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Homogenization-based optimization of wall thickness distribution for TPMS two-fluid heat exchangers

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ABSTRACT

Triply periodic minimal surface (TPMS) structures are attractive for high-performance heat exchangers (HXs) owing to their continuous flow pathways and high surface-area-to-volume ratios. In addition, spatially graded wall thickness offers further potential to enhance thermal-hydraulic performance. This study proposes an effective porous media model for TPMS two-fluid HXs, in which effective heat-transfer coefficients derived from unit-cell CFD simulations are incorporated to represent heat exchange between the fluid phases and the solid wall. The proposed model enables macroscopic evaluation of wall-thickness effects on HX performance without resolving the complex TPMS geometry, while reducing the computational cost by approximately 260 times compared with full-scale simulations. By integrating the model into a density-based topology optimization framework, a rapid and practical method for optimizing wall-thickness distribution in TPMS HXs is established. Full-scale numerical simulations of the optimized-thickness design for a gyroid two-fluid HX show a 12.2% improvement in the performance evaluation criterion (PEC) compared with the uniform-thickness design. The improvement arises from the optimized non-uniform wall thickness, which directs more flow toward the core ends and enhances velocity uniformity. As a result, heat transfer is enhanced at the core ends, leading to more effective utilization of the entire HX core and improved overall thermal performance.

1. Introduction

Rapid advances in science and technology have substantially increased the demand for effective thermal management solutions across various industrial sectors. Heat exchangers (HXs) are one of the most important components in thermal management, playing vital roles in a wide range of applications such as air conditioning, refrigeration, automotive cooling, and industrial process systems. Additionally, advances in additive manufacturing (AM) technology have enabled the fabrication of products with complex internal geometries that were once difficult to produce [1,2]. These advances in manufacturing capabilities have facilitated the practical implementation of lattice structures and TPMS structures, which offer superior mechanical properties [3,4].

TPMS-based HXs have attracted increasing attention owing to their high surface-area-to-volume ratio and continuous flow passages, which promote enhanced thermal and hydraulic performance [5–8]. Both numerical and experimental investigations have demonstrated that TPMS HXs can outperform conventional compact HX concepts, including printed circuit heat exchangers (PCHEs), in terms of heat transfer capability and overall thermal-hydraulic performance [9–17]. TPMS HXs have been further improved through geometric modifications such

as stretching and surface perturbations [18,19], as well as implicit-function-based TPMS designs [20,21]. In addition, optimization frameworks based on correlations or data-driven models have been proposed to facilitate design exploration [22,23]. Despite these advances, these approaches typically assume spatially uniform wall thickness and unit-cell size, leaving the potential of spatially varying TPMS geometries insufficiently explored.

To exploit such spatial variations, functionally graded TPMS structures have attracted growing interest due to their potential to enhance HX performance [24,25]. Recent studies have explored graded TPMS HXs using several strategies, including conformal designs to improve flow uniformity [26], radial grading of unit-cell size [27], and mathematically defined local morphological gradients [28,29]. Hybrid graded designs combining different TPMS cell types (e.g., gyroid and diamond) have also been proposed, showing improved overall thermal-hydraulic performance [30,31]. These studies demonstrate that graded TPMS designs offer a promising approach to both enhance heat transfer and hydraulic performance. However, most design methods for graded TPMS HXs rely on empirical or experimental approaches, and optimization strategies are still underdeveloped. Previous studies have shown

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Nomenclature**Roman symbols**

A	area [m^2]
L_{cell}	unit-cell size [m]
L_{HX}	HX length [m]
D_h	hydraulic diameter [m]
V	volume [m^3]
$\dot{V}$	volumetric flow rate [$\text{m}^3 \text{s}^{-1}$]
c	level-set parameter [m]
w	weighting factor in objective function [-]
Q	heat transfer rate [W]
Q^*	volumetric heat source term in the porous model [W m^{-3}]
Δp	pressure drop [Pa]
p	pressure [Pa]
U_{HX}	overall heat transfer coefficient [$\text{W m}^{-2} \text{K}^{-1}$]
ΔT	temperature difference [K]
T	temperature [K]
$\mathbf{u}$	velocity [m s^{-1}]
$\mathbf{U}$	Darcy velocity [m s^{-1}]
$\bar{U}$	mean velocity magnitude [m s^{-1}]
CV	coefficient of variation of velocity magnitude [-]
f_{low}	low-velocity area fraction [-]
Re	Reynolds number [-]
Pr	Prandtl number [-]
Nu	Nusselt number [-]
f	Fanning friction factor [-]
j	Colburn j factor [-]
PEC	performance evaluation criterion [-]
k	thermal conductivity [$\text{W m}^{-1} \text{K}^{-1}$]
k^*	effective thermal conductivity [$\text{W m}^{-1} \text{K}^{-1}$]
c_p	specific heat capacity [$\text{J kg}^{-1} \text{K}^{-1}$]
$\mathbf{q}$	heat flux [W m^{-2}]

Greek symbols

α	viscous resistance coefficient [Pa s m^{-2}]
β	inertial resistance coefficient [kg m^{-4}]
γ	design variable (unfiltered) [-]
$\hat{\gamma}$	design variable (filtered) [-]
ε_i	porosity of fluid phase i [-]
η	threshold parameter in f_{low} [-]
μ	viscosity [Pa s]
ρ	density [kg m^{-3}]

Subscripts

i	fluid index ($i = 1, 2$)
h	hot
c	cold
in	inlet
out	outlet
f	fluid
s	solid
w	wall
cell	unit cell
RVE	representative volume element
ave	averaged value

tot	total
0	reference value

Abbreviations

AM	additive manufacturing
HX	heat exchanger
LMTD	logarithmic mean temperature difference
NS	Navier–Stokes
PCHE	printed circuit heat exchanger
RVE	representative volume element
BC	boundary condition
TO	topology optimization
TPMS	triply periodic minimal surface

that wall thickness strongly influences thermal and hydraulic characteristics [32–34]. Therefore, determining the optimal wall thickness distributions is essential for further improving graded TPMS HX performance. From a macroscopic perspective, TPMS cells with different wall thicknesses can be treated as different materials. Consequently, the optimization of wall thickness distributions for TPMS HXs can be formulated as a material distribution problem.

Topology optimization (TO) [35] is a well-established method for optimizing material distribution. It offers a high degree of design freedom by representing the material distribution using design variables. TO has been actively studied for thermal-fluid systems [36, 37] and has recently been applied to two-fluid HXs [38]. For example, density-based [39,40] and level-set-based [41] TO methods have been developed for two-fluid HXs, both aiming to maximize heat transfer under pressure constraints while preventing fluid mixing. Recent studies have improved TO for two-fluid HXs by combining topology and shape optimization [42] and by adopting sparse narrow-band schemes that update only the fluid–solid interface for efficient large-scale optimization [43]. However, two-fluid HXs optimized using conventional TO are often unmanufacturable due to their complex geometries. In recent years, TO frameworks have been proposed that incorporate manufacturability constraints, and their feasibility has been demonstrated using AM and experimental testing [44,45]. Nevertheless, support-free fabrication of the resulting geometries often still requires post-processing, and simultaneously achieving high thermal–hydraulic performance and manufacturability remains challenging, especially given the design freedom inherent to TO. These limitations motivate the development of optimization methods for two-fluid HXs based on inherently manufacturable architectures, such as TPMS.

In recent years, optimization of TPMS structures has gained significant attention due to their manufacturability via AM. However, directly optimizing the detailed microscale features of TPMS geometries is computationally prohibitive due to their complexity. To address this challenge, researchers have employed a homogenization approach. Due to the periodic and spatially regular nature of TPMS structures, they can be represented as homogenized materials with equivalent effective properties. This method enables the prediction of macroscopic behavior without resolving detailed microscale geometries, substantially reducing computational cost. In addition, the effective properties can be linked to design variables of TO. Therefore, by treating TPMS structures with different wall thicknesses, cell sizes, or cell types as different homogenized materials and incorporating them into TO frameworks, we can effectively optimize graded TPMS structures. These approaches have been pioneered mainly in structural problems, including studies on wall thickness optimization [46–48], sequential optimization of cell size followed by wall thickness [49] and its extension to thermal conduction problems [50], optimization considering multiple TPMS cell types [51,52], and optimization including void phases [53], and

optimization of both density and orientation [54]. Taken together, these efforts highlight the potential of homogenization-based TO as a powerful tool for improving the functional performance of graded TPMS structures.

Recently, researchers have extended homogenization-based TO to thermal-fluid problems. For large-scale thermal-fluid systems, directly resolving detailed geometries requires an extremely large number of mesh elements, making full-scale simulations often computationally prohibitive. To overcome this limitation, porous media models have been widely adopted [55–62]. They simplify the heat exchange regions by treating them as porous media, reducing computational cost while maintaining sufficient accuracy in predicting pressure drop and heat transfer. These models also enable the application of homogenization-based TO to thermal-fluid systems with periodic microstructures. In previous studies, lattice structures with varying diameters and TPMS structures with varying wall thicknesses have been modeled as homogenized porous media with effective properties, and their diameter/thickness distributions have been optimized within a TO framework [63–65]. However, these studies focus on single-fluid convective systems, in which heat transfer is modeled primarily between the fluid and the solid structure. By contrast, two-fluid TPMS HXs involve more complex and strongly coupled heat-transfer processes: heat is exchanged between two moving fluids across partition walls while simultaneously being conducted through the solid structure. Accurately capturing this conjugate thermal coupling therefore requires a fundamentally different effective modeling approach, rather than a straightforward extension of existing single-fluid homogenization-based formulations. Given the high surface area and favorable thermal-fluid transport properties of TPMS structures, extending homogenization-based TO to two-fluid systems is an important engineering challenge.

In this paper, we propose an optimization method of wall thickness distribution for TPMS HXs based on an effective porous media model, which replaces the detailed microscale geometry with a homogenized porous-medium representation to reduce computational cost. Extending homogenization-based TO to two-fluid HXs requires an effective model that accurately represents the coupled heat transfer between the two fluids and the separating wall. To this end, we introduce effective heat transfer coefficients to characterize the thermal interaction between each fluid and the wall. The effective properties are obtained from finite element simulations of the gyroid unit cell, and their relationship with the design variables is approximated using polynomials. By integrating heat transfer prediction based on effective heat transfer coefficients with density-based TO, wall-thickness distribution optimization for TPMS HXs using an effective model becomes feasible. The optimization objective is defined as a weighted sum of the heat transfer rate and the pressure drop, and TO is conducted for multiple sets of weighting factors. The optimized designs are reconstructed into full-scale gyroid structures via a dehomogenization process, and their performance is rigorously assessed through high-fidelity simulations. Moreover, achieving a resolution sufficient to represent complex geometries within a direct TO framework would require extremely fine meshes, resulting in prohibitively high computational costs. By limiting the design space to TPMS architectures, the proposed method achieves manufacturability while maintaining high thermal-hydraulic performance and avoiding such computational costs, thereby circumventing the key limitations of direct TO-based two-fluid HX design approaches. The effectiveness and limitations of the proposed approach are examined through numerical case studies.

The remainder of this paper is organized as follows. In Section 2, we describe the effective porous media model for the TPMS two-fluid HX, the calculation methods for the effective material properties, and the full-scale analysis model. In Section 3, we present the optimization model and the formulation of the optimization problem. In Section 4, we validate the full-scale analysis model, verify the effective porous media model, and present the optimization and full-scale analysis results. In Section 5, we discuss the physical mechanisms underlying the performance improvement and the sources of discrepancies between the effective and full-scale models. Finally, Section 6 concludes the paper and outlines directions for future work.

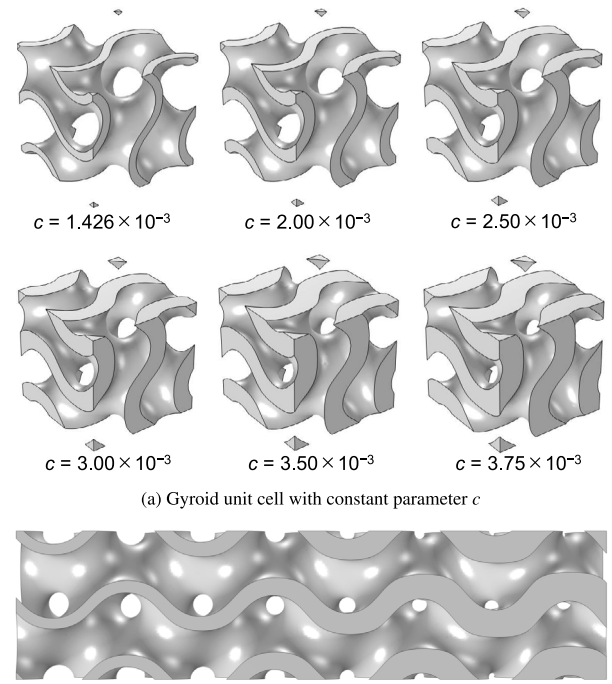


Fig. 1. Illustration of gyroid structures showing the effect of the parameter c on wall thickness for $L_{\text{cell}} = 4.60 \times 10^{-3}$ [m].

2. Numerical modeling

This section presents the mathematical foundations of the effective porous media model for TPMS HXs. The core concept of the proposed model is the incorporation of heat exchange between each fluid and the wall into the effective thermal formulation. The methodology for computing the effective properties and the metrics for evaluating full-scale analysis results are also described.

2.1. Gyroid design with controllable wall thickness

TPMS are periodic surfaces without self-intersections that extend infinitely along three principal axes [66]. The first example of TPMS was discovered by Schwarz in 1865 [67], and the gyroid structure, which is the focus of the present work, was discovered by Alan Schoen in 1970 [68]. The gyroid surface is represented by the following equation:

$$\sin X \cos Y + \sin Z \cos X + \sin Y \cos Z = c/L_{\text{cell}}, \quad (1)$$

where c is the level-set parameter, L_{cell} is the unit cell size, and $X = 2\pi x/L_{\text{cell}}$, $Y = 2\pi y/L_{\text{cell}}$, $Z = 2\pi z/L_{\text{cell}}$, where $\mathbf{x} = (x, y, z)$ represents a spatial position. When c is set to zero, the surface defined by Eq. (1) partitions the space into two regions of equal volume.

To apply gyroid structures to HXs, it is necessary to define solid domains. In this study, the solid domain of the gyroid structure, denoted as Ω_s , is defined using two level-set functions as follows [49]:

$$\begin{aligned} G_1(\mathbf{x}) &= c/L_{\text{cell}} + (\sin X \cos Y + \sin Z \cos X + \sin Y \cos Z), \\ G_2(\mathbf{x}) &= c/L_{\text{cell}} - (\sin X \cos Y + \sin Z \cos X + \sin Y \cos Z), \\ \Omega_s &= \{\mathbf{x} \mid G_1(\mathbf{x}) \geq 0, G_2(\mathbf{x}) \geq 0\}. \end{aligned} \quad (2)$$

These level-set surfaces are generated by translating the reference surface defined by $c = 0$ an equal distance in the positive and negative directions. These solid domains divide the space into two fluid regions of equal volume.

Fig. 1 illustrates the effect of parameter c on the gyroid structure. Fig. 1(a) shows a gyroid unit cell with a constant value of c , where it can be seen that the wall thickness increases as c increases. Fig. 1(b) shows a gyroid structure generated by continuously varying c spatially. As demonstrated in Fig. 1(b), smoothly varying c results in a corresponding continuous change in the wall thickness of the gyroid structure.

