New Interaction Equation for Plate Buckling†

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Abstract

This paper is concerned with proposal of an accurate elastic buckling interaction equation for simply supported rectangular plates subjected to five load components such as compression and bending in two directions and shear. In order to construct this interaction, data on buckling strength of plate under two load components partly calculated in the present study and partly obtained from existing ones are first taken, and then buckling interaction relationships for any two load components are developed. Based on these relationships, the new buckling interaction equation for five load components is theoretically derived. Assessment of accuracy is performed and it is found that the proposed equation has sufficient accuracy for practical designs. Some comparisons are also made with the interactions proposed by Lloyd's Register and Det Norske Veritas. The result indicates that the new interaction equation proposed in this paper yields better accuracy in safety side.

KEY WORDS: (Elastic Buckling) (Rectangular Plate) (Buckling Interaction Equation)

1. Introduction

Such many plate structures as ships are composed of plate elements, which may sustain mainly in-plane loads and therefore are designed to have sufficient in-plane stiffness and strength.

In these structures, plate buckling is one of the most important design criteria and buckling load may usually be obtained as an eigen-value solution of the governing equations for the plate. However, in many cases, it is very difficult to gain an exact solution for such complicated buckled shape as under any combinations of loading and different boundary condition. Even approximate solutions tend to require a considerable analytical and/or numerical efforts.

To estimate buckling strength for practical design, simple and accurate buckling interaction equations are of great interest to designers, and therefore, charts or simplified equations based on these solutions, included in many handbooks[1)], have been proposed. However, from the viewpoint of safety, high level accuracy has also been required in evaluation of buckling strength in the design stage.

In this paper, a new and accurate elastic buckling interaction equation is proposed for simply supported rectangular plates subjected to combined five load components such as compression and bending in two directions and shear.

In order to construct this interaction, buckling strengths are evaluated by using the incremental Galerkin method[2)]. and the finite element method in addition to the existing data, and then buckling interaction relationships under each combination of any two load components are developed.

Assessment of the accuracy is carried out making some comparisions with an interaction equation proposed recently by Lloyd's Register[3)] and a design procedure proposed by Det Norske Veritas[4)].

2. Methods Used in Evaluation of Plate Bucking Strength

Many different methods are available to evaluate the buckling strength of rectangular plate. Theories behind the methods can be found in many textbooks. In this section, ordinary formulations which were used partly for the existing data and partly for non-existing data, and newly developed methods used for cases of non-existing data are outlined.

2.1 Closed form solution by the energy method[5)]

In this method, the principle of minimum potential

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energy is used. When a rectangular plate under in-plane loads as shown in **Fig. 1**, buckles, total potential energy Π due to lateral deflection w may be obtained as follows;

$$\Pi = U - V \qquad (1)$$

where

U = strain energy stored in the plate due to buckling deformation

$$= \iint \frac{D}{2} [(\frac{\partial^2 w}{\partial x^2})^2 + 2(1-\nu)(\frac{\partial^2 w}{\partial x \partial y})^2 + 2\nu(\frac{\partial^2 w}{\partial x^2})(\frac{\partial^2 w}{\partial y^2}) + (\frac{\partial^2 w}{\partial y^2})^2]\, dxdy$$

V = loss of potential energy of external loads due to buckling deformation

$$= \iint \frac{t}{2} [\sigma_x (\frac{\partial w}{\partial x})^2 + 2\tau(\frac{\partial w}{\partial x})(\frac{\partial w}{\partial y}) + \sigma_y (\frac{\partial w}{\partial y})^2]\, dxdy$$

and $D = \frac{Et^3}{12(1-\nu^2)}$, ν = Poisson's ratio, t = plate thickness, E = modulus of elasticity and also σ_x, σ_y and τ $(=\tau_{xy})$ are stresses loaded at plate edges as shown in Fig. 1. It is the entire volume of the plate that should be integrated by the above equation. The principle of minimum potential energy gives the following buckling criterion;

$$\delta\Pi = 0 \qquad (2)$$

For simply supported rectangular plates, the following deflection function which is the form of a Fourier series may be assumed;

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi}{a} x \sin\frac{n\pi}{b} y \qquad (3)$$

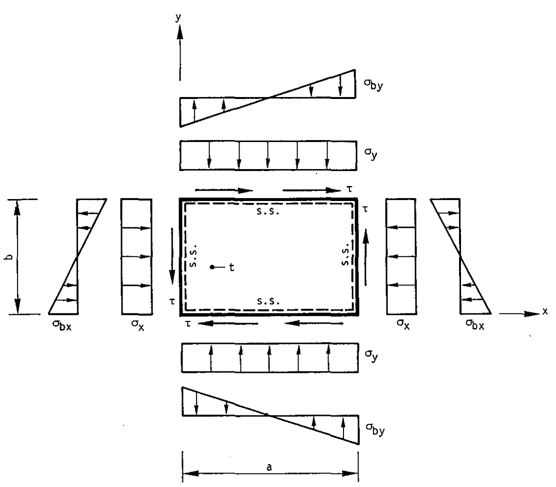

Fig. 1 Simply supported rectangular plate and applied loads

where

a, b = length and breadth of the plate, respectively

A_{mn} = coefficients

After substituting Eq.(3) into Eq.(2) and integrating it, the buckling load is determined from the following stationary condition, which represents simultaneous equations.

$$\frac{\partial \Pi}{\partial A_{mn}} = 0 \ (m, n = 1 \sim \infty) \qquad (2^*)$$

If the assumed deflection function is the exact one, the final solution is the same as one obtained from the solution of the differential equation. However, in general, it is very difficult to solve the homogeneous equation Eq.(2*) analytically. Therefore, numerical procedures are usually applied.

2.2 Incremental Galerkin method[2)]

When buckling load as an eigen-value is calculated by the previous method, it is generally necessary to take a large number of terms of the assumed deflection function in order to gain sufficient accuracy for such complicated shape of buckling as with shear buckling.

As for numerical techniques, they also require considerable computing time. These problems may also occur when post-buckling behaviour of plates with initial deflections is analyzed.

From the above viewpoints, the authors have developed the incremental Galerkin's method for analysis of large deflection behaviour of simply supported rectangular plates under in-plane loads. In this paper, this method is applied to establish non-exsisting data of buckling strength.

Large deflection theory of plates with initial deflection in the elastic range is governed by the following fundamental equilibrium and compatibility equations;

$$\nabla^4 w = \frac{t}{D} [\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 (w+w_0)}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 (w+w_0)}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 (w+w_0)}{\partial x \partial y}] \qquad (4)$$

$$\nabla^4 F = E [(\frac{\partial^2 w}{\partial x \partial y})^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w_0}{\partial x \partial y} - \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2}] \qquad (5)$$

where, F is a stress function satisfying the following relationships;

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \sigma_y = \frac{\partial^2 F}{\partial x^2}, \tau = -\frac{\partial^2 F}{\partial x \partial y}$$

and w = deflection due to external load, w_0 = initial deflection.

Equations (4) and (5) are nonlinear partial differential equations with regard to F and w, so that it is very difficult to solve the above equations directly when the plate buckles with complicated buckling shape.

Therefore, the fundamental equations are linearized by the incremental technique, as follows:

At the (n)th step of load increments, deflection and stress function may be expressed as follows;

$$w = w^{n-1} + \Delta w$$
$$F = F^{n-1} + \Delta F \qquad (6)$$

where,

w, F = total deflection and stress function for loading from zero to the (n)th step of load increments, respectively

w^{n-1}, F^{n-1} = total deflection and stress function for loading from zero to the $(n-1)$th step of load increments, respectively

Δw, ΔF = increments of deflection and stress function due to load increment in the (n)th loading step, respectively.

Substituting Eq.(6) into Eqs.(4) and (5), and neglecting the terms of Δw and ΔF of higher order than the 2nd order which are negligible when increment of load and deflection is very small, the following fundamental equations which are linear in an interval of a load increment are obtained:

$$\nabla^4(\Delta w) = \frac{t}{D}\Big[\frac{\partial^2 \Delta F}{\partial y^2}\frac{\partial^2 w_T}{\partial x^2} + \frac{\partial^2 \Delta F}{\partial x^2}\frac{\partial^2 w_T}{\partial y^2} + \frac{\partial^2 F^{n-1}}{\partial y^2}\frac{\partial^2 \Delta w}{\partial x^2} + \frac{\partial^2 F^{n-1}}{\partial x^2}\frac{\partial^2 \Delta w}{\partial y^2} - 2\frac{\partial^2 \Delta F}{\partial x \partial y}\frac{\partial^2 w_T}{\partial x \partial y} - 2\frac{\partial^2 F^{n-1}}{\partial x \partial y}\frac{\partial^2 \Delta w}{\partial x \partial y}\Big] \qquad (7)$$

$$\nabla^4(\Delta F) = E\Big[2\frac{\partial^2 w_T}{\partial x \partial y}\frac{\partial^2 \Delta w}{\partial x \partial y} - \frac{\partial^2 w_T}{\partial y^2}\frac{\partial^2 \Delta w}{\partial x^2} - \frac{\partial^2 w_T}{\partial x^2}\frac{\partial^2 \Delta w}{\partial y^2}\Big] \qquad (8)$$

where,

$$w_T = w^{n-1} + w_0$$

Assuming $\Delta w = \Sigma\,\Sigma \Delta A_{mn} \Phi_m(x)\cdot\Psi_n(y)$ as a deflection function which satisfies the boundary conditions and substituting this assumed deflection function into Eq.(8), ΔF can be expressed as a function of Δw. After substituting this obtained ΔF and ΔW into Eq.(7), Δw and ΔF can be obtained without difficulty by the Galerkin method. Ultimately, w and F are calculated as follows;

$$w = \sum_{i=1}^{n} \Delta w_i$$
$$F = \sum_{i=1}^{n} \Delta F_i$$

From the above, for simply supported rectangular plates with initial deflections, it may be assumed that shapes of deflection due to loading and initial deflection are;

$$w = \sum_m \sum_n A_{mn} \sin\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \qquad (9.a)$$

$$w_0 = \sum_m \sum_n A_{0mn} \sin\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \qquad (9.b)$$

and the boundary conditions are;

$$\left.\begin{aligned} &w = 0,\ \frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2} = 0 \ \text{ at } x = 0, a \\ &w = 0,\ \frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2} = 0 \ \text{ at } y = 0, b \end{aligned}\right\} \qquad (10.a)$$

$$\left.\begin{aligned} &\int_0^b \sigma_x t dy = P_x \\ &\int_0^a \sigma_y t dx = P_y \\ &\int_0^b \sigma_x t y dy = M_x \\ &\int_0^a \sigma_y t x dx = M_y \\ &\tau_{xy} = \tau \end{aligned}\right\} \qquad (10.b)$$

In this analysis, buckling load is obtained not as an eigen-value but as the load applied at the point when deflection in a plate with very small initial deflection increases rapidly as external load is applied incrementally.

