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Fractal Multi-Grid Method for Ultra Large Scale Mechanical and Thermal Simulations [†]

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Abstract

The finite element method is a powerful tool for predicting welding residual stresses and distortions. However, the phenomena in welding are three-dimensional in their nature. Generally, the three-dimensional thermal-elastic-plastic FE analysis requires very long computational time and large memory size. To overcome this problem, the idea of a fractal multi-grid method is proposed. Its potential capability is demonstrated through application to very simple mechanical and thermal problems.

KEY WORDS: (Fractal) (Multi-Grid) (Computational Time) (Degree of Freedom) (Ultra Large Scale) (Finite Element Method)

1. Introduction

Welding is one of the essential processes for assembling steel structures, such as ships and automobiles. However, it is impossible to avoid residual stress and the distortion due to the shrinkage produced in the vicinity of the weld line through the welding thermal cycle. The former may result in a reduction of the fatigue strength. The latter creates various problems during the assembly process, such as excessive gaps and misalignment between parts to be welded. To prevent or minimize these problems, the quantitative prediction and the effective control of the welding residual stress and deformation are necessary.

However, it is very costly and time consuming to carryout experiments or mockup tests in cases of large structures such as ships or pressure vessels. An alternative approach is computational analysis using finite element methods which is effective in solving nonlinear problems¹⁻³⁾. In general, the phenomena in welding are three-dimensional nonlinear transient problems which require very long computational times to analyze using FEM.

There are basically two approaches to tackle this problem. One is the elastic FE analysis using the concept of inherent strain or eigen strain⁴⁾. The other is to

develop a thermal-elastic-plastic FEM which can greatly reduce the computational time. To improve the speed of computation, we must take advantage of the characteristics of the welding problem. Noting the fact that the region which exhibits strong nonlinearity is limited to a very small area compared to the size of the model to be analyzed and the remaining part is mostly linear, the problem is transformed into the combination of a large linear problem and a small, but moving, strong nonlinear problem. When the structure to be analyzed is large, most of the computational time is used for solving large linear or quasi-linear problems. Thus, the authors developed a fractal multi-grid method which can solve large scale problems with more than one million degrees of freedom and its potential capability is examined.

2. Characteristics of Welding Phenomena

Most welded structures are plate structures. Dimension-wise, the size of the structure is about the order of 1 to 10 m and the thickness of the plate is in the order between 1 mm and 20 mm. The size of the weld pool which characterizes the local phenomena in the weld zone is roughly 10 mm in width and length and 2 to 5 mm in depth. This means that the size of the element in the weld joint should be in the order of 2 to 5 mm and this requires more than 100,000 elements to model the

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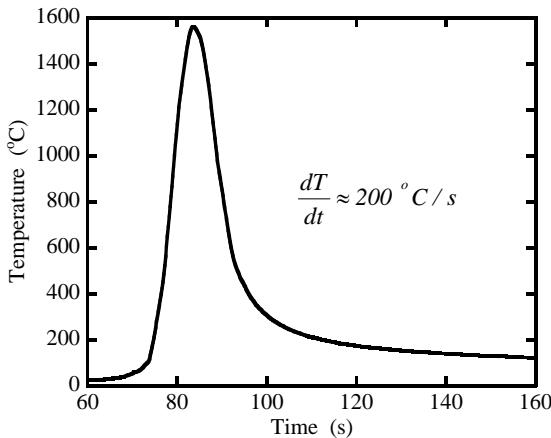


Fig.1 Transient temperature at a point on the welding line.

structure to be analyzed.

The temperature at the point located on the weld line changes with the movement of the welding torch as shown in **Fig.1**. The temperature changes drastically with the movement of the torch and its time rate is about the order of 200 °C/s. If it takes 120 seconds for the torch to travel from the starting end to the finishing end and the maximum temperature increment to achieve the convergence is 10 °C, $(200/10) \times 120 = 2400$ steps are required to complete the welding simulation. This means that a simultaneous equation with many degrees of freedom must be solved thousands of times to simulate welding.

Another characteristic of the welding problem is that the material properties such as the Young's modulus and the yield stress are temperature dependent and this is a major cause of strong nonlinearity of the problem. Thus, the welding can be considered as a problem with a nonlinear region moving with the torch. However the nonlinear region under high temperature is limited to a very small area compared with the whole structure.

