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A Generalized Framework for Inference-Based Diagnosis of Discrete Event Systems Capturing Both Disjunctive and Conjunctive Decision-Making

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***Abstract*—We have previously introduced an inference-based framework for decentralized decision-making, comprising of multiple observers, each with its own partial observation, where inferencing over the ambiguities of the self and the others is used to issue local decisions, and a global decision is taken to be the one possessing the minimum ambiguity level. In this setting, we previously introduced the notion of N-inference $\vee$-diagnosability to characterize the existence of a *disjunctive* decentralized diagnosis scheme so that any fault can be detected within a bounded delay, using at most N-levels of inferencing, by *one* of the diagnosers. While the *disjunctive* scheme relies on *one of the diagnosers making the failure decision*, the dual *conjunctive* scheme relies on *none of the diagnosers making the nonfailure decision*. It is known that the two schemes are incomparable, and in this paper we extend our earlier work to provide a more general framework, introducing the notion of N-inference diagnosability, capturing both disjunctive and conjunctive schemes. We also develop a method for verifying N-inference diagnosability, as well as discuss several of its useful properties.**

***Index Terms*—Ambiguity, decentralized diagnosis, discrete event system, inference diagnosability, inferencing.**

I. Introduction

DIAGNOSIS of systems is important for detecting the occurrence of any abnormal behaviors. For discrete event systems (DESs), there exists a long history of research on fault diagnosis (see for example [25]). When a fault occurs, it may not be observed owing to the partial observability of system behaviors, but its occurrence may be inferred through additional observations of the future behaviors since the post-fault behaviors may produce different observations from those produced by the pre-fault behaviors. If such resolution of fault can occur within a bounded delay, uniformly across all possible behaviors, the system is said to be diagnosable [13].

For large complex, physically distributed systems, a centralized fault detection scheme may be prohibitive, and instead, the task of fault detection may need to be performed by a set of local diagnosers that rely on their own subsets of accessible sensors to observe the system behavior. There are many prior papers dealing with decentralized/distributed diagnosis of DESs, including [1]–[4], [9]–[12], [14]–[16], [23], [24]. The notion of codiagnosability to capture the property that a fault can be detected by one of the local diagnosers within a uniformly bounded delay was formally introduced in [10]. This scheme, where at least one diagnoser issues the failure decision, is "disjunctive" in nature, and it turns out that a dual "conjunctive" scheme also exists, as proposed later in [24], where a fault can be ascertained when none of the diagnosers issue the nonfailure decision. These two dual schemes are incomparable in general [24].

A later development in decentralized diagnosis of DESs is the introduction of the framework for inferencing in decentralized decision-making. In this setting, each diagnoser uses not only the knowledge of its own observation, but also the inference about the possible knowledge of other diagnosers to arrive at its own local decision. A "winning" local decision (one needing the least level of inferencing) is used to issue a global decision (about whether or not a fault occurred in the past—See the first two paragraphs of Section IV for a formal definition, and (8) and (9) for the correctness requirements). While the initial work on inference-based decentralized diagnosis considered a single-level of such inferencing about one's knowledge and others' knowledge [24], a generalized framework to allow arbitrary-levels of inferencing was proposed in [8] for control and in [9] for diagnosis. While the framework for inference-based decentralized control [8] was general enough to capture both disjunctive and conjunctive decision-making, the one proposed in [9] for decentralized diagnosis was limited to only disjunctive decision-making.

The main contribution of the present work is to introduce a generalized framework for inference-based decentralized diagnosis that not only supports multi-levels of inferencing over self-knowledge and the inferred others' knowledge, but also supports *both* the disjunctive and conjunctive schemes of decision-making. The main difference from [9], which supports only the disjunctive scheme, lies in the identification of the *seed pair of failure and nonfailure behaviors* that must be disambiguated through inferencing. In [9], this seed pair was simply taken to be all failure versus all nonfailure behaviors. It turns out that this is bit strong, and instead only those failure behaviors that have allowed the execution of post-fault behaviors to a certain minimum number of steps (equaling the uniformly bounded delay of detection) must be disambiguated from the nonfailure behaviors. The generalization to capture both disjunctive and conjunctive cases requires new insights in correctly identifying the "seed" pair of languages

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that an inference-based decentralized diagnosis approach must discriminate in order to arrive at the desired correct decisions of disjunctive as well as conjunctive scenarios, and formally establishing the correctness of these new insights.

Another key contribution is demonstrating the existence of a delay bound of detection that can be used to perform an inference-based disambiguation analysis between the above mentioned seed pair of failure versus nonfailure behaviors. It is not known a priori as to how long, beyond the occurrence of faults, one might "ignore" before disambiguating with the nonfailure behaviors. We show the decidability of determining the existence of such a delay bound, needed for defining the seed pair of behaviors for which the inference-based disambiguation analysis should be performed.

Yet another contribution is the comparison of the proposed notion of diagnosability with the pre-existing notions of diagnosability from the literature, formally establishing the generality of the proposed notion, as shown in Fig. 1. Some other properties such as generality with respect to increasing levels of inferencing are also provided. The paper thus brings closure to an open problem of how to perform disjunctive as well as conjunctive decision-making for fault detection in an inference-based setting. The results presented in this paper were first reported at the authors' conference papers [19], [20] but without proofs. This paper provides additional insights, streamlines most of the presentation and algorithms, and includes correctness proofs.

II. Notation and Preliminaries

A deterministic automaton is a five-tuple $G = (Q, \Sigma, \delta, q_0, Q_m)$, where Q is the set of states, Σ is the finite set of events, $\delta : Q \times \Sigma \to Q$ is the partial transition function, $q_0 \in Q$ is the initial state, and $Q_m \subseteq Q$ is the set of marked states[1]. Let Σ^* be the set of all finite traces of elements of Σ, including the empty trace ε. The function δ can be generalized to $\delta : Q \times \Sigma^* \to Q$ in the usual manner. The generated language of G, denoted by $L(G)$, is defined as $L(G) = \{s \in \Sigma^* \mid \delta(q_0, s)!\}$, where $\delta(q_0, s)!$ denotes that $\delta(q_0, s)$ is defined.

Let $K \subseteq \Sigma^*$ be a language. The set of all prefixes of traces in K is denoted by $pr(K)$, i.e., $pr(K) = \{s \in \Sigma^* \mid \exists t \in \Sigma^* : st \in K\}$. If $K = pr(K)$, then K is said to be prefix-closed. For a trace $s \in \Sigma^*$, $pr(\{s\})$ is denoted by $pr(s)$. A prefix-closed language K is said to be *deadlock-free* if for any $s \in K$, $\{s\}\Sigma \cap K \neq \emptyset$. For each trace $s \in \Sigma^*$, $|s|$ denotes its length. For any $m \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of all nonnegative integers, let $\Sigma^{\geq m} := \{s \in \Sigma^* \mid |s| \geq m\}$ and $\Sigma^{\leq m} := \{s \in \Sigma^* \mid |s| \leq m\}$.

Let $I = \{1, 2, \ldots, n\}$ denote the index set of local diagnosers that perform the task of diagnosis. We assume that the limited sensing capabilities of the ith local diagnoser D_i ($i \in I$) can be represented by the local observation mask, $M_i : \Sigma \to \Delta_i \cup \{\varepsilon\}$, where Δ_i is the set of locally observed symbols. An event $\sigma \in \Sigma$ with $M_i(\sigma) = \varepsilon$ is said to be unobservable under M_i. The local observation mask M_i is extended to $M_i : \Sigma^* \to \Delta_i^*$ in the usual manner. Two traces $s, s' \in \Sigma^*$ with $M_i(s) = M_i(s')$ are said to be M_i-indistinguishable. In addition, the inverse map of M_i, denoted by $M_i^{-1} : \Delta_i^* \to 2^{\Sigma^*}$, is defined as $M_i^{-1}(t) = \{s \in \Sigma^* \mid M_i(s) = t\}$ for each $t \in \Delta_i^*$. For any languages $L \subseteq \Sigma^*$ and $L' \subseteq \Delta_i^*$, $M_i(L) \subseteq \Delta_i^*$ and $M_i^{-1}(L') \subseteq \Sigma^*$ are defined as $M_i(L) = \{M_i(s) \in \Delta_i^* \mid s \in L\}$ and $M_i^{-1}(L') = \{s \in \Sigma^* \mid M_i(s) \in L'\}$, respectively.

Let $L \neq \emptyset$ be a prefix-closed language that represents the generated language of a plant (system to be diagnosed) modeled as an automaton $G = (Q, \Sigma, \delta, q_0, Q)$, and $K \subseteq L$ be a nonempty prefix-closed language that represents a nonfailure specification language. Traces in $L - K$ are considered as failure traces and the task of diagnosis is to determine the execution of any trace in $L - K$ within an additional bounded number of system executions. Without loss of generality, the plant language L can be taken to be deadlock-free [9].

III. Existing Notions of Diagnosability for Decentralized Diagnosis

In this section, we review the existing notions of diagnosability of the pair (L, K) of languages for decentralized diagnosis. The notions of disjunctive and conjunctive-codiagnosability that do not involve inferencing were defined as follows.

Definition 1: [10], [24] The pair (L, K) of languages is said to be *disjunctive-codiagnosable* if

$$\exists m \in \mathbb{N}, \forall s \in L \cap (L - K)\Sigma^{\geq m}, \\ \exists i \in I : M_i^{-1} M_i(s) \cap L \subseteq L - K.$$

It is also said to be *conjunctive-codiagnosable* if

$$\exists m \in \mathbb{N}, \forall s \in K, \exists i \in I : M_i^{-1} M_i(s) \cap L \subseteq K\Sigma^{\leq m}.$$

The notion of disjunctive-codiagnosability (respectively, conjunctive-codiagnosability) requires that there should exist $m \in \mathbb{N}$ such that, for any failure trace $s \in L \cap (L - K)\Sigma^{\geq m}$ (respectively, nonfailure trace $s \in K$), there exists a local diagnoser that can distinguish s from any nonfailure trace in K (respectively, failure trace in $L \cap (L - K)\Sigma^{\geq m}$), where $L \cap (L - K)\Sigma^{\geq m}$ is the set of failure traces for which at least m events occurred after the occurrence of the failure.

These notions of codiagnosability were extended by introducing a single-level of inferencing as follows.

Definition 2: [24] The pair (L, K) of languages is said to be *conditionally disjunctive-codiagnosable* if

$$\exists m \in \mathbb{N}, \forall s \in L \cap (L - K)\Sigma^{\geq m}, \\ \exists i \in I, \forall u \in M_i^{-1} M_i(s) \cap K, \\ \exists j \in I : M_j^{-1} M_j(u) \cap L \subseteq K\Sigma^{\leq m}.$$

It is also said to be *conditionally conjunctive-codiagnosable* if

$$\exists m \in \mathbb{N}, \forall s \in K, \\ \exists i \in I, \forall u \in M_i^{-1} M_i(s) \cap L \cap (L - K)\Sigma^{\geq m}, \\ \exists j \in I : M_j^{-1} M_j(u) \cap L \subseteq L - K.$$

In our prior work, we have introduced a framework that allows multi-level inferencing over various local diagnosis decisions of varying ambiguity levels (see definition at the

[1] In this paper, an automaton is deterministic, unless otherwise stated.

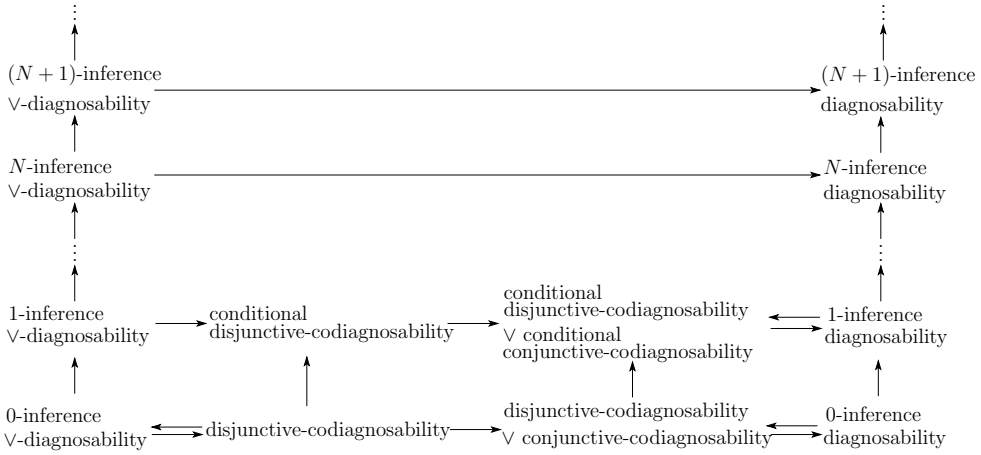


Fig. 1. A relation among notions of diagnosability for decentralized diagnosis.

beginning of Section IV, and further formulized in (7)), and defined the notion of N-inference $\vee$-diagnosability [9]. A monotonically decreasing sequence $\{(F_k, H_k)\}_{k\geq 0}$ of language pairs is defined as follows:

- Base step:
$$F_0 := L - K,\ H_0 := K.$$
- Induction step:
$$F_{k+1} := F_k \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(H_k) \right),$$
$$H_{k+1} := H_k \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(F_k) \right).$$

In the base step, $F_0 = L - K$ is the set of failure traces, and $H_0 = K$ is the set of nonfailure traces. In the induction step, F_{k+1} is a sublanguage of F_k consisting of traces for which there exists an M_i-indistinguishable trace in H_k for each $i \in I$. The sublanguage H_{k+1} of H_k can be understood in a similar fashion. Then, N-inference $\vee$-diagnosability is defined as follows.

Definition 3: [9] The pair (L, K) of languages is said to be *N-inference $\vee$-diagnosable* if there exists $m \in \mathbb{N}$ such that $F_{N+1} \cap (L \cap (L-K)\Sigma^{\geq m}) = \emptyset$.

As shown in Fig. 1, 0-inference $\vee$-diagnosability is equivalent to disjunctive-codiagnosability [9]. Since disjunctive-codiagnosability is incomparable with conjunctive-codiagnosability [24], the inference-based diagnosis framework of [9] does not capture the conjunctive scheme.

