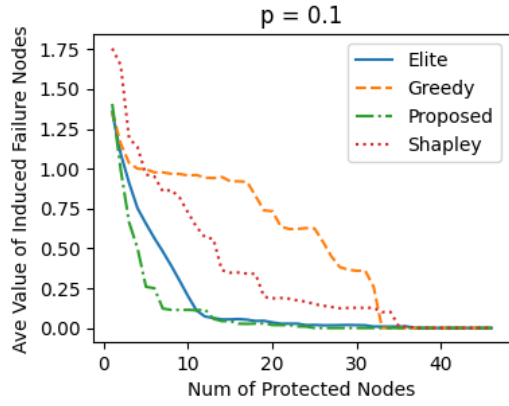


Figure 1. Average number of induced failure nodes as a function of the number of protected nodes (k) for Model 1

Figure 2. Average number of induced failure nodes as a function of the number of protected nodes (k) for Model 2

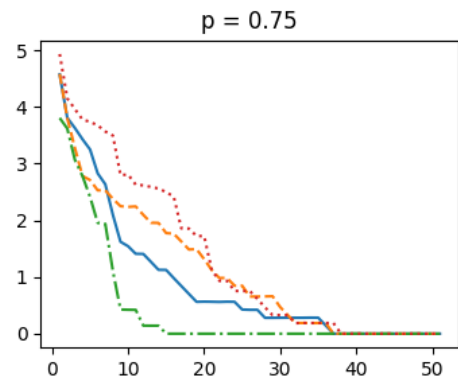
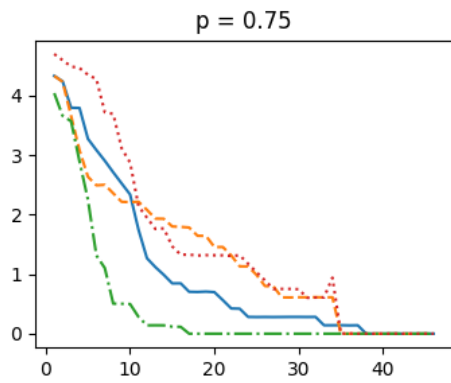
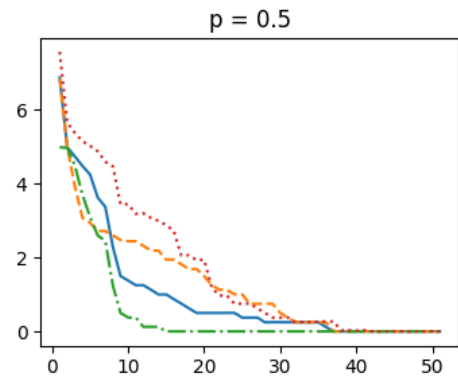
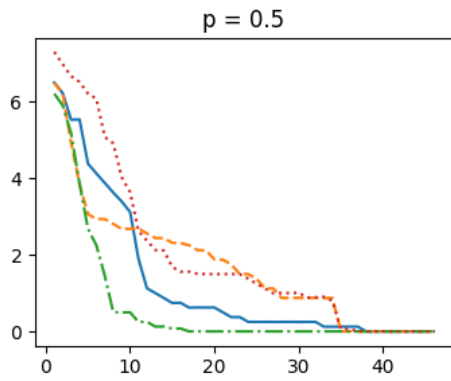
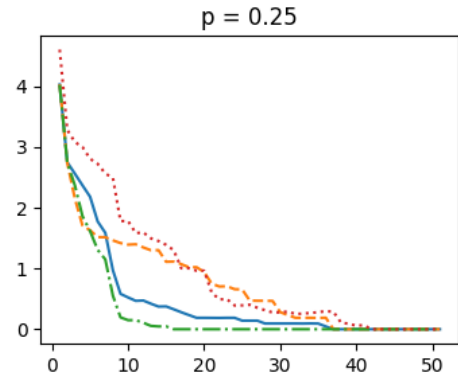
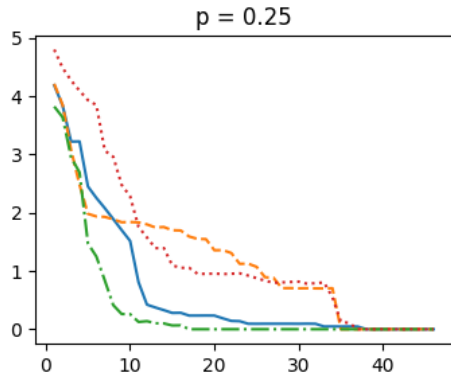
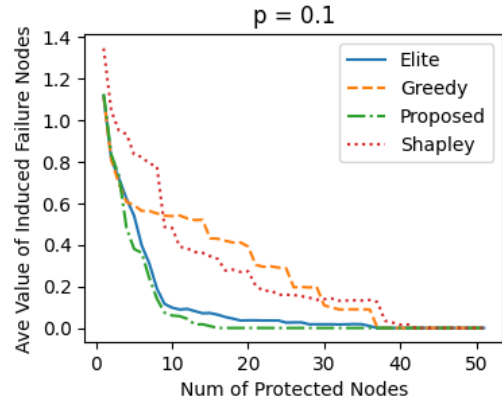
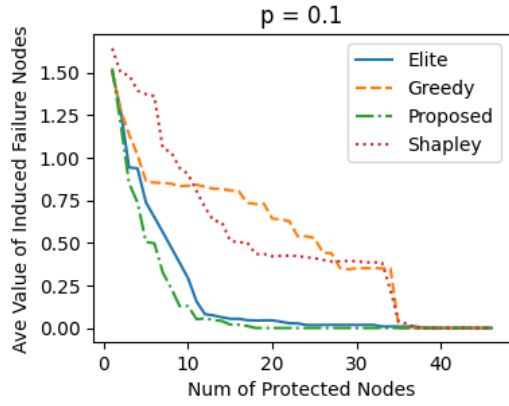


