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Author(s)	Takai, Shigemasa
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Robust Codiagnosability for Decentralized Diagnosis of Uncertain Discrete Event Systems

Shigemasa Takai

Abstract—We consider a robust decentralized diagnosis problem for uncertain discrete event systems. We assume that, instead of the single exact model, a set of multiple possible models, each of which has its own event set and own normal behavior, is given for the task of diagnosis. The robust decentralized diagnosis problem requires us to synthesize a decentralized diagnoser such that, for all possible models, it detects the occurrence of any failure string within a finite number of steps. To characterize the existence of such a robust decentralized diagnoser, we introduce a notion of robust codiagnosability of the set of possible models. Then, we develop a method for verifying robust codiagnosability effectively.

I. INTRODUCTION

For discrete event systems (DESs), a notion of diagnosability was introduced in [15] as a property which ensures that the occurrence of any failure string can be detected within a finite number of steps. In the diagnosis framework of [15], there is a single diagnoser that infers whether a failure string has occurred based on partial event observations. For networked systems including traffic control systems and manufacturing systems, a decentralized diagnosis scheme where multiple local diagnosers independently diagnose the system based on local event observations [5], [16], [12], [23], [7], [24], [19], [11], [20], [21] is more suitable than the centralized scheme. To guarantee the detection of the occurrence of any failure string in the decentralized scheme, the diagnosability property of [15] needs to be extended, and the extended property is referred to as codiagnosability. According to the architectures of decentralized diagnosis including the disjunctive architecture [5], [12], conjunctive architecture [23], conditional architecture [23], and inference-based architecture [7], [19], several notions of codiagnosability have been proposed.

In the existing works on failure diagnosis mentioned above, it is implicitly assumed that a single model of the system to be diagnosed has been identified. Model uncertainty is one of the important issues in model based approaches for control and diagnosis of DESs [9], [25], [8]. In [25], an algorithm was developed to determine the exact model of the system from the set of multiple possible models. If the uncertainty cannot be resolved to determine the exact model, then we need a scheme that allows the existence of the model uncertainty. In supervisory control of DESs [14], a robust supervisory control problem was formulated in [9], which

requires us to synthesize a single supervisor that achieves the specification for all possible models. In [9], it was assumed that all the possible models have the same event set and the specification language is a sublanguage of the marked language of each possible model. These assumptions were relaxed under full as well as partial event observations [1], [13].

Model uncertainty was also addressed in failure diagnosis of DESs. In [8], a learning-based approach for constructing a diagnoser was proposed. A robust failure diagnosis problem formulated in [17] requires us to synthesize a single diagnoser such that, for all possible models, it detects the occurrence of any failure string within a finite number of steps. The notion of robust diagnosability was introduced to characterize the existence of such a robust diagnoser, and a method for verifying robust diagnosability was developed [17]. In the setting of [17], while the event set is common among all possible models, each possible model has its own normal behavior. In [2], the robust diagnosability property of [17] was generalized by allowing each possible model to have its own event set. The generalized notion of robust diagnosability of [2] subsumes diagnosability against permanent sensor failures [4], [6] and diagnosability against intermittent sensor failures [3]. In addition, an algorithm was developed for verifying generalized robust diagnosability, which improves the computational complexity as compared with the verification method of [17]. Besides, in a similar setting to [17], a robust prognosis problem was solved in [18].

In this paper, we consider the robust diagnosis problem in the decentralized setting. As in [2], we allow each possible model to have its own event set and own normal behavior. We employ the disjunctive architecture [5], [12] and define a notion of robust codiagnosability by extending generalized robust diagnosability of [2]. The defined notion of robust codiagnosability can also be regarded as a generalization of disjunctive codiagnosability of [12]. We show that robust codiagnosability is a necessary and sufficient condition for the existence of a decentralized diagnoser such that, for all possible models, it detects the occurrence of any failure string within a finite number of steps. Furthermore, we develop a method for verifying robust codiagnosability effectively.

II. PRELIMINARIES

A. Automata and Languages

A DES to be diagnosed is modeled as a finite automaton $G = (Q, \Sigma, \delta, q_0)$, where Q is the finite set of states, Σ is the finite set of events, a partial function $\delta : Q \times \Sigma \rightarrow Q$

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Shigemasa Takai is with the Division of Electrical, Electronic and Information Engineering, The University of Osaka, Suita, Osaka 565-0871, Japan takai@eei.eng.osaka-u.ac.jp

is the transition function, and $q_0 \in Q$ is the initial state. A sequence $q^{(0)} \xrightarrow{\sigma^{(0)}} q^{(1)} \xrightarrow{\sigma^{(1)}} \dots q^{(k-1)} \xrightarrow{\sigma^{(k-1)}} q^{(k)}$ ($k \geq 1$) of transitions such that $\delta(q^{(h)}, \sigma^{(h)}) = q^{(h+1)}$ for each $h \in \{0, 1, \dots, k-1\}$ is called a path of G , and it is also called a cycle if $q^{(0)} = q^{(k)}$.

Σ^* denotes the set of all finite strings of elements of Σ , including the empty string ε . The transition function δ can be generalized to $\delta : Q \times \Sigma^* \rightarrow Q$ in the usual manner. For any $q \in Q$ and any $s \in \Sigma^*$, $\delta(q, s)!$ means that $\delta(q, s) \in Q$, that is, $\delta(q, s)$ is defined. Let $\mathbb{N}$ be the set of all nonnegative integers. For any $s \in \Sigma^*$, $|s| \in \mathbb{N}$ denotes its length. For any $s \in \Sigma^*$ and any $k \in \mathbb{N}$, s^k denotes k concatenations of s . The generated language of G , denoted by $L(G)$, is defined as $L(G) = \{s \in \Sigma^* \mid \delta(q_0, s)!\}$. For each $s \in L(G)$, the postlanguage of $L(G)$ after s is defined as $L(G)/s = \{t \in \Sigma^* \mid st \in L(G)\}$. For a language $K \subseteq \Sigma^*$, the set of all prefixes of strings in K is denoted by $pr(K)$, that is, $pr(K) = \{s \in \Sigma^* \mid (\exists t \in \Sigma^*) st \in K\}$. If $K = pr(K)$, then K is said to be (prefix-)closed.

