OUKA
The University of Osaka
Institutional Knowledge Archive

Title	Delay bound of inference-based decentralized diagnosis in discrete event systems
Author(s)	Takai, Shigemasa; Kumar, Ratnesh
Citation	2016 13th International Workshop on Discrete Event Systems, WODES 2016. 2016, p. 224-229
Version Type	AM
URL	https://hdl.handle.net/11094/106738
rights	© 2016 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.
Note	

The University of Osaka Institutional Knowledge Archive : OUKA

https://ir.library.osaka-u.ac.jp/

The University of Osaka

Delay Bound of Inference-Based Decentralized Diagnosis in Discrete Event Systems

Shigemasa Takai and Ratnesh Kumar

***Abstract*— We previously introduced an inference-based decentralized diagnosis framework for discrete event systems, where inferencing over the ambiguities of the self and the others is used to issue local decisions, and a global decision is taken to be the one with the least ambiguity level. In this setting, we introduced the notion of N-inference diagnosability to characterize the detection of any failure within a bounded delay, using at most N-levels of inferencing, and subsuming both disjunctive and conjunctive ways of decision fusions. In this paper, we compute the delay bound within which the occurrence of any failure can be detected for an N-inference diagnosable system. Computing the delay bound is important to execute mitigation actions in a timely manner, and is a figure of merit of a diagnosis scheme.**

I. Introduction

There exists a long history of research on fault diagnosis of discrete event systems (DESs) (see for example [4], [10]). For large complex, physically distributed systems, the task of fault detection may need to be performed by a set of local diagnosers that rely on their own subsets of accessible sensors to observe the system behaviors. There are many prior papers dealing with decentralized diagnosis, including [1–3], [7]. A decentralized diagnosis scheme was introduced in [3], where a fault is detectable if and only if it can be detected by one of the local diagnosers within a uniformly bounded delay, and the notion of codiagnosability was introduced to capture this property. This scheme, where at least one diagnoser issues the failure decision, is "disjunctive" in nature, and it turns out that a dual "conjunctive" scheme, as proposed later in [7], also exists where a fault is detected when none of the diagnosers issue a nonfailure decision. These two dual schemes are incomparable in general [7].

A later development in decentralized diagnosis of DESs is the introduction of the framework for inferencing in decentralized decision-making. In this setting, each diagnoser uses not only its own knowledge of the system behaviors, but also the inference about the possible knowledge of other diagnosers to arrive at its own local decision. A "winning" local decision (namely, the one needing the least level of inferencing) is used to issue a global decision. While the initial work on inference-based decentralized diagnosis considered a single-level of such inferencing [7], a generalized framework to allow arbitrary-levels of inferencing was proposed in [2] for the disjunctive scheme. Recently, we have introduced a framework for inference-based decentralized diagnosis that not only supports multi-levels of inferencing, but also supports both the disjunctive and conjunctive schemes of decision-making [6].

For a diagnosable system, the delay bound is defined as the minimum number of steps within which the detection of any failure is guaranteed. Knowing the delay bound is important to execute mitigation actions in a timely manner, and is a figure of merit of a diagnosis scheme. Algorithms for computing the delay bound are reported in the literature for various schemes: disjunctive [3], conjunctive [8], and conditional disjunctive [9]. The contribution of this paper is to provide the computation of the delay bound of diagnosis in the inference-based framework of [6], generalizing the existing schemes. It is expected that the delay bound will be smaller compared to the existing decidable schemes, all of which are subsumed by our framework.

II. Notation and Preliminaries

An automaton is a five-tuple $G = (Q, \Sigma, \delta, q_0, Q_m)$, where Q is the set of states, Σ is the finite set of events, $\delta : Q \times \Sigma \to Q$ is the partial transition function, $q_0 \in Q$ is the initial state, and $Q_m \subseteq Q$ is the set of marked states. Let Σ^* be the set of all finite traces of elements of Σ, including the empty trace ε. The function δ can be generalized to $\delta : Q \times \Sigma^* \to Q$ in the usual manner. The generated language of G, denoted by $L(G)$, is defined as $L(G) = \{s \in \Sigma^* \mid \delta(q_0, s)!\}$, where $\delta(q_0, s)!$ denotes that $\delta(q_0, s)$ is defined.

Let $K \subseteq \Sigma^*$ be a language. The set of all prefixes of traces in K is denoted by $pr(K)$. If $K = pr(K)$, then K is said to be (prefix-)closed. A closed language K is said to be *deadlock-free* if for any $s \in K$, $\{s\}\Sigma \cap K \neq \emptyset$. For each trace $s \in \Sigma^*$, $|s|$ denotes its length. For any $m \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of all nonnegative integers, let $\Sigma^{\geq m} := \{s \in \Sigma^* \mid |s| \geq m\}$ and $\Sigma^{\leq m} := \{s \in \Sigma^* \mid |s| \leq m\}$.

Let $I = \{1, 2, \ldots, n\}$ denote the index set of local diagnosers that perform the task of diagnosis. We assume that the limited sensing capabilities of the ith local diagnoser D_i ($i \in I$) can be represented by the local observation mask, $M_i : \Sigma \to \Delta_i \cup \{\varepsilon\}$, where Δ_i is the set of locally observed symbols. The local observation mask M_i is extended to $M_i : \Sigma^* \to \Delta_i^*$ in the usual manner. For any languages $L \subseteq \Sigma^*$ and $L' \subseteq \Delta_i^*$, $M_i(L) \subseteq \Delta_i^*$ and $M_i^{-1}(L') \subseteq \Sigma^*$ are defined as $M_i(L) = \{M_i(s) \in \Delta_i^* \mid s \in L\}$ and $M_i^{-1}(L') = \{s \in \Sigma^* \mid M_i(s) \in L'\}$, respectively.