The accuracy of the solution also depends on the number of terms taken in deflection series in Eq.(9) and how accurately they can express the buckling shape. However, much more terms may be taken in this method since the linearized fundamental equation gives linear homogeneous equations that are solved directly without difficulty.

Furthermore, buckling load can easily be evaluated with good accuracy if small initial deflection is taken.

If a larger number of terms of deflection series are taken, it converges from upper bound value to exact one.

2.3 Finite element method[6)]

According to the theorem of minimum potential energy mentioned in sec. 2.1, the fundamental equation for determination of buckling load by FEM can be derived: A rectangular plate is subdivided into finite elements and deflection of the total plate due to buckling is expressed as an assembly of the deflection of each element;

that is,

$$w = [A_b] \{w_n\} \tag{11}$$

where,

$[A_b]$ = shape function

$\{w_n\}$ = nodal displacement

After substituting Eq.(11) into Eq.(1), partial differentiation with respect to the nodal displacements is performed, leading to stationary conditions;

$$0 = (\Sigma\ [K_{bb}] + \Sigma\ [K_I])\{w_n\} \tag{12}$$

where,

$[K_{bb}]$ ≑ bending stiffness matrix of a finite element

$[K_I]$ = stability coefficient matrix of a finite element, which is a linear function of stresses

Σ = assembly of all elements

Equation (12) has a trivial solution $\{w_n\} = 0$. Non-trivial solution exists only if the following equation is satisfied;

$$|\Sigma [K_{bb}] + \Sigma [K_I]| = 0 \tag{13}$$

In the above procedure, when a combination of stresses associated with $[K_I]$ is reached such that, Eq.(13) is satisfied the buckling load is determined as an eigen-value.

The accuracy of buckling load determined by this method depends on the assumed shape function, the number of element and the complexity of the buckling shape.

3. New Buckling Interaction Equation

In this paper, a simply supported rectangular plate which has a, b and t that are length, breadth and thickness of the plate respectively, is considered as shown in Fig. 1. When the plate is subjected to combined in-plane loads, such as uniform compression and bending in two directions and uniform shear, a new buckling interaction equation is derived.

3.1 Buckling interaction equation for two load components

As the first step for constructing a new interaction equation, the interaction between each pair of load components such as σ_x and σ_y, σ_{bx} and σ_{by}, etc. is considered.

Figure 2 to 11 represent these interaction relationships for different values of aspect ratio of the plate. The x axis can always be chosen parallel to the longer side of the plate, so that a/b becomes greater than 1.0 and it is unnecessary to consider plates with aspect ratio a/b smaller than 1.0 in constructing interaction relationships.

Interaction relationships presented in Refs. 1, 5, 7 and 8 are reproduced in these figures except Figs. 5 and 7, which are constructed using the incremental Galerkin method, and Fig. 11, which is partly calculated by the finite element method. The sources of these figures are shown in **Table 1**.

Since the accuracy of these buckling loads expressed by solid lines in Figs. 2 to 11 is sufficient, they are considered as reference solutions for comparison of accuracy with buckling load obtained by different formulae.

The interaction equations formulated in this paper are also plotted in Figs. 2 to 11. Subscript cr in the following interaction equations means buckling strength of plates under a single load (see appendix).

Table 1 Sources of elastic buckling interaction relationships between different load components

	σ_x	σ_y	σ_{bx}	σ_{by}	τ
σ_x					
σ_y	Calculated by Energy Method				
σ_{bx}	Ref. 7	Ref. 7			
σ_{by}	Calculated by Inc. Energy Method & Ref.7	Refs. 5 and 7	Calculated by Inc. Energy Method		
τ	Ref. 7	Ref. 8	Ref. 7	Calculated by F.E.M. and Ref. 7	

(1) Biaxial compressions, σ_x and σ_y

Figure 2 shows the buckling interaction relationships of plates with different aspect ratios subjected to biaxial compressions. These relations are obtained by the energy method. Only one term of the deflection series, Eq.(3), in each direction is necessary and the solution in this case is exact.

The interaction relationship is represented by a straight line for $1 \leqslant a/b \leqslant \sqrt{2}$, where buckling occurs in one-half buckling wave. When a/b is greater than $\sqrt{2}$ the buckling mode changes into multi-half buckling waves depending or a/b and σ_x/σ_y.

For any aspect ratio, a/b, the number of half buckling waves decreases as σ_x/σ_y decreases until it reaches one-half buckling wave for small values of σ_x/σ_y.

The interaction relationship in this case is represented by a group of intersecting straight lines, as shown in Fig. 2. Each straight line corresponds to a buckling mode and the intersection points (knuckles) indicate transition of the buckling mode.

As a/b increases, the buckling interaction relationship between σ_x and σ_y becomes more convex with more knuckles on it, as shown in Fig. 2. This also implies that the interaction between σ_x and σ_y becomes less pronounced. If σ_x and σ_y are applied to a rectangular plate, the exact solution may be used to represent the buckling interaction relationship. However, a continuous equation is preferable in order to derive an interaction equation for general load components.

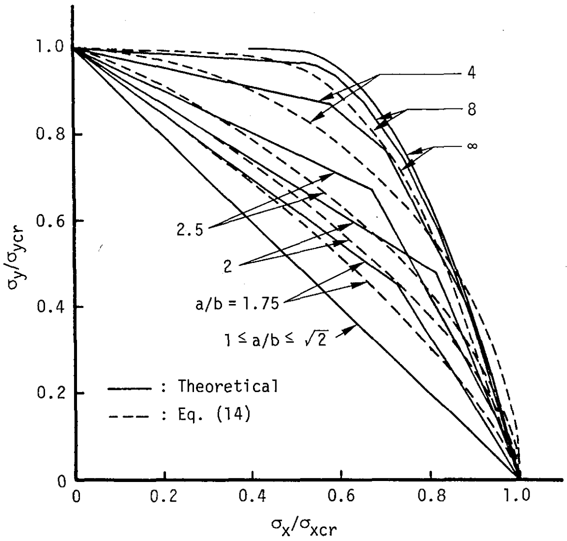

Fig. 2 Buckling interaction between σ_x and σ_y

Although it is difficult to express accurately the exact solution for all a/b ratios by a single continuous equation, the following equation gives a good, rather slightly conservative, approximation of the exact solution, as shown by the dotted lines in Fig. 2.

$$\left(\frac{\sigma_x}{\sigma_{xcr}}\right)^{\alpha_1} + \left(\frac{\sigma_y}{\sigma_{ycr}}\right)^{\alpha_2} - 1 = 0 \qquad (14)$$

where,

$\alpha_1 = \alpha_2 = 1.0 \qquad 1 \leqslant \beta \leqslant \sqrt{2}$

$\alpha_1 = 1.0$, $\alpha_2 = 1.110\beta - 0.569 \qquad \sqrt{2} < \beta \leqslant 2$

$\alpha_1 = 0.450\beta + 0.10$, $\alpha_2 = 0.125\beta + 1.40 \qquad 2 < \beta \leqslant 4$

$\alpha_1 = 0.20\beta^2 - 1.60\beta + 5.10$, $\alpha_2 = -0.30\beta + 3.10 \qquad 4 < \beta \leqslant 8$

$\alpha_1 = 5.10$, $\alpha_2 = 0.70 \qquad 8 < \beta$

$\beta = a/b$

(2) Longitudinal compression σ_x and longitudinal bending σ_{bx}

Figure 3 shows the buckling interaction relationships of plates with different aspect ratios subjected to axial compression σ_x in the x direction and in-plane bending σ_{bx} in the same direction. These relations are obtained using the energy method. Although the solution in this case is approximate (upper bound), it should be quite accurate because only a few terms (2 or 3 terms) of the deflection series are required for convergence.

As may be seen from Fig. 3, this relationship is insensitive to the a/b ratios and may be represented accurately by the following equation, which is also plotted by a dotted line in the figure.

