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Fractal Multi-Grid Method for Ultra Large Scale Mechanical and Thermal Simulations†

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Abstract

The finite element method is a powerful tool to predict not only the mechanical behavior of structures but also the residual stresses and distortions caused by welding. In most problems with engineering interest, the phenomena are three-dimensional in their nature. Generally, the three-dimensional elastic-plastic FE analysis requires very long computational time and large memory size. To overcome this problem, an idea of Fractal Multi-Grid method is proposed. Its potential capability is demonstrated through application to very simple mechanical problems.

KEY WORDS: (Fractal), (Multi-Grid), (Ultra Large Scale Simulation), (Mechanical), (Thermal)

1. Introduction

Nonlinear transient three dimensional FE simulation requires very long computational time. To improve the speed of computation, we must take advantage of the characteristics of the problem. Noting the fact that the region which exhibits strong nonlinearity in welding or crack propagation problem is limited to a very small area compared with the size of the model to be analyzed and the remaining part is mostly linear or weakly nonlinear, the problem can be transformed into a combination of a large linear problem and a small, but moving, strong nonlinear problem. Based on this idea, the authors proposed an Iterative Substructure Method (ISM)[1]. When the structure to be analyzed is large, most of the computational time is used for solving the large linear or quasi-linear problem. This can be greatly reduced by saving the stiffness matrix after forward elimination. When the stiffness matrix is unchanged, the solution of the simultaneous equation can be obtained by the backward substitution using the same matrix after forward elimination. Since the ISM uses the Skyline method to solve the large linear problem, the maximum degrees of freedom which can be handled by one PC is about 50,000. Thus, the authors developed a Fractal Multi-Grid (FMG) method which can solve large scale problems with more than one million degrees of freedom. The advantage of this method is that the computing time and the required memory size is nearly proportional to the degrees of freedom. Its potential capability is demonstrated through two and three dimensional elastic problems as well as plate bending problems.

2. Fractal Multi-Grid Method

Noting that the size of the nonlinear region is small, the key for improving computational speed is how to solve the large quasi-linear problem efficiently both in computational speed and memory saving. There are several possibilities, such as the ICCG (Incomplete Cholesky Conjugate Gradient) method, the multifrontal method2) and the multi-grid method3). All of the three methods are superior to the Skyline method in memory saving. The parallel and grid computings are also promising.

The idea of the Fractal Multi-Grid method is illustrated using a two dimensional simple elastic problem as shown in Fig. 1. A square sheet is stretched at its four vertices. When the model is subdivided into 8 × 8 elements, the deformation is computed by solving the basic cells consist of 2 × 2 elements under the prescribed displacements at four vertices as shown in Fig. 2. Such a basic operation is repeated hierarchically from the top level to the lowest level. In this process, the continuity between the neighboring cells is ignored as illustrated in Fig. 3. The continuity of the displacement can be

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recovered at the lowest level by interpolating the displacements at two nodes sharing the same cell boundary. In this way, the continuity of the traction is not guaranteed. It is retained through the iteration. Since all the governing equations and boundary conditions must be satisfied at the lowest or the finest level, the error in the stress field is evaluated at the lowest level and it is transferred to the upper level. The detail of the scheme is as follows,

Step-(1) Solve the cell at the 1st-level under the given boundary condition, not necessarily prescribed displacements at four vertices.

Step-(2) Use the displacements at 9 nodes obtained for the 1st-level as the prescribed displacements at four vertices, compute displacements of four cells belonging to the 2nd-level.

Step-(3) Repeat the same computation as in Step-(2) until the lowest level.

Step-(4) Force the continuity of the displacements at four nodes on the edge of the cell in the lowest level by interpolating the displacements at the two nodes belonging to the two cells sharing the same edge.

Step-(5) Compute the unbalanced force at the lowest level.

Step-(6) Redistribute the unbalanced force at the four middle nodes and one center node of the cell to the four nodes at vertices according to the following rule.

middle node: Redistribute one half of the unbalanced force to the two corner nodes on the same edge.

center node: Redistribute one fourth of the unbalanced force to the four corner nodes of the cell.

Step-(7) Split the above unbalanced nodal forces to the two nodes belonging to the cells sharing the same edge according to the ratio of the stiffness.

