Title	Analytical Calculation Method of Restraint Stresses and Strains due to Slit Weld in Rectangular Plates(WELDING MECHANICS, STRENGTH AND DESIGN)
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Citation	Transactions of JWRI. 1982, 11(1), p. 105-113
Version Type	VoR
URL	https://doi.org/10.18910/10762
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Analytical Calculation Method of Restraint Stresses and Strains due to Slit Weld in Rectangular Plates†

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Abstract

There are many metallurgical and dynamical factors which influence initiation of cold cracking at weld joints. However, when welding conditions are specified, the initiation of cold cracking may be predicted by the stress and strain induced at a point where the initiation is expected. In this respect, the stress and strain are the important information to prevent cold cracking from the dynamical point of view.

When a welded joint is simple such as of one dimensional constraint state, the restraint stress of a weld joint can be estimated with a fairly good accuracy by the conventional concept of restraint intensity. On the other hand, restraint stresses and strains of two dimensional constraint state are produced by not only shrinkage of weld metal but also thermal deformation of the plate. In this reason, it is difficult to predict them simply from the geometric configuration of a joint without considering this thermal deformation.

In this paper, thermal elastic-plastic behaviors in a slit type welded joint is idealized and an analytical calculation method for the stresses and strains is developed. And, it is shown that the restraint stresses and strains in any size of slit joints can be calculated simply by this method.

KEY WORDS: (Restraint Stress) (Restraint Strain) (Analytical Calculation Method) (Inherent Shrinkage) (Slit Weld)

1. Introduction

When a welded joint is dynamically in one dimensional constraint state, restraint stresses produced in the welded metal can be accurately predicted by the restraint intensity in the welded joint. On the other hand, it is hard to predict restraint stresses by the restraint intensity in the two dimensional constraint state such as of slit weld, circular patch weld, etc. of which restraint of thermal expansion and contraction varies along the weld line.

In this relation, the authors have made a series of researches to clarify the characteristics of restraint stresses and strains in two dimensional constraint state. In this paper, an analysis model is set by idealization of thermal elastic-plastic phenomenon due to slit weld in a rectangular plate (**Fig. 1**) which is the basic example showing two dimensional constraint state. By use of this model, an analytical calculation method is proposed to evaluate restraint stresses and strains produced perpendicular to the weld line of weld metal including the effects of the change of specimen size and heat input.

2. Modelling of Thermal Elastic-Plastic Phenomenon and Assumption of Analysis

In experiments, mechanism of restraint stresses and strains produced by first pass welding of the slit in a finite rectangular plate shown in Fig. 1 is rather complicated. Therefore, it is clarified by thermal elastic-plastic analysis in consideration of the effect of moving heat source[1].

To begin with, the weld metal and the base metal are assumed to be dynamically separated and to expand and contract independently.

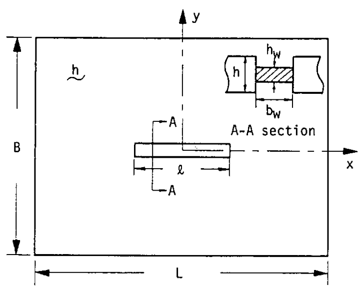

Fig. 1 Slit weld specimen

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† Received on March 31, 1982

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Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka, Japan

(1) In this case, weld metal contracts freely from the melting point to room temperature. The process of its free contraction is divided by the rigidity recovery temperature T_m into two stages. In the first stage which is from the melting point to T_m, weld metal shows no reaction to external force and in the second stage which is below T_m, it starts to show apparent reaction.

(2) On the other hand, the base metal which is not so simple as the weld metal shows two dynamical stages of deforming behavior. Firstly, in the heating stage, thermal deformation of the base metal behaves in the direction to close the groove. This is called the groove closing displacement. In the next stage, with the lapse of time, heat concentrated in the vicinity of the groove spreads and the temperature of the base metal becomes uniform, so that the groove and its vicinity get cooled and begin to contract to restore its original form from the deformed one at the first stage. This is the second stage.

When the weld metal and the base plate are taken into separate accounts as stated above, each deformation appear to be simple. However, the real thermal elastic-plastic phenomenon is very complicated owing to the dynamical interaction between the weld metal and the base plate.

In the real phenomenon, the above mentioned each stage of both the weld metal and the base plate takes place approximately at a time. In the first stage, the weld metal is dynamically in the melting state and does not resist any thermal deformation of the base plate. As a result, the edges of the slit of the base plate deform freely to close the groove owing to its thermal deformation alone. This closing groove displacement grows approximately maximum along the slit line when the weld metal reaches T_m at the slit end. After that, the weld metal and the base plate step into the second stage at a same time and show their sequential behaviors in each way.

In the second stage, the weld metal in the slit is cooled to the rigidity recovery temperature, T_m, and acts with the base plate together. The weld metal contracts itself as to close the groove and at the same time resists the restoration of the groove. This produces the reaction of the base plate in the second stage, that is, the groove cannot recover its original form from the deformed one. As a result, restraint stresses and strains are produced in the weld metal due to its contraction and resistance to restoration of the groove, that is the reaction of the base plate in the second stage.

In this kind of dynamical behaviors, when the weld metal is cooled to the rigidity recovery temperature, the summation of the groove closing displacement of the base plate and the shrinkage of the weld metal due to its cooling to the rigidity recovery temperature can be considered to be the inherent shrinkage which is the source of restraint stresses and strains. According to the result of thermal elastic-plastic analysis, the groove closing displacement of the base plate is the major component of its inherent shrinkage and the shrinkage of the weld metal is so little that can be ignored. Henceforth, the groove closing displacement of the base plate is regarded as the approximate inherent shrinkage.