IV. Inference-based Decentralized Diagnosis

Let $C = \{0, 1, \phi\}$ be the set of diagnosis decisions, where "0" represents a nonfailure decision, "1" represents a failure decision, and "ϕ" represents an unsure decision. Each inference-based local diagnoser D_i is defined as a map $D_i : M_i(L) \to C \times \mathbb{N}$ [9], where for each $s \in L$,

$$D_i(M_i(s)) = (c_i(M_i(s)), n_i(M_i(s))).$$

Here $c_i(M_i(s)) \in C$ denotes the diagnosis decision of D_i following an observation $M_i(s) \in M_i(L)$, and $n_i(M_i(s)) \in \mathbb{N}$ denotes the ambiguity level of the diagnosis decision of D_i. Let $n(s) \in \mathbb{N}$ be the minimum ambiguity level of local decisions [9], i.e., $n(s) := \min_{i\in I} n_i(M_i(s))$.

The decentralized diagnoser $\{D_i\}_{i\in I}$ that consists of local diagnosers D_i ($i \in I$) issues global diagnosis decisions. Formally, $\{D_i\}_{i\in I}$ is defined as a map $\{D_i\}_{i\in I} : L \to C$. For each $s \in L$, the diagnosis decision $\{D_i\}_{i\in I}(s)$ is given as follows [9]:

$$\{D_i\}_{i\in I}(s) = \begin{cases} 1, & \text{if } \forall i \in I : n_i(M_i(s)) = n(s) \\ & \quad \Rightarrow c_i(M_i(s)) = 1 \\ 0, & \text{if } \forall i \in I : n_i(M_i(s)) = n(s) \\ & \quad \Rightarrow c_i(M_i(s)) = 0 \\ \phi, & \text{otherwise.} \end{cases}$$

The global diagnosis decision is taken to be the same as a local diagnosis decision possessing the minimum ambiguity level.

A useful notion of a decentralized diagnoser is the boundedness of the ambiguity level of its decisions. Intuitively, a decentralized diagnoser is said to be N-inferring if

- all failure decisions for failure traces or all nonfailure or unsure decisions for nonfailure traces have ambiguity levels below N, and
- for a sufficiently long failure trace or a nonfailure trace, there exists a "winning" local decision if the ambiguity level is at most N.

Definition 4: A decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ is said to be *N-inferring* if the following two conditions hold:

1) Either
$$\forall s \in L - K : \{D_i\}_{i\in I}(s) = 1 \Rightarrow n(s) \leq N, \quad (1)$$
or
$$\forall s \in K : \{D_i\}_{i\in I}(s) \neq 1 \Rightarrow n(s) \leq N. \quad (2)$$
2) There exists $m \in \mathbb{N}$ such that
$$\forall s \in (L \cap (L-K)\Sigma^{\geq m}) \cup K : \\ n(s) \leq N \Rightarrow \{D_i\}_{i\in I}(s) \neq \phi. \quad (3)$$

V. Generalized Framework for Inference-based Decentralized Diagnosis

Let $N \in \mathbb{N}$ be a given nonnegative integer. (N represents a parameter of inference diagnosability and is elaborated later.) Given a plant language L, a nonfailure specification language $K \subseteq L$, and a nonnegative integer $m \in \mathbb{N}$, we inductively define a monotonically decreasing sequence $\{(F_k(m), H_k(m))\}_{k\geq 0}$ of language pairs as follows:

- Base step:

$$F_0(m) := L \cap (L - K)\Sigma^{\geq m},\ H_0(m) := K.$$

- Induction step:

$$\begin{aligned} F_{k+1}(m) &:= F_k(m) \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(H_k(m)) \right), \\ H_{k+1}(m) &:= H_k(m) \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(F_k(m)) \right). \end{aligned}$$

Remark 1: Note that the sequence $\{(F_k(m), H_k(m))\}_{k\geq 0}$ is fundamentally different from the one defined in [9], and is needed to provide a generalized framework for inference-based diagnosis capturing both disjunctive and conjunctive strategies. The sequence will be reduced to the one in [9] by setting $m = 0$, but that will reduce the fusion mechanism to purely disjunctive, i.e., the conjunctive decision fusion will be lost.

Then, we have the following definition of N-inference diagnosability.

Definition 5: The pair (L, K) of languages is said to be *N-inference diagnosable* if there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$.

Assume that the pair (L, K) of languages is N-inference diagnosable so that there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$. Note that the former implies $H_{N+2}(m) = \emptyset$, whereas the latter implies $F_{N+2}(m) = \emptyset$. Knowing that (L, K) is N-inference diagnosable, a local diagnoser can compute its diagnosis decision and associate an ambiguity level with it as follows. Let $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$ for some $m \in \mathbb{N}$. For each $s \in L$, the ith local diagnoser D_i computes

$$n_i^f(M_i(s)) := \min\{k \in \mathbb{N} \mid M_i(s) \notin M_i(H_k(m))\}, \quad (4)$$

$$n_i^h(M_i(s)) := \min\{k \in \mathbb{N} \mid M_i(s) \notin M_i(F_k(m))\}. \quad (5)$$

Note that since $F_{N+1}(m) = H_{N+2}(m) = \emptyset$ or $H_{N+1}(m) = F_{N+2}(m) = \emptyset$, both $n_i^f(M_i(s))$ and $n_i^h(M_i(s))$ are bounded above by $N + 2$. Here $n_i^f(M_i(s))$ represents the ambiguity level of a failure decision contemplated by the ith diagnoser following the observation $M_i(s)$. It denotes the minimum index k such that the observation $M_i(s)$ does not match with the observation of any trace in $H_k(m)$. Similarly, the notation $n_i^h(M_i(s))$ represents the ambiguity level of a nonfailure decision contemplated by the ith diagnoser following the observation $M_i(s)$.

For a local diagnoser $D_i : M_i(L) \to C \times \mathbb{N}$, its diagnosis decision and ambiguity level following an observation $M_i(s) \in M_i(L)$, i.e., $D_i(M_i(s)) = (c_i(M_i(s)), n_i(M_i(s)))$, are determined as follows [9]:

$$c_i(M_i(s)) = \begin{cases} 1, & \text{if } n_i^f(M_i(s)) < n_i^h(M_i(s)) \\ 0, & \text{if } n_i^h(M_i(s)) < n_i^f(M_i(s)) \\ \phi, & \text{if } n_i^f(M_i(s)) = n_i^h(M_i(s)) \end{cases} \quad (6)$$

and

$$n_i(M_i(s)) = \min\{n_i^f(M_i(s)), n_i^h(M_i(s))\}. \quad (7)$$

I.e., the diagnosis decision with the smaller ambiguity level is chosen.

Remark 2: In summary, the decentralized diagnosis scheme for an N-inference diagnosable pair (L, K) of languages can be implemented as follows. (A decidable method of checking N-inference diagnosability is presented in Section VII.) For such a pair (L, K), there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$. A method of computing such a minimum integer m has recently been developed in [21]. The language pairs $(F_k(m), H_k(m))$ $(k = 0, 1, \ldots, N+1)$ are computed offline and then each of the ith local decision-makers, i.e., diagnosers is equipped with the set $\{(M_i(F_k(m)), M_i(H_k(m)))\}_{0\leq k\leq N+1}$ of language pairs. When the plant executes a trace $s \in L$, it is observed as the trace $M_i(s)$ at the ith local site. Using (4) and (5), the ith local diagnoser computes the values $n_i^f(M_i(s))$ and $n_i^h(M_i(s))$. When $n_i^f(M_i(s))$ (respectively, $n_i^h(M_i(s))$) is smaller, the ith local diagnoser issues a failure (respectively, nonfailure) decision with the ambiguity level $n_i^f(M_i(s))$ (respectively, $n_i^h(M_i(s))$), whereas when the two values are the same, the unsure decision with the ambiguity level $n_i^f(M_i(s)) = n_i^h(M_i(s))$ is issued. Since the ith local diagnoser has access to the set $\{(M_i(F_k(m)), M_i(H_k(m)))\}_{0\leq k\leq N+1}$ of language pairs, it only needs to perform certain membership-check and minimization operations of (4) and (5) to determine its diagnosis decision for each observation $M_i(s) \in M_i(L)$. Thus, similar to the online diagnosis scheme proposed in [10], the online complexity is linear in the sizes of the computed languages in $\{(M_i(F_k(m)), M_i(H_k(m)))\}_{0\leq k\leq N+1}$. On the other hand, the offline complexity of computing the said languages is dominated by the complexity of computing the delay bound m (which needs to be computed first). This has the same complexity as that of verifying N-inference diagnosability, discussed in Remark 5, that checks for the *existence* of a finite delay bound m; the computation of an *exact* delay bound m requires only certain additional linear complexity computations (e.g., summation, minimization, and maximization) along the paths of an acyclic automaton structure introduced in the verification steps given in Section VII.

All local decisions are collected at a central decision fusion unit, where a global decision is always taken to be a winning local decision, i.e., a local decision possessing the minimum ambiguity level. N-inference diagnosability guarantees that each fault is detected within the delay bound m (soundness), and the failure decision is never issued for a nonfailure trace (completeness).

First, we show that N-inference diagnosability is a sufficient condition for the existence of an N-inferring decentralized diagnoser with no missed and incorrect detections. This requires

the following lemma, which can be proved in the same way as Lemma 1 of [9].

Lemma 1: For any $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$, consider the decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ that consists of the local diagnosers $D_i : M_i(L) \to C \times \mathbb{N}$ $(i \in I)$ given by (4)–(7). Then,

$$\forall s \in F_0(m) : s \in F_{n(s)}(m),$$

and

$$\forall s \in H_0(m) : s \in H_{n(s)}(m).$$

The following lemma shows that the decentralized diagnoser for which the local diagnosers are given by (4)–(7) is N-inferring and guarantees that there are no missed and incorrect detections.

Lemma 2: If the pair (L, K) of languages is N-inference diagnosable, then there exists an N-inferring decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ that satisfies

$$\exists m \in \mathbb{N}, \forall s \in L \cap (L - K)\Sigma^{\geq m} : \{D_i\}_{i\in I}(s) = 1, \quad (8)$$

$$\forall s \in K : \{D_i\}_{i\in I}(s) \neq 1. \quad (9)$$

Proof: Since (L, K) is N-inference diagnosable, there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$. For such m, we consider the decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ that consists of the local diagnosers $D_i : M_i(L) \to C \times \mathbb{N}$ $(i \in I)$ given by (4)–(7).

First, we show that $\{D_i\}_{i\in I}$ is N-inferring. To prove the first condition of Definition 4, we consider the case that $F_{N+1}(m) = \emptyset$. Consider any $s \in L - K$ such that $\{D_i\}_{i\in I}(s) = 1$. For any $i \in I$ such that $n_i(M_i(s)) = n(s)$, we have $c_i(M_i(s)) = 1$, which implies that $n_i(M_i(s)) = n_i^f(M_i(s)) < n_i^h(M_i(s))$. In addition, since $F_{N+1}(m) = \emptyset$, we have $M_i(s) \notin M_i(F_{N+1}(m))$, which implies that $n_i^h(M_i(s)) \leq N + 1$. It follows that $n(s) = n_i(M_i(s)) \leq N$. Thus, (1) holds.

We also consider the case that $H_{N+1}(m) = \emptyset$. Consider any $s \in K$ such that $\{D_i\}_{i\in I}(s) \neq 1$. Since $s \in K$, we have $s \in H_0(m)$. There exists $l \in \mathbb{N}$ such that $0 \leq l < N + 1$, $s \in H_l(m)$, and $s \notin H_{l+1}(m)$. Since $s \in H_l(m)$ and $s \notin H_{l+1}(m)$, there exists $j \in I$ such that $s \notin M_j^{-1}M_j(F_l(m))$. It follows that $M_j(s) \notin M_j(F_l(m))$. We have $n(s) \leq n_j(M_j(s)) \leq n_j^h(M_j(s)) \leq l < N + 1$, which implies that $n(s) \leq N$. Thus, (2) holds.

To prove the second condition of Definition 4, we consider any $s \in (L \cap (L - K)\Sigma^{\geq m}) \cup K$ such that $n(s) \leq N$. For the sake of contradiction, we suppose that $\{D_i\}_{i\in I}(s) = \phi$, i.e., there exist $i, j \in I$ such that $n(s) = n_i(M_i(s)) = n_j(M_j(s))$, $c_i(M_i(s)) \neq 1$, and $c_j(M_j(s)) \neq 0$. We have two cases that $s \in L \cap (L - K)\Sigma^{\geq m}$ and $s \in K$.

We consider the case that $s \in L \cap (L - K)\Sigma^{\geq m} = F_0(m)$. It follows from $c_i(M_i(s)) \neq 1$ that $n(s) = n_i(M_i(s)) = n_i^h(M_i(s))$, which implies that $M_i(s) \notin M_i(F_{n(s)}(m))$. It follows that $s \notin F_{n(s)}(m)$. This contradicts Lemma 1.

We also consider the case that $s \in K = H_0(m)$. It follows from $c_j(M_j(s)) \neq 0$ that $n(s) = n_j(M_j(s)) = n_j^f(M_j(s))$, which implies that $M_j(s) \notin M_j(H_{n(s)}(m))$. It follows that $s \notin H_{n(s)}(m)$. This contradicts Lemma 1.

Next, we prove that $\{D_i\}_{i\in I}$ satisfies (8). For the sake of contradiction, we suppose that there exists $s \in L \cap (L - K)\Sigma^{\geq m} = F_0(m)$ such that $\{D_i\}_{i\in I}(s) \neq 1$. Then, there exists $i \in I$ such that $n(s) = n_i(M_i(s))$ and $c_i(M_i(s)) \neq 1$. We have $n(s) = n_i^h(M_i(s))$, which implies that $M_i(s) \notin M_i(F_{n(s)}(m))$. It follows that $s \notin F_{n(s)}(m)$. This contradicts Lemma 1.

