Commodity Taxation and Economic Efficiency*
—An Estimation of CGE Models

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Abstract

The Ramsey rule describes a system of optimal commodity taxation, in which the relative changes in compensated demands caused by extra commodity taxes are the same for all goods to minimize the deadweight loss. Although it contradicts the optimal commodity taxation theory, the uniform commodity taxation is widely implemented in practice. This paper applies Dynamic Computable General Equilibrium Models to investigate the influences of uniform commodity taxation on labor and capital's prices and supplies, and consumer's utility level. We give a comparison to labor and capital taxation. It shows that increases in commodity taxation do much more harm to economic efficiency than labor and capital taxation.

Keywords : Optimal commodity taxation, Uniform commodity taxation, Economic efficiency
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1.Introduction

The arguments on commodity taxation and economic efficiency have existed over a long time in the literature. Diamond and Mirrlees (1971) demonstrated that production efficiency is desirable even if a full Pareto optimum is not achieved; the presence of optimal commodity taxes implies the desirability of aggregate production efficiency, although it is weak. Coady and Drèze (2000) presented a "generalized Ramsey rule" for optimum taxation, giving one principle, that is, under specific distributional assumptions, "radial" reforms in the space of shadow taxes (i.e. small tax changes that bring consumer prices closer to shadow prices) are welfare improving. Asano and Fukushima (2000) evaluated the optimal tax equlibrium in relation to the uniform and the lump sum tax equilibria arguing that the deadweight losses of uniform and optimal taxation are quite small and very close to each other, the optimal rates are strikingly close to uniformity. However, Holcombe (2002) suggests that it is appropriate to apply the conventional inverse elasticity rules to predict the optimal tax structure.

The arguments can be divided into two groups in theory. One is to stress the non-uniform taxes system based on the Ramsey rule arguing that, in order to raise certain revenue by proportionate taxes on some of or all uses of income, "decrement of utility" is minimized when goods and service are taxed in inverse proportion to their elasticities of demand. Another one advocates a system of uniform taxation in order to obtain some administrative simplicity and mostly the economic efficiency, arguing that the Ramsey rule has little practical significance and the uniform taxes are not necessarily to bring out larger deadweight loss. In practice, although it contradicts optimal commodity taxation theory, the uniform commodity taxation is widely implemented. In many countries, standard proportional rates of 3-22% are set up to tax on the goods and service with some relief rates. Japan is practicing a uniform rate of 5% with no relief rates.

In this paper, we apply Dynamic Computable General Equilibrium Models to investigate the effects of uniform commodity taxation on economic efficiency by utilizing the Japanese data. We give a comparison to labor and capital taxation. It shows that increases in commodity taxation do much more harm to economic efficiency than labor and capital taxation. The outline of this paper is as follows. In section 2, we clarify the existing theory on optimal commodity taxation. Section 3 provides MPSGE (Mathematical Programming System for General Equilibrium) analysis. Section 4 describes the simulations and results. The last concludes.

2. Theory on Optimal Commodity Taxation

The theory on optimal commodity taxation includes the classical Ramsey rule, the Ramsey rule with many households and the Ramsey rule with decreasing returns to scale. It is necessary to review them, because they originally put an emphasis on the relationship between commodity taxation and economic efficiency, which is the subject of this paper. The former two are clarified as follows (the third one is too complicate, but not common, so we omit it).

A. The Classical Ramsey Rule

The conventional wisdom of optimal commodity taxation is the Ramsey rule, which is one most enduring and unquestioned concept of optimal taxation.

(1) Derivation

We first describe how the Ramsey rule is derived.

The optimal tax problem can be summarized by the maximization

$$MaxV(q_1 \cdots q_n, w, I)$$

$$q_i = p_i + \tau_i\ ,\quad p_i = c^i w, i = 1, \cdots, n \tag{1}$$

$$s.t. \sum_{i=1}^{n} \tau_i x = R \tag{2}$$

where V is the indirect utility function, $q_1 \cdots q_n$ are the consumer prices of good $1,\cdots,n$, c^i is a coefficient that describes the labour input necessary to produce one unit of good i, w is the wage rate, I is the lump-sum income, p_i is the pre-tax consumer price, x is the consumer demands, τ is the tax rate, R is the amount revenue to be raised.