In this study, functionally graded gyroid two-fluid HXs are designed by spatially varying c within the gyroid HX core domain. This approach enables the optimization of wall thickness distribution, thereby improving the thermal and hydraulic performance of the gyroid HXs.

2.2. Effective porous media model for TPMS two-fluid HXs

Direct optimization of TPMS HX structures is challenging due to the high computational cost associated with their complex internal geometry. To address this issue, a porous media approximation is effective, as it can significantly reduce the computational cost. Toward this goal, previous studies have utilized the periodicity of TPMS structures to construct effective porous media models by treating them as homogeneous porous media [65]. Such models enable the prediction of physical behavior without explicitly resolving the complex internal geometry, thereby substantially reducing computational cost. However, an effective model applicable to the optimization of two-fluid TPMS HXs has not yet been established. In this study, we propose an effective porous media model that incorporates heat exchange between each fluid and the solid wall to overcome this limitation.

This study employs two types of models: an effective porous media model for optimization calculations, and a Navier–Stokes (NS) model for full-scale analysis and the computation of effective properties. The following assumptions apply to both models:

- (1) The flow is steady and incompressible.
- (2) The flow is laminar.
- (3) The material properties of both the fluid and the solid are constant and temperature-independent.
- (4) The hot and cold fluids are assumed to be the same fluid.
- (5) Thermal radiation effects are not considered.

Additionally, the effective porous media model incorporates the following additional assumption:

- (6) The effective properties are assumed to be isotropic.

Based on these assumptions, each model is developed and analyzed.

Flow through porous media is typically described by Darcy's law, which accounts only for viscous losses. However, when the flow velocity is relatively high or when the medium has a complex internal geometry, such as in TPMS structures, inertial effects cannot be neglected. In such cases, the Brinkman–Forchheimer model [69,70], which extends the Brinkman equation [71] by incorporating the Forchheimer term [72], is commonly used. This model includes the Darcy resistance, viscous diffusion, and inertial loss terms, making it suitable as an effective porous media model for TPMS structures. In this study, the porous flow model for the TPMS two-fluid HXs is expressed as follows:

$$\begin{aligned} \nabla \cdot \mathbf{U}_i &= 0, \\ \frac{\rho}{\varepsilon_i} (\mathbf{U}_i \cdot \nabla) \frac{\mathbf{U}_i}{\varepsilon_i} &= \nabla \cdot [-p_i \mathbf{I} + \mathbf{K}] - \alpha_i \mathbf{U}_i - \beta_i |\mathbf{U}_i| \mathbf{U}_i, \\ \mathbf{K} &= \frac{\mu}{\varepsilon_i} (\nabla \mathbf{U}_i + (\nabla \mathbf{U}_i)^T) - \frac{2}{3} \frac{\mu}{\varepsilon_i} (\nabla \cdot \mathbf{U}_i) \mathbf{I}, \end{aligned} \quad (3)$$

where $\mathbf{U}_i$ is the Darcy velocity vector of fluid i ($i = 1, 2$), defined as the volumetric flow rate per unit cross-sectional area within the porous medium. The parameters ε_i , p_i , α_i , and β_i denote the porosity, pressure, viscous resistance coefficient, and inertial resistance coefficient of fluid i , respectively. The viscosity and fluid density are represented by μ

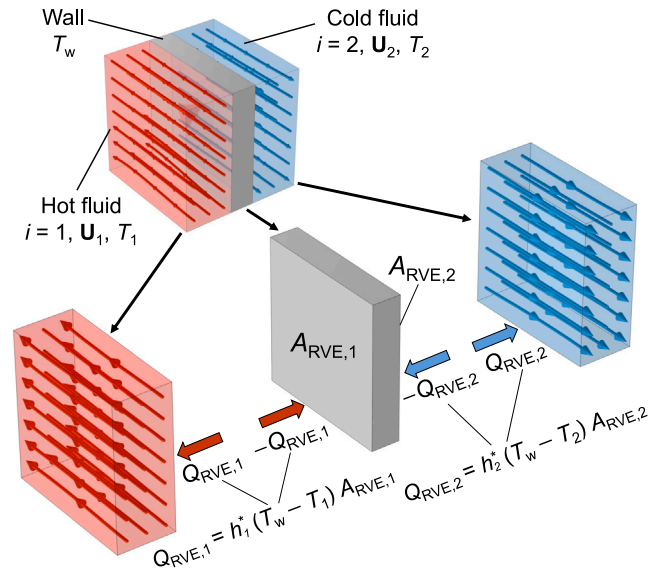


Fig. 2. Schematic of heat exchange between fluid i and the wall within a representative volume element (RVE).

and ρ , respectively. Using this model enables efficient and reasonable prediction of the macroscopic flow behavior through TPMS structures, without directly resolving the complex geometry.

Heat exchange between fluids and solids must be incorporated into the effective thermal model when constructing a porous media model for TPMS two-fluid HXs. To construct the effective thermal model, the heat exchange within a representative volume element (RVE) of a two-fluid HX is expressed using representative values. Fig. 2 shows a schematic illustration of the heat exchange within the RVE. As shown in Fig. 2, both the hot and cold fluids exchange heat with the wall. The total heat exchange of fluid i within the RVE, denoted as $Q_{RVE,i}$, can be microscopically expressed as follows:

$$Q_{RVE,i} = \int_{\Gamma_i} h_i(\mathbf{x}) (\tilde{T}_w(\mathbf{x}) - \tilde{T}_i(\mathbf{x})) dA, \quad (4)$$

where Γ_i denotes the wall boundary in contact with fluid i , and $h_i(\mathbf{x})$ represents the local heat transfer coefficient between the wall and fluid i . $\tilde{T}_w(\mathbf{x})$ and $\tilde{T}_i(\mathbf{x})$ denote the wall and fluid temperatures, respectively. By appropriately selecting the RVE to capture the average flow and temperature behavior, Eq. (4) can be approximated using representative values within the RVE as follows:

$$Q_{RVE,i} = h_i^* (T_w - T_i) A_{RVE,i} \quad (5)$$

where h_i^* is the effective heat transfer coefficient of fluid i , T_w is volume-averaged wall temperature, T_i is volume-averaged temperature of fluid i , and $A_{RVE,i}$ is the wall–fluid i interfacial area in the RVE. The effective heat transfer coefficient h_i^* can be defined as:

$$h_i^* = \frac{\int_{\Gamma_i} \mathbf{q}_i \cdot \mathbf{n}_i dA}{A_{RVE,i} (T_w - T_i)}, \quad (6)$$

where $\mathbf{q}_i$ is the heat flux vector on the wall surface in contact with fluid i , and $\mathbf{n}_i$ is the outward unit normal vector on the wall surface Γ_i with respect to the fluid domain. Furthermore, dividing Eq. (5) by the volume of the RVE, V_{RVE} , yields the volumetric heat exchange rate:

$$Q_i^* = h_i^* (T_w - T_i) A_{RVE,i} / V_{RVE}. \quad (7)$$

By incorporating this volumetric heat source term, the effective thermal models for both fluid i and the solid, accounting for the heat exchange effects, are formulated as follows:

$$\rho c_p \mathbf{U}_i \cdot \nabla T_i + \nabla \cdot (-k_{f,i}^* \nabla T_i) = Q_i^*, \quad (8)$$

$$\nabla \cdot (-k_s^* \nabla T_w) = -Q_1^* - Q_2^*, \quad (9)$$

where c_p is the specific heat capacity of the fluid, $k_{f,i}^*$ is the effective thermal conductivity of fluid i , and k_s^* is the effective thermal conductivity of the solid. The two fluid temperatures T_1 and T_2 and the wall temperature T_w are treated as independent nodal field variables and are solved in a fully coupled manner via the volumetric heat source terms Q_i^* in Eqs. (8) and (9).

Porous media modeling of microchannel structures, in which heat exchange between fluid and solid is represented as a volumetric heat source term, has also been proposed in previous studies focusing on compact tube HXs [61] and TPMS HXs [62]. In these studies, one of the two fluids was treated as a solid domain to simplify the porous media model. Moreover, the heat transfer coefficient for fluid–solid surface was prescribed based on existing correlations. In contrast, the porous media model developed in the present study adopts an RVE approach, wherein both fluids are explicitly modeled. Furthermore, the effective heat transfer coefficient used in the model is derived from microscale RVE simulations, enabling a more accurate evaluation of the heat exchange between each fluid and the solid.

By solving the coupled equations, Eqs. (3), (8) and (9), the flow and heat transfer behavior in TPMS two-fluid HXs can be predicted at a reasonable computational cost. This effective porous media model is not limited to TPMS structures but can also be applied to porous media modeling of two-fluid HXs with other periodic architectures.

2.3. Computing effective properties

To solve the effective porous media model constructed in Section 2.2, it is necessary to compute the effective properties. The effective properties include porosity ϵ_i , surface area within the RVE $A_{RVE,i}$, viscous resistance coefficient α_i , inertial resistance coefficient β_i , effective thermal conductivity of fluid $k_{f,i}^*$, effective thermal conductivity of solid k_s^* , and effective heat transfer coefficient h_i^* . These effective properties are calculated based on the finite element method. In simulations that do not employ the effective porous medium model (i.e., the RVE and full-scale simulations), a conjugate heat transfer problem is solved by coupling the fluid flow and energy equation in the fluid with heat conduction in the solid:

$$\nabla \cdot \mathbf{u} = 0, \quad (10a)$$

$$\rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mu \nabla \cdot [\nabla \mathbf{u} + (\nabla \mathbf{u})^T], \quad (10b)$$

$$\rho c_p \mathbf{u} \cdot \nabla T = \nabla \cdot (k_f \nabla T) \quad (\text{fluid}), \quad (10c)$$

$$0 = \nabla \cdot (k_s \nabla T) \quad (\text{solid}). \quad (10d)$$

At the fluid–solid interface, both temperature and normal heat flux are continuous. Here, k_f and k_s denote the thermal conductivities of the fluid and solid, respectively. Eqs. (10) are used to compute the effective properties.

In this study, we focus on the periodicity of TPMS structures and quantitatively evaluate the corresponding effective properties through numerical homogenization based on the RVE method. The RVE is defined as a region that statistically represents the entire material [73]. For TPMS structures, a unit cell with periodic boundary conditions (BCs) is widely adopted as a RVE for evaluating effective properties [54, 74, 75]. In particular, Gajetti et al. derived Darcy–Forchheimer correlations as functions of porosity from unit-cell simulations for several TPMS geometries, including the gyroid, and validated the resulting homogenized models against full-scale numerical simulations [75]. Accordingly, the gyroid unit cell with periodic BCs is adopted as the RVE in this study.

The gyroid structure defined by Eq. (2) partitions the space into two geometrically identical regions with equal volume fractions, and the material properties of the hot and cold fluids are identical in this study. Therefore, the porosity ϵ_i , the wall–fluid i interfacial area in the RVE $A_{RVE,i}$, the viscous resistance coefficient α_i , the inertial resistance

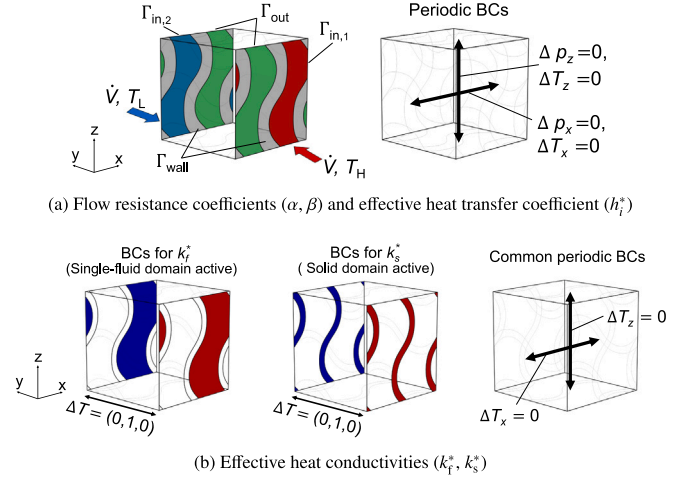


Fig. 3. BCs for the RVE used to evaluate effective properties.

coefficient β_i , and the effective thermal conductivity $k_{f,i}^*$ are the same for fluid 1 and fluid 2. Accordingly, in this study, the subscript i for these effective properties is omitted, and they are hereafter denoted as ϵ , A_{RVE} , α , β , k^* . Regarding the effective heat transfer coefficient h_i^* , its value depends on the magnitude of the Darcy velocity $|\mathbf{U}|$, so h_1^* and h_2^* may differ at the same location.

The porosity ϵ is defined as the ratio of the volume of fluid i to the total volume of the RVE:

$$\epsilon = \frac{V_{f,i}}{V_{RVE}} = \frac{V_{f,i}}{L_{cell}^3}, \quad (11)$$

where $V_{f,i}$ represents the volume of each fluid region within the RVE and V_{RVE} is the total volume of the RVE. The contact surface area between the wall and each fluid in the RVE, A_{RVE} , is computed directly from the geometry of the gyroid unit cell.

To compute the coefficients α , β , and the effective heat transfer coefficient h_i^* , conjugate thermofluid simulations are conducted on the RVE. Fig. 3(a) illustrates the BCs applied to the RVE for evaluating these effective properties. A counterflow configuration is assumed in this study: the hot fluid enters through the inlet surface $\Gamma_{in,1}$ with temperature $T = T_H$ and volumetric flow rate $\dot{V}$, while the cold fluid enters through the inlet surface $\Gamma_{in,2}$ with temperature $T = T_L$ and the same flow rate $\dot{V}$. Both fluids exit through the outlet surface Γ_{out} , where a pressure BC $p = 0$ is imposed. The wall surface Γ_{wall} , located on the same plane as the inlet surfaces, is treated as adiabatic. To capture the average flow behavior, fully developed flow conditions are imposed at the inlet and outlet, and periodic BCs are applied to the lateral surfaces. Simulations are performed for various gyroid unit cells with different wall thicknesses and a range of flow rates. The wall thickness is controlled by varying the level-set parameter c , which defines the gyroid structure based on Eq. (2).

The pressure drop per unit length, $-\Delta p/L_{cell}$, is described as a function of the Darcy velocity component U aligned with the pressure drop direction, the viscous resistance coefficient α , and the inertial resistance coefficient β . Specifically, the coefficients α and β are determined by fitting the relationship between the pressure gradient and U using the following equation:

$$-\frac{\Delta p}{L_{cell}} = -\alpha U - \beta U^2. \quad (12)$$

This equation corresponds to the Darcy–Forchheimer model, which accounts for both viscous and inertial effects in porous media flow [70].

The effective heat transfer coefficient h_i^* can be calculated using Eq. (6), and its value depends on the wall thickness and the Darcy velocity magnitude. As illustrated in Fig. 3(a), the same volumetric flow

rate $\dot{V}$ is imposed for both fluid phases in the RVE. Accordingly, when constructing the interpolation function for h_i^* , the condition $|\mathbf{U}_1| = |\mathbf{U}_2|$ is assumed. Under the symmetry and identical-property assumptions described above, the two fluid phases are expected to exhibit equivalent heat transfer characteristics at a given Darcy velocity magnitude $|\mathbf{U}|$. Therefore, h_i^* is described using an identical interpolation functional for $i = 1, 2$. In this study, thermofluid simulations using the RVE are performed to calculate the effective heat transfer coefficient h_i^* for each combination of wall thickness and flow rate. The resulting h_i^* values are then fitted as a function of wall thickness and the Darcy velocity magnitude $|\mathbf{U}_i|$ to construct the interpolation function.