Finally, we prove that $\{D_i\}_{i\in I}$ satisfies (9). For the sake of contradiction, we suppose that there exists $s \in K = H_0(m)$ such that $\{D_i\}_{i\in I}(s) = 1$. Then, there exists $i \in I$ such that $n(s) = n_i(M_i(s))$ and $c_i(M_i(s)) = 1$. We have $n(s) = n_i^f(M_i(s))$, which implies that $M_i(s) \notin M_i(H_{n(s)}(m))$. It follows that $s \notin H_{n(s)}(m)$. This contradicts Lemma 1. ■

Next, we have the following lemma which states that N-inference diagnosability is a necessary condition for the existence of an N-inferring decentralized diagnoser with no missed and incorrect detections.

Lemma 3: If there exists an N-inferring decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ that satisfies (8) and (9), then the pair (L, K) of languages is N-inference diagnosable.

Proof: Since $\{D_i\}_{i\in I}$ satisfies (8), there exists $m \in \mathbb{N}$ such that

$$\forall s \in L \cap (L - K)\Sigma^{\geq m} : \{D_i\}_{i\in I}(s) = 1. \quad (10)$$

For the sake of contradiction, we suppose that (L, K) is not N-inference diagnosable. Then, we have $F_{N+1}(m) \neq \emptyset$ and $H_{N+1}(m) \neq \emptyset$. Since $\{D_i\}_{i\in I}$ is N-inferring, we have the two cases that a) (1) holds and b) (2) holds.

First, we show that in the case a), there exist $s_0, s_1, \ldots, s_l \in L$ and $i_0, i_1, \ldots, i_{l-1} \in I$ such that

- $s_j \in \begin{cases} F_{n(s_j)+1}(m), & \text{if } j \text{ is an even number} \\ H_{n(s_j)+1}(m), & \text{if } j \text{ is an odd number} \end{cases}$ $(j = 0, 1, \ldots, l)$,
- $n(s_j) > n(s_{j+1})$ $(j = 0, 1, \ldots, l - 1)$,
- $n(s_l) = 0$,
- $M_{i_j}(s_j) = M_{i_j}(s_{j+1})$ $(j = 0, 1, \ldots, l - 1)$.

We can pick up $s_0 \in F_{N+1}(m) \neq \emptyset$. Since $s_0 \in F_0(m) = L \cap (L - K)\Sigma^{\geq m}$, we have by (10) that $\{D_i\}_{i\in I}(s_0) = 1$. It follows from (1) that $n(s_0) \leq N$. Then, we have $s_0 \in F_{N+1}(m) \subseteq F_{n(s_0)+1}(m)$. In addition, there exists $i_0 \in I$ such that $n_{i_0}(M_{i_0}(s_0)) = n(s_0)$ and $c_{i_0}(M_{i_0}(s_0)) = 1$.

Since $s_0 \in F_{n(s_0)+1}(m)$, we have $s_0 \in M_{i_0}^{-1}M_{i_0}(H_{n(s_0)}(m))$. Then, we can pick up $s_1 \in H_{n(s_0)}(m)$ such that $M_{i_0}(s_0) = M_{i_0}(s_1)$. Since $s_1 \in H_0(m) = K$, it follows from (9) that $\{D_i\}_{i\in I}(s_1) \neq 1$. There exists $i_1 \in I$ such that $n_{i_1}(M_{i_1}(s_1)) = n(s_1)$ and $c_{i_1}(M_{i_1}(s_1)) \neq 1$. Since

$$n(s_1) \leq n_{i_0}(M_{i_0}(s_1)) = n_{i_0}(M_{i_0}(s_0)) = n(s_0) \leq N,$$

we have by the second condition of Definition 4 that $\{D_i\}_{i\in I}(s_1) = 0$, i.e.,

$$\forall i \in I : n_i(M_i(s_1)) = n(s_1) \Rightarrow c_i(M_i(s_1)) = 0.$$

It follows from $c_{i_0}(M_{i_0}(s_1)) = c_{i_0}(M_{i_0}(s_0)) = 1$ that

$$n(s_1) < n_{i_0}(M_{i_0}(s_1)) = n_{i_0}(M_{i_0}(s_0)) = n(s_0).$$

Thus, we have $s_1 \in H_{n(s_0)}(m) \subseteq H_{n(s_1)+1}(m)$.

Since $s_1 \in H_{n(s_1)+1}(m)$, we have $s_1 \in M_{i_1}^{-1}M_{i_1}(F_{n(s_1)}(m))$. Then, we can pick up $s_2 \in F_{n(s_1)}(m)$ such that $M_{i_1}(s_1) = M_{i_1}(s_2)$. Since $s_2 \in F_0(m) = L \cap (L-K)\Sigma^{\geq m}$, we have $\{D_i\}_{i\in I}(s_2) = 1$. There exists $i_2 \in I$ such that $n_{i_2}(M_{i_2}(s_2)) = n(s_2)$ and $c_{i_2}(M_{i_2}(s_2)) = 1$. Moreover, since $c_{i_1}(M_{i_1}(s_2)) = c_{i_1}(M_{i_1}(s_1)) \neq 1$, we have

$$n(s_2) < n_{i_1}(M_{i_1}(s_2)) = n_{i_1}(M_{i_1}(s_1)) = n(s_1).$$

Thus, we have $s_2 \in F_{n(s_1)}(m) \subseteq F_{n(s_2)+1}(m)$.

By repeating this procedure, we can obtain $s_0, s_1, \ldots, s_l \in L$ and $i_0, i_1, \ldots, i_{l-1} \in I$ that satisfy the above four conditions.

Similarly, we can show that in the case b), there exist $s_0, s_1, \ldots, s_l \in L$ and $i_0, i_1, \ldots, i_{l-1} \in I$ such that

- $s_j \in \begin{cases} H_{n(s_j)+1}(m), & \text{if } j \text{ is an even number} \\ F_{n(s_j)+1}(m), & \text{if } j \text{ is an odd number} \end{cases}$ $(j = 0, 1, \ldots, l)$,
- $n(s_j) > n(s_{j+1})$ $(j = 0, 1, \ldots, l-1)$,
- $n(s_l) = 0$,
- $M_{i_j}(s_j) = M_{i_j}(s_{j+1})$ $(j = 0, 1, \ldots, l-1)$.

In the both cases a) and b), we have $s_l \in F_1(m)$ or $s_l \in H_1(m)$ for $s_l \in L$ with $n(s_l) = 0$.

First, we consider the case that $s_l \in F_1(m)$. Since $s_l \in F_0(m) = L \cap (L-K)\Sigma^{\geq m}$, we have $\{D_i\}_{i\in I}(s_l) = 1$. There exists $i \in I$ such that $0 = n(s_l) = n_i(M_i(s_l))$ and $c_i(M_i(s_l)) = 1$. Since $s_l \in M_i^{-1}M_i(H_0(m))$, there exists $s_{l_i} \in H_0(m) = K$ such that $M_i(s_l) = M_i(s_{l_i})$. We have $\{D_i\}_{i\in I}(s_{l_i}) \neq 1$. Since $0 = n_i(M_i(s_l)) = n_i(M_i(s_{l_i}))$, we have $0 = n(s_{l_i}) = n_i(M_i(s_{l_i})) \leq N$. By the second condition of Definition 4, we have $\{D_i\}_{i\in I}(s_{l_i}) = 0$. Since $n(s_{l_i}) = n_i(M_i(s_{l_i}))$, we have $c_i(M_i(s_{l_i})) = 0$. This contradicts $c_i(M_i(s_{l_i})) = c_i(M_i(s_l)) = 1$.

Next, we consider the case that $s_l \in H_1(m)$. Since $s_l \in H_0(m) = K$, we have $\{S_i\}_{i\in I}(s_l) \neq 1$. There exists $i \in I$ such that $0 = n(s_l) = n_i(M_i(s_l))$ and $c_i(M_i(s_l)) \neq 1$. Since $s_l \in M_i^{-1}M_i(F_0(m))$, there exists $s_{l_i} \in F_0(m) = L \cap (L-K)\Sigma^{\geq m}$ such that $M_i(s_l) = M_i(s_{l_i})$. We have $\{D_i\}_{i\in I}(s_{l_i}) = 1$. Since $0 = n_i(M_i(s_l)) = n_i(M_i(s_{l_i}))$, we have $0 = n(s_{l_i}) = n_i(M_i(s_{l_i}))$, which implies together with $\{D_i\}_{i\in I}(s_{l_i}) = 1$ that $c_i(M_i(s_l)) = c_i(M_i(s_{l_i})) = 1$. This contradicts $c_i(M_i(s_l)) \neq 1$. ∎

The following theorem, which follows from Lemmas 2 and 3, shows that N-inference diagnosability is a necessary and sufficient condition for the existence of an N-inferring decentralized diagnoser with no missed and incorrect detections.

Theorem 1: There exists an N-inferring decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ that satisfies (8) and (9) if and only if the pair (L, K) of languages is N-inference diagnosable.

Example 1: We consider a plant modeled by the finite automaton G shown in Fig. 2(a). Let $n = 2$, $\Delta_1 = \{a, a', a'', c, d, e\}$, $\Delta_2 = \{b, b', b'', c, d, e\}$, and

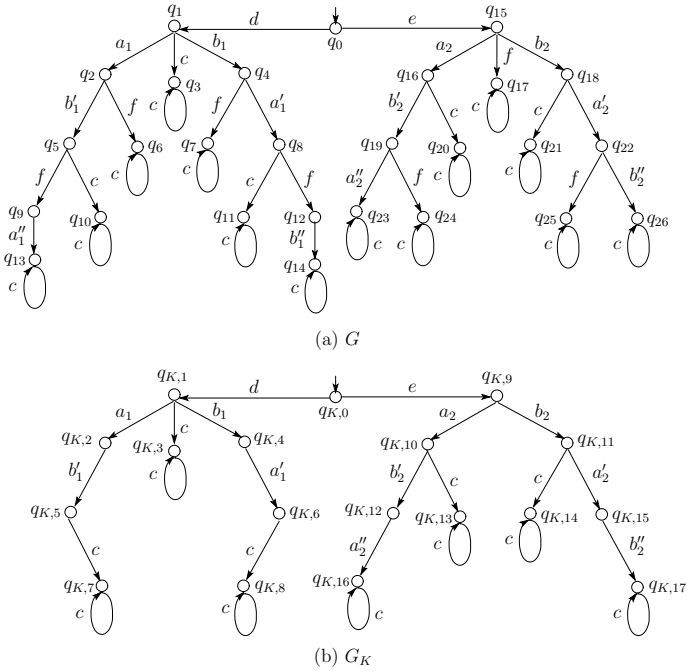


Fig. 2. Automata G and G_K of Example 1.

$$M_1(\sigma) = \begin{cases} a, & \text{if } \sigma \in \{a_1, a_2\} \\ a', & \text{if } \sigma \in \{a'_1, a'_2\} \\ a'', & \text{if } \sigma \in \{a''_1, a''_2\} \\ \sigma, & \text{if } \sigma \in \{c, d, e\} \\ \varepsilon, & \text{otherwise,} \end{cases}$$

$$M_2(\sigma) = \begin{cases} b, & \text{if } \sigma \in \{b_1, b_2\} \\ b', & \text{if } \sigma \in \{b'_1, b'_2\} \\ b'', & \text{if } \sigma \in \{b''_1, b''_2\} \\ \sigma, & \text{if } \sigma \in \{c, d, e\} \\ \varepsilon, & \text{otherwise.} \end{cases}$$

In addition, let $K \subseteq L$ be a prefix-closed language generated by the finite automaton G_K shown in Fig. 2(b). I.e., the event f denotes the occurrence of a failure.

First, we show that (L, K) is 3-inference diagnosable. We consider any $m \in \mathbb{N}$ such that $m \geq 1$. Initially, we have

$$\begin{aligned} F_0(m) &= d(a_1 f c^m c^* + b_1 f c^m c^* + a_1 b'_1 f a''_1 c^{m-1} c^* \\ &\quad + b_1 a'_1 f b''_1 c^{m-1} c^*) \\ &\quad + e(f c^m c^* + a_2 b'_2 f c^m c^* + b_2 a'_2 f c^m c^*), \\ H_0(m) &= pr(d(c^+ + a_1 b'_1 c^+ + b_1 a'_1 c^+) \\ &\quad + e(a_2 c^+ + a_2 b'_2 a''_2 c^+ + b_2 c^+ + b_2 a'_2 b''_2 c^+)). \end{aligned}$$

Since

$$\begin{aligned}
M_1(F_0(m)) &= d(ac^mc^* + c^mc^* + aa''c^{m-1}c^* \\
&\quad + a'c^{m-1}c^*) \\
&\quad + e(c^mc^* + ac^mc^* + a'c^mc^*), \\
M_2(F_0(m)) &= d(c^mc^* + bc^mc^* + b'c^{m-1}c^* \\
&\quad + bb''c^{m-1}c^*) \\
&\quad + e(c^mc^* + b'c^mc^* + bc^mc^*), \\
M_1(H_0(m)) &= pr(d(c^+ + ac^+ + a'c^+) \\
&\quad + e(ac^+ + aa''c^+ + c^+ + a'c^+)), \\
M_2(H_0(m)) &= pr(d(c^+ + b'c^+ + bc^+) \\
&\quad + e(c^+ + b'c^+ + bc^+ + bb''c^+)),
\end{aligned}$$

we have

$$\begin{aligned}
F_1(m) &= F_0(m) \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(H_0(m)) \right) \\
&= d(a_1fc^mc^* + b_1fc^mc^*) \\
&\quad + e(fc^mc^* + a_2b_2'fc^mc^* + b_2a_2'fc^mc^*), \\
H_1(m) &= H_0(m) \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(F_0(m)) \right) \\
&= d(c^mc^* + a_1b_1'c^mc^* + b_1a_1'c^mc^*) \\
&\quad + e(a_2c^mc^* + b_2c^mc^*).
\end{aligned}$$

Moreover, by the iterative computations, we obtain

$$\begin{aligned}
F_2(m) &= d(a_1fc^mc^* + b_1fc^mc^*) + efc^mc^*, \\
H_2(m) &= dc^mc^* + e(a_2c^mc^* + b_2c^mc^*), \\
F_3(m) &= efc^mc^*, \\
H_3(m) &= dc^mc^*,
\end{aligned}$$

and finally, we have $F_4(m) = H_4(m) = \emptyset$, which implies that (L, K) is 3-inference diagnosable.

