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Author(s)	Matsunawa, Akira; Takemata, Hiroyuki; Okamoto, Ikuo
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Analysis of Temperature Field with Equiradial Cooling Boundary around Moving Heat Sources †
–Heat Conduction Analysis for Estimation of Thermal Hysteresis during Underwater Welding–

Akira MATSUNAWA *, Hiroyuki TAKEMATA** and Ikuo OKAMOTO***

Abstract

The paper describes the method of solving partial differential equation of heat conduction for moving source under the boundary condition that the equiradial surface of finite length around a moving heat source, i.e., semi-spherical surface for a point source and cylindrical surface for a linear one, is kept constant temperature, supposing to estimate thermal field in underwater welding by local cavity methods. Analyses are conducted for both point and linear heat sources and rigorous solutions are obtained, and their results are compared in detail with Rosenthal's solutions. Furthermore, the cooling rate at particular temperature range, which is practically useful for the prediction of hardness of welded part of steel plate, are analysed using the obtained solutions and the effect of the radius of cooling boundary on the rate are clarified for various welding variables.

KEY WORDS: (Heat Flow) (Temperature) (Cooling) (Underwater Welding) (Hardness) (Theoretical Investigations) (Thermal Cycling)

1. Introduction

In underwater arc welding with wet or semi-wet processes, there often arise metallurgical problems such as hydrogen absorption and increase in cooling rate which results in hardened weld structures as well as bringing many kinds of weld defects. One of the authors (A.M.) has developed an underwater welding process with fluid stabilized local cavity in order to maintain a stable arc and to solve or minimize the above unfavourable matters.[1),2)] In the TIG and MIG melt run weldings with this process, it was experimentally verified that the hydrogen absorption was less than that in carefully controlled manual arc welding with a low hydrogen covered electrode in usual air atmosphere. Another important feature of the process is that the cooling rate of base metal can be adequately controlled by selecting the cavity size. The relation between cooling rate and cavity diameter is, however, much dependent on various welding parameters such as welding speed, heat input, etc., but the problem has not been fully analysed theoretically and experimentally.

The paper describes an analytical method to evaluate the temperature field in underwater welding by a circular local cavity method. Here, for the simplicity of mathematical analysis, an approach is made for a moving heat source either of point and linear one under the condition that the equiradial boundary of constant temperature around a source moves with the same speed with that of heat source, which is the first order approximation of thermal hysteresis during the underwater welding under the question.

2. Modelling for Analysis

2.1. Nomenclature

The following nomenclature will be used throughout this paper;

- g : heat input [J/s]
- g' : heat input per unit length ($=g/h$) [J/s·m]
- h : thickness of plate [m]
- v : speed of source [m/s]
- t : time [s]
- T : temperature [C]
- T_0: temperature of cooling boundary [C]
- T_f: reference temperature (fusion temperature, for example) [C]
- K : heat conductivity [J/s·m·C]
- $1/2\lambda$: thermal diffusivity [m^2/s]

† Received on April 5, 1980
* Associate Professor
** Graduate Student (Presently, Japan Steel Works, Co., Ltd.)
*** Professor

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R : radius of cooling boundary [m]
n : non-dimensional heat input $(=\lambda v q / 2\pi K (T_f - T_0))$
θ : normalized temperature $(=(T - T_0)/(T_f - T_0))$
Λ : non-dimensional radius of cooling boundary $(=\lambda v R)$
ρ : normalized radius $(=r/R)$
H : non-dimensional plate thickness $(=\lambda v h)$

2.2. Assumption and Boundary Condition

In order to obtain analytical solutions of differential equation of heat conduction, the following assumption and boundary condition are adopted here;

1) Physical properties of the plate, i.e., K and λ are independent on the temperature and position.
2) The moving speed v and rate of heat input q are constant.
3) Heat losses from surface by convection and radiation are ommitted.
4) A cooling boundary of constant temperature $(T=T_0)$ is always placed at the position of equiradial distance R from moving heat source and the temperature is kept at T_0 all over the place of $r \geqq R$. Namely, the temperature rise is zero, as shown in **Figure 1**, on and outside of the semi-spherical surface for a point source and cylindrical surface for a linear source, in which the center of circular boundary coincides with the source position.

The basic partial differential equation of heat conduction is expressed as well known as the following form in the fixed co-ordinate (x, y, z).

$$2\lambda \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \qquad (1)$$

Supposing that a point heat source q is supplied on the surface of semi-infinite plate and moves along the x-axis with the constant speed of v, one can rewrite the Equation (1) into the folowing form using the moving co-ordinate (ξ, y, z) whose origin is taken at the heat source.