The Lagrange-function is given by

$$L = V(q_1 \cdots q_n, w, I) + \lambda\left[\ \sum_{i=1}^{n} \tau_i x_i - R\ \right] \tag{3}$$

$$\frac{\partial L}{\partial \tau_k} = \frac{\partial V}{\partial \tau_k} + \lambda\left[\ x_k + \sum_{i=1}^{n} \tau_i \frac{\partial x_i}{\partial q_k}\ \right] = 0 \tag{4}$$

(where $\frac{\partial V}{\partial q_k} \equiv \frac{\partial V}{\partial \tau_k}, \frac{\partial x_i}{\partial q_k} \equiv \frac{\partial x_i}{\partial \tau_k}$ have been used)

$$\frac{\partial V}{\partial \tau_k} = -\lambda\left[\ x_k + \sum_{i=1}^{n} \tau_i \frac{\partial x_i}{\partial q_k}\ \right] \tag{5}$$

Following Roy's identity

$$\frac{\partial V}{\partial \tau_k}=-\frac{\partial V}{\partial I}x_k=-\alpha x_k \tag{6}$$

where I is the lump-sum income, α is the marginal utility of income.
Substituting (6) into (5)

$$\alpha x_k=\lambda[\ x_k+\sum_{i=1}^{n}\tau_i\frac{\partial x_i}{\partial q_k}\ ] \tag{7}$$

$$\sum_{i=1}^{n}\tau_i\frac{\partial x_i}{\partial q_k}=-\left[\frac{\lambda-\alpha}{\lambda}\right]x_k \tag{8}$$

Utilizing the Slutsky equation

$$\frac{\partial x_i}{\partial q_k}=S_{ik}-x_k\frac{\partial x_i}{\partial I} \tag{9}$$

Substituting (9) into (8)

$$\sum_{i=1}^{n}\tau_i\left[S_{ik}-x_k\frac{\partial x_i}{\partial I}\right]=-\left[\frac{\lambda-\alpha}{\lambda}\right]x_k \tag{10}$$

$$\sum_{i=1}^{n}\tau_i S_{ik}=-\left[1-\frac{\alpha}{\lambda}+\sum_{i=1}^{n}\tau_i\frac{\partial x_i}{\partial I}\right]x_k \tag{11}$$

$$\sum_{i=1}^{n}\tau_i S_{ik}=-\theta x_k,\ \ (\theta=\left[1-\frac{\alpha}{\lambda}+\sum_{i=1}^{n}\tau_i\frac{\partial x_i}{\partial I}\right]) \tag{12}$$

Utilizing $S_{ik}=S_{ki}$ (The Slutsky-matrix is symmetric)

$$\sum_{i=1}^{n}\tau_i^* S_{ki}=-\theta x_k^* \tag{13}$$

Equation (12) must equal (13). This is the Ramsey rule describing a system of optimal commodity taxes, in which the relative changes in compensated demand caused by extra commodity taxes are the same for all goods to minimize the deadweight loss.

(2) Special Case: the Inverse Elasticity Rule

First, for sake of simplicity, we assume that the lump-sum income is zero and taxable goods are just two, goods 1 and 2. The Ramsey rule can therefore be considered that the deadweight loss caused by levied taxes must be equal for goods 1 and 2 in order to minimize total deadweight loss. That is, (14) must be equal to (15).

$$\tau_1\frac{\partial x_1}{\partial q_1}=-\left[\frac{\lambda-\alpha}{\lambda}\right]x_1 \tag{14}$$

$$\tau_2 \frac{\partial x_2}{\partial q_2} = -\left[\frac{\lambda - \alpha}{\lambda}\right] x_2 \tag{15}$$

Thus, the Ramsey rule can be described as

$$\frac{\tau_1 \partial x_1 \big/ \partial q_1}{\tau_2 \partial x_2 \big/ \partial q_2} = \frac{x_1}{x_2} \tag{16}$$

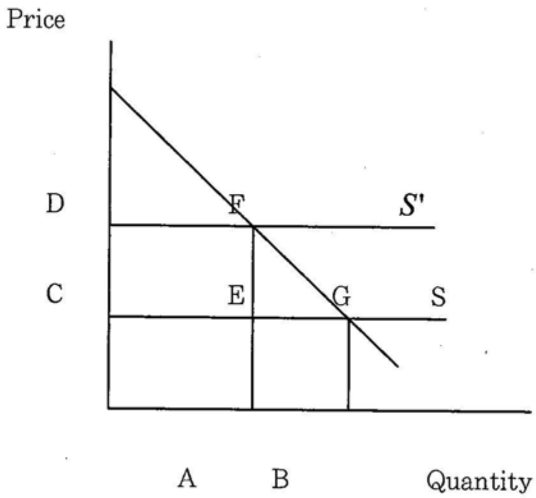
(16) can be changed into

$$\frac{\dfrac{\partial x_1}{\partial q_1} / x_1}{\dfrac{\partial x_2}{\partial q_2} / x_2} = \frac{\tau_2}{\tau_1} \tag{17}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\tau_2}{\tau_1} \tag{18}$$