This work was supported in part by the National Science Foundation under Grants NSF-CCF-1331390 and NSFECCS-1509420 and by JSPS KAKENHI Grant Number 15K06140.

S. Takai is with the Division of Electrical, Electronic and Information Engineering, Osaka University, Suita, Osaka 565-0871, Japan, e-mail: takai@eei.eng.osaka-u.ac.jp.

R. Kumar is with the Department of Electrical and Computer Engineering, Iowa State University, Ames, IA 50011-3060 USA, e-mail: rkumar@iastate.edu.

Let $L \neq \emptyset$ be a closed language that represents the generated language of a plant (system to be diagnosed) modeled as an automaton G, and $K \subseteq L$ be a nonempty closed language that represents a nonfailure specification. Traces in $L - K$ are considered as failure traces and the task of diagnosis is to determine the execution of any trace in $L - K$ within an additional bounded number of system executions. Without loss of generality, the plant language L can be taken to be deadlock-free [2].

In what follows, we let $G = (Q, \Sigma, \delta, q_0, Q)$ be the finite plant model with $L(G) = L$ and $G_K = (Q_K, \Sigma, \delta_K, q_{K,0}, Q_K)$ be a finite generator of the nonfailure specification language, i.e., $L(G_K) = K$. We augment the generator G_K by adding a dump state $d \notin Q_K$. Formally, the augmented generator is defined as $\tilde{G}_K = (\tilde{Q}_K, \Sigma, \tilde{\delta}_K, q_{K,0}, \tilde{Q}_K)$, where $\tilde{Q}_K = Q_K \cup \{d\}$, and the state transition function $\tilde{\delta}_K : \tilde{Q}_K \times \Sigma \to \tilde{Q}_K$ is defined as

$$\tilde{\delta}_K(\tilde{q}_K, \sigma) = \begin{cases} \delta_K(\tilde{q}_K, \sigma), & \text{if } \tilde{q}_K \in Q_K \wedge \delta_K(\tilde{q}_K, \sigma)! \\ d, & \text{otherwise.} \end{cases}$$

Then we have $L(\tilde{G}_K) = \Sigma^*$. The refinement of the plant with respect to the specification is obtained by taking the synchronous product: $G \parallel \tilde{G}_K = (Q \times \tilde{Q}_K, \Sigma, \xi, (q_0, q_{K,0}), Q \times \tilde{Q}_K)$. Then we have $L(G \| \tilde{G}_K) = L(G) \cap L(\tilde{G}_K) = L \cap \Sigma^* = L$. For each $s \in L$, $s \in L - K$ if and only if $G || \tilde{G}_K$ reaches a state whose second element is the dump state d by the execution of s.

III. Inference-Based Diagnosis Framework

A. Existence Condition of N-Inferring Diagnosers

Let $C = \{0, 1, \phi\}$ be the set of diagnosis decisions, where "0" represents a nonfailure decision, "1" represents a failure decision, and "ϕ" represents an unsure decision. Each inference-based local diagnoser D_i is defined as a map $D_i : M_i(L) \to C \times \mathbb{N}$ [2], where for each $s \in L$, $D_i(M_i(s)) = (c_i(M_i(s)), n_i(M_i(s)))$. Here $c_i(M_i(s)) \in C$ denotes the diagnosis decision of D_i following an observation $M_i(s) \in M_i(L)$, and $n_i(M_i(s)) \in \mathbb{N}$ denotes the ambiguity level of the diagnosis decision of D_i. Let $n(s)$ be the minimum ambiguity level of local decisions [2], i.e., $n(s) := \min_{i \in I} n_i(M_i(s))$.

The decentralized diagnoser $\{D_i\}_{i \in I}$ that consists of local diagnosers D_i ($i \in I$) issues global diagnosis decisions. Formally, $\{D_i\}_{i \in I}$ is defined as a map $\{D_i\}_{i \in I} : L \to C$. For each $s \in L$, the diagnosis decision $\{D_i\}_{i \in I}(s)$ is given as follows [2]:

$$\{D_i\}_{i \in I}(s) = \begin{cases} 1, & \text{if } \forall i \in I : n_i(M_i(s)) = n(s) \\ & \quad \Rightarrow c_i(M_i(s)) = 1 \\ 0, & \text{if } \forall i \in I : n_i(M_i(s)) = n(s) \\ & \quad \Rightarrow c_i(M_i(s)) = 0 \\ \phi, & \text{otherwise.} \end{cases}$$

The global diagnosis decision is taken to be the same as a local diagnosis decision possessing the minimum level of ambiguity.

Let $N \in \mathbb{N}$ be a given nonnegative integer. A decentralized diagnoser $\{D_i\}_{i \in I} : L \to C$ is said to be N-*inferring* [6] if the following two conditions hold:

1) Either

$$\forall s \in L - K : \{D_i\}_{i \in I}(s) = 1 \Rightarrow n(s) \leq N, \quad (1)$$

or

$$\forall s \in K : \{D_i\}_{i \in I}(s) \neq 1 \Rightarrow n(s) \leq N. \quad (2)$$

2) There exists $m \in \mathbb{N}$ such that

$$\forall s \in (L \cap (L - K)\Sigma^{\geq m}) \cup K : \\ n(s) \leq N \Rightarrow \{D_i\}_{i \in I}(s) \neq \phi. \quad (3)$$

Given a plant language L, a nonfailure specification language $K \subseteq L$, and a nonnegative integer $m \in \mathbb{N}$, we inductively define a monotonically decreasing sequence $\{(F_k(m), H_k(m))\}_{k \geq 0}$ of language pairs as follows [6]:

- Base step:

$$F_0(m) := L \cap (L - K)\Sigma^{\geq m}, \; H_0(m) := K.$$

- Induction step:

$$F_{k+1}(m) := F_k(m) \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(H_k(m)) \right),$$
$$H_{k+1}(m) := H_k(m) \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(F_k(m)) \right).$$

Then we have the following definition of N-inference diagnosability.

