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Mechanical Characteristics of Repair Welds in a Thick Plate (Report II)[†]

— Validity of Two-dimensional Plane-deformation Analysis —

You Chul KIM*, Teruhisa YAMAKITA**, Han Sur BANG*** and Yukio UEDA****

Abstract

As repair welding is performed in a limited range, the mechanical behavior is three-dimensional problem. Then, a huge amount of computing time and expense are required for carrying out three-dimensional thermo-elasto-plastic analysis. So, it is important to study possibility of replacing three-dimensional problem with one- or two-dimensional one by keeping the mechanical condition of three-dimensional problem.

In the region which undergoes the temperature history of a quasi-stationary state, the magnitude and the distribution of residual stress and plastic strain are same at each cross section along weld line if the rigidity to thermal expansion and shrinkage is uniform. The shorter repair weld length is and the slower travel speed is, the more the stress component σ_y (perpendicular to weld line) increases. In addition, the slower travel speed is, the more the plastic strain component ϵ_z^p (along weld line) decreases. Only when weld length is long and there is no temperature gradient along weld line, three-dimensional problems can, in the strict sense of the production mechanism, be replaced by two-dimensional (plane-deformation) problems. Besides, as magnitude of residual stress is limited by the yield strength of material, it can be accurately estimated by plane-deformation analysis. If travel speed is slower, it needs to be noted to underestimate the stress component σ_y in plane-deformation analysis. Whereas, since the magnitude and distribution of plastic strain are largely affected by the mechanical boundary conditions, estimation of plastic strain is less accurate than that of residual stress in plane-deformation analysis. In this study, if the region cooled down up to 400°C experiences the temperature history of a quasi-stationary state (the weld length; $l \geq 0.2m$, at least the middle parts of model undergoes a quasi-stationary state of temperature), under the condition of travel speed $v \geq 0.0055m/s$, the magnitude and the distribution of welding residual stress and plastic strain can be estimated by plane-deformation analysis. Only the distribution and the magnitude of residual stress can be estimated under the condition of travel speed $v \geq 0.0017m/s$.

KEY WORDS : (Repair Welding) (Transient Stress-strain) (Residual Stress-strain) (Three-dimensional Analysis) (Finite Element Method)

1. Introduction

Investigating the soundness of repair welded joint from a mechanical point of view, it is important to know the magnitude and the distribution of welding residual stress and plastic strain produced in repair welding.

As repair welding is performed in a limited portion, the mechanical behavior is three-dimensional problem. The characteristics of the distribution of welding residual stress and plastic strain in repair welding and their production mechanism were elucidated based on the results of three-dimensional thermo-elasto-plastic analysis by the finite element method¹⁾. Then, a huge amount of computing time and expense are required for carrying out three-dimensional thermo-elasto-plastic analysis. So, it is important not only for reducing expenses but also for studying

the various fundamental behaviors of mechanical problems to study possibility of replacing three-dimensional problem with one- or two-dimensional one by keeping the mechanical condition of three-dimensional problem.

In this paper, for studying the effects of travel speed and weld length which influence on the magnitude and the distribution of residual stress and plastic strain produced in repair welding in a thick plate, three-dimensional thermo-elasto-plastic analysis is performed on a series of models for repair welding. At the same time, two-dimensional plane-deformation thermo-elasto-plastic analysis is carried out. Comparing the results of these two analyses, the necessary mechanical conditions are investigated for replacing three-dimensional problem with two-dimensional (plane-deformation) one.

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2. Essential Differences of Three-dimensional Thermo-elasto-plastic Problem and Two-dimensional One Based on Temperature and Modelling

2.1 Essential differences of three-dimensional thermo-elasto-plastic problem and two-dimensional (plane-deformation) one

Considering, from a mechanical point of view, the essential features of three-dimensional thermo-elasto-plastic problem and two-dimensional plane-deformation one, the distribution of temperature in welding problem (it means external loads in strength of structural problem) and restriction to thermal expansion and contraction have much value. Restriction of thermal expansion and contraction is severe since repair welding is carried out in a limited portion. A series of models for this study is assumed that repair welding is performed to the portion which has enough stiffness and restriction is uniform along weld line (Fig.1). The essential differences between three-dimensional problem and plane-deformation one are examined by paying attention to the characteristics of the distribution of temperature.

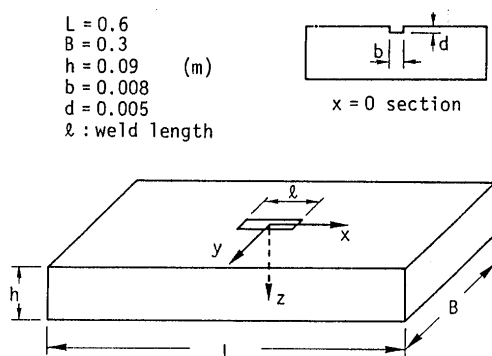


Fig. 1 Model for analysis

Weld starting and finishing edges closely exist in repair welding. Moreover, the temperature gradient occurs along weld line by moving heat source. In case of plane-deformation problem applying instantaneous heat source, no temperature gradient along weld line exists. The difference of two problems based on the characteristics of temperature is found by whether or not the temperature gradient including the effect of weld starting and finishing edges exist. If the temperature gradient occurs, the mechanical boundary condition is changed by reason that the period up to recovering mechanical stiffness of weld metal is different on weld line. The difference of the transient mechanical condition largely influences on the process of production of stress and plastic strain. Those are

essential differences between three-dimensional problem and plane-deformation one based on the distribution of temperature from a mechanical point of view.

