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A Generalized Inference-Based Prognosis Framework for Discrete Event Systems

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Abstract: We previously introduced an inference-based framework for decentralized prognosis of discrete event systems, and formulated the notion of N-inference $\vee$-prognosability to characterize the existence of a *disjunctive* decentralized prognosis scheme such that any fault can be predicted prior to its occurrence by at least one of the prognosers, using at most N-levels of inferencing. While the *disjunctive* scheme relies on *one of the prognosers making a positive decision*, the dual *conjunctive* scheme relies on *none of the prognosers making a negative decision*. It is known that the two schemes are incomparable, and in this paper we extend our earlier work to provide a more general framework, by introducing a notion of N-inference prognosability, that captures both disjunctive and conjunctive schemes. We also develop a method for verifying N-inference prognosability.

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Keywords: Discrete event system, Decentralized prognosis, Inference prognosability

1. INTRODUCTION

The task of failure prognosis is to predict an impending failure prior to its occurrence. For discrete event systems, a notion of uniformly-bounded predictability of failures was formulated in Genc and Lafortune (2009) as follows: Each failure trace must possess a nonfailure prefix such that any indistinguishable trace has the property that a failure is inevitable within a uniformly bounded number of steps. While the existence of a uniform bound for the delay of failure detection is essential for defining diagnosability (otherwise a diagnoser may end up waiting for an arbitrarily long period before diagnosing a failure), the existence of a uniform bound within which a failure is guaranteed to occur is not essential for defining prognosability. This observation led us to weaken the definition of prognosability in Kumar and Takai (2010) as follows: Each failure trace must possess a nonfailure prefix such that for any indistinguishable trace, a failure in future is guaranteed.

For large, physically distributed systems, decentralized prognosis is employed, where multiple local prognosers that rely on their own subsets of accessible sensors make local prognosis decisions that are fused to decide a global one. See for example Kumar and Takai (2010), Khoumsi and Chakib (2012), and Yin and Li (2016). The notion of coprognosability that captures the property that any fault can be predicted by one of the local prognosers was introduced in Kumar and Takai (2010). This scheme, where at least one prognoser issues a positive decision, is "disjunctive" in nature. In contrast, a dual "conjunctive" scheme, that is incomparable to the disjunctive one, was later proposed in Khoumsi and Chakib (2012), where a fault is predicted when none of the prognosers issue a negative decision. These notions of coprognosability were extended to guarantee that the occurrence of any fault can be predicted k steps before (Khoumsi and Chakib (2012); Yin and Li (2016)).

A general framework for inference-based decentralized decision-making was introduced in Kumar and Takai (2007) and adopted to the case of diagnosis in Kumar and Takai (2009) and Takai and Kumar (2017), and to prognosis in Takai and Kumar (2011). In the inference-based setting, each prognoser uses not only its own knowledge of the system behaviors, but also the inference about the possible knowledge of the system behaviors of other prognosers to arrive at its own local decision. The "winning" local decision (namely, the one needing the least level of inferencing) is set as the global decision. While the inference-based frameworks for control (Kumar and Takai (2007)) and diagnosis (Takai and Kumar (2017)) were general enough to capture both disjunctive and conjunctive decision-making, the work of Takai and Kumar (2011) for prognosis employs only the disjunctive scheme, and requires that the indicator traces (namely, traces for which a fault is inevitable) be discriminated from the nonindicator ones (namely, the complement of indicator traces).

The main contribution of the present work is to introduce a generalized framework for inference-based decentralized prognosis that supports *both* the disjunctive and conjunctive schemes of decision-making. It is shown that this requires that only the boundary nonfaulty traces (those for which some one step extension results in a fault) be discriminated from the nonindicator ones. This is a weaker

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requirement than the one stated in the last paragraph for disjunctive prognosis. A notion of N-inference prognosability is formulated and its necessity and sufficiency are proved. In addition, a method for verifying N-inference prognosability is developed.

2. NOTATION AND PRELIMINARIES

An automaton G is defined as $G = (Q, \Sigma, \delta, q_0, Q_m)$, where Q is the set of states, Σ is the finite set of events, $\delta : Q \times \Sigma \to Q$ is the partial transition function, $q_0 \in Q$ is the initial state, and $Q_m \subseteq Q$ is the set of marked states. Let Σ^* be the set of all finite traces of elements of Σ, including the empty trace ε. The function δ can be generalized to $\delta : Q \times \Sigma^* \to Q$ in the usual manner. The generated and marked languages of G, denoted by $L(G)$ and $L_m(G)$, respectively, are defined as $L(G) = \{s \in \Sigma^* \mid \delta(q_0, s)!\}$ and $L_m(G) = \{s \in \Sigma^* \mid \delta(q_0, s) \in Q_m\}$, where $\delta(q, s)!$ denotes that $\delta(q, s)$ is defined for each $q \in Q$ and each $s \in \Sigma^*$. For each trace $s \in \Sigma^*$, $|s|$ denotes its length. For any $m \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of all nonnegative integers, let $\Sigma^{\geq m} := \{s \in \Sigma^* \mid |s| \geq m\}$.

