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Effect of Residual Stress and Mechanical Heterogeneity on Brittle Fracture Strength of Welded Joints†

Kunihiko SATOH* and Masao TOYODA**

Abstract

Effects of residual stress and mechanical heterogeneity caused due to welding on brittle fracture initiation in welded joints are analytically investigated by using fracture mechanics. The residual stress redistribution due to the existence of a crack is considered. The fracture strength can be approximately evaluated by the calculation neglecting the redistribution of residual stress with the exception of the much lower stress level. Effects of mechanical heterogeneity on brittle fracture are equivalent in calculations to those of residual stress. The transition temperature \( T_i \) is changed by the factors such as residual stress, mechanical heterogeneity and fracture toughness. The elevation of temperature \( \Delta T_i \) due to residual stress is predominant at lower residual stress levels.

1. Introduction

Behaviors of brittle fracture initiation in welded joints are affected with several factors which are different from factors in homogeneous materials. The main factors of these are welding residual stress and fracture toughness ahead of (welding) crack tip. In addition to them, mechanical heterogeneity caused due to welding is one of factors which play an essential role in case of brittle fracture in welded joints. Effect of residual stress on brittle fracture initiation in welded structure was already discussed in terms of fracture mechanics by Sakai et al.\(^9\) and Kanazawa et. al.\(^9\). However, the investigations have never been done on the effects of mechanical heterogeneity on brittle fracture or composite effects of those factors. Recently, Koshiga et. al.\(^8\) proposed the technique of calculation in which was considered effects of residual stress and fracture toughness in order to discuss effects of stress-relieving heat treatment on brittle fracture initiation in welded structure. The basic concept employed for discussion in the present paper is that concept proposed by Koshiga, and the brittle fracture strength of welded joints having residual stress and mechanical heterogeneity was calculated. Furthermore, the authors try to take a clue in considering composite effects of those factors.

2. Basic conditions for calculating brittle fracture strength

2.1 Distributions of residual stress

In this paper, the welded plates including a crack (or notch) under tension parallale to a weld line, so called “Wells-Kihara Test” are supposed as shown in Fig. 1, the residual stress distributions are approximated by step-like (rectangle) form or parabolic form in which the tensile residual stress is distributed within \( 2x_0 \) mm across the weld, and the rest \( -W-2x_0 \) is uniformly distributed compressive residual stress with its magnitude equal to balance the tension. If the width of welded plates is infinite and the co-ordinate \( x-x_0 \) at which the residual stress equal to zero is unchangeable, the residual stress distribution \( \sigma_r(x) \) in a infinite plate \( W \to \infty \) is given as follows. For parabolic form

\[
\begin{align*}
\sigma_r(x) &= (\sigma_r)_f \left( 1 - \frac{x^2}{x_0^2} \right) \quad (|x| \leq x_0) \\
\sigma_r(x) &= 0 \quad (|x| > x_0)
\end{align*}
\]

and for step-like (rectangular) form

\[
\begin{align*}
\sigma_r(x) &= \sigma_r \quad (|x| \leq x_0) \\
\sigma_r(x) &= 0 \quad (|x| > x_0)
\end{align*}
\]

If the crack (or notch) exists in welded joints, the residual stress distribution becomes different from the

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distributions as shown in Fig. 1. The residual stress redistributes when the transverse notch is made after welding. When the welding crack causes at low temperature or the fatigue crack propagates, the residual stress also redistributes. The redistribution of the residual stress in case of existence of crack can be calculated using Dugdale model, as stated afterwards.

2. 2 Mechanical heterogeneity of welded joints

In calculating the brittle fracture strength at low stress level, it is sufficient to consider only heterogeneity of yield stress $\sigma_y$. Parts in vicinity of weld line is given thermal-cycles in welding. The relationship between the yield stress $\sigma_y$ after the thermal-cycles and the peak temperature of it were given by Suzuki's experiments. If the peak temperature of parts in vicinity of weld line is known, yield stress $\sigma_y$ of that parts is estimated from these experimented data. Figure 2 shows the relation between the peak temperature $T_a$ (for $q/vh=6500$ cal/cm$^2$) and $\sigma_y$ estimated from $T_a$ using Suzuki's data (curve 1) and distance from weld line. In Fig. 2, the dotted line is the distribution of $\sigma_y$ assumed in the later calculation of fracture strength of welded joints.

![Fig. 2. Distribution of yield stress across a weld line](image)

2. 3 Relationship between yield stress $\sigma_y$ and temperature

It is generally recognized that relation between $\sigma_y$ and temperature $T'(K)$ is given as follows.

$$\sigma_y = \sigma_{y0} \exp \left\{ D(1/T - 1/273) \right\}$$

Where $\sigma_{y0}$ is yield stress at sub-zero and $D$ is a material constant. The $D$-value is given in Fig. 3 as a function of yield stress at room temperature ($\approx \sigma_{y0}$) using data published in EW-Committee of Japan Welding Engineering Society.