According to the mentioned above, as the temperature gradient occurs along weld line under the condition of slow travel speed and short weld length, it may be difficult to apply three-dimensional problem to plane-deformation one. Based on the results by two analyses, possibility whether three-dimensional problem can be replaced by plane-deformation one and the necessary mechanical condition are investigated hereinafter.

2.2 Modelling

The model and coordinate system for analysis of repair welding are shown in Fig.1. The single-pass submerged arc welding, in which heat input $Q=34\text{kJ/cm}$ and thermal efficient $\eta=0.9$, is performed with travel speed v in a slit (length ℓ , breadth $b=8$, depth $d=5(\text{mm})$) on the middle part of model (length $L=600$, breadth $B=300$, thickness $h=90(\text{mm})$) along x-direction as shown in Fig.1. The initial temperature (a room temperature) is 15°C . Material of the specimen is mild steel as shown in Fig.2. The models used in this study are shown in Table 1.

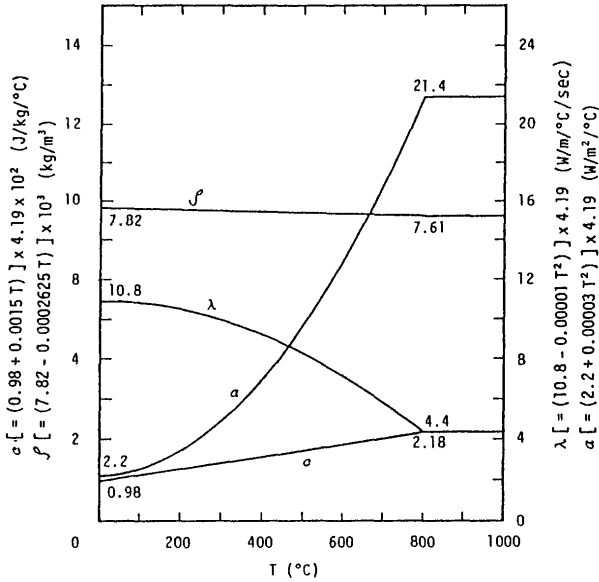
3. Results of Thermo-elasto-plastic Analysis and Consideration

The effects of repair weld length and travel speed, which affect the magnitude and the distribution of welding residual stress and plastic strain produced in repair welding, are considered based on the results of three-dimensional thermo-elasto-plastic analysis.

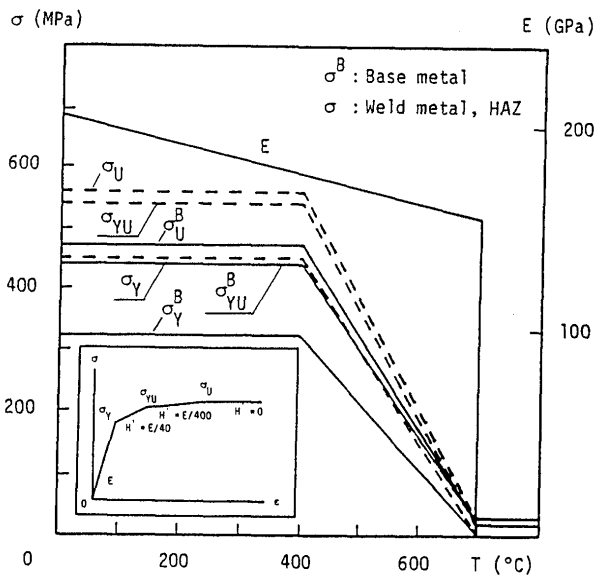
Moreover, welding residual stress and plastic strain obtained by plane-deformation thermo-elasto-plastic analysis are compared with those by three-dimensional one, and the mechanical significance of plane-deformation analysis is studied.

Three-dimensional unstationary thermal conduction analysis is carried out¹⁾ on the assumed models as shown in Table 1, and then three-dimensional thermal stress analysis is performed with the results of that. For plane-deformation analysis, the middle cross section ($x=0$, yz -plane) of specimens is selected and thermal stress analysis is carried out with temperature of three-dimensional analysis under the assumption of continuum of temperature at middle cross section. The distribution of temperature of each model (model M1F, M3F and M3S) along weld line at the time when weld metal of middle of specimen is cooled up to 500°C are shown in Fig.3. Besides, M6I is the model which repair weld length ($\ell=600\text{ mm}$) extends from edge to edge in x-direction. The instan-

taneous heat source (assuming that travel speed is infinite) is applied to it, so that each cross section (yz -plane) of middle parts of model M6I experiences the same thermal history.