Let $K \subseteq \Sigma^*$ be a language. The set of all prefixes of traces in K is denoted by $pr(K)$, i.e., $pr(K) = \{s \in \Sigma^* \mid \exists t \in \Sigma^* : st \in K\}$. If $K = pr(K)$, then K is said to be prefix-closed. A prefix-closed language K is said to be *deadlock-free* if for any $s \in K$, $\{s\}\Sigma \cap K \neq \emptyset$. For a trace $s \in \Sigma^*$, $pr(\{s\})$ is denoted by $pr(s)$, and the language K after s, denoted by $K\backslash s$, is defined as $K\backslash s = \{t \in \Sigma^* \mid st \in K\}$.

Let $I = \{1, 2, \ldots, n\}$ denote the index set of local prognosers that perform the task of prognosis without sharing their observations. We assume that the limited sensing capabilities of the ith local prognoser P_i ($i \in I$) can be represented as the local observation mask, $M_i : \Sigma \to \Delta_i \cup \{\varepsilon\}$, where Δ_i is the set of locally observed symbols. The local observation mask M_i is extended to $M_i : \Sigma^* \to \Delta_i^*$ in the usual manner. Two traces $s, s' \in \Sigma^*$ with $M_i(s) = M_i(s')$ are said to be M_i-indistinguishable. The inverse map of M_i, denoted by $M_i^{-1} : \Delta_i^* \to 2^{\Sigma^*}$, is defined as $M_i^{-1}(t) = \{s \in \Sigma^* \mid M_i(s) = t\}$ for each $t \in \Delta_i^*$. For any languages $L \subseteq \Sigma^*$ and $L' \subseteq \Delta_i^*$, $M_i(L) \subseteq \Delta_i^*$ and $M_i^{-1}(L') \subseteq \Sigma^*$ are defined as $M_i(L) = \{M_i(s) \in \Delta_i^* \mid s \in L\}$ and $M_i^{-1}(L') = \{s \in \Sigma^* \mid M_i(s) \in L'\}$, respectively.

Let $L \neq \emptyset$ be a prefix-closed language representing the generated language of a plant modeled as an automaton $G = (Q, \Sigma, \delta, q_0, Q)$, and $K \subseteq L$ be a nonempty prefix-closed language representing a nonfailure specification. Traces in $L - K$ are considered failure traces and the task of prognosis is to predict the execution of any trace in $L - K$. Without loss of generality, the plant language L can be taken to be deadlock-free (Kumar and Takai (2010)).

3. EXISTING NOTIONS OF PROGNOSABILITY FOR DECENTRALIZED PROGNOSIS

In this section, we review the existing notions of prognosability of the pair (L, K) of languages for decentralized prognosis. Notions of *boundary* traces (for which a failure can occur in a next step), *indicator* traces (for which a failure in future is guaranteed), and *nonindicator* traces (that are not indicator traces) are defined as follows:

Definition 1. (Kumar and Takai (2010)) For the pair (L, K) of languages, we define the set of

- *boundary* traces of K with respect to L as
$$\partial_L(K) := \{s \in K \mid \{s\}\Sigma \cap (L - K) \neq \emptyset\},$$
- *indicator* traces of K with respect to L as
$$\Im_L(K) := \{s \in K \mid \exists m \in \mathbb{N} : \{s\}\Sigma^{\geq m} \cap L \subseteq L - K\},$$
- *nonindicator* traces of K with respect to L as
$$\begin{aligned}\Upsilon_L(K) &:= K - \Im_L(K) \\ &= \{s \in K \mid \forall m \in \mathbb{N} : \{s\}\Sigma^{\geq m} \cap K \neq \emptyset\}.\end{aligned}$$

The existing notions of disjunctive and conjunctive-coprognosability that do not involve inferencing were defined as follows:

Definition 2. (Kumar and Takai (2010); Khoumsi and Chakib (2012)) The pair (L, K) of languages is said to be *disjunctive-coprognosable* if
$$\partial_L(K) \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(\Upsilon_L(K))\right) = \emptyset.$$
It is also said to be *conjunctive-coprognosable* if
$$\Upsilon_L(K) \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(\partial_L(K))\right) = \emptyset.$$

In our prior work, we have introduced a framework that allows multi-level inferencing over various local prognosis decisions of varying ambiguity levels, and defined a notion of N-inference $\vee$-prognosability (Takai and Kumar (2011)). A monotonically decreasing sequence $\{(\Im_k, \Upsilon_k^{\Im})\}_{k \geq 0}$ of language pairs is defined as follows:

- Base step:
$$\Im_0 := \Im_L(K),\ \Upsilon_0^{\Im} := \Upsilon_L(K).$$
- Induction step:
$$\Im_{k+1} := \Im_k \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(\Upsilon_k^{\Im})\right),$$
$$\Upsilon_{k+1}^{\Im} := \Upsilon_k^{\Im} \cap \left(\bigcap_{i \in I} M_i^{-1} M_i(\Im_k)\right).$$

Then N-inference $\vee$-prognosability is defined as follows:

Definition 3. (Takai and Kumar (2011)) The pair (L, K) of languages is said to be *N-inference $\vee$-prognosable* if
$$\partial_L(K) \subseteq \Im_L(K) - \Im_{N+1}.$$

It was shown in Takai and Kumar (2011) that 0-inference $\vee$-prognosability is equivalent to disjunctive-coprognosability. Since disjunctive-coprognosability is incomparable with conjunctive-coprognosability (Khoumsi and Chakib (2012)), the inference-based prognosis framework of Takai and Kumar (2011) does not capture the conjunctive scheme.