Definition 1: [6] The pair (L, K) of languages is said to be N-*inference diagnosable* if there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$.

The following theorem shows that N-inference diagnosability is a necessary and sufficient condition for the existence of an N-inferring decentralized diagnoser with no missed and incorrect detections.

Theorem 1: [6] There exists an N-inferring decentralized diagnoser $\{D_i\}_{i \in I} : L \to C$ that satisfies

$$\exists m \in \mathbb{N}, \forall s \in L \cap (L - K)\Sigma^{\geq m} : \{D_i\}_{i \in I}(s) = 1, \quad (4)$$
$$\forall s \in K : \{D_i\}_{i \in I}(s) \neq 1 \quad (5)$$

if and only if the pair (L, K) of languages is N-inference diagnosable.

Assume that the pair (L, K) of languages is N-inference diagnosable so that there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$. Then a local diagnoser can compute its diagnosis decision and associate a level of ambiguity with it as follows. Let $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$ for some $m \in \mathbb{N}$. For each $s \in L$, the ith local diagnoser D_i computes

$$n_i^f(M_i(s)) := \min\{k \in \mathbb{N} \mid M_i(s) \notin M_i(H_k(m))\}, \quad (6)$$
$$n_i^h(M_i(s)) := \min\{k \in \mathbb{N} \mid M_i(s) \notin M_i(F_k(m))\}. \quad (7)$$

For a local diagnoser $D_i : M_i(L) \to C \times \mathbb{N}$, its diagnosis decision and ambiguity level following an observation

$M_i(s) \in M_i(L)$, i.e., $D_i(M_i(s)) = (c_i(M_i(s)), n_i(M_i(s)))$, is determined as follows [2]:

$$c_i(M_i(s)) = \begin{cases} 1, & \text{if } n_i^f(M_i(s)) < n_i^h(M_i(s)) \\ 0, & \text{if } n_i^h(M_i(s)) < n_i^f(M_i(s)) \\ \phi, & \text{if } n_i^f(M_i(s)) = n_i^h(M_i(s)), \end{cases} \quad (8)$$

$$n_i(M_i(s)) = \min\{n_i^f(M_i(s)), n_i^h(M_i(s))\}. \quad (9)$$

It was shown in [6] that the decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ for which the local diagnosers are given by (6)–(9) is N-inferring and satisfies

$$\forall s \in L \cap (L - K)\Sigma^{\geq m} : \{D_i\}_{i\in I}(s) = 1 \quad (10)$$

and (5). I.e., any failure can be detected by the decentralized diagnoser $\{D_i\}_{i\in I} : L \to C$ within m steps.

B. Testing N-Inference Diagnosability

Here we present our construction of testing automata reported in [5] that were used for checking N-inference diagnosability. The same automata, with additional counting, introduced in the next section, will be used for the delay bound computation.

Violation of N-inference diagnosability of (L, K) requires that for any $m \in \mathbb{N}$, $F_{N+1}(m) \neq \emptyset$ and $H_{N+1}(m) \neq \emptyset$. For simplicity of presentation, we consider the case of two local diagnosers, i.e., $I = \{1, 2\}$. The results continue to hold for an arbitrary number of local diagnosers. As shown in the following proposition, the nonemptiness of $F_{N+1}(m)$ and $H_{N+1}(m)$ can be characterized as the existence of certain $(2N + 3)$ traces in L.

Proposition 1: [5, Proposition 1] Consider the pair (L, K) of languages and any $m \in \mathbb{N}$.

1) $F_{N+1}(m) \neq \emptyset$ if and only if there exist $(2N+3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ such that
   - $s_0 \in F_0(m)$,
   - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\}$:
   $$s_{ii'} \in \begin{cases} H_0(m), & \text{if } i' \text{ is an even number} \\ F_0(m), & \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i \in I : M_i(s_0) = M_i(s_{i0})$,
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_2(s_{1i'}) = M_2(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(s_{1i'}) = M_1(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_1(s_{2i'}) = M_1(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(s_{2i'}) = M_2(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$
2) $H_{N+1}(m) \neq \emptyset$ if and only if there exist $(2N+3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ such that
   - $s_0 \in H_0(m)$,
   - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\}$:
   $$s_{ii'} \in \begin{cases} F_0(m), & \text{if } i' \text{ is an even number} \\ H_0(m), & \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i \in I : M_i(s_0) = M_i(s_{i0})$,
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_2(s_{1i'}) = M_2(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(s_{1i'}) = M_1(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
   - $\forall i' \in \{0, 1, \ldots, N-1\}$:
   $$\begin{cases} M_1(s_{2i'}) = M_1(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(s_{2i'}) = M_2(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$

Employing Proposition 1, we first provide a construction to test the nonemptiness of F_{N+1} for any $m \in \mathbb{N}$. We define, respectively, the index set I_N of $(2N + 3)$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ of Proposition 1 and I_{FN} of among those that are failure traces $s_0, s_{11}, s_{13}, \ldots, s_{1l}, s_{21}, s_{23}, \ldots, s_{2l} \in L - K$, where

$$l = \begin{cases} N-1, & \text{if } N \text{ is an even number} \\ N, & \text{if } N \text{ is an odd number,} \end{cases}$$

as follows:

$$I_N = \{0, 10, 11, \ldots, 1N, 20, 21, \ldots, 2N\},$$
$$I_{FN} = \{0, 11, 13, \ldots, 1l, 21, 23, \ldots, 2l\}.$$