(a) Physical properties



(b) Mechanical properties

Fig. 2 Temperature dependence of physical and mechanical properties

Table 1 Weld length and welding conditions

	Weld length (m)	Heat input (MJ/m)	Travel speed (m/s)
M1F	0.1	3.4	0.0055
M3F	0.3	3.4	0.0055
M3S	0.3	3.4	0.0017
M6I	0.6	3.4	∞

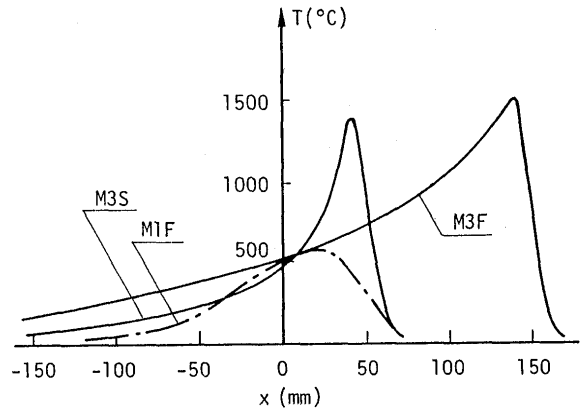


Fig. 3 Temperature distributions along weld line

3.1 Effects of repair weld length and travel speed on welding residual stress and plastic strain by three-dimensional analysis

The distribution of welding residual stress and plastic strain produced in weld metal along weld line, are shown in Fig.4. All of the models shows the same behavior, and then the order¹⁾ of mechanical severity is $x \rightarrow y \rightarrow z$ -direction. The distributions of welding residual stress and plastic strain through thickness of the middle cross section are shown in Fig.5. The effects of repair weld length and travel speed are considered below paying attention to the results of the analyses.

(a) Effect of repair weld length

The results of three-dimensional analysis for model M1F and M3F, which have different weld length with same travel speed, are compared to consider the effect of repair weld length. The distributions of welding residual stress (Fig.4(a)) produced in weld metal along weld line are not influenced by repair weld length but the magnitude of residual stress can be affected by it, and then the residual stress component, σ_y , perpendicular to weld line of model M1F is larger than that of model M3F, because repair weld length of model M1F is shorter than that of model M3F. The transient stress components of weld metal ($y=3, z=8(\text{mm})$) in the middle ($x=0$) of repair welds are shown in Fig.6 (a) and (b). The difference of σ_y between two models appears through the whole of the temperature history but considerably distinguishes in the range of low temperatures. This phenomenon in the range of low temperatures depends on severity of restriction of contraction in y -direction. Therefore, it is considered that σ_y of model M1F became large, for the reason that in general restriction of y -direction is severe²⁾ in model M1F having shorter repair weld length than model M3F.

Besides, paying attention to plastic strain, the region where the residual plastic strain components; ϵ_x^p (along

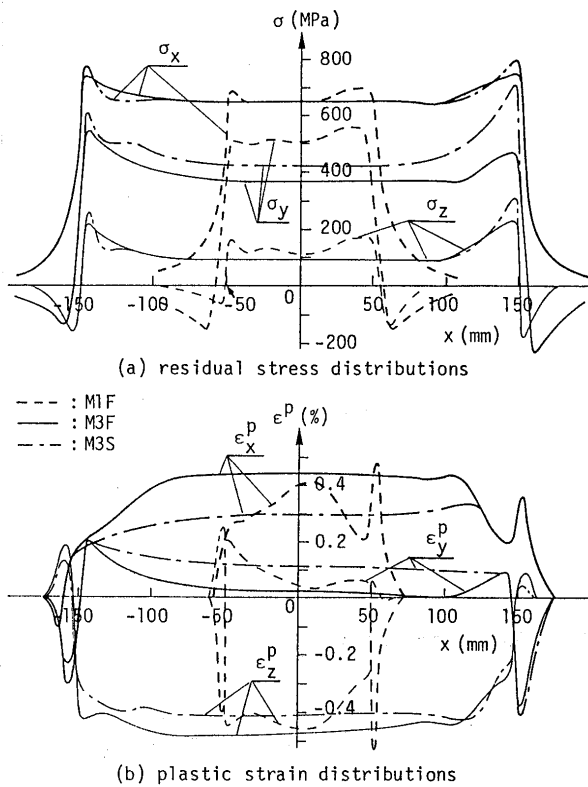


Fig. 4 Distributions of welding residual stress and plastic strain along weld line (three-dimensional thermo-elasto-plastic analysis)

weld line), ϵ_y^p (perpendicular to weld line) and ϵ_z^p (in thickness direction) are uniformly distributed does not exist in model M1F of which repair weld length is shorter than that of model M3F (Fig.4(b)). However, the process of production of plastic strain between two models is almost same (Fig.6(a'), (b')), provided repair weld length is as long as that of model M1F.