Step-(8) Compute the correction to the nodal displacement of the cell using the redistributed unbalanced force with the prescribed correction displacements at the four corner nodes.

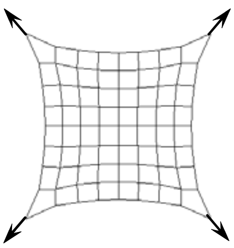

Fig. 1 Elastic sheet stretched at four corners.

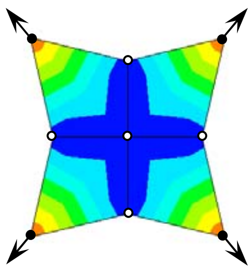

Fig. 2 Basic cell consists of 4 elements.

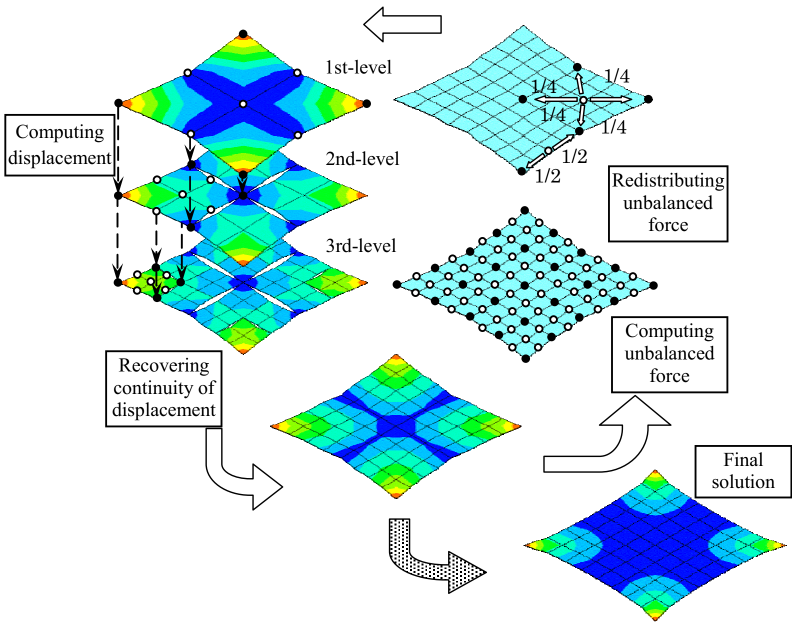

Fig. 3 Procedure of computation.

Step-(9) Repeat the Steps-(2) through (8) until the norm of the unbalanced force at the lowest level becomes small enough.

The details of the computational scheme may be different when the problem to be solved is different, such as for heterogeneous or nonlinear problems. If the problem is a linear isotropic two or three dimensional problem and the regular uniform mesh is used, the stiffness matrix of all cells becomes the same. Therefore, it is not necessary to save the stiffness matrices of all cells. Once the inverse of the stiffness matrix of a cell is computed, it can be used repeatedly. From the aspect of memory space, the required memory size is only three or four times the degrees of freedom.

3. Example Problems

3.1 Elastic square sheet stretched at four vertices

One of the example problems is the elastic square sheet stretched at four vertices as shown in Fig. 1. This problem is solved by FMG with the basic solution procedure. Figure 4 shows the convergence of the norm of unbalanced force with the iteration for 6 cases, in which the number of hierarchies is from 2 to 7. The degrees of freedom (DOF) for the 7-level is 33,282. The rate of convergence becomes slightly smaller as the number of hierarchies increases but good convergence is observed generally. The relation between the computing time and the degrees of freedom is summarized in Fig. 5. The computing time to achieve the relative error of 10^{-3} to 10^{-8} is plotted. The error in this figure is the relative value which is normalized by that of the first iteration. As seen from the slope of the curve, the computational time increases almost linearly with the degrees of freedom up to the 12-level. In the case of the conventional direct solution method, the computational time is proportional to $n^{2.0}$ (for two dimensional problem) or $n^{2.3}$ (for three dimensional problem). Though the computational time increases almost linearly with the degrees of freedom up to the 12-level, its slope becomes larger when the level is higher than 12. This is partly because the time to access the disc becomes dominant in computation.