Letting $m = 1$, we synthesize the decentralized diagnoser using (4)–(7). The local decisions of D_1 and D_2 computed using (4)–(7) are shown in Table I. For example, $D_1(da'c)$ is computed as follows. Since $da'c \in M_1(H_1(1)) - M_1(H_2(1))$, we have by (4) that $n_1^f(da'c) = 2$. On the other hand, since $da'c \in M_1(F_0(1)) - M_1(F_1(1))$, we have by (5) that $n_1^h(da'c) = 1$. It follows that $1 = n_1^h(da'c) < n_1^f(da'c) = 2$. By (6) and (7), we have $c_1(da'c) = 0$ and $n_1(da'c) = 1$, which implies that D_1 makes a nonfailure decision following the observation $da'c \in M_1(L)$ with the ambiguity level 1.

Then, the global diagnosis decisions of the decentralized diagnoser $\{D_i\}_{i\in I}$ are computed as shown in Table II. For example, $\{D_i\}_{i\in I}(da_1fc)$ is computed as follows. Since $2 = n_1(M_1(da_1fc)) < n_2(M_2(da_1fc)) = 3$ and $c_1(M_1(da_1fc)) = 1$, we have $n(da_1fc) = 2$ and $\{D_i\}_{i\in I}(da_1fc) = 1$.

By Tables I and II, we can verify that $\{D_i\}_{i\in I}$ is a 3-inferring decentralized diagnoser that satisfies (8) and (9).

We present an example of the local and global decision making during the online diagnosis procedure. We assume that the plant G executes a failure trace $da_1b_1'fa_1'' \in L - K$. Then the local diagnoser D_1 (respectively, D_2) observes $M_1(da_1b_1'fa_1'') = daa''$ (respectively, $M_2(da_1b_1'fa_1'') = db'$).

TABLE I
LOCAL DECISIONS OF D_1 AND D_2.

$t \in M_1(L)$	$n_1^f(t)$	$n_1^h(t)$	$c_1(t)$	$n_1(t)$
$t \in \{\varepsilon, d, e\}$	1	0	0	0
$t \in dc^+$	4	3	0	3
$t = da$	1	0	0	0
$t \in dac^+$	2	3	1	2
$t = da'$	1	1	ϕ	1
$t \in da'c^+$	2	1	0	1
$t \in daa''c^*$	0	1	1	0
$t \in ec^+$	3	4	1	3
$t = ea$	1	0	0	0
$t \in eac^+$	3	2	0	2
$t = ea'$	1	0	0	0
$t \in ea'c^+$	1	2	1	1
$t \in eaa''c^*$	1	0	0	0
$t \in M_2(L)$	$n_2^f(t)$	$n_2^h(t)$	$c_2(t)$	$n_2(t)$
$t \in \{\varepsilon, d, e\}$	1	0	0	0
$t \in dc^+$	4	3	0	3
$t = db$	1	0	0	0
$t \in dbc^+$	2	3	1	2
$t = db'$	1	1	ϕ	1
$t \in db'c^+$	2	1	0	1
$t \in dbb''c^*$	0	1	1	0
$t \in ec^+$	3	4	1	3
$t = eb$	1	0	0	0
$t \in ebc^+$	3	2	0	2
$t = eb'$	1	0	0	0
$t \in eb'c^+$	1	2	1	1
$t \in ebb''c^*$	1	0	0	0

TABLE II
GLOBAL DECISIONS OF $\{D_i\}_{i\in I}$.

$s \in L$	$n(s)$	$\{D_i\}_{i\in I}(s)$
$s \in pr(d(a_1(f + b_1'f) + b_1(f + a_1'f)))$	0	0
$s \in dc^+$	3	0
$s \in d(a_1fc^+ + b_1fc^+)$	2	1
$s \in d(a_1b_1'c^+ + b_1a_1'c^+)$	1	0
$s \in d(a_1b_1'fa_1''c^* + b_1a_1'fb_1''c^*)$	0	1
$s \in pr(e(f + a_2b_2'f + b_2a_2'f))$	0	0
$s \in efc^+$	3	1
$s \in e(a_2c^+ + b_2c^+)$	2	0
$s \in e(a_2b_2'fc^+ + b_2a_2'fc^+)$	1	1
$s \in e(a_2b_2'a_2''c^* + b_2a_2'b_2''c^*)$	0	0

The local decision making of D_1 and D_2 during the execution of $da_1b_1'fa_1'' \in L - K$ is performed as follows:

$$\begin{aligned}
D_1(\varepsilon) &= (c_1(\varepsilon), n_1(\varepsilon)) = (0, 0) \\
&\xrightarrow{d} D_1(d) = (c_1(d), n_1(d)) = (0, 0) \\
&\xrightarrow{a} D_1(da) = (c_1(da), n_1(da)) = (0, 0) \\
&\xrightarrow{a''} D_1(daa'') = (c_1(daa''), n_1(daa'')) = (1, 0), \\
D_2(\varepsilon) &= (c_2(\varepsilon), n_2(\varepsilon)) = (0, 0) \\
&\xrightarrow{d} D_2(d) = (c_2(d), n_2(d)) = (0, 0) \\
&\xrightarrow{b'} D_2(db') = (c_2(db'), n_2(db')) = (\phi, 1).
\end{aligned}$$

Then the global decision is computed at the central decision fusion unit as follows:

$$\begin{aligned}
\{D_i\}_{i\in I}(\varepsilon) = 0 &\xrightarrow{d} \{D_i\}_{i\in I}(d) = 0 \\
&\xrightarrow{a_1} \{D_i\}_{i\in I}(da_1) = 0 \xrightarrow{b_1'} \{D_i\}_{i\in I}(da_1b_1') = 0 \\
&\xrightarrow{f} \{D_i\}_{i\in I}(da_1b_1'f) = 0 \xrightarrow{a_1''} \{D_i\}_{i\in I}(da_1b_1'fa_1'') = 1.
\end{aligned}$$

The occurrence of the failure event f is detected after a_1'' is executed.

Remark 3: In [3], a multi-decision inference-based architecture where several inference-based decentralized diagnosers work in parallel was proposed, and a certain diagnosability property was defined based on N-inference $\vee$-diagnosability of [9]. The multi-decision architecture can be generalized following the framework of this paper. Note that, owing to the undecidability of joint-observability [22], the multi-decision framework is in general undecidable (so stringent decidable conditions have been proposed). In contrast, as we show below, the inference-based framework is decidable for any specified level of inferencing. (See Section VII below.)

VI. Properties of N-Inference Diagnosability

In this section, we study various properties of N-inference diagnosable systems. We begin by establishing that the classes of N-inference diagnosable systems form a monotonically increasing sequence as a function of N. Since the sequence $\{(F_k(m), H_k(m))\}_{k\geq 0}$ of language pairs is monotonically decreasing for any $m \in \mathbb{N}$, the following result is easily obtained (the proof is omitted).

Theorem 2: For any $N \in \mathbb{N}$, if the pair (L, K) of languages is N-inference diagnosable, then it is $(N+1)$-inference diagnosable.

The converse relation of Theorem 2 need not hold. For example, (L, K) of Example 1 is 3-inference diagnosable but not 2-inference diagnosable.

Next, we show that 0-inference diagnosability subsumes both disjunctive-codiagnosability and conjunctive-codiagnosability [10], [24], and 1-inference diagnosability subsumes both conditional disjunctive-codiagnosability and conditional conjunctive-codiagnosability [24].

Theorem 3: The pair (L, K) of languages is conjunctive-codiagnosable if and only if there exists $m \in \mathbb{N}$ such that $H_1(m) = \emptyset$.

Proof: ($\Rightarrow$) We prove the contrapositive. We suppose that $H_1(m) \neq \emptyset$ for any $m \in \mathbb{N}$. We consider any $s \in H_1(m) \subseteq H_0(m) = K$. Then, we have $s \in M_i^{-1}M_i(F_0(m))$ for all $i \in I$. There exists $v_i \in F_0(m) = L \cap (L-K)\Sigma^{\geq m}$ such that $M_i(s) = M_i(v_i)$. Since $v_i \in L \cap (L-K)\Sigma^{\geq m}$, we can write $v_i = t_i u_i \in L - K$, where $t_i \in L - K$ and $|u_i| \geq m$. We have $v_i \in M_i^{-1}M_i(s) \cap L$ and $v_i \notin K\Sigma^{\leq m}$ for all $i \in I$. Thus, (L, K) is not conjunctive-codiagnosable.

($\Leftarrow$) We assume that there exists $m \in \mathbb{N}$ such that $H_1(m) = \emptyset$. Consider any $s \in K = H_0(m)$. Since $H_1(m) = \emptyset$ and $s \in H_0(m)$, there exists $i \in I$ such that $s \notin M_i^{-1}M_i(F_0(m))$. It follows that $M_i(s) \notin M_i(F_0(m))$. For any $u \in M_i^{-1}M_i(s) \cap L$, we have $M_i(u) = M_i(s) \notin M_i(F_0(m))$, which implies that $u \notin F_0(m) = L \cap (L-K)\Sigma^{\geq m}$, i.e., $u \in K\Sigma^{\leq m}$. Thus, (L, K) is conjunctive-codiagnosable. ∎

We can prove the following theorem in a similar way to Theorem 3.

Theorem 4: The pair (L, K) of languages is disjunctive-codiagnosable if and only if there exists $m \in \mathbb{N}$ such that $F_1(m) = \emptyset$.

The following corollary follows from Theorems 3 and 4.

Corollary 1: The pair (L, K) of languages is 0-inference diagnosable if and only if it is disjunctive-codiagnosable or conjunctive-codiagnosable.

Theorem 5: The pair (L, K) of languages is conditionally conjunctive-codiagnosable if and only if there exists $m \in \mathbb{N}$ such that $H_2(m) = \emptyset$.

Proof: ($\Rightarrow$) We prove the contrapositive. We suppose that $H_2(m) \neq \emptyset$ for any $m \in \mathbb{N}$. We consider any $s \in H_2(m) \subseteq H_0(m) = K$. Since $s \in H_2(m)$, we have $s \in M_i^{-1}M_i(F_1(m))$ for all $i \in I$. There exists $u_i \in F_1(m) \subseteq F_0(m) = L \cap (L-K)\Sigma^{\geq m}$ such that $M_i(s) = M_i(u_i)$. It follows that $u_i \in M_i^{-1}M_i(s) \cap L \cap (L-K)\Sigma^{\geq m}$ for all $i \in I$. Moreover, since $u_i \in F_1(m)$, we have $u_i \in M_j^{-1}M_j(H_0(m))$ for all $j \in I$. There exists $v_{ij} \in H_0(m) = K$ such that $M_j(u_i) = M_j(v_{ij})$. It follows that $v_{ij} \in M_j^{-1}M_j(u_i) \cap L$ and $v_{ij} \notin L - K$ for all $j \in I$. Thus, (L, K) is not conditionally conjunctive-codiagnosable.

($\Leftarrow$) We prove the contrapositive. We suppose that (L, K) is not conditionally conjunctive-codiagnosable. Then, for any $m \in \mathbb{N}$, there exists $s \in K = H_0(m)$ such that

$$\forall i \in I, \exists u_i \in M_i^{-1}M_i(s) \cap L \cap (L-K)\Sigma^{\geq m},$$
$$\forall j \in I : M_j^{-1}M_j(u_i) \cap L \not\subseteq L - K.$$

Since $s \in H_0(m)$ and $s \in M_i^{-1}M_i(u_i) \subseteq M_i^{-1}M_i(L \cap (L-K)\Sigma^{\geq m}) = M_i^{-1}M_i(F_0(m))$ for all $i \in I$, we have $s \in H_1(m)$. In addition, for each $j \in I$, since $M_j^{-1}M_j(u_i) \cap L \not\subseteq L - K$, there exists $v_{ij} \in M_j^{-1}M_j(u_i) \cap K = M_j^{-1}M_j(u_i) \cap H_0(m)$. Since $u_i \in L \cap (L-K)\Sigma^{\geq m} = F_0(m)$ and $u_i \in M_j^{-1}M_j(v_{ij}) \subseteq M_j^{-1}M_j(H_0(m))$ for all $j \in I$, we have $u_i \in F_1(m)$. It follows that $s \in M_i^{-1}M_i(u_i) \subseteq M_i^{-1}M_i(F_1(m))$ for all $i \in I$, which implies together with $s \in H_1(m)$ that $s \in H_2(m) \neq \emptyset$. ∎

We can prove the following theorem in a similar way to Theorem 5.

Theorem 6: The pair (L, K) of languages is conditionally disjunctive-codiagnosable if and only if there exists $m \in \mathbb{N}$ such that $F_2(m) = \emptyset$.

The following corollary follows from Theorems 5 and 6.

Corollary 2: The pair (L, K) of languages is 1-inference diagnosable if and only if it is conditionally disjunctive-codiagnosable or conditionally conjunctive-codiagnosable.

Moreover, we show that N-inference diagnosability is weaker than N-inference $\vee$-diagnosability introduced in [9]. We need the following lemma.

Lemma 4: For any $k, m \in \mathbb{N}$, $F_k(m) \subseteq F_k(0)$ and $H_k(m) \subseteq H_k(0)$ hold.

Proof: We consider any $m \in \mathbb{N}$. We use induction to prove that $F_k(m) \subseteq F_k(0)$ and $H_k(m) \subseteq H_k(0)$ for any $k \in \mathbb{N}$.

For the base step, we have

$$F_0(m) = L \cap (L-K)\Sigma^{\geq m} \subseteq L \cap (L-K)\Sigma^{\geq 0} = F_0(0)$$

and

$$H_0(m) = K = H_0(0).$$

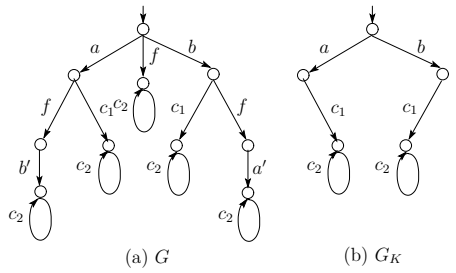


Fig. 3. Automata G and G_K of Theorem 7.