4. INFERENCE-BASED PROGNOSIS

Let $C = \{1, 0, \phi\}$ be the set of prognosis decisions, where "1" represents a positive decision, "0" represents a negative decision, and "ϕ" represents an unsure decision. Each inference-based local prognoser P_i is defined as a

map $P_i : M_i(K) \to C \times \mathbb{N}$ (Takai and Kumar (2011)), where for each $s \in K$, $P_i(M_i(s)) = (c_i(M_i(s)), n_i(M_i(s)))$. Here $c_i(M_i(s)) \in C$ denotes the prognosis decision of P_i following an observation $M_i(s) \in M_i(K)$, and $n_i(M_i(s)) \in \mathbb{N}$ denotes the ambiguity level of the prognosis decision of P_i. Let $n(s) \in \mathbb{N}$ be the minimum ambiguity level of local decisions (Takai and Kumar (2011)), i.e., $n(s) = \min_{i\in I} n_i(M_i(s))$.

The decentralized prognoser $\{P_i\}_{i\in I}$ that consists of local prognosers P_i $(i \in I)$ issues global prognosis decisions. Formally, $\{P_i\}_{i\in I}$ is defined as a map $\{P_i\}_{i\in I} : K \to C$. For each $s \in K$, the prognosis decision $\{P_i\}_{i\in I}(s)$ is given as follows (Takai and Kumar (2011)):

$$\{P_i\}_{i\in I}(s) = \begin{cases} 1, \text{ if } \forall i \in I : n_i(M_i(s)) = n(s) \\ \qquad \Rightarrow c_i(M_i(s)) = 1 \\ 0, \text{ if } \forall i \in I : n_i(M_i(s)) = n(s) \\ \qquad \Rightarrow c_i(M_i(s)) = 0 \\ \phi, \text{ otherwise.} \end{cases}$$

A useful notion of a decentralized prognoser is the boundedness of the ambiguity level of its decisions, defined as follows:

Definition 4. A decentralized prognoser $\{P_i\}_{i\in I} : K \to C$ is said to be *N-inferring* if the following two conditions hold:

- Either
$$\forall s \in \partial_L(K) : \{P_i\}_{i\in I}(s) = 1 \Rightarrow n(s) \le N,$$
or
$$\forall s \in \Upsilon_L(K) : \{P_i\}_{i\in I}(s) \neq 1 \Rightarrow n(s) \le N.$$
- $\forall s \in \partial_L(K) \cup \Upsilon_L(K) : n(s) \le N \Rightarrow \{P_i\}_{i\in I}(s) \neq \phi.$

5. GENERALIZED FRAMEWORK FOR INFERENCE-BASED PROGNOSIS

Let $N \in \mathbb{N}$ be a given nonnegative integer. (N represents a parameter of inference prognosability and is elaborated later.) Given a plant language L and a nonfailure specification language $K \subseteq L$, we inductively define a monotonically decreasing sequence $\{(\partial_k, \Upsilon_k^\partial)\}_{k\ge 0}$ of language pairs as follows:

- Base step:
$$\partial_0 := \partial_L(K), \ \Upsilon_0^\partial := \Upsilon_L(K).$$
- Induction step:
$$\partial_{k+1} := \partial_k \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(\Upsilon_k^\partial) \right),$$
$$\Upsilon_{k+1}^\partial := \Upsilon_k^\partial \cap \left(\bigcap_{i\in I} M_i^{-1} M_i(\partial_k) \right).$$

Remark 5. It is clear from the definition of the sequence $\{(\partial_k, \Upsilon_k^\partial)\}_{k\ge 0}$, which can be obtained by replacing $\Im_L(K)$ by $\partial_L(K)$ in the definition of the earlier sequence $\{(\Im_k, \Upsilon_k^\Im)\}_{k\ge 0}$ utilized in the case of N-inference $\vee$-diagnosability, that the generalized framework seeks to discriminate between the boundary and nonindicator traces, which is a weaker requirement than that of discriminating between the indicator and nonindicator traces. The definitions of disjunctive coprognosability as well as conjunctive coprognosability also seek to discriminate between the boundary and nonindicator traces (see Definition 2), but without using any inferencing.

Then we have the following definition of N-inference prognosability.

Definition 6. The pair (L, K) of languages is said to be *N-inference prognosable* if $\partial_{N+1} = \emptyset$ or $\Upsilon_{N+1}^\partial = \emptyset$.

Remark 7. Since $\partial_0 = \partial_L(K)$ and $\Upsilon_0^\partial = \Upsilon_L(K)$, 0-inference prognosability subsumes both disjunctive-coprognosability and conjunctive-coprognosability.

As shown in the following proposition, N-inference prognosability is weaker than N-inference $\vee$-prognosability introduced in Takai and Kumar (2011).

Proposition 8. For any $N \in \mathbb{N}$, if the pair (L, K) of languages is N-inference $\vee$-prognosable, then it is N-inference prognosable.

The converse relation of Proposition 8 need not hold.