3.2 Plate modeled using solid elements

When a coarse global mesh is employed, the

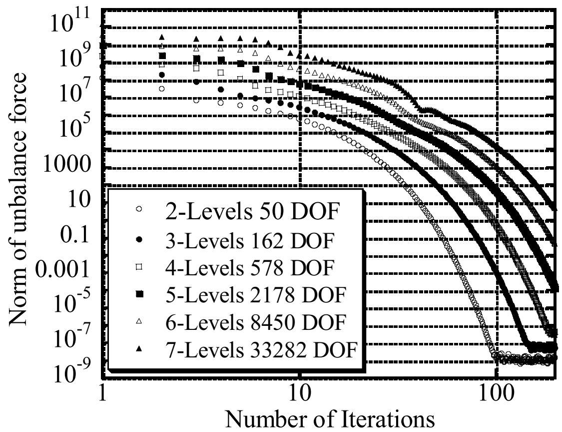

Fig. 4 Convergence of norm of unbalanced force.

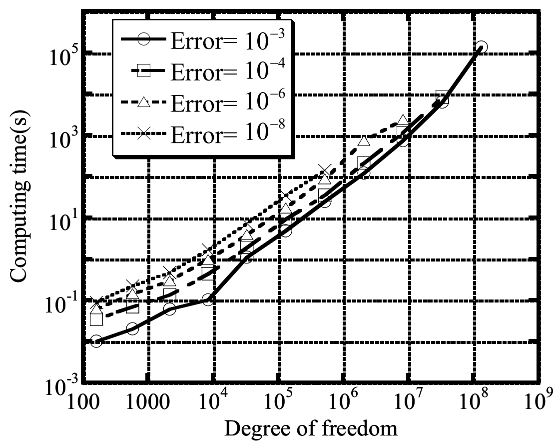

Fig. 5 Relation between computing time and degree of freedom.

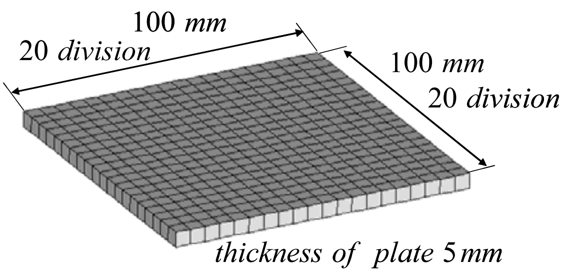

Fig. 6 Coarse global mesh of a square plate.

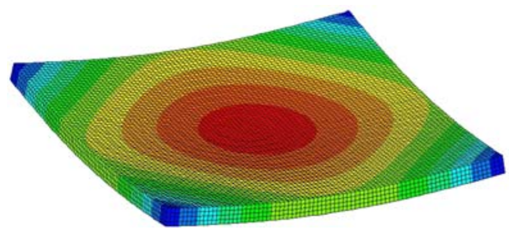

Fig. 7 Deflection of plate.

deformation of a plate can be analyzed. Figure 6 shows the coarse global mesh of a square plate. The size of the plate is 100 mm in width and length and 5 mm in thickness. The plate is divided into a 20 x 20 mesh. Each element is further divided into FMG of two-level or subdivided into 64 elements. Thus, the total number of elements becomes 25,600 and the degrees of freedom are 98,415. The plate is assumed to be simply supported at four corners and the center of the plate is pulled down 0.0001 mm. The computed deflection is shown in Fig. 7 using contour lines.

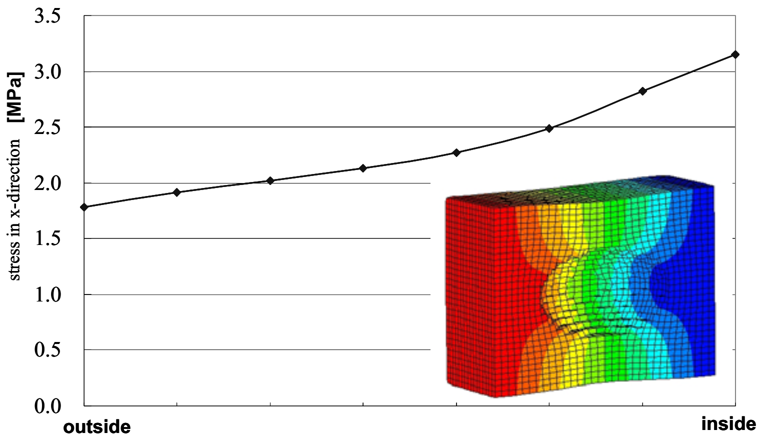

Fig. 8 Stress distribution through wall thickness in a cube with spherical cavity.