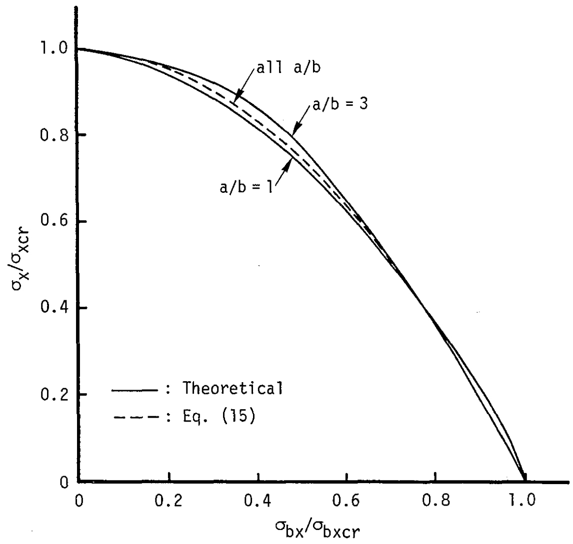

Fig. 3 Buckling interaction between σ_x and σ_{bx}

$$\frac{\sigma_x}{\sigma_{xcr}} + \left(\frac{\sigma_{bx}}{\sigma_{bxcr}}\right)^2 - 1 = 0 \qquad (15)$$

(3) Transverse compression σ_y and longitudinal bending σ_{bx}

The buckling interaction relationships in this case are also obtained by the energy method and presented in **Fig. 4**. As in case (2), only a few terms of the deflection series are necessary to obtain convergence and the solutions may be quite accurate. These relations are well approximated by the following equation (see Fig. 4):

$$\left(\frac{\sigma_y}{\sigma_{ycr}}\right)^{\alpha_3} + \left(\frac{\sigma_{bx}}{\sigma_{bxcr}}\right)^{\alpha_4} - 1 = 0 \qquad (16)$$

where,

$\alpha_3 = \alpha_4 = 1.50\beta - 0.30 \qquad 1 \leqslant \beta \leqslant 1.6$

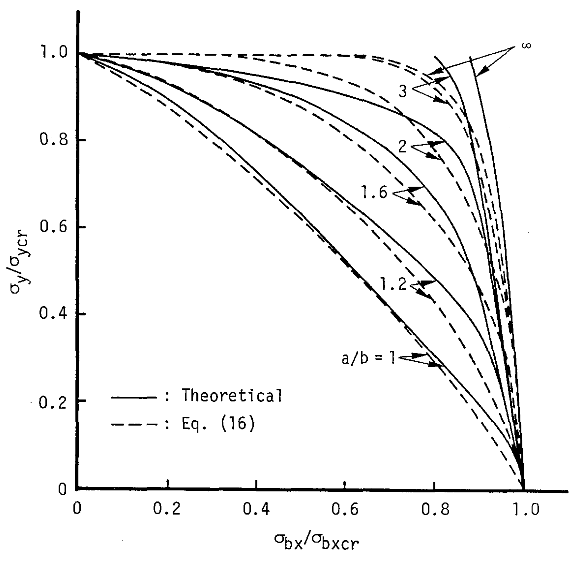

Fig. 4 Buckling interaction between σ_y and σ_{bx}

$$\left.\begin{array}{l}\alpha_3 = -0.625\beta + 3.10 \\ \alpha_4 = 6.25\beta - 7.90\end{array}\right\} 1.6 < \beta \leqslant 3.2$$

$$\left.\begin{array}{l}\alpha_3 = 1.1 \\ \alpha_4 = 12.1\end{array}\right\} 3.2 < \beta$$

(4) Longitudinal compression σ_x and transverse bending σ_{by}

The buckling load for small a/b ratios may be calculated accurately using 3 terms of the deflection series. As a/b increases, more terms would be necessary to obtain accurate solutions by the energy method.

The buckling interaction relationships presented in **Fig. 5** for $a/b = 1$ and 1.25 are calculated by the energy method and those for $a/b = 3$ and 5 by the incremental Galerkin method using a larger number of terms of the deflection series (7 to 11 terms).

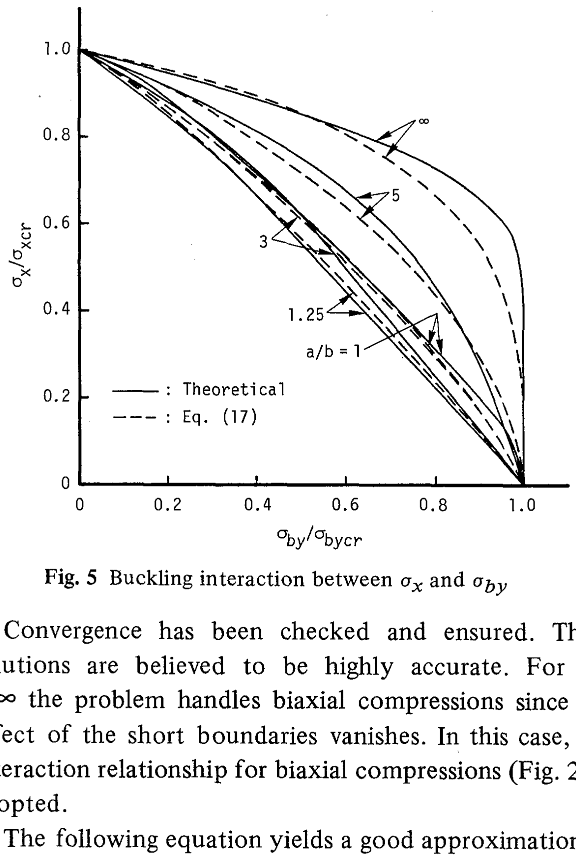

Fig. 5 Buckling interaction between σ_x and σ_{by}

Convergence has been checked and ensured. These solutions are believed to be highly accurate. For $a/b = \infty$ the problem handles biaxial compressions since the effect of the short boundaries vanishes. In this case, the interaction relationship for biaxial compressions (Fig. 2) is adopted.

The following equation yields a good approximation to these relationships as may be seen in Fig. 5.

$$\left(\frac{\sigma_x}{\sigma_{xcr}}\right)^{\alpha_5} + \left(\frac{\sigma_{by}}{\sigma_{bycr}}\right)^{\alpha_6} - 1 = 0 \qquad (17)$$

where,

$$\left.\begin{array}{l}\alpha_5 = 0.930\beta^2 - 2.890\beta + 3.160 \\ \alpha_6 = 1.20\end{array}\right\} 1 \leqslant \beta \leqslant 2$$

$$\left.\begin{array}{l}\alpha_5 = 0.066\beta^2 - 0.246\beta + 1.328 \\ \alpha_6 = 1.20\end{array}\right\} 2 < \beta \leqslant 5$$

$$\left.\begin{array}{l}\alpha_5 = 1.117\beta - 3.837 \\ \alpha_6 = -0.167\beta + 2.035\end{array}\right\} 5 < \beta \leqslant 8$$

$$\left.\begin{array}{l}\alpha_5 = 5.10 \\ \alpha_6 = 0.70\end{array}\right\} 8 < \beta$$

(5) Transverse compression σ_y and transverse bending σ_{by}

The buckling shape in this case may be represented accurately by a few terms of the deflection series. The energy method is used with enough accuracy in deriving the buckling interaction relationships for $a/b = 1$, 1.25 and 5. These relationships are shown in **Fig. 6**.

When a/b approaches infinitely, σ_{by} may be replaced by transversely uniform compression as stated in (4). The relationship between σ_y and σ_{by} becomes a straight line as shown in Fig. 6.

The following equation yields a good approximation to these relationships as may be seen in Fig. 6.

$$\left(\frac{\sigma_y}{\sigma_{ycr}}\right)^{\alpha_7} + \left(\frac{\sigma_{by}}{\sigma_{bycr}}\right)^{\alpha_8} - 1 = 0 \qquad (18)$$

where,

$$\left.\begin{array}{l}\alpha_7 = 1.0 \\ \alpha_8 = (14.0 - \beta)/6.5\end{array}\right\} 1 \leqslant \beta \leqslant 7.5$$

$$\left.\begin{array}{l}\alpha_7 = 1.0 \\ \alpha_8 = 1.0\end{array}\right\} 7.5 < \beta$$

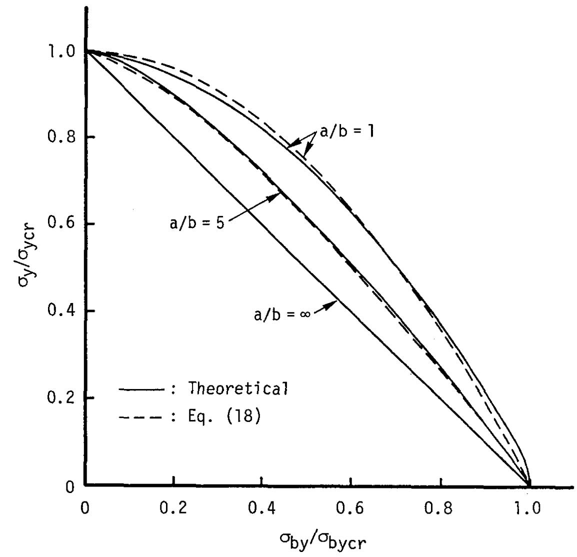

Fig. 6 Buckling interaction between σ_y and σ_{by}

(6) Biaxial bendings σ_{bx} and σ_{by}

The buckling shape associated with this load combination requires many terms of the deflection series in order to obtain accurate solutions. It is not appropriate to use the energy method in this case.

The interaction relationships presented in **Fig. 7** are obtained by the incremental Galerkin method using 9 to 22 terms of the deflection series with convergence checked and ensured.

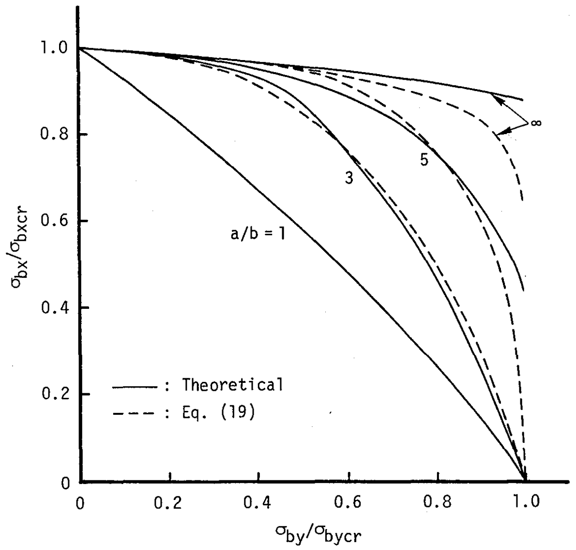

Fig. 7 Buckling interaction between σ_{bx} and σ_{by}

These relations may be well approximated by the following equation as may be seen in Fig. 7.

$$\left(\frac{\sigma_{bx}}{\sigma_{bxcr}}\right)^{\alpha_9} + \left(\frac{\sigma_{by}}{\sigma_{bycr}}\right)^{\alpha_{10}} - 1 = 0 \qquad (19)$$

where,

$\alpha_9 = 0.050\beta + 1.080$, $\alpha_{10} = 0.268\beta - 1.248/\beta + 2.112$ $\quad 1 \leqslant \beta \leqslant 3$

$\alpha_9 = 0.146\beta^2 - 0.533\beta + 1.515$, $\alpha_{10} = 0.268\beta - 1.248/\beta + 2.112$ $\quad 3 < \beta \leqslant 5$

$\alpha_9 = 3.20\beta - 13.50$, $\alpha_{10} = -0.70\beta + 6.70$ $\quad 5 < \beta \leqslant 8$

$\alpha_9 = 12.10$, $\alpha_{10} = 1.10$ $\quad 8 < \beta$

(7) Longitudinal compression σ_x and uniform shear τ

Figures 8 to **11** represent the buckling interaction relationships between shear τ and each of the other four load components considered in this paper. Solutions, under the influence of shear stress, require much more terms of the solution series to obtain convergence specially for higher aspect ratios.

