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# Diquarks study from lattice QCD using the extended HALQCD method

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## 1. Introduction

Diquarks are colored composite objects formed by two quarks. In nature, observing isolated diquarks is forbidden by the color confinement of Quantum Chromodynamics (QCD). Despite this restriction, the diquark structure remains a subject of intense interest in hadron physics, frequently serving as a foundational building block in phenomenological models to describe conventional hadrons and exotic states, such as tetraquarks and pentaquarks [1]. Because direct experimental isolation of diquarks is impossible due to confinement, first-principles studies via lattice QCD have emerged as an indispensable tool for investigating their properties.

Within the lattice QCD community, previous investigations into diquark properties generally fall into three categories. The first approach extracts the diquark's effective mass directly from the diquark two-point correlation functions employing a specific gauge-fixing condition, such as Landau gauge [2–4]. While stable effective mass plateaus are observed, this method does not rigorously address the presence or nature of a bound-state pole in the gauge-dependent propagator. The second approach investigates diquark properties within gauge invariant baryon states. Because baryons are color-singlet states, thus having gauge-invariant properties [5–8]. In practice, these authors introduce a static heavy quark to neutralize the color charge of the diquark, studying the resulting heavy-light-light baryonic system. Although this strategy fully respects  $SU(3)$  gauge invariance, the extracted mass differences between various diquark channels remain inherently contaminated by the interaction energy between the static quark and the

diquark. In the third category, charmed baryons (such as the  $\Lambda_c$  and  $\Sigma_c$ ) are treated as charm quark–diquark systems, and the extended HAL QCD potential method is applied to study the system's dynamics [9–11]. Within this framework, the interaction potential between the charm quark and the diquark can be successfully isolated from the total binding energy, allowing the diquark mass to be extracted. However, this requires pre-determining the charm quark mass from charmonium spectroscopy, which introduces systematic ambiguities and uncertainties into the final diquark mass.

In this study, we combine the advantages of these strategies by neutralizing the color charge of the diquark with a static quark with infinite mass and applying the extended HAL QCD method to extract the diquark mass and the quark–diquark potential. By adopting this approach, we can eliminate the systematic ambiguities associated with a finite charm quark mass while successfully isolating the static quark–diquark interaction energy from the diquark mass. In this article, we present our preliminary results for a baryonic system composed of a static heavy quark and a scalar ( $J^P = 0^+$ ) diquark.

## 2. Extended HALQCD potential method

In this study, the angular momentum of our baryonic states can essentially be specified by the orbital angular momentum,  $L$  of the diquark relative to the static quark because of two reasons: First reason is that this study only covers the scalar diquark with  $J^\pi = 0^+$ . Second reason is that the spin of the static quark is completely decoupled and excluded from the rest of the baryonic system. This is because the static quark

corresponds to the heavy quark limit of the charm quark (Heavy Quark Spin Symmetry). Hence, our baryonic state can be denoted as  $|B(L)\rangle$ , where  $L$  is the orbital angular momentum between the diquark and the static quark.

We now define the equal-time Nambu-Bethe-Salpeter (NBS) wavefunction for our systems, this system consists of a static quark and a scalar diquark:

$$\psi_L(\mathbf{r}) \equiv \langle 0 | D_c(\mathbf{x}) Q_c(\mathbf{y}) | B(L) \rangle, \quad r \equiv x - y,$$

where  $Q_c(x)$  is the static quark field with color index  $c$ .  $D_c(x)$  is the composite scalar diquark field defined as

$$D_c(x) \equiv \epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x),$$

with  $C \equiv i\gamma^2\gamma^0$  being the charge conjugation matrix.

Next, we demand this NBS wavefunction to satisfy the following Schrodinger equation:

$$\left( -\frac{\nabla^2}{2m_D} + \hat{V} \right) \psi_L(\mathbf{r}) = (\varepsilon_L - m_D),$$

where  $\varepsilon_L$  denotes the energy of the quark-diquark system and  $m_D$  denotes the diquark mass. The NBS wavefunction is extracted from the quark-diquark four-point correlator. For positive time separations  $t > 0$ , our quark-diquark four-point correlator is written as

$$\begin{aligned} C(\mathbf{r}, t; t_{\text{src}}) &\equiv \frac{1}{V} \sum_{\mathbf{\Delta}} \langle 0 | D_c(\mathbf{r} + \mathbf{\Delta}, t) Q_c(\mathbf{\Delta}, t) \cdot \mathcal{J}^\dagger(t_{\text{src}}) | 0 \rangle \\ &= \frac{1}{V} \sum_{\mathbf{\Delta}} \sum_n \langle 0 | D_c(\mathbf{r} + \mathbf{\Delta}, t) Q_c(\mathbf{\Delta}, t) | n \rangle \\ &\quad \cdot \langle n | \mathcal{J}^\dagger(t_{\text{src}}) | 0 \rangle \cdot e^{-E_n(t-t_{\text{src}})} \end{aligned}$$

where  $V$  is the spatial volume, and the sum over  $\mathbf{\Delta}$  projects to zero spatial momentum. The states  $|n\rangle$  and energies  $E_n$  denote the  $n$ -th eigenstate and eigenenergy of the QCD Hamiltonian, respectively.  $\mathcal{J}(t_{\text{src}})$  denotes the source operator. We will examine the wall and the exponentially smeared sources with several smearing size to check the ground-state saturation in this report.