The effective thermal conductivities of the fluid and solid, k_f^* and k_s^* , are evaluated as follows. Fig. 3(b) illustrates the BCs applied to the RVE for evaluating these effective properties. To evaluate the effective thermal conductivity, a unit temperature difference ($\Delta T = 1$) is imposed between two opposing faces of the target region in the RVE, and the heat conduction equation (Eq. (10d)) is solved. When computing the effective thermal conductivity of the fluid, the unit temperature difference is applied to only one of the fluid regions. Isothermal BCs are applied on the surfaces where the temperature difference is imposed. Periodic BCs are applied on the surrounding surfaces perpendicular to the isothermal surfaces, and adiabatic BCs are applied on the remaining surfaces. Under these BCs, the effective thermal conductivities of the fluid and solid are calculated based on Fourier's law $\mathbf{q} = -k\nabla T$ using the following equations:

$$\frac{1}{A_{\text{cell}}} \int_{\Psi_f} \mathbf{q} \cdot \mathbf{n} dA = -k_f^* \frac{\Delta T}{L_{\text{cell}}}, \quad (13)$$

$$\frac{1}{A_{\text{cell}}} \int_{\Psi_s} \mathbf{q} \cdot \mathbf{n} dA = -k_s^* \frac{\Delta T}{L_{\text{cell}}}, \quad (14)$$

where A_{cell} is the cross-sectional area of the unit cell ($A_{\text{cell}} = L_{\text{cell}}^2$), Ψ_f and Ψ_s denote the surfaces on the high-temperature side where the unit temperature difference is applied in the fluid and solid regions, respectively, $\mathbf{q}$ is the heat flux vector, $\mathbf{n}$ is the outward unit normal vector on the surface.

2.4. Full-scale performance evaluation

In this study, commonly used dimensionless performance metrics are introduced to quantitatively evaluate the heat transfer, flow resistance, and overall performance of multiple HX designs. The actual velocity and temperature distributions within the full-scale two-fluid HX are obtained through numerical simulations based on the governing equations (Eq. (10)). Using these simulation results, the velocity, temperature, and pressure distributions at the inlet and outlet are integrated to evaluate the heat transfer rate Q and the pressure drop Δp of the HX according to the following equations:

$$Q_i = \left| \int_{\Gamma_{\text{out},i}} \rho c_p T \mathbf{u} \cdot \mathbf{n} dA - \int_{\Gamma_{\text{in},i}} \rho c_p T \mathbf{u} \cdot \mathbf{n} dA \right|, \quad (15)$$

$$Q = \frac{Q_1 + Q_2}{2},$$

$$\Delta p_i = \frac{1}{A_{\text{in},i}} \int_{\Gamma_{\text{in},i}} p dA - \frac{1}{A_{\text{out},i}} \int_{\Gamma_{\text{out},i}} p dA, \quad (16)$$

$$\Delta p = \frac{\Delta p_1 + \Delta p_2}{2}.$$

Based on these metrics, dimensionless performance metrics are defined.

The Reynolds number Re is introduced as a dimensionless parameter representing the flow state:

$$\text{Re} = \frac{\rho u_{\text{in}} D_h}{\mu}, \quad (17)$$

where u_{in} is the inlet velocity and D_h is the hydraulic diameter. The hydraulic diameter D_h is calculated as follows [10,21,30]:

$$D_h = \frac{4V_{\text{f,tot}}}{A_{\text{tot}}}, \quad (18)$$

where $V_{\text{f,tot}}$ is the total fluid volume in the core region of the HX and A_{tot} is the total heat exchange surface area.

The Fanning friction factor f , representing the flow resistance, is calculated from the pressure drop as follows:

$$f = \frac{D_h \Delta p}{2 \rho u_{\text{in}}^2 L_{\text{HX}}}, \quad (19)$$

where L_{HX} is the geometric length of the HX core in the bulk-flow (streamwise) direction.

For the heat transfer performance, the overall heat transfer coefficient U_{HX} is calculated using the logarithmic mean temperature difference ΔT_{LMTD} :

$$\Delta T_{\text{LMTD}} = \frac{(T_{h,\text{in}} - T_{c,\text{out}}) - (T_{h,\text{out}} - T_{c,\text{in}})}{\ln \left(\frac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}} \right)}, \quad (20)$$

$$U_{\text{HX}} = \frac{Q}{A_{\text{tot}} \Delta T_{\text{LMTD}}}. \quad (21)$$

As a dimensionless parameter characterizing convective heat transfer, the Nusselt number Nu is introduced:

$$\text{Nu} = \frac{U_{\text{HX}} D_h}{k_f}. \quad (22)$$

Furthermore, to evaluate the convective heat transfer performance normalized by flow conditions and fluid properties, the Colburn j factor is introduced:

$$j = \frac{\text{Nu}}{\text{Re} \text{Pr}^{1/3}}, \quad (23)$$

where Pr is the Prandtl number. This provides a standardized basis for comparing the heat transfer performance of different TPMS HXs.

In this study, the Performance Evaluation Criterion (PEC) [76–78] is introduced to evaluate the overall thermal-hydraulic performance of TPMS HXs, considering both heat transfer and flow resistance:

$$\text{PEC} = \frac{j/j_0}{f/f_0}, \quad (24)$$

where j_0 and f_0 are the reference values of the Colburn j factor and the Fanning friction factor f , respectively. PEC provides a single dimensionless metric that balances heat transfer enhancement against pressure drop, enabling quantitative comparison among TPMS HX designs. PEC is used to identify the most efficient design by considering the trade-off between heat transfer rate and pressure drop.

3. Optimization problem

3.1. Formulation basics for topology optimization of TPMS structures

TO, originally proposed by Bendsoe and Kikuchi [35], enables the optimization of material distribution in a fixed design domain under a high degree of design freedom. In TO, the material distribution within a fixed design domain D is defined by a characteristic function $\chi_{\Omega} : D \rightarrow \{0, 1\}$ representing the presence or absence of material. This function generally takes discrete values: 0 for void regions and 1 for material regions. In contrast, TPMS structures are characterized by level set functions, where local wall thickness is controlled by the level set parameter c . However, restricting to binary values, such as in characteristic functions, does not allow for the representation of intermediate thicknesses.

To overcome this limitation, the density method [79], which defines design variables as continuous values between 0 and 1, is effective. By employing a continuous density function to control the wall thickness of TPMS structures, flexible thickness variations, including intermediate values, can be realized. Furthermore, if the parameter controlling the wall thickness of TPMS structures is defined spatially discretely, it may cause abrupt thickness transitions and geometric discontinuities that degrade performance and complicate manufacturing. To overcome this issue, applying filtering to the density function is effective. Spatial

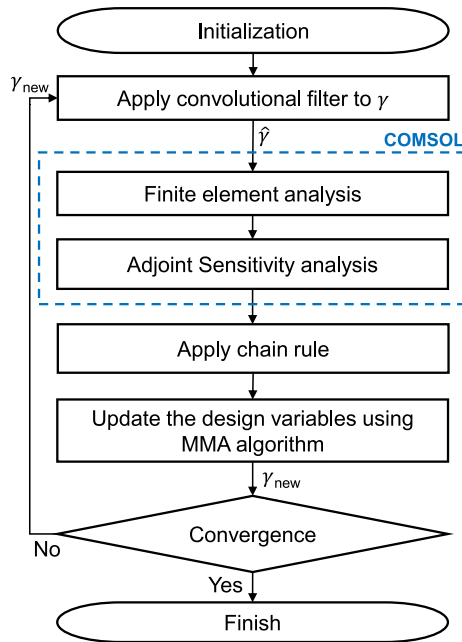


Fig. 4. Flowchart of the optimization procedure.

smoothing of the design variables through filtering enables continuous and smooth control of the wall thickness distribution. This filtering approach enables the realization of graded TPMS structures that are both physically feasible and manufacturable.

In this study, a convolutional filter [80,81] is applied to the density function γ , generating a spatially smooth design variable field $\hat{\gamma}$. This field does not directly define material presence, but serves as a wall thickness control parameter linked to the level set parameter c , which governs the local wall thickness of the TPMS structure. Here, the design variable field and the level set parameter distribution c are related by the following relationship:

$$\hat{\gamma} = \frac{c(\mathbf{x}) - c_{\min}}{c_{\max} - c_{\min}}, \quad (25)$$

where $c_{\min}$ and $c_{\max}$ denote the minimum and maximum values of c , respectively.

In this study, the design variable distribution $\hat{\gamma}$ is optimized based on the density method, and effective properties are interpolated as functions of the design variable $\hat{\gamma}$. This framework enables the optimization of the wall thickness distribution in TPMS two-fluid HXs. Gradients of the objective function with respect to the original design variable γ are obtained by applying the chain rule.

The overall optimization procedure is summarized in Fig. 4. At each iteration, MATLAB applies a convolutional filter to the design variables γ to obtain $\hat{\gamma}$. The filtered field $\hat{\gamma}$ is passed to COMSOL Multiphysics via LiveLink™ for MATLAB, where finite element analysis and adjoint sensitivity analysis are performed to compute the sensitivity with respect to the filtered design variables $\hat{\gamma}$. The computed sensitivities are returned to MATLAB, and the sensitivity with respect to the original design variables γ is obtained using the chain rule. The design variables γ are then updated using the method of moving asymptotes (MMA) [82]. In this study, Svanberg's publicly available MATLAB implementation of MMA distributed by SMOPTIT (GNU GPLv3) [83] is used, specifically the MATLAB files `mmasub.m` and `subsol.v.m`. This loop is repeated until the prescribed convergence criterion is satisfied.

3.2. Problem formulation

In this study, the effective porous media model proposed in Section 2.2 is adopted as the physical model for wall-thickness optimization. The governing equations for the velocity field, the fluid energy

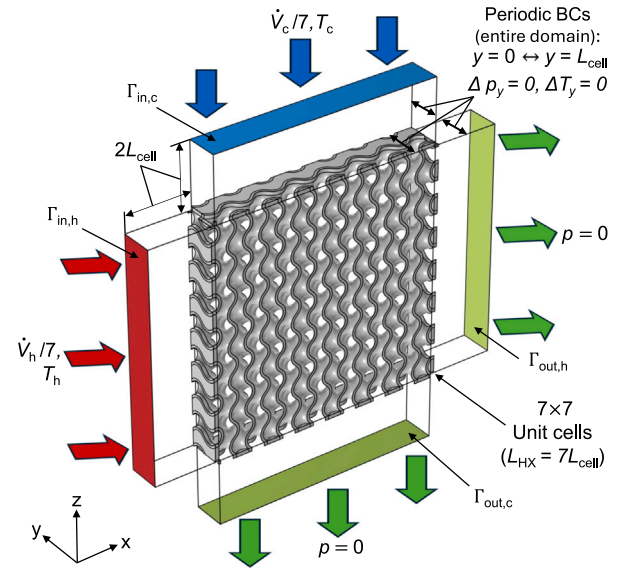


Fig. 5. Computational domain and BCs for numerical model validation.

Table 1
Material properties used for validation.

Parameter	Solid	Fluid
Thermal conductivity [W/(m K)]	0.18	0.631
Specific heat capacity [J/(kg K)]	2450	4178
Density [kg/m³]	1181	992.2
Viscosity [Pa s]		6.53×10^{-4}

equation, and the solid heat conduction equation are given by Eqs. (3), (8), and (9), respectively.

In HX design, it is crucial to consider both heat transfer performance and pressure drop. In this study, an objective function that incorporates these two factors with a weighting factor w is employed. The optimization problem is formulated as follows:

$$\begin{aligned} &\text{Find } \gamma \\ &\text{minimize } J = -Q_{\text{ave}} + w\Delta p_{\text{ave}} \\ &\text{subject to } 0 \leq \gamma(\mathbf{x}) \leq 1, \quad \forall \mathbf{x} \in D \end{aligned} \quad (26)$$

The average heat transfer rate Q_{ave} is defined as the average of the absolute values of the heat transfer amounts calculated from the temperature differences between the inlet and outlet of fluids 1 and 2:

$$Q_{\text{ave}} = \frac{\rho c_p}{2} \left(\left| \int_{\Gamma_{\text{out},1}} T_1 \mathbf{U}_1 \cdot \mathbf{n} dA - \int_{\Gamma_{\text{in},1}} T_1 \mathbf{U}_1 \cdot \mathbf{n} dA \right| + \left| \int_{\Gamma_{\text{out},2}} T_2 \mathbf{U}_2 \cdot \mathbf{n} dA - \int_{\Gamma_{\text{in},2}} T_2 \mathbf{U}_2 \cdot \mathbf{n} dA \right| \right), \quad (27)$$

where $\Gamma_{\text{in},i}$ and $\Gamma_{\text{out},i}$ denote the inlet and outlet surfaces of fluid i , respectively. The average pressure drop Δp_{ave} is defined as the average of the pressure differences between the inlet and outlet of fluids 1 and 2:

$$\Delta p_{\text{ave}} = \frac{1}{2} \left(\frac{1}{A_{\text{in},1}} \int_{\Gamma_{\text{in},1}} p_1 dA - \frac{1}{A_{\text{out},1}} \int_{\Gamma_{\text{out},1}} p_1 dA + \frac{1}{A_{\text{in},2}} \int_{\Gamma_{\text{in},2}} p_2 dA - \frac{1}{A_{\text{out},2}} \int_{\Gamma_{\text{out},2}} p_2 dA \right). \quad (28)$$

This study performs optimization with different weighting factors w to obtain multiple design solutions reflecting the trade-off between heat transfer performance and pressure drop.

Table 2
Results of the mesh independence study.

Mesh	Number of elements	Q [W]	Error in Q [%]	$\Delta p/L_{HX}$ [Pa/m]	Error in $\Delta p/L_{HX}$ [%]
Mesh 1	3,167,846	148.48	3.51	57.86	2.94
Mesh 2	4,810,819	145.61	1.52	56.98	1.38
Mesh 3	7,130,430	144.41	0.68	56.61	0.71
Mesh 4	10,865,567	143.74	0.21	56.38	0.30
Mesh 5	14,463,139	143.57	0.09	56.25	0.07
Mesh 6	18,356,949	143.44	Baseline	56.21	Baseline

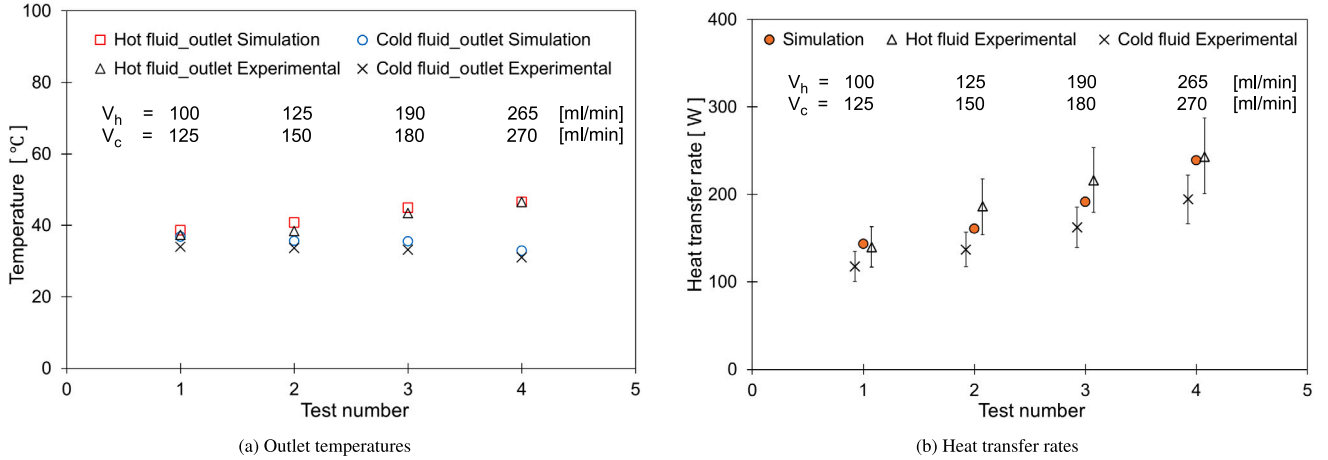


Fig. 6. Comparison of numerical and experimental results for validation.