Hereinafter, in order to investigate the effect of repair weld length which affects stress and plastic strain, the transient temperature in weld metal along weld line calculated by three-dimensional unstationary heat conduction analysis (considering the effect of moving heat source) are illustrated as the solid line in Fig.7. The dotted line also expresses the distribution curve of temperatures in a quasi-stationary state obtained from the analysis for infinite weld length. The range, which lies between two chain lines at weld starting and finishing edge, represents a quasi-stationary state of temperature during cooling down up to temperature of the longitudinal axis (Fig.7). That range does not receive the effect of weld starting and finishing edge, so that the temperature history is equal to that of infinite weld length. The range, which experiences a quasi-stationary state of temperature during cooling down up to a room temperature, undergoes the same loading history. Consequently, even if the temperature gradient exists along welded line, welding residual stress

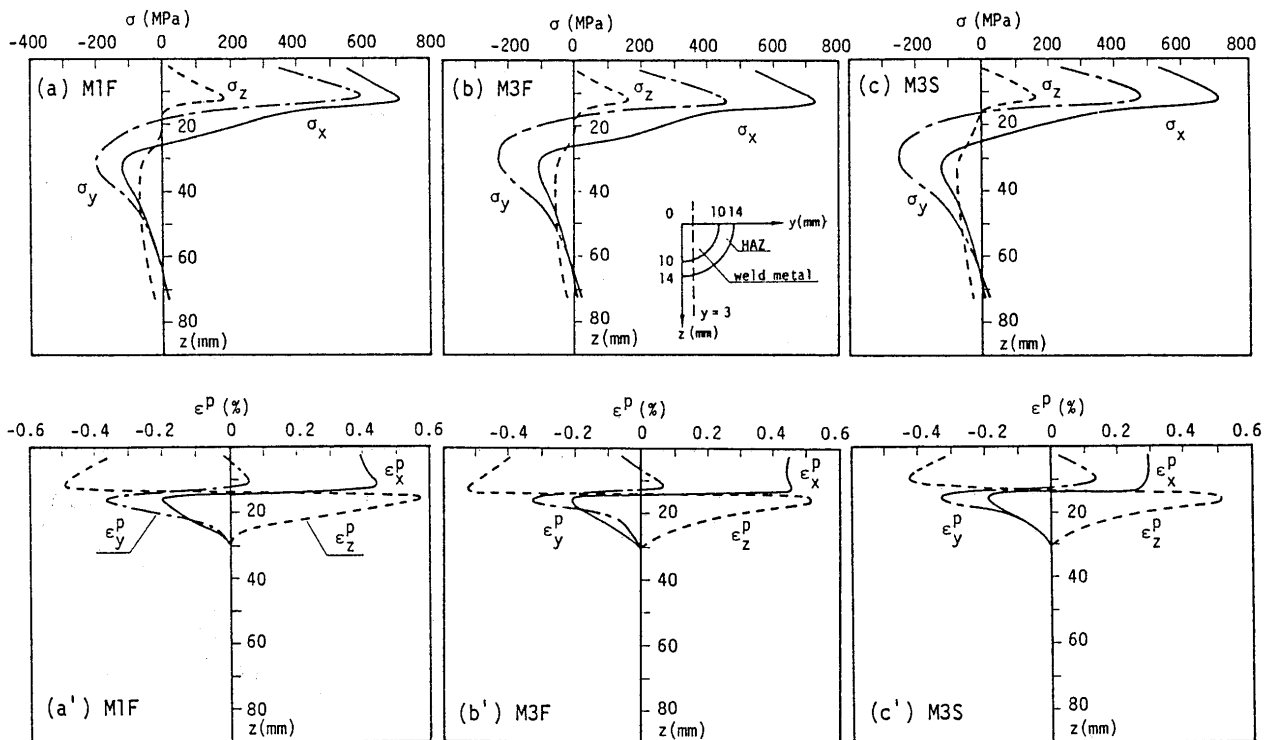


Fig. 5 Distributions of welding residual stress and plastic strain through thickness (three-dimensional thermo-elasto-plastic analysis)