According to the standard FE solution procedure, the whole system is solved as a nonlinear problem even when the nonlinear region is very small. This is the reason why unrealistically large computational time is required. This limits the application of the nonlinear welding simulation to practical problems in industry.

3. Separating Problem into Linear and Nonlinear Problems

As discussed, the welding is characterized as a large problem with a small moving nonlinear region. If the problem is separated into a large but linear problem and a small moving nonlinear problem, the computational time can be reduced. Based on this idea, the authors developed an iterative substructure method⁵⁾ which may be categorized as a domain decomposition method⁶⁾. One

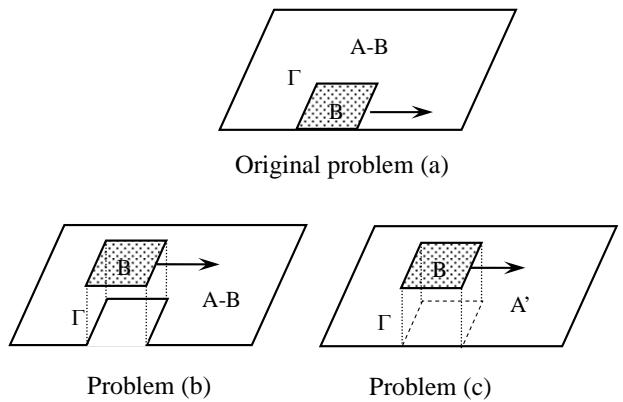


Fig.2 Basic idea of iterative substructure method.

straight forward method is to cutout the nonlinear region B from the structure A as the problem (b) in **Fig.2**. Γ is the boundary between regions A-B and B. In this way, the remaining linear region A-B changes its shape with the movement of the nonlinear region B. This requires the solving of a large linear problem with changing stiffness matrix. To avoid this, whole model A' is used instead of A-B in the iterative substructure method as the problem (c) in **Fig.2**. The model A' is assumed to be subjected to the same loading condition as A but it has the stiffness of the whole model in the past and the stiffness is kept unchanged until updated. Thus the solution time can be saved if the matrix after the forward elimination is stored and repeatedly used. The stiffness of the model A' is updated when it is necessary to maintain good convergence of the solution.

4. Fractal Multi-Grid Method

Though the computational time is greatly reduced by the iterative substructure method, the size of the problem which can be handled is limited by that of available memory. When Pentium 4 (3.8 GHz, 2 GB) is used, 20,000 elements is the upper limit of the problem which can be handled. To analyze welding problems with practical interest, such as weld joints in nuclear power plants and those of automobiles, more than 100,000 elements are required to model the details of the structure.

Noting that the size of the nonlinear region is small, the key for improving computational speed is to solve the large quasi-linear problem efficiently both in computational speed and memory saving. There are several possibilities, such as ICCG (Incomplete Cholesky Conjugate Gradient) method, multifrontal method⁷⁾ and multi-grid method⁸⁾. All of the three methods are superior to the skyline method in memory saving. Also, parallel and grid computings are promising possibilities.

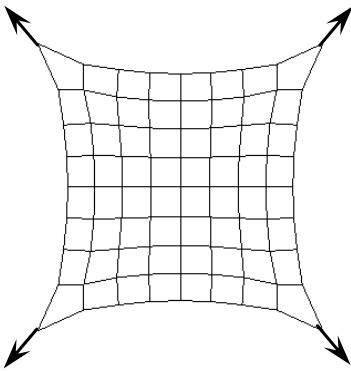


Fig.3 Elastic sheet stretched at four corners.

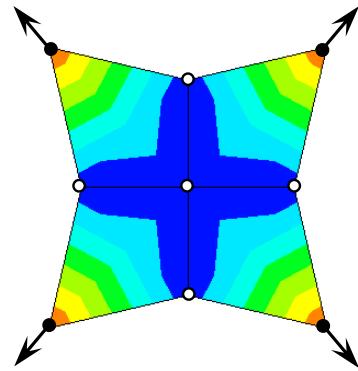


Fig.4 Basic cell consists of 4 elements.

