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EXISTENCE OF SOLUTIONS OF SOME NONLINEAR WAVE EQUATIONS

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0. Introduction and Theorem

Let H be a real Hilbert space and A be a positive self adjoint operator in H. Let ϕ be a lower semi continuous proper convex function from H to $(-\infty,\infty]$ and $\partial\phi$ be the subdifferential of ϕ. Then we shall consider the following equation

$$
(0.1)\qquad \begin{cases} \dfrac{d^2}{dt^2}u+Au+\partial\phi u \ni f(\cdot, u) \\ u(0)=a\,, \quad \dfrac{d}{dt}u(0)=b \qquad \text{on} \quad [0, T] \end{cases}
$$

where T is a positive number.

The above equation was studied in Schatzman [3], [4], [5] and Maruo [2]. In this paper we prove the existence of a solution of the problem (0.1) under certain assumptions which are somewhat weaker than those of Schatzman [5] and Maruo [2].

In [5] Schatzman showed the existence and uniqueness of a solution of the following nonlinear wave equation

$$
(0.2)\qquad \begin{cases} \left(\dfrac{\partial^2}{\partial t^2}u-\dfrac{\partial^2}{\partial x^2}u\right)(u-r)=0\,, \quad \dfrac{\partial^2}{\partial t^2}u-\dfrac{\partial^2}{\partial x^2}u \geqq 0 \\ \text{in the sense of distributions in} \quad [0, 1]\times[0, T], \\ u(x, t)\geqq r(x)\,, \quad u(x, 0)=u_0(x)\,, \quad \text{for} \quad x\in[0, 1]\,, \\ \dfrac{\partial}{\partial t}u(x, t)=u_1(x) \quad \text{a.e.} \quad \text{in } [0, 1]\,, \\ u(0, t)=u(1, t)=0 \qquad \text{for} \quad t\in[0, T]\,, \end{cases}
$$

where r is a continuous given function such that $r(0)<0$, $r(1)<0$ and $\dfrac{d^2}{dx^2}r(x)\geqq 0$ (in the distribution sense). Set $K=\{f\in L_2(0, 1);\, f(x)\geqq r(x)\}$. The equation (0.2) is rewritten as the following equation in $L_2(0, 1)$

$$(0.3)\qquad \begin{cases} \dfrac{d^2}{dt^2}u+Au+\partial I_K\, u\ni 0 \\ u(0)=u_0\,,\ \ \dfrac{d}{dt}u(0)=u_1 \end{cases}$$

where $I(u)=\begin{cases} 0 & \text{if } u\in K \\ \infty & \text{if } u\notin K \end{cases}$ and

$A=-\dfrac{d^2}{dx^2}$ (Dirichlet problem).

We will show that we can apply our main theorem to this equation if only r is a continuous function satisfying $r(0)<0$, $r(1)<0$ to deduce the existence of a solution of (0.3). The solution of (0.3), however, does not always satisfy the locally energy conserving condition (see [5]). Hence we cannot get the uniqueness of a solution.

Let Ω be a domain in R^n with smooth boundary and consider the case $\phi(u)=\int_\Omega |u(x)|^{p+1}dx$, $p>1$. Then the equation (0.1) represents a nonlinear Klein-Gordon equation. It will be shown that if $(n+2)>p(n-2)$, the result of this paper can be applied to (0.1) in this case. Note that when $n=3$ this inequality is satisfied for $p=3$.

Now we state notations which will be used throughout this paper. The inner product of H is denoted by $(\cdot,\ \cdot)$. When S is a Banach space, its norm is denoted by $|\cdot|_S$. We denote by V the domain of $A^{1/2}$ endowed with the graph norm. By $\partial\phi_\lambda$ and ϕ_λ we denote the Yosida approximations of $\partial\phi$ and ϕ respectively (i.e. $\partial\phi_\lambda x=\lambda^{-1}(1-J_\lambda)^{-1}x$ and $\phi_\lambda(x)=(2\lambda)^{-1}|x-J_\lambda x|_H^2+\phi(J_\lambda x)$ where $J_\lambda=(1+\lambda\partial\phi)^{-1}$).