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 2\lambda\left(-v \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial t}\right) \qquad (2)$$

where, $\xi = x - vt$.

In case of the quasi-stationary state, i.e., $\partial T/\partial t = 0$, the equation is reduced to

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -2\lambda v \frac{\partial T}{\partial \xi} \qquad (3).$$

Taking

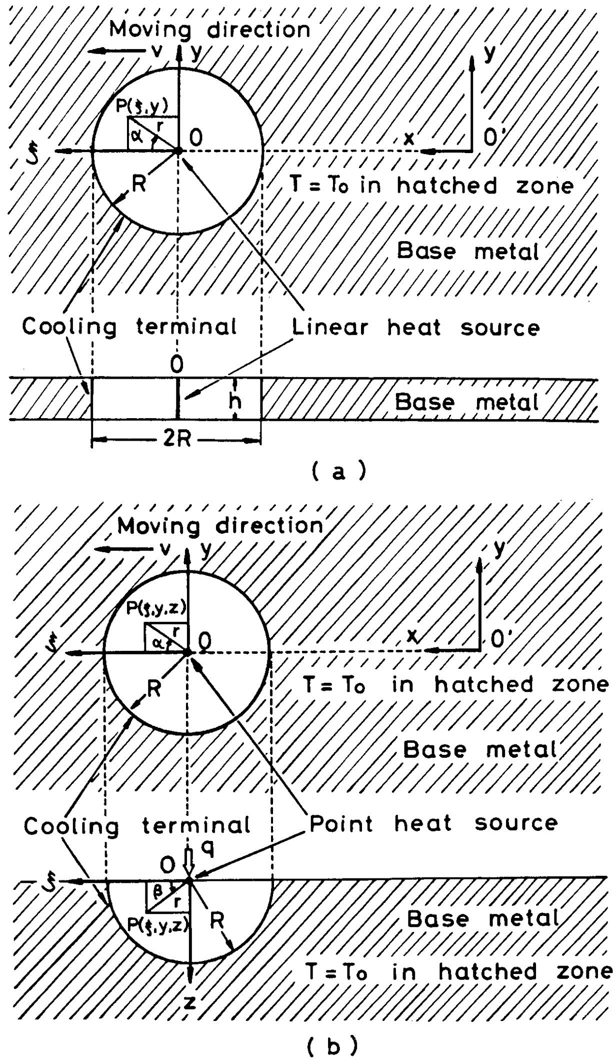

Fig. 1 Heat conduction modelling with equiradial cooling boundary around moving heat sources
(a) Three dimensional heat flow
(b) Two dimensional heat flow

$$T - T_0 = e^{-\lambda v \xi} \cdot \Phi(\xi, y, z) \qquad (4)$$

after Rosenthal's method[3], the Equation (4) becomes

$$\frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} - (\lambda v)^2 \Phi = 0 \qquad (5).$$

Therefore, the present question is to solve the Equation (5) under the above stated condition of 4).

3. Solution of Temperature Field with Symmetrical Cooling Boundary around a Moving Heat Source

3.1. Point Heat Source (Three Dimensional Heat Flow)

The previous equation (5) has a complete symmetrical form with respect to each axis of coordinate. Then, employing the polar coordinate (r, α, β) and also considering the spherical symmetricity of the problem under question, the partial differential equation of (5) is converted to the following differential equation;

$$\frac{d^2(r\Phi)}{dr^2} - (\lambda v)^2 (r\Phi) = 0 \qquad (6)$$

where, $r = (\xi^2 + y^2 + z^2)^{1/2}$
$\xi = r \cos a \cos \beta$
$y = r \sin a \cos \beta$
$z = r \sin \beta.$

From the Equations (6) and (4), the general solution of temperature becomes

$$T - T_0 = \frac{1}{r} exp(-\lambda vr \cos a \cos \beta) \times [C_1 exp(\lambda vr) + C_2 exp(-\lambda vr)] \quad (7).$$