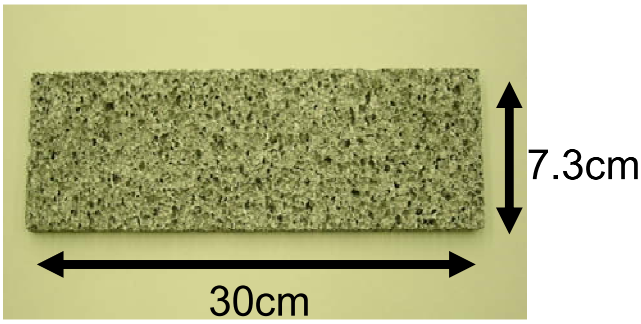

Fig. 9 Formed aluminum alloy.

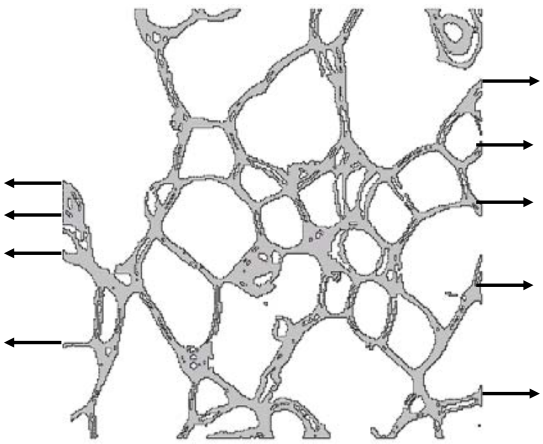

Fig. 10 X-ray CT scan image and applied displacements.

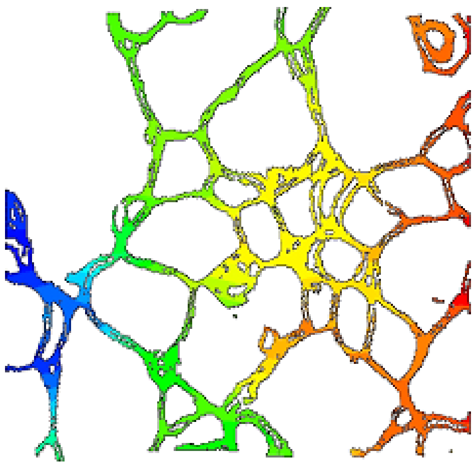

Fig. 11 Displacement in tensile direction.

3.3 Cube with a spherical cavity

The proposed FMG method can be regarded also as a method to model the space which contains a physical model to be analyzed. A simple example is uni-axial stretch of a cube which has a spherical cavity at the center as shown in Fig. 8. The stress distribution through the wall of the cube is shown in the figure as an example. The cubic space can be carved in arbitrary way no matter from inside or outside. If the information from the CT scan of the specimen to be analyzed is available, the specimen can be reproduced in the space formed by the FMG method without complex computation.

Figure 9 shows the formed aluminum alloy. Its X-ray CT scan image is shown in Fig. 10. The deformation under the applied forced stretch at both edges is computed using the CT scan image directly. The problem is idealized as a two dimensional problem and solved by the proposed FMG. Figure 11 shows the computed displacement in the stretching direction.

4. Conclusions

The problems with practical interest in industry are generally large in size and complex in geometry. To solve such large problems, the authors proposed a Fractal Multi-Grid method. Its effectiveness and potential usefulness are demonstrated through several examples and the following conclusions are drawn.

(1) The computing time is roughly proportional to the degrees of freedom.
(2) Though the rate of convergence becomes small when the hierarchical level becomes large, the convergence is generally good and acceptable accuracy can be obtained in 200 iterations or so.
(3) Ultra large problems such as a problem with 134,250,498 degrees of freedom can be solved by the FMG method.
(4) Using a coarse global mesh, a model with arbitrary geometry such as a plate can be analyzed by the FMG method.
(5) Specimens with arbitrary geometry in a cubic space can be simulated by the FMG method.

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