In this paper, the above stated phenomena are idealized and a model is set for analysis as follows.

1) In disregard of the shrinkage of the weld metal, the groove closing displacement which is the thermal deformation of the base plate at the rigidity recovery temperature of the weld metal is set as the inherent shrinkage. Stresses and strains of the weld metal produced by the inherent shrinkage are considered to coincide with restraint stresses and strains produced by the slit weld in a finite rectangular plate.

2) The effect of moving heat source is also disregarded. In a relatively thin plate of which plate thickness (h) is smaller than the critical plate thickness (h_{cr}), ($h \leq h_{cr}$), temperature at the slit is assumed to be uniform in the thickness direction and the heat input is regarded as given by an instantaneous plane heat source. On the other hand, when plate thickness is larger ($h > h_{cr}$), the instantaneous linear heat source is assumed to be along the center line of the plate thickness. In this case, temperature distribution is not uniform in the plate thickness direction but it should be three dimensional one.

3) Thermal deformation is two dimensional in relatively thinner plates ($h \leq h_{cr}$), but it brings about a three dimensional problem in thicker plates ($h > h_{cr}$). For this, highly accurate simplification is presented in Ref. 2). In accordance with it, the three dimensional temperature distribution due to the instantaneous linear heat source placed along the center line of the plate is dealt as a two dimensional problem in which temperature at the center of plate thickness is assumed to uniformly distribute in the plate thickness direction.

4) Inherent shrinkage due to the slit weld which is regarded as an instantaneous heat source is considered to be the groove closing displacement at the rigidity recovery temperature at the center of its plate thickness in the middle of the slit.

5) It is assumed that the base plate is completely elastic and the weld metal which shows complete

elastic-plastic behaviors obeys the Tresca's yield conditions. Moreover, the breadth of the weld metal is presumed to be very small in comparison with the breadth or length of the finite rectangular base plate. Therefore, if restraint stresses are elastic, extension of the weld metal is ignored because it is extremely small. On the contrary, if they reach the yield stress, the weld metal extends freely because of its complete plasticity but is restricted in magnitude by deformation of the base plate.

3. Analytical Calculation Method for Inherent Shrinkage

3.1 Distribution of temperature by instantaneous heat source

An infinite plate is placed with an instantaneous plane heat source of heat input q (cal/mm²) or an instantaneous linear heat source of q (cal/mm) in the center of its plate thickness for the slit length of l. If dependency of the physical constant of the material on temperature and thermal reflections from the both surfaces are disregarded, the temperature distribution at the center of plate thickness in the middle of the slit at the rigidity recovery temperature, T_m, is given by the following equation.

$$T-T_i=\frac{T_m-T_i}{2}\exp\left[-\pi\left(\frac{Y}{2}\frac{l}{h'_{cr}}\right)^2\right]\cdot \left[\Phi\left\{\frac{\sqrt{\pi}}{2}\frac{l}{h'_{cr}}(1+X)\right\}+\Phi\left\{\frac{\sqrt{\pi}}{2}\frac{l}{h'_{cr}}(1-X)\right\}\right] \quad (1)$$

where,

$X=2x/l$, $Y=2y/l$, l; length of the slit

$\Phi(u)=\frac{2}{\sqrt{\pi}}\int_0^u e^{-u^2}du$; error function

$h'_{cr}=h_{cr}^2/h$ in the case of $h\leq h_{cr}$

$h'_{cr}=h_{cr}$ in the case of $h>h_{cr}$

$h_{cr}=\sqrt{\frac{q}{c\rho(T_m-T_i)}}$; critical plate thickness (mm)

c; specific heat (cal/g·°C), ρ; density (g/mm³), T_m; rigidity recovery temperature (°C), T_i; initial temperature (°C)

On the other hand, a finite rectangular plate which satisfies such ratios as, $h'_{cr}/l\leq(1/2)(B/l)$, $h'_{cr}/l\leq(1/2)(L/l-1)$, is influenced extremely little by thermal reflection from the plate boundary. Therefore, it is considered possible to apply Eq. (1) to finite rectangular plates which satisfy the above conditions.

3.2 Calculation method of inherent shrinkage

Analysis of inherent shrinkage S_T which is the source of restraint stresses and strains due to slit weld in a finite rectangular plate is equivalent to an accurate evaluation of the thermal deformation of the base plate at the groove due to the temperature distribution in Eq. (1), or the groove closing displacement of the base plate.

The analysis is composed of application of an analytical solution for an infinite plate and the correcting calculation to make it applicable to a finite plate. The analytical procedure is shown in **Fig. 2-1.** The inherent shrinkage S_T is obtained as a sum of the groove closing displacement S_T^∞ for an infinite plate produced at the rigidity recovery temperature in the middle of the slit (summation of Fig. 2-1(b) and (c), which results in (c')) and the additional deformation ΔS_T produced by modifying into a finite plate (Fig. 2-1(d) results in (d')). For convenience of such summation of various solutions as this one, concrete calculation methods are proposed below.