4. Numerical examples

In this study, both the full-scale simulations and the effective porous media simulations are performed in COMSOL Multiphysics (version 6.2), whereas the optimization loop (filtering and design-variable updates) is implemented in MATLAB (version 2022b).

For the flow equations, different discretization schemes are employed for the effective porous media and full-scale simulations. A Taylor–Hood element pair is used in the effective porous media simulations, with quadratic (P2) elements for velocity and linear (P1) elements for pressure. In the full-scale simulations, a stabilized equal-order element pair (P1–P1) is employed for the NS equations to improve computational efficiency, with streamline and crosswind diffusion stabilization as implemented in COMSOL Multiphysics. All temperature fields in both simulations are discretized using linear (P1) elements. The discretization schemes and mesh settings for the effective porous media and full-scale simulations are summarized below.

Effective porous media simulations. A structured hexahedral mesh with uniform element size is used for the effective porous media simulations. Each element has a side length of $L_{cell}/2$.

Full-scale simulations. The full-scale simulations are performed on the reconstructed gyroid geometry obtained via the dehomogenization procedure, and the TPMS surface is meshed using a boundary-fitted mesh. A boundary-layer mesh is introduced near the solid walls, resulting in a hybrid mesh dominated by tetrahedral and prismatic elements (with pyramidal elements in transition regions).

4.1. Validation of numerical simulations

To ensure the reliability of the numerical simulation results, a validation study was conducted based on a reference experiment. Since this study focuses on laminar flow conditions and a gyroid structure, the work by Dixit et al. [12] was selected for validation, as it reports detailed experimental data on a gyroid two-fluid HX under such conditions. In their study, the thermal performance of a HX composed of a gyroid structure with a cell size of 4.6 mm and a porosity of 0.8 was

experimentally evaluated. The geometry consisted of seven repeated unit cells in all three spatial directions.

In contrast, to reduce computational cost, the numerical model used for validation was simplified to a single unit cell in the y -direction, as shown in Fig. 5. The gyroid geometry was adjusted to achieve a total fluid porosity of $\epsilon_{tot} = 0.8$ (i.e., $\epsilon_{tot} = \epsilon_1 + \epsilon_2$) and was constructed using Eq. (2) with $L_{cell} = 4.6 \times 10^{-3}$ m and $c = 1.426 \times 10^{-3}$ m. The solid domain was assumed to be composed of a polymer, and its material properties were taken from Dixit et al. Water was used as the working fluid. The reference temperature for the fluid properties was defined as the average of the inlet temperatures of the hot and cold streams, which was 313.15 K. The properties at this temperature were used. The material properties employed in the validation are summarized in Table 1. To enable a meaningful comparison, the inlet conditions were matched to those used in Dixit et al. However, to account for the reduction of the HX thickness to one-seventh of the original geometry, the volumetric flow rates in the simulation were scaled accordingly to one-seventh of the experimental values. The inlet temperatures of the hot and cold fluids were set to $T_h = 333.15$ K and $T_c = 293.15$ K, respectively. As BCs, the inlet and outlet faces were prescribed with the specified flow conditions, as illustrated in Fig. 5. The surfaces normal to the y -direction ($y = 0$ and $y = L_{cell}$) were treated as periodic boundaries to approximate the flow behavior in the central region of the full-scale HX. The remaining external surfaces of the computational domain were assumed to be adiabatic and subject to no-slip conditions.

Initially, a mesh independence study was conducted using the present simulation model. Several meshing schemes were evaluated by varying the minimum cell size and the number of boundary layer elements, and their effects on the pressure drop and heat transfer rate were assessed. The study was carried out at volumetric flow rates of $\dot{V}_h = 100$ mL/min (1.67×10^{-6} m³/s) and $\dot{V}_c = 125$ mL/min (2.08×10^{-6} m³/s). The results are summarized in Table 2. The heat transfer rate Q is multiplied by seven to account for the reduced computational domain, which corresponds to one-seventh of the original heat-exchange region. As the number of elements increases, both Q and $\Delta p/L_{HX}$ converge to constant values. In this study, Mesh 4 was selected for the subsequent numerical analyses by considering the balance between computational

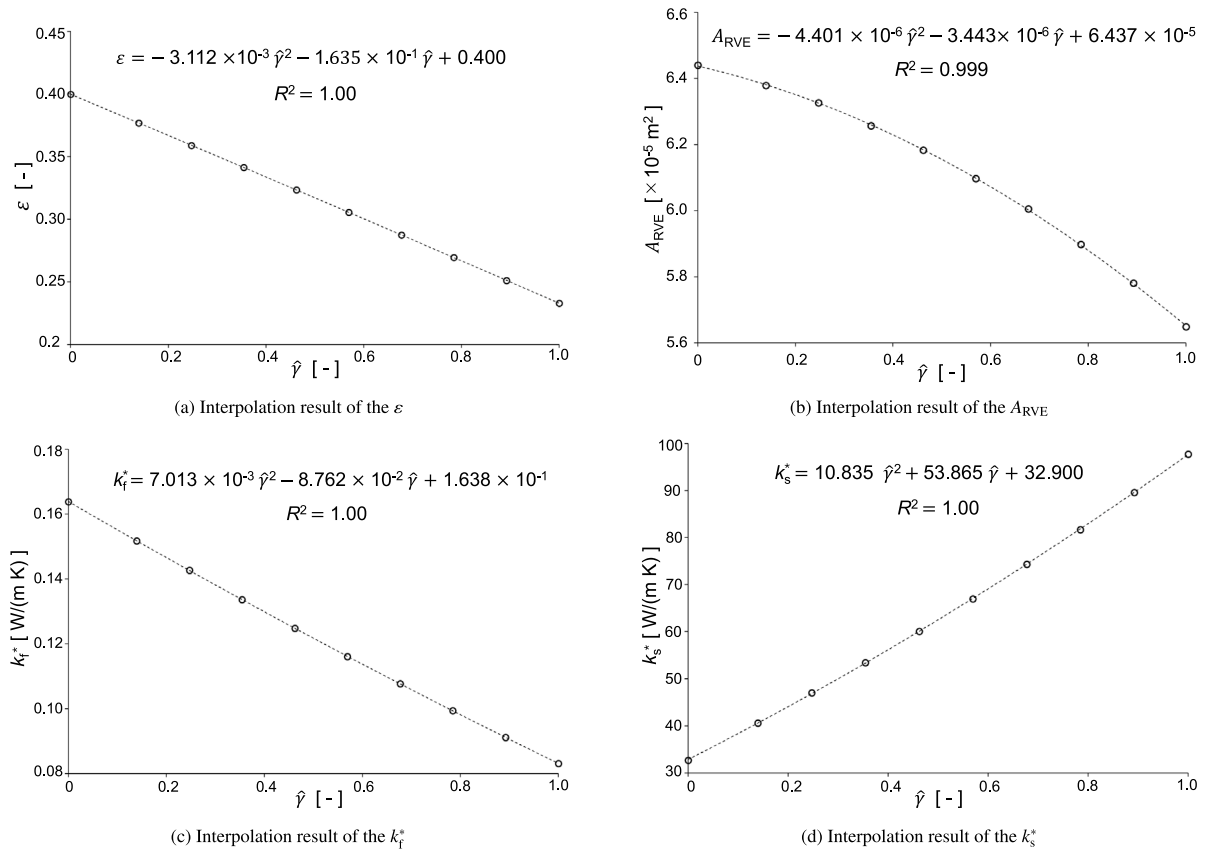


Fig. 7. Interpolation functions of effective properties dependent on design variable.

Table 3

Material properties used for the optimization.

Parameter	Solid	Fluid
Thermal conductivity [W/(m K)]	237	0.631
Specific heat capacity [J/(kg K)]	900	4178
Density [kg/m ³]	2700	992.2
Viscosity [Pa s]		6.53×10^{-4}

accuracy and cost. With this mesh configuration, the minimum element size and the number of boundary layer mesh layers are 9.6×10^{-5} m and 4, respectively. All numerical results in this paper, except those based on the effective model, are obtained using this mesh configuration.

Subsequently, validation was conducted by comparing the numerical results with the experimental data reported by Dixit et al. Fig. 6(a) presents the comparison of outlet temperatures. While some discrepancies were observed, the overall trends reasonably agreed with the experimental data. Fig. 6(b) shows the comparison of heat transfer rates. In the experimental results, a difference in heat transfer rate of approximately 22 to 55 W was observed between the hot and cold sides, which is due to environmental heat losses and other unavoidable experimental uncertainties. The heat transfer rates obtained from the numerical simulation fell within this range, indicating reasonable consistency. Although certain deviations between the experimental and numerical results were observed, these can be attributed to measurement errors in the experiments, unavoidable heat losses, and simplifications in the numerical model. Therefore, the present numerical model is considered sufficiently accurate for gyroid HXs under laminar flow conditions and is applicable to the subsequent analyses.

4.2. Computation of effective properties

The effective properties are computed based on the method described in Section 2.3. In this study, optimization is conducted for a gyroid two-fluid HX composed of aluminum as the solid material and water as the working fluid. The material properties are defined at a reference temperature of 313.15 K, which corresponds to the average of the inlet temperatures of the hot and cold fluids. The material properties employed in the optimization are summarized in Table 3.

In this study, the unit cell length is set to $L_{\text{cell}} = 4.6 \times 10^{-3}$ m, and the level-set parameter is bounded by $c_{\text{min}} = 1.426 \times 10^{-3}$ m and $c_{\text{max}} = 3.75 \times 10^{-3}$ m. To derive interpolation functions for the effective properties, 10 unit cells with different wall thicknesses are analyzed with $c = 1.426, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, 3.25, 3.5, 3.75$ ($\times 10^{-3}$). The effective properties of each unit cell are computed and then expressed as functions of the design variable $\hat{\gamma}$ via polynomial interpolation.

Figs. 7 and 8 present the computed effective properties. Fig. 7 illustrates the interpolation results for ε , A_{RVE} , k_f^* and k_s^* , each fitted with a quadratic function based on the data obtained from individual unit cells. Fig. 8 shows the interpolation results for α , β , and h_i^* . The inlet temperatures of the RVE were set to $T_H = 314.15$ K and $T_L = 312.15$ K, and the volumetric flow rate $\dot{V}$ was varied from 1.0×10^{-8} to 2.5×10^{-6} m³/s. Since α , β , and h_i^* depend not only on wall thickness but also on inlet flow rate, polynomial interpolation functions were constructed using all available data, covering 10 wall thicknesses and 16 flow rates for each thickness. The volumetric flow rate was sampled at $\dot{V} = \{1.0, 2.5, 5.0, 7.5\} \times 10^{-8}$, $\{1.0, 2.0, 4.0, 6.0, 8.0\} \times 10^{-7}$, and $\{1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5\} \times 10^{-6}$ m³/s. The interpolation functions shown in Figs. 8(a) and 8(b) were constructed in MATLAB. For Fig. 8(a), the pressure drop was fitted as a function of the design variable $\hat{\gamma}$ and the Darcy velocity U by a constrained least-squares procedure using fmincon. To ensure physically consistent trends, the

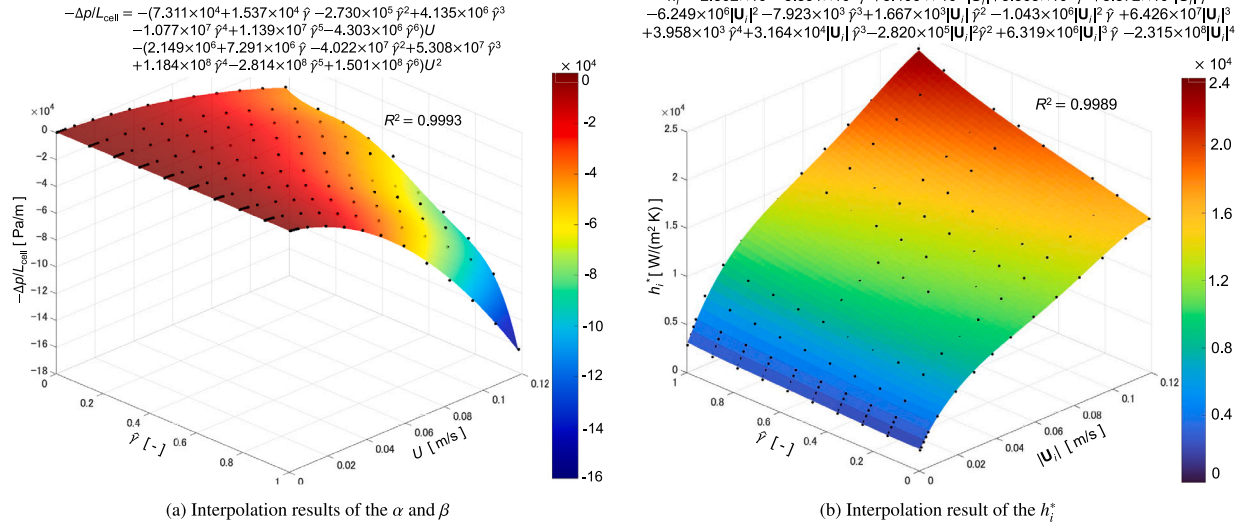


Fig. 8. Interpolation functions of effective properties dependent on design variable and Darcy velocity.

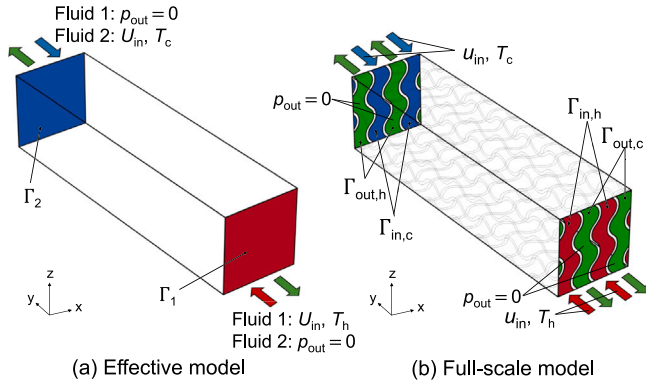


Fig. 9. Analysis model used for verification of the effective porous media model.

fitted surface was constrained such that $\Delta p/L_{\text{cell}}$ increases with both $\hat{\gamma}$ and U :

$$\frac{\partial(\Delta p/L_{\text{cell}})}{\partial \hat{\gamma}} \geq 0, \quad \frac{\partial(\Delta p/L_{\text{cell}})}{\partial U} \geq 0. \quad (29)$$

The effective properties α and β were then determined from the fitted pressure-drop surface using Eq. (12). For Fig. 8(b), the effective heat transfer coefficient h_t^* was fitted using the MATLAB Curve Fitting Toolbox function `fit` with a two-variable polynomial model ('poly44'). The resulting interpolation surfaces closely reproduce the discrete data over the entire design range.