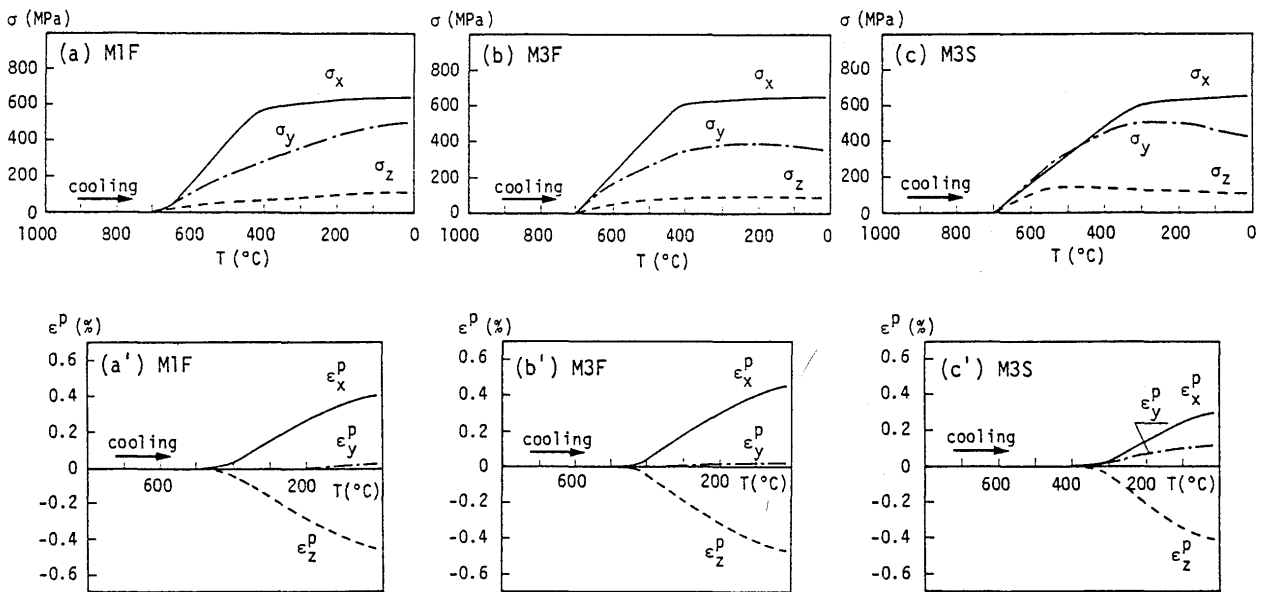


Fig. 6 Transient stress and plastic strain in weld metal (at $y=3, z=8(\text{mm})$) (three-dimensional thermo-elasto-plastic analysis)

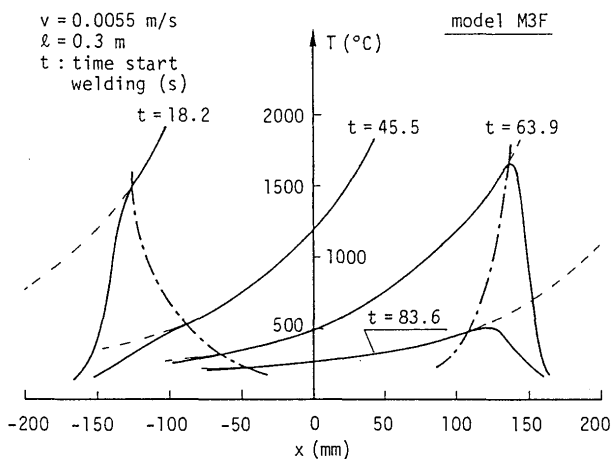


Fig. 7 Comparison of distributions of temperature along weld line between infinite weld length and finite one

and plastic strain uniformly produce along weld line under the condition that stiffness as to thermal expansion and contraction is uniform.

(b) Effect of travel speed

To compare the results of the analysis for model M3F and M3S having different travel speed with same repair weld length, the effect of travel speed is examined concerning with stress and plastic strain. Observing welding residual stress (Fig.4(a)), the difference of travel speed which affects the distribution of residual stress slightly appears on the arc starting and finishing edges. Beside, the difference of it represents as the magnitude of the stress component σ_y perpendicular to weld line. The transient stress components in weld metal ($y=3, z=8(\text{mm})$)

of model M3F and M3S are shown in Fig.6(b), (c). All of the stress components σ_x (along weld line), σ_y (perpendicular to weld line) and σ_z (in thickness direction) are greatly influenced by the difference of travel speed in comparative high temperature. However, at a room temperature only the stress component σ_y receives the effect.

Besides, considering the distributions of plastic strain as shown in Fig.4(b), all of the plastic strain components considerably undergo the effect of travel speed, so that the magnitude of ϵ_x^p of model M3F with fast travel speed is larger than that of model M3S with slow travel speed but the magnitude of ϵ_y^p of model M3F is smaller than that of model M3S. The transient plastic strain components in weld metal ($y=3, z=8(\text{mm})$) of model M3F and M3S are shown in Fig.6(b'), (c'), and thus the process of production of residual stress is different in two models (Fig.6 (b), (c)) but that of plastic strain has the same tendency between two models.

Considering model M6I with instantaneous heat source (travel speed $v=\infty$), the magnitude and the distribution of welding residual stress and plastic strain obtained by three-dimensional analysis, and those produced in the region where does not receive the edge effect, agree with those by plane-deformation analysis. The results of three-dimensional analysis for model M6I are referred to Fig.8(b), (b') and Fig.9(b), (b').

Comparing the results of model M6I, M3F and M3S, the faster travel speed is, the smaller σ_y is, the larger ϵ_x^p is, and the smaller ϵ_y^p is.