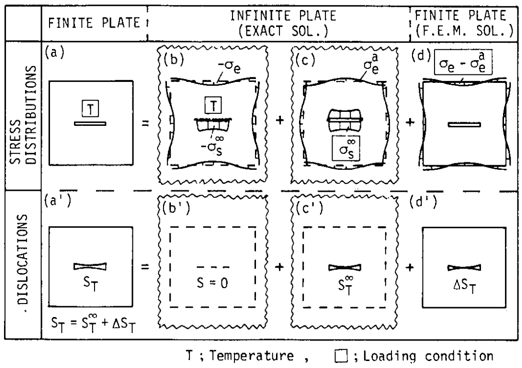

Fig. 2-1 Procedure for calculation of inherent shrinkage: S_T

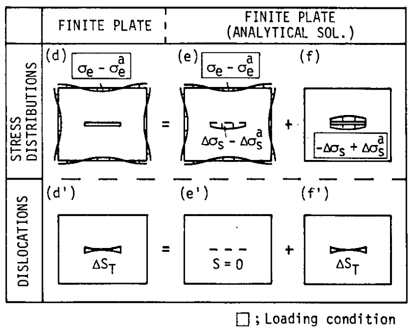

Fig. 2-2 Procedure for calculation of additional inherent shrinkage: ΔS_T

[Analysis (A)] As in Fig. 2-1(b), temperature distribution of Eq. (1) is imposed in an infinite plate. Resulting stresses $\{-\sigma\}$ produced at an arbitrary point (x, y) can by given by the following equations based on the theory of two dimensional thermal elasticity[3].

$$\left.\begin{aligned}\sigma_x&=\frac{1}{2}\int_{-1}^{1}(\cos^2\theta\ \sigma_1+\sin^2\theta\ \sigma_2)dX'\\ \sigma_y&=\frac{1}{2}\int_{-1}^{1}(\sin^2\theta\ \sigma_1+\cos^2\theta\ \sigma_2)dX'\\ \tau_{xy}&=\frac{1}{2}\int_{-1}^{1}(\cos\theta\ \sin\theta\ \sigma_1-\sin\theta\ \cos\theta\ \sigma_2)dX'\end{aligned}\right\}\quad(2)$$

where,

$$\sigma_1=\{E\alpha(T_m-T_i)l/h'_{cr}\}[-(1/2\pi)(h'_{cr}/r)^2\cdot\{1-\exp(-\pi(r/h'_{cr})^2\}]$$
$$\sigma_2=\{E\alpha(T_m-T_i)l/h'_{cr}\}[\exp(-\pi(r/h'_{cr})^2)+(1/2\pi)(h'_{cr}/r)^2\{1-\exp(-\pi(r/h'_{cr})^2\}]$$
$$r^2=(l/2)^2\{(X-X')^2+Y^2\},\ \cos\theta=(X-X')/(2r/l),$$
$$\sin\theta=Y/(2r/l),\ X=2x/l,\ X'=2x'/l,\ Y=2y/l$$

α; coefficient of linear expansion (1/°C),
E; Young's modulus (kg/mm^2)

From the above, a stress in y direction $-\sigma_s^\infty$ on the heat source line ($-1\leq X\leq 1$, $Y=0$) (corresponds to the slit line) is obtained. The stress of the opposite sign, σ_s^∞, is expressed by the next equation.

$$\sigma_s^\infty(X)=\frac{E}{\pi}\frac{\alpha(T_m-T_i)h'_{cr}}{l}\left[\frac{1-\exp\{(-\pi/4)(1+X)^2(l/h'_{cr})^2\}}{1+X}+\frac{1-\exp\{(-\pi/4)(1-X)^2(l/h'_{cr})^2\}}{1-X}\right]\quad(3)$$

[Analysis (B)] As in Fig. 2-1(c), the closing displacement at the slit, S_T^∞, produced by the temperature distribution of Eq. (1) in an infinite plate with a slit is obtained as follows. In an infinite plate without slit of which thermal stresses are obtained by [Analysis (A)], a slit on its heat source line is made in order to make a free boundary. For analysis, the existing stresses along slit lines, σ_s^∞, obtained in [Analysis (A)] are applied to a stress-free infinite plate with a slit in the opposite direction. The resulting additional stresses in the plate are $\{\sigma^a\}$ and the resulting displacement of the slit lines is S_T^∞[4].

Here, the conditions for free edges of a slit are satisfied by superposing the solutions by [Analyses (A) and (B)], and the infinite plate is subjected to $\{-\sigma\}+\{\sigma^a\}$. S_T^∞ and $\{\sigma^a\}$ are given in the following equations by use of Westergaard's stress function[5].

$$S_T^\infty(X)=\frac{2l}{\pi E}\int_{-1}^{1}\sigma_s^\infty(X')\log\left\{\left|\frac{1-XX'}{X-X'}\right|+\sqrt{\left(\frac{1-XX'}{X-X'}\right)^2-1}\right\}dX'\quad(4)$$

$$\begin{Bmatrix}\sigma_x^a\\ \sigma_y^a\\ \tau_{xy}^a\end{Bmatrix}=\frac{l}{2}\int_{-1}^{1}\begin{Bmatrix}R_eZ_I-yI_mZ'_I\\ R_eZ_I+yI_mZ'_I\\ -yR_eZ'_I\end{Bmatrix}\sigma_s^\infty(X')dX'\quad(5)$$

where,

$$Z_I=\sqrt{1-(X')^2}/[(\pi/2)l(z-X')\sqrt{z^2-1}],$$
$$Z'_I=dZ_I/dz,\ z=2x/l+i(2y/l)=X+iY,\ i=\sqrt{-1}$$

R_e, I_m; the real and the imaginary parts of complex function

[Analysis (C)] The resultant stresses $\{-\sigma\}+\{\sigma^a\}$ in an infinite plate with a slit and its displacement at the slit lines are obtained by superposition of the solutions by [Analyses (A) and (B)]. In order to apply these solutions to a finite plate, the boundary conditions along the perimeters of a finite rectangular plate, $x=\pm L/2$, $y=\pm B/2$, need to be satisfied.