4.3. Verification of effective porous medium model

The effective porous media model proposed in Section 2.2 and the interpolation functions for the effective properties derived in Section 4.2 are verified here. Numerical simulations using both the effective porous media model and the full-scale model are performed over the same analysis domain. The validity of the constructed optimization model is assessed by comparing the pressure drop and heat transfer rate obtained from both models. The analysis domain used for verification is illustrated in Fig. 9. In this model, a uniform distribution of the design variable $\hat{\gamma}$ is applied in the effective model, whereas the full-scale model is configured with a gyroid structure having a uniform wall thickness corresponding to the given design variable. Counter-flow conditions are

assumed, with a hot fluid inlet temperature of $T_h = 333.15$ K and a cold fluid inlet temperature of $T_c = 293.15$ K. No-slip and adiabatic BCs are applied to all surfaces except the inlet and outlet. In the full-scale model, fully developed flow conditions are imposed at the inlet and outlet boundaries. In the effective model, the Darcy velocity at the inlet is adjusted to match the volumetric flow rate of the full-scale simulation.

To evaluate the accuracy of the effective model, comparisons with the full-scale model were performed over a range of inlet velocities ($u_{\text{in}} = 0.01$ – 0.06 m/s). Fig. 10 shows a comparison between the full-scale results and the effective model predictions, where the surfaces represent the full-scale simulations and the points denote the effective model. Fig. 11 shows parity plots comparing the two models. Overall, the effective model consistently reproduces the trends observed in the full-scale results across the entire range of flow rates and wall thicknesses. In particular, as shown in Fig. 11(a), the heat transfer rate Q shows excellent agreement, with a coefficient of determination of $R^2 = 0.999$.

As shown in Fig. 10(b), the deviation in the predicted pressure drop increases as the inlet velocity deviates from approximately $u_{\text{in}} = 0.03$ m/s. This behavior occurs because the resistance coefficients $\alpha(\hat{\gamma})$ and $\beta(\hat{\gamma})$ are interpolated solely as functions of the wall thickness $\hat{\gamma}$. Because the interpolation expressions depend only on the wall thickness, the model cannot fully capture the change in the relative contributions of viscous and inertial effects associated with variations in the velocity scale within the gyroid structure. Nevertheless, as shown in Fig. 11(b), the effective model predicts the pressure drop with reasonable overall accuracy, with a coefficient of determination of $R^2 = 0.884$.

The purpose of the effective model is not to exactly replace high-fidelity simulations, but to provide a reduced-order model for efficient performance evaluation during the optimization process. Therefore, it is important that the model accurately captures the relative performance trends associated with variations in the design variables and flow conditions. The present results demonstrate that the dependence of both the heat transfer rate Q and the pressure drop Δp on the wall thickness is consistently reproduced across multiple flow-rate conditions. Consequently, the constructed effective model can guide the optimization toward improving the objective function, indicating its suitability for wall-thickness distribution optimization of gyroid HXs.

4.4. Computational efficiency and applicability

The computation times reported in this section correspond to analyses performed using a 2.45 GHz AMD EPYC 7763 64-core processor with 512 GB RAM.

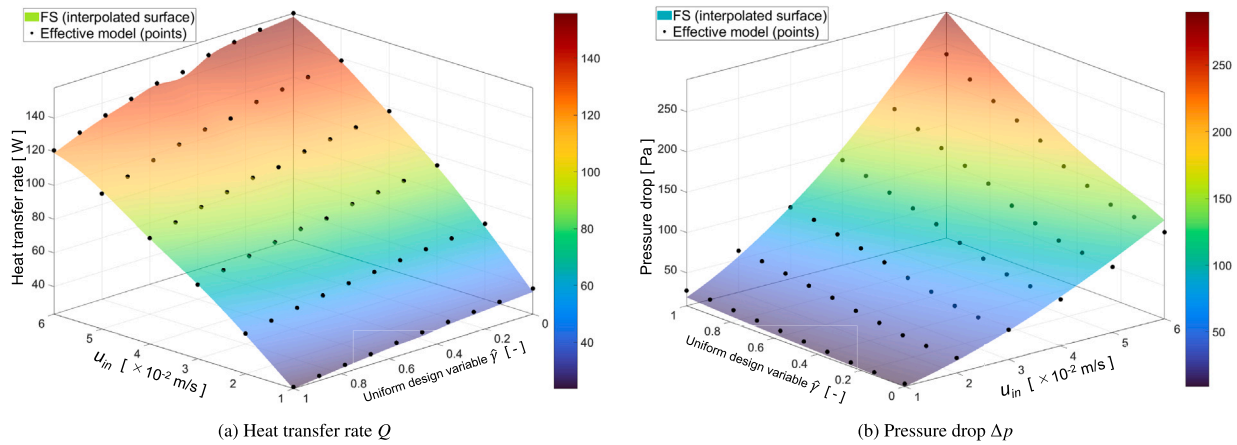


Fig. 10. Comparison between full-scale (FS) results and effective model predictions over multiple flow rates and wall thicknesses.

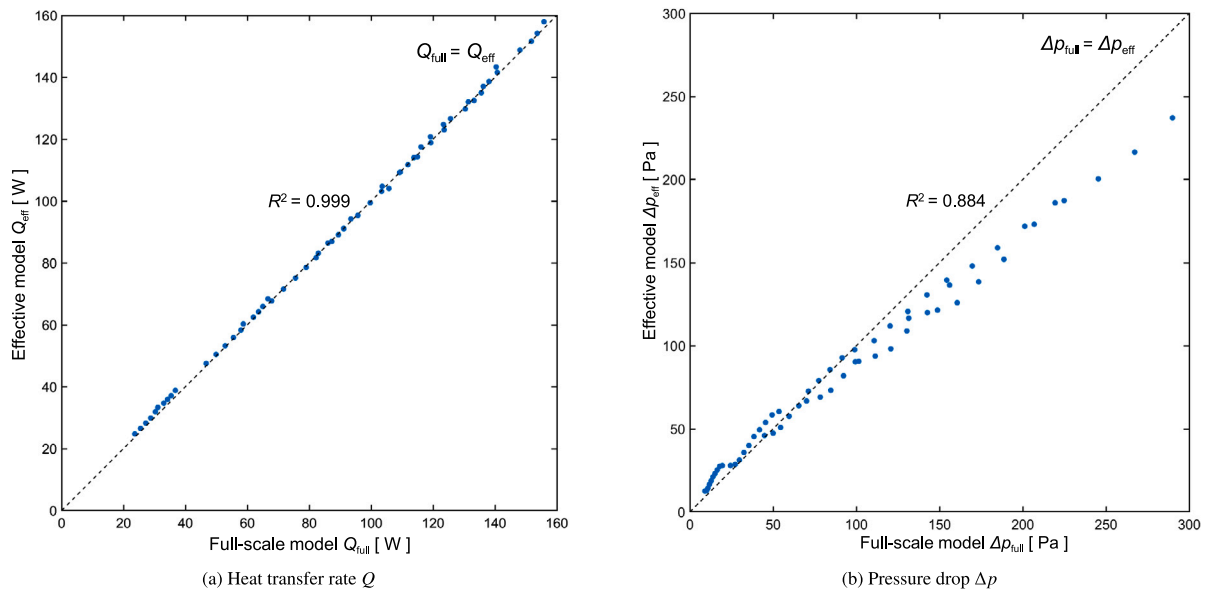


Fig. 11. Parity plots comparing full-scale results and effective model predictions.

Using the analysis model employed for verification in Section 4.2 (Fig. 9), the computational costs of the full-scale and effective models were compared. For a uniform thickness distribution of $c = 1.426 \times 10^{-3}$, the full-scale model contained 6,691,417 mesh elements and required 8115 s for analysis. In contrast, the effective model, with 256 mesh elements, required only 31 s, making it approximately 260 times faster than the full-scale simulation. These results demonstrate that the proposed effective model can drastically reduce computational cost while maintaining reasonable accuracy.

In this study, numerical analyses were performed for a sufficient number of RVEs to ensure that the resulting interpolation functions provide adequate predictive accuracy. Specifically, ten RVEs with different thicknesses were prepared, and each was analyzed under 16 flow rate patterns. For example, analyzing 16 flow rate patterns for a gyroid unit cell with $c = 1.426 \times 10^{-3}$ and 227,471 mesh elements under the BCs shown in Fig. 3(a) required 2423 s. Consequently, analyzing all ten RVEs to construct the interpolation functions would require approximately seven hours. This time can be further reduced by parallel computing. While reducing the number of thickness variations or flow rate patterns can further lower computational cost, it may compromise the predictive accuracy of the effective model. Since optimization involves multiple iterative calculations, performing optimization using the full-scale model would require an impractically high computational

cost and take an excessively long time. Considering this, the computational cost for constructing the interpolation functions of the effective properties is reasonable.

In the present study, the hot and cold fluids were assumed to be identical to clarify the effect of wall-thickness optimization. It should be noted that the effective properties, including the effective heat transfer coefficient, are identified from RVE simulations under specific fluid/solid properties. Therefore, when the working fluids and/or solid material are changed, the effective properties should be re-identified through additional RVE analyses under the new conditions. In the case of dissimilar fluids, the effective properties for each fluid should be determined separately according to their respective thermophysical properties. Accordingly, although the interpolation functions derived in Section 4.2 are limited to the conditions considered in this study, the proposed method can be extended to other TPMS structures, materials, and unit cell sizes. While this study focused on counter-flow configurations, the method can also be applied to cross-flow or parallel-flow problems by appropriately modifying the RVE BCs. Consequently, the proposed method can be widely applied to HXs with various TPMS types and flow configurations, providing a foundation for developing simplified models and optimization frameworks.

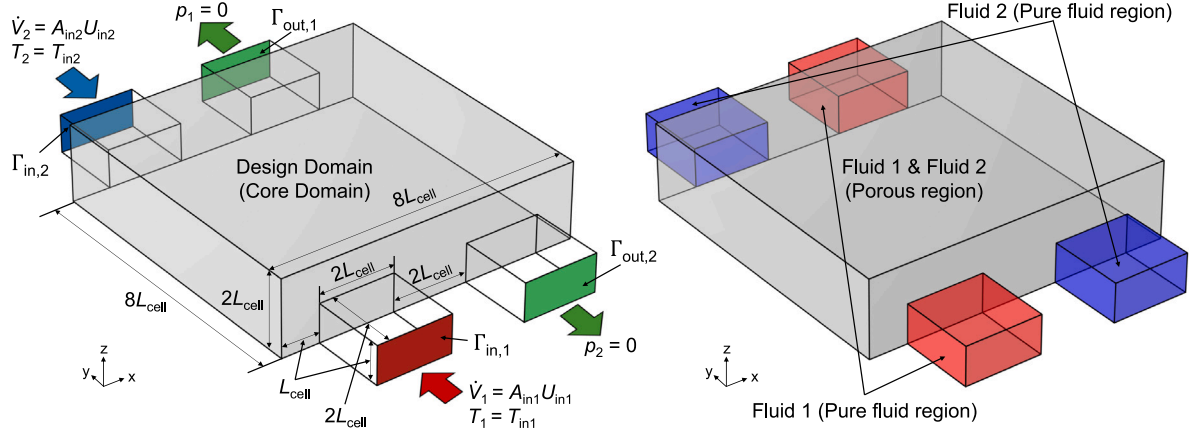


Fig. 12. Problem setup for wall-thickness optimization of the gyroid two-fluid HX.

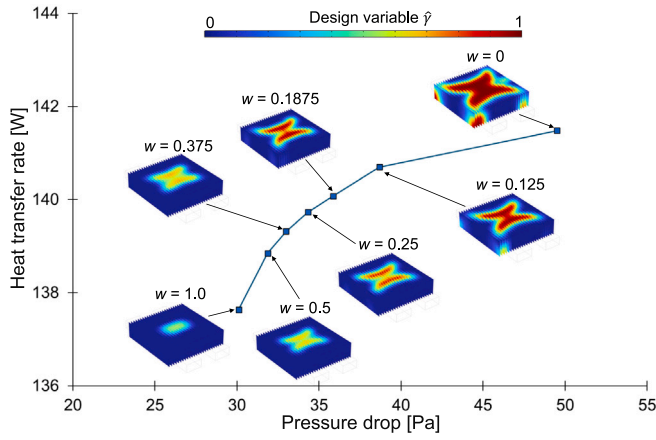


Fig. 13. Optimization results for different values of the weighting factor w .

4.5. Optimization results

The wall-thickness distribution of the gyroid two-fluid HXs was optimized according to the procedure described in Section 3.1. In this study, a counterflow configuration was assumed, and the optimization was performed based on the problem setup shown in Fig. 12. Fluid 1 was designated as the hot fluid and Fluid 2 as the cold fluid. The inlet temperatures were set to $T_{in1} = 333.15$ K and $T_{in2} = 293.15$ K, respectively. The inlet flow rates $\dot{V}_i$ for fluids 1 and 2 were set as BCs. These flow rates are given by $\dot{V}_i = 2L_{cell}^2 U_{ini}$, where L_{cell} is the length of the unit cell, which is 4.6×10^{-3} m, and U_{ini} denotes the average inlet velocity of fluid i . In this study, the flow rates $\dot{V}_1$ and $\dot{V}_2$ corresponding to $U_{in1} = U_{in2} = 0.03$ m/s were used. Adiabatic and no-slip BCs were applied to all surfaces except the inlet and outlet. The gyroid structures were embedded only within the core domain of the HX, while the other domains were treated as pure fluid regions. To obtain multiple optimized solutions, the optimization was performed multiple times with different values of the weighting factor w . As the initial configuration, the design variable $\hat{\gamma}$ was uniformly set to 0.5, and the optimization was performed over 50 iterations. The effective porous media simulations used for optimization employed a structured hexahedral mesh with 1152 elements in total.

Fig. 13 presents the optimized results corresponding to each value of w . All values shown in Fig. 13 were evaluated using the effective porous media model. As the value of the weighting factor w increased, the regions with larger thicknesses decreased, leading to a corresponding reduction in pressure drop. Fig. 14 shows, at the cross-section $z =$

$L_{cell}/2$, the design-variable distribution (a), the hot-fluid Darcy velocity distribution for the entire cross-section (b) and within $L_{cell} \leq y \leq 7L_{cell}$ (c), and the hot-fluid temperature distribution (d), for both the initial configuration ($\hat{\gamma} = 0.5$) and the optimized structure ($w = 0.25$). In each panel, the left half corresponds to the uniform-thickness design, and the right half to the optimized-thickness design.

Fig. 15 presents the distributions of the volumetric heat source terms Q_1^* and Q_2^* for both the initial configuration ($\hat{\gamma} = 0.5$) and the optimized structure ($w = 0.25$). Since the temperature of the hot fluid T_1 is generally higher than the wall temperature T_w , the hot fluid transfers heat to the wall in most of the core domain. In this study, the volumetric heat source term is defined as the heat received by the fluid per unit volume. Therefore, Q_1^* is generally negative, and $-Q_1^*$ serves as an indicator of the magnitude of heat exchange. Negative values of $-Q_1^*$ or Q_2^* appear in localized regions. For the hot fluid, these negative values arise when its temperature falls below the wall temperature after passing near the inlet of the cold fluid. Such occurrences are consistent with the physical behavior of the system.

The total heat transfer increases from 135.7 W (uniform-thickness, $\hat{\gamma} = 0.5$) to 139.7 W (optimized-thickness, $w = 0.25$). The underlying mechanisms are discussed in Section 5.

4.6. Full-scale performance evaluation results

Dehomogenization and full-scale numerical analyses were conducted in COMSOL to further demonstrate the effectiveness of the optimized wall-thickness distribution. Using the optimized distribution $\hat{\gamma}(\mathbf{x})$, the corresponding level-set parameter distribution $c(\mathbf{x})$ was reconstructed, and the gyroid geometry was generated based on the level-set formulation (Eq. (2)). The reconstructed geometries are presented in Fig. 16. Fig. 16(a) shows the uniform-thickness design, whereas Fig. 16(b) shows the optimized design with spatially varying thickness.