Substituting the following boundary condition;

$$\begin{cases} T = T_0 \quad \text{at} \quad r = R \\ -2\pi rK(\partial T/\partial r) \to q \quad \text{as} \quad r \to 0 \end{cases} \qquad (8),$$

the solution of temperature at an arbitrary point (r, a, β) within the region bounded by a spherical cooling terminal can be expressed in the perfect form as

$$T - T_0 = \frac{q}{2\pi K}\frac{1}{r} exp[-\lambda vr(1 + \cos a \cos \beta)] \times \frac{1 - exp[-2\lambda v(R - r)]}{1 - exp[-2\lambda vR]} \qquad (9),$$

or in non-dimensional expression [4] after the definition in **2.1.**,

$$\frac{\theta}{n} = \frac{1}{\rho\Lambda} exp[-\rho\Lambda(1 + \cos a \cos \beta)] \frac{1 - exp[-2\Lambda(1-\rho)]}{1 - exp[-2\Lambda]} \qquad (10).$$

If the last term is denoted as

$$\gamma \equiv \frac{1 - exp[-2\Lambda(1-\rho)]}{1 - exp[-2\Lambda]} \qquad (11),$$

it is obvious that

$$\lim_{\Lambda\to\infty} r = 1 \text{ for } 0 \leqq \rho < 1$$

and the Equation (10) completely coincides with the cooling boundary is palced at infinity and is included pressed as

$$\frac{\theta}{n} = \frac{1}{\lambda vr} exp[-\lambda vr(1 + \cos a \cos \beta)] \qquad (12),$$

since $\rho\Lambda = (r/R)(\lambda vR) = \lambda vr$ Namely, the solution obtained by Rosenthal is corresponded when the circular cooling boundary is placed at infinity and is included mathematically in the solution (10) in which the cooling terminal is positioned at finite distance from source. It is clear, therefore, that γ defined by equation (11) means the cooling term and represents the degree of deviation in temperature from Rosenthal's solution.

In **Figures 2** and **3** are shown the behaviour of cooling

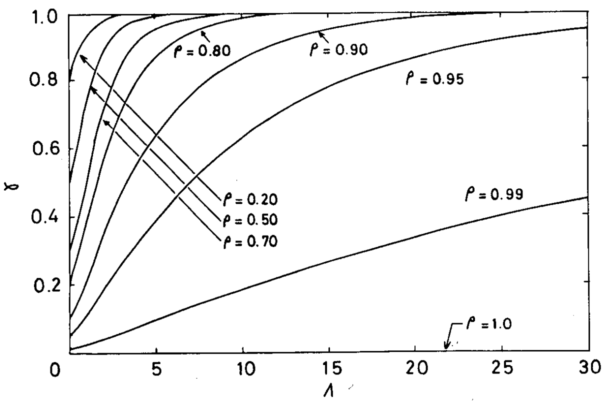

Fig. 2 Behaviour of cooling term γ against radius of cooling boundary (Three dimensional heat flow)

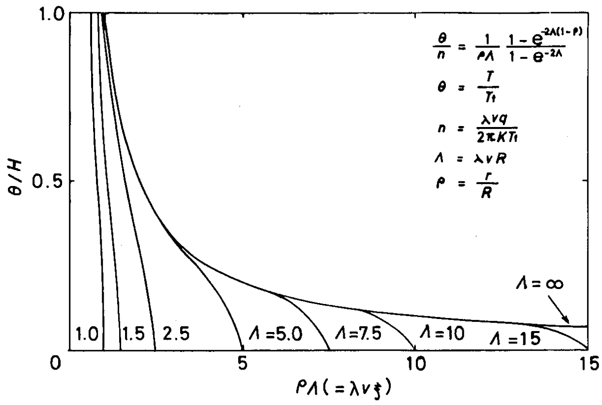

Fig. 3 Temperature distribution along moving axis behind heat source at various cooling boundary radius (Two dimensional heat flow; T_0 is assumed to be 0 C.)

term γ against the change in radius of cooling boundary Λ and the temperature distribution along the rear axis of source movement for various Λ-values. Temperature distribution near the source is not influenced by the cooling terminal, i.e., $\gamma \cong 1$, but obeys the Rosenthal's solution

($\Lambda = \infty$). In the region far from the source, however, temperature decreases rapidly apart from the Rosenthal's solution due to the effect of cooling terminal. Naturally, the effect reaches as far as to the higher temperature region at center, when the Λ-value becomes small.