For any subset $\Sigma' \subseteq \Sigma$, $P_{\Sigma, \Sigma'} : \Sigma^* \rightarrow \Sigma'^*$ is the natural projection from Σ^* to Σ'^* , which is inductively defined as $P_{\Sigma, \Sigma'}(\varepsilon) = \varepsilon$ and

$$P_{\Sigma, \Sigma'}(s\sigma) = \begin{cases} P_{\Sigma, \Sigma'}(s)\sigma, & \text{if } \sigma \in \Sigma' \\ P_{\Sigma, \Sigma'}(s), & \text{otherwise} \end{cases}$$

for any $s \in \Sigma^*$ and any $\sigma \in \Sigma$. $P_{\Sigma, \Sigma'} : \Sigma^* \rightarrow \Sigma'^*$ can be extended to $P_{\Sigma, \Sigma'} : 2^{\Sigma^*} \rightarrow 2^{\Sigma'^*}$, where 2^{Σ^*} is the power set of Σ^* , such that $P_{\Sigma, \Sigma'}(K) = \{P_{\Sigma, \Sigma'}(s) \in \Sigma'^* \mid s \in K\}$ for any $K \in 2^{\Sigma^*}$. The inverse projection $P_{\Sigma, \Sigma'}^{-1} : \Sigma'^* \rightarrow 2^{\Sigma^*}$ is defined as $P_{\Sigma, \Sigma'}^{-1}(s') = \{s \in \Sigma^* \mid P_{\Sigma, \Sigma'}(s) = s'\}$ for any $s' \in \Sigma'^*$.

B. Diagnosability for Centralized Diagnosis

In the setting of centralized diagnosis, a single diagnoser diagnoses the system G based on the partial event observations. To model partial event observations, the event set Σ is partitioned into the observable event set Σ_o and the unobservable event set Σ_{uo} . When the system G generates a string $s \in L(G)$, a diagnoser observes the observable event string $P_{\Sigma, \Sigma_o}(s) \in \Sigma_o^*$. For any $s, s' \in L(G)$, if $P_{\Sigma, \Sigma_o}(s) = P_{\Sigma, \Sigma_o}(s')$, then they are said to be indistinguishable (under P_{Σ, Σ_o}). Formally, a diagnoser is modeled as a function $D : P_{\Sigma, \Sigma_o}(L(G)) \rightarrow \{1, 0\}$, where 1 and 0 are diagnosis decisions. For any $s \in L(G)$, $D(P_{\Sigma, \Sigma_o}(s)) = 1$ (respectively, $D(P_{\Sigma, \Sigma_o}(s)) = 0$) means that the occurrence of a failure has been (respectively, has not been) detected after $P_{\Sigma, \Sigma_o}(s)$ is observed by the diagnoser D .

The normal behavior of the system G is assumed to be described by a nonempty closed sublanguage $K \subseteq L(G)$. The occurrence of an event $\sigma \in \Sigma$ such that $s\sigma \in L(G) - K$ for some $s \in K$ models a failure. A string in K (respectively, $L(G) - K$) is called a normal (respectively, failure) string. For the sake of simplicity, we assume in the remainder of the paper that the system G is deadlock free, that is, for any $s \in L(G)$, there exists $\sigma \in \Sigma$ such that $s\sigma \in L(G)$. However, as in [12], we do not impose the assumption that there does not exist a cycle by unobservable events.

The system G is said to be diagnosable with respect to a nonempty closed sublanguage $K \subseteq L(G)$ [15] if

$$\begin{aligned} &(\exists m \in \mathbb{N})(\forall s \in L(G) - K)(\forall t \in L(G)/s) \\ &|t| \geq m \\ &\Rightarrow [(\forall u \in L(G)) P_{\Sigma, \Sigma_o}(st) = P_{\Sigma, \Sigma_o}(u) \Rightarrow u \notin K]. \end{aligned}$$

The diagnosability property serves as a necessary and sufficient condition for the existence of a diagnoser $D : P(L(G)) \rightarrow \{1, 0\}$ that satisfies

$$\begin{aligned} &(\exists m \in \mathbb{N})(\forall s \in L(G) - K)(\forall t \in L(G)/s) \\ &|t| \geq m \Rightarrow D(P_{\Sigma, \Sigma_o}(st)) = 1 \end{aligned} \quad (1)$$

and

$$(\forall s \in K) D(P_{\Sigma, \Sigma_o}(s)) = 0. \quad (2)$$

The conditions (1) and (2) mean that the occurrence of any failure string can be detected correctly within a finite number of steps.

C. Codiagnosability for Decentralized Diagnosis

In the decentralized setting, multiple local diagnosers diagnose the system G based on local event observations. Let $I = \{1, 2, \dots, n\}$, where $n \geq 2$, be the index set of local diagnosers. For each $i \in I$, the locally observable and locally unobservable event sets of the i th local diagnoser are denoted by Σ_o^i and Σ_{uo}^i , respectively, where $\Sigma_{uo}^i = \Sigma - \Sigma_o^i$. Then, the i th local diagnoser is formally represented by a function $D_i : P_{\Sigma, \Sigma_o^i}(L(G)) \rightarrow \{1, 0\}$. A decentralized diagnoser is the set $\{D_i : P_{\Sigma, \Sigma_o^i}(L(G)) \rightarrow \{1, 0\} \mid i \in I\}$ of local diagnosers and it issues the diagnosis decision according to the local diagnosis decisions of local diagnosers. In this paper, we employ the disjunctive architecture [5], [12] where if at least one local diagnoser issues the diagnosis decision 1, then the decision of the decentralized diagnoser is 1; otherwise the decision of the decentralized diagnoser is 0.