Next we shall introduce the assumptions.
Let X_1 and X_2 be real Banach spaces.

Assumption 1. The following inclusion relations hold:

$$V\subset X_1\subset H\subset X_2 \quad\text{and}\quad X_1\subset\{\text{the dual space of } X_2\}$$

where each inclusion mapping is continuous. Moreover X_1 is separable and the inclusion mapping from V to X_1 is compact. H is dense in X_2.

Assumption 2. There exists $z\in V$ such that

$$(\partial\phi_\lambda x,\ x-z)\geqq c_1|\partial\phi_\lambda x|_{X_2}-c_2$$

for $x\in V$, $|x|_V\leqq R$ and $|\phi(x)|\leqq R$ where c_1 and c_2 are positive constants depending only on R and z.

Assumption 3. The continuous function f from $[0,\ T]\times H$ to H satisfies for any $t \in [0, T]$ and $x, y \in H$

$$|f(t, x)-f(t, y)|_H \leqq h(t)|x-y|_H,$$

$$\left|\frac{d}{dt} f(t, x)\right|_H \leqq h(t)\,(1+|x|_H)$$

where h is a function belonging to $L_1(0, T)$.

ASSUMPTION 4. The closure of $D(\phi) \cap V$ in H is equal to the closure of $D(\phi)$ in H.

Clearly V and X_1 are dense in H. By assumption H is dense in X_2. We use the same notation $(\cdot, \cdot)$ as the inner product of H to denote the pairings between V, X_1, X_2 and their corresponding duals.

Now we define the solution of (0.1).

DEFINITION. We say that a function $u \in C([0, T]; X_1) \cap W^1_\infty(0, T; H)$ is a solution of the equation (0.1) when it satisfies the following requirements;

1) For any $t \in [0, T]$ $u(t) \in D(\phi) \cap V$.

2) There exist weak right and left derivatives $\frac{d^\pm}{dt} u(t) \in H$ for any $t \in [0, T]$. Moreover for any $t \in [0, T]$

$$\left|\frac{d^\pm}{dt} u(t)\right|_H^2 + |u(t)|_V^2 + 2\phi(u(t)) \leqq |b|_H^2 + |a|_V^2 + 2\phi(a)$$
$$+2\int_0^T \left(f(s, u(s)), \frac{d}{ds} u(s)\right) ds$$

(with necessary modifications at 0 and T).

3) There exists a linear functional F on $C([0, T]; X_1)$ such that

$$F(v-u) \leqq \int_0^T \phi(v(s))\, ds - \int_0^T \phi(u(s))\, ds$$

for any $v \in C([0, T]; X_1)$ and

$$\int_0^T \left(\frac{d}{ds} u(s), \frac{d}{ds} v(s)\right) ds + \int_0^T (f(s, u(s)) - Au(s), v(s))\, ds$$
$$+(b, v(0)) - \left(\frac{d^-}{dt} u(T), v(T)\right) = F(v)$$

for any $v \in C([0, T]; X_1) \cap L_1(0, T; V) \cap W^1_\infty(0, T; H)$.

4) The initial conditions are satisfied in the following sense

$$u(0) = a,\ b - \frac{d}{dt} u(0) \in \partial I_{K_0} a$$

where K_0 is the closure of the domain of ϕ, I_{K_0} is the indicator function of

K_0 and ∂I_{K_0} is the subdifferential of I_{K_0}.

We state the theorem.

Theorem. *Let a and b be given elements satisfying*

$$a \in V \cap D(\phi), \quad b \in H.$$

Then under the assumptions 1, 2, 3 *and* 4 *we have at least one solution of* (0.1).

To prove the above theorem we consider the following approximate equations for $\lambda > 0$

$$\begin{cases} \dfrac{d^2}{dt^2} u_\lambda + A u_\lambda + \partial \phi_\lambda u_\lambda = f(\cdot, u_\lambda) \\ u_\lambda(0) = a, \quad \dfrac{d}{dt} u_\lambda(0) = b. \end{cases} \tag{0.4}$$

In the next section using a method similar to that of [2] we shall investigate the convergence of the solutions of the approximate equations (0.4). In section 2 we prove the theorem. In section 3 we show some examples.