Equation (18) is the special case of the Ramsey rule, the most popular wisdom of optimal taxation, which states that under some assumptions, the "decrement of utility" (other expressions such as deadweight loss or inefficiency effect of extra taxes or discouragement of taxable commodity) is minimized when goods are taxed in inverse proportion to their elasticities of demand. Figure A provides an impressionable interpretation for it.

Figure A A Simple Interpretation of the Ramsey Rule



We first give some assumptions: ①The elasticity of supply is zero; ②The income effect caused by the change of price is neglected; ③ There is no effect of cross-price. Then supply curve becomes level. Due to the introduction of commodity taxes, the supply curve shifts from S to S', the deadweight loss L and tax revenue R to be raised are given by

$$L=\frac{1}{2}EG\cdot EF=\frac{1}{2}\varepsilon\tau q\cdot\tau p=\frac{1}{2}\varepsilon\tau^2 pq \tag{19}$$

$$R=CD\cdot CE=\tau p\cdot(q-\varepsilon\tau q)=\tau pq-\varepsilon\tau^2 pq \tag{20}$$

$$\frac{\partial L}{\partial\tau}=\varepsilon\tau\cdot pq \tag{21}$$

$$\frac{\partial R}{\partial\tau}=(1-2\varepsilon\tau)pq \tag{22}$$

$$\frac{\partial L}{\partial R}=\frac{\varepsilon\tau}{1-2\varepsilon\tau} \tag{23}$$

$$\frac{\partial L_1}{\partial R_1}=\frac{\varepsilon_1\tau_1}{1-2\varepsilon_1\tau_1}=\frac{\partial L_2}{\partial R_2}=\frac{\varepsilon_2\tau_2}{1-2\varepsilon_2\tau_2} \tag{24}$$

$$\frac{\varepsilon_1}{\varepsilon_2}=\frac{\tau_2}{\tau_1} \tag{25}$$

The result (25) is the same as (18).

B. Ramsey Rule with Many Households

The original literature on the extension of the Ramsey rule with single household to many households is by Diamond and Mirrlees (1971). This extension is as follows.

The optimal commodity taxes problem is given by

$$Max_{(t_1,\cdots,t_{n-1})}W(V^1,\cdots,V^H)$$

$$\text{s.t.}\ \sum_{i=1}^{n-1}\sum_{h=1}^{H}\tau_i x_i^h=R \tag{26}$$

$$\text{argmax}\ \ U^h(x^h,\ell)\ \ \text{s.t.}\ \sum_{i=1}^{n-1}q_i x_i=w\ell^h+I^h \tag{27}$$

where W is the indirect function including the indirect functions U^h of H households.

The Lagrange function is

$$L=W(V^1,\ldots,V^H+\lambda[\ \sum_{i=1}^{n-1}\sum_{h=1}^{H}\tau_i x_i^h-R\ \ ] \tag{28}$$

$$\frac{\partial L}{\partial q_k}=\sum_{h=1}^{H}\frac{\partial W}{\partial V^h}\frac{\partial V^h}{\partial q_k}+\lambda\left[\sum_{h=1}^{H}x_k^h+\sum_{i=1}^{n-1}\sum_{h=1}^{H}\tau_i\frac{\partial x_i^h}{\partial q_k}\right]=0 \tag{29}$$

Utilizing Roy's identity ($\frac{\partial V^h}{\partial q_k} = -\alpha^h x_k^h$)

$$\sum_{h=1}^{H} \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial q_k} = -\sum_{h=1}^{H} \frac{\partial W}{\partial V^h} \alpha^h x_k^h \tag{30}$$

Now utilizing the social marginal utility of income ($\beta^h = \frac{\partial W}{\partial V^h} \alpha^h$)

$$\sum_{h=1}^{H} \beta^h x_k^h = \lambda \left[\sum_{h=1}^{H} x_k^h + \sum_{i=1}^{n-1} \sum_{h=1}^{H} \tau_i \frac{\partial x_i^h}{\partial q_k} \right] \tag{31}$$