The assumption of constant temperature on the semi-spherical surface of radius R (see the assumption 4) in the section 2.2.) is equivalent to the fact that the heat sink of finite intensity is distributed on the boundary surface. The sink intensity is easily obtained from equations (9) or (10) in either dimensional or nondimensional form.

$$-K\left(\frac{\partial T}{\partial r}\right)_{r=R} = \frac{\lambda v q \; exp\;[-\lambda v R\,(1 + cos\,a\,cos\,\beta)]}{\pi R\,[1 - exp\,(-2\lambda v R)]} \quad (13)$$

$$-\frac{1}{n}\left(\frac{\partial \theta}{\partial(\rho\Lambda)}\right)_{\rho=1} = \frac{2\;exp\;[-\Lambda\,(1 + cos\,a\,cos\,\beta)]}{\Lambda\,(1 - exp\,[-2\Lambda\,])} \quad (14)$$

Figure 4 shows the distribution of heat sink intensity on the surface of semi-spherical cooling boundary. The

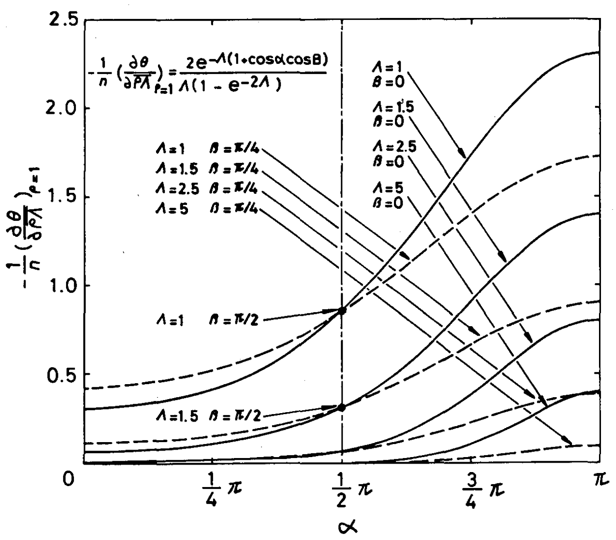

Fig. 4 Distribution of heat sink intensity on the surface of cooling boundary (Three dimensional heat flow)

sink intensity shows, as seen in the figure, the maximum value at the rear end on the moving axis (i.e., $a = \pi$, $\beta = 0$ & $\lambda v \xi = -\Lambda$), and it decreases remarkably with the reduction of a (or increase in β). Furthermore, the intensity naturally reduces with increase in Λ and it is small enough so as to be practically negligible except in the region of rear part of source on the plate surface, if Λ is greater than 5.

3.2. Linear Heat Source (Two Dimensional Heat Flow)

As there is no heat flow in the z-direction in case of two dimensional flow, the Equation (4) can be expressed as follow using the cylindrical coordinate (r, a);

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r}\frac{d\Phi}{dr} - (\lambda v)^2\,\Phi = 0 \quad (15)$$

Since the boundary condition in this case is

$$\begin{cases} T = T_0 \quad \text{at} \quad r = R \\ -2\pi r K\left(\frac{\partial T}{\partial r}\right) \to q' = q/h \quad \text{as} \quad r \to 0 \end{cases} \quad (16),$$

the objective solution of equation (15) is

$$T - T_0 = \frac{q}{2\pi h K} exp\;[-\lambda v r\,cos\,a\,] \cdot K_0\,(\lambda v r) \times \left[1 - \frac{K_0\,(\lambda v R)}{I_0\,(\lambda v R)} \cdot \frac{I_0\,(\lambda v r)}{K_0\,(\lambda v r)}\right] \quad (17)$$

or in dimensionless form

$$\frac{\theta}{n} = \frac{1}{H\Lambda}\;exp\;[-\rho\Lambda cos\,a\,] \cdot K_0\,(\rho\Lambda) \times \left[1 - \frac{K_0\,(\Lambda)}{I_0\,(\Lambda)} \cdot \frac{I_0\,(\rho\Lambda)}{K_0\,(\rho\Lambda)}\right] \quad (18)$$

Here, K_0 and I_0 are the modified Bessel functions of 0 order and H $(=\lambda v h)$ is the nondimensional plate thickness.