The following theorem shows that N-inference prognosability is a necessary and sufficient condition for the existence of an N-inferring decentralized prognoser with no missed and incorrect detections.

Theorem 9. There exists an N-inferring decentralized prognoser $\{P_i\}_{i\in I} : K \to C$ that satisfies

$$\forall s \in L - K, \exists t \in pr(s) \cap K, \\ \forall u \in (pr(s) \cap K)\backslash t : \{P_i\}_{i\in I}(tu) = 1, \quad (1)$$

and

$$\forall s \in \Upsilon_L(K) : \{P_i\}_{i\in I}(s) \neq 1. \quad (2)$$

if and only if the pair (L, K) of languages is N-inference prognosable.

Assume that the pair (L, K) of languages is N-inference prognosable so that $\partial_{N+1} = \emptyset$ or $\Upsilon_{N+1}^\partial = \emptyset$. Then a local prognoser can compute its prognosis decision and associate an ambiguity level with it as follows. For each $s \in K$, the ith local prognoser P_i computes

$$n_i^\partial(M_i(s)) := \min\{k \in \mathbb{N} \mid M_i(s) \notin M_i(\Upsilon_k^\partial)\}, \quad (3)$$
$$n_i^\Upsilon(M_i(s)) := \min\{k \in \mathbb{N} \mid M_i(s) \notin M_i(\partial_k)\}. \quad (4)$$

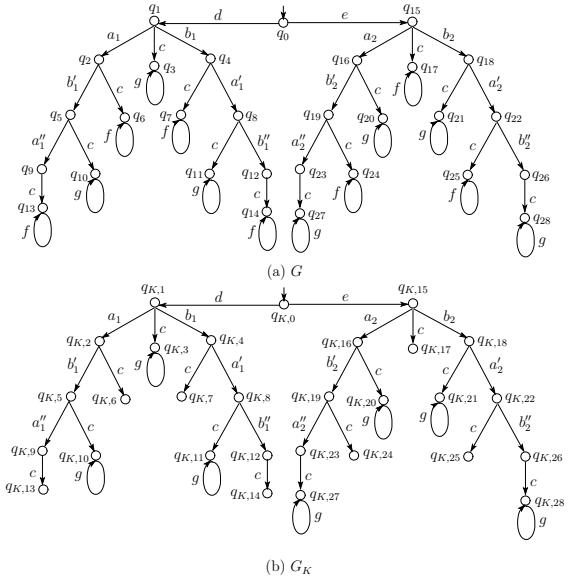
Note that since $\partial_{N+1} = \emptyset$ and $\Upsilon_{N+1}^\partial = \emptyset$ imply $\Upsilon_{N+2}^\partial = \emptyset$ and $\partial_{N+2} = \emptyset$, respectively, both $n_i^\partial(M_i(s))$ and $n_i^\Upsilon(M_i(s))$ are bounded above by $N + 2$. Here $n_i^\partial(M_i(s))$ represents the ambiguity level of a positive decision contemplated by the ith prognoser following an observation $M_i(s)$. Similarly, the notation $n_i^\Upsilon(M_i(s))$ represents the ambiguity level of a negative decision contemplated by the ith prognoser following an observation $M_i(s)$.

For a local prognoser $P_i : M_i(K) \to C \times \mathbb{N}$, its prognosis decision and ambiguity level following an observation $M_i(s) \in M_i(K)$, i.e., $P_i(M_i(s)) = (c_i(M_i(s)), n_i(M_i(s)))$, are determined as follows (Takai and Kumar (2011)):

$$c_i(M_i(s)) = \begin{cases} 1, \text{ if } n_i^\partial(M_i(s)) < n_i^\Upsilon(M_i(s)) \\ 0, \text{ if } n_i^\Upsilon(M_i(s)) < n_i^\partial(M_i(s)) \\ \phi, \text{ if } n_i^\partial(M_i(s)) = n_i^\Upsilon(M_i(s)) \end{cases} \quad (5)$$

and

$$n_i(M_i(s)) = \min\{n_i^\partial(M_i(s)), n_i^\Upsilon(M_i(s))\}, \quad (6)$$



(a) G

(b) G_K

Fig. 1. Automata G and G_K of Example 10.

i.e., the prognosis decision with the smaller ambiguity level is chosen. It can be verified that the decentralized prognoser $\{P_i\}_{i\in I}$ for which the local prognosers are given by (3)–(6) is N-inferring and satisfies (1) and (2).

Example 10. We consider a plant modeled by a finite automaton G shown in Fig. 1(a). Let $n = 2$, $\Delta_1 = \{a, a', a'', c, d, e\}$, $\Delta_2 = \{b, b', b'', c, d, e\}$, and

$$M_1(\sigma) = \begin{cases} a, & \text{if } \sigma \in \{a_1, a_2\} \\ a', & \text{if } \sigma \in \{a'_1, a'_2\} \\ a'', & \text{if } \sigma \in \{a''_1, a''_2\} \\ \sigma, & \text{if } \sigma \in \{c, d, e\} \\ \varepsilon, & \text{otherwise,} \end{cases}$$

$$M_2(\sigma) = \begin{cases} b, & \text{if } \sigma \in \{b_1, b_2\} \\ b', & \text{if } \sigma \in \{b'_1, b'_2\} \\ b'', & \text{if } \sigma \in \{b''_1, b''_2\} \\ \sigma, & \text{if } \sigma \in \{c, d, e\} \\ \varepsilon, & \text{otherwise.} \end{cases}$$

In addition, let $K \subseteq L$ be a prefix-closed language generated by a finite automaton G_K shown in Fig. 1(b), i.e., the event f denotes the occurrence of a failure.