Solutions presented in these figures are obtained by the energy method. They usually tend to be less accurate than the cases of normal stresses. However, they are considered to have reasonable accuracy for practical purposes. Although available solutions do not cover the whole range of a/b ratios, they cover the more used range.

These relationships may be approximated reasonably and rather conservatively by the following equations, which are plotted in respective figures.

The buckling interaction relationship between longitudinal compression σ_x and uniform shear τ is expressed as follows (see **Fig. 8**):

$$\frac{\sigma_x}{\sigma_{xcr}} + \left(\frac{\tau}{\tau_{cr}}\right)^{\alpha_{11}} - 1 = 0 \qquad (20)$$

where,

$\alpha_{11} = -0.160\beta^2 + 1.080\beta + 1.082 \quad 1 \leqslant \beta \leqslant 3.2$

$\alpha_{11} = 2.90 \quad 3.2 < \beta$

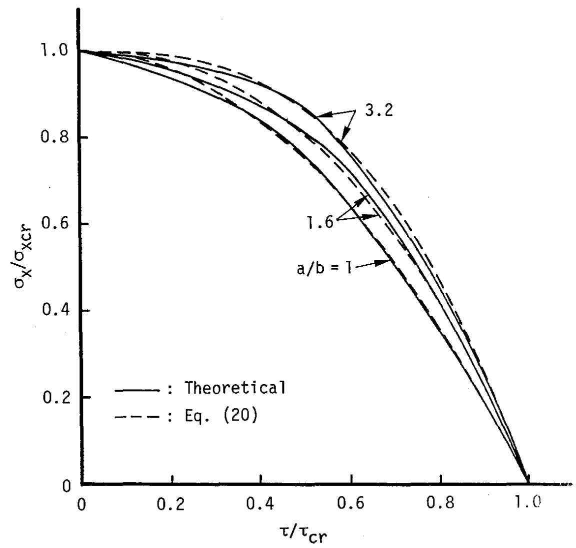

Fig. 8 Buckling interaction between σ_x and τ

(8) Transverse compression σ_y and uniform shear τ

In the same manner as stated in (7), the buckling equation between transverse compression σ_y and uniform shear τ is expressed as (see **Fig. 9**) :

$$\frac{\sigma_y}{\sigma_{ycr}} + \left(\frac{\tau}{\tau_{cr}}\right)^{\alpha_{12}} - 1 = 0 \qquad (21)$$

where,

$\alpha_{12} = 0.10\beta + 1.90 \quad 1 \leqslant \beta \leqslant 2$

$\alpha_{12} = 0.70\beta + 0.70 \quad 2 < \beta \leqslant 6$

$\alpha_{12} = 4.90 \quad 6 < \beta$

(9) Longitudinal bending σ_{bx} and uniform shear τ

As may be seen from **Fig. 10**, the interaction relation is

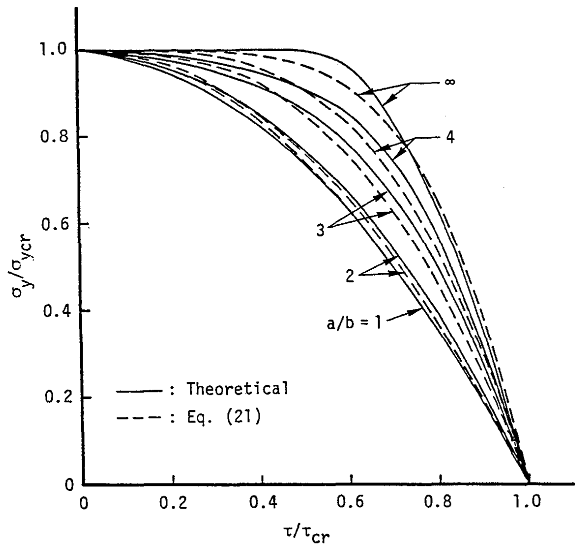

Fig. 9 Buckling interaction between σ_y and τ

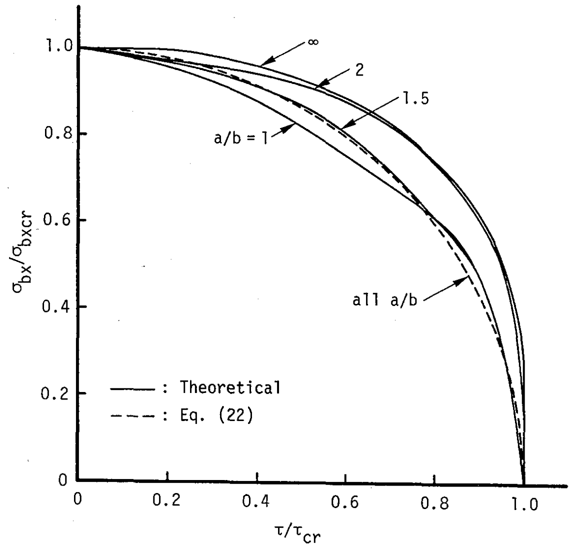

Fig. 10 Buckling interaction between σ_{bx} and τ

insensitive to the a/b ratio and may be approximated accurately by the following equation, which is also plotted in the figure.

$$\left(\frac{\sigma_{bx}}{\sigma_{bxcr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 - 1 = 0 \tag{22}$$

(10) Transverse bending σ_{by} and uniform shear τ

In case of transverse in-plane bending σ_{by} and uniform shear τ, which may be an important combination in design of deep girder, a series of eigen value analysis for the plate with $a/b = 3$ is carried out by the finite element method to extend the available range of a/b ratio.

However, these relationships may be insensitive to a/b ratio, and may be approximated resonably by the following equation (see **Fig. 11**).

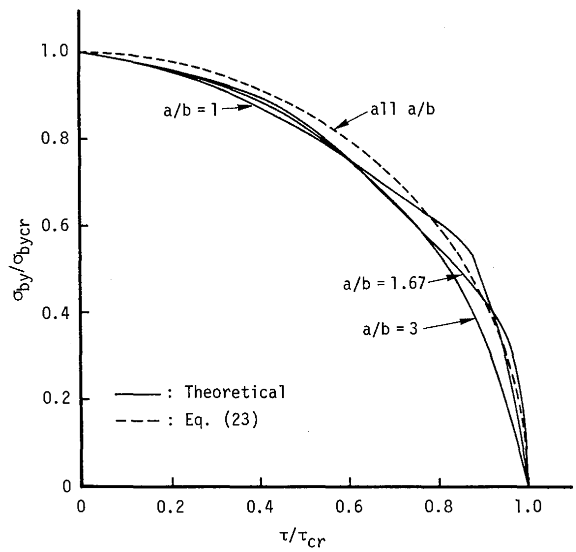

Fig. 11 Buckling interaction between σ_{by} and τ

$$\left(\frac{\sigma_{by}}{\sigma_{bycr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 - 1 = 0 \tag{23}$$

3.2 Buckling interaction for five load components

A general interaction relationship among the five load components is derived on the basis of the interaction relationships between two load components as presented in sec. 3.1.

At the first step, a set of two interaction relationships between two load components are chosen so that one of the load components is common in both relationships. The two relationships are combined to obtain, a new relationship among three load components.

By carrying on this procedure furthermore, the interation relationship among the five load components is obtained.

In the following, the procedure for finding out the interaction among five load components is presented in detail.

(1) Interaction among three load components

Equations (14), (15) and (16) are respectively the interactions between σ_x and σ_y, σ_x and σ_{bx}, and σ_y and σ_{bx}. Making combination of these relationships, an interaction relationship among three load components, σ_x, σ_y and σ_{bx} is first derived. Its schematic representation is shown in **Fig. 12**.

Let's consider the plate that buckles under these three load components which are denoted by σ_x, σ_y and σ_{bx}^*, respectively.

When no transverse compression is applied $\sigma_y = 0$, the interaction among σ_x, σ_{bx} and σ_y, corresponds to that between σ_x and σ_{bx}, Eq.(15). In this case, the critical value σ_x^* of the longitudinal compressive stress σ_x which together with σ_{bx}^* causes buckling of the plate may be obtained by Eq.(15) as follows:

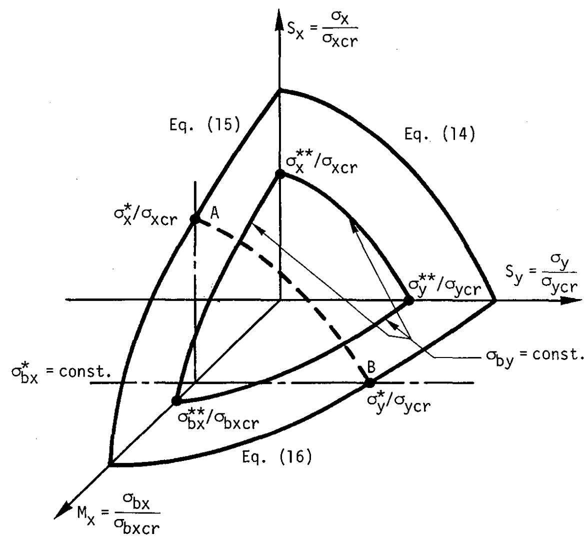

Fig. 12 Combining buckling interaction equations

$$\sigma_x^* = \sigma_{xcr}\,[1-(\sigma_{bx}^*/\sigma_{bxcr})^2] \qquad (24)$$

σ_x^* is represented by point A in the $S_x - M_x$ plane in Fig. 12. Similarly, when no longitudinal compression is applied $\sigma_x = 0$, the transverse compressive stress σ_y^* which causes buckling of the plate may be obtained by Eq.(16).

$$\sigma_y^* = \sigma_{ycr}[1-(\sigma_{bx}^*/\sigma_{bxcr})^{\alpha_4}]^{1/\alpha_3} \qquad (25)$$

σ_y^* is represented by point B in the $S_y - M_x$ plane in Fig. 12. It may be assumed that a similar relation to that of Eq.(14) exists between σ_x and σ_y in any plane of $M_x = \sigma_{bx}/\sigma_{bxcr}$ = constant.