In other words, the resultant stresses $\{-\sigma\}+\{\sigma^a\}$ have values of $\{-\sigma_e\}+\{\sigma_e^a\}$ along the lines to be perimeters, which have to vanish by cutting the plate at the perimeters. Analytically, the stresses $\{-\sigma_e\}+\{\sigma_e^a\}$ are applied in the opposite direction on the perimeters of boundary of a finite rectangular plate with a slit, as shown in Fig. 2-1 (d). Followingly, the additional displacement ΔS_T (Fig. 2-1 (d')) occurs at the slit lines (**Fig. 2-2**). This solution is superposed with those by [Analyses (A) and (B)], so that the free boundary conditions are fully satisfied as in Fig. 2-1 (a) and (a').

As a result, the final displacement along the slit lines (inherent shrinkage) S_T is attained as,

$$S_T=S_T^\infty+\Delta S_T\quad(6)$$

Although the solution of [Analysis (C)], that is ΔS_T, can be directly calculated by the aid of the finite element method, an analytical calculation method is presented in **appendix 1** with the purpose of simplifying calculations of variation of the sizes of test specimens and heat input.

4. Analytical Calculation Method for Restraint Stresses and Strains Based on Inherent Shrinkage

As stated in Chapter 2, restraint stresses σ^r due to the slit weld are calculated as ones produced by application of inherent shrinkage S_T along the slit line. In this case, the weld metal is subjected to elastic, elastic-plastic, or fully plastic stress distribution depending upon the magnitude of the inherent shrinkage and the ratios of a rectangular plate. These stress distributions which can be directly calculated by the aid of the finite element method will be evaluated here by the following analytical method.

4.1 Elastic restraint stresses and strains in the weld metal

If the plate thickness is thin and the slit line is very long in comparison to h_{cr}, the restraint stresses and strains in the weld metal are elastic. According to assumption 5), it is only the base plate that deforms elastically in reaction to the inherent shrinkage. Therefore, restraint stresses (stresses of the base plate produced along the slit line), σ_s^r (**Fig. 3**(a)), are given as elastic stresses produced by application of S_T along slit lines of a rectangular plate (Fig. 3(a′)). Figure 3 shows the procedure for this analysis.

In the analysis, the components of inherent shrinkage $S_T = S_T^\infty + \Delta S_T$ will be given separately along slit line.

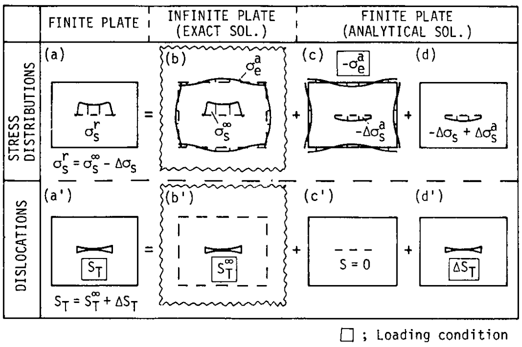

Fig. 3 Procedure for calculation of restraint stress: σ_s^r

[Analysis (A)] As in Fig. 3(b′), the first component S_T^∞ of S_T is applied along the slit line of an infinite plate. The resultant stresses σ_s^∞ are expressed by Eq. (3) of [Analysis (A)] in Chapter 3. That is, the displacement resulted from release of σ_s^∞ produced in an infinite plate due to the temperature distribution of Eq. (1) by making a slit is S_T^∞ ([Analysis (B)] in Chapter 3).

[Analysis (B)] In the state of maintaining S_T^∞ given to slit lines, a finite rectangular plate is cut out from an infinite plate. Consequently, the perimeters of the rectangular plate becomes a free boundary. Analytically speaking, stresses $\{\sigma_e^a\}$ produced at perimeters ($x=\pm L/2$, $y=\pm B/2$) of an infinite plate are applied in a rectangular plate without slit in the opposite sign direction, as shown in Fig. 3(c). The resultant stresses produced along the slit line are $-\Delta\sigma_s^a$. As a matter of course, S_T^∞ does not vary.

Restraint stresses $(\sigma_s^\infty - \Delta\sigma_s^a)$ produced by applying S_T^∞ along the slit line of a rectangular plate are obtained by superposing solutions of [Analyses (A) and (B)].

[Analysis (C)] If the second component ΔS_T of S_T is imposed along the slit line (Fig. 3(d′)), the restraint stresses turn to be $(-\Delta\sigma_s + \Delta\sigma_s^a)$ as shown in Fig. 2-2 (f). The method of this analysis is already shown in Section 3.2 (Fig. 2-2).