As stated in Section 3.1, a convolutional filter was applied to the design variable field. Accordingly, the optimized design variable distribution $\hat{\gamma}(\mathbf{x})$ is spatially smooth. Since $\hat{\gamma}$ is linearly mapped to the level-set parameter $c(\mathbf{x})$ of the gyroid surface via Eq. (25), the reconstructed $c(\mathbf{x})$ likewise forms a spatially continuous field. As a result, the dehomogenized gyroid geometry exhibits smooth thickness transitions without geometric discontinuities. For the optimized design, a continuous thickness transition is observed, particularly in the central region of the core.

To prevent mixing between the hot and cold fluids, 0.5 mm-thick solid walls were introduced at the interface between the pure fluid regions and the HX core region. The overall mesh of the full-scale model is shown in Fig. 17(a), while the mesh of the mixing-prevention walls is illustrated in Fig. 17(b). No geometric issues (e.g., self-intersections

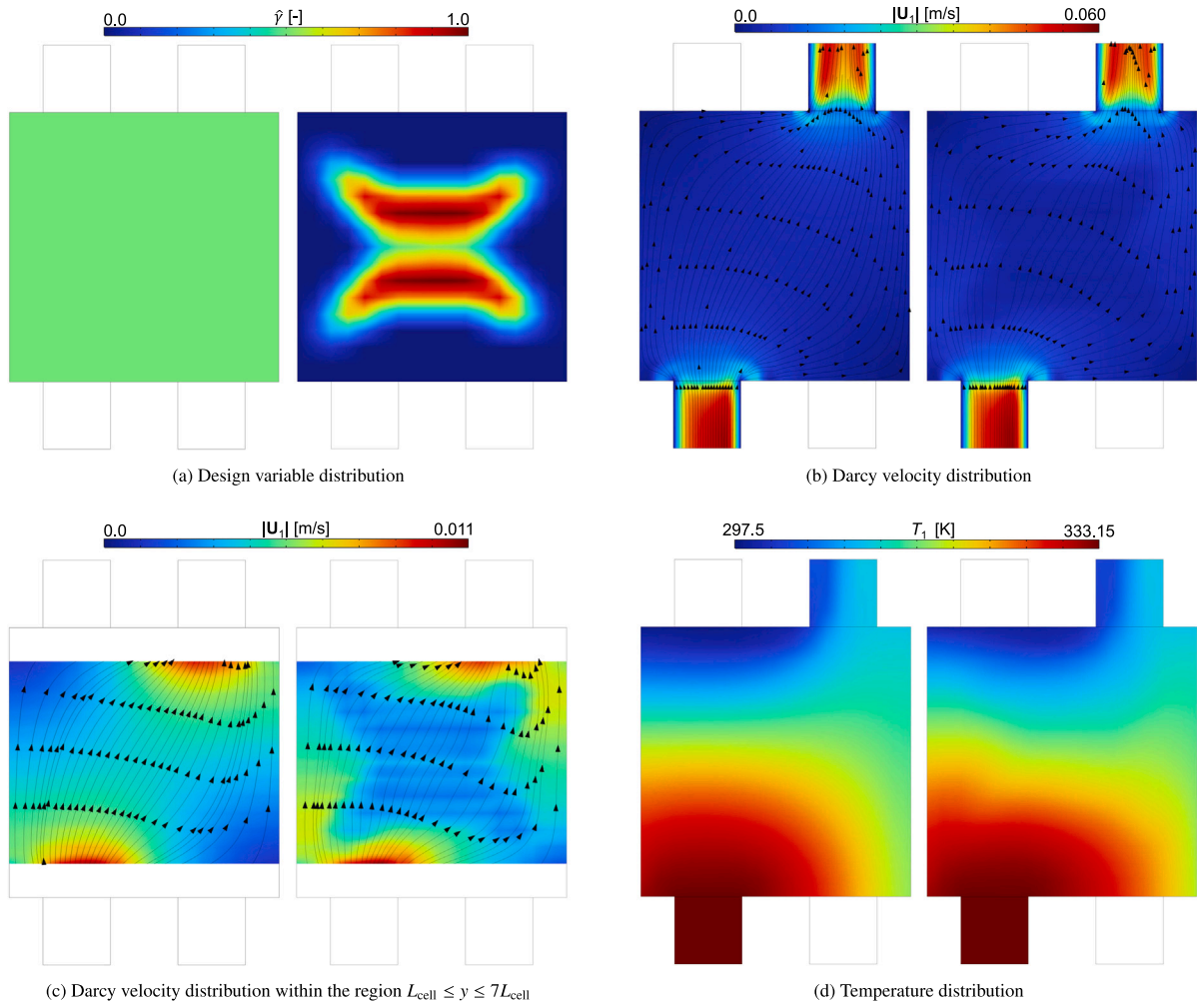


Fig. 14. Distributions at the cross-section $z = L_{\text{cell}}/2$ focusing on the hot fluid for both the initial (left) and optimized (right) designs.

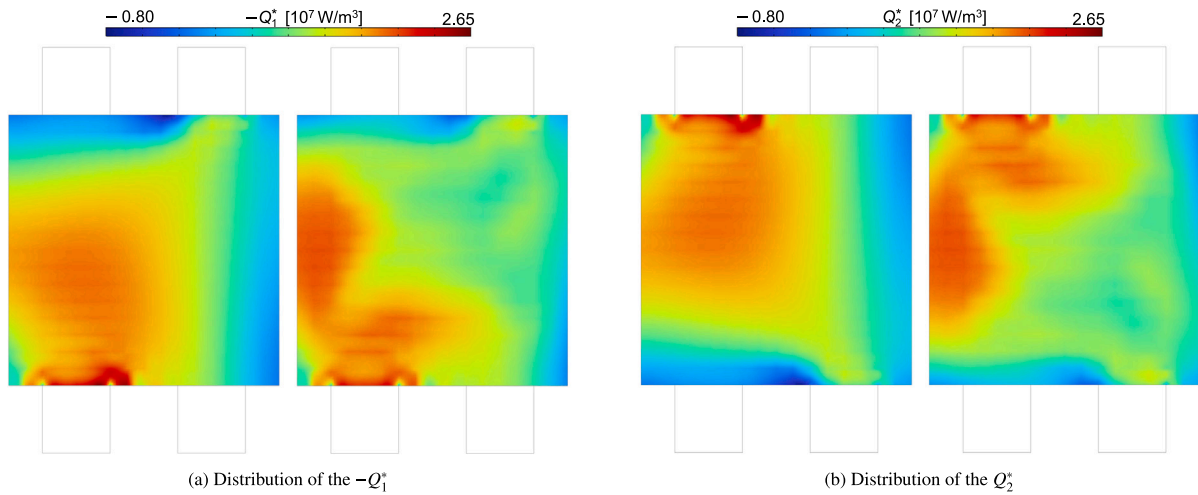


Fig. 15. Distributions of the volumetric heat source terms at the cross-section $z = L_{\text{cell}}/2$ for both the initial (left) and optimized (right) designs.

or discontinuities) were observed in the reconstructed structures, and meshing was completed without manual geometric modification. The resulting full-scale models were discretized using a body-fitted mesh with boundary-layer refinement, comprising approximately 1.97×10^7 elements for the uniform-thickness design ($c = 1.426 \times 10^{-3}$) and 2.26×10^7 elements for the optimized-thickness design ($w = 0.25$).

In the full-scale simulations, the conjugate heat transfer problem defined by Eqs. (10) is solved. Based on Eq. (17), Re ranges from 90 to 110, reflecting slight variations in the hydraulic diameter due to wall-thickness changes. Gyroid structures form a periodic porous medium with bicontinuous and highly interconnected pore networks. Previous studies on porous media flows indicate that the onset of unsteadiness or

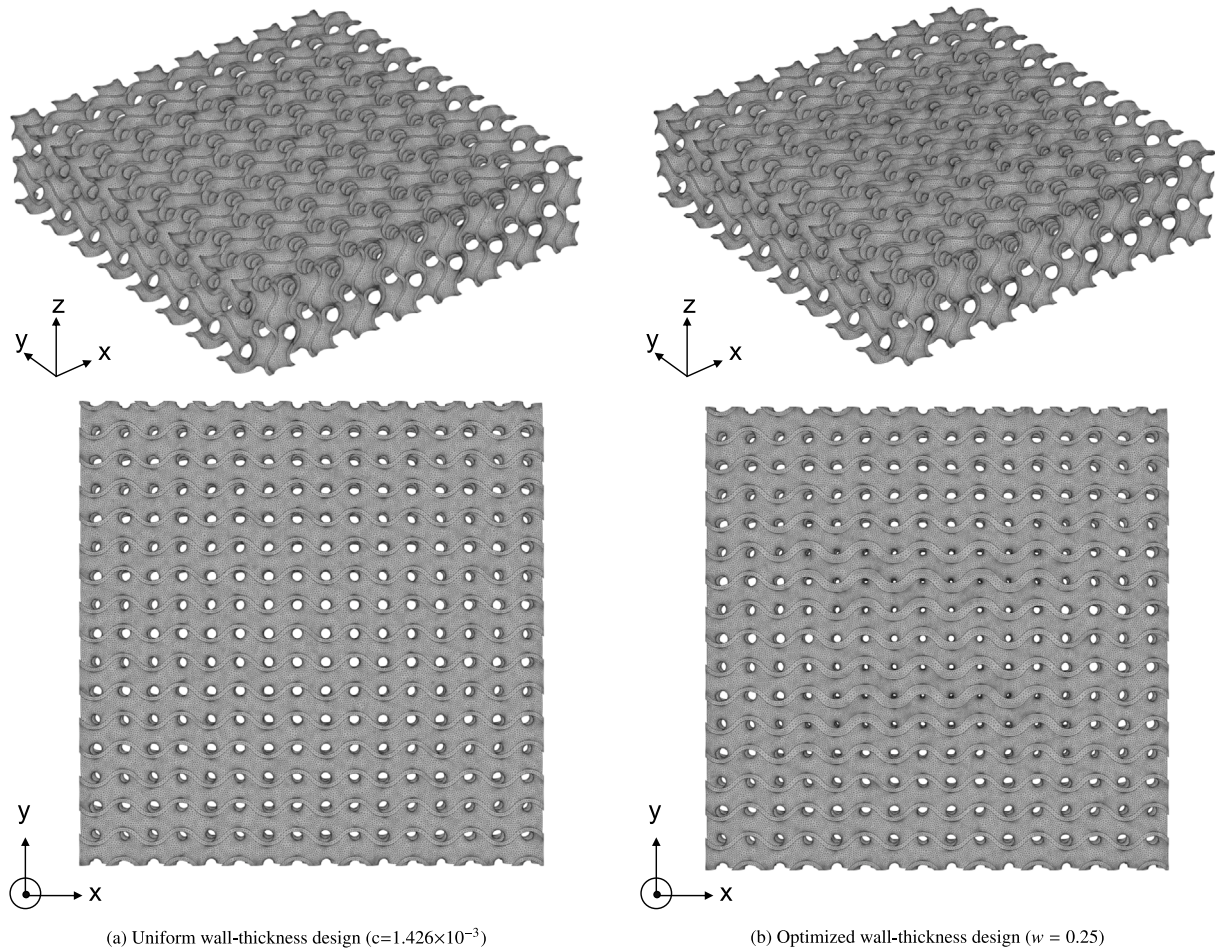


Fig. 16. Reconstructed gyroid geometries obtained through the dehomogenization process.

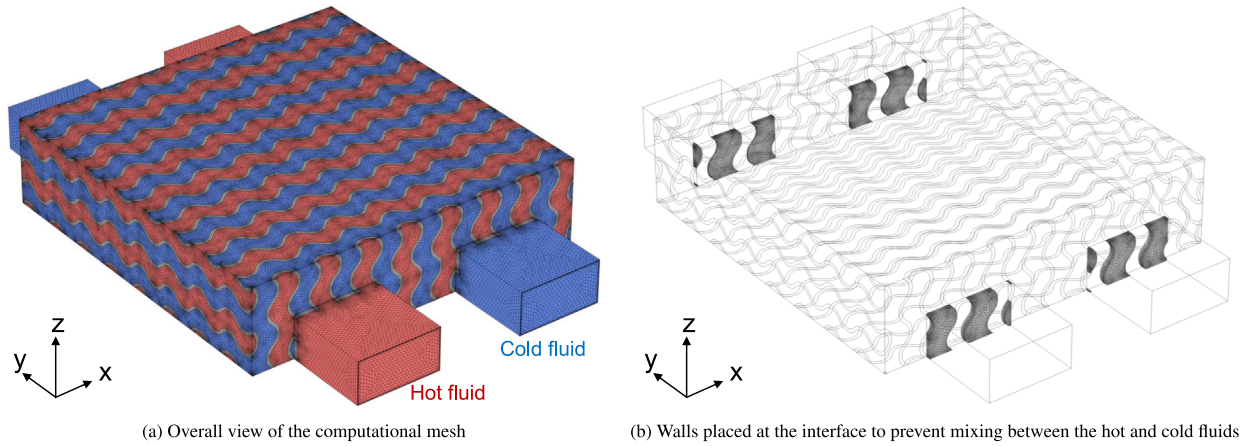


Fig. 17. Full-scale mesh of the gyroid HX (uniform wall-thickness design: $c = 1.426 \times 10^{-3}$).

transition toward turbulence typically occurs at Re of order 10^2 [84,85]. For example, Dybbs and Edwards [84] reported that steady laminar flow persists up to approximately $Re \approx 150$ in canonical porous media before the first evidence of flow unsteadiness appears. Therefore, the Re in the present simulations is within a range generally associated with laminar flow in porous-media-type structures. Hence, the laminar-flow assumption adopted in this study is reasonable.

Fig. 18 shows the results of the full-scale analysis. The gyroid HXs with optimized thickness distribution exhibited improved heat

exchange performance compared to those with uniform thickness distribution. For example, the optimized design with $w = 0.25$ has a porosity of $\epsilon = 0.361$. For reference, a uniform-thickness gyroid with $c = 1.98 \times 10^{-3}$ would yield the same porosity. Although the uniform-thickness design with the thinnest wall ($c = 1.426 \times 10^{-3}$) exhibits the lowest pressure drop, the optimized designs provide higher heat transfer rates at comparable pressure drops. This indicates that locally varying the gyroid HX thickness using the proposed method is more effective than a uniform increase in thickness for enhancing heat transfer. Within the optimized designs, the pressure drop decreases as the weighting factor

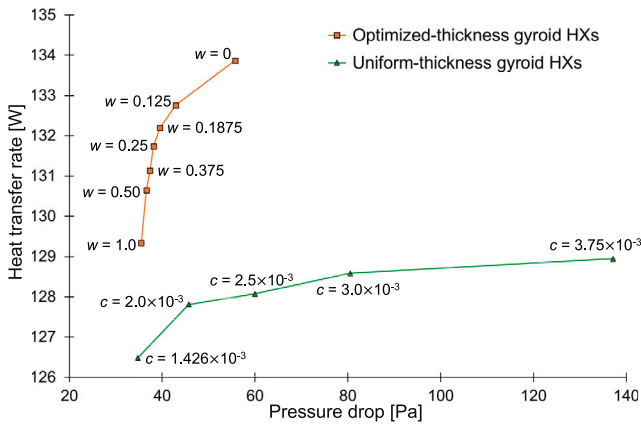


Fig. 18. Full-scale simulation results for gyroid HXs with uniform and optimized thickness distribution.

w increases, demonstrating that the optimization using the effective model appropriately captures the trade-off between heat transfer and pressure drop.