First, we show that (L, K) is 3-inference prognosable. Initially, we have $\partial_0 = d(a_1c + b_1c + a_1b'_1a''_1c + b_1a'_1b''_1c) + e(c + a_2b'_2c + b_2a'_2c)$ and $\Upsilon^{\partial}_0 = d(cg^* + a_1b'_1cg^* + b_1a'_1cg^*) + e(a_2cg^* + b_2cg^* + a_2b'_2a''_2(\varepsilon + cg^*) + b_2a'_2b''_2(\varepsilon + cg^*))$. By the iterative computations, we obtain $\partial_3 = ec$ and $\Upsilon^{\partial}_3 = dcg^*$, and finally, $\partial_4 = \Upsilon^{\partial}_4 = \emptyset$, which implies that (L, K) is 3-inference prognosable.

The local decisions of P_1 and P_2 are computed by (3)–(6). Using these local decisions, the global decisions of the decentralized prognoser $\{P_i\}_{i\in I}$ are computed as shown in Table 1. By Table 1, we can verify that $\{P_i\}_{i\in I}$ is a 3-inferring decentralized prognoser that satisfies (1) and (2).

6. VERIFICATION OF N-INFERENCE PROGNOSABILITY

In this section, we show how to verify N-inference prognosability for a pair (L, K) of regular languages. Since

Table 1. Global decisions of $\{P_i\}_{i\in I}$.

$s \in K$	$n(s)$	$\{P_i\}_{i\in I}(s)$
$s \in pr(d(a_1b'_1a''_1 + b_1a'_1b''_1))$	0	ϕ
$s \in dcg^*$	3	0
$s \in d(a_1c + b_1c)$	2	1
$s \in d(a_1b'_1cg^* + b_1a'_1cg^*)$	1	0
$s \in d(a_1b'_1a''_1c + b_1a'_1b''_1c)$	0	1
$s \in pr(e(a_2b'_2 + b_2a'_2))$	0	ϕ
$s = ec$	3	1
$s \in e(a_2cg^* + b_2cg^*)$	2	0
$s \in e(a_2b'_2c + b_2a'_2c)$	1	1
$s \in e(a_2b'_2a''_2(\varepsilon + cg^*) + b_2a'_2b''_2(\varepsilon + cg^*))$	0	0

0-inference prognosability is the same as disjunctive-coprognosability or conjunctive-coprognosability, for which verification methods have already been established, we consider the case of $N \geq 1$. For simplicity of presentation, we consider the case of two local prognosers, i.e., $I = \{1, 2\}$. The results continue to hold for an arbitrary number of local prognosers.

Violation of N-inference prognosability of (L, K) requires that $\partial_{N+1} \neq \emptyset$ and $\Upsilon^{\partial}_{N+1} \neq \emptyset$. As shown in the following proposition, the nonemptiness of ∂_{N+1} and $\Upsilon^{\partial}_{N+1}$ can be characterized by the existence of certain $2N + 3$ traces.

Proposition 11. Consider the pair (L, K) of languages.

- $\partial_{N+1} \neq \emptyset$ if and only if there exist $2N + 3$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in K$ such that
  - $s_0 \in \partial_0$,
  - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\}$:
    $$s_{ii'} \in \begin{cases} \Upsilon^{\partial}_0, & \text{if } i' \text{ is an even number} \\ \partial_0, & \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i \in I : M_i(s_0) = M_i(s_{i0})$,
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
    $$\begin{cases} M_2(s_{1i'}) = M_2(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(s_{1i'}) = M_1(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
    $$\begin{cases} M_1(s_{2i'}) = M_1(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(s_{2i'}) = M_2(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$
- $\Upsilon^{\partial}_{N+1} \neq \emptyset$ if and only if there exist $2N + 3$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N} \in K$ such that
  - $s_0 \in \Upsilon^{\partial}_0$,
  - $\forall i \in I, \forall i' \in \{0, 1, \ldots, N\}$:
    $$s_{ii'} \in \begin{cases} \partial_0, & \text{if } i' \text{ is an even number} \\ \Upsilon^{\partial}_0, & \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i \in I : M_i(s_0) = M_i(s_{i0})$,
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
    $$\begin{cases} M_2(s_{1i'}) = M_2(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_1(s_{1i'}) = M_1(s_{1(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number,} \end{cases}$$
  - $\forall i' \in \{0, 1, \ldots, N-1\}$:
    $$\begin{cases} M_1(s_{2i'}) = M_1(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an even number} \\ M_2(s_{2i'}) = M_2(s_{2(i'+1)}), \\ \quad \text{if } i' \text{ is an odd number.} \end{cases}$$

We develop a method for verifying N-inference prognosability based on Proposition 11. We first test the emptiness of ∂_{N+1}. For this, we construct an automaton that accepts all $2N+3$ traces satisfying the five conditions of the first part of Proposition 11. We begin by refining the nonfailure specification model with respect to the plant model by taking the synchronous product. Let $G=(Q,\Sigma,\delta,q_0,Q)$ be a finite plant model with $L(G)=L$ and $G_K=(Q_K,\Sigma,\delta_K,q_{K,0},Q_K)$ be a finite generator of the nonfailure specification language, i.e., $L(G_K)=K$. The synchronous product $G \parallel G_K$ is denoted by $G \parallel G_K = (Q\times Q_K,\Sigma,\xi,(q_0,q_{K,0}),Q\times Q_K)$.