Using the notation for these two index sets, we build a generator of all traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in L$ that satisfy the last three of the five conditions of the first part of Proposition 1. We define $T_{FN} = (R_{FN}, \Sigma_{FN}, \alpha_{FN}, r_{FN,0}, R_{FN,m})$ as follows [5]:

- The state set R_{FN} is defined as $R_{FN} = (Q \times \tilde{Q}_K) \times Q_{10} \times Q_{11} \times \cdots \times Q_{1N} \times Q_{20} \times Q_{21} \times \cdots \times Q_{2N}$, where
$$Q_{ii'} = \begin{cases} Q_K, & \text{if } i' \text{ is an even number} \\ Q \times \tilde{Q}_K, & \text{if } i' \text{ is an odd number} \end{cases}$$
for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.
- The initial state $r_{FN,0} \in R_{FN}$ is given as
$$r_{FN,0} = ((q_0, q_{K,0}), q_{10,0}, q_{11,0}, \ldots, q_{1N,0}, q_{20,0}, q_{21,0}, \ldots, q_{2N,0}),$$
where
$$q_{ii',0} = \begin{cases} q_{K,0}, & \text{if } i' \text{ is an even number} \\ (q_0, q_{K,0}), & \text{if } i' \text{ is an odd number} \end{cases}$$
for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.
- The set $R_{FN,m}$ of marked states is defined as $R_{FN,m} = \{r_{FN} \in R_{FN} \mid \forall i_{FN} \in I_{FN} : r_{FN}(i_{FN}) \in Q \times \{d\}\}$, where, for each
$$r_{FN} = ((q, q_K), q_{10}, q_{11}, \ldots, q_{1N}, q_{20}, q_{21}, \ldots, q_{2N})$$
in R_{FN}, we let $r_{FN}(0) = (q, q_K)$ and $r_{FN}(ii') = q_{ii'}$ for each $i \in I$ and each $i' \in \{0, 1, \ldots, N\}$.

- The event set Σ_{FN} is defined as

$$\Sigma_{FN} = \underbrace{(\Sigma \cup \{\varepsilon\}) \times (\Sigma \cup \{\varepsilon\}) \times \cdots \times (\Sigma \cup \{\varepsilon\})}_{(2N+3) \text{ times}} - \{(\varepsilon, \varepsilon, \ldots, \varepsilon)\}.$$

- For each $r_{FN} = ((q, q_K), q_{10}, \ldots, q_{1N}, q_{20}, \ldots, q_{2N})$ and each $\sigma_{FN} = (\sigma, \sigma_{10}, \ldots, \sigma_{1N}, \sigma_{20}, \ldots, \sigma_{2N})$, $\alpha_{FN}(r_{FN}, \sigma_{FN})!$ if the following five conditions are satisfied:
  - $\sigma \neq \varepsilon \Rightarrow \xi((q, q_K), \sigma)!$,
  - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\} : \sigma_{ii'} \neq \varepsilon \Rightarrow$

$$\begin{cases} \delta_K(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an even number} \\ \xi(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an odd number,} \end{cases}$$

  - $\forall i \in I : M_i(\sigma) = M_i(\sigma_{i0})$,
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:

$$\begin{cases} M_2(\sigma_{1i'}) = M_2(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(\sigma_{1i'}) = M_1(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$

  - $\forall i' \in \{0, 1, \ldots, N-1\}$:

$$\begin{cases} M_1(\sigma_{2i'}) = M_1(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(\sigma_{2i'}) = M_2(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$

  If $\alpha_{FN}(r_{FN}, \sigma_{FN})!$, then

$$\alpha_{FN}(r_{FN}, \sigma_{FN}) = ((q', q'_K), q'_{10}, q'_{11}, \ldots, q'_{1N}, q'_{20}, q'_{21}, \ldots, q'_{2N}),$$

  where

$$(q', q'_K) = \begin{cases} \xi((q, q_K), \sigma), & \text{if } \sigma \neq \varepsilon \\ (q, q_K), & \text{otherwise} \end{cases}$$

$$q'_{ii'} = \begin{cases} \delta_K(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an even number}] \\ \xi(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an odd number}] \\ q_{ii'}, \\ \quad \text{otherwise.} \end{cases}$$

For any $\sigma_{FN} = (\sigma, \sigma_{10}, \sigma_{11}, \ldots, \sigma_{1N}, \sigma_{20}, \sigma_{21}, \ldots, \sigma_{2N})$ in Σ_{FN}, we let $\sigma_{FN}(0) = \sigma$ and $\sigma_{FN}(ii') = \sigma_{ii'}$ for each $i \in I$ and each $i' \in \{0, 1, \ldots, N\}$.