The system G is said to be codiagnosable with respect to a nonempty closed sublanguage $K \subseteq L(G)$ [12] if

$$\begin{aligned} &(\exists m \in \mathbb{N})(\forall s \in L(G) - K)(\forall t \in L(G)/s) \\ &|t| \geq m \Rightarrow [(\exists i \in I)(\forall u \in L(G)) \\ &P_{\Sigma, \Sigma_o^i}(st) = P_{\Sigma, \Sigma_o^i}(u) \Rightarrow u \notin K]. \end{aligned}$$

The codiagnosability property is a necessary and sufficient condition for the existence of a decentralized diagnoser $\{D_i : P_{\Sigma, \Sigma_o^i}(L(G)) \rightarrow \{1, 0\} \mid i \in I\}$ that satisfies

$$\begin{aligned} &(\exists m \in \mathbb{N})(\forall s \in L(G) - K)(\forall t \in L(G)/s) \\ &|t| \geq m \Rightarrow [(\exists i \in I) D_i(P_{\Sigma, \Sigma_o^i}(st)) = 1] \end{aligned} \quad (3)$$

and

$$(\forall s \in K)(\forall i \in I) D_i(P_{\Sigma, \Sigma_o^i}(s)) = 0. \quad (4)$$

III. ROBUST DIAGNOSABILITY

We assume that, instead of the single exact model $G = (Q, \Sigma, \delta, q_0)$, a set $\{G_j = (Q_j, \Sigma_j, \delta_j, q_{j,0}) \mid j \in J\}$ of multiple possible models with possibly different event sets, where $J = \{1, 2, \dots, l\}$ is the index set, is given for the task of diagnosis [2]. For each possible model G_j ($j \in J$), its normal behavior is described by a nonempty closed sublanguage $K_j \subseteq L(G_j)$, and its own observable and unobservable event sets are denoted by $\Sigma_{j,o}$ and $\Sigma_{j,uo}$, respectively, where $\Sigma_{j,uo} = \Sigma_j - \Sigma_{j,o}$. Note that, for any $j, j' \in J$, $K_j = K_{j'}$ and $\Sigma_{j,o} = \Sigma_{j',o}$ are not required.

For the set $\{G_j \mid j \in J\}$ of possible models, a diagnoser is modeled as a function $D : \bigcup_{j \in J} P_{\Sigma_j, \Sigma_{j,o}}(L(G_j)) \rightarrow \{1, 0\}$ and it is required to satisfy

$$\begin{aligned} (\forall j \in J) \\ (\exists m_j \in \mathbb{N})(\forall s_j \in L(G_j) - K_j)(\forall t_j \in L(G_j)/s_j) \\ |t_j| \geq m_j \Rightarrow D(P_{\Sigma_j, \Sigma_{j,o}}(s_j t_j)) = 1 \end{aligned} \quad (5)$$

and

$$(\forall j \in J)(\forall s_j \in K_j)D(P_{\Sigma_j, \Sigma_{j,o}}(s_j)) = 0 \quad (6)$$

for the set $\{K_j \mid j \in J\}$ of nonempty closed sublanguages $K_j \subseteq L(G_j)$. A diagnoser that satisfies (5) and (6) is robust in the sense that, for each possible model, it can detect the occurrence of any failure string correctly within a finite number of steps.

The following definition of robust diagnosability is taken from [2]¹:

Definition 1: The set $\{G_j \mid j \in J\}$ of possible models is said to be robustly diagnosable with respect to the set $\{K_j \mid j \in J\}$ of nonempty closed sublanguages $K_j \subseteq L(G_j)$ if

$$\begin{aligned} (\forall j \in J) \\ (\exists m_j \in \mathbb{N})(\forall s_j \in L(G_j) - K_j)(\forall t_j \in L(G_j)/s_j) \\ |t_j| \geq m_j \\ \Rightarrow [(\forall j' \in J)(\forall u_{j'} \in L(G_{j'})) \\ P_{\Sigma_j, \Sigma_{j,o}}(s_j t_j) = P_{\Sigma_{j'}, \Sigma_{j',o}}(u_{j'}) \Rightarrow u_{j'} \notin K_{j'}]. \end{aligned}$$

As shown in the following proposition, which can be proved in the same way as Theorem 6 of [17], robust diagnosability is a necessary and sufficient condition for the existence of a diagnoser $D : \bigcup_{j \in J} P_{\Sigma_j, \Sigma_{j,o}}(L(G_j)) \rightarrow \{1, 0\}$ that satisfies (5) and (6).

Proposition 1: For the set $\{G_j \mid j \in J\}$ of possible models, there exists a diagnoser $D : \bigcup_{j \in J} P_{\Sigma_j, \Sigma_{j,o}}(L(G_j)) \rightarrow \{1, 0\}$ that satisfies (5) and (6) for the set $\{K_j \mid j \in J\}$ of nonempty closed sublanguages $K_j \subseteq L(G_j)$ if and only if $\{G_j \mid j \in J\}$ is robustly diagnosable with respect to $\{K_j \mid j \in J\}$.