In the large- $t$  limit, the four-point correlator is dominated by the ground state, and the S-wave NBS wavefunction  $\psi_s(\mathbf{r})$  is obtained as

$$\psi_s(\mathbf{r}) \propto C(\mathbf{r}, t; t_{\text{src}}) \quad \text{for large } t$$

The extracted NBS wavefunction corresponds to the S-wave since we are using the wall and exponentially smeared source.

Next, we define

$$\tilde{V}_0(\mathbf{r}) \equiv \frac{\nabla^2 \psi_s(\mathbf{r})}{\psi_s(\mathbf{r})},$$

which we refer to as the prepotential. Using the Schrödinger equation, the prepotential is related to the potential as

$$\tilde{V}_0(\mathbf{r}) = 2m_D[V_0(\mathbf{r}) + m_D - \varepsilon_S],$$

where  $\varepsilon_S \equiv \varepsilon_{L=S}$ . Similarly, for the P-wave sector, the Schrödinger equation can be written as

$$(-\nabla^2 + \tilde{V}_0(r))\psi_P(\mathbf{r}) = \Delta\tilde{E}\psi_P(\mathbf{r}),$$

where  $\varepsilon_P \equiv \varepsilon_{L=P}$  and  $\Delta\tilde{E} \equiv 2m_D(\varepsilon_P - \varepsilon_S)$ . We will refer to this equation as “pre-Schrödinger equation”.

We determine the diquark mass  $m_D$  by requiring consistency between the baryonic masses obtained from the two-point correlators and those from the Schrödinger equation. This is done by solving the “pre-Schrödinger” equation using the prepotential  $\tilde{V}_0(\mathbf{r})$  in the P-wave sector to extract  $\Delta\tilde{E}$ . Then,  $m_D$  is determined by

$$m_D = \frac{\Delta\tilde{E}}{2(\varepsilon_P - \varepsilon_S)},$$

using the values of  $\varepsilon_S$  and  $\varepsilon_P$  extracted from two-point correlators. The quark-diquark potential is reconstructed as

$$V_0(r) = \frac{1}{2m_D} \tilde{V}_0(r) - m_D + \varepsilon_S.$$

after obtaining  $m_D$ .

### 3. Numerical Result

#### 3.1 Lattice Setup

Our lattice QCD calculations are performed using the

(2+1) - flavor ( $ud + s$ ) gauge configurations generated by the PACS-CS Collaboration [12]. The ensemble consists of 399 configurations on a lattice of size,  $L^3 \times T = 32^3 \times 64$ , generated with the Iwasaki gauge

action at  $\beta = \frac{6}{g^2} = 1.9$  and the nonperturbatively

$O(a)$ -improved Wilson quark action at  $(\kappa_{ud}, \kappa_s) = (0.13700, 0.13640)$  and  $C_{SW} = 1.715$ . This setup leads to a lattice spacing of  $a \approx 0.0907(13)$  fm ( $a^{-1} \approx 2.176(31)$  GeV), a spatial lattice size of  $La = 32a \approx 2.902$  fm, a pion mass of  $m_\pi \approx 700$  MeV, and a nucleon mass of  $m_N \approx 1.583(5)$  GeV. Coulomb gauge fixing is applied to fix the gauge. The wall source and exponentially smeared source with smearing function,  $f(\mathbf{r}) = \exp(-\frac{|\mathbf{r}|}{b})$  with various smearing radii,  $b$  is employed. To reduce statistical fluctuations, hypercubic (HYP) smearing with HYP1 parameter set [13] is applied to the gauge links entering the static-quark propagators (Wilson lines). Various number of source points is employed to improve the statistical precision which will be discussed later in this article. Further noise reduction is achieved by averaging over the cubic rotational symmetry of the lattice as well as by utilizing time reversal and charge-conjugation symmetries. Statistical uncertainties are estimated using the jackknife method with a bin size of 15 configurations.

### 3.2 Two-point correlators and effective masses

We first calculate the effective masses for various baryonic systems with different diquark by performing a naïve exponential fit on the two-point correlator,

$$G(t) = \langle J_R(\vec{x}, t) J_R^\dagger(\vec{x}, 0) \rangle$$

where  $J_R(x) \equiv \epsilon_{abc} (\tilde{u}_a^T(x) C \Gamma \tilde{d}_b(x)) \tilde{Q}_c(x)$  denotes the interpolating operator for the baryonic state where  $u$ ,  $d$ , and  $\tilde{Q}$  denote the spatially smeared quark fields,  $C$  denotes the charge conjugation matrix and  $a, b, c$  indicates the color indices. At the source time slice ( $t = 0$ ), these smeared fields are constructed by convoluting the local quark fields  $q \in \{u, d, Q\}$  with a

function  $f(x)$ , thus we can write  $\tilde{q}(t) = \sum_{\mathbf{x}} q(\mathbf{x}, t_0) \cdot f(\mathbf{x})$ . We apply  $f(\mathbf{x}) = 1$  (Wall) and  $f(\mathbf{x}) = \exp(-|\mathbf{x}|/a)$  (SRC01) in this report. The static quark propagator is replaced by a temporal Wilson line.