To accept traces satisfying all five of the conditions in the first part of Proposition 1 for some $m \in \mathbb{N}$, we build a nondeterministic acyclic automaton over the set of maximal strongly connected components (max-SCCs) of T_{FN} [5]: $\mathcal{T}_{FN} = (V_{FN}, \Sigma_{FN}, \beta_{FN}, V_{FN,0}, V_{FN,m})$, where

- The state set V_{FN} is defined as $V_{FN} = \{V_{FN,0}, V_{FN,1}, \ldots, V_{FN,|V_{FN}|-1}\}$, where, for any $k \in \{0, 1, \ldots, |V_{FN}| - 1\}$, $V_{FN,k}$ is a max-SCC of T_{FN}. Without loss of generality, we assume that $r_{FN,0} \in V_{FN,0}$.
- The set $V_{FN,m}$ of marked states is defined as $V_{FN,m} = \{V_{FN,k} \in V_{FN} \mid V_{FN,k} \cap R_{FN,m} \neq \emptyset\}$.
- The nondeterministic state transition function $\beta_{FN} : V_{FN} \times \Sigma_{FN} \to 2^{V_{FN}}$ is defined as

$$\beta_{FN}(V_{FN,k}, \sigma_{FN}) = \{V_{FN,k'} \in V_{FN} \mid k \neq k' \wedge [\exists r_{FN} \in V_{FN,k}, \exists r'_{FN} \in V_{FN,k'} : \alpha_{FN}(r_{FN}, \sigma_{FN}) = r'_{FN}]\}$$

  for each $V_{FN,k} \in V_{FN}$ and each $\sigma_{FN} \in \Sigma_{FN}$.

A *labeling* function $J_{FN} : V_{FN} \to 2^{I_{FN}}$ is defined as

$$J_{FN}(V_{FN,k}) = \{i_{FN} \in I_{FN} \mid [\exists r_{FN} \in V_{FN,k} : r_{FN}(i_{FN}) \in Q \times \{d\}] \wedge [\exists r_{FN}, r'_{FN} \in V_{FN,k}, \exists \sigma_{FN} \in \Sigma_{FN} : \alpha_{FN}(r_{FN}, \sigma_{FN}) = r'_{FN} \wedge \sigma_{FN}(i_{FN}) \neq \varepsilon]\}$$

for each $V_{FN,k} \in V_{FN}$. Then, by the construction, $i_{FN} \in J_{FN}(V_{FN,k})$ means that $s_{i_{FN}}$ can be extended to an arbitrarily long failure trace in a max-SCC $V_{FN,k}$. Therefore, we can test whether $F_{N+1}(m) \neq \emptyset$ for any $m \in \mathbb{N}$ as shown in the following proposition.

Proposition 2: [5, Proposition 3] Consider the pair (L, K) of regular languages. Then, $F_{N+1}(m) \neq \emptyset$ for any $m \in \mathbb{N}$ if and only if there exists a path $V_{FN,0} = V_{FN,k_0} \xrightarrow{\sigma_{FN}^{(k_0)}} V_{FN,k_1} \xrightarrow{\sigma_{FN}^{(k_1)}} \cdots \xrightarrow{\sigma_{FN}^{(k_{h-1})}} V_{FN,k_h} \in V_{FN,m}$ in the acyclic automaton $\mathcal{T}_{FN}$ such that

$$\bigcup_{k \in \{k_0, k_1, \ldots, k_h\}} J_{FN}(V_{FN,k}) = I_{FN}. \tag{11}$$

Dually to check the nonemptiness of $H_{N+1}(m)$ for any $m \in \mathbb{N}$, we construct T_{HN} and $\mathcal{T}_{HN}$ dual to T_{FN} and $\mathcal{T}_{FN}$, respectively. Among the $(2N+3)$ traces of the second part of Proposition 1, $s_{10}, s_{12}, \ldots, s_{1l}, s_{20}, s_{22}, \ldots, s_{2l} \in L - K$ are failure traces, where

$$l = \begin{cases} N, & \text{if } N \text{ is an even number} \\ N-1, & \text{if } N \text{ is an odd number,} \end{cases}$$

and their index set is denoted by

$$I_{HN} = \{10, 12, \ldots, 1l, 20, 22, \ldots, 2l\}.$$

Employing this index set, we define a generator of all $(2N+3)$ traces satisfying the last three of the five conditions of the second part of Proposition 1, $T_{HN} = (R_{HN}, \Sigma_{HN}, \alpha_{HN}, r_{HN,0}, R_{HN,m})$, as follows [5]:

- The state set R_{HN} is defined as $R_{HN} = Q_K \times Q_{10} \times Q_{11} \times \cdots \times Q_{1N} \times Q_{20} \times Q_{21} \times \cdots \times Q_{2N}$, where

$$Q_{ii'} = \begin{cases} Q \times \tilde{Q}_K, & \text{if } i' \text{ is an even number} \\ Q_K, & \text{if } i' \text{ is an odd number} \end{cases}$$

  for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.
- The initial state $r_{HN,0} \in R_{HN}$ is given as $r_{HN,0} = (q_{K,0}, q_{10,0}, q_{11,0}, \ldots, q_{1N,0}, q_{20,0}, q_{21,0}, \ldots, q_{2N,0})$, where

$$q_{ii',0} = \begin{cases} (q_0, q_{K,0}), & \text{if } i' \text{ is an even number} \\ q_{K,0}, & \text{if } i' \text{ is an odd number} \end{cases}$$

for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.