IV. ROBUST CODIAGNOSABILITY

We introduce a notion of robust codiagnosability by extending robust diagnosability to decentralized diagnosis. For each possible model $G_j = (Q_j, \Sigma_j, \delta_j, q_{j,0})$ ($j \in J$), the locally observable and locally unobservable event sets of the

i th local diagnoser ($i \in I$) are denoted by $\Sigma_{j,o}^i$ and $\Sigma_{j,uo}^i$, respectively, where $\Sigma_{j,uo}^i = \Sigma_j - \Sigma_{j,o}^i$. Then, for the set $\{G_j \mid j \in J\}$ of possible models, the i th local diagnoser is represented by a function $D_i : \bigcup_{j \in J} P_{\Sigma_j, \Sigma_{j,o}^i}(L(G_j)) \rightarrow \{1, 0\}$.

We formulate a robust decentralized diagnosis problem as follows:

Problem 1: For the set $\{G_j = (Q_j, \Sigma_j, \delta_j, q_{j,0}) \mid j \in J\}$ of possible models and the set $\{K_j \mid j \in J\}$ of nonempty closed sublanguages $K_j \subseteq L(G_j)$, a robust decentralized diagnosis problem is a problem of synthesizing a decentralized diagnoser $\{D_i : \bigcup_{j \in J} P_{\Sigma_j, \Sigma_{j,o}^i}(L(G_j)) \rightarrow \{1, 0\} \mid i \in I\}$ that satisfies

$$\begin{aligned} (\forall j \in J) \\ (\exists m_j \in \mathbb{N})(\forall s_j \in L(G_j) - K_j)(\forall t_j \in L(G_j)/s_j) \\ |t_j| \geq m_j \Rightarrow [(\exists i \in I)D_i(P_{\Sigma_j, \Sigma_{j,o}^i}(s_j t_j)) = 1] \end{aligned} \quad (7)$$

and

$$(\forall j \in J)(\forall s_j \in K_j)(\forall i \in I)D_i(P_{\Sigma_j, \Sigma_{j,o}^i}(s_j)) = 0. \quad (8)$$

To characterize the existence of a decentralized diagnoser $\{D_i \mid i \in I\}$ that satisfies (7) and (8), a notion of robust codiagnosability is defined as follows:

Definition 2: The set $\{G_j \mid j \in J\}$ of possible models is said to be robustly codiagnosable with respect to the set $\{K_j \mid j \in J\}$ of nonempty closed sublanguages $K_j \subseteq L(G_j)$ if

$$\begin{aligned} (\forall j \in J) \\ (\exists m_j \in \mathbb{N})(\forall s_j \in L(G_j) - K_j)(\forall t_j \in L(G_j)/s_j) \\ |t_j| \geq m_j \\ \Rightarrow [(\exists i \in I)(\forall j' \in J)(\forall u_{j'} \in L(G_{j'})) \\ P_{\Sigma_j, \Sigma_{j,o}^i}(s_j t_j) = P_{\Sigma_{j'}, \Sigma_{j',o}^i}(u_{j'}) \Rightarrow u_{j'} \notin K_{j'}]. \end{aligned}$$

The robust codiagnosability property requires that, for each possible model G_j ($j \in J$), any sufficiently long failure string should be distinguished from a normal string of any possible model $G_{j'}$ ($j' \in J$) by at least one local diagnoser.

Remark 1: In the special case where the index set I (respectively, J) is a singleton, robust codiagnosability is reduced to robust diagnosability of Definition 1 (respectively, disjunctive codiagnosability of [12]).

The following theorem, which extends Proposition 1 to decentralized diagnosis, justifies the definition of robust codiagnosability.

Theorem 1: For the set $\{G_j \mid j \in J\}$ of possible models, there exists a decentralized diagnoser $\{D_i : \bigcup_{j \in J} P_{\Sigma_j, \Sigma_{j,o}^i}(L(G_j)) \rightarrow \{1, 0\} \mid i \in I\}$ that satisfies (7) and (8) for the set $\{K_j \mid j \in J\}$ of nonempty closed sublanguages $K_j \subseteq L(G_j)$ if and only if $\{G_j \mid j \in J\}$ is robustly codiagnosable with respect to $\{K_j \mid j \in J\}$.

By Theorem 1, if the set $\{G_j \mid j \in J\}$ of possible models is robustly codiagnosable with respect to $\{K_j \mid j \in J\}$, then the decentralized diagnoser $\{D_i \mid i \in I\}$, where, for

¹The notion of robust diagnosability of Definition 1 is referred to as general robust diagnosability in [2]

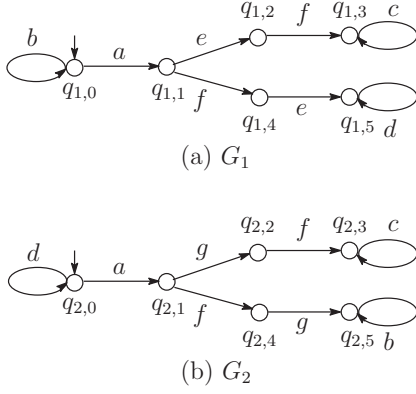


Fig. 1. (a) Finite automaton G_1 . (b) Finite automaton G_2 .

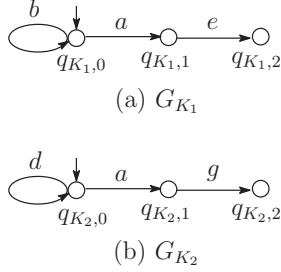


Fig. 2. (a) Finite automaton G_{K_1} . (b) Finite automaton G_{K_2} .

each $i \in I$, D_i is given as

$$D_i(s_{io}) = \begin{cases} 1, & \text{if } (\forall j \in J) s_{io} \in P_{\Sigma_j, \Sigma_{j,o}^i}(L(G_j)) \\ & \Rightarrow P_{\Sigma_j, \Sigma_{j,o}^i}^{-1}(s_{io}) \cap K_j = \emptyset \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

for any $s_{io} \in \bigcup_{j \in J} P_{\Sigma_j, \Sigma_{j,o}^i}(L(G_j))$, solves the robust decentralized diagnosis problem.