Thus obtained solutions of [Analyses (A), (B) and (C)] are superposed so that the restraint stresses σ_s^r produced along the slit line are as,

$$\sigma_s^r = \sigma_s^\infty - \Delta\sigma_s \tag{7}$$

Each term of the right side are given by Eq. (3) and Eq. (A3–1), respectively. As for restraint stresses σ_w in weld metal, with h, the plate thickness of a base plate, and h_w, the throat thickness of the weld metal, they are expressed as,

$$\sigma_w = \sigma_s^r(h/h_w) = (\sigma_s^\infty - \Delta\sigma_s)(h/h_w) \tag{8}$$

In addition, restraint strains ε_w produced at the same time in the weld metal are elastic ones ε_w^e and can be given by the following equation.

$$\varepsilon_w = \varepsilon_w^e = \sigma_w / E \tag{9}$$

Distribution of the restraint strains is the same as that of the restraint stresses.

4.2 Restraint stresses and strains in the weld metal with partial plasticity

Based on the analytical result of elastic-plastic behaviors of a rectangular plate with a slit under the action of inherent shrinkage S_T by the aid of the finite element method, elastic-plastic behaviors are idealized and its analytical calculation method will be developed in the following.

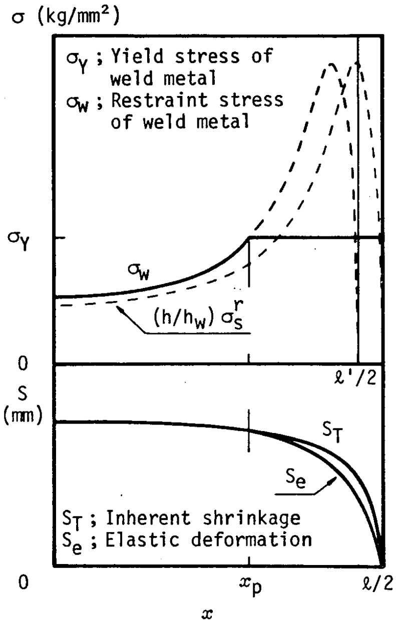

Fig. 4 Simplified elastic-plastic analysis of restraint stress

It is general that when increase elastic restraint stresses in the weld metal, plastic portion develops partially from slit ends to the center where stresses are less. The resulting elastic-plastic stress distribution is shown in **Fig. 4**. As is the two dimensional problem, elastic stress value changes due to any partial plastification, no matter determined inherent shrinkage S_T is imposed regardless whether stress distribution is elastic or elastic-plastic. It is recognized from the calculated result by the aid of the finite element method that the form of stress distribution in the remaining elastic part is approximately the same as one calculated by Eq. (8) regarding that the slit length l is shortened to l'. Based on the calculated results, l' is determined as an first approximation in the followings.

$$l'=\zeta l \tag{10}$$

where,

$\zeta=0.0024\,\sigma_Y+0.765 \qquad (30\leq\sigma_Y\leq98)$

$\zeta=1.0 \qquad (\sigma_Y>98)$

σ_Y; yield stress in the weld metal (kg/mm^2)

Stress σ_w in the weld metal of this analysis model equals the yield stress σ_Y at the point of $x=\pm x_p$ beyond which elastic stresses do not occur. The point x_p at which Eq. (8) makes $\sigma_w=\sigma_Y$ on the assumption that $l\rightarrow l'$ can be given by Eq. (11).

$$\frac{2x_p}{l'}=X_p\simeq\pm\sqrt{1-\frac{(2E/\pi)\{\alpha(T_m-T_i)h'_{cr}/l'\}}{\sigma_Y(h_w/h)+\Delta\sigma_s}} \tag{11}$$

As a result, elastic-plastic stress distribution in the weld metal can be obtained. If elastic-plastic stresses are accurately obtained as an first approximation, deformation S_e (elastic deformation of the base plate) produced along the slit line by applying σ_w for throat thickness along the slit line should coincide with inherent shrinkage S_T in the elastic portions ($|X|<X_p$). Therefore, if they do not coincide, l' has to be adjusted so that $S_e=S_T$ in the elastic portions. The calculation method of S_e is shown in **appendix 4**.

From the above, if the weld metal is subjected to partial plastification, its restraint stresses σ_w and strains ε_w perpendicular to the weld line are as follows.

$$\left.\begin{aligned}\sigma_w(X_1)=&\frac{E}{\pi}\,\frac{\alpha(T_m-T_i)h'_{cr}}{l'}\,\frac{h}{h_w}\cdot\\&\left[\frac{1-\exp\{(-\pi/4)(1+X_1)^2(l'/h'_{cr})^2\}}{1+X_1}\right.\\&\left.+\frac{1-\exp\{(-\pi/4)(1-X_1)^2(l'/h'_{cr})^2\}}{1-X_1}\right]\\&-\Delta\sigma_s(B/l',\,L/l')\cdot(h/h_w)\qquad(|X|<X_p)\\\sigma_w=\sigma_Y,&\qquad\qquad(X_p\leq|X|\leq1)\end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned}&\varepsilon_w=\varepsilon^e_w=\sigma_w(X_1)/E\qquad(|X|<X_p)\\&\varepsilon_w=\varepsilon^e_w+\varepsilon^p_w=\sigma_Y/E+(S_T-S_e)/b_w\\&\qquad\qquad(X_p\leq|X|\leq1)\end{aligned}\right\} \tag{13}$$

where,

$X_1=2x/l',\ X=2x/l$

ε^p_w; plastic strains in weld metal, b_w; root gap (mm)

4.3 Generally yielded restraint stresses and strains in the weld metal

When the weld metal yields completely, restraint stress σ_w produced in the weld metal equal yield stress σ_Y. In this case, S_e is given as the displacement (elastic deformation of the base plate) produced by imposition of the yield stress on the throat thickness h_w along the slit line. This S_e can be calculated by use of restraint intensity R_p (expressed by the highly accurate approximate equation)[6] of a finite rectangular plate by applying uniform loading along the slit line as follows.