Figure 3. Average number of induced failure nodes as a function of the number of protected nodes (k) for Model 3

Figure 4. Average number of induced failure nodes as a function of the number of protected nodes (k) for Model 4

The most relevant study to ours is [2], which formulates the hardening problem based on the IIM. In that study, it is assumed that the initially failed nodes are known in advance of selecting which nodes to harden. Accordingly, their objective is to minimize the number of subsequent node failures induced by these known failures. In contrast, our study does not rely on this assumption; instead, we model each node as failing with a certain probability.

The hardening problem addressed in this study can be viewed as an instance of identifying the most important nodes within a network. A node that is crucial in operating the network is often called *critical*. Identifying critical nodes is of practical importance, as it can help prevent failure propagation and improve system reliability and failure tolerance [17]. Studies on critical node identification for interdependent networks include [18]–[24]. These studies used different approaches from each other based on different interdependent network models.

There have only been a few exceptional studies that utilize cooperative game theory in the context of network reliability. These examples include [25]–[27]. Suri et al. [28] and Aadithya et al. [29] used Shapley value to determine the relative influence of each node in a network. To the best of our knowledge, no studies have utilized Shapley value to solve reliability issues in interdependent networks, except for our studies, including earlier ones [30], [31].

Apart from reliability context, Shapley value is used for quantifying the influence of nodes in a complex network, such as a social network, in, for example, [28], [29]. Again, all these examples we know of do not consider interdependent networks.

In general, the complexity of the computation of Shapley value is NP-hard. Known polynomial-time computation algorithms exist only under special conditions. Therefore, approximation approaches have been widely studied (e.g., [32], [33]). In this study, we used a simple Monte Carlo method to approximate the Shapley value. We have yet to apply these approximation approaches to our context, but we plan to explore them in the future.

7. CONCLUSION

To solve the hardening problem in interdependent networks, we proposed using the Shapley value to quantify the significance of individual nodes. This is enabled by treating the failure propagation as a coalitional game where initially failed nodes collaborate to impact the network. Once the significance is evaluated for all nodes, the top k nodes are selected to be hardened. Experimental results demonstrated that this method outperformed other methods.

Possible future work includes the following:

- (1) Improve the algorithm for approximating Shapley value in terms of speed and error.
- (2) Adapt the proposed approach to different failure assumptions, such as non-independent and identically distributed (non-IID) failures.

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