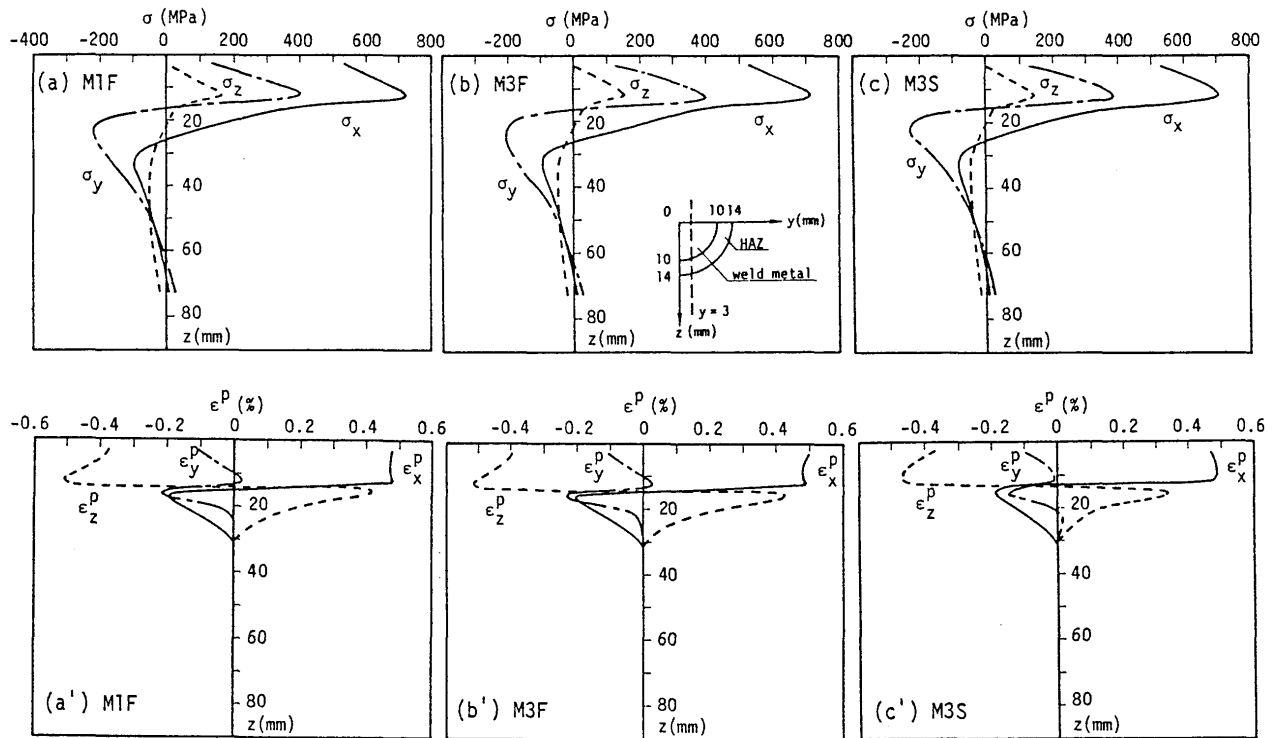


Fig. 8 Distributions of welding residual stress and plastic strain through thickness (two-dimensional plane-deformation thermo-elasto-plastic analysis)

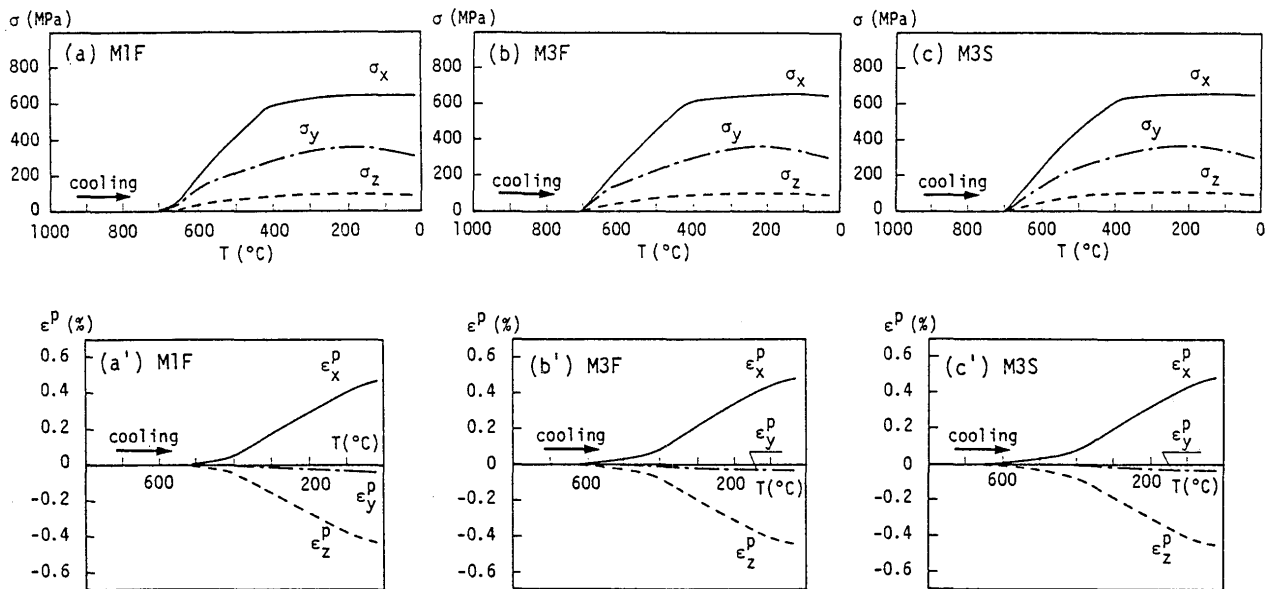


Fig. 9 Transient stress and plastic strain in weld metal (at y=3, z=8(mm)) (two-dimensional plane-deformation thermo-elasto-plastic analysis)

In case that heat in-input per unit length is same, the slower travel speed is, the shorter the distance (Fig.2) to the mechanical melting region ahead of the observing point becomes, therefore, plastic strain goes on receiving the effect of melting portion until the observing point cools down up to a room temperature. If the distance to

the melting portion near the observing point is short and melting region exist for a long time, mechanical restriction is considered to be weak because the melting portion does not restrict for the contraction of the observing point. Therefore, the stress components σ_x and σ_y experience the same history until the observing point reaches to low

temperature under slow travel speed as shown in Fig.6(c), and after disappearing the effect of melting region, mechanical restraint along weld line is severest regardless of travel speed. Moreover, the magnitude of each stress component is limited by the yield strength (yield surface) of material. By the results, as the stress component σ_y is large under the condition of slow travel speed and then the beginning of plasticity is delayed, the plastic strain component ε_y^p is considered to be small.