The effective mass is defined as follows:

$$m_{c\Gamma}(t) \equiv a^{-1} \log[G_\Gamma(t)/G_\Gamma(t+1)]$$

where  $a$  is the lattice spacing. In this report, we focus only on 2 baryonic states with  $J^\pi = 0^+, 1^-$ . They are constructed by choosing  $\Gamma = \gamma_5$  and  $\gamma_5 \gamma_i$  respectively. These states are interpreted as the  $0^+$  diquark in  $S$ -wave and  $P$ -wave respectively. They will be used in the extended HALQCD method to obtain the scalar diquark mass later.

The diquark mass is calculated from this equation,

$$m_D = \frac{\Delta \bar{E}}{2(\varepsilon_P - \varepsilon_S)} \text{ so it can be easily understood that the}$$

diquark mass is sensitive to the value of effective masses for the  $P$ -wave and  $S$ -wave states. Hence, identifying a stable plateau in the effective mass plot is crucial to extract an accurate ground-state effective mass. To ensure optimal overlap with the desired ground state, we employ and compare several different source operators. The results are shown in Figure 1.

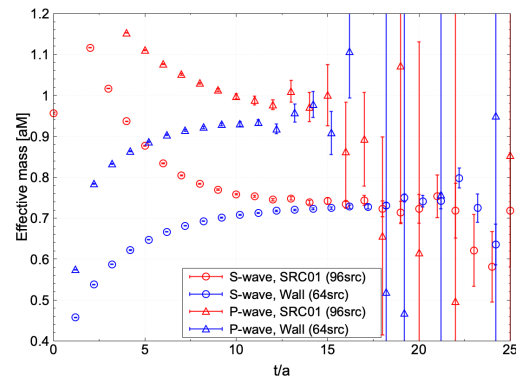


Figure 1 : Effective mass plot of of  $S$ -wave (circle) and  $P$ -wave (triangle) against  $t/a$  in lattice unit. The “Wall” denotes wall-source and “SRC01” denotes exponentially-smeared source with  $f(\mathbf{r}) = \exp(-\frac{|\mathbf{r}|}{a})$ . 96src and 64src is the number of source points used.

As seen in Figure 1, ground-state saturation is not fully

extracted in the P-wave sector (triangle), as overlapping plateaus are not achieved yet. By the definition of our effective masses mentioned earlier, the effective mass plateaus extracted from different source operators should overlap as shown in the S-wave sector (circle); an insufficient convergence among these operators indicates that contamination from excited states is not negligible yet. The choice of source operator is critical in this context, as different operators exhibit different ground-state convergence speeds. We also demonstrated the improvement in the stability of the effective mass plateau with an increasing number of source points, as shown in Figure 2. Specifically, a significantly more stable plateau can be extracted when employing 96 source points.

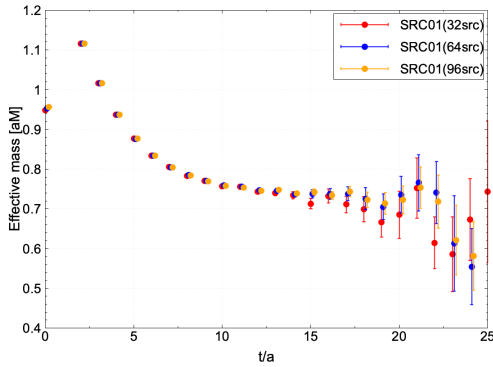


Figure 2 : Effective mass plot of S-wave against  $t/a$  in lattice unit. A more stable plateau can be obtained when the number of source points increased.

The present results were obtained using the computational resources allocated by D3 Center, Cybermedia Center, The University of Osaka last year. To refine these findings and improve our statistical precision, future work will be carried out by increasing the number of source points or implementing a wider variety of source operators. Specifically, we will implement multiple exponentially smeared sources with different smearing radii to select the operator that optimizes ground-state saturation.

#### 4. Conclusion

In this work, we presented a lattice QCD study of baryonic systems composed of a diquark and a static quark, aiming to determine both the diquark mass and the quark–diquark potential. We investigated the systematic relationship between effective mass convergence and the choice of source operators. Our current findings indicate that enhancing statistical precision and optimizing ground-state saturation are crucial next steps. Securing reliable effective mass plateau is essential before proceeding to the next phase of this project: extracting the diquark mass and the quark–diquark potential using the extended HAL QCD method.

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