The cooling term in case of two dimensional flow is

$$\gamma' = 1 - \frac{K_0(\Lambda)}{I_0(\Lambda)} \cdot \frac{I_0\,(\rho\Lambda)}{K_0(\rho\Lambda)} \quad (19)$$

and its change with Λ is shown in **Figure 5** by solid lines, in which is also indicated the behaviour of cooling term

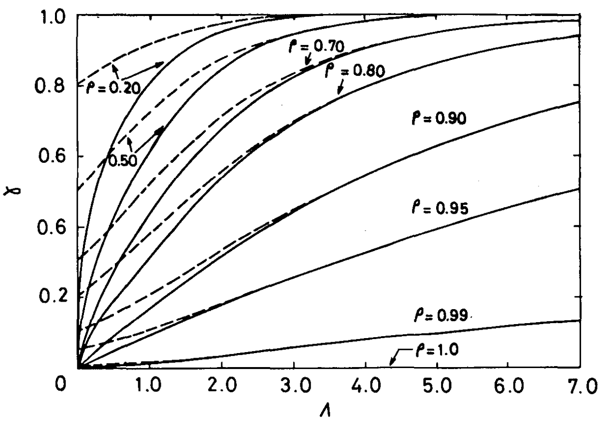

Fig. 5 Behaviour of cooling term γ' against radius of cooling boundary in case of two dimensional heat flow (Broken line shows the case of three dimensional flow)

in three dimensional flow by broken lines. Since the divergence of heat flux in two dimensional flow is less than that in three dimensional case, the influence of cooling terminal is greater, particularlly in case of smaller values of Λ, in the two dimension. However, in the region of larger values of Λ, the values of γ and γ' agree quite well each other and hence the equation (18) can be expressed with good approximation, if Λ is larger than 4, as

$$\frac{\theta}{n} = \frac{1}{H\Lambda}\ exp\ [-\rho\Lambda \cos a] \cdot K_0(\rho\Lambda) \times \left[\frac{1-exp\ [-2\Lambda(1-\rho)]}{1-exp\ (-2\Lambda)}\right] \text{ for } \Lambda > 4 \quad (20).$$

It is evident that equations (17), (18) and (20) completely include the Rosenthal's solution of two dimensional flow which corresponds the condition of infinite value of Λ.

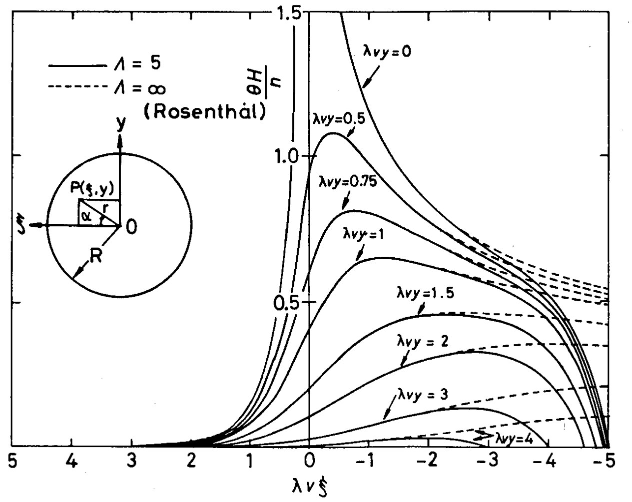

Fig. 6 Temperature distribution inside the region bounded by circular cooling boundary (Two dimensional flow)

Figure 6 shows an example of calculated temperature distribution by equation (16) and in **Figure 7** are represented the effect of radius of cooling boundary and heat input on the calculated bead width and pool length.

The heat sink intensity on the cylindrical surface of cooling terminal can be readily calculated as the same manner with the previous three dimensional case and the results are;

$$-K(\frac{\partial T}{\partial r})_{r=R} = \frac{\lambda v q}{2\pi h}\ exp\ (-\lambda v R \cos a) \cdot K_1(\lambda v R) \times \left[1 + \frac{K_0(\lambda v R)\ \ I_1(\lambda v R)}{I_0(\lambda v R) \cdot K_1(\lambda v R)}\right] \quad (21),$$

or

$$-\frac{1}{n}\left(\frac{\partial\theta}{\partial(\rho\Lambda)}\right)_{\rho=1} = \frac{1}{H}\exp(-\Lambda \cos a)\ K_1(\Lambda) \times \left[1 + \frac{K_0(\Lambda) \cdot I_1(\Lambda)}{I_0(\Lambda) \cdot K_1(\Lambda)}\right] \quad (22),$$

where, K_1 and I_1 are the modified Bessel functions of the first order.