Based on the results of three-dimensional analysis, the effect of travel speed and repair weld length, which affect the magnitude and the distribution of welding residual stress and plastic strain, are examined. Changing travel speed and weld length, only the magnitude of the stress component σ_y largely receives the effect of them. However, all plastic strain components considerably sustain the effect of them.

The distinction of travel speed and repair weld length makes the temperature gradient along weld line. The gradient is changed the mechanical condition of weld metal and the vicinity, and largely affects the process of production of stress and plastic strain. The produced stress is limited by the yield condition of material regardless of severity of mechanical restriction but plastic strain greatly undergoes the effect of severity of mechanical restriction. As the result, it is considered that plastic strain is greatly differed.

3.2 Consideration and comparison of results of three-dimensional thermo-elasto-plastic analysis and two-dimensional plane-deformation one

Comparing the results of three-dimensional analysis with those of two-dimensional plane-deformation analysis, the differences of the mechanical condition based on stress and plastic strain are noticed only at the middle cross section ($x=0$, yz -plane) of repair welding. The distributions of welding residual stress and plastic strain through thickness obtained by two-dimensional plane-deformation analysis are shown in Fig.8, and then transient stress and plastic strain of weld metal ($y=3$, $z=8$ (mm)) are shown in Fig.9.

(a) Case of short repair weld length with fast travel speed (model M1F)

The welding residual stress components σ_x and σ_z obtained by plane-deformation analysis (Fig.8(a)) agree with those by three-dimensional analysis (Fig.5(a)), but only the magnitude of σ_y is greatly different in the weld metal. Namely, the magnitude of σ_y of three-dimensional analysis is greater than that by plane-deformation analysis.

Moreover, noting the plastic strain components obtained by both analyses (Figs.5(a') and 8(a')), the dis-

tribution of those are almost same. However, on the magnitude of plastic strain, especially the strain component ε_x^p obtained by plane-deformation analysis in weld metal is larger than that by three-dimensional analysis, but the component ε_y^p obtained by plane-deformation analysis is smaller than that by three-dimensional analysis.

(b) Case of long repair weld length with fast travel speed (model M3F)

As similar to model M1F having short repair weld length, the residual stress components σ_x and σ_z obtained by plane-deformation analysis (Fig.8(b)) agree with those of three-dimensional analysis (Fig.5(b)), but only the magnitude of stress component σ_y obtained by three-dimensional analysis is larger than that by plane-deformation analysis.

Observing the plastic strain components, the component ε_y^p obtained by three-dimensional analysis is larger than that by plane-deformation analysis (Fig.8(b')), however, the component ε_x^p is smaller than it.

(c) Case of long repair weld length with slow travel speed (model M3S)

The residual stress components σ_x and σ_z obtained by plane-deformation analysis (Fig.8(c)) and three-dimensional analysis (Fig.5(c)) have a good coincidence, but only the magnitude of σ_y obtained by plane-deformation analysis is smaller than that by three-dimensional analysis.

Observing the plastic strain components, the magnitude of ε_x^p obtained by three-dimensional analysis (Fig.5(c')) is smaller than that by plane-deformation analysis (Fig.8(c')), but ε_y^p is larger.

(d) Case of long repair weld length with instantaneous heat source (model M6I)

Based on the results of three-dimensional analysis, the magnitude and the distribution of transient/residual stress and plastic strain produced in the region, where does not reach the effect of edges, agree with those by plane-deformation analysis. From the above results, when repair weld length is long and there is no temperature gradient along weld line, it is found that three-dimensional problem can be strictly replaced by two-dimensional plane-deformation problem including the production mechanism of welding residual stress and plastic strain.

Meanwhile, in model M1F, M3F and M3S, the process of production of stress is different each other. However, observing the residual stress components; σ_x and σ_z obtained by three-dimensional analysis and plane-deformation analysis coincide whereas only the magnitude of the component σ_y by three-dimensional analysis is larger. That tendency appears under the condition of slower travel speed and shorter repair weld length. The process of production of plastic strain has the same tendency,

however, the magnitude of plastic strain is greatly different between the results of three-dimensional analysis and that of plane-deformation analysis. The slower travel speed is, the more considerable tendency is. As explained in 3.1(a) and (b), the difference of the results of two analyses is caused by the distinction of the mechanical boundary condition due to the temperature gradient along weld line.