Example 1: We consider two possible models G_1 and G_2 shown in Fig. 1(a) and (b), respectively, where $\Sigma_1 = \{a, b, c, d, e, f\}$ and $\Sigma_2 = \{a, b, c, d, f, g\}$. Let $I = \{1, 2\}$, $\Sigma_{1,o}^1 = \{a, d\}$ and $\Sigma_{1,o}^2 = \{a, b, c\}$ for G_1 , and $\Sigma_{2,o}^1 = \{a, b\}$ and $\Sigma_{2,o}^2 = \{a, c, d\}$ for G_2 . We assume that the nonempty closed sublanguages $K_1 \subseteq L(G_1)$ and $K_2 \subseteq L(G_2)$ that model the normal behaviors of G_1 and G_2 , respectively, are generated by the finite automata G_{K_1} and G_{K_2} shown in Fig. 2 (a) and (b), respectively. That is, a failure is modeled by the occurrence of the event f .

To establish robust codiagnosability of $\{G_1, G_2\}$ with respect to $\{K_1, K_2\}$, we first show that, for G_1 and K_1 ,

$$\begin{aligned} & (\forall s_1 \in L(G_1) - K_1)(\forall t_1 \in L(G_1)/s_1) \\ & |t_1| \geq 2 \\ & \Rightarrow [(\exists i \in I)(\forall j \in J)(\forall u_j \in L(G_j)) \\ & P_{\Sigma_1, \Sigma_{1,o}^i}(s_1 t_1) = P_{\Sigma_j, \Sigma_{j,o}^i}(u_j) \Rightarrow u_j \notin K_j]. \end{aligned}$$

For any $s_1 \in L(G_1) - K_1$ and any $t_1 \in L(G_1)/s_1$ with $|t_1| \geq 2$, $s_1 t_1$ can be written as $s_1 t_1 = b^h a e f c^k$, where $h \geq 0$ and $k \geq 2$, or $s_1 t_1 = b^h a f e d^k$, where $h \geq 0$ and

$k \geq 1$. We consider the case where $s_1 t_1 = b^h a e f c^k$. For any $u_1 \in L(G_1)$, if $P_{\Sigma_1, \Sigma_{1,o}^1}(u_1) = P_{\Sigma_1, \Sigma_{1,o}^2}(s_1 t_1) = b^h a c^k$, then $u_1 = s_1 t_1 = b^h a e f c^k \notin K_1$. In addition, if $h > 0$ then, for any $u_2 \in L(G_2)$, $P_{\Sigma_2, \Sigma_{2,o}^2}(u_2) \neq P_{\Sigma_1, \Sigma_{1,o}^2}(s_1 t_1) = b^h a c^k$. In the case of $h = 0$, for any $u_2 \in L(G_2)$, if $P_{\Sigma_2, \Sigma_{2,o}^2}(u_2) = P_{\Sigma_1, \Sigma_{1,o}^2}(s_1 t_1) = a c^k$ then $u_2 = a g f c^k \notin K_2$. That is, the occurrence of f in $s_1 t_1 = b^h a e f c^k$ can be detected by the second local diagnoser based on the local observation $b^h a c^k$. We also consider the case where $s_1 t_1 = b^h a f e d^k$. For any $u_1 \in L(G_1)$, if $P_{\Sigma_1, \Sigma_{1,o}^1}(u_1) = P_{\Sigma_1, \Sigma_{1,o}^2}(s_1 t_1) = a d^k$, then $u_1 \in \{b^{h'} a f e d^k \mid h' \geq 0\}$, which implies $u_1 \notin K_1$. In addition, for any $u_2 \in L(G_2)$, $P_{\Sigma_2, \Sigma_{2,o}^1}(u_2) \neq P_{\Sigma_1, \Sigma_{1,o}^1}(s_1 t_1) = a d^k$. That is, the occurrence of f in $s_1 t_1 = b^h a f e d^k$ can be detected by the first local diagnoser based on the local observation $a d^k$. Similarly, we can show that, for G_2 and K_2

$$\begin{aligned} & (\forall s_2 \in L(G_2) - K_2)(\forall t_2 \in L(G_2)/s_2) \\ & |t_2| \geq 2 \\ & \Rightarrow [(\exists i \in I)(\forall j \in J)(\forall u_j \in L(G_j)) \\ & P_{\Sigma_2, \Sigma_{2,o}^i}(s_2 t_2) = P_{\Sigma_j, \Sigma_{j,o}^i}(u_j) \Rightarrow u_j \notin K_j]. \end{aligned}$$

For any $s_2 \in L(G_2) - K_2$ and any $t_2 \in L(G_2)/s_2$ with $|t_2| \geq 2$, $s_2 t_2$ can be written as $s_2 t_2 = d^h a g f c^k$, where $h \geq 0$ and $k \geq 2$, or $s_2 t_2 = d^h a f g b^k$, where $h \geq 0$ and $k \geq 1$. The occurrence of f in $s_2 t_2 = d^h a g f c^k$ (respectively, $s_2 t_2 = d^h a f g b^k$) can be detected by the second (respectively, first) local diagnoser based on the local observation $d^h a c^k$ (respectively, $a b^k$). Thus, $\{G_1, G_2\}$ is robustly codiagnosable with respect to $\{K_1, K_2\}$. According to (9), the first and second local diagnosers D_1 and D_2 , respectively, are synthesized as

