First Order Approximation Approach toward Welding Design Optimization[†]
— Preliminary Study —

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Abstract

It is pointed out in this paper that first order approximation technique might be one of the most convenient and effective technique to solve decision-making problems related to optimization of welding conditions in design and fabrication. Although most of the decision-making problems encountered in welding are those of discrete variables, only few techniques are available to solve such discrete variable problems. First order approximation technique described herein is considered to be one of the simplest and have wide applicability, and the case of decreasing stress concentration factor in a cruciform welded joint subjected to bending is described by way of illustration of the effectiveness of this approach.

1. Introduction

This report points out that first order approximation technique is expected to be one of the simplest and most effective technique for solving decision-making problems related to optimization of welding conditions in design and fabrication. Although the discussion here is limited to the optimization problems in stress analysis of welded joints, first order approximation technique can easily be extended to other areas in welding.

What troubles us most when we are to apply any one of the traditional optimization techniques to welding is the fact that most of the traditional optimization techniques are based on the assumption that the design variables are continuous. (The term design here is used in the broader sense of the word, including fabrication) But apart from continuous stochastic variables which come in at the stage of fabrication, the design variables in welding are mostly discrete. Take for example the problem of selection of the thickness of a primary or secondary plate in a fillet welded joint. As in most cases the thickness will be selected among the standard thicknesses in the code for an economic reason, the design variable, thickness, should be considered as discrete. Further if we take another example in operation, the shape of a bead which is known to have much influence on fatigue strength is determined by such factors as welding position or welding rod etc. These conditions are originally discrete. Therefore, most of the welding design variables are discrete. But optimization techniques which can deal with discrete variables are quite

few, and further, most of these techniques are now in the stage of being developed. These techniques need, however, considerable amount of the expertise and are quite difficult to understand for non-professionals in this field. The traditional techniques and these new techniques are developed to seek 'global minimum' solutions to optimization problems. But if we look at the practical aspect of design, the primary requirement would be to improve the present design, if any, even though the solution may be only 'local minimum' and not 'global minimum'.

The first order approximation technique described herein is not a technique to seek a 'global minimum' solution at a stretch, but a practical simple technique which indicates guidelines for the selection of design variables among many combinations to improve the present design, although it might be minor.

2. First Order Approximation Technique

If we denote the object function by f and the design variables by $x_1, x_2, x_3, ..., x_n$,

$$f = f(x_1, x_2, ..., x_n)$$

If f is expanded in a Taylor series with respect to its design point and is approximated to the first order by neglecting higher order terms.,

$$f = f^n + \frac{\partial f}{\partial x_1} \delta x_1 + \frac{\partial f}{\partial x_2} \delta x_2 + \frac{\partial f}{\partial x_3} \delta x_3 ...... + \frac{\partial f}{\partial x_n} \delta x_n$$

where f^n denotes the value of f at its design point and partial derivatives are evaluated at each design point.

The meaning of the partial derivative $\dfrac{\partial f}{\partial x_i}$ in the above expression is that if the design variable x_i is changed by the amount x_i in the neighbourhood of its design point, it indicates how the object function f changes with it. i.e., the partial derivative $\dfrac{\partial f}{\partial x_i}$ represents the effect of the change of x_i upon f. Therefore, if we evaluate the partial derivative $\dfrac{\partial f}{\partial x_i}$ at each design point with respect to each design variable, it will be easily known how we should change the design variable to improve the present design.

3. Illustration; Cruciform Joint subjected to Pure Bending

Here, the case of stress analysis of a cruciform joint subjected to pure bending will be described by way of illustration. In the following, the stress concentration factor α at the weld toe is chosen as an object function, and the problem will be how we can decrease α, no matter how small the improvement might be. Note that it is different from the minimization of α.
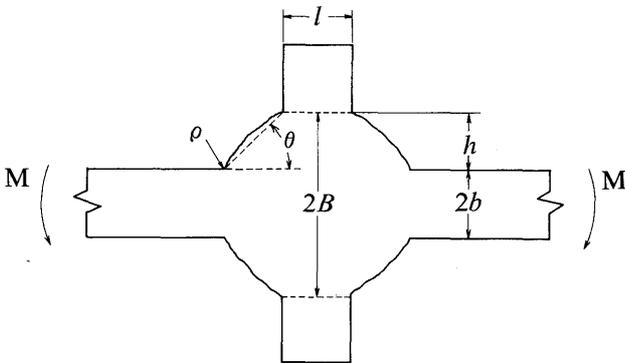