Replacing σ_{xcr} and σ_{ycr} in Eq.(14) by σ_x^* and σ_y^* respectively, the following buckling interaction relationship among σ_x, σ_y and σ_{bx} may be obtained;

$$(\sigma_x/C_1\sigma_{xcr})^{\alpha_1} + (\sigma_y/C_2\sigma_{ycr})^{\alpha_2} - 1 = 0 \qquad (26)$$

where,

$$C_1 = 1 - (\sigma_{bx}/\sigma_{bxcr})^2$$
$$C_2 = [1 - (\sigma_{bx}/\sigma_{bxcr})^{\alpha_4}]^{1/\alpha_3}$$

(2) Interaction among four load components

In a similar way, Eqs.(17), (18) and (19) may be combined with Eq.(26) to obtain a relationship among σ_x, σ_y, σ_{bx} and σ_{by} as follows:

Under any value of σ_{by}^{**} which is smaller than σ_{bycr}, the longitudinal compression σ_x^{**} which causes buckling of the plate may be obtained by Eq.(17) when $\sigma_y = \sigma_{bx} = 0$.

$$\sigma_x^{**} = \sigma_{xcr}\,[1-(\sigma_{by}^{**}/\sigma_{bycr})^{\alpha_6}]^{1/\alpha_5} \qquad (27)$$

Under the same value of σ_{by}^{**}, the transverse compression σ_y^{**} which causes buckling of the plate may also be obtained by Eq.(18) when $\sigma_x = \sigma_{bx} = 0$

$$\sigma_y^{**} = \sigma_{ycr}\,[1-(\sigma_{by}^{**}/\sigma_{bycr})^{\alpha_8}]^{1/\alpha_7} \qquad (28)$$

Under the same value of σ_{by}^{**}, the longitudinal bending σ_{bx}^{**} which causes buckling of the plate may be obtained by Eq.(19) when $\sigma_x = \sigma_y = 0$

$$\sigma_{bx}^{**} = \sigma_{bxcr}\,[1-(\sigma_{by}^{**}/\sigma_{bycr})^{\alpha_{10}}]^{1/\alpha_9} \qquad (29)$$

σ_x^{**}, σ_y^{**} and σ_{bx}^{**} in the above equations are represented in Fig. 12.and they are substituted in σ_{xcr}, σ_{ycr} and σ_{bxcr} in Eq.(26), respectively.

Assuming that the relation among σ_x, σ_y and σ_{bx} for σ_{by}^{**} = constant is similar to that for $\sigma_{by} = 0$. The following interaction relationship among σ_x, σ_y, σ_{bx} and σ_{by} may be obtained by substituting σ_x^{**}, σ_y^{**} and σ_{cx}^{**} for σ_{xcr}, σ_{ycr} and σ_{bxcr} in Eq.(26).

$$(\sigma_x/C_3C_4\sigma_{xcr})^{\alpha_1} + (\sigma_y/C_5C_6\sigma_{ycr})^{\alpha_2} - 1 = 0 \quad (30)$$

where,

$$C_3 = [1-(\sigma_{by}/\sigma_{bycr})^{\alpha_6}]^{1/\alpha_5}$$
$$C_4 = 1-(\sigma_{bx}/C_7\sigma_{bxcr})^2$$
$$C_5 = [1-(\sigma_{by}/\sigma_{bycr})^{\alpha_8}]^{1/\alpha_7}$$
$$C_6 = [1-(\sigma_{bx}/C_7\sigma_{bxcr})^{\alpha_4}]^{1/\alpha_3}$$
$$C_7 = [1-(\sigma_{by}/\sigma_{bycr})^{\alpha_{10}}]^{1/\alpha_9}$$

(3) Interaction among five load components

Finally, Eqs.(20), (21), (22) and (23) are combined with Eq.(30) as follows:

Under any constant value of uniform shear τ^{***} which is smaller than τ_{cr}, σ_x^{***} which causes buckling of the plate may be obtained by Eq.(20) when the other load components vanish;

$$\sigma_x^{***} = \sigma_{xcr}\,[1-(\tau^{***}/\tau_{cr})^{\alpha_{11}} \qquad (31)$$

Similarly, σ_y^{***}, σ_{bx}^{***} and σ_{by}^{***} may be obtained by Eqs.(21), (22) and (23) repectively.

$$\sigma_y^{***} = \sigma_{ycr}[1-(\tau^{***}/\tau_{cr})^{\alpha_{12}}] \qquad (32)$$

$$\sigma_{bx}^{***} = \sigma_{bxcr}\,[1-(\tau^{***}/\tau_{cr})^2]^{0.5} \qquad (33)$$

$$\sigma_{by}^{***} = \sigma_{bycr}\,[1-(\tau^{***}/\tau_{cr})^2]^{0.5} \qquad (34)$$

Assuming that the interaction among σ_x, σ_y, σ_{bx} and σ_{by} for any value of shear τ^{***} = constant is similar to that for $\tau = 0$, the following interaction equation is obtained by substituting σ_x^{***}, σ_y^{***}, σ_{bx}^{***} and σ_{by}^{***} for σ_{xcr}, σ_{ycr}, σ_{bxcr} and σ_{bycr} repectively into Eq.(30).

$$\Gamma_B = (C_8C_9C_{10})^{\alpha_1}(C_{11}C_{12}C_{13})^{\alpha_2}\,[(\sigma_x/C_8C_9C_{10}\sigma_{xcr})^{\alpha_1} + (\sigma_y/C_{11}C_{12}C_{13}\sigma_{ycr})^{\alpha_2}-1] \qquad (35.a)$$

where,

$$C_8 = 1-(\tau/\tau_{cr})^{\alpha_{11}}$$
$$C_9 = [1-(\sigma_{by}/C_{14}\sigma_{bycr})^{\alpha_6}]^{1/\alpha_5}$$
$$C_{10} = 1-(\sigma_{bx}/C_{14}C_{15}\sigma_{bxcr})^2$$
$$C_{11} = 1-(\tau/\tau_{cr})^{\alpha_{12}}$$
$$C_{12} = [1-(\sigma_{by}/C_{14}\sigma_{bycr})^{\alpha_8}]^{1/\alpha_7}$$
$$C_{13} = [1-(\sigma_{bx}/C_{14}C_{15}\sigma_{bxcr})^{\alpha_4}]^{1/\alpha_3}$$
$$C_{14} = [1-(\tau/\tau_{cr})^2]^{0.5}$$
$$C_{15} = [1-(\sigma_{by}/C_{14}\sigma_{bycr})^{\alpha_{10}}]^{1/\alpha_9}$$

Consequently, the buckling interaction equation among five load components is obtained as follows:

$$\Gamma_B = 0 \qquad (35.b)$$

Coefficients α_1 to α_{12} included in the above equation are given with Eqs.(14) to (21). If Γ_B is smaller than zero with applied stresses substituted into Eq.(35), the plate does not buckle. If the applied stresses are gradually increased until they satisfy Eq.(35.b), the plate buckles.

3.3 Method of solution of proposed equation

At first sight, Eq.(35) seems somewhat complicated to solve analytically. However, it is not difficult to solve it numerically. Procedure of solution may differ according to the particular problem in hand, such as finding out the buckling stresses in case of proportionally increasing load, or finding out the critical value of a load component which increases gradually while other load components are kept constant, or finding out the critical thickness of the plate under a given load, etc.

However, in the following, general procedure is proposed to solve a general problem. Many other problems may be treated in a similar way.

(a) Definition of problem

As shown in Fig. 1, a simply supported rectangular plate ($a \times b \times t$) is subjected to a set of initial stresses composed of σ_x°, σ_y°, σ_{bx}°, σ_{by}°, τ° and a gradually increasing load which produces additional stress $\eta\sigma_x$, $\eta\sigma_y$, $\eta\sigma_{bx}$, $\eta\sigma_{by}$, $\eta\tau$, where η is a load factor.

It is required to find out the critical value of load factor η with which the plate just buckles. In this problem, by setting non-existing load components at zero or equating the initial load components to zero, a wide variety of problems may be formulated.