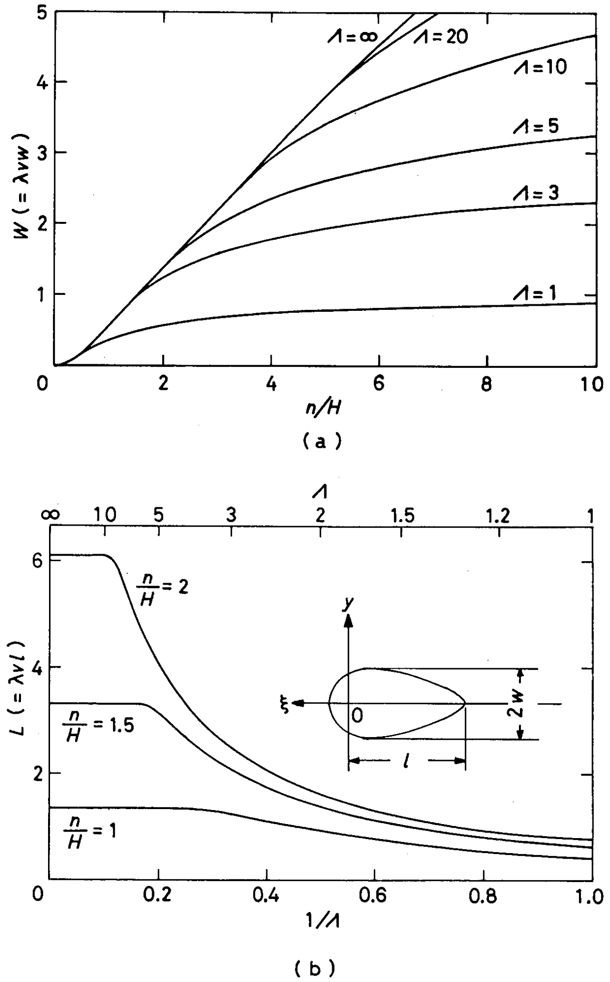

Fig. 7 Effect of heat input and radius of cooling boundary on bead parameters
(a) Bead width
(b) Pool length

In **Figure 8** is shown the intensity distribution of heat sink computed from equation (22). Overall tendency is similar to that of point heat source shown in the previous Figure 4, but the rate of intensity decrease with increase in Λ is rather dull compared with that in three dimensional case, which is primarilly caused by the fact that divergence of flux is smaller in linear heat source.

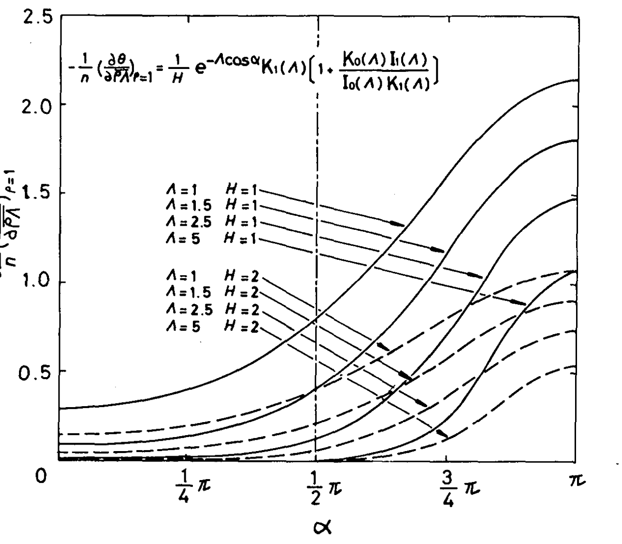

Fig. 8 Distribution of heat sink intensity on the surface of cooling boundary (Two dimensional flow)

4. Average Cooling Rate at Particular Temperature Range

In underwater welding by local cavity methods, there usually takes place more or less the metallurgical deterioration of welded part induced by the influence of environmental water. In welding of steel, for example, there often arises the problems such as decrease in ductility, delayed fracture (stress corrosion cracking), etc., at the remarkably hardened part. Hence, the hardness has been often taken as an important measure of welding fabrication.[5), 6)]

Cooling rate is, therefore, one of the important factor to be controlled in underwater welding processes, since the hardness of steel plate has a strong dependence on the cooling rate at particular range of temperature. It is widely recognized that the average cooling rate from 800 C ($\theta = 800/1{,}500 = 0.533$) to 500 C ($\theta = 0.333$) determines the hardness of steel plate. According to a recent work by Arata et al.[7)], the hardness of the weld metal can be predicted in satisfactory accuracy if the cooling rate in the above temperature range and the chemical compostion of steel are known.