Substituting the Slutsky equation ($\frac{\partial x_i^h}{\partial q_k} = S_{ik}^h - x_k^h \frac{\partial x_i^h}{\partial I^h}$) into (31)

$$\frac{\partial L}{\partial q_k} = \sum_{h=1}^{H} \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial q_k} + \lambda \left[\sum_{h=1}^{H} x_k^h + \sum_{i=1}^{n-1} \sum_{h=1}^{H} \tau_i \frac{\partial x_i^h}{\partial q_k} \right] = 0 \tag{32}$$

$$\frac{\sum_{i=1}^{n-1} \sum_{h=1}^{H} \tau_i S_{ki}^h}{\sum_{h=1}^{H} x_k^h} = \frac{1}{\lambda} \frac{\sum_{h=1}^{H} \beta^h x_k^h}{\sum_{h=1}^{H} x_k^h} - 1 + \frac{\sum_{h=1}^{H} \left[\sum_{i=1}^{n-1} \tau_i \frac{\partial x_i^h}{\partial I^h} \right] x_k^h}{\sum_{h=1}^{H} x_k^h} \tag{33}$$

$$\lambda \sum_{i=1}^{n-1} \sum_{h=1}^{H} \tau_i S_{ki}^h = \sum_{h=1}^{H} x_k^h (\beta^h + \lambda \sum_{h=1}^{H} \tau_i \frac{\partial x_i^h}{\partial I}) - \lambda \sum_{h=1}^{H} x_k^h \tag{34}$$

(Denoting $\sum_{i=1}^{n-1} \sum_{h=1}^{H} \tau_i S_{ki}^h = \Delta X_k^s, \beta^h + \lambda \sum_{h=1}^{H} \tau_i \frac{\partial x_i^h}{\partial I} = \gamma^h, \sum_{h=1}^{H} x_k^h = X_k$)

$$\frac{\Delta X_k^s}{X_k} = \frac{\sum_{h=1}^{H} x_k^h \gamma^h}{\lambda X_k} - 1 \tag{35}$$

It shows that the approximate relative change in compensated demand of good k caused by the introduction of the commodity taxes system should be less if individuals associated with high social marginal utility of income consume much of the good.

C. Some Argument on Ramsey Rule

The Ramsey rule requires equal "discouragement" of each taxable commodity, and commodities with inelastic demands require higher taxes to achieve the same discouragement. Clearly, commodities with relatively inelastic demands should be taxed heavily conflicts the consideration of equity. This problem maybe originated from the premise condition under which the Ramsey rule is derived. The derivation of the Ramsey rule considers the task of raising certain revenue is one and only one premise condition. Dixit (1970) intended to extend the Ramsey rule, taking account of possibility that some commodities may be un-taxable as Lerner (1970) mentioned in detail, in which the consumer's budget constraint is one more premise condition, although eventually being in favor of the Ramsey rule. The Ramsey rule maybe considers the task of raising certain revenue is more important than economic efficiency. In many countries, although with regressive characteristic, it is widely implemented to levy most uses of income including inelastic goods and services such as food, medical and so on by setting up a uniform taxation rate, which conflicts the Ramsey rule, to ensure that government can raise certain revenue. Because high elastic goods cannot ensure raising certain revenue due to their high elasticities of demand.

By considering cost-benefit analyses, the Ramsey rule is also un-practical. In the US, total commodity taxes collections comprise only 3.5% of total tax receipts. In Japan, total commodity taxes collections comprise average 9% of total taxes revenues before 1997 and 19% after 1997, due to a rise of commodity tax rate from 3% to 5% in 1997. Those in other OECD countries are about 11%. To raise these relative small amounts of revenue, the government may be reluctant to set up a taxes system in order to follow the Ramsey rule even if the elasticities of demand for different thousands of goods and service can be calculated. In fact, the calculation of these elasticities is almost impossible. Thus, in many countries, a standard uniform taxation is levied on most commodities including relatively low elastic goods and service. At the same time, heavy taxation is levied on goods such as tobacco, alcohol et al. with relatively high elasticities to partly take equity into consideration to some extent.

Because of the existence of the divergence between theory and practice, or maybe there is no substantial difference between them, it may be interesting to give a further empirical study on commodity taxation, especially its relation with economic efficiency. In the following, we focus on the relationship of uniform commodity taxation and economic efficiency.

3. Dynamic Computable General Equilibrium Models

We utilize Dynamic Computable General Equilibrium Models (DCGEM), which are solved by *MPSGE* (Mathematical Programming System for General Equilibrium) analysis.