The following definition identifies boundary states (from which a failure can happen in a next step) and nonindicator states (from which a failure is not inevitable).

Definition 12. For a finite plant model $G=(Q,\Sigma,\delta,q_0,Q)$ and a finite generator $G_K=(Q_K,\Sigma,\delta_K,q_{K,0},Q_K)$ of the nonfailure specification language $K\subseteq L$,

- the set $\partial(Q\times Q_K)\subseteq Q\times Q_K$ of *boundary* states is the set of all states (q,q_K) of $G\parallel G_K$ such that there exists $\sigma\in\Sigma$ with $\delta(q,\sigma)!$ and $\neg\delta_K(q_K,\sigma)!$, where $\neg\delta_K(q_K,\sigma)!$ denotes the negation of $\delta_K(q_K,\sigma)!$, and
- the set of $\Upsilon(Q_K)\subseteq Q_K$ of *nonindicator* states is the set of all states of G_K from which a cycle in G_K can be reached.

The index set of $2N+3$ traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N}\in K$ of Proposition 11 is denoted by

$$I_N=\{0,10,11,\ldots,1N,20,21,\ldots,2N\}.$$

Among these $2N+3$ traces, $s_0, s_{11}, s_{13}, \ldots, s_{1l}, s_{21}, s_{23}, \ldots, s_{2l}\in K$ are boundary traces, where

$$l=\begin{cases} N-1, & \text{if } N \text{ is an even number}\\ N, & \text{if } N \text{ is an odd number,}\end{cases}$$

and their index set is denoted by

$$I_{N,odd}=\{0,11,13,\ldots,1l,21,23,\ldots,2l\}.$$

Other traces are nonindicator ones. As a finite acceptor of all traces $s_0, s_{10}, s_{11}, \ldots, s_{1N}, s_{20}, s_{21}, \ldots, s_{2N}\in K$ that satisfy the five conditions of the first part of Proposition 11, we define

$$T_{\partial N}=(R_{\partial N},\Sigma_{\partial N},\alpha_{\partial N},r_{\partial N,0},R_{\partial N,m})$$

as follows:

- The state set $R_{\partial N}$ is defined as $R_{\partial N}=(Q\times Q_K)\times Q_{10}\times Q_{11}\times\cdots\times Q_{1N}\times Q_{20}\times Q_{21}\times\cdots\times Q_{2N}$, where
$$Q_{ii'}=\begin{cases} Q_K, & \text{if } i' \text{ is an even number}\\ Q\times Q_K, & \text{if } i' \text{ is an odd number}\end{cases}$$
for any $i\in I$ and any $i'\in\{0,1,\ldots,N\}$.
- The initial state $r_{\partial N,0}\in R_{\partial N}$ is given as
$$r_{\partial N,0}=((q_0,q_{K,0}),q_{10,0},q_{11,0},\ldots,q_{1N,0},q_{20,0},q_{21,0},\ldots,q_{2N,0}),$$
where
$$q_{ii',0}=\begin{cases} q_{K,0}, & \text{if } i' \text{ is an even number}\\ (q_0,q_{K,0}), & \text{if } i' \text{ is an odd number}\end{cases}$$
for any $i\in I$ and any $i'\in\{0,1,\ldots,N\}$.
- The set $R_{\partial N,m}$ of marked states is defined as
$$R_{\partial N,m}=\{r_{\partial N}\in R_{\partial N}\mid [\forall i_N\in I_{N,odd}: r_{\partial N}(i_N)\in\partial(Q\times Q_K)] \wedge[\forall i_N\in I_N-I_{N,odd}: r_{\partial N}(i_N)\in\Upsilon(Q_K)]\},$$
where, for each
$r_{\partial N}=((q,q_K),q_{10},q_{11},\ldots,q_{1N},q_{20},q_{21},\ldots,q_{2N})$
in $R_{\partial N}$, we let $r_{\partial N}(0)=(q,q_K)$ and $r_{\partial N}(ii')=q_{ii'}$ for each $i\in I$ and each $i'\in\{0,1,\ldots,N\}$.
- The event set $\Sigma_{\partial N}$ is defined as
$$\Sigma_{\partial N}=\underbrace{(\Sigma\cup\{\varepsilon\})\times(\Sigma\cup\{\varepsilon\})\times\cdots\times(\Sigma\cup\{\varepsilon\})}_{(2N+3)\text{ times}}-\{(\varepsilon,\varepsilon,\ldots,\varepsilon)\}.$$
- For each
$r_{\partial N}=((q,q_K),q_{10},\ldots,q_{1N},q_{20},\ldots,q_{2N})\in R_{\partial N}$
and each
$\sigma_{\partial N}=(\sigma,\sigma_{10},\ldots,\sigma_{1N},\sigma_{20},\ldots,\sigma_{2N})\in\Sigma_{\partial N}$,
$\alpha_{\partial N}(r_{\partial N},\sigma_{\partial N})!$ if the following five conditions are satisfied:
  - $\sigma\neq\varepsilon\Rightarrow\xi((q,q_K),\sigma)!$,
  - $\forall i\in I,\forall i'\in\{0,1,\ldots,N\}:\sigma_{ii'}\neq\varepsilon\Rightarrow$
$$\begin{cases}\delta_K(q_{ii'},\sigma_{ii'})!, & \text{if } i' \text{ is an even number}\\ \xi(q_{ii'},\sigma_{ii'})!, & \text{if } i' \text{ is an odd number,}\end{cases}$$
  - $\forall i\in I: M_i(\sigma)=M_i(\sigma_{i0})$,
  - $\forall i'\in\{0,1,\ldots,N-1\}$:
$$\begin{cases} M_2(\sigma_{1i'})=M_2(\sigma_{1(i'+1)}), & \text{if } i' \text{ is an even number}\\ M_1(\sigma_{1i'})=M_1(\sigma_{1(i'+1)}), & \text{if } i' \text{ is an odd number,}\end{cases}$$
  - $\forall i'\in\{0,1,\ldots,N-1\}$:
$$\begin{cases} M_1(\sigma_{2i'})=M_1(\sigma_{2(i'+1)}), & \text{if } i' \text{ is an even number}\\ M_2(\sigma_{2i'})=M_2(\sigma_{2(i'+1)}), & \text{if } i' \text{ is an odd number.}\end{cases}$$