The temperature change with time at a certain point (x, y, z) in the fixed coordinate is related as following with the temperature gradient at the corresponding point (ξ, y, z) in the moving coordinate.

$$\frac{\partial T(x,y,z,t)}{\partial t} = \frac{\partial T(\xi,y,z)}{\partial \xi}\,\frac{\partial \xi}{\partial t} = -\nu\,\frac{\partial T(\xi,y,z)}{\partial \xi} \tag{23}$$

The above cooling rate can be expressed in dimensioless form by the following way.

$$\frac{\partial T}{\partial t} = (T_f - T_o)\frac{\partial \theta}{\partial t} = -\nu\frac{\partial T}{\partial \xi} = -\lambda\nu^2(T_f - T_o)\frac{\partial \theta}{\partial(\lambda\nu\xi)}$$

From the second and forth terms,

$$\frac{1}{\lambda\nu^2}\,\frac{\partial\theta}{\partial t} = -\frac{\partial\theta}{\partial(\lambda\nu\xi)} = -\frac{1}{\Lambda}\,\frac{\partial\theta}{\partial\rho_\xi}$$

where, $\rho_\xi = (\xi/R)$.

Here, the factor $(1/\lambda\nu^2)$ having the unit of time is the parameter for nondimensional transformation.

The average cooling rate at the particular temperature range $\Delta\theta = \theta_1 - \theta_2$ is thus defined as follow.

$$\frac{1}{\lambda\nu^2}\left(\frac{\Delta\theta}{\Delta t}\right)_{\theta_1\to\theta_2} = -\frac{\theta_1 - \theta_2}{\lambda\nu(\xi_1 - \xi_2)}$$

$$= -\frac{1}{\Lambda}\,\frac{\theta_1 - \theta_2}{(\rho_{\xi_1} - \rho_{\xi_2})} \tag{25}$$

Notations ρ_{ξ_1} (or ξ_1) and ρ_{ξ_2} (or ξ_2) in the above equations are the distances in ξ -direction from the source where θ adopts θ_1 and θ_2 respectively.

As has been already described in the previous Figures 4 and 8, the most remarkably cooled place is that where the heat sink intensity becomes maximum, namely at the rear part of source on the moving line (ξ-axis). In the next, therefore, will be described the average rate at this part.

In case of three dimensional heat flow, the temperature distribution along the ξ-axis behind source becomes as follow from the previous Equations (8) or (9).

$$T - T_0 = \frac{q}{2\pi K}\,\frac{1}{\xi}\,\frac{1 - exp[-2\lambda\nu(R - \xi)]}{1 - exp(-2\lambda\nu R)} \tag{26}$$

or

$$\frac{\theta}{n} = \frac{1}{\rho_\xi\Lambda}\,\frac{1 - exp[-2\Lambda(1 - \rho_\xi)]}{1 - exp(-2\Lambda)} \tag{27}$$

In the above equations, however, ρ_ξ (or ξ) can not be solved algebratically for a given θ, and hence one has to determine the solution by a numerical method such as Newton-Raphson method. On the other hand in the case of Rosenthal's solution,

$$\frac{\theta}{n} = \frac{1}{\lambda\nu\xi}$$

and one can readily calculate the average cooling rate as

$$\frac{1}{\lambda\nu^2}\left(\frac{\Delta\theta}{\Delta t}\right)_{\theta_1\to\theta_2} = -\frac{\theta_1 - \theta_2}{\lambda\nu(r_1 - r_2)} = \frac{\theta_1\theta_2}{n} \tag{28}.$$

Namely, the non-dimensional cooling rate between θ_1 and θ_2 is reverse proportional to the dimensionless heat input.

Figures 9 and **10** show the numerically calculated results of average cooling rate vs. radius of cooling

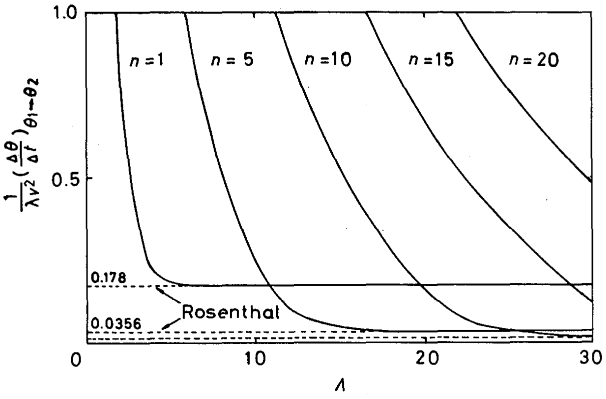

Fig. 9 Effect of cooling boundary radius on the cooling rate along moving axis at particular temperature range (θ_1 = 800/1,500 = 0.533 and θ_2= 500/1,500 = 0.333) (Three dimensional heat flow)