![Fig. 3. The $D$-value representing temperature dependency of yield stress for several kinds of steel and weld metal.](image)

2. 4 Criterion for brittle fracture initiation and relationship between fracture toughness and temperature

The current engineering fracture criteria $K_c$, $\Phi_e$ and $\rho_e^*$ for brittle fracture initiation are critically reviewed analytically and experimentally by Kanazawa et al., and it is concluded that the fracture initiation in existence of initial stress ahead of the crack tip due to welding, pre-loading etc. is explained equally well by $\Phi_e$ or $\rho_e^*$ criterion, but not by $K_c$ criterion. In this paper, the basic concept employed expeditiously for discussion is that brittle fracture should initiate when tensile yield zone $\rho^*$ formed ahead of a pre-existing crack attains a critical size $\rho_e^*$ depending on the material, because the authors adopt the experimental equations showed by Koshiga et al. as the relationship between fracture toughness and temperature.

The relationship between the $\rho_e^*$-value and temperature was given by Koshiga et al. as follows

$$\rho_e^* = \alpha(T/100)^{\beta}$$

where $\alpha$ is a material constant.

For the case when the residual stress is equal to zero, the tensile yield zone $\rho^*$ in a infinite plate subjected to uniform tensile stress $\sigma$ is determined by

$$\sigma_y \cos^{-1} \left( \frac{c}{a} \right) = \frac{\pi}{2} \rho^*$$

where $a = c + \rho^*$

Denoting the temperature at which the fracture stress $\sigma_f = 1/2 \sigma_{y0}$ by $(T_{100})_{\rho_e}$, from eqs, (3), (4), (5), it follows that

$$\frac{\sigma}{\rho^*} = \left[ \left( \frac{T_{100}}{100} \right) \frac{100}{100} \right]^{\beta} \sec \left( \frac{\pi}{4} \exp \left( \frac{D}{(T_{100})_{\rho_e}} \right) \right) - 1 \right]^{-1} = 1$$

Figure 4 illustrates the relationships between $(T_{100})_{\rho_e}$.
Effect of Residual Stress and Mechanical Heterogeneity

![Graph showing relation between $\alpha/c$ and $(T_{10})_{4^0}$](image)

$\frac{\alpha}{c} = 0.98 \exp \left[ -\frac{(T_{10})_{4^0}(^\circ K)}{40} \right] \quad (D=115°K)$

$\frac{\alpha}{c} = 2.58 \exp \left[ -\frac{(T_{10})_{4^0}(^\circ K)}{35} \right] \quad (D=70°K)$

Fig. 4. Relation between $\alpha/c$ and $(T_{10})_{4^0}$.

and $\alpha/c$. The dotted curves ①, ② in Fig. 4 represent the relations when $D=115°$ (K) and $D=70°$ (K) respectively. These relationships approximate to the relationships shown by the solid lines in Fig. 4 which are given as follows.

\[
\frac{\alpha}{c} = 0.98 \exp \left[ -\frac{(T_{10})_{4^0}(^\circ K)}{40} \right] \quad (D=115°K) \tag{6-2}
\]

\[
\frac{\alpha}{c} = 2.58 \exp \left[ -\frac{(T_{10})_{4^0}(^\circ K)}{35} \right] \quad (D=70°K) \tag{6-2}
\]

On the following calculations for brittle fracture strength of welded joints having residual stress and mechanical heterogeneity, the authors employ the $(T_{10})_{4^0}$-value as the parameter representing the fracture toughness.

Experiencing the Dugdale's idea, the $\rho^*$-value in a infinite plate having residual stress and mechanical heterogeneity can be easily determined. The residual stress $\sigma_r(x)$ and yield stress $\sigma_y(x)$ (as shown schematically in Fig. 5 (a)) are distributed on crack line. In this case, using the Dugdale model (as shown in Fig. 5 (b)), the $\rho^*$-value in a infinite plate subject to uniform tensile stress $\sigma$ is determined by

\[
- \left[ \int_{-a}^{a/2} + \int_{a/2}^{a} \right] \frac{\sigma_y(x)}{\sqrt{a^2-x^2}} \, dx = 0,
\]

\[
\rho^* = \frac{c + \rho^*}{\cos \left( \frac{c}{\rho^*} \right)} \int_{0}^{a} \frac{T_{10}(x)}{\sqrt{a^2-x^2}} \, dx
\]

\[
a = c + \rho^*
\]