- The set $R_{HN,m}$ of marked states is defined as $R_{HN,m} = \{r_{HN} \in R_{HN} \mid \forall i_{HN} \in I_{HN} : r_{HN}(i_{HN}) \in Q \times \{d\}\}$, where, for each $r_{HN} \in R_{HN}$, $r_{HN}(i_N)$ ($i_N \in I_N$) are defined in the same way as $r_{FN}(i_N)$, where $r_{FN} \in R_{FN}$.
- The event set Σ_{HN} is defined as $\Sigma_{HN} = \Sigma_{FN}$.
- For each $r_{HN} = (q_K, q_{10}, \ldots, q_{1N}, q_{20}, \ldots, q_{2N})$ and each $\sigma_{HN} = (\sigma, \sigma_{10}, \ldots, \sigma_{1N}, \sigma_{20}, \ldots, \sigma_{2N})$, $\alpha_{HN}(r_{HN}, \sigma_{HN})!$ if the following five conditions are satisfied:
  - $\sigma \neq \varepsilon \Rightarrow \delta_K(q_K, \sigma)!$,
  - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\} : \sigma_{ii'} \neq \varepsilon \Rightarrow$
$$\begin{cases} \xi(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an even number} \\ \delta_K(q_{ii'}, \sigma_{ii'})!, & \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i \in I : M_i(\sigma) = M_i(\sigma_{i0})$,
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
$$\begin{cases} M_2(\sigma_{1i'}) = M_2(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(\sigma_{1i'}) = M_1(\sigma_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
$$\begin{cases} M_1(\sigma_{2i'}) = M_1(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(\sigma_{2i'}) = M_2(\sigma_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$

  If $\alpha_{HN}(r_{HN}, \sigma_{HN})!$, then
$$\begin{aligned} &\alpha_{HN}(r_{HN}, \sigma_{HN}) \\ &\quad = (q'_K, q'_{10}, q'_{11}, \ldots, q'_{1N}, q'_{20}, q'_{21}, \ldots, q'_{2N}), \end{aligned}$$
  where
$$q'_K = \begin{cases} \delta_K(q_K, \sigma), & \text{if } \sigma \neq \varepsilon \\ q_K, & \text{otherwise} \end{cases}$$
  and
$$q'_{ii'} = \begin{cases} \xi(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an even number}] \\ \delta_K(q_{ii'}, \sigma_{ii'}), \\ \quad \text{if } \sigma_{ii'} \neq \varepsilon \wedge [i' \text{ is an odd number}] \\ q_{ii'}, \\ \quad \text{otherwise.} \end{cases}$$

For each $\sigma_{HN} \in \Sigma_{HN}$, $\sigma_{HN}(i_N)$ ($i_N \in I_N$) is defined in the same way as $\sigma_{FN}(i_N)$, where $\sigma_{FN} \in \Sigma_{FN}$.

Using T_{HN}, we construct a nondeterministic acyclic automaton $\mathcal{T}_{HN} = (V_{HN}, \Sigma_{HN}, \beta_{HN}, V_{HN,0}, V_{HN,m})$ over the set of max-SCCs of T_{HN} as follows [5]:

- The state set V_{HN} is defined as $V_{HN} = \{V_{HN,0}, V_{HN,1}, \ldots, V_{HN,|V_{HN}|-1}\}$, where, for any $k \in \{0, 1, \ldots, |V_{HN}| - 1\}$, $V_{HN,k}$ is a max-SCC of T_{HN}. Without loss of generality, we assume that $r_{HN,0} \in V_{HN,0}$.
- The set $V_{HN,m}$ of marked states is defined as $V_{HN,m} = \{V_{HN,k} \in V_{HN} \mid V_{HN,k} \cap R_{HN,m} \neq \emptyset\}$.
- The nondeterministic state transition function $\beta_{HN} : V_{HN} \times \Sigma_{HN} \to 2^{V_{HN}}$ is defined as
$$\begin{aligned} &\beta_{HN}(V_{HN,k}, \sigma_{HN}) \\ &\quad = \{V_{HN,k'} \in V_{HN} \mid k \neq k' \\ &\qquad \wedge [\exists r_{HN} \in V_{HN,k}, \exists r'_{HN} \in V_{HN,k'} : \\ &\qquad\quad \alpha_{HN}(r_{HN}, \sigma_{HN}) = r'_{HN}]\} \end{aligned}$$
  for each $V_{HN,k} \in V_{HN}$ and each $\sigma_{HN} \in \Sigma_{HN}$.

A labeling function $J_{HN} : V_{HN} \to 2^{I_{HN}}$ is defined as
$$\begin{aligned} &J_{HN}(V_{HN,k}) \\ &\quad = \{i_{HN} \in I_{HN} \mid [\exists r_{HN} \in V_{HN,k} : \\ &\qquad r_{HN}(i_{HN}) \in Q \times \{d\}] \\ &\qquad \wedge [\exists r_{HN}, r'_{HN} \in V_{HN,k}, \exists \sigma_{HN} \in \Sigma_{HN} : \\ &\qquad\quad \alpha_{HN}(r_{HN}, \sigma_{HN}) = r'_{HN} \wedge \sigma_{HN}(i_{HN}) \neq \varepsilon]\} \end{aligned}$$
for each $V_{HN,k} \in V_{HN}$.

Similar to Proposition 2, we have the following result.

Proposition 3: [5, Proposition 5] Consider the pair (L, K) of regular languages. Then, $H_{N+1}(m) \neq \emptyset$ for any $m \in \mathbb{N}$ if and only if there exists a path $V_{HN,0} = V_{HN,k_0} \xrightarrow{\sigma_{HN}^{(k_0)}} V_{HN,k_1} \xrightarrow{\sigma_{HN}^{(k_1)}} \cdots \xrightarrow{\sigma_{HN}^{(k_{h-1})}} V_{HN,k_h} \in V_{HN,m}$ in the acyclic automaton $\mathcal{T}_{HN}$ such that
$$\bigcup_{k \in \{k_0, k_1, \ldots, k_h\}} J_{HN}(V_{HN,k}) = I_{HN}. \tag{12}$$

IV. Computation of Delay Bound

The delay bound is defined as the minimum number of steps within which the detection of any failure is guaranteed. In this section, we show how to compute the delay bound in the inference-based diagnosis framework. We assume that the pair (L, K) of languages is N-inference diagnosable. Then there exists $m \in \mathbb{N}$ such that $F_{N+1}(m) = \emptyset$ or $H_{N+1}(m) = \emptyset$. Let m_N^* be such a minimum integer m, i.e.,
$$m_N^* = \min\{m \in \mathbb{N} \mid F_{N+1}(m) = \emptyset \vee H_{N+1}(m) = \emptyset\}.$$
For any $m \geq m_N^*$, the decentralized diagnoser $\{D_i\}_{i \in I} : L \to C$ for which the local diagnosers are given by (6)–(9) can detect any failure within m steps. Hence, m_N^* can be considered as the delay bound.