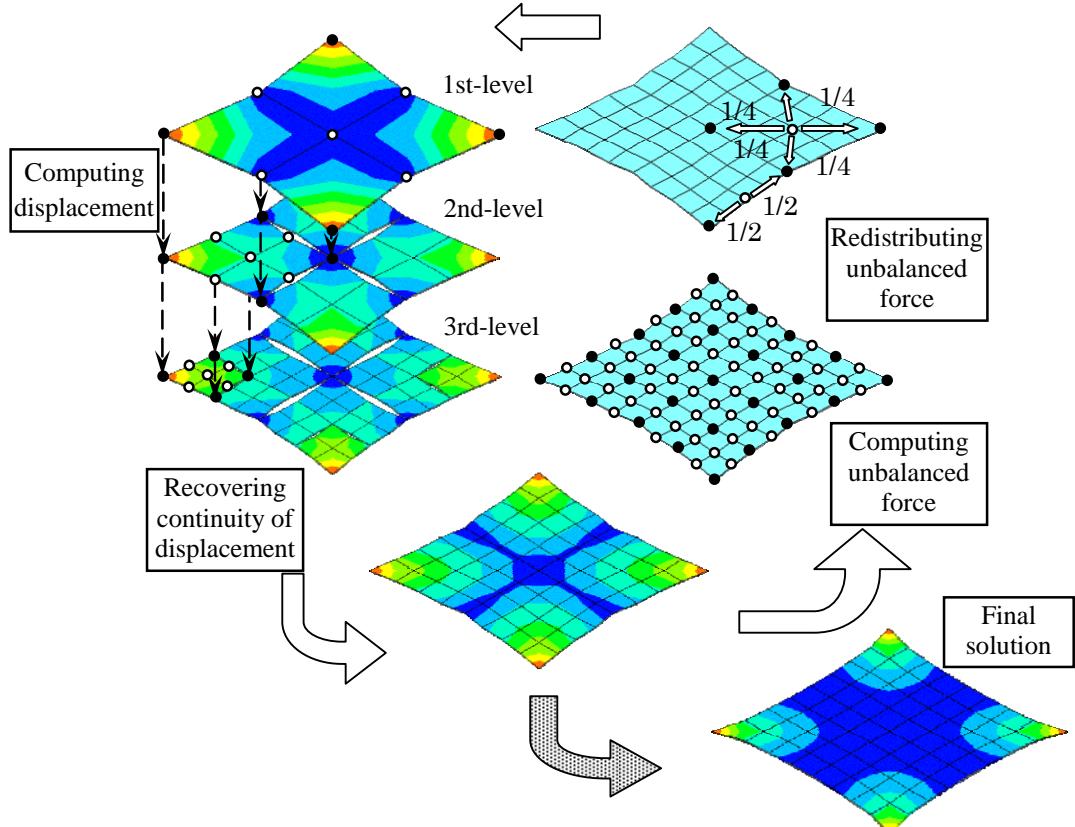


Fig.5 Procedure of computation.

Since the proposed method takes advantage of solving the same matrix repeatedly, ICCG may not be suitable. As one of the methods for solving large problems, the authors developed a fractal multi-grid method which may be categorized as a geometrical multi-grid method and tested its potential capabilities for two and three dimensional elastic problems, linear elastic plate bending and three dimensional transient thermal conduction. The idea of the fractal multi-grid method is illustrated using a two dimensional simple elastic problem shown in **Fig.3**. A square sheet is stretched at its four corners. When the model is subdivided into 8×8 elements, the deformation and the stress are computed by solving basic cells consist of 2×2 elements under the

prescribed displacements at four corners as shown in **Fig.4**. Such a basic operation is repeated hierarchically from the top level to the lowest level. In this process, the continuity between the neighboring cells is ignored as illustrated in **Fig.5**. The continuity of the displacement can be recovered at the lowest level by interpolating the displacements at two nodes sharing the same cell boundary. In this way, the continuity of the traction is not guaranteed. It is retained through the iteration. Since all the governing equations and boundary conditions must be satisfied at the lowest or the finest level, the error in stress field is evaluated at the lowest level and it is transferred to the upper level.

The detail of the scheme is the following.

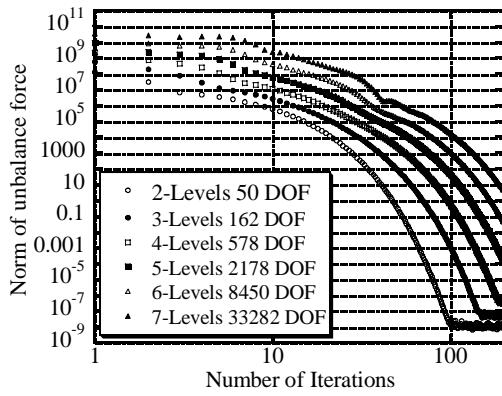


Fig.6 Convergence of norm of unbalanced nodal force.