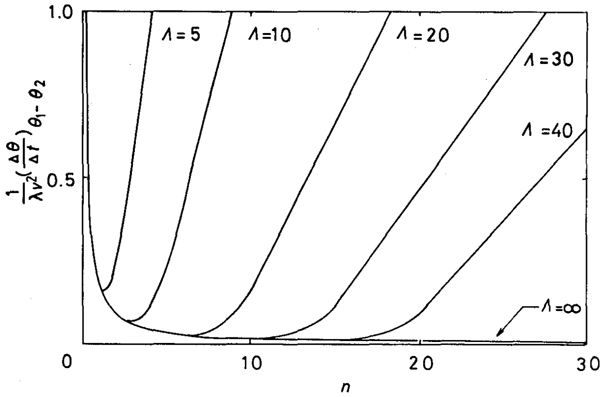

Fig. 10 Effect of heat input on cooling rate between θ_1 and θ_2 (Three dimensional flow)

boundary and heat input. In the caluclation, θ_1, and θ_2 have been chosen 0.533 (= 800 C/1,500 C) and 0.333 (= 500 C/1,500 C) respectively, considering the hardness of steel. As seen in Figure 9, the cooling rate decreases rapidly and gradually reaches the Rosenthal's case when the radius of cooling boundary is increased under the condition of constant heat input. While, as seen in Figure 10, when the heat input is increased under the constant cooling radius, the rate reduces along that of Rosenthal's case, but it rises remarkably in the range above a certain critical heat input.

The cooling rate in case of linear heat source can be also calculated in the similar way, and **Figure 11** shows the results along the moving axis and fusion line (weld bond line). The tendency is quite similar to that of previous point heat source, but the effect of cooling boundary on the cooling rate is more eminent

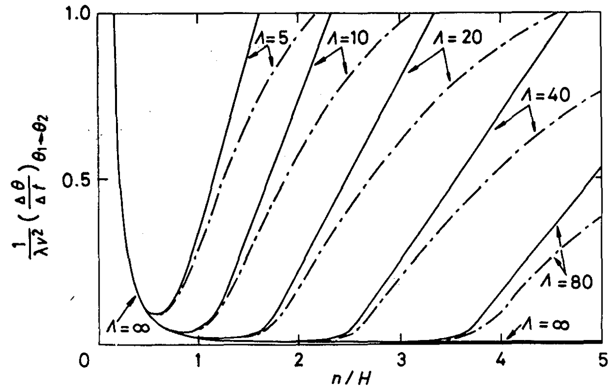

Fig. 11 Effect of heat input on cooling rate between θ_1 and θ_2 in case of two dimensional heat flow (Solid line: along moving axis, Broken line: along fusion boundary)

than that in three dimensional case due to the intrinsic characteristic of two dimensional conduction.

The above stated analysis leads one to recall the importance of selecting optimum relations between cavity radius (radius of cooling boundary) and welding parameters (heat input, welding speed, plate thickness, etc.) in the underwater welding using a local cavity method, or otherwise one may obtain very hardened weld even under the same condition with that of welding in air.

5. Concluding Remarks

In the above have been described the method of solving temperature filed when a equiradial cooling boundary around either a point or linear heat source moves with the source, which is aimed to estimate the thermal hysteresis during underwater welding by a circular local cavity formation method. The analysis has been conducted, however, by using very simplified models for the sake of mathematical treatment just like as other heat conduction investigations. So far the models employed here are concerned, the authors have proved the existence of rigorous solutions, but they have neglected the following matters which may be important factors in fusion welding. They are firstly the temperature dependence of thermal properties of materials, secondly the absorption and release of latent heat at melting and solidification, and thirdly the convection heat transfer due to liquid metal flow in a weld pool. Several reports have been published on the former two subjects[8)–10)], and thus it is possible to compare the differences between calculations when one considers these two factors or not. About the last factor of convective heat transfer, however, quantitative works have not been fully conducted yet but it remains as an important item of future investigations. In addition to the above stated general matters, the assumption of cooling boundary in this analytical model is equivalent with that the heat transfer coefficient between the plate surface and environmental water outside the cavity and also the heat conductivity of plate outside the cooling boundary are both supposed to be infinity, when one contrasts the practical underwater welding with a circular local cavity. It is, therefore, necessary to carry on further experimental verifications how much extent the theoretical analysis in this paper can well estimate the thermal fields in practical processes. On these matters will be precisely explained in future reports.

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