(b) Method of solution

For any value of η, the stresses acting on the plate (σ_x, σ_y, σ_{bx}, σ_{by}, τ) may be calculated as follows:

$$\left.\begin{aligned} \sigma_x &= \sigma_x^{\circ} + \eta\sigma_x \\ \sigma_y &= \sigma_y^{\circ} + \eta\sigma_y \\ \sigma_{bx} &= \sigma_{bx}^{\circ} + \eta\sigma_{bx} \\ \sigma_{by} &= \sigma_{by}^{\circ} + \eta\sigma_{by} \\ \tau &= \tau^{\circ} + \eta\tau \end{aligned}\right\} \quad (36)$$

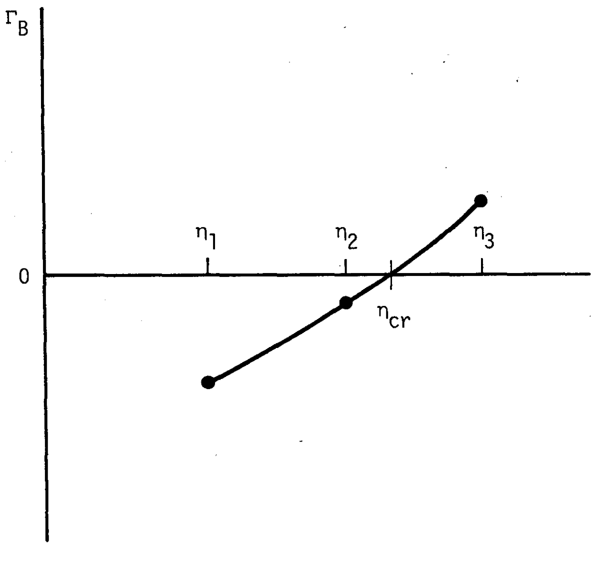

Fig. 13 Evaluation of η_{cr} by plotting Γ_B against η_i

The solution may proceed as follows:

1. Evaluate first the pure buckling stress such as σ_{xcr}, σ_{ycr}, σ_{bxcr}, σ_{bycr} and τ_{cr}, when a rectangular plate is subjected to a single load (See Appendix)
2. Compute the values of α_1 to α_{12}
3. Substitute the initial load into Eq.(35) and calculate Γ_B to see that the plate does not buckle under the initial load (Γ_B is negative)
4. Guided by Γ_B obtained in 3, assume two or three values of load factor, η_1, η_2 and η_3, substitute them in Eq.(36) and put the resulting stresses corresponding to each η into Eq.(35). Then, calculate values of Γ_B.
5. Plot Γ_B against η as shown in **Fig. 13** and find out the value of η at which Γ_B becomes equal to zero. This η may be denoted by η_{cr} and is the critical value of the load factor when the plate just buckles.

4. Assessment of Accuracy of Proposed Interaction Equation

4.1 Buckling interaction equation for two load components

As shown above, a buckling interaction equation is derived for a simply supported rectangular plate subjected to five load components. From this equation, Eq.(35), the interaction equation between each two load components, Eqs.(14) to (23) can easily be derived.

As may be seen in Figs. 2 to 11, these equations are accurate enough to be applied to practical design. For infinitely long plates, $a/b = \infty$, the buckling interactions seem to change suddenly, therefore the accuracy of the new proposed interaction equation may be reduced. However, for practical aspect ratios, the accuracy is good enough and the estimated buckling load tends to be conservative.

4.2 Buckling interaction equation for three load components

Further assessment of accuracy is carried out for cases with three load components, with four or five load components.

As mentioned above, it was assumed that the interaction relationship among three load components is similar to that between two load components, as shown in Fig. 12. This assumption does not have much effect on accuracy for practical aspect ratios of the plate and a similar accuracy may be expected as shown later.

In the following, the accuracy of buckling load subjected to combined three load components is investigated by comparison between estimated values, Eq.(35) and values calculated by either the incremental Galerkin method or the energy method.

(a) Biaxial compressions σ_x and σ_y and shear τ

Figure 14 compares the buckling load of a rectangular plate with different aspect ratios estimated by Eq.(35) and that calculated by the incremental Galerkin method[2] for a plate subjected to biaxial compression and shear, of which the load ratio is $\sigma_x/\sigma_{xcr} = \sigma_y/\sigma_{ycr} = \tau/\tau_{cr}$. They show good agreement.

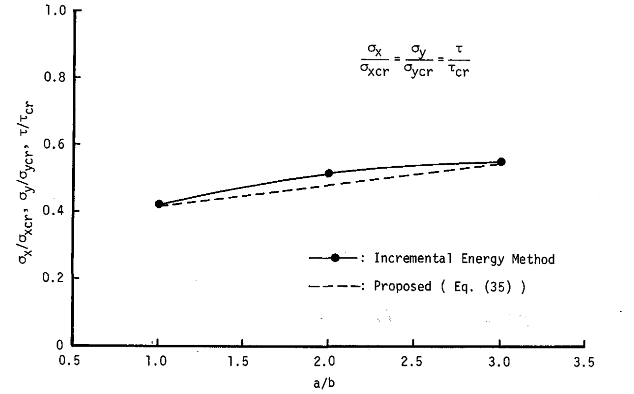

Fig. 14 Comparision between proposed equation, Eq.(35) and theoretical buckling load of rectangular plates subjected to σ_x, σ_y and τ

(b) Biaxial compressions σ_x and σ_y and longitudinal bending σ_{bx}

Figures 15.a, b and **c** show the buckling interaction relationships of rectangular plates with different aspect ratios subjected to biaxial compressions and in-plane bending in the longitudinal direction, of which the load ratios $\sigma_{bx}/\sigma_{bxcr}$ with respect to σ_y/σ_{ycr} are varied from 0.5, 1.0 to 2.0. The results by Eq.(35) are compared with those by the energy method[7]. The buckling loads estimated by Eq.(35) are seen to be accurate enough.

(c) Transverse compression σ_y, longitudinal bending σ_{bx} and shear τ

The buckling interaction relationship of an infinitely long plate subjected to transverse compression, longitudinal in-plane bending and shear is presented in Figs. 16.a and b.

In **Fig. 16.a**, τ/τ_{cr}—$\sigma_{bx}/\sigma_{bxcr}$ relationship are presented for two cases of constant transverse compression, of which σ_y/σ_{ycr} is 0.5 and 0.8. On the other hand, the σ_y/σ_{ycr} against τ/τ_{cr} relationship are shown in **Fig. 16.b** for four cases of constant in-plane bending, of which $\sigma_{bx}/\sigma_{bxcr}$ is 0.0, 0.5, 0.8 and 0.9.

The solid lines show the result calculated by the energy method[7], while the dotted ones show those by Eq.(35). It may be seen from the above two figures that Eq.(35) is accurate enough for engineering purposes.

5. Elastic — Plastic Buckling

In the previous section, elastic buckling of the plate subjected to combined five load components has been discussed.

If the plate is perfectly flat, that is it has no initial

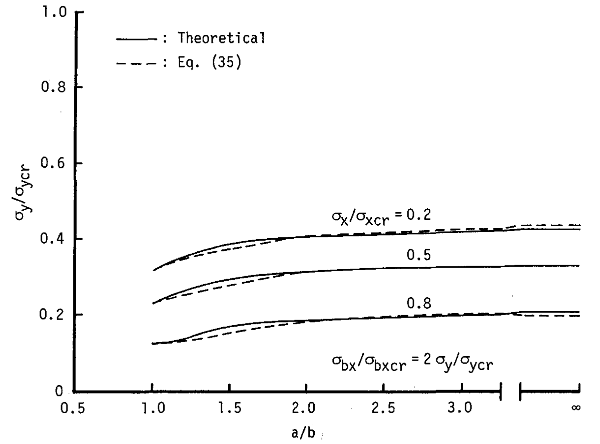

Fig. 15.a Comparision between proposed equation, Eq.(35) and theoretical buckling load of rectangular plates subjected to σ_x, σ_y and σ_{bx}

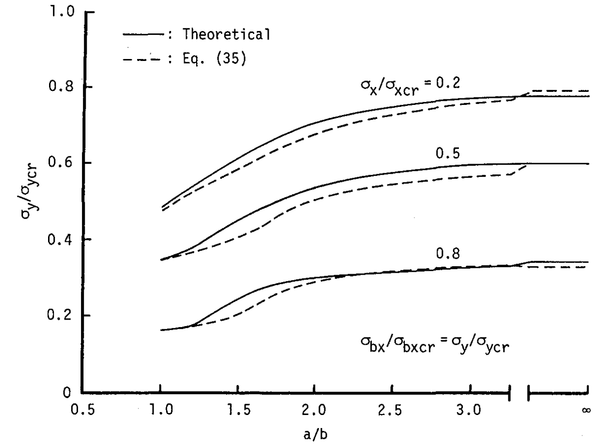

Fig. 15.b Comparision between proposed equation, Eq.(35) and theoretical buckling load of rectangular plates subjected to σ_x, σ_y and σ_{bx}

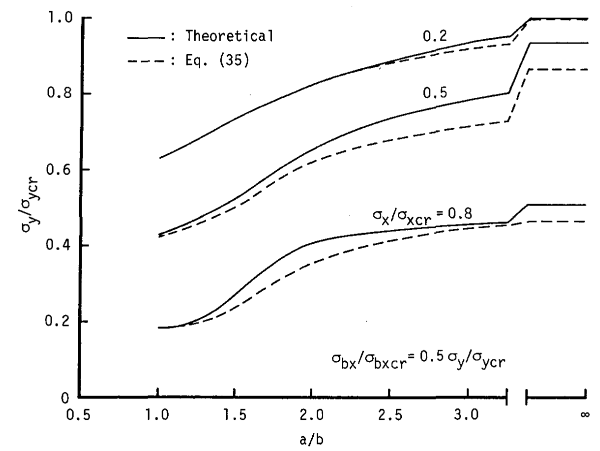

Fig. 15.c Comparision between proposed equation, Eq.(35) and theoretical buckling load of rectangular plates subjected to σ_x, σ_y and σ_{bx}

imperfections, the plate will buckle elastically or plastically under uniform in-plane compression or uniform shear. Elastic — plastic buckling may not occure in this case but under in-plane bending, elastic — plastic buckling may take place in the plate because the plastic

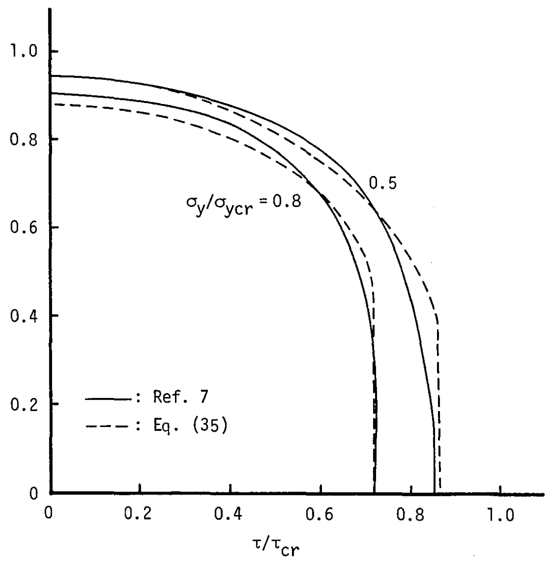

Fig. 16.a Comparision between proposed equation, Eq.(35) and theoretical buckling interaction relationship of infinite plate subjected to σ_y, σ_{bx} and τ