- Step-(1) Solve the cell at the 1st-level under the given boundary condition, not necessarily prescribed displacements at four corners.
- Step-(2) Use the displacements at 9 nodes obtained for the 1st-level as the prescribed displacements at four corners, compute displacements of four cells belonging to the 2nd-level.
- Step-(3) Repeat the same computation as in Step-(2) until the lowest level.
- Step-(4) Force the continuity of the displacements at four nodes on the edge of the cell in the lowest level by interpolating the displacements at the two nodes belonging to the two cells sharing the same edge.
- Step-(5) Compute the unbalanced force at the lowest level.
- Step-(6) Redistribute the unbalanced force at the four middle nodes and one center node of the cell to the four corner nodes according to the following rule.
 - Middle node: Redistribute one half of the unbalanced force to the two corner nodes on the same edge.
 - Center node: Redistribute one fourth of the unbalanced force to the four corner nodes of the cell.
- Step-(7) Spread the above unbalanced nodal forces to the two nodes belonging to the cells sharing the same edge according to the ratio of the stiffness.
- Step-(8) Compute the correction to the nodal displacement of the cell using the redistributed unbalanced force with the prescribed correction displacements at the four corner nodes.
- Step-(9) Repeat the Steps-(2) through (8) until the norm of the unbalance force at the lowest level becomes small enough.

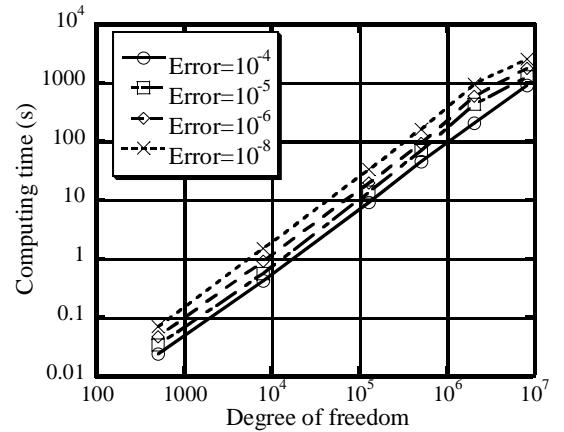


Fig.7 Relation between degree of freedom and computation time.

The details of the computational scheme may be different when the problem to be solved is different, such as heterogeneous or nonlinear problems. If the problem is a linear isotropic two or three dimensional problem and the regular uniform mesh is used, the stiffness matrix of all cells becomes the same. Therefore, it is not necessary to save the stiffness matrixes of all cells. Once the inverse of the stiffness matrix of a cell is computed, it can be used repeatedly. From the aspect of memory space, the required memory size is only three or four times the degree of freedom.

5. Example Problems

Though further improvements are necessary for practical application in welding problems, potential capability of the proposed method is tested using very simple mechanical and thermal problems.

5.1 Stretching of square elastic sheet

One of the example problems is the square elastic sheet stretched at four corners as shown in Fig.3. **Figure 6** shows the convergence of the norm of unbalance force with the iteration for 6 cases in which the number of hierarchy is 2, 3, 4, 5, 6 and 7. The number of degree of freedom for 7-levels is 33,282. The rate of convergence becomes slightly smaller as the number of hierarchy increases but good convergence is observed generally. The relation between the computing time and the degree of freedom is summarized in **Fig.7**. The computing time to achieve the relative error of 10^{-4} to 10^{-8} is plotted. The error in this figure is the relative value which is normalized by that of the first iteration. As seen from the slope of the curve, the computational time increases almost linearly with the degree of freedom n . In the case of conventional direct solution method, the computational time is proportional to $n^{2.0}$ (for two

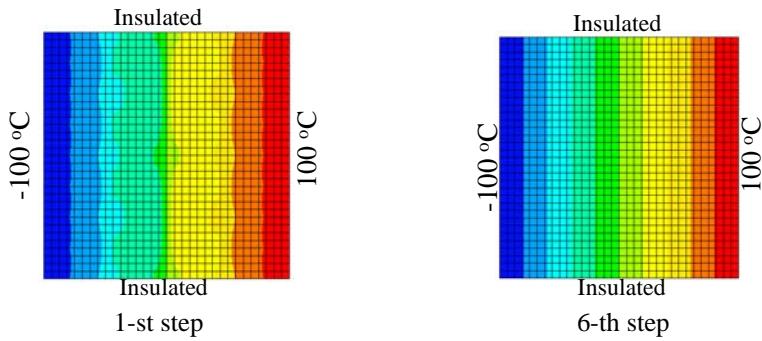


Fig.8 Convergence of temperature distribution with iteration.