  If $\alpha_{\partial N}(r_{\partial N},\sigma_{\partial N})!$, then
$$\alpha_{\partial N}(r_{\partial N},\sigma_{\partial N})=((q',q'_K),q'_{10},q'_{11},\ldots,q'_{1N},q'_{20},q'_{21},\ldots,q'_{2N}),$$
where
$$(q',q'_K)=\begin{cases}\xi((q,q_K),\sigma), & \text{if } \sigma\neq\varepsilon\\ (q,q_K), & \text{otherwise}\end{cases}$$
and
$$q'_{ii'}=\begin{cases}\delta_K(q_{ii'},\sigma_{ii'}), & \text{if } \sigma_{ii'}\neq\varepsilon\wedge[i' \text{ is an even number}]\\ \xi(q_{ii'},\sigma_{ii'}), & \text{if } \sigma_{ii'}\neq\varepsilon\wedge[i' \text{ is an odd number}]\\ q_{ii'}, & \text{otherwise.}\end{cases}$$

For each $\sigma_{\partial N}=(\sigma,\sigma_{10},\sigma_{11},\ldots,\sigma_{1N},\sigma_{20},\sigma_{21},\ldots,\sigma_{2N})$ in $\Sigma_{\partial N}$, we let $\sigma_{\partial N}(0)=\sigma$ and $\sigma_{\partial N}(ii')=\sigma_{ii'}$ for each $i\in I$ and each $i'\in\{0,1,\ldots,N\}$. Furthermore, for a trace $s_{\partial N}=\sigma_{\partial N,1}\sigma_{\partial N,2}\cdots\sigma_{\partial N,l}\in\Sigma^*_{\partial N}$ $(l\geq 1)$, we let $s_{\partial N}(i_N)=\sigma_{\partial N,1}(i_N)\sigma_{\partial N,2}(i_N)\cdots\sigma_{\partial N,l}(i_N)$ for each $i_N\in I_N$. In addition, let $\varepsilon(i_N)=\varepsilon$ for each $i_N\in I_N$.

The following proposition shows a necessary and sufficient condition for the set ∂_{N+1} to be empty.

Proposition 13. Consider the pair (L,K) of regular languages. Then, $\partial_{N+1}=\emptyset$ if and only if $L_m(T_{\partial N})=\emptyset$.

Dually to check the emptiness of $\Upsilon^{\partial}_{N+1}$, we define

$$T_{\Upsilon N}=(R_{\Upsilon N},\Sigma_{\Upsilon N},\alpha_{\Upsilon N},r_{\Upsilon N,0},R_{\Upsilon N,m})$$

as follows:

Table 2. Complexity for constructing automata $T_{\partial N}$ and $T_{\Upsilon N}$.