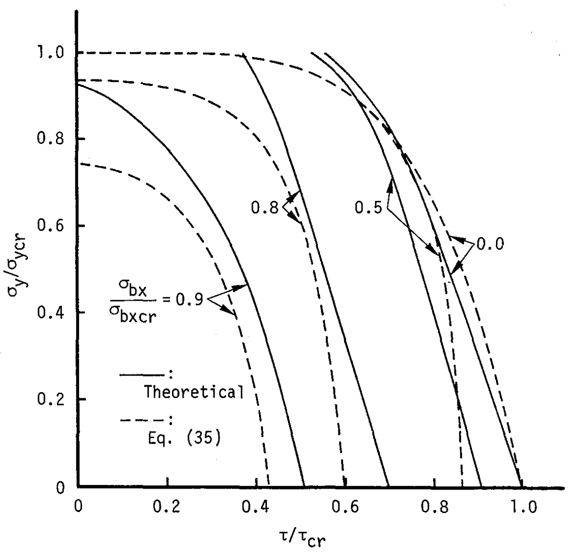

Fig. 16.b Comparision between proposed equation, Eq.(35) and theoretical buckling interaction relationship of infinite plate subjected to σ_y, σ_{bx} and τ

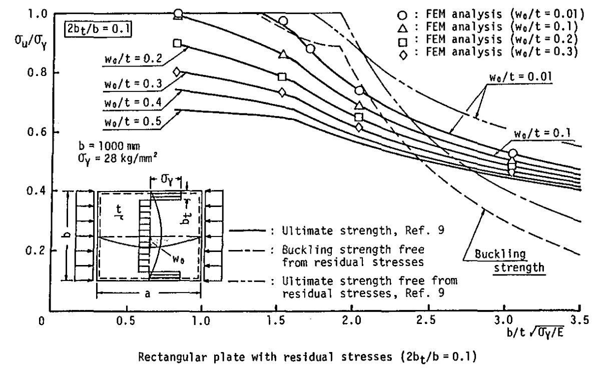

Fig. 17 Ultimate strength of a rectangular plate with initial imperfections due to welding

range is expanded from the edges to the inside as bending moment increases.

Results of theoretical analyses of buckling and ultimate strengths of a rectangular plate are shown with respect to its slenderness ratio $\lambda = b/t\sqrt{\sigma_0/E}$, ($\sigma_0$ = yield stress) in Fig. 17[9]. The plate has different values of unitial deflection and residual stresses.

According to **Fig. 17**, if the plate has no initial imperfections, it reaches ultimate limit state after elastic buckling or fails by plastic collapse.

In general, a plate with initial deflection and welding residual stress may not exhibit clear buckling phenomenon and reaches its ultimate strength state, as the load increases. Thin plates, in which the slenderness ratio is greater than about 2.4, the ultimate strength is higher than the buckling strength of a similar flat plate and the later is regarded as a failure condition.

On the other hand, in case of thicker plates, of which slenderness ratio is smaller than 2.4, initial imperfections cause a plate to fail by plastic collapse below elastic or plastic buckling strength. This type of behaviour is a phenomenon of plastic collapse. The nature of this behaviour is different from that arising in elastic buckling. Therefore, buckling strength is not likely to be valid for such thick plates as a failure condition.

From the above discussion, it may be better to treat plastic collpase phenomenon separately rather than to make corrections to the elastic buckling relationships. This paper concentrates on elastic buckling, while interaction relationships for plastic collpase are planned for future research.

6. Buckling Interaction Equations Proposed by Lloyd's Register and Det Norske Veritas and Their Accuracy

6.1 Comparison with an interaction equation proposed by Lloyd's Register

Lloyd's Register[3] proposed the following buckling interaction equation for a simply supported rectangular plate subjected to five load components.

$$\left(\frac{\sigma_x}{\sigma_{xcr}}\right)^\alpha + \left(\frac{\sigma_y}{\sigma_{ycr}}\right)^\alpha + \left(\frac{\sigma_{bx}}{\sigma_{bxcr}}\right)^2 + \left(\frac{\sigma_{by}}{\sigma_{bycr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1 \qquad (37)$$

where,

$\alpha = 0.6 + 0.4/\beta \qquad 0.3 \leqslant \beta \leqslant 1.0$
$\alpha = 0.6 + 0.4\beta \qquad 1.0 \leqslant \beta \leqslant 3.5$
$\beta = a/b$

It is should be noted that the above equation is valid for $1/3.5 \leqslant a/b \leqslant 3.5$. In case of two load components, this equation yields good approximation in some cases. However, in some other cases, it yields estimation on the non-conservative side of the theoretical relationships as may be seen in **Figs. 18.b,d,e** and **f.**

Equation (35), proposed in this paper, is plotted in the same figures for comparison with Lloyd's equation. In case of in-plane bending and shear, both equations yield the same result as shown in Figs. 10 and 11. Eq.(35), however, yields better results for all other load combinations.

For combination of more than two load components, Lloyd's equation just adds nondimensional load components. Such direct addition may need justification.

6.2 Comparison with an interaction equation proposed by Det Norske Veritas

DnV[4)] proposed the following equation as a buckling interaction relationship for rectangular plates

$$\eta = \sqrt{\eta^2{}_{cs} + 2(\tfrac{b}{a})^2 \eta_{cs}\eta_c + \eta^2{}_c} \qquad (38)$$

where,

η = usage factor and when the factor of safety is considered to be more than 1.0, η is regarded as 1.0

η_{cs} = usage factor for the combination of shear and compression in one direction

For thin plates ($1.4 < \lambda$, which is modified reduced slenderness ratio and defined in 3.3 of Ref.4)),

$$\eta_{cs} = \frac{1+\psi}{4}\frac{\sigma}{\sigma_{cr}} + \sqrt{(\frac{3-\psi}{4}\frac{\sigma}{\sigma_{cr}})^2 + (\frac{\tau}{\tau_{cr}})^2} \qquad (39)$$

where,

η_c = usage factor for compression in the other direction, i.e. σ/σ_{cr} in the other direction

σ = reference stress for compression in any one direction

σ_{cr} = critical value of the reference stress

ψ = stress multiplier, the ratio of stresses at plate edges as shown in **Fig. 19** and is used to combine axial compression in one direction and in-plane bending in the same direction

DnV specifies values for σ_{cr} as follows:

$$\sigma_{cr} = C\frac{\pi^2 E}{12(1-\nu^2)}(\frac{t}{b})^2 = C\sigma_E \qquad (40)$$

where, in the x direction,

$$C = \frac{8.4}{\psi + 1.1} \qquad 0 < \psi \leqslant 1$$

$$C = 7.6 - 6.4\psi + 10\psi^2 \quad -1 \leqslant \psi < 0$$

and in the y direction,

$$C = [1 + (\frac{b}{a})^2]^2 \frac{2.1}{\psi + 1.1} \qquad 0 < \psi \leqslant 1$$

$$C = (1+\psi)C_a - \psi C_b + 10\psi(1+\psi)(\frac{b}{a})^2 \qquad -1 < \psi < 0$$