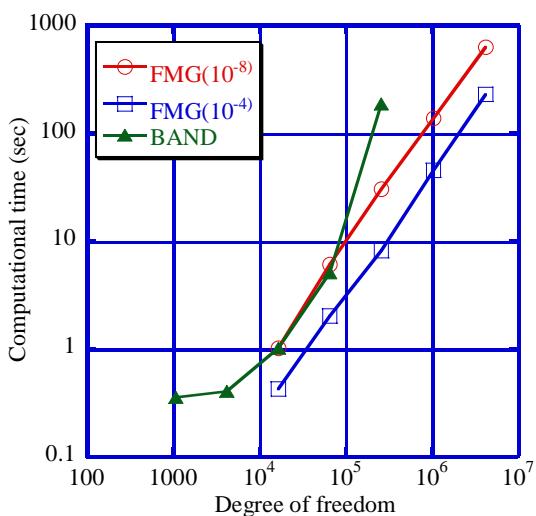


Fig.9 Comparison of computational time between FMG and Band method (2-D steady thermal conduction).

dimensional problem) or $n^{2.3}$ (for three dimensional problem).

5.2 Simple steady and transient thermal conduction problem

The next example is a simple two dimensional steady heat conduction problem as shown in **Fig.8**. The model is a square plate where all surfaces are insulated except for the right and left edges facing each other where the temperatures are given. The temperature field at the 1st step and the 3rd step in iteration are shown in **Fig.8**. The mesh division in this case is 32×32 . As seen from the figure, the convergence is very fast. The relation between the degree of freedom n and the computational time is plotted in **Fig.9**. The same problem is solved using the conventional direct method (Band method) and plotted for comparison. In case of two dimensional steady problem, the proposed FMG method is superior to the Band method. The maximum size of the problem which the Band method can handle is 200,000. It becomes 4,000,000 when FMG method is used. The same comparison is made for three

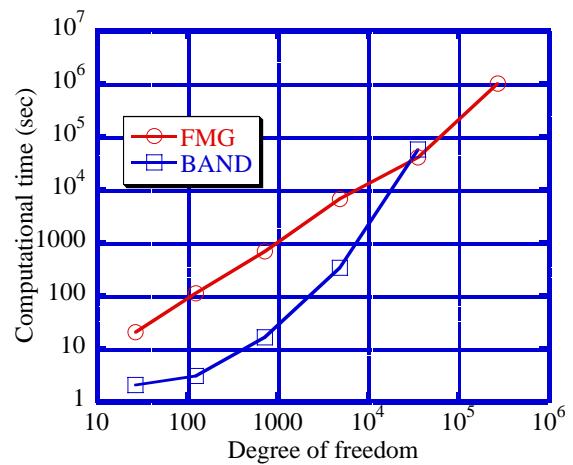


Fig.10 Comparison of computational time between FMG and Band method (3-D transient thermal conduction).

dimensional transient thermal conduction problem. As shown in **Fig.10**, the FMG method becomes faster than the Band method when the degrees of freedom exceed 36,000.

6. Conclusions

As discussed in this report, the practical welding problems are very large and highly nonlinear transient problems. It is necessary to develop fast and memory saving schemes to encourage the FEM simulation of practical welding problems in industry. Noting that the welding problem is a mostly linear problem with a small nonlinear region moving with the torch, the problem can be separated into large quasi-linear problem and small but moving nonlinear problem. As a method to solve large linear problem, the authors proposed a fractal multi-grid method and demonstrated its potential capability in both mechanical and thermal problems. As discussed in this report, by relaxing the continuity, the possible choice of the solution scheme can be greatly expanded. It may be worthwhile to look into the potential of such a method for solving large scale welding problems.

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