A. MPSGE Static Analysis

We intend to find an equilibrium in the economy, which consists of two economic agents:

producers and consumers. Producers are the firms that take initial endowments of production factors of Capital (K) and Labor (L) from the consumers as inputs of production and convert them into outputs. There are four sectors: Agriculture, Energy, Manufacture, and Service. They produce goods and service of X_1, X_2, X_3, X_4, respectively, at the given technology levels. For simplicity, we assume there is a single representative consumer. The consumer derives his income I from the sales of his endowments. Then he purchases his preferred choice of goods and service and obtains utility from consumption of them. We want to determine the prices and quantities which maximize producers' profits and consumer's utility.

$$\max W(X_1, X_2, X_3, X_4)$$

$$\text{s.t. } \sum_{i=1}^{4} P_{X_i} X_i = P_K K + P_L L \tag{36}$$

$$X_i = F_i(K_{X_i}, L_{X_i}), i = 1,2,3,4 \tag{37}$$

$$K = \sum_{i=1}^{4} K_{X_i} \tag{38}$$

$$L = \sum_{i=1}^{4} L_{X_i} \tag{39}$$

where W is a utility function; P_{X_i} are the prices of goods and service X_i; P_K is the price of capital K; P_L is the price of labor L; K_{X_i} and L_{X_i} are capital and labor used in the production sector X_i. This is a standard microeconomic textbook optimization problem, and a usual technique for finding the solution is the method of Lagrange multipliers. This problem can be solved in *GAMS* (General Algebraic Modeling System) as a non-linear programming (*NLP*). There are some cases (such as a presence of several consumers, taxes, or other distortions) where it is not possible to solve the problem of finding a market equilibrium as an optimization problem. Then the problem could be approached in a different way. It can be turned into a Mixed Complimentarity Problem (*MCP*) and solved as a system of non-linear equations. *NLP* problems are a subset of *MCP* and *MPSGE* finds an equilibrium as a solution to *MCP*. The nonlinear complimentarity problem (*NCP*), given a nonlinear function $F: R^n \rightarrow R^n$, is to find an $x \in R^n$ such that

$$NCP(F): F(x) \geq 0, x \geq 0, x^T f(x) = 0 \tag{40}$$

In the case when we have not just one x, but a vector of $\overline{x} = (x_1, x_2, \cdots, x_n)$, then there is a system of n equations like $\overline{x} f(\overline{x}) = 0$, which forms a *MCP* problem. The word "mixed" in *MCP* reflects the fact that the solution is a mix of equalities $f(x) = 0$ and inequalities $f(x) > 0$.

Mathiesen (1985) suggested that the Arrow-Debreu general equilibrium model can be formulated as *MCP*, with a set of three non-negative variables of prices, quantities (they are called as activity levels in *MPSGE*) and income levels in solving *MCP*. An equilibrium in the variables satisfies a system of three inequalities: zero profit condition, market clearance condition, and income balance condition. To illustrate the equilibrium conditions, now introduce an additional production sector U representing utility derived from the consumption of X_i. Accordingly, P_U is a price of the output of the production sector U. An activity level Y is a vector with the following components $Y=(X_i,U)$. We assume that activity levels X_i and U are positive (i.e., the condition $Y \geq 0$ is satisfied with the strict inequality as $Y>0$)

First, consider zero profit conditions. Zero profit condition requires that no producer earns "excess" profits (i.e. the value of inputs must be equal to or greater than the value of outputs).

$$-\Pi_i(p_i)=C_i(P_L,P_K)-R_i(p_i)\geq 0, i=1,2,3,4 \tag{41}$$

$$-(P_U-e(P_{X_i}))=0 \tag{42}$$

where Π is the unit profit function with constant returns to scale, C_i and R_i are unit cost and revenue functions for X_i (cost of production of one unit of a good under the factor prices w and r); e is a unit cost (expenditure) function for U (cost of buying one unit of utility under the prices of P_{X_i}).

Second, the market clearance condition can be written as

$$\sum_i y_i \frac{\partial \Pi_i(p_i)}{\partial p_i} \geq \sum d_i(p,M) \tag{43}$$

which means that, at equilibrium prices and quantities, the net supply of goods and service i by the constant returns to scale production sectors should be equal to or greater than the aggregate final demand for i by households and governments, given market prices p_i and income levels I. Market clearance condition requires that if prices are positive then supply should be equal to demand. It is represented by the following equations

$$X_i=\frac{\partial e}{\partial P_{X_i}}U, i=1,2,3.4 \tag{44}$$

$$W=\frac{I}{P_U} \tag{45}$$

$$L=\sum_{i=1}^{4}\frac{\partial C_{X_i}}{\partial P_L} \tag{46}$$

$$K=\sum_{i=1}^{4}\frac{\partial C_{X_i}}{\partial P_K} \tag{47}$$

On the demand side, partial derivatives of $\partial e/\partial P_i$, $\partial C_i/\partial P_L$ and $\partial C_i/\partial P_K$ represent compensated demand functions (by Shephard's lemma).