$$\begin{aligned}S_e&=\sigma_Y h_w/R_p(X)\\&=(\sigma_Y/E)(h_w/h)\{2l/(1-\beta_p)\}\sqrt{1-X^2}\end{aligned} \tag{14}$$

where,

$\beta_p=0.6/(L/l)^n+0.75/(B/l)^{1.82}$

$n=5.8/(B/l)^2+2.2 \qquad (B/l\geq1.8,\ L/l\geq1.5)$

The above equation indicates an ellipse. Accordingly, restraint strains ε_w of the weld metal produced perpendicular to the weld line are given by the following equation.

$$\varepsilon_w=\varepsilon^e_w+\varepsilon^p_w=\sigma_Y/E+(S_T-S_e)/b_w \tag{15}$$

5. Examples

Figure 5 shows inherent shrinkage S_T and elastic deformation S_e of the base plate produced in the specimen with the same size ratios ($B/l=1.875$, $L/l=2.5$) as those of the y-slit weld cracking test specimen in two cases of the slit length $l=80$ (ratio of y-slit weld cracking test specimen) and 240 mm, fixing the other conditions as plate thickness $h=20$ mm, throat thickness $h_w=5$ mm, root gap $b_w=4$ mm, heat input $Q=17{,}000$ J/cm ($h_{cr}=17.54$ mm), rigidity recovery temperature $T_m=700$°C, initial temperature $T_i=15$°C, linear expansion coefficient $\alpha=1.2\times10^{-5}$ 1/°C, specific heat $c=0.188$ cal/g·°C, density $\rho=7.66$ g/cm^3, yield stress of weld metal $\sigma_Y=50$ kg/mm^2, and Young's modulus $E=21{,}000$ kg/mm^2. **Figure 6** shows restraint stresses (residual stresses) σ_w of the weld metal perpendicular to the weld line and its restraint strains (residual plastic strains) ε^p_w.

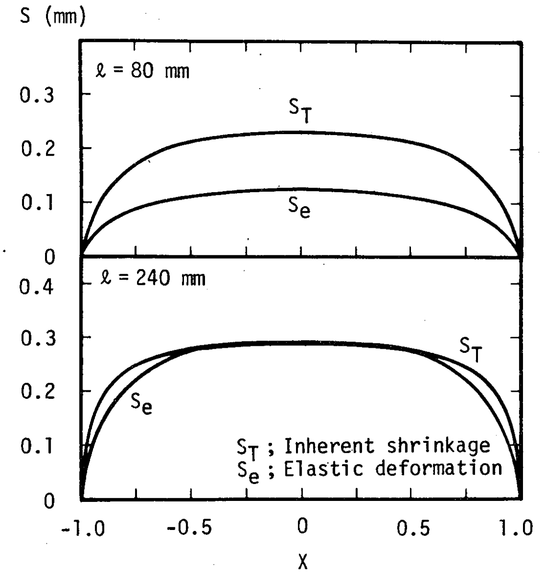

Fig. 5 Distribution of inherent shrinkage: S_T and elastic deformation: S_e along slit

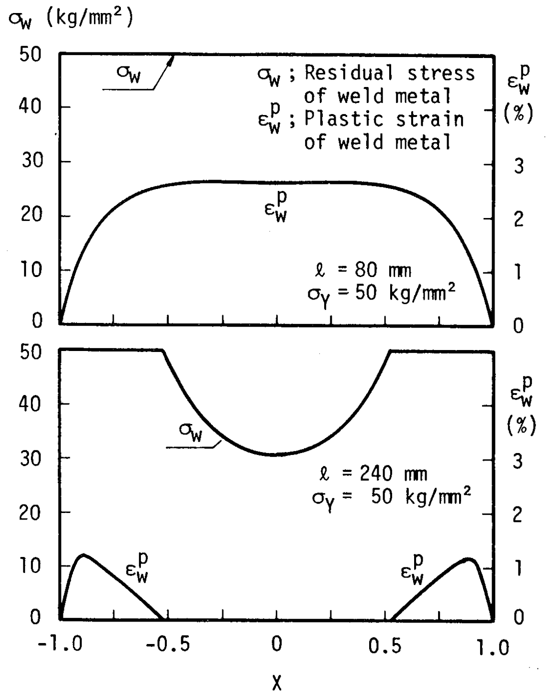

Fig. 6 Distribution of residual stress and plastic strain along slit

6. Conclusions

In this report, slit weld of a finite rectangular plate is chosen as the example of two dimensional constraint state and its elastic-plastic phenomenon is modelled so that an analytical method is presented to calculate restraint stresses and strains of the weld metal produced perpendicular to the weld line for arbitrary combination of specimen ratios and heat input. The main results are as follows.

(1) Restraint stresses and strains due to slit weld directly depend upon groove closing displacement (inherent shrinkage) along the slit line on the occasion that the weld metal recovers its rigidity. An accurate analytical method for this inherent shrinkage is shown by superposing the analytical solution for an infinite plate and the solution for the effect of a finite plate by the aid of the finite element method.

(2) Based on the calculation result by the finite element method, an analytical calculation method for the effect of a finite plate is shown in appendix 1. As a result, with determined size ratios of a slit specimen and under determined welding conditions, inherent shrinkage can be easily calculated without aid of the finite element method.