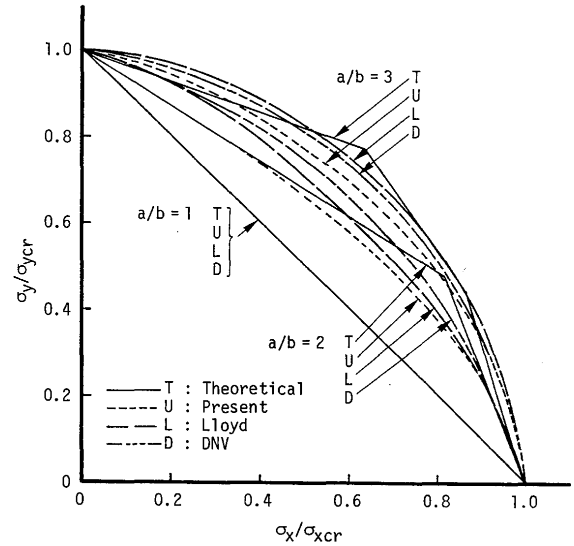

Fig. 18.a Accuracy of buckling interaction equations for σ_x and σ_y

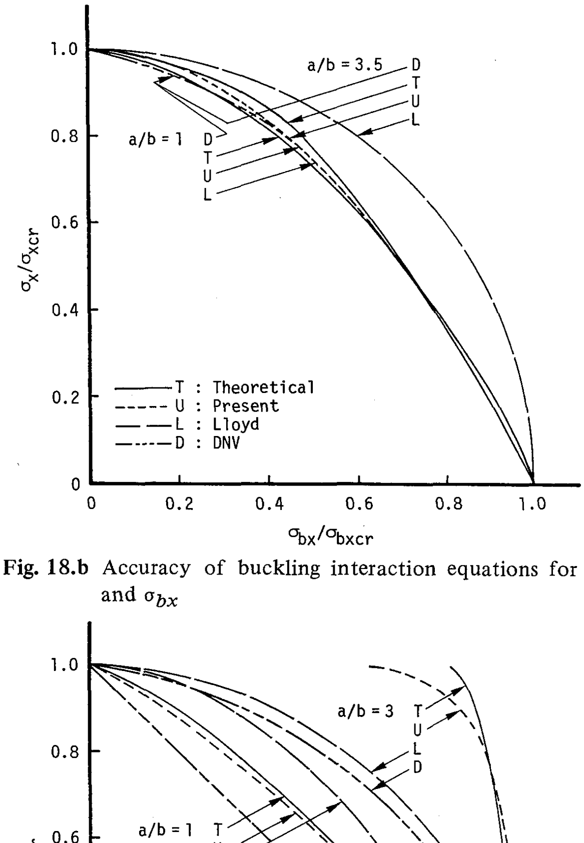

Fig. 18.b Accuracy of buckling interaction equations for σ_x and σ_{bx}

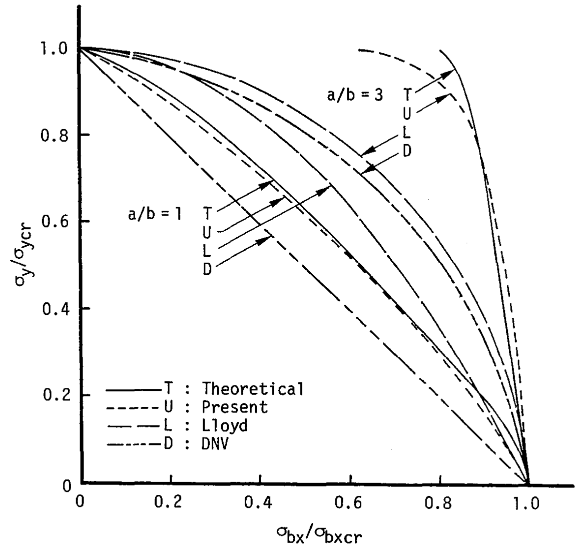

Fig. 18.c Accuracy of buckling interaction equations for σ_y and σ_{bx}

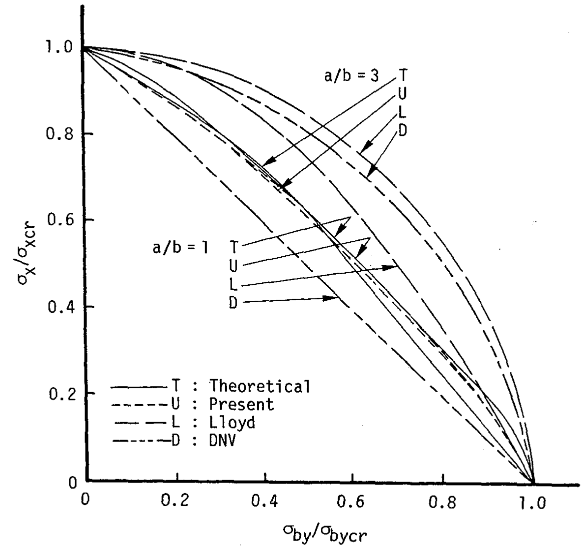

Fig. 18.d Accuracy of buckling interaction equations for σ_x and σ_{by}

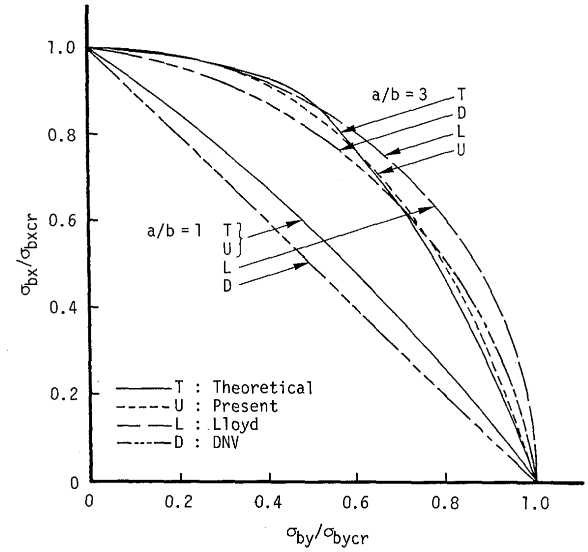

Fig. 18.f Accuracy of buckling interaction equations for σ_{bx} and σ_{by}

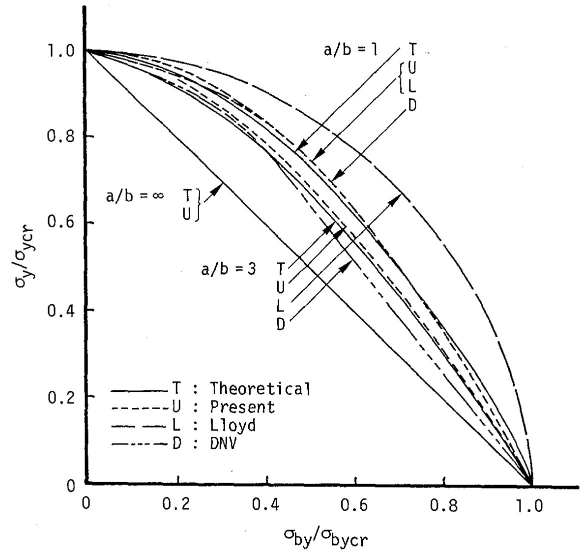

Fig. 18.e Accuracy of buckling interaction equations for σ_y and σ_{by}

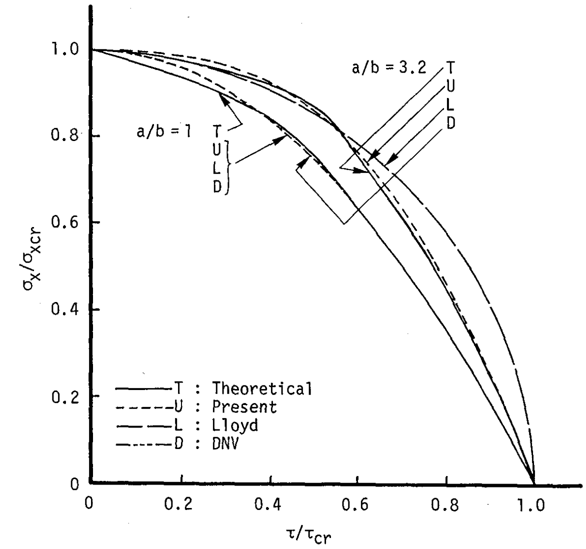

Fig. 18.g Accuracy of buckling interaction equations for σ_x and τ

$$C_a = 1.91\ [1 + (\frac{b}{a})^2]^2$$

$$C_b = 24(\frac{b}{a})^2 \qquad \frac{a}{b} \leqslant 1.5$$

$$C_b = 2+16(\frac{b}{a})^2 + 8(\frac{b}{a})^4 \qquad \frac{a}{b} > 1.5$$

This relationships are also plotted in Figs. 18.a to *h* together with Eq.(35) proposed in this paper. Eq.(38) proposed by DnV makes accurate estimation of buckling strength in some cases. However, it underestimates buckling strength greatly in some other cases of complicated interaction.

7. Conclusion

In this paper, a new elastic buckling interaction equa-

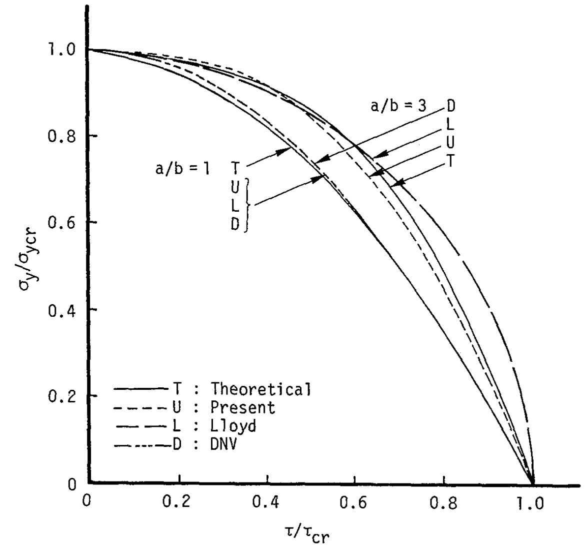

Fig. 18.h Accuracy of buckling interaction equations for σ_y and τ

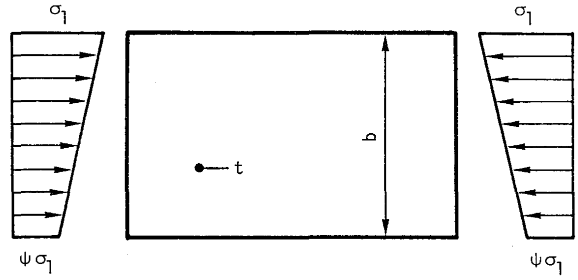

Fig. 19 Stress multiplier ψ in Eq.(39)

tion for simply supported rectangular plates subjected to five load components is proposed. In order to formulate this equation, buckling strengths of a plate under two load components are calculated, in addition to the existing data, and buckling interaction equations under each two load components were developed. Based on these equations, a new buckling interaction equation for five load components is theoretically derived.

A method of solution to the proposed equation is presented and assessment of its accuracy is performed. It is found that the proposed equation has sufficient accuracy for practical purposes.

Some comparisions with an interaction equation proposed by Lloyd's Register and a design procedure proposed by Det Norske Veritas are also carried out.

The result indicates that the new elastic buckling interaction equation proposed in this paper yields better accuracy on the safe side.

Appendix

Buckling strength for simply supported rectangular plates under a single load component can be calculated as follows;

$$\sigma_{xcr} = k_x \sigma_E, \quad \sigma_{ycr} = k_y \sigma_E$$
$$\sigma_{bxcr} = k_{bx} \sigma_E, \quad \sigma_{bycr} = k_{by} \sigma_E$$
$$\tau_{cr} = k_\tau \sigma_E$$

where,

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

E = modulus of elasticity, ν = Poisson's ratio

and coefficients k_x, k_y, k_{bx}, k_{by} and k_τ may be calculated for rectangular plates ($a/b \geqslant 1$) as follows:

$$\left.\begin{aligned} k_x &= (m/\beta + \beta/m)^2 \\ k_y &= (1.0 + 1.0/\beta^2)^2 \end{aligned}\right\} \quad \text{(exact solution)}$$

$$\left.\begin{aligned} k_{bx} &= 23.9 \\ k_{by} &= 23.9 \quad 1 \leqslant \beta \leqslant 1.5 \\ &= 15.87+1.87\beta^2+8.6/\beta^2 \quad \beta>1.5 \end{aligned}\right\} \quad \text{(approximate formula)}$$

$$k_\tau = 5.34 + 4.0/\beta^2 \quad \text{(approximate formula)}$$

where,

$\beta = a/b$

m = buckling wave number in the x direction and the minimum integer to satisfy the condition $\beta \leqslant \sqrt{m(m+1)}$

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