1. Convergence of approximate solutions

In this section under the assumptions 1, 2, 3 and 4 we shall study the convergence of the solutions of (0.4). In what follows let initial values a and b belong to $V \cap D(\phi)$ and H respectively.
First we show some properties of the approximate solutions.

Lemma 1. *For any* $\lambda > 0$ *we have solutions of the problem* (0.4) *such that*

$$u_\lambda \in C([0, T]; H) \cap L_\infty(0, T; V) \cap W^1_\infty(0, T; H) \cap W^2_\infty(0, T; V^*)$$

where V^* is the dual space of V.

Proof. See p. 289 Barbu [1].

Lemma 2. *We hold the following equality and inequality*

1) $$\left|\frac{d}{dt} u_\lambda(t)\right|_H^2 + |u_\lambda(t)|_V^2 + 2\phi_\lambda(u_\lambda(t)) = |b|_H^2 + |a|_V^2 + 2\phi_\lambda(a) + 2\int_0^t \left(f(s, u_\lambda(s)), \frac{d}{ds} u_\lambda(s)\right) ds$$

2) $$\left|\frac{d}{dt} u_\lambda(t)\right|_H^2 + |u_\lambda(t)|_V^2 + 2\phi_\lambda(u_\lambda(t)) \leqq C_1(|b|_H^2 + |a|_H^2 + |a|_V^2 + 1)$$

where C_1 is a constant depending only on h and T.

Proof. See Lemma 2.2 in [2].

Lemma 3. *There exists a constant independent of λ such that*

$$\int_0^T |\partial\phi_\lambda u_\lambda(s)|_{X_2}\, ds \leqq Constant.$$

Proof. In the inequality of assumption 2 we put $x=u_\lambda(t)$. From Lemma 2 the constants c_1 and c_2 are independent of λ. Replacing $\partial\phi_\lambda u$ by $-(u_\lambda''+Au_\lambda-f(\cdot, u_\lambda))$, integrating over $[0, T]$, using the integration by parts and noting 2) of Lemma 2 we get the conclusion of the Lemma.

Lemma 4. *We have a continuous function u from $[0, T]$ to H such that a subsequence $\{u_{\lambda_j}\}$ of the sequence $\{u_\lambda\}$ converges uniformly to u in H as $\lambda_j \to 0$.*

Proof. In view of 2) of Lemma 2 $|u_\lambda(t)|_V$ is uniformly bounded. Hence from the assumption 1 we know that $\{u_\lambda(t)\}$ is a relatively compact subset of H for any $t\in[0, T]$. From 2) of Lemma 2 $\{u_\lambda\}$ is uniformly continuous. Thus using Ascoli-Arzela's theorem this lemma is proved.

For simplicity we denote this subsequence by $\{u_\lambda\}$.

Lemma 5. *There exists a subsequence $\{\lambda_j\}$ of $\{\lambda\}$ such that $\{u_{\lambda_j}\}$ converges to u in $C([0, T]; X_1)$, $\left\{\frac{d}{dt}u_{\lambda_j}\right\}$ converges to $\frac{d}{dt}u$ in weak*-$L_\infty(0, T; H)$ and $\{u_{\lambda_j}(t)\}$ weakly converges to $u(t)$ in V for any $t\in[0, T]$. Hence we know that*

$$u\in C([0, T]; X_1)\cap W_\infty^1(0, T; H)$$

and

$$u(t)\in D(\phi)\cap V \qquad \textit{for any} \quad t\in[0, T].$$

Proof. From 2) of Lemma 2 it is easy to prove that some subsequence $\left\{\frac{d}{dt}u_{\lambda_j}\right\}$ converges to $\frac{d}{dt}u$ in weak*-$L_\infty(0, T; H)$. Since $\{u_{\lambda_j}(t)\}$ is bounded in V it follows from Lemma 4 that $\{u_{\lambda_j}(t)\}$ weakly converges to $u(t)$ in V. Next we assume that there exists a sequence $\{t_i\}_{i=1}^\infty$ such that

$$(1.1) \qquad \begin{cases} \lim_{i\to\infty} t_i = t_\infty, \\ |u(t_i)-u(t_\infty)|_{X_1} \geqq \delta_0 > 0 \quad \text{and} \quad t_i\in[0, T] \\ \text{for } i=1, 2, \cdots. \end{cases}$$