$$D_1(s_{1o}) = \begin{cases} 1, & \text{if } s_{1o} \in \{a b^k \mid k \geq 1\} \cup \{a d^k \mid k \geq 1\} \\ 0, & \text{otherwise} \end{cases}$$

for any $s_{1o} \in \bigcup_{j \in \{1,2\}} P_{\Sigma_j, \Sigma_{j,o}^1}(L(G_j))$ and

$$D_2(s_{2o}) = \begin{cases} 1, & \text{if } s_{2o} \in \{b^h a c^k \mid h \geq 0 \wedge k \geq 1\} \\ & \cup \{d^h a c^k \mid h \geq 0 \wedge k \geq 1\} \\ 0, & \text{otherwise} \end{cases}$$

for any $s_{2o} \in \bigcup_{j \in \{1,2\}} P_{\Sigma_j, \Sigma_{j,o}^2}(L(G_j))$. For the set $\{G_1, G_2\}$ of possible models and the set $\{K_1, K_2\}$ of nonempty closed sublanguages, the decentralized diagnoser $\{D_1, D_2\}$ satisfies (7) and (8).

V. VERIFICATION OF ROBUST CODIAGNOSABILITY

We present how to verify robust codiagnosability of the set $\{G_j \mid j \in J\}$ of possible models with respect to the set $\{K_j \mid j \in J\}$ of nonempty closed regular sublanguages. For each $j \in J$, since K_j is regular, there exists a finite automaton $G_{K_j} = (Q_{K_j}, \Sigma_j, \delta_{K_j}, q_{K_j,0})$ such that $L(G_{K_j}) = K_j$. By adding a dump state, denoted by $q_{d_j} \notin Q_{K_j}$, we augment the finite automaton G_{K_j} as

$$\tilde{G}_{K_j} = (\tilde{Q}_{K_j}, \Sigma_j, \tilde{\delta}_{K_j}, q_{K_j,0}),$$

where the state set $\tilde{Q}_{K_j}$ is given as $\tilde{Q}_{K_j} = Q_{K_j} \cup \{q_{d_j}\}$, and the transition function $\tilde{\delta}_{K_j} : \tilde{Q}_{K_j} \times \Sigma_j \rightarrow \tilde{Q}_{K_j}$ is defined as

$$\begin{aligned} \tilde{\delta}_{K_j}(\tilde{q}_{K_j}, \sigma_j) &= \begin{cases} \delta_{K_j}(\tilde{q}_{K_j}, \sigma_j), & \text{if } \tilde{q}_{K_j} \in Q_{K_j} \wedge \delta_{K_j}(\tilde{q}_{K_j}, \sigma_j)! \\ q_{d_j}, & \text{otherwise} \end{cases} \end{aligned}$$

for each $\tilde{q}_{K_j} \in \tilde{Q}_{K_j}$ and each $\sigma_j \in \Sigma_j$. By the definition of $\tilde{\delta}_{K_j}$, we have $L(\tilde{G}_{K_j}) = \Sigma_j^*$. In addition, for any $s_j \in \Sigma_j^*$, $s_j \notin K_j$ if and only if $\tilde{\delta}_{K_j}(q_{K_j,0}, s_j) = q_{d_j}$. To characterize a failure string in $L(G_j) - K_j$, we construct the synchronous composition

$$G_j \parallel \tilde{G}_{K_j} = (Q_j \times \tilde{Q}_{K_j}, \Sigma_j, \alpha_j, (q_{j,0}, q_{K_j,0}))$$

of G_j and $\tilde{G}_{K_j}$, where the transition function $\alpha_j : (Q_j \times \tilde{Q}_{K_j}) \times \Sigma_j \rightarrow (Q_j \times \tilde{Q}_{K_j})$ is defined as

$$\begin{aligned} \alpha_j((q_j, \tilde{q}_{K_j}), \sigma_j) &= \begin{cases} (\delta_j(q_j, \sigma_j), \tilde{\delta}_{K_j}(\tilde{q}_{K_j}, \sigma_j)), & \text{if } \delta_j(q_j, \sigma_j)! \\ \text{undefined}, & \text{otherwise} \end{cases} \end{aligned}$$

for each $(q_j, \tilde{q}_{K_j}) \in Q_j \times \tilde{Q}_{K_j}$ and each $\sigma_j \in \Sigma_j$. By the definition of α_j , the generated language $L(G_j \parallel \tilde{G}_{K_j})$ of $G_j \parallel \tilde{G}_{K_j}$ is given as $L(G_j \parallel \tilde{G}_{K_j}) = L(G_j)$. Then, for any $s_j \in L(G_j)$, s_j is a failure string in $L(G_j) - K_j$ if and only if the state reached by the execution of s_j in $G_j \parallel \tilde{G}_{K_j}$ is in $Q_j \times \{q_{d_j}\}$, that is, its second element is the dump state q_{d_j} .

To verify robust codiagnosability of $\{G_j \mid j \in J\}$ with respect to the set $\{K_j \mid j \in J\}$, for each $j \in J$ and each $\phi_j = (j, j_1, j_2, \dots, j_n) \in \Phi_j$, where

$$\Phi_j = \{j\} \times \underbrace{J \times J \times \dots \times J}_{n \text{ times}}$$

we construct a finite automaton, called a verifier,

$$V_{\phi_j} = (X_{\phi_j}, \Sigma_{\phi_j}, \delta_{\phi_j}, x_{\phi_j,0}) \quad (10)$$

by composing $G_j \parallel \tilde{G}_{K_j}$, which is used to trace a failure string of G_j , and the n finite automata $G_{K_{j_1}}, G_{K_{j_2}}, \dots, G_{K_{j_n}}$, which are used to trace normal strings of G_{j_i} ($i = 1, 2, \dots, n$) as follows:

- The state set X_{ϕ_j} is given as

$$X_{\phi_j} = (Q_j \times \tilde{Q}_{K_j}) \times \left(\prod_{i=1}^n Q_{K_{j_i}} \right).$$

- The initial state $x_{\phi_j,0} \in X_{\phi_j}$ is given as

$$x_{\phi_j,0} = ((q_{j,0}, q_{K_j,0}), q_{K_{j_1},0}, q_{K_{j_2},0}, \dots, q_{K_{j_n},0}).$$

- The event set Σ_{ϕ_j} is a subset of $\hat{\Sigma}_{\phi_j}$ defined as

$$\hat{\Sigma}_{\phi_j} = \bar{\Sigma}_j \times \left(\prod_{i=1}^n \bar{\Sigma}_{j_i} \right)$$

where $\bar{\Sigma}_{j'} = \Sigma_{j'} \cup \{\varepsilon\}$ for any $j' \in J$, and it is given as

$$\begin{aligned} \Sigma_{\phi_j} &= \{(\sigma_j, \bar{\sigma}_{j_1}, \bar{\sigma}_{j_2}, \dots, \bar{\sigma}_{j_n}) \in \hat{\Sigma}_{\phi_j} \mid \\ &\quad \sigma_j \in \Sigma_j \wedge [(\forall i \in I) \sigma_j \in \Sigma_{j,o}^i \Rightarrow \Sigma_{j_i,o}^i] \\ &\quad \wedge [(\forall i \in I) \bar{\sigma}_{j_i} = e_j(i, \sigma_j)]\} \\ &\quad \cup \{(\varepsilon, \bar{\sigma}_{j_1}, \bar{\sigma}_{j_2}, \dots, \bar{\sigma}_{j_n}) \in \hat{\Sigma}_{\phi_j} \mid \\ &\quad (\exists i \in I) (\exists \sigma_{j_i} \in \Sigma_{j_i,uo}^i) \\ &\quad (\forall i' \in I) \bar{\sigma}_{j_{i'}} = e_{i,uo}(i', \sigma_{j_i})\}, \end{aligned}$$

where

$$e_j(i, \sigma_j) = \begin{cases} \sigma_j, & \text{if } \sigma_j \in \Sigma_{j,o}^i \\ \varepsilon, & \text{otherwise} \end{cases}$$

and

$$e_{i,uo}(i', \sigma_{j_i}) = \begin{cases} \sigma_{j_i}, & \text{if } i' = i \\ \varepsilon, & \text{otherwise.} \end{cases}$$

For any $(\sigma_j, \bar{\sigma}_{j_1}, \bar{\sigma}_{j_2}, \dots, \bar{\sigma}_{j_n}) \in \Sigma_{\phi_j}$, where $\sigma_j \in \Sigma_j$, the condition $(\forall i \in I) \sigma_j \in \Sigma_{j,o}^i \Rightarrow \Sigma_{j_i,o}^i$ means that, for any $i \in I$, if σ_j is locally observable by the i th local diagnoser in G_j , it should also be locally observable in G_{j_i} . Then, $\bar{\sigma}_{j_i}$ is σ_j if σ_j is locally observable by the i th local diagnoser in G_j ; it is ε , otherwise. Also, for any $(\varepsilon, \bar{\sigma}_{j_1}, \bar{\sigma}_{j_2}, \dots, \bar{\sigma}_{j_n}) \in \Sigma_{\phi_j}$, there exists $i \in I$ such that $\bar{\sigma}_{j_i} = \sigma_{j_i}$ for some locally unobservable event $\sigma_{j_i} \in \Sigma_{j_i,uo}^i$ and $\bar{\sigma}_{j_{i'}} = \varepsilon$ for any $i' \in I$ with $i' \neq i$. Therefore, any element $(\bar{\sigma}_j, \bar{\sigma}_{j_1}, \bar{\sigma}_{j_2}, \dots, \bar{\sigma}_{j_n})$ of Σ_{ϕ_j} satisfies

$$\bar{\sigma}_j \neq \varepsilon \vee [(\exists i \in I) \bar{\sigma}_{j_i} \neq \varepsilon]$$

and

$$(\forall i \in I) P_{\Sigma_j, \Sigma_{j,o}^i}(\bar{\sigma}_j) = P_{\Sigma_{j_i}, \Sigma_{j_i,o}^i}(\bar{\sigma}_{j_i}).$$

- For any $x_{\phi_j} = ((q_j, \tilde{q}_{K_j}), q_{K_{j_1}}, q_{K_{j_2}}, \dots, q_{K_{j_n}}) \in X_{\phi_j}$ and any $\sigma_{\phi_j} = (\bar{\sigma}_j, \bar{\sigma}_{j_1}, \bar{\sigma}_{j_2}, \dots, \bar{\sigma}_{j_n}) \in \Sigma_{\phi_j}$, $\delta_{\phi_j}(x_{\phi_j}, \sigma_{\phi_j})!$ if and only if the following two conditions hold:

- $\bar{\sigma}_j \neq \varepsilon \Rightarrow \alpha_j((q_j, \tilde{q}_{K_j}), \bar{\sigma}_j)!$,
- $(\forall i \in I) \bar{\sigma}_{j_i} \neq \varepsilon \Rightarrow \delta_{K_{j_i}}(q_{K_{j_i}}, \bar{\sigma}_{j_i})!$.