The PEC, a widely used performance metric for HXs that considers both heat transfer and pressure drop, was used to identify the best-performing design. Fig. 19 presents dimensionless performance factors for gyroid HXs with uniform-thickness and optimized-thickness distributions. Fig. 19(a) shows the scatter plot of normalized j versus normalized f . Fig. 19(b) presents the corresponding PEC values for each design. The baseline values of j_0 and f_0 were calculated for the uniform-thickness design ($c = 1.426 \times 10^{-3}$), which has the thinnest wall across the entire core domain.

For the uniform-thickness design, Fig. 19(a) shows that increasing the wall thickness raises j/j_0 to a maximum of 1.17, while f/f_0 increases to 2.62. This is because a uniform increase in wall thickness reduces the effective flow cross-sectional area, thereby increasing the mean flow velocity and enhancing the heat transfer coefficient, which leads to greater heat exchange. However, the accompanying increase in flow resistance results in a significant rise in pressure loss. Consequently, from the perspective of PEC, which reflects the trade-off between heat transfer and pressure drop, the HX with the thinnest wall ($c = 1.426 \times 10^{-3}$) was optimal among the uniform-thickness designs. This result indicates that, in uniform-thickness designs, reducing the wall thickness effectively enhances HX performance by simultaneously decreasing both flow resistance and conductive thermal resistance.

In contrast, as shown in Fig. 19(b), the optimized-thickness designs achieved higher PEC values than the uniform-thickness design for all weighting factors w except $w = 0$. As an exception, for $w = 0$, the optimization did not account for pressure drop, resulting in a lower PEC value. This confirms that the optimized-thickness designs achieve a superior trade-off between heat transfer and pressure drop. The design optimized with $w = 0.25$ achieved the highest PEC (1.122), a 12.2% improvement over the best-performing uniform-thickness design ($c = 1.426 \times 10^{-3}$), indicating an improved balance between heat transfer and pressure drop. Heat transfer rates and pressure drops for the uniform-thickness ($c = 1.426 \times 10^{-3}$) and optimized-thickness ($w = 0.25$) designs were 126.5 W and 131.7 W, and 34.7 Pa and 38.1 Pa, respectively. The volumetric heat densities for the designs with uniform and optimized thickness were 1.015×10^7 W/m³ and 1.057×10^7 W/m³, respectively, showing a 4.16% improvement through optimization.

These results demonstrate that introducing the optimized non-uniform wall thickness distribution improves the efficiency of gyroid HXs.

4.7. Flow and thermal field analysis

The physical fields obtained from full-scale simulations of the optimized and baseline designs are compared to evaluate the impact of the optimized thickness distribution. Fig. 20 shows the physical fields at $z = L_{\text{cell}}/2$ for both the uniform-thickness ($c = 1.426 \times 10^{-3}$, left) and optimized-thickness ($w = 0.25$, right) designs, including the velocity distributions in (a) and (b), the pressure distribution in (c), and the temperature distribution in (d).

As shown in Fig. 20(b), the optimized-thickness distribution leads to higher velocities near the left and right edges of the HX core region. A similar trend is observed in Fig. 14(c), which shows the Darcy velocity distribution within the region $L_{\text{cell}} \leq y \leq 7L_{\text{cell}}$. To clearly illustrate the differences in velocity distribution, Fig. 21 presents the velocity magnitude along the lines at $x = 0.5L_{\text{cell}}$, $x = 4.0L_{\text{cell}}$, and $x = 7.5L_{\text{cell}}$ within the cross-section. Fig. 21(b) shows no significant difference in velocity between the two designs at the center of the HX ($x = 4.0L_{\text{cell}}$). In contrast, near the edges of the heat exchange region (Figs. 21(a) and 21(c)), the optimized-thickness design exhibits higher velocity magnitudes than the uniform-thickness design.

These changes reflect a redistribution of the flow within the core, accompanied by a reduction in low-velocity regions and improved flow uniformity. To quantitatively evaluate this effect, the velocity uniformity on the cross-section at $z = L_{\text{cell}}/2$ is assessed using two metrics: the low-velocity area fraction f_{low} and the coefficient of variation (CV). The f_{low} represents the portion of the fluid cross-sectional area where the flow velocity is below η times the mean velocity $\bar{U}$, and is defined using the indicator function $I(\mathbf{u}(\mathbf{x}))$ as follows:

$$f_{\text{low}} = \frac{\int_S I(\mathbf{u}(\mathbf{x})) dS}{\int_S dS}, \quad (30)$$

where

$$I(\mathbf{u}(\mathbf{x})) = \begin{cases} 1, & \text{if } |\mathbf{u}(\mathbf{x})| \leq \eta \bar{U}, \\ 0, & \text{otherwise,} \end{cases} \quad (31)$$

and $\bar{U}$ is the mean velocity over the cross-section:

$$\bar{U} = \frac{\int_S |\mathbf{u}(\mathbf{x})| dS}{\int_S dS}. \quad (32)$$

The CV quantifies the overall velocity variability across the fluid region S and is defined as:

$$\text{CV} = \frac{\sqrt{\frac{1}{\int_S dS} \int_S (|\mathbf{u}(\mathbf{x})| - \bar{U})^2 dS}}{\bar{U}}. \quad (33)$$

The CV, defined as the velocity standard deviation σ_u normalized by the mean velocity $\bar{U}$, offers a consistent measure of velocity variability independent of the mean flow magnitude.

Both metrics are evaluated for the uniform-thickness and optimized-thickness designs. The mean velocities $\bar{U}$ were 0.0225 m/s for the uniform-thickness design and 0.0250 m/s for the optimized-thickness design. Despite the same inlet flow rate, the optimized design exhibits a higher mean velocity owing to a local reduction in the effective flow area associated with the optimized geometry. The calculated low-velocity area fraction f_{low} is shown in Fig. 22. Here, the values were computed for η ranging from 0.3 to 0.05. The corresponding improvement rates are also indicated in the figure and defined as:

$$\text{Improvement rate} = \left(1 - \frac{f_{\text{low,opt}}}{f_{\text{low,uni}}} \right) \times 100\%, \quad (34)$$

where $f_{\text{low,opt}}$ and $f_{\text{low,uni}}$ denote the low-velocity area fractions for the optimized-thickness and uniform-thickness designs, respectively. As

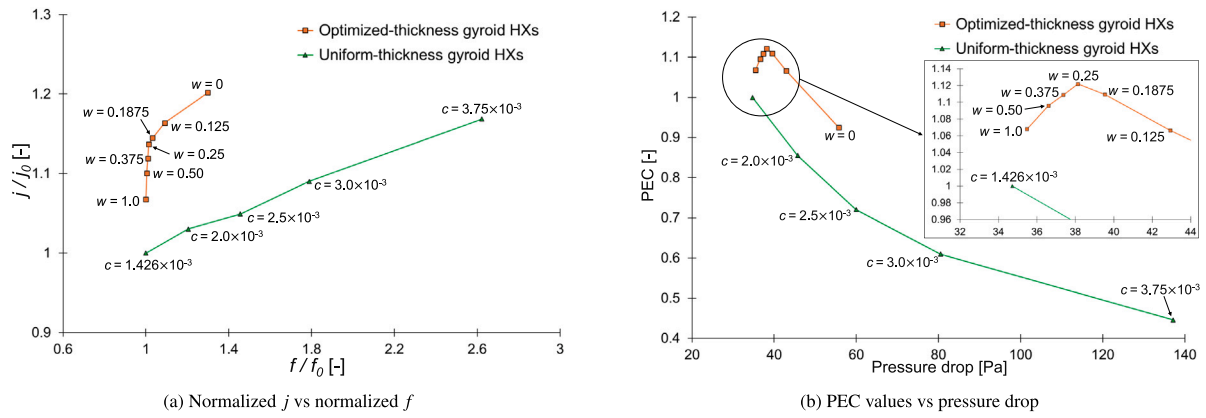


Fig. 19. Dimensionless performance factors for gyroid HXs with uniform and optimized wall thickness distribution.

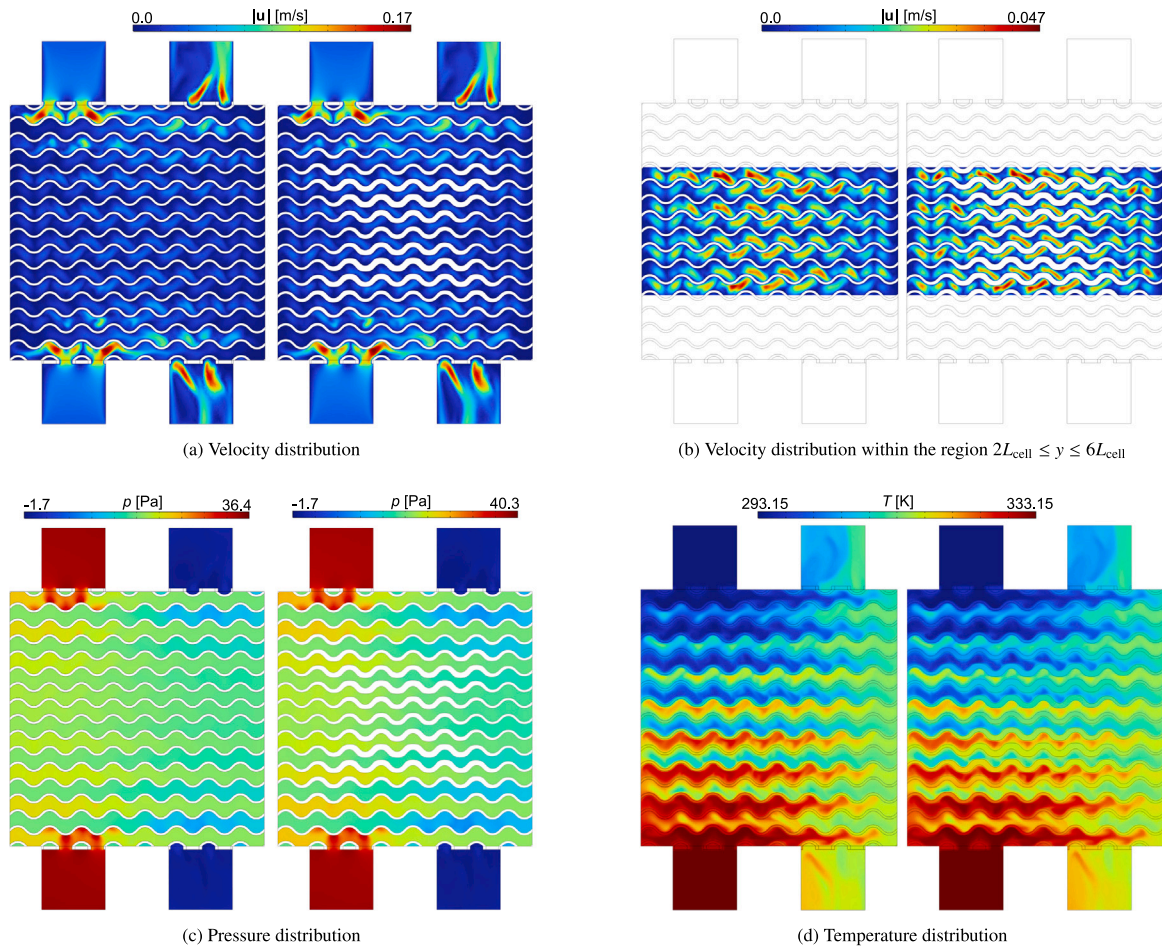


Fig. 20. Physical fields at the cross-section $z = L_{\text{cell}}/2$ for the uniform-thickness (left, $c = 1.426 \times 10^{-3}$) and optimized-thickness (right, $w = 0.25$) gyroid HXs.

shown in Fig. 22, f_{low} is smaller for the optimized-thickness design for all values of η , with improvement rates ranging from 10.8% to 15.3%. These results quantitatively demonstrate that the optimized thickness distribution effectively reduces low-velocity regions. Additionally, the CV was calculated as 1.063 for the uniform-thickness design and 0.975 for the optimized-thickness design, representing an 8.3% reduction in velocity variability. These findings demonstrate that the optimized-thickness distribution reduces low-velocity regions and enhances velocity uniformity.

To further examine the thermal characteristics of the optimized design, the wall heat flux distributions are visualized and compared

between the uniform and optimized thickness designs. Fig. 23 shows the heat flux distributions on the wall in contact with the hot fluid within the region $0.25L_{\text{cell}} \leq z \leq 0.75L_{\text{cell}}$ for both the uniform- and optimized-thickness designs. As shown in Fig. 21, the optimized-thickness distribution produces higher velocities near the edges of the core region. Accordingly, Fig. 23 presents the heat flux distributions specifically in these edge regions ($0 \leq x \leq L_{\text{cell}}$, $7L_{\text{cell}} \leq x \leq 8L_{\text{cell}}$). A comparison between the uniform- and optimized-thickness designs confirms that the optimized distribution results in higher heat flux values at the edges. A similar trend is observed in Fig. 15, which shows the volumetric heat source term distributions obtained from the

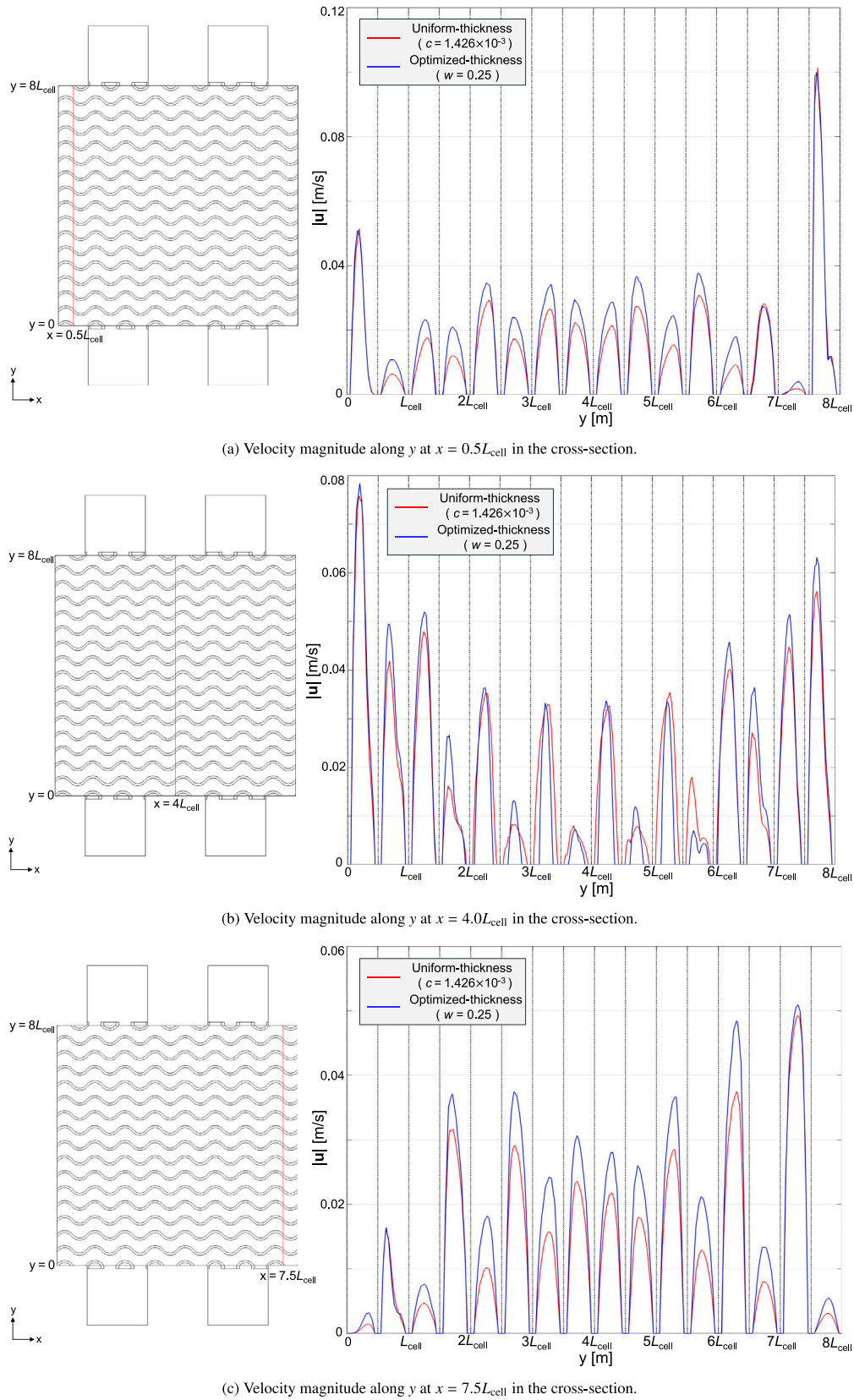


Fig. 21. Velocity magnitude profiles at different positions in the cross-section.