Let $\mathbb{N}_{FN}$ and $\mathbb{N}_{HN}$ be the sets of delays under which (L, K) is "N-inference disjunctive and conjunctive diagnosable", respectively, i.e., $\mathbb{N}_{FN} = \{m \in \mathbb{N} \mid F_{N+1}(m) = \emptyset\}$ and $\mathbb{N}_{HN} = \{m \in \mathbb{N} \mid H_{N+1}(m) = \emptyset\}$. Then we can define the minimum delays for the disjunctive and conjunctive cases as
$$\begin{aligned} m_{FN}^* &= \begin{cases} \min \mathbb{N}_{FN}, & \text{if } \mathbb{N}_{FN} \neq \emptyset \\ \text{undefined}, & \text{otherwise,} \end{cases} \\ m_{HN}^* &= \begin{cases} \min \mathbb{N}_{HN}, & \text{if } \mathbb{N}_{HN} \neq \emptyset \\ \text{undefined}, & \text{otherwise,} \end{cases} \end{aligned}$$
respectively, and the overall minimum delay as
$$m_N^* = \begin{cases} \min\{m_{FN}^*, m_{HN}^*\}, & \text{if } \mathbb{N}_{FN} \neq \emptyset \wedge \mathbb{N}_{HN} \neq \emptyset \\ m_{FN}^*, & \text{if } \mathbb{N}_{FN} \neq \emptyset \wedge \mathbb{N}_{HN} = \emptyset \\ m_{HN}^*, & \text{if } \mathbb{N}_{FN} = \emptyset \wedge \mathbb{N}_{HN} \neq \emptyset. \end{cases}$$

To compute m^*_N, we develop methods for computing m^*_{FN} and m^*_{HN} when $\mathbb{N}_{FN} \neq \emptyset$ and $\mathbb{N}_{HN} \neq \emptyset$, respectively.

Note that m^*_{FN} is the minimum number of post-fault events the system must execute for a fault to be diagnosed. We use the acyclic automaton $\mathcal{T}_{FN}$ for computing m^*_{FN} when $\mathbb{N}_{FN} \neq \emptyset$. Under this condition, we have $F_{N+1}(m^*_{FN}) = \emptyset$. Then from the inverse of Proposition 2, it holds for any path, in the set $\mathcal{P}_{FN}$ of all paths from the initial state to final states of the form $V_{FN,0} = V_{FN,k_0} \xrightarrow{\sigma^{(k_0)}_{FN}} V_{FN,k_1} \xrightarrow{\sigma^{(k_1)}_{FN}} \cdots \xrightarrow{\sigma^{(k_{h-1})}_{FN}} V_{FN,k_h} \in V_{FN,m}$, that $\bigcup_{k\in\{k_0,k_1,\ldots,k_h\}} J_{FN}(V_{FN,k}) \neq I_{FN}$. Then for a corresponding index $i_{FN} \in I_{FN} - \bigcup_{k\in\{k_0,k_1,\ldots,k_h\}} J_{FN}(V_{FN,k})$, it holds that only the inter-max-SCC transitions in T_{FN} (that are also the transitions of $\mathcal{T}_{FN}$) witness a post-fault event, whereas the intra-max-SCC transitions in T_{FN} (max-SCCs of T_{FN} are states in $\mathcal{T}_{FN}$) are simply ε-transitions (otherwise that index would already be included in $\bigcup_{k\in\{k_0,k_1,\ldots,k_h\}} J_{FN}(V_{FN,k})$ by the definition of J_{FN}). Then for each such index, each transition along a path $p_{FN} \in \mathcal{P}_{FN}$ may contribute to a post-fault event count depending on whether or not that transition is a non-ε-transition for that index. Accordingly, for each path $p_{FN} : V_{FN,0} = V_{FN,k_0} \xrightarrow{\sigma^{(k_0)}_{FN}} V_{FN,k_1} \xrightarrow{\sigma^{(k_1)}_{FN}} \cdots \xrightarrow{\sigma^{(k_{h-1})}_{FN}} V_{FN,k_h} \in V_{FN,m}$ in $\mathcal{P}_{FN}$ and each index $i_{FN} \in I_{FN} - \bigcup_{k\in\{k_0,k_1,\ldots,k_h\}} J_{FN}(V_{FN,k})$, we define an index-specific post-fault event count of each transition of the path as follows: For each $j \in \{1, 2, \ldots, h\}$,

$$w_{FN,i_{FN}}(V_{FN,k_{j-1}}, \sigma^{(k_{j-1})}_{FN}, V_{FN,k_j}) := \begin{cases} 1, & \text{if } [\sigma^{(k_{j-1})}_{FN}(i_{FN}) \neq \varepsilon] \\ & \wedge[\exists r_{FN} \in V_{FN,k_{j-1}}, \exists r'_{FN} \in V_{FN,k_j} : \\ & \quad \alpha_{FN}(r_{FN}, \sigma^{(k_{j-1})}_{FN}) = r'_{FN} \\ & \quad \wedge r'_{FN}(i_{FN}) \in Q \times \{d\}] \\ 0, & \text{otherwise.} \end{cases}$$