However, the magnitude and the distribution of transient/residual stress and plastic strain obtained by plane-deformation analysis as to four models (model M1F, M3F, M3S and M6I) are almost same (Figs.8 and 9). That is considered as the reason why the cross sections experienced a quasi-stationary state of temperature in moving heat source undergo nearly the same temperature history³⁾ as the case of supplying infinite weld length with instantaneous heat source at least in cooling stage. Therefore, temperature, for calculating thermal stress of two-dimensional plane-deformation problem, can be obtained by not only three-dimensional analysis but also two-dimensional one assumed no heat transfer to the both sides of surface (yz -plane) on the section.

4. Mechanical Condition Necessary for Replacing Three-dimensional Problem with Two-dimensional (Plane-deformation) Problem

As mentioned in 3.1(a), even if travel speed is finite, welding residual stress and plastic strain are uniformly produced along weld line so long as in the region which experienced the temperature history of a quasi-stationary state and which stiffness to thermal expansion and contraction of welded parts is uniform. The welding residual stress component σ_y and the plastic strain components ε_x^p and ε_y^p obtained by three-dimensional analysis are considerably different from those by plane-deformation analysis under the condition of slow travel speed, even though temperature undergoes a quasi-stationary state (ref. 3.2(d)).

Hereinafter, based on the features of temperature, the necessary mechanical condition to be satisfied for estimating the magnitude and the distribution of welding residual stress and plastic strain by plane-deformation analysis, is considered.

From the results of a series of analyses in this study, if the region cooled down up to 400°C experiences the temperature history of a quasi-stationary state under the condition of travel speed $v \geq 0.0055\text{m/s}$, the magnitude and the distribution of welding residual stress and plastic strain can be estimated by two-dimensional plane-deformation analysis. Only the distribution and the magnitude of residual stress can be estimated under the condi-

tion of travel speed $v=0.0017\text{m/s}$.

The distance experienced a quasi-stationary state of temperature from weld starting to finishing edges is about within 100mm respectively from both edges regardless of travel speed. Moreover, the length received the edge effect is defined about 100mm by analysis of model M6I.

Without regard to restriction of weld starting and finishing edges, the magnitude and the distribution of welding residual stress, in the region which experienced a quasi-stationary state of temperature cooling down up to a room temperature (400°C in this analysis) under actual travel speed, can be predicted by two-dimensional plane-deformation analysis.

The computing time (CPU-time; Computer SX-1 of Osaka University is used) for calculating three-dimensional thermo-elasto-plastic analysis and plane-deformation one is compared in Table 2, and the CPU-time of plane-deformation analysis is about 1/1000 of three-dimensional one.

Table 2 Comparison of CPU-time of three-dimensional and plane-deformation analysis

	No. of unknown		CPU time (s)	
	3D	PLD	3D	PLD
M1F	8089	173	36000	29
M3F	9133	173	48000	42
M3S	11743	173	49000	27
M6I	6174	173	9000	16

3D : Three-dimensional analysis
PLD : Plane-deformation analysis

5. Conclusion

In this paper, a series of models for repair welding in a thick plate is assumed, and then three-dimensional thermo-elasto-plastic analysis and two-dimensional plane-deformation one are performed.

The summary of the main conclusion is as follows:

From the results of three-dimensional thermo-elasto-plastic analysis;

- (1) In the region which undergoes the temperature history of a quasi-stationary state, whether there is the temperature gradient along weld line or not, the magnitude and the distribution of welding residual stress and plastic strain are same at each cross section along weld line if the rigidity to thermal expansion and shrinkage is uniform.
- (2) The shorter repair weld length is and the slower travel speed is, the more the welding residual stress component of perpendicular to weld line, σ_y , increases. In addition, the slower travel speed is, the more plastication delays, so that the plastic strain component

along weld line, ε_x^p , decreases.

From the comparison between the results of three-dimensional analysis and two-dimensional plane-deformation one;

- (3) Only when the weld length is long and there is no temperature gradient along weld line, three-dimensional problems can, in the strict sense of the production mechanism of stress and plastic strain by welding, be replaced by two-dimensional (plane-deformation) ones. Besides, as residual stress is limited by the yield strength of material, it can be accurately estimated by plane-deformation analysis. If travel speed is slower, it needs to be noted to underestimate the stress component σ_y by plane-deformation analysis. Whereas, since the magnitude and the distribution of plastic strain are largely affected by the mechanical boundary conditions, the estimation of plastic strain is less accurate than that of residual stress by plane-deformation analysis.
- (4) The distribution of temperature for plane-deformation thermo-elasto-plastic stress analysis can be given from two-dimensional analysis which assumes no thermal conduction from the cross section.
- (5) From a series of results of analyses in this study, if the

region cooled down up to 400°C experiences the temperature history of a quasi-stationary state (the weld length; $l \geq 0.2\text{m}$, at least the middle parts of model undergoes a quasi-stationary state of temperature), under the condition of travel speed $v \geq 0.0055\text{m/s}$, the magnitude and the distribution of welding residual stress and plastic strain can be estimated by two-dimensional plane-deformation analysis. Only the distribution and the magnitude of residual stress can be estimated under the condition of travel speed $v \geq 0.0017\text{m/s}$.

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