Third, the income balance condition requires that for each agent (including any government entities) value of income must equal the value of factor endowments and tax revenue. For the consumer,

$$P_L L + P_K K = I \tag{48}$$

MPSGE would solve above thirteen equations for thirteen unknowns: activity levels (quantities) of $X_i (i = 1,2,3,4)$ and U , prices of $P_{X_i} (i = 1,2,3,4), P_U, P_L, P_K$ and income I . It should be noted that equilibrium conditions are hard to formulate in an explicit form for certain problems. The good news for a *MPSGE* user is that one does not need to spend any time deriving the equilibrium conditions. *MPSGE* builds them for you automatically. (*MPSGE* and its algebraic form *MCP* can be seen in Shoven and Whalley (1984) Rutherford's (1997)). It should be noted that *MPSGE* is a tool for the formulation but not for the solution of complimentarity problems. In order to run *MPSGE* program, *MCP* solvers should be used. *GAMS* has two *MCP* solvers: *MILES* and *PATH*. In Tom's words, *MILES* is his abandoned child, which leaves *PATH* as the first choice.

B. Dynamic Analysis

In a dynamic economy, a representative consumer maximizes the present value of his lifetime utility

$$\max \sum_{t=1}^{\infty} (\frac{1}{1+\rho})^t U(c_t)$$

$$\text{s.t. } c_t = F(K_t, L_t) - I_t \tag{49}$$

$$K_{t+1} = K_t(1-\delta) + I_t \tag{50}$$

where t - time periods, ρ - individual time-preference parameter, U - utility function, c_t -consumption in period t, K is capital and F represents production function. The consumer faces the following constraints. First, total output produced in the economy is divided to consumption and investment, I_t. Second, capital depreciates at the rate δ.

We intend to find the first-order conditions. The Lagrangian is:

$$\begin{aligned} \Im = (\frac{1}{1+\rho})^t U(c_t) + \lambda_1(F(K_t, L_t) - I_t - c_t) + \lambda_2(K_{t-1}(1-\delta) \\ + I_{t-1} - K_t) + \lambda_3(K_t(1-\delta) + I_t - K_{t+1}) \end{aligned} \tag{51}$$

The first-order conditions are:

$$\frac{\partial \Im}{\partial c_t} = (\frac{1}{1+\rho})^t \frac{\partial U(c_t)}{\partial c_t} - \lambda_1 = 0 \tag{52}$$

$$\frac{\partial \Im}{\partial K_t} = \lambda_1 (\frac{\partial F}{\partial K_t}) - \lambda_2 + \lambda_3 (1-\delta) = 0 \tag{53}$$

$$\frac{\partial \Im}{\partial I_t} = -\lambda_1 + \lambda_3 = 0 \tag{54}$$

Now recall that the Lagrange multiplier shows the sensitivity to changes in the constraint. In economic terms it is a measure of value of the scarce resources in the problem under consideration. In a usual utility maximization problem (maximization of utility subject to a budget constraint), the Lagrange multiplier can be interpreted as the marginal utility of (budget) money. In a usual cost minimization problem the Lagrange multiplier can be interpreted as a marginal cost of production (or the internal value, or imputed value, or more frequently, the shadow price). Mathematically, all these problems are identical. What is different is our interpretation. So in the Lagrange optimization the multuplier shows the shadow price (or marginal cost). Under constant returns to scale and perfect competition assumptions, price equals marginal cost. Therefore, we can rewrite the first-order conditions as:

$$P_t = (\frac{1}{1+\rho})^t \frac{\partial U(c_t)}{\partial c_t} \tag{55}$$

$$PK_t = (1-\delta) PK_{t+1} + P_t \frac{\partial F(K_t, L_t)}{\partial K_t} \tag{56}$$

$$P_t = PK_{t+1} \tag{57}$$

where P_t, PK_t and PK_{t+1} are the values of the corresponding Lagrange multipliers. They can be interpreted as the price of output, the price of capital today, and the price of capital tomorrow, respectively. The utility maximization problem above is formulated as a *NLP* problem.