	Complexity of constructing $T_{\partial N}$
Arbitrary even N	$O(\lvert Q\rvert^{1+\sum_{k=1}^{N/2} n(n-1)^{2k-1}} \cdot (\lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O(\lvert Q\rvert^{O(n(n-1)^{N-1})} \cdot (\lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{O(n(n-1)^N)})$
Arbitrary odd N	$O(\lvert Q\rvert^{1+\sum_{k=1}^{(N+1)/2} n(n-1)^{2k-1}} \cdot (\lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O((\lvert Q\rvert \cdot \lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{O(n(n-1)^N)})$
	Complexity of constructing $T_{\Upsilon N}$
Arbitrary even N	$O(\lvert Q\rvert^{\sum_{k=0}^{N/2} n(n-1)^{2k}} \cdot (\lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O((\lvert Q\rvert \cdot \lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{O(n(n-1)^N)})$
Arbitrary odd N	$O(\lvert Q\rvert^{\sum_{k=0}^{(N-1)/2} n(n-1)^{2k}} \cdot (\lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{1+\sum_{k=0}^{N} n(n-1)^k}) = O(\lvert Q\rvert^{O(n(n-1)^{N-1})} \cdot (\lvert Q_K\rvert \cdot \lvert\Sigma\rvert)^{O(n(n-1)^N)})$

- The state set $R_{\Upsilon N}$ is defined as $R_{\Upsilon N} = Q_K \times Q_{10} \times Q_{11} \times \cdots \times Q_{1N} \times Q_{20} \times Q_{21} \times \cdots \times Q_{2N}$, where
$$Q_{ii'} = \begin{cases} Q \times Q_K, & \text{if } i' \text{ is an even number} \\ Q_K, & \text{if } i' \text{ is an odd number} \end{cases}$$
for any $i \in I$ and any $i' \in \{0, 1, \ldots, N\}$.
- The initial state $r_{\Upsilon N,0} \in R_{\Upsilon N}$ is given in a similar way to the initial state $r_{\partial N,0} \in R_{\partial N}$ of $T_{\partial N}$.
- The set $R_{\Upsilon N,m}$ of marked states is defined as
$$R_{\Upsilon N,m} = \{r_{\Upsilon N} \in R_{\Upsilon N} \mid [\forall i_N \in I_{N,odd} : r_{\Upsilon N}(i_N) \in \Upsilon(Q_K)] \wedge [\forall i_N \in I_N - I_{N,odd} : r_{\Upsilon N}(i_N) \in \partial(Q \times Q_K)]\},$$
where, for each $r_{\Upsilon N} \in R_{\Upsilon N}$, $r_{\Upsilon N}(i_N)$ $(i_N \in I_N)$ is defined in the same way as $r_{\partial N}(i_N)$, where $r_{\partial N} \in R_{\partial N}$.
- The event set $\Sigma_{\Upsilon N}$ is defined as $\Sigma_{\Upsilon N} = \Sigma_{\partial N}$.
- The state transition function $\alpha_{\Upsilon N} : R_{\Upsilon N} \times \Sigma_{\Upsilon N} \to R_{\Upsilon N}$ is defined in a similar way to the state transition function $\alpha_{\partial N} : R_{\partial N} \times \Sigma_{\partial N} \to R_{\partial N}$ of $T_{\partial N}$.

Similar to Proposition 13, we have the following result.

Proposition 14. Consider the pair (L, K) of regular languages. Then, $\Upsilon^{\partial}_{N+1} = \emptyset$ if and only if $L_m(T_{\Upsilon N}) = \emptyset$.

Then we have the following theorem.

Theorem 15. The pair (L, K) of regular languages is N-inference prognosable if and only if $L_m(T_{\partial N}) = \emptyset$ or $L_m(T_{\Upsilon N}) = \emptyset$.

Remark 16. To verify N-inference prognosability of the pair (L, K) of regular languages using Theorem 15, we need to construct the finite automata $T_{\partial N}$ and $T_{\Upsilon N}$. By the definitions of $T_{\partial N}$ and $T_{\Upsilon N}$, the complexity of constructing them is shown in Table 2.

The following example illustrates the verification result of Theorem 15.

Example 17. We revisit the setting of Example 10. We verify whether (L, K) is 2-inference prognosable using Theorem 15. First, we construct the finite automaton $T_{\partial 2}$ to verify whether $L_m(T_{\partial 2}) = \emptyset$. A part of $T_{\partial 2}$ is shown in Fig. 2. Since a marked state $((q_{17}, q_{K,17}), q_{K,21}, (q_{25}, q_{K,25}), q_{K,28}, q_{K,20}, (q_{24}, q_{K,24}), q_{K,27})$ is reachable in $T_{\partial 2}$, we have $L_m(T_{\partial 2}) \neq \emptyset$. Similarly, we can verify that $L_m(T_{\Upsilon 2}) \neq \emptyset$ for the finite automaton $T_{\Upsilon 2}$. Thus, by Theorem 15, we can conclude that (L, K) is not 2-inference prognosable.

7. CONCLUSION

A framework for inference-based decentralized prognosis was proposed, which supports *multi-level inferencing* for *disjunctive* as well as *conjunctive* decision making. The notion of N-inference prognosability was introduced and its necessity and sufficiency were proved. A method for verifying N-inference prognosability was developed. The proposed framework can be extended to address the problem of predicting the occurrence of any fault at least k steps in advance.

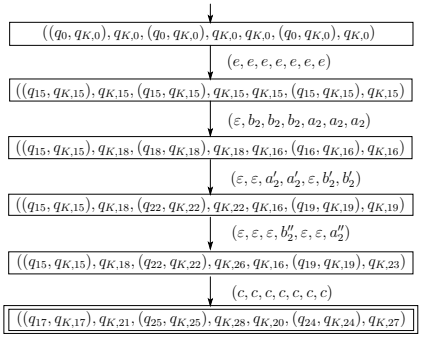


Fig. 2. A part of the automaton $T_{\partial 2}$.

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