If $\delta_{\phi_j}(x_{\phi_j}, \sigma_{\phi_j})!$, then

$$\delta_{\phi_j}(x_{\phi_j}, \sigma_{\phi_j}) = ((q'_j, \tilde{q}'_{K_j}), q'_{K_{j_1}}, q'_{K_{j_2}}, \dots, q'_{K_{j_n}}),$$

where

$$(q'_j, \tilde{q}'_{K_j}) = \begin{cases} \alpha_j((q_j, \tilde{q}_{K_j}), \bar{\sigma}_j), & \text{if } \bar{\sigma}_j \neq \varepsilon \\ (q_j, \tilde{q}_{K_j}), & \text{otherwise} \end{cases}$$

and, for each $i \in I$,

$$q'_{K_{j_i}} = \begin{cases} \delta_{K_{j_i}}(q_{K_{j_i}}, \bar{\sigma}_{j_i}), & \text{if } \bar{\sigma}_{j_i} \neq \varepsilon \\ q_{K_{j_i}}, & \text{otherwise.} \end{cases}$$

Let $\pi : \Sigma_{\phi_j}^* \rightarrow \Sigma_j^*$ and $\pi_i : \Sigma_{\phi_j}^* \rightarrow \Sigma_{j_i}^*$ for each $i \in I$ be projection functions defined as follows:

- $\pi(\varepsilon) = \varepsilon$, $\pi_i(\varepsilon) = \varepsilon$,
- $(\forall s_{\phi_j} \in \Sigma_{\phi_j}^*) (\forall \sigma_{\phi_j} = (\bar{\sigma}_j, \bar{\sigma}_{j_1}, \bar{\sigma}_{j_2}, \dots, \bar{\sigma}_{j_n}) \in \Sigma_{\phi_j})$
 $\pi(s_{\phi_j} \sigma_{\phi_j}) = \pi(s_{\phi_j}) \bar{\sigma}_j$, $\pi_i(s_{\phi_j} \sigma_{\phi_j}) = \pi_i(s_{\phi_j}) \bar{\sigma}_{j_i}$.

The following theorem shows a method for verifying robust codiagnosability of $\{G_j \mid j \in J\}$ with respect to $\{K_j \mid j \in J\}$.

Theorem 2: The set $\{G_j \mid j \in J\}$ of possible models is not robustly codiagnosable with respect to the set $\{K_j \mid j \in J\}$ of nonempty closed regular sublanguages $K_j \subseteq L(G_j)$ generated by the finite automata G_{K_j} if and only if there exist $j \in J$ and $\phi_j = (j, j_1, j_2, \dots, j_n) \in \Phi_j$ such that, in the verifier V_{ϕ_j} defined as (10), there exists a reachable cycle $x_{\phi_j}^{(1)} \xrightarrow{\sigma_{\phi_j}^{(1)}} x_{\phi_j}^{(2)} \xrightarrow{\sigma_{\phi_j}^{(2)}} \dots \xrightarrow{\sigma_{\phi_j}^{(k-1)}} x_{\phi_j}^{(k)} \xrightarrow{\sigma_{\phi_j}^{(k)}} x_{\phi_j}^{(1)}$ such that

$$(\exists h \in \{1, 2, \dots, k\}) \pi(\sigma_{\phi_j}^{(h)}) \neq \varepsilon \wedge \tilde{q}_{K_j}^{(h)} = q_{d_j}, \quad (11)$$

where $x_{\phi_j}^{(h)} = ((q_j^{(h)}, \tilde{q}_{K_j}^{(h)}), q_{K_{j_1}}^{(h)}, q_{K_{j_2}}^{(h)}, \dots, q_{K_{j_n}}^{(h)})$.

Remark 2: Robust codiagnosability of the set $\{G_j \mid j \in J\}$ of possible models with respect to the set $\{K_j \mid j \in J\}$ of nonempty closed regular sublanguages $K_j \subseteq L(G_j)$ generated by the finite automata G_{K_j} can be verified using Theorem 2. For each $j \in J$ and each $\phi_j = (j, j_1, j_2, \dots, j_n) \in \Phi_j$, we need to construct the verifier V_{ϕ_j} and verify the existence of a reachable cycle that satisfies (11).

The number of states of the verifier V_{ϕ_j} is $|Q_j| \times (|Q_{K_j}| + 1) \times (\prod_{i=1}^n |Q_{K_{j_i}}|)$. Since $|\Sigma_{\phi_j}| \leq |\Sigma_j| + (\sum_{i=1}^n |\Sigma_{j_i}|)$, the number of transitions in V_{ϕ_j} is at most $|Q_j| \times (|Q_{K_j}| + 1) \times (\prod_{i=1}^n |Q_{K_{j_i}}|) \times \{|\Sigma_j| + (\sum_{i=1}^n |\Sigma_{j_i}|)\}$. Since the number of verifiers is l^{n+1} , the computational complexity of verifying robust codiagnosability using Theorem 2 is $O(l^{n+1} \times |Q_{\max}| \times |Q_{K_{\max}}|^{n+1} \times n |\Sigma_{\max}|)$, where $|Q_{\max}| = \max\{|Q_j| \mid j \in J\}$, $|Q_{K_{\max}}| = \max\{|Q_{K_j}| \mid j \in J\}$, and $|\Sigma_{\max}| = \max\{|\Sigma_j| \mid j \in J\}$. That is, similar to the verification algorithm of [2], the computational complexity is polynomial in the number of possible models. However, as in the verification of disjunctive codiagnosability [12], [10], it is exponential in the number of local diagnosers.

VI. CONCLUSIONS

In this paper, we studied robust decentralized diagnosis of DESs in the case that, instead of the single exact model, a set of multiple possible models, each of which has its own event set and own normal behavior, is given. By extending the notion of generalized robust diagnosability of [2], we defined a notion of robust codiagnosability, which serves as a necessary and sufficient condition for the existence of a decentralized diagnoser such that, for all possible models, it detects the occurrence of any failure string within a finite number of steps. We also presented how to verify robust codiagnosability effectively. Computing a delay bound of any failure string can be detected, is currently under investigation.

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