As has already been discussed, *MPSGE* solves it as an *MCP* problem. Let RK_t and W_t represent rental rate of capital and wage rate in period t. Denote a unit cost function as $C(RK_t, W_t)$ (Unit cost function is a solution of the problem: $\min(W_t L_t + RK_t K_t)$ s.t. $F(K_t, L_t) = 1$) , and a demand function as $D(P_t, M)$ (Demand function is a solution of the problem: $\max \sum (1/(1+\rho))^t U(c_t)$ s.t. $\sum P_t c_t = M$) , where M is consumer's income. Then *MCP* can be formulated as follows.

Zero profit conditions:

$$P_t \geq PK_{t+1}, I_t \geq 0, I_t(P_t - PK_{t+1}) = 0 \tag{58}$$

$$PK_t \geq RK_t + (1-\delta)PK_{t+1}, K_t \geq 0, K_t(PK_t - RK_t - (1-\delta)PK_{t+1}) = 0 \tag{59}$$

$$C(RK_t, W_t) \geq P_t, Y_t \geq 0, Y_t(CRK_t, W_t) - P_t = 0 \tag{60}$$

Market clearance conditions:

$$Y_t \geq D(P_t, M), P_t \geq 0, P_t(Y_t - D(P_t, M) + I_t) = 0 \tag{61}$$

$$L_t \geq Y_t \frac{\partial C(RK_t, W_t)}{\partial W_t}, W_t \geq 0, W_t(L_t - Y_t \frac{\partial C(RK_t, W_t)}{\partial W_t}) = 0 \tag{62}$$

$$K_t \geq Y_t \frac{\partial C(RK_t, W_t)}{\partial RK_t}, RK_t \geq 0, RK_t(K_t - Y_t \frac{\partial C(RK_t, W_t)}{\partial RK_t}) = 0 \tag{63}$$

Income balance condition:

$$M = PK_0 + \sum_{t=0}^{\infty} W_t L_t, M > 0 \tag{64}$$

As noted before, one does not need to program these equilibrium conditions explicitly. *MPSGE* constructs them automatically.

4. Simulations

In the following, we first give the benchmark solution to the models, and then perform some simulations to investigate the influences of uniform commodity taxation on economy by comparing to labor and capital taxation

A. Procedures

We first create the benchmark *SAM* (Social Accounting Matrices) data, which are from the Japanese *I/O* table and *SNA* of 2000. Table 1 describes the *SAM* data.

Insert Table 1 here

Second, we give some changes in the static model as follows.

(1) Assume the population and productivity grows at rate g (0.01), the capital depreciation rate δ (0.05), and the interest rate r (0.05).

(2) Set the time from 2000 through 2050

(3) Declare two parameters and their growth rates

$QREF(t)$ is an index of economic activity which represents the reference quantities path, and $PREF(t)$ presents value prices index which represents the reference prices path.

$$QREF(t) = (1+g)^{**}(ORD(t)-1) \tag{65}$$

$$PREF(t) = 1/(1+r)^{**}(ORD(t)-1) \tag{66}$$

where $ORD(t)-1$ is an exponent to represent the fact that in the base year. $QREF$ and $PREF$ are equal to 1 and grow thereafter.

(4) Calibrate initial steady state capital stock

$$K_0 = R_0/(r+\delta) \tag{67}$$

where K_0 denotes initial capital stock, R_0 denotes base year capital income.

(5) Adjust investment and consumption to be consistent with steady state

$$I_0 = (\delta+g)K_0 \tag{68}$$

$$I_0 = \frac{(\delta+g)R_0}{r+\delta} \tag{69}$$

(6) Introduce two more production blocks: capital accumulation, K, and investment, I.

(7) Introduce two more production blocks: capital accumulation, K, and investment, I, and change production blocks for them.

Rewrite the first-order conditions for capital and investment as.

$$PK_t = (1-\delta)PK_{t+1} + RK_t \tag{70}$$

$$PK_{t+1} = P_t \tag{71}$$

where PK and RK represent two prices for capital: purchase price and rental price. We assume a constant interest rate r, so all future prices (including price of labor and capital) in terms of present value are:

$$P_{t+1} = \frac{P_t}{1+r} \tag{72}$$

Equation (71) can be rearranged using equation (72) for PK:

$$PK_t = (1+r)P_t \tag{73}$$

Substitution of equation (73) for PK_t and equation (71) for PK_{t+1} into (70) leads to:

$$(1+r)P_t = (1-\delta)P_t + RK_t \tag{74}$$

$$RK_t = (\delta+r)P_t \tag{75}$$

The reference quantity for RK_t is $K_0(r+\delta)$. The dynamic model would not calibrate without this reference quantity. The reason for this is the fact that RK is used in several production blocks: X, Y and K, and we need to reflect the relationship between RK_t and P_t given by equation (75).