For the induction step, we suppose that $F_k(m) \subseteq F_k(0)$ and $H_k(m) \subseteq H_k(0)$. Then, we have

$$\begin{aligned} F_{k+1}(m) &= F_k(m) \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(H_k(m)) \right) \\ &\subseteq F_k(0) \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(H_k(0)) \right) \\ &= F_{k+1}(0). \end{aligned}$$

Similarly, we have $H_{k+1}(m) \subseteq H_{k+1}(0)$. ■

Theorem 7: For any $N \in \mathbb{N}$, if the pair (L, K) of languages is N-inference $\vee$-diagnosable, then it is N-inference diagnosable. The converse need not hold.

Proof: Since (L, K) is N-inference $\vee$-diagnosable, there exists $m \in \mathbb{N}$ such that $F_{N+1}(0) \cap F_0(m) = \emptyset$. By Lemma 4, we have

$$F_{N+1}(m) = F_{N+1}(m) \cap F_0(m) \subseteq F_{N+1}(0) \cap F_0(m) = \emptyset,$$

which implies that (L, K) is N-inference diagnosable.

To show that the converse need not hold, we consider a plant modeled by the finite automaton G shown in Fig. 3(a), which is taken from [24]. Let $n = 2$, $\Delta_1 = \{a, a', c\}$, $\Delta_2 = \{b, b', c\}$, and

$$M_1(\sigma) = \begin{cases} c, & \text{if } \sigma \in \{c_1, c_2\} \\ \sigma, & \text{if } \sigma \in \{a, a'\} \\ \varepsilon, & \text{otherwise,} \end{cases}$$

$$M_2(\sigma) = \begin{cases} c, & \text{if } \sigma \in \{c_1, c_2\} \\ \sigma, & \text{if } \sigma \in \{b, b'\} \\ \varepsilon, & \text{otherwise.} \end{cases}$$

We assume that $K \subseteq L$ is generated by the finite automaton G_K shown in Fig. 3(b).

As shown in Example 5 of [24], (L, K) is conditionally conjunctive-codiagnosable but not conditionally disjunctive-codiagnosable. By Corollary 2, it is 1-inferece diagnosable. However, since 1-inferece $\vee$-diagnosability is stronger than conditionally disjunctive-codiagnosability [9], (L, K) is not 1-inferece $\vee$-diagnosable. ■

Remark 4: The notion of decentralized diagnosability was introduced to study a distributed diagnosis problem and its undecidability property was established in [14]. It was shown in [17] that, for any $N \in \mathbb{N}$, N-inference $\vee$-diagnosability is strictly stronger than decentralized diagnosability. Analogously, we can show that N-inference diagnosability is also strictly stronger than decentralized diagnosability.

VII. Verification of N-Inference Diagnosability

In this section, we establish the decidability of checking N-inference diagnosability for a pair (L, K) of regular languages. By Corollary 1, 0-inference diagnosability is the same as disjunctive-codiagnosability or conjunctive-codiagnosability, and their verification has already been shown to be decidable. So we consider the case of $N \geq 1$. For simplicity of presentation, we consider the case of two local diagnosers, i.e., $I = \{1, 2\}$. The results continue to hold for an arbitrary number of local diagnosers.

Violation of N-inference diagnosability of (L, K) requires that for any $m \in \mathbb{N}$, $F_{N+1}(m) \neq \emptyset$ and $H_{N+1}(m) \neq \emptyset$. Our key insight is to show that the nonemptiness of $F_{N+1}(m)$ and $H_{N+1}(m)$ can be characterized as the existence of certain $(2N + 3)$ traces in L.

Proposition 1: Consider the pair (L, K) of languages and any $m \in \mathbb{N}$.

1) $F_{N+1}(m) \neq \emptyset$ if and only if there exist $(2N+3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ such that
   - $s_0 \in F_0(m)$,
   - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\}$:
   $$s_{ii'} \in \begin{cases} H_0(m), & \text{if } i' \text{ is an even number} \\ F_0(m), & \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i \in I : M_i(s_0) = M_i(s_{i0})$,
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_2(s_{1i'}) = M_2(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(s_{1i'}) = M_1(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_1(s_{2i'}) = M_1(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(s_{2i'}) = M_2(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$
2) $H_{N+1}(m) \neq \emptyset$ if and only if there exist $(2N+3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ such that
   - $s_0 \in H_0(m)$,
   - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\}$:
   $$s_{ii'} \in \begin{cases} F_0(m), & \text{if } i' \text{ is an even number} \\ H_0(m), & \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i \in I : M_i(s_0) = M_i(s_{i0})$,
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_2(s_{1i'}) = M_2(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(s_{1i'}) = M_1(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_1(s_{2i'}) = M_1(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(s_{2i'}) = M_2(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$

Proof: We only prove the first part. The second part can be proved analogously.

($\Rightarrow$) Since $F_{N+1}(m) \neq \emptyset$, we can pick up $s_0 \in F_{N+1}(m)$. We have $s_0 \in F_{N+1}(m) \subseteq F_0(m)$. Since

$$s_0 \in F_{N+1}(m) \subseteq \bigcap_{i\in I} M_i^{-1} M_i(H_N(m)),$$

there exist $s_{10}, s_{20} \in H_N(m) \subseteq H_0(m)$ such that $M_1(s_0) = M_1(s_{10})$ and $M_2(s_0) = M_2(s_{20})$, respectively. Also, since

$$s_{10}, s_{20} \in H_N(m) \subseteq \bigcap_{i\in I} M_i^{-1} M_i(F_{N-1}(m)),$$

there exist $s_{11}, s_{21} \in F_{N-1}(m) \subseteq F_0(m)$ such that $M_2(s_{10}) = M_2(s_{11})$ and $M_1(s_{20}) = M_1(s_{21})$, respectively. In addition, since

$$s_{11}, s_{21} \in F_{N-1}(m) \subseteq \bigcap_{i\in I} M_i^{-1} M_i(H_{N-2}(m)),$$

there exist $s_{12}, s_{22} \in H_{N-2}(m) \subseteq H_0(m)$ such that $M_1(s_{11}) = M_1(s_{12})$ and $M_2(s_{21}) = M_2(s_{22})$, respectively. By repeating this procedure, we can obtain $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ that satisfy the five conditions of the first part of the lemma.

($\Leftarrow$) We consider $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ that satisfy the five conditions of the first part of the lemma. We prove $F_{N+1}(m) \neq \emptyset$ by showing that $s_0 \in F_{N+1}(m)$.

First, we show that $s_0 \in F_1(m)$ and

$$\forall i \in I, \forall i' \in \{0, 1, \ldots, N-1\} : \\ s_{ii'} \in \begin{cases} H_1(m), & \text{if } i' \text{ is an even number} \\ F_1(m), & \text{if } i' \text{ is an odd number.} \end{cases}$$

Since $s_0 \in F_0(m)$, $M_i(s_0) = M_i(s_{i0})$, and $s_{i0} \in H_0(m)$ for each $i \in \{1, 2\}$, we have $s_0 \in F_1(m)$. In addition, for s_{10}, since $M_1(s_{10}) = M_1(s_0)$, $M_2(s_{10}) = M_2(s_{11})$, $s_{10} \in H_0(m)$, and $s_0, s_{11} \in F_0(m)$, we have $s_{10} \in H_1(m)$. Analogously, we have $s_{20} \in H_1(m)$. Furthermore, for $s_{1i'}$ ($i' \in \{1, 2, \ldots, N-1\}$), if i' is an even number, then $M_1(s_{1i'}) = M_1(s_{1(i'-1)})$, $M_2(s_{1i'}) = M_2(s_{1(i'+1)})$, $s_{1i'} \in H_0(m)$, and $s_{1(i'-1)}, s_{1(i'+1)} \in F_0(m)$, which implies that $s_{1i'} \in H_1(m)$. If i' is an odd number, then $M_2(s_{1i'}) = M_2(s_{1(i'-1)})$, $M_1(s_{1i'}) = M_1(s_{1(i'+1)})$, $s_{1i'} \in F_0(m)$, and $s_{1(i'-1)}, s_{1(i'+1)} \in H_0(m)$, which implies that $s_{1i'} \in F_1(m)$. Analogously, we have

$$s_{2i'} \in \begin{cases} H_1(m), & \text{if } i' \text{ is an even number} \\ F_1(m), & \text{if } i' \text{ is an odd number.} \end{cases}$$

In a similar way, we can show that $s_0 \in F_2(m)$ and

$$\forall i \in I, \forall i' \in \{0, 1, \ldots, N-2\} : \\ s_{ii'} \in \begin{cases} H_2(m), & \text{if } i' \text{ is an even number} \\ F_2(m), & \text{if } i' \text{ is an odd number.} \end{cases}$$

By repeating the above procedure, we can obtain $s_0 \in F_N(m)$ and $s_{10}, s_{20} \in H_N(m)$. Since $M_i(s_0) = M_i(s_{i0})$ for each $i \in \{1, 2\}$, we have $s_0 \in F_{N+1}(m)$. ∎

Note that from the proof of Proposition 1, in case of $F_{N+1}(m) \neq \emptyset$, for each i, $s_{ii'} \in H_{N-i'}(m)$ for even i' and $s_{ii'} \in F_{N-i'}(s)$ for odd i'. The dual holds when $H_{N+1}(m) \neq \emptyset$: for each i, $s_{ii'} \in F_{N-i'}(m)$ for even i' and $s_{ii'} \in H_{N-i'}(m)$ for odd i'. In other words, lower-indexed traces are members of higher-indexed, smaller-sized, failure/nonfailure sets.

We develop a method for verifying N-inference diagnosability based on Proposition 1. We first test the nonemptiness of $F_{N+1}(m)$ for any $m \in \mathbb{N}$. (The nonemptiness of $H_{N+1}(m)$ for any $m \in \mathbb{N}$ can be checked similarly.) For this, we construct an automaton that generates the $(2N+3)$ traces satisfying the last three of the five conditions of the first part of Proposition 1. (Note that these conditions do not involve m which is an unknown parameter.) We begin by refining the plant model with respect to the nonfailure specification model. Let $G = (Q, \Sigma, \delta, q_0, Q)$ be the finite plant model with $L(G) = L$ and $G_K = (Q_K, \Sigma, \delta_K, q_{K,0}, Q_K)$ be a finite generator of the nonfailure specification language, i.e., $L(G_K) = K$. We augment the generator G_K by adding a dump state $d \notin Q_K$. Formally, the augmented generator is defined as $\tilde{G}_K = (\tilde{Q}_K, \Sigma, \tilde{\delta}_K, q_{K,0}, \tilde{Q}_K)$, where $\tilde{Q}_K = Q_K \cup \{d\}$, and the state transition function $\tilde{\delta}_K : \tilde{Q}_K \times \Sigma \to \tilde{Q}_K$ is defined as

$$\tilde{\delta}_K(\tilde{q}_K, \sigma) = \begin{cases} \delta_K(\tilde{q}_K, \sigma), & \text{if } \tilde{q}_K \in Q_K \wedge \delta_K(\tilde{q}_K, \sigma)! \\ d, & \text{otherwise.} \end{cases}$$

Then, we have $L(\tilde{G}_K) = \Sigma^*$. The refinement of the plant with respect to the specification is obtained by taking the synchronous product [7]: $G \parallel \tilde{G}_K = (Q \times \tilde{Q}_K, \Sigma, \xi, (q_0, q_{K,0}), Q \times \tilde{Q}_K)$. Then, we have $L(G \| \tilde{G}_K) = L(G) \cap L(\tilde{G}_K) = L \cap \Sigma^* = L$. For each $s \in L$, $s \in L - K$ if and only if $G \| \tilde{G}_K$ reaches a state whose second coordinate is the dump state d by the execution of s.

The index set of $(2N+3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ of Proposition 1 is denoted by

$$I_N = \{0, 10, 11, \ldots, 1N, 20, 21, \ldots, 2N\}.$$

Among these $(2N+3)$ traces, $s_0, s_{11}, s_{13}, \ldots, s_{1l}, s_{21}, s_{23}, \ldots, s_{2l} \in L - K$ are failure traces, where

$$l = \begin{cases} N-1, & \text{if } N \text{ is an even number} \\ N, & \text{if } N \text{ is an odd number,} \end{cases}$$

and their index set is denoted by

$$I_{FN} = \{0, 11, 13, \ldots, 1l, 21, 23, \ldots, 2l\}.$$

Having introduced the above notations for the two index sets, we are ready to build a generator of all traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ that satisfy the last three of the five conditions of the first part of Proposition 1. We define

$$T_{FN} = (R_{FN}, \Sigma_{FN}, \alpha_{FN}, r_{FN,0}, R_{FN,m})$$

as follows:

- The state set R_{FN} is defined as $R_{FN} = (Q \times \tilde{Q}_K) \times Q_{10} \times Q_{11} \times \cdots \times Q_{1N} \times Q_{20} \times Q_{21} \times \cdots \times Q_{2N}$, where

$$Q_{ii'} = \begin{cases} Q_K, & \text{if } i' \text{ is an even number} \\ Q \times \tilde{Q}_K, & \text{if } i' \text{ is an odd number} \end{cases}$$

for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.