effective-model simulations. Even in the effective model, the optimized-thickness design exhibits larger heat source term magnitudes near the core edges, consistent with the full-scale simulation results. This

increase is consistent with the locally higher velocities observed in these regions. The overall heat transfer coefficient U_{HX} , calculated from Eq. (21), provides a quantitative measure of the heat transfer

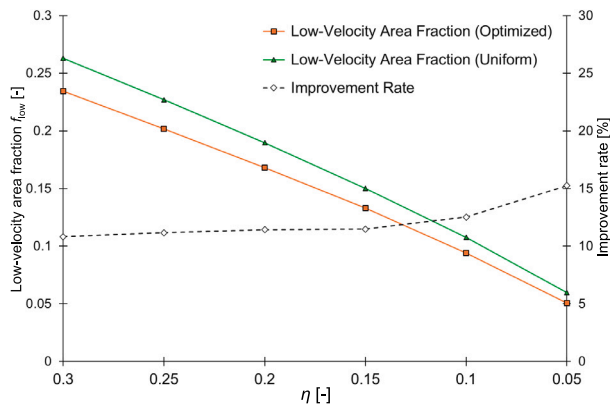


Fig. 22. Low-velocity area fraction f_{low} for uniform and optimized-thickness designs at $z = L_{cell}/2$, with improvement rate.

performance. For the optimized thickness distribution, U_{HX} reaches 577 W/(m² K), representing an approximately 14% increase compared to $U_{HX} = 508$ W/(m² K) for the uniform-thickness design.

5. Discussion

Building on the flow and thermal field analyses in Section 4.7, we discuss the mechanisms underlying the performance improvement induced by the optimized thickness distribution. The causes of the remaining discrepancies between the effective-model predictions and the full-scale simulations are also discussed.

As shown in Section 4.7, the optimized thickness distribution improves flow uniformity by reducing low-velocity regions, thereby enhancing overall thermal performance. More generally, the results indicate that spatially tailoring wall thickness to impose localized hydraulic resistance can serve as a passive flow-redistribution strategy. When the hydraulic resistance is nearly spatially uniform, the flow tends to follow a more direct inlet-to-outlet pathway, which can leave parts of the core underutilized for heat exchange. Introducing localized resistance can steer the flow toward such regions, thereby improving the global heat-transfer effectiveness. In the literature on conventional HXs, non-uniform velocity distributions have been shown to lead to performance degradation [86–88]. Similarly, several studies on TPMS HXs have shown that improving velocity uniformity can enhance overall heat transfer efficiency, as it promotes more effective heat exchange throughout the entire HX [26–29,62]. In the optimization context, TPMS heat-sink studies [65] have similarly shown that optimizing a spatially varying porosity distribution (corresponding to local wall-thickness) can improve flow uniformity and thereby enhance overall thermal performance. In summary, the performance-enhancement mechanism observed here is consistent with trends reported in prior studies, providing a physical rationale for the optimized designs.

The causes of the discrepancy in pressure drop between the effective model and the full-scale simulations are discussed below. A comparison of Figs. 13 and 18 shows that the full-scale simulation results in a higher pressure drop than the effective model. In the full-scale simulation, solid walls were introduced to prevent mixing of the hot and cold flow streams. These walls narrow the flow paths near the inlets and outlets, forcing the fluid to converge into localized regions, as seen in the velocity distribution in Fig. 20(a). This leads to an increase in pressure loss due to flow acceleration and redirection associated with the localized flow concentration. The effective model cannot capture such geometrical and flow-concentration effects because the model assumes spatially averaged behavior and a more uniform flow field. Furthermore, in the verification case of Section 4.3, these solid walls were not included in the full-scale model, and the flow configuration

closely resembled that assumed in the effective model. As a result, the discrepancy in pressure drop between the effective and full-scale models was relatively small in verification. Therefore, the presence of solid walls, not considered in the effective model, appears to be the primary cause of the pressure drop discrepancy between the effective and full-scale analyses of the optimized HX designs. In addition, the inherent modeling assumptions of the effective porous medium formulation may also contribute to the discrepancy. As discussed in Section 4.3, the resistance coefficients $\alpha(\hat{\gamma})$ and $\beta(\hat{\gamma})$ are interpolated solely as functions of the wall thickness. Consequently, the model cannot fully capture changes in the relative contributions of viscous and inertial effects associated with variations in the local velocity scale within the gyroid structure, which may lead to additional discrepancies in the predicted pressure drop. Moreover, spatial variations in the local cross-sectional area may introduce additional acceleration and deceleration losses and thus contribute to minor discrepancies; however, this effect is expected to be secondary because the thickness distribution varies smoothly.

Here, the causes of the discrepancy in heat transfer rate are discussed. A comparison of Figs. 14(d) and 20(d) shows that the overall temperature distributions of the hot fluid are qualitatively consistent between the two models. However, the integrated heat transfer predicted by the effective model is higher than that obtained from the full-scale simulation (Figs. 13 and 18). As described in Section 2.3, the effective heat transfer coefficient is derived under the assumption of counter-flow within the RVE. Accordingly, the effective model assumes a counter-flow configuration throughout the entire core domain. This assumption was valid in the verification case in Section 4.3, where the full-scale geometry also maintained a counter-flow layout, leading to good agreement between the two models. In a counter-flow configuration, the hot fluid inlet aligns with the cold fluid outlet, maintaining a large temperature difference and ensuring efficient heat transfer. However, within the optimization problem setup, there are regions where the flow is partially parallel-flow or cross-flow. In these regions, the fluid temperatures tend to equalize more rapidly, reducing the local temperature gradient and thereby decreasing heat transfer efficiency. Consequently, the full-scale simulation yields a lower heat transfer rate than the effective model, which is attributable to the effective model's assumption of ideal counter-flow.

Although quantitative discrepancies remain due to inherent assumptions in the numerical homogenization and dehomogenization procedures, such discrepancies are difficult to fully eliminate. Nevertheless, the effective model captures the dominant trends observed in the full-scale analysis, namely the edgewise velocity enhancement and the associated increase in heat transfer. Taken together, these results suggest that the proposed effective model provides a reliable reduced-order framework for the optimization of TPMS HXs. By substantially reducing computational cost while preserving the dominant physical trends, the model enables the efficient generation of optimized-thickness HXs that outperform uniform-thickness designs. Overall, these findings confirm the practical applicability of the proposed TO framework for the optimization of gyroid two-fluid HXs.

6. Conclusion

TPMS HXs exhibit excellent thermo-hydraulic performance and are considered promising alternatives to conventional compact HXs such as PCHEs due to their high surface-area-to-volume ratio and continuous flow channels. In this study, an optimization method of wall thickness distribution for TPMS two-fluid HXs was proposed to improve their overall thermo-hydraulic performance. To enable optimization with reasonable computational cost, an effective porous media model was developed for TPMS HXs. An effective heat transfer coefficient was introduced to represent the heat exchange between the fluid and the solid wall, and this heat exchange was implemented as a volumetric heat source term. The proposed effective model was applied to gyroid

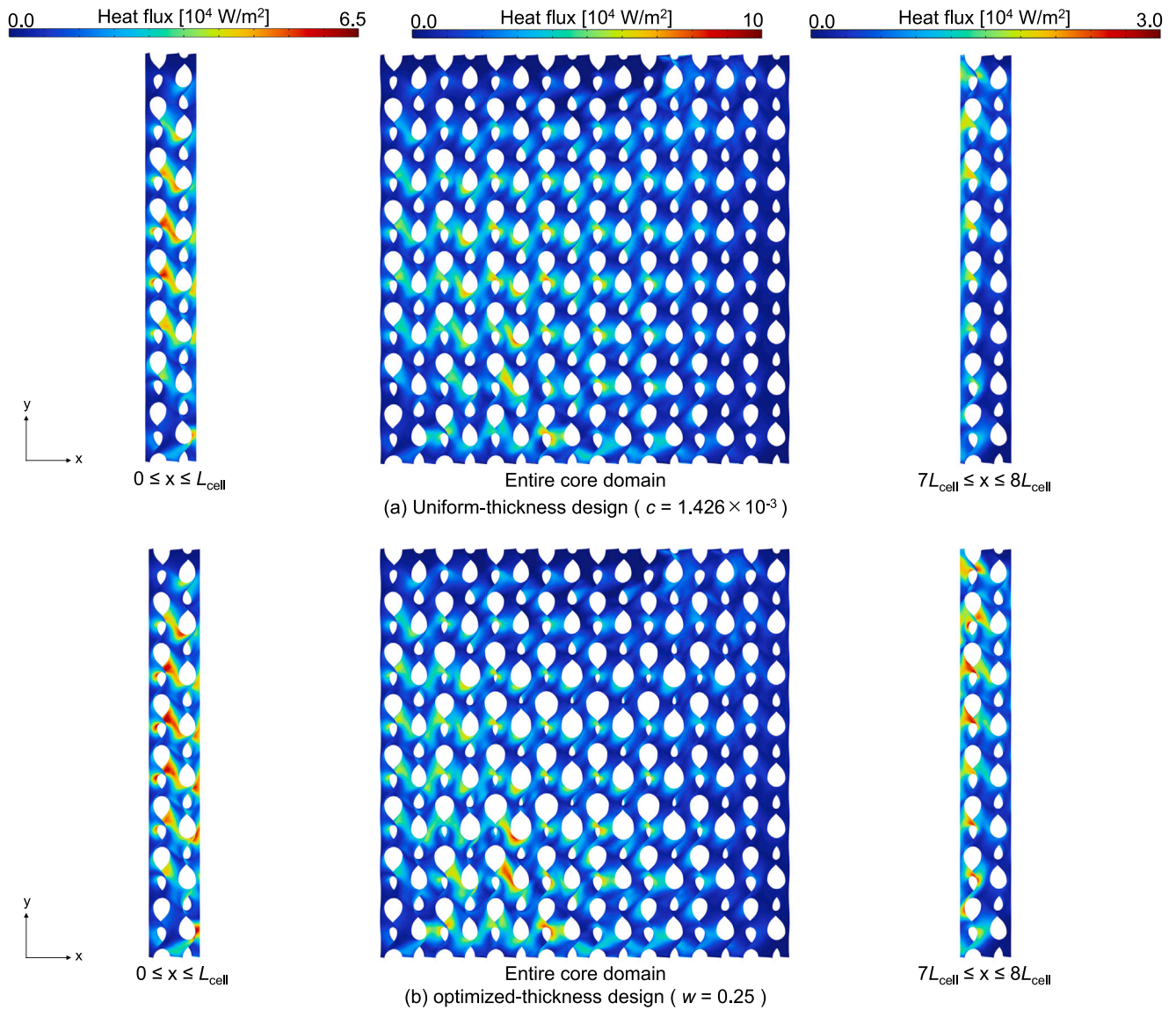


Fig. 23. Heat flux distribution for uniform-thickness (top, $c = 1.426 \times 10^{-3}$) and optimized-thickness (bottom, $w = 0.25$) gyroid HXs.

two-fluid HXs, and the wall thickness distribution was optimized considering both pressure drop and heat transfer rate. Furthermore, multiple optimized design variable fields were dehomogenized to reconstruct the corresponding gyroid HXs, and full-scale numerical analyses were conducted to demonstrate the effectiveness of the proposed method. The main findings of this study are summarized as follows:

- The proposed effective model for TPMS HXs was validated through comparisons with full-scale simulations over a range of inlet velocities. The model consistently reproduced the trends in both the heat transfer rate and pressure drop observed in the full-scale simulations. In particular, the heat transfer rate showed excellent agreement with the full-scale results ($R^2 = 0.999$), while the pressure drop was predicted with reasonable accuracy ($R^2 = 0.884$). These results indicate that the effective model reliably captures the performance trends of the full-scale model and provides a reliable framework for the wall-thickness optimization of gyroid HXs.
- Optimization of the wall-thickness distribution of the gyroid HXs using the effective model yielded designs with thicker walls near the central region of the core domain. The increased central thickness redirects the flow toward the lateral regions, suppressing

low-velocity regions and improving velocity uniformity. Full-scale simulations confirmed this behavior, showing higher flow velocities near the core edges than in the uniform-thickness design and enhanced local heat transfer. Consequently, the optimized configuration utilizes the entire HX core domain more effectively, leading to improved thermal performance.

- Full-scale simulations of the gyroid HX with an optimized-thickness distribution demonstrated superior thermo-hydraulic performance compared with the uniform-thickness design. Specifically, the optimized design with a weighting factor of $w = 0.25$ achieved a maximum PEC of 1.122, corresponding to a 12.2% improvement in overall performance relative to the uniform-thickness gyroid HX when accounting for the trade-off between heat transfer rate and pressure drop. In addition, the volumetric heat density increased by 4.16%, further confirming the improved thermal performance of the optimized design.

This study demonstrated the effectiveness of the proposed optimization method in improving the overall performance of gyroid HXs. The framework can be extended to various TPMS structures by calculating effective properties based on the corresponding TPMS unit cell. Nonetheless, some error due to the porous media approximation is

inevitable, and this study focused solely on wall thickness as the design variable, leaving room for further improvement. Future work will focus on improving the predictive accuracy of the effective model and extending the optimization framework to additional design variables, such as unit cell size and multiple TPMS types, enabling more advanced optimizations and further enhancing the performance of TPMS HXs. In addition, TPMS channels may exhibit transitional or turbulent flow under practical operating conditions. Additive-manufacturing-induced surface roughness, geometric variations, and tortuous flow paths can promote flow instabilities and laminar–turbulent transition. For higher-Re applications, incorporating turbulence modeling into the effective porous media model (e.g., a RANS model such as the SST k - ω model) will be an important step toward expanding the applicability of the proposed method. Furthermore, fabricating gyroid HXs with the optimized wall thickness and experimentally validating their performance remain important objectives for future research.

CRedit authorship contribution statement

Kaito Ohtani: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Hiroki Kawabe:** Writing – review & editing, Validation, Software, Investigation. **Kentaro Yaji:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition, Conceptualization. **Kikuo Fujita:** Writing – review & editing, Supervision. **Vikrant Aute:** Writing – review & editing, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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