Production block I introduces equation (71).

(8) Change the reference quantities in the demand and utility blocks

(9) Introduce terminal constraint

An important characteristic of the dynamic problem is a treatment of capital in the last period of modeling. We cannot solve numerically for an infinite number of periods, hence, some adjustments are needed for approximation of finite horizon model to the infinite horizon choices. Special procedures should be introduced for the terminal capital, otherwise, all capital would be consumed in the last period, and nothing would be invested. The infinite horizon consumer problem can be partitioned at year T with a single state variable K_{T+1}, linking the two problems. This means that if you set the terminal capital stock to the appropriate value, then the time path of consumer demands will be identical to the infinite horizon values.

Following Rutherford (1997), we give a constraint on the growth rate of investment in the terminal.

$$\frac{I_T}{I_{T-1}} = \frac{Y_T}{Y_{T-1}} = \frac{UTIL_{T-1}}{UTIL_{T-1}} \tag{76}$$

where T is a terminal period. The advantage of the usage of this constraint is that it imposes balanced growth in the terminal period but does not require that the model achieve steady-state growth. The meaning of the constraint is that investment in a terminal period should grow at the same rate as output and utility.

(10) Make initial values assignment

The initial values are assigned to reflect present values (prices are adjusted for $PREF(t)$ and growth in quantities (quantities are adjusted for $QREF(t)$).

B. Result

The solution listing shows that all of marginal values are equal to zero, activity levels are rising over time and prices are falling over time. After benchmark replication, one can run counterfactual experiments, such as introduction of taxes, change in growth rates, elasticities, etc. For the purpose of the present paper, we introduce taxes on commodity, labor and capital. We simulate four cases: (1) Commodity tax 5%, (2) Labor tax 5% and capital tax 5%, (3) Commodity tax 10%, (4) Labor tax 10% and capital tax 10%. The result suggests that there are little changes in price and supply of labor for the four cases. It also suggests that increases in taxes make capital price increase, while capital supply and consumer utility decrease greatly. We further compare the influences of different tax increases on prices and supplies of labor and capital, and utility of consumer. We find that increases in all of the taxes bring about increasing changes in capital prices, the influences of labor and capital taxes are far greater than those of the commodity taxes, although the former is negative in the beginning. We also find that, increases in all of the taxes affect the prices and supplies of labor negatively in the beginning; and the effect of commodity tax 5% becomes the same as that of labor and capital taxes 10% in a long term; commodity tax 10% brings about the greatest changes in labor prices and supplies. Finally, we find that increases in all of the taxes bring about decreasing changes in capital supplies and consumer utilities, the negative influences of the commodity taxes are far greater than those of labor and capital taxes.

5. Conclusion

The Ramsey rule describes a system of optimal commodity taxation, in which the relative changes in compensated demands caused by extra commodity taxes are the same for all goods to minimize the deadweight losses. Although it contradicts optimal commodity taxation theory the uniform commodity taxation is widely implemented in practice. We tried to investigate the influence of uniform commodity taxation on prices and supplies of labor and capital, and utility of consumer. We gave a comparison to labor and capital taxation by utilizing Dynamic Computable General Equilibrium Models. It shows that increases in commodity taxation do much more harm to economic efficiency than labor and capital taxation.

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Table 1 Social Accounting Matrix Data

Output	agr	energy	mfg	service	capital	Labor	hcons	gcons	Invest	export	import	taxm	output
agr	1.558	0.0007	8.426	1.498			3.966		0.967	0.072	1.974	0.144	14.370
energy	0.270	9.885	7.485	13.918			10.103		0.016	0.316	8.544	1.051	32.398
mfg	2.260	0.276	117.974	59.197			60.738	4.247	35.267	46.255	30.557	2.632	293.026
service	2.207	6.425	57.501	150.525			225.354	117.288	58.210	10.844	9.250	0.009	619.095
capital	6.168	7.211	33.213	143.282									189.874
labor	1.373	2.803	58.486	232.100									294.762
hcons					173.641	266.788							440.429
taxi	0.534	5.796	9.941	18.577	16.233	27.974						3.836	82.889
Saving							140.268	-38.646	-94.46	-7.162			0
output	14.369	32.398	293.025	619.095	189.874	294.762	440.429	82.889	0	50.325	50.325	3.836	