- The initial state $r_{FN,0} \in R_{FN}$ is given as

$$r_{FN,0} = ((q_0, q_{K,0}), q_{10,0}, q_{11,0}, \ldots, q_{1N,0}, q_{20,0}, q_{21,0}, \ldots, q_{2N,0}),$$

where

$$q_{ii',0} = \begin{cases} q_{K,0}, & \text{if } i' \text{ is an even number} \\ (q_0, q_{K,0}), & \text{if } i' \text{ is an odd number} \end{cases}$$

for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.
- The set $R_{FN,m}$ of marked states is defined as

$$R_{FN,m} = \{r_{FN} \in R_{FN} \mid \forall i_{FN} \in I_{FN} : r_{FN}(i_{FN}) \in Q \times \{d\}\},$$

where, for each

$$r_{FN} = ((q, q_K), q_{10}, q_{11}, \ldots, q_{1N}, q_{20}, q_{21}, \ldots, q_{2N})$$

in R_{FN}, we let $r_{FN}(0) = (q, q_K)$ and $r_{FN}(ii') = q_{ii'}$ for each $i \in I$ and each $i' \in \{0, 1, \ldots, N\}$.
- The event set Σ_{FN} is defined as

$$\Sigma_{FN} = \underbrace{(\Sigma \cup \{\varepsilon\}) \times (\Sigma \cup \{\varepsilon\}) \times \cdots \times (\Sigma \cup \{\varepsilon\})}_{(2N+3) \text{ times}} - \{(\varepsilon, \varepsilon, \ldots, \varepsilon)\}.$$

- For each $r_{FN} = ((q, q_K), q_{10}, \ldots, q_{1N}, q_{20}, \ldots, q_{2N})$ and each $\sigma_{FN} = (\sigma, \sigma_{10}, \ldots, \sigma_{1N}, \sigma_{20}, \ldots, \sigma_{2N})$, $\alpha_{FN}(r_{FN}, \sigma_{FN})!$ if the following five conditions are satisfied:
  - $\sigma \neq \varepsilon \Rightarrow \xi((q, q_K), \sigma)!$,
  - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\} : \sigma_{ii'} \neq \varepsilon \Rightarrow$

$$\begin{cases} \delta_K(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an even number} \\ \xi(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an odd number,} \end{cases}$$

  - $\forall i \in I : M_i(\sigma) = M_i(\sigma_{i0})$,
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:

$$\begin{cases} M_2(\sigma_{1i'}) = M_2(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(\sigma_{1i'}) = M_1(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$

  - $\forall i' \in \{0, 1, \ldots, N-1\}$:

$$\begin{cases} M_1(\sigma_{2i'}) = M_1(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(\sigma_{2i'}) = M_2(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$

If $\alpha_{FN}(r_{FN}, \sigma_{FN})!$, then

$$\alpha_{FN}(r_{FN}, \sigma_{FN}) = ((q', q'_K), q'_{10}, q'_{11}, \ldots, q'_{1N}, q'_{20}, q'_{21}, \ldots, q'_{2N}),$$

where

$$(q', q'_K) = \begin{cases} \xi((q, q_K), \sigma), & \text{if } \sigma \neq \varepsilon \\ (q, q_K), & \text{otherwise} \end{cases}$$

and

$$q'_{ii'} = \begin{cases} \delta_K(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an even number}] \\ \xi(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an odd number}] \\ q_{ii'}, \\ \quad \text{otherwise.} \end{cases}$$

For each $\sigma_{FN} = (\sigma, \sigma_{10}, \sigma_{11}, \ldots, \sigma_{1N}, \sigma_{20}, \sigma_{21}, \ldots, \sigma_{2N})$ in Σ_{FN}, we let $\sigma_{FN}(0) = \sigma$ and $\sigma_{FN}(ii') = \sigma_{ii'}$ for each $i \in I$ and each $i' \in \{0, 1, \ldots, N\}$. Furthermore, for a trace $s_{FN} = \sigma_{FN,1}\sigma_{FN,2}\cdots\sigma_{FN,l} \in \Sigma^*_{FN}$ ($l \geq 1$), we let $s_{FN}(i_N) = \sigma_{FN,1}(i_N)\sigma_{FN,2}(i_N)\cdots\sigma_{FN,l}(i_N)$ for each $i_N \in I_N$. In addition, let $s_{FN}(i_N) = \varepsilon$ for each $i_N \in I_N$ if $s_{FN} = \varepsilon$.

From the construction of T_{FN}, the following proposition is obtained (the proof is omitted).

Proposition 2: Consider any $(2N + 3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ such that $s_{i_N} \in K$ for each $i_N \in I_N - I_{FN}$. Then these $(2N + 3)$ traces satisfy the last three of the five conditions of the first part of Proposition 1 if and only if there exists $s_{FN} \in L(T_{FN})$ such that $s_{FN}(i_N) = s_{i_N}$ for each $i_N \in I_N$.

Now that we have constructed a generator of all $(2N + 3)$ traces that satisfy the last three of the five conditions of the first part of Proposition 1, we next extend the construction that can be used to satisfy all the five conditions. Since the parameter $m \in \mathbb{N}$ is not known a priori, we must check $F_{N+1}(m) \neq \emptyset$ for *any* m (to help establish non-diagnosability). m represents the number of steps of post-fault executions, and in fact T_{FN} accepts traces s_{FN} such that $s_{FN}(0) \in F_{N+1}(m)$ with $m = 0$. To allow an arbitrary number of post-fault executions (equivalently, an arbitrary value of m), the post-fault extensions in T_{FN} must visit cycles leading to an arbitrary growth in the number m of post-fault executions. Note that a single cycle may not elongate all $(2l + 1)$ failure traces $s_{i_{FN}}$ ($i_{FN} = 0, 11, 13, \ldots, 1l, 21, 23, \ldots, 2l$) because some of these trace elements may witness only ε-transitions along that cycle. Hence a multitude of cycles may need to be executed sequentially to elongate all the $(2l + 1)$ failure trace elements. To keep track of which cycles elongate which of the trace elements (by executing at least one non-ε-transition), we collapse all the maximal strongly connected components (max-SCCs) of T_{FN} into individual nodes, labeling those nodes with the trace elements that witness at least one non-ε-transition in the corresponding max-SCCs, and build a nondeterministic acyclic automaton over the node set of max-SCCs, $\mathcal{T}_{FN} = (V_{FN}, \Sigma_{FN}, \beta_{FN}, V_{FN,0}, V_{FN,m})$, as follows:

- The state set V_{FN} is defined as $V_{FN} = \{V_{FN,0}, V_{FN,1}, \ldots, V_{FN,|V_{FN}|-1}\}$, where, for any $k \in \{0, 1, \ldots, |V_{FN}| - 1\}$, $V_{FN,k}$ is a max-SCC of T_{FN}. Without loss of generality, we assume that $r_{FN,0} \in V_{FN,0}$.
- The set $V_{FN,m}$ of marked states is defined as $V_{FN,m} = \{V_{FN,k} \in V_{FN} \mid V_{FN,k} \cap R_{FN,m} \neq \emptyset\}$.
- The nondeterministic state transition function β_{FN} :

$V_{FN} \times \Sigma_{FN} \to 2^{V_{FN}}$ is defined as

$$\begin{aligned}&\beta_{FN}(V_{FN,k}, \sigma_{FN})\\ &\quad = \{V_{FN,k'} \in V_{FN} \mid k \neq k'\\ &\qquad \wedge [\exists r_{FN} \in V_{FN,k}, \exists r'_{FN} \in V_{FN,k'} :\\ &\qquad\quad \alpha_{FN}(r_{FN}, \sigma_{FN}) = r'_{FN}]\}\end{aligned}$$

for each $V_{FN,k} \in V_{FN}$ and each $\sigma_{FN} \in \Sigma_{FN}$.

A *labeling* function $J_{FN} : V_{FN} \to 2^{I_{FN}}$ is defined as

$$\begin{aligned}&J_{FN}(V_{FN,k})\\ &\quad = \{i_{FN} \in I_{FN} \mid [\exists r_{FN} \in V_{FN,k} :\\ &\qquad r_{FN}(i_{FN}) \in Q \times \{d\}]\\ &\qquad \wedge [\exists r_{FN}, r'_{FN} \in V_{FN,k}, \exists \sigma_{FN} \in \Sigma_{FN} :\\ &\qquad\quad \alpha_{FN}(r_{FN}, \sigma_{FN}) = r'_{FN} \wedge \sigma_{FN}(i_{FN}) \neq \varepsilon]\}\end{aligned}$$

for each $V_{FN,k} \in V_{FN}$. Then by the construction, $i_{FN} \in J_{FN}(V_{FN,k})$ means that $s_{i_{FN}}$ can be extended to an arbitrarily long failure trace in a max-SCC $V_{FN,k}$. Therefore, we can test whether $F_{N+1}(m) \neq \emptyset$ for any $m \in \mathbb{N}$ by verifying the existence of a path $V_{FN,0} = V_{FN,k_0} \overset{\sigma_{FN}^{(k_0)}}{\to} V_{FN,k_1} \overset{\sigma_{FN}^{(k_1)}}{\to} \cdots \overset{\sigma_{FN}^{(k_{h-1})}}{\to} V_{FN,k_h} \in V_{FN,m}$ in $\mathcal{T}_{FN}$ such that the union of labels of V_{FN,k_j} $(j = 0, 1, \ldots, h)$ is equal to the index set I_{FN} of all $(2l+1)$ failure traces. This is formalized by the following proposition. (The proof is found in the appendix.)

Proposition 3: Consider the pair (L, K) of regular languages. Then, $F_{N+1}(m) \neq \emptyset$ for any $m \in \mathbb{N}$ if and only if there exists a path $V_{FN,0} = V_{FN,k_0} \overset{\sigma_{FN}^{(k_0)}}{\to} V_{FN,k_1} \overset{\sigma_{FN}^{(k_1)}}{\to} \cdots \overset{\sigma_{FN}^{(k_{h-1})}}{\to} V_{FN,k_h} \in V_{FN,m}$ in the acyclic automaton $\mathcal{T}_{FN}$ such that

$$\bigcup_{k \in \{k_0, k_1, \ldots, k_h\}} J_{FN}(V_{FN,k}) = I_{FN}. \tag{11}$$

Dually to check the nonemptiness of $H_{N+1}(m)$ for any $m \in \mathbb{N}$, we construct T_{HN} and $\mathcal{T}_{HN}$ dual to T_{FN} and $\mathcal{T}_{FN}$, respectively. Among the $(2N+3)$ traces of the second part of Proposition 1, $s_{10}, s_{12}, \ldots, s_{1l}, s_{20}, s_{22}, \ldots, s_{2l} \in L - K$ are failure traces, where

$$l = \begin{cases} N, & \text{if } N \text{ is an even number} \\ N-1, & \text{if } N \text{ is an odd number,} \end{cases}$$

and their index set is denoted by

$$I_{HN} = \{10, 12, \ldots, 1l, 20, 22, \ldots, 2l\}.$$

Then we define $T_{HN} = (R_{HN}, \Sigma_{HN}, \alpha_{HN}, r_{HN,0}, R_{HN,m})$ as follows:

- The state set R_{HN} is defined as $R_{HN} = Q_K \times Q_{10} \times Q_{11} \times \cdots \times Q_{1N} \times Q_{20} \times Q_{21} \times \cdots \times Q_{2N}$, where
$$Q_{ii'} = \begin{cases} Q \times \tilde{Q}_K, & \text{if } i' \text{ is an even number} \\ Q_K, & \text{if } i' \text{ is an odd number} \end{cases}$$
for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.
- The initial state $r_{HN,0} \in R_{HN}$ is given as
$$\begin{aligned} r_{HN,0} \;=\; &(q_{K,0}, q_{10,0}, q_{11,0}, \ldots, q_{1N,0},\\ &q_{20,0}, q_{21,0}, \ldots, q_{2N,0}), \end{aligned}$$
where
$$q_{ii',0} = \begin{cases} (q_0, q_{K,0}), & \text{if } i' \text{ is an even number} \\ q_{K,0}, & \text{if } i' \text{ is an odd number} \end{cases}$$
for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.
- The set $R_{HN,m}$ of marked states is defined as
$$\begin{aligned} R_{HN,m} \;=\; &\{r_{HN} \in R_{HN} \mid \forall i_{HN} \in I_{HN} :\\ &r_{HN}(i_{HN}) \in Q \times \{d\}\}, \end{aligned}$$
where, for each $r_{HN} \in R_{HN}$, $r_{HN}(i_N)$ $(i_N \in I_N)$ is defined in the same way as $r_{FN}(i_N)$, where $r_{FN} \in R_{FN}$.
- The event set Σ_{HN} is defined as $\Sigma_{HN} = \Sigma_{FN}$.
- For each $r_{HN} = (q_K, q_{10}, \ldots, q_{1N}, q_{20}, \ldots, q_{2N})$ and each $\sigma_{HN} = (\sigma, \sigma_{10}, \ldots, \sigma_{1N}, \sigma_{20}, \ldots, \sigma_{2N})$, $\alpha_{HN}(r_{HN}, \sigma_{HN})!$ if the following five conditions are satisfied:
  - $\sigma \neq \varepsilon \Rightarrow \delta_K(q_K, \sigma)!$,
  - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\} : \sigma_{ii'} \neq \varepsilon \Rightarrow$
$$\begin{cases} \xi(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an even number} \\ \delta_K(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i \in I : M_i(\sigma) = M_i(\sigma_{i0})$,
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
$$\begin{cases} M_2(\sigma_{1i'}) = M_2(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(\sigma_{1i'}) = M_1(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
$$\begin{cases} M_1(\sigma_{2i'}) = M_1(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(\sigma_{2i'}) = M_2(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$

  If $\alpha_{HN}(r_{HN}, \sigma_{HN})!$, then
$$\begin{aligned}&\alpha_{HN}(r_{HN}, \sigma_{HN})\\ &\quad = (q'_K, q'_{10}, q'_{11}, \ldots, q'_{1N}, q'_{20}, q'_{21}, \ldots, q'_{2N}),\end{aligned}$$
  where
$$q'_K = \begin{cases} \delta_K(q_K, \sigma), & \text{if } \sigma \neq \varepsilon \\ q_K, & \text{otherwise} \end{cases}$$
  and
$$q'_{ii'} = \begin{cases} \xi(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an even number}] \\ \delta_K(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an odd number}] \\ q_{ii'}, \\ \quad \text{otherwise.} \end{cases}$$

For each $\sigma_{HN} \in \Sigma_{HN}$ and $s_{HN} \in \Sigma_{HN}^*$, $\sigma_{HN}(i_N)$ and $s_{HN}(i_N)$ $(i_N \in I_N)$ are defined in the same way as $\sigma_{FN}(i_N)$ and $s_{FN}(i_N)$, respectively, where $\sigma_{FN} \in \Sigma_{FN}$ and $s_{FN} \in \Sigma_{FN}^*$.

From the construction of T_{HN}, the following proposition is obtained (the proof is omitted).

Proposition 4: Consider any $(2N+3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ such that

$s_{i_N} \in K$ for each $i_N \in I_N - I_{HN}$. Then these $(2N+3)$ traces satisfy the last three of the five conditions of the second part of Proposition 1 if and only if there exists $s_{HN} \in L(T_{HN})$ such that $s_{HN}(i_N) = s_{i_N}$ for each $i_N \in I_N$.