Since $|u(t_i)|_V$ is bounded there exists a subsequence $\{u(t_{i_j})\}$ which converges to some element w of X_1. On the other hand u is continuous in H. Hence $w=u(t_\infty)$. The above results contradict (1.1). Thus u is continuous in X_1. Combining that for any $t\in[0, T]$ $\{u_{\lambda_j}(t)\}$ is a relatively compact set in X_1, that $\{u_{\lambda_j}\}$ is uniformly convergent to u in H and that u is continuous in X_1 we can prove that $\{u_{\lambda_j}\}$ is uniformly convergent to u in X_1.

For simplicity we denote the subsequence $\{\lambda_j\}$ by $\{\lambda\}$.

We denote $\int_0^t \partial\phi_\lambda u_\lambda(t)\,dt$ by $\rho_\lambda(t)$ for any $t\in[0, T]$. Then ρ_λ belongs to $W^1_\infty([0, T]; H)$.

Lemma 6. *There exists a subsequence $\{\lambda_j\}\subset\{\lambda\}$ such that for $\alpha\in X_1$ and $t\in[0, T]$, $\{(\rho_{\lambda_j}(t), \alpha)\}$ converges.*

Proof. From Lemma 3 for any $\alpha\in X_1$ we know that the total variation of the function $\{(\rho_\lambda(t), \alpha)\}$ on $[0, T]$ is uniformly bounded in λ. Noting that X_1 is separable and using Helly's choice theorem and the diagonal method we have a subsequence $\{\lambda_j\}$ such that

$$\lim_{\lambda_j\to 0} (\rho_{\lambda_j}(t), \alpha) \quad \text{exists for any} \quad \alpha\in X_1\,.$$

For simplicity we denote $\{\lambda_j\}$ by $\{\lambda\}$.

Put

$$\int_0^t (\partial\phi_\lambda u_\lambda(s), v(s))\,ds = F_{\lambda,t}(v) \quad \text{for } v\in C([0, T]; X_1) \text{ and } t\in[0, T]\,.$$

Lemma 7. *For each $t\in[0, T]$ there exists a linear continuous functional F_t on $C([0, T]; X_1)$ such that*

$$\lim_{\lambda\to 0} F_{\lambda,t}(v) = F_t(v) \textit{ for any } v\in C([0, T]; X_1)\,.$$

Proof. Combining Lemmas 3 and 6 and approximating v by step functions we can prove this lemma.

Lemma 8. *For any $v\in C([0, T]; X_1)$ there exist*

$$\textit{right} \lim_{t\to t_0} F_t(v) \quad \textit{and} \quad \textit{left} \lim_{t\to t_0} F_t(v)$$

where $t_0\in[0, T]$ (with necessary modifications at 0 and T).

Proof. From Lemma 7 it follows

$$|F_t(v)-F_s(s)| \leqq \lim_{\lambda\to 0} \int_s^t |\partial\phi_\lambda u_\lambda(\tau)|_{X_2}\,d\tau\cdot \sup_{0<\tau<T} |v(\tau)|_{X_1}\,.$$

Combining this inequality with Lemma 3 we see that $F_t(v)$ is of bounded variation. Thus the lemma is proved.

We put

$$\lim_{t\to T} F_t(v) = F(v)\,.$$

Lemma 9. *For any $t\in[0, T]$ there exist weak right and left derivatives $\frac{d^{\pm}}{dt}u(t)$ in H (with necessary modifications at* 0 *and T).*

Proof. Let v be an arbitrary element of $C^1([0, T]; X_1)\cap C([0, T]; V)$. Forming the inner product of (0.4) and v and integrating by parts we get

$$\text{(1.2)}\qquad \Big(\frac{d}{dt}u_\lambda(t), v(t)\Big) = (b, v(0)) + \int_0^t \Big(\frac{d}{ds}u_\lambda(s), \frac{d}{ds}v(s)\Big)\,ds + \int_0^t (f(s, u_\lambda(s)) - Au_\lambda(s), v(s))\,ds - F_{\lambda,t}(v)\,.$$