Using the index-specific post-fault event count of transitions, we define an index-specific post-fault event count for the path p_{FN} by simply adding the index-specific counts along the transitions of p_{FN}:

$$w_{FN,i_{FN}}(p_{FN}) := \sum_{j=1}^{h} w_{FN,i_{FN}}(V_{FN,k_{j-1}}, \sigma^{(k_{j-1})}_{FN}, V_{FN,k_j}).$$

Next, a minimum among all indices in $I_{FN} - \bigcup_{k\in\{k_0,k_1,\ldots,k_h\}} J_{FN}(V_{FN,k})$ is taken to determine the post-fault event count across all such indices for the path p_{FN}:

$$w_{FN}(p_{FN}) := \min_{i_{FN} \in I_{FN} - \cup_{k\in\{k_0,\ldots,k_h\}} J_{FN}(V_{FN,k})} w_{FN,i_{FN}}(p_{FN}).$$

Finally a maximum of the counts along all paths in $\mathcal{P}_{FN}$ is taken to obtain the required delay bound of diagnosis:

$$w_{FN} := \begin{cases} \max_{p_{FN}\in\mathcal{P}_{FN}} w_{FN}(p_{FN}), & \text{if } \mathcal{P}_{FN} \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}$$

Since $\mathcal{P}_{FN}$ is finite, w_{FN} is effectively computable.

Theorem 2: Assume that $\mathbb{N}_{FN} \neq \emptyset$. Then, $m^*_{FN} = w_{FN}$.

Dually we can use $\mathcal{T}_{HN}$ to compute m^*_{HN} when $\mathbb{N}_{HN} \neq \emptyset$. Simply replace the subscript "FN" by "HN" in the computation of m^*_{FN} given earlier (such as in the definitions of the various post-fault event counts). Then similar to Theorem 2, we have the following result.

Theorem 3: Assume that $\mathbb{N}_{HN} \neq \emptyset$. Then, $m^*_{HN} = w_{HN}$.

Remark 1: To compute the delay bound m^*_N using Theorems 2 and 3, we first need to construct the finite automata T_{FN} and T_{HN}. By the definitions of T_{FN} and T_{HN}, the complexity of constructing those is exponential with respect to N and n. Next, we construct the acyclic automata $\mathcal{T}_{FN}$ and $\mathcal{T}_{HN}$ by identifying all max-SCCs of T_{FN} and T_{HN}. This complexity is $O((|R_{FN}| + |R_{HN}|) \times |\Sigma|^{2N+3})$. Then, m^*_N can be obtained by exploring all paths of $\mathcal{T}_{FN}$ and $\mathcal{T}_{HN}$ that end with marked states.

V. Conclusion

In [6], we presented a framework for inference-based decentralized diagnosis, supporting multi-level inferencing for disjunctive as well as conjunctive decision making, and introduced the notion of N-inference diagnosability to characterize the class of diagnosable systems. This paper supplements the work of [6] by providing a method to compute the delay bound of diagnosis for N-inference diagnosable systems. It is expected that this delay bound will be smaller compared to the one achieved under any other framework subsumed by the N-inference diagnosability (such as disjunctive and/or conjunctive diagnosability, with or without conditional decisions).

References

[1] R. Debouk, S. Lafortune, and D. Teneketzis, "Coordinated decentralized protocols for failure diagnosis of discrete event systems," *Discrete Event Dyna. Syst.: Theory Appl.*, vol. 10, no. 1&2, pp. 33–86, 2000.

[2] R. Kumar and S. Takai, "Inference-based ambiguity management in decentralized decision-making: Decentralized diagnosis of discrete-event systems," *IEEE Trans. Autom. Sci. Eng.*, vol. 6, no. 3, pp. 479–491, 2009.

[3] W. Qiu and R. Kumar, "Decentralized failure diagnosis of discrete event systems," *IEEE Trans. Syst., Man, Cybern., Part A: Syst. Humans*, vol. 36, no. 2, pp. 384–395, 2006.

[4] M. Sampath, R. Sengupta, S. Lafortune, K. Sinnamohideen, and D. Teneketzis, "Diagnosability of discrete-event systems," *IEEE Trans. Autom. Control*, vol. 40, no. 9, pp. 1555–1575, 1995.

[5] S. Takai and R. Kumar, "Verification of generalized inference diagnosability for decentralized diagnosis of discrete event systems," in *Proc. 20th IEEE Int. Conf. Emerging Tech. Factory Autom.*, 2015.

[6] S. Takai and R. Kumar, "A generalized inference-based diagnosis framework for discrete event systems capturing both disjunctive and conjunctive decision-making," in *Proc. 54th IEEE Conf. Decision and Control*, pp. 3522–3527, 2015.

[7] Y. Wang, T.-S. Yoo, and S. Lafortune, "Diagnosis of discrete event systems using decentralized architectures," *Discrete Event Dyna. Syst.: Theory Appl.*, vol. 17, no. 2, pp. 233–263, 2007.

[8] T. Yamamoto and S. Takai, "Reliable decentralized diagnosis of discrete event systems using the conjunctive architecture," *IEICE Trans. Fundamentals*, vol. E97-A, no. 7, pp. 1605–1614, 2014.

[9] S. Yokota and S. Takai, "Computation of the delay bound in decentralized diagnosis of discrete event systems with conditional decisions," in *Proc. 53rd IEEE Conf. Decision and Control*, pp. 5961–5966, 2014.

[10] J. Zaytoon and S. Lafortune, "Overview of fault diagnosis methods for Discrete Event Systems," *Annual Reviews in Control*, vol. 37, no. 2, pp. 308–320, Dec. 2013.