Using T_{HN}, we construct a nondeterministic acyclic automaton $\mathcal{T}_{HN} = (V_{HN}, \Sigma_{HN}, \beta_{HN}, V_{HN,0}, V_{HN,m})$ over the node set of max-SCCs of T_{HN} as follows:

- The state set V_{HN} is defined as $V_{HN} = \{V_{HN,0}, V_{HN,1}, \ldots, V_{HN,|V_{HN}|-1}\}$, where, for any $k \in \{0,1,\ldots,|V_{HN}|-1\}$, $V_{HN,k}$ is a max-SCC of T_{HN}. Without loss of generality, we assume that $r_{HN,0} \in V_{HN,0}$.
- The set $V_{HN,m}$ of marked states is defined as $V_{HN,m} = \{V_{HN,k} \in V_{HN} \mid V_{HN,k} \cap R_{HN,m} \neq \emptyset\}$.
- The nondeterministic state transition function $\beta_{HN} : V_{HN} \times \Sigma_{HN} \to 2^{V_{HN}}$ is defined as

$$\begin{aligned}&\beta_{HN}(V_{HN,k}, \sigma_{HN})\\ &\quad = \{V_{HN,k'} \in V_{HN} \mid k \neq k'\\ &\qquad \wedge[\exists r_{HN} \in V_{HN,k}, \exists r'_{HN} \in V_{HN,k'} :\\ &\qquad\quad \alpha_{HN}(r_{HN}, \sigma_{HN}) = r'_{HN}]\}\end{aligned}$$

for each $V_{HN,k} \in V_{HN}$ and each $\sigma_{HN} \in \Sigma_{HN}$.

A labeling function $J_{HN} : V_{HN} \to 2^{I_{HN}}$ is defined as

$$\begin{aligned}&J_{HN}(V_{HN,k})\\ &\quad = \{i_{HN} \in I_{HN} \mid [\exists r_{HN} \in V_{HN,k} :\\ &\qquad r_{HN}(i_{HN}) \in Q \times \{d\}]\\ &\qquad \wedge[\exists r_{HN}, r'_{HN} \in V_{HN,k}, \exists \sigma_{HN} \in \Sigma_{HN} :\\ &\qquad\quad \alpha_{HN}(r_{HN}, \sigma_{HN}) = r'_{HN} \wedge \sigma_{HN}(i_{HN}) \neq \varepsilon]\}\end{aligned}$$

for each $V_{HN,k} \in V_{HN}$.

Similar to Proposition 3, we have the following result (the proof is similar and omitted).

Proposition 5: Consider the pair (L, K) of regular languages. Then, $H_{N+1}(m) \neq \emptyset$ for any $m \in \mathbb{N}$ if and only if there exists a path $V_{HN,0} = V_{HN,k_0} \overset{\sigma_{HN}^{(k_0)}}{\to} V_{HN,k_1} \overset{\sigma_{HN}^{(k_1)}}{\to} \cdots \overset{\sigma_{HN}^{(k_{h-1})}}{\to} V_{HN,k_h} \in V_{HN,m}$ in the acyclic automaton $\mathcal{T}_{HN}$ such that

$$\bigcup_{k \in \{k_0, k_1, \ldots, k_h\}} J_{HN}(V_{HN,k}) = I_{HN}. \tag{12}$$

Then we have the following theorem that can be used to decidably test N-inference diagnosability.

Theorem 8: The pair (L, K) of regular languages is not N-inference diagnosable if and only if

- there exists a path $V_{FN,0} = V_{FN,k_0} \overset{\sigma_{FN}^{(k_0)}}{\to} V_{FN,k_1} \overset{\sigma_{FN}^{(k_1)}}{\to} \cdots \overset{\sigma_{FN}^{(k_{h-1})}}{\to} V_{FN,k_h} \in V_{FN,m}$ in the acyclic automaton $\mathcal{T}_{FN}$ that satisfies (11), and
- there exists a path $V_{HN,0} = V_{HN,k_0} \overset{\sigma_{HN}^{(k_0)}}{\to} V_{HN,k_1} \overset{\sigma_{HN}^{(k_1)}}{\to} \cdots \overset{\sigma_{HN}^{(k_{h-1})}}{\to} V_{HN,k_h} \in V_{HN,m}$ in the acyclic automaton $\mathcal{T}_{HN}$ that satisfies (12).

Remark 5: To verify N-inference diagnosability of the pair (L, K) of regular languages using Theorem 8, we first need to construct the finite automata T_{FN} and T_{HN}. By the definitions of T_{FN} and T_{HN}, the complexity of constructing them is shown in Table III. Next, we construct the nondeterministic acyclic automata $\mathcal{T}_{FN}$ and $\mathcal{T}_{HN}$ by identifying all max-SCCs of T_{FN} and T_{HN}. This complexity is $O((|R_{FN}| + |R_{HN}|) \times |\Sigma|^{1+\sum_{k=0}^{N} n(n-1)^k})$, where

$$|R_{FN}| = \begin{cases} O(|Q|^{1+\sum_{k=1}^{N/2} n(n-1)^{2k-1}} \cdot |Q_K|^{1+\sum_{k=0}^{N} n(n-1)^k}), \\ \quad \text{if } N \text{ is an even number} \\ O(|Q|^{1+\sum_{k=1}^{(N+1)/2} n(n-1)^{2k-1}} \cdot |Q_K|^{1+\sum_{k=0}^{N} n(n-1)^k}), \\ \quad \text{if } N \text{ is an odd number,} \end{cases}$$

$$|R_{HN}| = \begin{cases} O(|Q|^{\sum_{k=0}^{N/2} n(n-1)^{2k}} \cdot |Q_K|^{1+\sum_{k=0}^{N} n(n-1)^k}), \\ \quad \text{if } N \text{ is an even number} \\ O(|Q|^{\sum_{k=0}^{(N-1)/2} n(n-1)^{2k}} \cdot |Q_K|^{1+\sum_{k=0}^{N} n(n-1)^k}), \\ \quad \text{if } N \text{ is an odd number.} \end{cases}$$

Then, N-inference diagnosability of (L, K) can be verified by exploring all paths of $\mathcal{T}_{FN}$ and $\mathcal{T}_{HN}$ that end with marked states. It should be further noted that the verification is an offline activity (and so more manageable), while, and as noted in Remark 2, online activity for making a diagnosis decision requires membership-checks in certain languages that are computed offline, plus certain minimization computations, and these are similarly also manageable.

The following example illustrates the verification result of Theorem 8.

Example 2: We revisit the setting of Example 1. We verify whether (L, K) is 2-inference diagnosable using Theorem 8.

First, we construct the finite automaton T_{F2} to verify whether there exists $m \in \mathbb{N}$ such that $F_3(m) = \emptyset$. A part of T_{F2} is shown in Fig. 4. Using T_{F2}, the acyclic automaton $\mathcal{T}_{F2}$ is constructed. The corresponding part of $\mathcal{T}_{F2}$ is shown in Fig. 5. Since

$$\begin{aligned}&((q_{17}, d), q_{K,14}, (q_{25}, d), q_{K,17}, q_{K,13}, (q_{24}, d), q_{K,16})\\ &\overset{(c,c,c,c,c,c,c)}{\to}\\ &((q_{17}, d), q_{K,14}, (q_{25}, d), q_{K,17}, q_{K,13}, (q_{24}, d), q_{K,16})\end{aligned}$$

in T_{F2}, we have

$$\begin{aligned}&J_{F2}(\{((q_{17}, d), q_{K,14}, (q_{25}, d), q_{K,17}, q_{K,13}, (q_{24}, d), q_{K,16})\})\\ &\qquad = \{0, 11, 21\} = I_{F2}.\end{aligned}$$

Thus, the path shown in Fig. 5 satisfies (11) in Proposition 3, which implies that $F_3(m) \neq \emptyset$ for any $m \in \mathbb{N}$. Similarly, we can verify by Proposition 5 that $H_3(m) \neq \emptyset$ for any $m \in \mathbb{N}$. Thus, by Theorem 8, we can conclude that (L, K) is not 2-inference diagnosable.

VIII. Conclusion

A framework for inference-based decentralized decision-making for fault detection has been proposed, which supports *multi-level inferencing* for *disjunctive* as well as *conjunctive* decision making. The notion of N-inference diagnosability was introduced and its necessity and sufficiency were established. A thorough comparison with the existing notions

TABLE III
COMPLEXITY FOR CONSTRUCTING AUTOMATA T_{FN} AND T_{HN}.

	Complexity of constructing T_{FN}
Arbitrary even N	$O(\|Q\|^{1+\sum_{k=1}^{N/2} n(n-1)^{2k-1}} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O(\|Q\|^{O(n(n-1)^{N-1})} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{O(n(n-1)^N)})$
Arbitrary odd N	$O(\|Q\|^{1+\sum_{k=1}^{(N+1)/2} n(n-1)^{2k-1}} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O((\|Q\| \cdot \|Q_K\| \cdot \|\Sigma\|)^{O(n(n-1)^N)})$
$N = 1$	$O(\|Q\|^{n^2-n+1} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{n^2+1})$
$N = 2$	$O(\|Q\|^{n^2-n+1} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{n^2+n(n-1)^2+1})$
	Complexity of constructing T_{HN}
Arbitrary even N	$O(\|Q\|^{\sum_{k=0}^{N/2} n(n-1)^{2k}} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O((\|Q\| \cdot \|Q_K\| \cdot \|\Sigma\|)^{O(n(n-1)^N)})$
Arbitrary odd N	$O(\|Q\|^{\sum_{k=0}^{(N-1)/2} n(n-1)^{2k}} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O(\|Q\|^{O(n(n-1)^{N-1})} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{O(n(n-1)^N)})$
$N = 1$	$O(\|Q\|^{n} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{n^2+1})$
$N = 2$	$O(\|Q\|^{n+n(n-1)^2} \cdot (\|Q_K\| \cdot \|\Sigma\|)^{n^2+n(n-1)^2+1})$

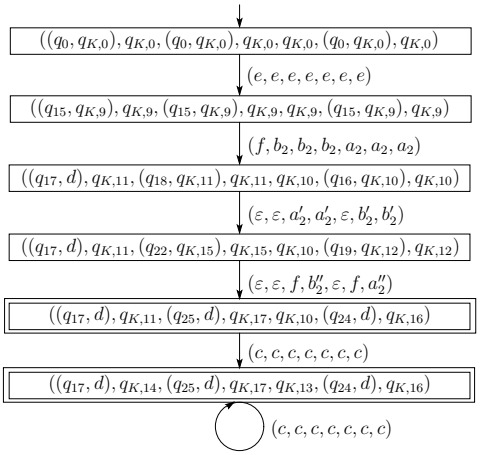


Fig. 4. A part of the automaton T_{F2}.

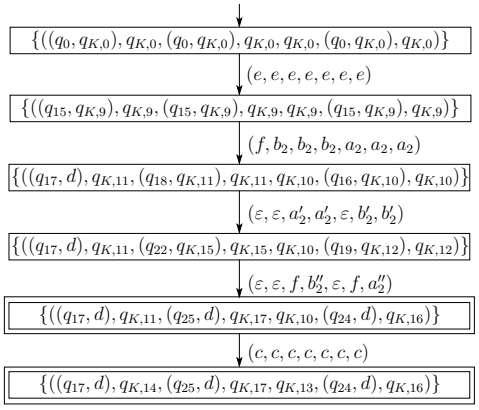


Fig. 5. A part of the automaton $\mathcal{T}_{F2}$.

of decentralized diagnosability properties was performed, and the generality of N-inference diagnosability was established. Properties of N-inference diagnosability were also explored. The paper also showed that N-inference diagnosability is decidable by presenting a verification algorithm. The paper also describes an online approach for implementing the proposed diagnosis scheme, which utilizes the offline computations of certain languages (see Remark 2). The complexity of the online diagnosis step is linear in the sizes of the offline computed languages. The complexity of the offline computed languages could be made more efficient through their structured representations, such as BDDs, and can be a subject of future research.

The paper opens up avenue for establishing similar frameworks for inference-based decentralized decision making for other tasks such as prognosis [5], [6], [18], which provide direction for future research within the proposed framework.

APPENDIX A
PROOF OF PROPOSITION 3

($\Rightarrow$) We consider any $m \in \mathbb{N}$ such that $m > |R_{FN}|$, where $|R_{FN}|$ denotes the number of states of T_{FN}. By the first part of Proposition 1, there exist $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ that satisfy the five conditions of the first part of Proposition 1.

Since $s_0 \in F_0(m) = L \cap (L-K)\Sigma^{\geq m}$, there exist $t_0 \in L-K$ and $u_0 \in \Sigma^{\geq m}$ such that $t_0 u_0 = s_0$. In addition, for each index $ii' \in I_{FN} - \{0\}$, since i' is an odd number, we have $s_{ii'} \in F_0(m)$. There exist $t_{ii'} \in L-K$ and $u_{ii'} \in \Sigma^{\geq m}$ such that $t_{ii'} u_{ii'} = s_{ii'}$. Moreover, for each index $ii' \in I_N - I_{FN}$, since i' is an even number, we have $s_{ii'} \in H_0(m) = K$.