(3) Restraint stresses and strains of the weld metal can be obtained from the direct elastic-plastic analysis by applying the above stated inherent shrinkage to the weld metal. In this case, if the specimen ratios of $B/l \geq 1.8$ and $L/l \geq 2.0$ are fulfiled, without elastic-plastic analysis, restraint stresses of the weld metal are classified into three groups such as elastic, elastic-plastic, and fully plastic, so that restraint stresses and strains can be analytically calculated by the proposed method.

Appendix 1 Analytical calculation method for additional deformation ΔS_T

As is stated in [Analysis (C)] of Section 3.2, additional deformation ΔS_T which can be directly calculated as the finite element solution can be also analytically calculated as indicated below and its analytical procedure is shown in Fig. 2-2.

[Analysis (A)] As in Fig. 2-2(e), stresses $\{-\sigma_e\}+\{\sigma_e^a\}$ produced along the perimeters of a finite plate ($x=\pm L/2$, $y=\pm B/2$) are applied to the boundary of a rectangular plate without slit in the opposite direction. Accordingly, stresses $(\Delta\sigma_s-\Delta\sigma_s^a)$ are produced along the slit line in the rectangular plate.

[Analysis (B)] Stresses along the slit line are obtained in [Analysis (A)]. Displacement resulted from release of these stresses by making a slit is ΔS_T (Fig. 2-2 (f')). That is, ΔS_T in [Analysis (C)] of Section 3.2 can be obtained by superposing the solutions of [Analyses (A) and (B)] (Fig. 2-2 (d')).

As is predicted from Saint-Venant's principle, stresses $(\Delta\sigma_s-\Delta\sigma_s^a)$ along the slit line which are stated in [Analysis (A)] distribute uniformly if the plate is large to a certain degree. According to the result of calculation, stresses at the slit ends and those at the middle of the slit are considered to be uniform in the ratio of over 0.9 and in order to fulfil this, the specimen size ratios have to be $B/l \geq 1.8$ and $L/l \geq 2.0$. Deformation ΔS_T produced along the slit line under this condition can be calculated applying restraint intensity R_p (Eq. (14)) of a finite plate under uniform loading as follows.

$$\Delta S_T = (\Delta\sigma_s^a - \Delta\sigma_s)h/R_p(X)$$
$$= 2l(\Delta\sigma_s^a - \Delta\sigma_s)\sqrt{1-X^2}/\{E(1-\beta_p)\} \quad \text{(A1-1)}$$
$$(B/l \geq 1.8,\ L/l \geq 2.0)$$

The above equation indicates an ellipse. Although $\Delta\sigma_s^a$ and $\Delta\sigma_s$ can be obtained by the direct finite element analysis, an analytical calculation method of $\Delta\sigma_s^a$ is shown in **appendix 2** and an approximate expression of $\Delta\sigma_s$ formulated by the aid of the minimum square method for the calculated result of the finite element method in **appendix 3**. Consequently, as $\Delta\sigma_s^a$ and $\Delta\sigma_s$ can be easily calculated by Eqs. (A2-5) and (A3-1) respectively, ΔS_T can be calculated by Eq. (A1-1) for various size ratios of specimens and welding conditions.

Appendix 2 Analytical calculation method of $\Delta\sigma_s^a$

According to reference 6)*), if stresses $\{\sigma_e^{p_0}\}$ produced in the location of an infinite plate corresponding to the boundary of a finite plate under uniform loading P_0 along the slit line are imposed in the opposite direction along the boundary of a rectangular plate without slit under no loading, its reaction stresses $\Delta\sigma_s^{p_0}$ produced in the location corresponding to the slit are given by the following equation.

$$\Delta\sigma_s^{p_0} = \beta_p \cdot P_0 \quad \text{(A2-1)}$$

Based on this idea, correction factor β_T to a finite plate from an infinite plate which is imposed along the slit line with thermal elastic stresses σ_s^∞ calculated by Eq. (3) is formulated, so that the reaction stresses $\Delta\sigma_s^a$ produced along the slit line under the effect of a rectangular plate are expressed as follows.

$$\Delta\sigma_s^a = \beta_T \sigma_{s0}^\infty \quad \text{(A2-2)}$$

In the above Eq. (A2-2), σ_{s0}^∞ is the value of thermal elastic stresses at the center of the slit line for an infinite plate.

$$\sigma_{s0}^\infty = \sigma_s^\infty(X=0) = (2E/\pi)\{\alpha(T_m - T_i)h'_{cr}/l\} \cdot$$
$$[1 - \exp\{(-\pi/4)(l/h'_{cr})^2\}]$$

The ratio of correction factors β_p and β_T, which appear in Eqs. (A2-1) and (A2-2) is investigated by the finite element method. As indicated in **Fig. A2-1** by the mark「●」, it is almost constant under any value of B/l and functions of only h'_{cr}/l. Therefore, it can be expressed with the size ratios of the specimen and h'_{cr}, function of heat input, as follows.

$$\beta_T(B/l,\ L/l,\ h'_{cr}/l) = \beta_p(B/l,\ L/l) \times a(h'_{cr}/l) \quad \text{(A2-3)}$$

Integral values of dislocations produced along the slit line under the above two loading conditions are denoted by A_p and A_T, respectively. A_T/A_p is the function of only h'_{cr}/l and can be approximately expressed as follows.

$$\beta_T/\beta_p = A_T/A_p = a(h'_{cr}/l) \quad \text{(A2-4)}$$

where,

$$A_T/A_p = \int_{-1}^{1} S_T^\infty dX / [2\alpha(T_m - T_i)h'_{cr}(1 - \exp\{(-\pi/4) \cdot (l/h'_{cr})^2\})]$$
$$A_p = \pi l^2/(2E)$$

The solid lines in Fig. A2-1 indicate thus estimated values of β_T/β_p which coincide well with ones obtained by the finite element method (β_{δ_0}/β_p in the figure shows the relation of correction factors under uniform displacement along the slit line).