From Lemmas 4, 5 and 7, the right side of the above equality converges for any $t\in[0, T]$ as $\lambda\to 0$. Since $\Big\{\frac{d}{dt}u_\lambda(t)\Big\}$ is uniformly bounded in H in view of Lemma 2, it follows that $\Big\{\frac{d}{dt}u_\lambda(t)\Big\}$ converges weakly in H for any $t\in[0, T]$. Put

$$Y_0 = \{t\in[0, T];\ \text{weak}\lim_{\lambda\to 0}\frac{d}{dt}u_\lambda(t) = \frac{d}{dt}u(t)\}\,.$$

Put $v(t)=v_0\in V$ in (1.2). We know that the total variation on Y_0 of the right side of (1.2) is uniformly bounded for λ. Hence the total variation on Y_0 of $\Big(\frac{d}{dt}u(t), v_0\Big)$ is bounded. Thus using that V is dense in H we have the existence of

$$\text{weak left}\lim_{t\in Y_0, t\to t_0}\frac{d}{dt}u(t) \quad\text{and}\quad \text{weak right}\lim_{t\in Y_0, t\to t_0}\frac{d}{dt}u(t)\,.$$

Therefore this lemma is proved.

Lemma 10. *Let $v\in C([0, T]; X_1)$ and $v(t)\in D(\phi)$ for a.e $t\in[0, T]$. Then it follows*

$$\overline{\lim_{t\to T}}\,\overline{\lim_{\lambda\to 0}}\int_0^t (\phi_\lambda(v(s)) - \phi_\lambda(u_\lambda(s)))\,ds \leqq \int_0^T (\phi(v(s)) - \phi(u(s)))\,ds\,.$$

Proof. From Lemma 4 and 5 the sequence $\{J_\lambda u_\lambda(t)\}$ converges to $u(t)$. Since ϕ is lower semi continuous it follows

$$\underline{\lim_{\lambda\to 0}}\,\phi_\lambda(u_\lambda(t)) \geqq \underline{\lim_{\lambda\to 0}}\,\phi(J_\lambda u_\lambda(t)) \geqq \phi(u(t))\,.$$

From Theorem 2.2 in [1] (p. 57) we have

$$\phi_\lambda(v(t)) \leqq \phi(v(t)) \text{ and } \lim_{\lambda\to 0} \phi_\lambda(v(t)) = \phi(v(t)).$$

Combining the above two results and Fatou's lemma we can prove our assertion.

Lemma 11. *The function u satisfies the initial conditions in the sense stated in* 4) *of Definition.*

Proof. For any $v \in D(\phi) \cap V$ from (1.2) it follows

$$\begin{aligned}&\left(\frac{d}{dt}u(t), v-u(t)\right)-(b, v-a)\\ &= \int_0^t (f(s, u(s))-Au(s), v-u(s))\, ds - \int_0^t \left(\frac{d}{ds}u(s), \frac{d}{ds}u(s)\right) ds\\ &-F_t(v-u) \equiv I_1 - I_2 + I_3.\end{aligned}$$

The left side of the above tends to $\left(\frac{d^+}{dt}u(0)-b,\ v-a\right)$ as $t \to 0$. From $u \in L_\infty(0, T; V) \cap W^1_\infty(0, T; H)$ we have

$$\lim_{t\to 0} I_1 = 0 \text{ and } \lim_{t\to 0} I_2 = 0.$$

On the other hand from Lemmas 5 and 7 it follows

$$F_t(v-u) = \lim_{\lambda\to 0} F_{\lambda,t}(v-u_\lambda).$$

Hence arguing as in the proof of Lemma 10

$$\varliminf_{t\to 0} F_t(v-u) \leqq \varlimsup_{t\to 0} \lim_{\lambda\to 0} \int_0^t (\phi_\lambda(v)-\phi_\lambda(u_\lambda(s)))\, ds = 0.$$

Thus

$$\left(\frac{d^+}{dt}u(0)-b,\ v-a\right) \geqq 0 \quad \text{for any} \quad v \in D(\phi) \cap V.$$

Therefore using the assumption 4 we obtain

$$b - \frac{d^+}{dt}u(0) \in \partial I_{K_0} a.$$

2. The proof of Theorem

Combining the definition of the subdifferential and Lemma 5, 7 and 10 we have the first half of 3) in Definition of the solution. From Lemmas 4, 5, 7

(1.2), 1) and the second half of 3) follow. Combining 2) of Lemmas 2, 5 and 9 we have 2) in Definition. From Lemma 11 we know 4). Thus the proof of the theorem is complete.