By Proposition 2, there exists $s_{FN} := \sigma_{FN}^{(0)} \sigma_{FN}^{(1)} \cdots \sigma_{FN}^{(l-1)} \in L(T_{FN})$ ($l \geq 1$) such that $s_{FN}(i_N) = s_{i_N}$ for any $i_N \in I_N$. For each $i_{FN} \in I_{FN}$, there exists $l_{i_{FN}}$ ($0 \leq l_{i_{FN}} \leq l-1$) such that $t_{i_{FN}} = \sigma_{FN}^{(0)}(i_{FN}) \sigma_{FN}^{(1)}(i_{FN}) \cdots \sigma_{FN}^{(l_{i_{FN}})}(i_{FN})$ and $u_{i_{FN}} = \sigma_{FN}^{(l_{i_{FN}}+1)}(i_{FN}) \sigma_{FN}^{(l_{i_{FN}}+2)}(i_{FN}) \cdots \sigma_{FN}^{(l-1)}(i_{FN})$. We consider the path $r_{FN,0} = r_{FN}^{(0)} \xrightarrow{\sigma_{FN}^{(0)}} r_{FN}^{(1)} \xrightarrow{\sigma_{FN}^{(1)}} \cdots \xrightarrow{\sigma_{FN}^{(l-1)}} r_{FN}^{(l)}$ in T_{FN} obtained by executing s_{FN}. For each $i_{FN} \in I_{FN}$, since $|u_{i_{FN}}| \geq m > |R_{FN}|$, there exist $j_{i_{FN}}, h_{i_{FN}}, j'_{i_{FN}} \in \mathbb{N}$ ($l_{i_{FN}} < j_{i_{FN}} \leq h_{i_{FN}} < j'_{i_{FN}} \leq l$) such that $r_{FN}^{(j_{i_{FN}})} = r_{FN}^{(j'_{i_{FN}})}$ and $\sigma_{FN}^{(h_{i_{FN}})}(i_{FN}) \neq \varepsilon$. Since $t_{i_{FN}} \in L-K$ and $t_{i_{FN}} = \sigma_{FN}^{(0)}(i_{FN}) \sigma_{FN}^{(1)}(i_{FN}) \cdots \sigma_{FN}^{(l_{i_{FN}})}(i_{FN}) \in pr(\sigma_{FN}^{(0)}(i_{FN}) \sigma_{FN}^{(1)}(i_{FN}) \cdots \sigma_{FN}^{(h_{i_{FN}}-1)}(i_{FN}))$, we have

$$r_{FN}^{(h_{i_{FN}})}(i_{FN}) = \xi((q_0, q_{K,0}), \sigma_{FN}^{(0)}(i_{FN}) \sigma_{FN}^{(1)}(i_{FN}) \cdots \sigma_{FN}^{(h_{i_{FN}}-1)}(i_{FN})) \in Q \times \{d\}.$$

It follows that $r_{FN}^{(l)} \in R_{FN,m}$.

For some $a_0, a_1, \ldots, a_{h-1} \in \{0, 1, \ldots, l\}$ such that $a_0 < a_1 < \cdots < a_{h-1}$, there exists a path $V_{FN,0} = V_{FN,k_0} \xrightarrow{\sigma_{FN}^{(a_0)}}$

$V_{FN,k_1} \overset{\sigma_{FN}^{(a_1)}}{\to} \cdots \overset{\sigma_{FN}^{(a_{h-1})}}{\to} V_{FN,k_h} \in V_{FN,m}$ that satisfies $\{r_{FN}^{(0)},\ldots,r_{FN}^{(a_0)}\} \subseteq V_{FN,k_0}, \{r_{FN}^{(a_0+1)},\ldots,r_{FN}^{(a_1)}\} \subseteq V_{FN,k_1},\ldots,\{r_{FN}^{(a_{h-1}+1)},\ldots,r_{FN}^{(l)}\} \subseteq V_{FN,k_h}$ in the acyclic automaton $\mathcal{T}_{FN}$. For each $i_{FN} \in I_{FN}$, states $r_{FN}^{(j_{i_{FN}})} = r_{FN}^{(j'_{i_{FN}})}$, $r_{FN}^{(h_{i_{FN}})}$, and $r_{FN}^{(h_{i_{FN}}+1)}$ are included in the same max-SCC in V_{FN}, which implies that there exists V_{FN,k_j} ($j \in \{0,1,\ldots,h\}$) such that $i_{FN} \in J_{FN}(V_{FN,k_j})$. Hence, the path $V_{FN,0} = V_{FN,k_0} \overset{\sigma_{FN}^{(a_0)}}{\to} V_{FN,k_1} \overset{\sigma_{FN}^{(a_1)}}{\to} \cdots \overset{\sigma_{FN}^{(a_{h-1})}}{\to} V_{FN,k_h}$ satisfies (11).

($\Leftarrow$) In the acyclic automaton $\mathcal{T}_{FN}$, there exists a path $V_{FN,0} = V_{FN,k_0} \overset{\sigma_{FN}^{(k_0)}}{\to} V_{FN,k_1} \overset{\sigma_{FN}^{(k_1)}}{\to} \cdots \overset{\sigma_{FN}^{(k_{h-1})}}{\to} V_{FN,k_h} \in V_{FN,m}$ that satisfies (11). For each $i_{FN} \in I_{FN}$, there exists $V_{FN,k_{j_{i_{FN}}}}$ ($j_{i_{FN}} \in \{0,1,\ldots,h\}$) such that

$$\begin{aligned}&[\exists r_{FN} \in V_{FN,k_{j_{i_{FN}}}} : r_{FN}(i_{FN}) \in Q \times \{d\}]\\ &\quad \wedge [\exists r_{FN}, r'_{FN} \in V_{FN,k_{j_{i_{FN}}}}, \exists \sigma_{FN} \in \Sigma_{FN} :\\ &\qquad \alpha_{FN}(r_{FN},\sigma_{FN}) = r'_{FN} \wedge \sigma_{FN}(i_{FN}) \neq \varepsilon]. \quad (13)\end{aligned}$$

Then, there exists a path $r_{FN,0} = r_{FN}^{(0)} \overset{\sigma_{FN}^{(0)}}{\to} r_{FN}^{(1)} \overset{\sigma_{FN}^{(1)}}{\to} \cdots \overset{\sigma_{FN}^{(l-1)}}{\to} r_{FN}^{(l)}$ in T_{FN} such that

- for some $a_0, a_1, \ldots, a_{h-1}$ such that $a_0 < a_1 < \cdots < a_{h-1}$, $\{r_{FN}^{(0)},\ldots,r_{FN}^{(a_0)}\} \subseteq V_{FN,k_0}$, $\{r_{FN}^{(a_0+1)},\ldots,r_{FN}^{(a_1)}\} \subseteq V_{FN,k_1},\ldots,\{r_{FN}^{(a_{h-1}+1)},\ldots,r_{FN}^{(l)}\} \subseteq V_{FN,k_h}$, and
- for each $i_{FN} \in I_{FN}$, there exist $j_{i_{FN}}, h_{i_{FN}}, j'_{i_{FN}} \in \mathbb{N}$ such that $0 < j_{i_{FN}} \leq h_{i_{FN}} < j'_{i_{FN}} \leq l$, $r_{FN}^{(j_{i_{FN}})} = r_{FN}^{(j'_{i_{FN}})}$, $r_{FN}^{(h_{i_{FN}})}(i_{FN}) \in Q \times \{d\}$, and $\sigma_{FN}^{(h_{i_{FN}})}(i_{FN}) \neq \varepsilon$.

For $I_{FN} = \{0, 11, 13, \ldots, 1l, 21, 23, \ldots, 2l\}$, we assume without loss of generality that $j_0 \leq j_{11} \leq j_{13} \leq \cdots \leq j_{1l} \leq j_{21} \leq j_{23} \leq \cdots \leq j_{2l}$.

For each $i_{FN} \in I_{FN}$, let $u_{FN,i_{FN}} = \sigma_{FN}^{(j_{i_{FN}})}\sigma_{FN}^{(j_{i_{FN}}+1)}\cdots\sigma_{FN}^{(j'_{i_{FN}}-1)}$. In addition, let

$$\begin{aligned}
v_{FN} &= \sigma_{FN}^{(0)}\sigma_{FN}^{(1)}\cdots\sigma_{FN}^{(j_0-1)},\\
v_{FN,0} &= \begin{cases}\varepsilon, & \text{if } j'_0 = j_{11} \vee j_0 = j_{11}\\ \sigma_{FN}^{(j_0)}\sigma_{FN}^{(j_0+1)}\cdots\sigma_{FN}^{(j_{11}-1)}, & \text{otherwise,}\end{cases}\\
v_{FN,ii'} &= \begin{cases}\varepsilon, & \text{if } j'_{ii'} = j_{i(i'+2)} \vee j_{ii'} = j_{i(i'+2)}\\ \sigma_{FN}^{(j_{ii'})}\sigma_{FN}^{(j_{ii'}+1)}\cdots\sigma_{FN}^{(j_{i(i'+2)}-1)}, & \text{otherwise,}\end{cases}\\
&\quad (i \in \{1,2\}, i' \in \{1,3,\ldots,l-2\})\\
v_{FN,1l} &= \begin{cases}\varepsilon, & \text{if } j'_{1l} = j_{21} \vee j_{1l} = j_{21}\\ \sigma_{FN}^{(j_{1l})}\sigma_{FN}^{(j_{1l}+1)}\cdots\sigma_{FN}^{(j_{21}-1)}, & \text{otherwise,}\end{cases}\\
v_{FN,2l} &= \begin{cases}\varepsilon, & \text{if } l = j'_{2l}\\ \sigma_{FN}^{(j_{2l})}\sigma_{FN}^{(j_{2l}+1)}\cdots\sigma_{FN}^{(l-1)}, & \text{otherwise.}\end{cases}
\end{aligned}$$

Then, we have

$$\begin{aligned}
\alpha_{FN}(r_{FN}^{(0)}, v_{FN}) &= r_{FN}^{(j_0)},\\
\alpha_{FN}(r_{FN}^{(j'_{i_{FN}})}, v_{FN,i_{FN}}) &= \alpha_{FN}(r_{FN}^{(j_{i_{FN}})}, v_{FN,i_{FN}})\\
&= \begin{cases} r_{FN}^{(j_{i(i'+2)})}, & \text{if } i_{FN} \in \{11,13,\ldots,\\ & 1(l-2),21,23,\ldots,\\ & 2(l-2)\}\\ r_{FN}^{(j_{11})}, & \text{if } i_{FN} = 0\\ r_{FN}^{(j_{21})}, & \text{if } i_{FN} = 1l\\ r_{FN}^{(l)}, & \text{if } i_{FN} = 2l,\end{cases}\\
\alpha_{FN}(r_{FN}^{(j_{i_{FN}})}, u_{FN,i_{FN}}) &= r_{FN}^{(j'_{i_{FN}})} \ (\forall i_{FN} \in I_{FN}).
\end{aligned}$$

For any $m \in N$, we consider a trace

$$\begin{aligned}
s_{FN} := {} & v_{FN}u_{FN,0}^m v_{FN,0}u_{FN,11}^m v_{FN,11}u_{FN,13}^m v_{FN,13}\cdots\\
& u_{FN,1l}^m v_{FN,1l}u_{FN,21}^m v_{FN,21}u_{FN,23}^m v_{FN,23}\cdots\\
& u_{FN,2l}^m v_{FN,2l} \in L(T_{FN}).
\end{aligned}$$

For each $i_{FN} \in I_{FN}$, let

$$t_{i_{FN}} := \begin{cases} v_{FN}(0), & \text{if } i_{FN} = 0\\ (v_{FN}u_{FN,0}^m v_{FN,0})(11), & \text{if } i_{FN} = 11\\ (v_{FN}u_{FN,0}^m v_{FN,0}u_{FN,11}^m v_{FN,11}\cdots & \\ \ u_{FN,1(i'-2)}^m v_{FN,1(i'-2)})(1i'), & \text{if } i_{FN} = 1i' \ (i' \in \{3,5,\ldots,l\})\\ (v_{FN}u_{FN,0}^m v_{FN,0}u_{FN,11}^m v_{FN,11}\cdots & \\ \ u_{FN,1l}^m v_{FN,1l})(21), & \text{if } i_{FN} = 21\\ (v_{FN}u_{FN,0}^m v_{FN,0}u_{FN,11}^m v_{FN,11}\cdots & \\ \ u_{FN,1l}^m v_{FN,1l}u_{FN,21}^m v_{FN,21}\cdots & \\ \ u_{FN,2(i'-2)}^m v_{FN,2(i'-2)})(2i'), & \text{if } i_{FN} = 2i' \ (i' \in \{3,5,\ldots,l\}),\end{cases}$$

and

$$u_{i_{FN}} := \begin{cases} (u_{FN,0}^m v_{FN,0}u_{FN,11}^m v_{FN,11}\cdots u_{FN,1l}^m v_{FN,1l} & \\ \ u_{FN,21}^m v_{FN,21}\cdots u_{FN,2l}^m v_{FN,2l})(0), & \text{if } i_{FN} = 0\\ (u_{FN,1i'}^m v_{FN,1i'}\cdots u_{FN,1l}^m v_{FN,1l} & \\ \ u_{FN,21}^m v_{FN,21}\cdots u_{FN,2l}^m v_{FN,2l})(1i'), & \text{if } i_{FN} = 1i' \ (i' \in \{1,3,5,\ldots,l\})\\ (u_{FN,2i'}^m v_{FN,2i'}\cdots u_{FN,2l}^m v_{FN,2l})(2i'), & \text{if } i_{FN} = 2i' \ (i' \in \{1,3,5,\ldots,l\}).\end{cases}$$

Then, since $\sigma_{FN}^{(h_{i_{FN}})}(i_{FN}) \neq \varepsilon$, we have $|u_{i_{FN}}| \geq m$. Once the state in $\tilde{G}_K$ reaches the dump state d, it remains there. Since $r^{(h_{i_{FN}})}(i_{FN}) \in Q \times \{d\}$, we have $\xi((q_0,q_{K,0}),t_{i_{FN}}) = r^{(j_{i_{FN}})}(i_{FN}) = r^{(j'_{i_{FN}})}(i_{FN}) = Q \times \{d\}$, which implies that $t_{i_{FN}} \in L - K$. Since $t_{i_{FN}} \in L - K$ and $|u_{i_{FN}}| \geq m$, we have $t_{i_{FN}}u_{i_{FN}} \in L \cap (L-K)\Sigma^{\geq m} = F_0(m)$. In addition, each element of $I_N - I_{FN}$ is of the form ii'', where $i \in \{1,2\}$ and

i'' is an even number. We have by the definition of T_{FN} that $s_{FN}(ii'') \in K = H_0(m)$.

For each $i_{FN} \in I_{FN}$, we have $s_{FN}(i_{FN}) = t_{i_{FN}} u_{i_{FN}}$. Letting $s_0 = s_{FN}(0)$ and $s_{ii'} = s_{FN}(ii')$ for each $i \in \{1, 2\}$ and $i' \in \{0, 1, \ldots, N\}$, the first two of the five conditions of the first part of Proposition 1 are satisfied. Furthermore, by Proposition 2, the last three of the five conditions of the first part of Proposition 1 are also satisfied. Thus, by the first part of Proposition 1, we have $F_{N+1}(m) \neq \emptyset$.

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