Fig. 1 Cruciform joint subjected to pure bending

The stress concentration factor α at the weld toe in a cruciform joint subjected to pure bending is given by the following approximate form[1],

$$\alpha=1+\frac{1-e^{-0.9\sqrt{B/h}\,\theta}}{1-e^{-0.9\frac{\pi}{2}\sqrt{B/h}}}(\alpha_0-1)$$

$$\alpha_0=1+\sqrt{\tanh\left\{2\left(\frac{l}{2B}+\frac{\rho}{2b}\right)\right\}\times}$$

$$\tanh\left\{\frac{(B/b-1)^{1/4}}{1-\rho/2b}\right\}\times$$

$$\left\{\frac{0.13+0.65(1-\rho/2b)^4}{(\rho/2b)^{1/3}}\right\}$$

where ρ=radius of curvature at the weld toe, θ= flank angle h=leg length, $2b$=thickness of a primary

plate, l=thickness of a secondary plate

The following discussion is based on the data obtained by Hijikata et al[2]. The mean values of ρ and θ in their experiments are obtained as ρ=0.55, θ=0.74 for an as weld material, and ρ=7.27, θ=0.74 for a toe dressed material. First, the difference between an as weld material and a toe dressed material is examined for the case of l=25, h=12.5, b=12.5. The partial derivatives for an as weld material are obtained as,

$$\frac{\partial\alpha}{\partial\theta}=1.716$$

$$\frac{\partial\alpha}{\partial\rho}=-0.852$$

$$\frac{\partial\alpha}{\partial l}=0.608$$

$$\frac{\partial\alpha}{\partial h}=0.001$$

$$\frac{\partial\alpha}{\partial b}=0.188$$

It is easily understood from this that to decrease α effectively, we can either decrease θ, or increase ρ. In the actual welding, ρ and θ are closely correlated, but the discussion above only indicates a design policy. It is also observed that the change in leg length does not have appreciable effect in decreasing α. It is also known that the selection of the thickness of a secondary plate is just as important as the consideration for ρ.

Let us assume that the toe is dressed according to the design policy obtained based on the above data. Then,

$$\frac{\partial\alpha}{\partial\theta}=0.040$$

$$\frac{\partial\alpha}{\partial\rho}=-0.131$$

$$\frac{\partial\alpha}{\partial l}=0.0004$$

$$\frac{\partial\alpha}{\partial h}=0.001$$

$$\frac{\partial\alpha}{\partial b}=0.080$$

are obtained for the partial derivatives of a toe dressed material. It is easily known that the effect of all the factors are greatly reduced, and that if there are to be any point to be improved in the design, it is ρ, and that the benefit by changing other factors is negligibly small, so that the design policy is to choose

some way of increasing ρ.

As discussed above, it is possible to make decisions or choose design policies for better improved design, based on the partial derivative values. These partial derivative values are, however, evaluated at their design points, and the change in design point is quite large as can be seen in the case of $\rho=0.55$ of a as weld material to $\rho=7.27$ of a toe dressed material. Therefore, rigorously speaking, the same trend is not guaranteed mathematically for a wide variation of design points. But in welding, the number of selections are limited, and if it is noted that it is most important how we can make decisions or choose design policies for better improved design in less time and trouble, the present first order approximation technique could be used as an approximate technique.

4. Summary

The effectiveness and simplicity of the first order approximation technique in decision-making problems for better improved welding design and fabrication is discussed. The problem of reduction of the stress concentration factor at the weld toe of a cruciform joint subjected to pure bending is discussed for illustration. First order approximation technique is considered to be one of the most effective, simple, and practical technique, at least for welding design. Another advantage of the first order approximation technique is that the partial derivative values are easily evaluated even in the case where an object function f is not given in an explicit form. (Take for example α for an object function, then the partial derivatives can be easily evaluated using a finite element method.) The greatest advantage may be that the effects of all the factors come into the analysis as linear summations, so that we can easily separate the factors which change by changing the welding design from factors which do not change. And it could be further added that this first order approximation technique can be easily extended to the reliability-based optimum welding design[3].

References
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