3. Examples

Example 1. Let Ω be a bounded domain in R^n with a smooth boundary. Set

$$
\begin{aligned}
&H = L_2(\Omega),\ X_1 = L_{p+1}(\Omega),\ X_2 = L_{(p+1)/p}(\Omega)\,, \\
&A = -\Delta \text{ (Dirichlet problem) and} \\
&\phi(u) = \int_\Omega |u(x)|^{p+1}\, dx
\end{aligned}
$$

where $p>1$.
Then we know $\partial\phi u=(p+1)|u|^{p-1}u$.
Putting $w_\lambda=(1+\lambda\partial\phi)^{-1}f$ we have $\partial\phi_\lambda(f)=\partial\phi(w_\lambda)$, $|w_\lambda(x)|\leqq|f(x)|$ and $w_\lambda(x)\cdot f(x)\geqq 0$. Hence it follows that

$$(\partial\phi_\lambda f, f) = (p+1)\int_\Omega |w_\lambda(x)|^p|f(x)|\,dx \geqq (p+1)\int_\Omega |w_\lambda(x)|^{p+1}\,dx$$

and

$$|\partial\phi_\lambda f|_{X_2} = (p+1)\left(\int_\Omega |w_\lambda(x)|^{p+1}\,dx\right)^{p/(p+1)}.$$

Then we have

$$(\partial\phi_\lambda f, f) \geqq (p+1)^{-1/p}|\partial\phi_\lambda f|_{X_2}^{(p+1)/p} > (p+1)^{-1/p}(|\partial\phi_\lambda f|_{X_2}-1)\,.$$

Thus the assumption 2 is satisfied.
If $(n+2)>p(n-2)$ using Sobolev's lemma we know that the assumption 1 is satisfied. Since $A=-\Delta$ (Dirichlet problem) it is easy to show the assumption 4.

Example 2. Put $H=L_2(0, 1)$, $X_1=C([0, 1])$, $X_2=L_1(0, 1)$ and $A=-\dfrac{d^2}{dx^2}$ (Dirichlet problem).
Let r be a continuous function on $[0, 1]$ such that $r(0)<0$ and $r(1)<0$. Set

$$K = \{f\in L_2(0, 1);\, f(x)\geqq r(x) \text{ a.e } x\in[0, 1]\}\,.$$

Let $\phi=I_K$ which is the indicator function of K. From Sobolev's lemma the assumptions 1 and 4 follow. We choose a function $\theta\in C^1([0, 1])$ such that $\theta(0)=\theta(1)=0$ and $\theta(x)-r(x)\geqq\delta_0>0$ for any $x\in[0, 1]$.
Since

$$\partial\phi_\lambda f(x) = \begin{cases} 0 & \text{if } f(x) \geqq r(x) \\ \lambda^{-1}(f(x)-r(x)) & \text{if } f(x) < r(x)\,, \end{cases}$$

and $f(x)<r(x)$ implies

$$\theta(x)-f(x)>\theta(x)-r(x)\geqq\delta_0$$

we have

$$(\partial\phi_\lambda f, f-\theta)\geqq\delta_0|\partial\phi_\lambda f|_{X_2}\,.$$

Hence the assumption 2 holds with $z=\theta$, $c_1=\delta_0$ and $c_2=0$.

EXAMPLE 3. Let K be a closed convex set in H with inner points and $X_1=H=X_2$. Let A be a positive self adjoint operator in H and V be Domain $(A^{1/2})$ endowed with the graph norm of $A^{1/2}$. If an inclution mapping from V to H is compact it follows that the assumption 1 holds. From Lemma 2.3 in [2] we have the assumption 2.

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