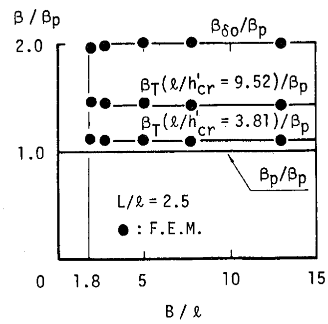

Fig. A2-1 Effect of geometrical shape on correction factor: β_T

As a result, $\Delta\sigma_s^a$ can be easily obtained by the following equation for various size ratios of specimens and heat input without the aid of the finite element method.

$$\Delta\sigma_s^a = \beta_p \times a(h'_{cr}/l) \times \sigma_{s0}^\infty$$
$$= \beta_p(E/\pi l)\int_{-1}^{1} S_T^\infty dX \quad \text{(A2-5)}$$
$$(B/l \geq 1.8,\ L/l \geq 2.0)$$

Appendix 3 Formulation of $\Delta\sigma_s$

In order to facilitate quantitative evaluation of $\Delta\sigma_s$, its finite element solution for ratios of $B/l \geq 1.8$ and $L/l \geq 2.0$ is formulated by the minimum square method.

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*) $\Delta\sigma_s^{p_0}$ was σ_{se}^p in Ref. 6).

$$\left.\begin{array}{l} \Delta\sigma_s/[(E/\pi)\{\alpha(T_m-T_i)h'_{cr}/l\}]=1.25/(L/l)^{2.15} \\ \quad [2\leq L/l<0.9675(B/l)^{0.8186}] \\ \Delta\sigma_s/[E/\pi)\{\alpha(T_m-T_i)h'_{cr}/l\}]=1.36/(B/l)^{1.76} \\ \quad [L/l\geq 0.9675(B/l)^{0.8186}] \end{array}\right\} \quad \text{(A3-1)}$$

Appendix 4 Analytical calculation method for S_e in the partially plastic weld metal

Elastic deformation S_e of a finite rectangular plate due to application of elastic-plastic stresses σ_w (Eq. (12)) mentioned in Section 4.2 along the slit line is analyzed. The procedure for this analysis is shown in **Fig. A4-1**. Abbreviating details, only the results of each procedure are described here.

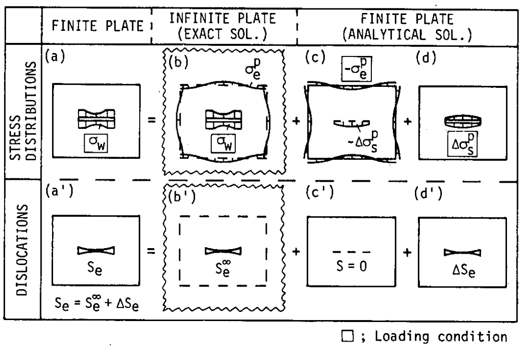

Fig. A4-1 Procedure for calculation of elastic deformation: S_e

[Analysis (A)] Elastic-plastic stresses σ_w are imposed along the slit line of an infinite plate with a slit (Fig. A4-1 (b)). The resultant stresses and displacement along the slit line are $\{\sigma^p\}$ and S_e^∞, respectively (Fig. A4-1 (b')). S_e^∞ can be analytically calculated by Eq. (4) substituting $(h_w/h)\sigma_w$ (Eq. (12)) for σ_s^∞.

[Analysis (B)] Stresses $\{\sigma_e^p\}$ produced along the boundary of a rectangular plate without slit are applied in the opposite direction. As a result, stresses $-\Delta\sigma_s^p$ are produced along the slit line (Fig. A4-1 (c)). $-\Delta\sigma_s^p$ can be obtained in the same manner as in appendix 2 by substituting S_e^∞ calculated in [Analysis (A)] for S_T^∞ in Eq. (A2-5).

$$\Delta\sigma_s^p=\beta_p(E/\pi l)\int_{-1}^{1} S_e^\infty dX \quad \text{(A4-1)}$$

[Analysis (C)] Stresses $\Delta\sigma_s^p$ obtained in [Analysis (B)] are imposed along the slit line of a rectangular plate with a slit. As a result, displacement ΔS_e are produced along the slit line (Fig. A4-1 (d')). As $\Delta\sigma_s^p$ are approximately uniform along the slit line with the ratios of $B/l\geq 1.8$ and $L/l\geq 1.5$, ΔS_e can be calculated as follows.

$$\Delta S_e=\Delta\sigma_s^p h/R_p(X) \quad (B/l\geq 1.8,\ L/l\geq 1.5) \quad \text{(A4-2)}$$

Consequently, free boundary conditions of a rectangular plate are fully satisfied by superposing [Analyses (A), (B) and (C)] and the resultant elastic deformation S_e (Fig. A4-1 (a')) along the slit line can be given by the following equation.

$$S_e=S_e^\infty+\Delta S_e \quad \text{(A4-3)}$$

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