3. Effects of residual stress on brittle fracture strength

3.1 Effect of form of residual stress distribution

Brittle fracture strength was calculated in the two cases of residual stress distribution, or rectangular type and parabolic type as shown in Fig. 1. In this case, the redistribution of residual stress due to the existence of the crack was neglected and it is assumed that materials are homogeneous and isotropic. The residual stress distribution $\sigma_r(x)$ was already given in eqs. (1) and (2), and the $\rho^*$-value in this case is given by following equations from eq. (7).

\[
\rho^* = \frac{c}{\cos \left( \frac{c}{\rho^*} \right)} \int_{0}^{a} \frac{T_{10}(x)}{\sqrt{a^2-x^2}} \, dx
\]

\[
\rho^* = \frac{c + \rho^*}{\cos \left( \frac{c}{\rho^*} \right)} \int_{0}^{a} \frac{T_{10}(x)}{\sqrt{a^2-x^2}} \, dx
\]

$T_{10}(x) = \sigma + \sigma_r(x)$

The relations between $\rho^*$-value and tensile stress $\sigma$ in
this two type is given as follows.

(1) Parabolic type

(i) \( a = c + \rho \) \( \leq x_x, \sigma + (\sigma_r)_r = \sigma_r \)  
(Where, \((\sigma_r)_r\) is the peak value of tensile residual stress.)

\[
\sigma_r \cos^{-1} \left( \frac{c}{a} \right) = \frac{\pi}{2} \left[ \sigma + (\sigma_r)_r \left( 1 - \frac{a^2}{2x^2_0} \right) \right] (9)
\]

(ii) \( a < x_x, \sigma + (\sigma_r)_r > \sigma_r \)

\[
\sigma_r \cos^{-1} \left( \frac{c}{a} \right) = \left[ \sigma_r - \sigma - (\sigma_r)_r \left( 1 - \frac{a^2}{2x^2_0} \right) \right]
\times \sin^{-1} \left( \frac{x_x}{a} \right) + \frac{\pi}{2} \left[ \sigma + (\sigma_r)_r \left( 1 - \frac{a^2}{2x^2_0} \right) \right]
\times \frac{(\sigma_r)_r}{x_x} \sqrt{\left( \frac{x_x}{x_0} \right)^2 - \left( \frac{x_r}{x_0} \right)^2} (10)
\]

where, \( x_r = x_x \left[ 1 - \frac{\sigma_r - \sigma}{(\sigma_r)_r} \right] \)

(iii) \( a > x_x, \sigma + (\sigma_r)_r \leq \sigma_r \)

\[
\sigma_r \cos^{-1} \left( \frac{c}{a} \right) = \frac{\pi}{2} \sigma_r \left( 1 - \frac{a^2}{2x^2_0} \right)
\times \sin^{-1} \left( \frac{x_x}{a} \right) - \frac{(\sigma_r)_r}{2} \sqrt{\left( \frac{x_x}{x_0} \right)^2 - 1} (11)
\]

(iv) \( a > x_x, \sigma + (\sigma_r)_r > \sigma_r \)

\[
\sigma_r \cos^{-1} \left( \frac{c}{a} \right) = \left[ \sigma_r - \sigma - (\sigma_r)_r \left( 1 - \frac{a^2}{2x^2_0} \right) \right]
\times \sin^{-1} \left( \frac{x_x}{a} \right) - \frac{\pi}{4} (\sigma_r)_r \left( \frac{a}{x_0} \right)^2
\times (\sigma_r)_r \sin^{-1} \left( \frac{x_x}{a} \right) - \frac{(\sigma_r)_r}{2} \frac{x_r}{x_0}
\times \sqrt{\left( \frac{x_x}{x_0} \right)^2 - \left( \frac{x_r}{x_0} \right)^2} (12)
\]

(2) Rectangular type

(i) \( a \leq x_x \)

\[
\sigma = \frac{2}{\pi} \sigma_r \cos^{-1} \left( \frac{c}{a} \right) - \sigma_r (13)
\]

(ii) \( a > x_x, \sigma + \sigma_r \leq \sigma_r \)

\[
\sigma = \frac{2}{\pi} \left[ \sigma_r \cos^{-1} \left( \frac{c}{a} \right) + \sigma_r \right]
\times \cos^{-1} \left( \frac{x_x}{a} \right) - \sigma_r (14)
\]

(iii) \( a > x_x, \sigma + \sigma_r > \sigma_r \)

\[
\sigma = \sigma_r \left[ \cos^{-1} \left( \frac{c}{a} \right) + \cos^{-1} \left( \frac{x_x}{a} \right) - \frac{\pi}{2} \right]
\times \cos^{-1} \left( \frac{x_x}{a} \right) (15)
\]

If the relationships between \( \sigma_r \) and \( \rho^*_r \)-value and temperature are given as shown in eq. (3) and eq. (4) respectively, the fracture strength of plates having residual stress can be calculated. Figure 6 shows the relationships between fracture strength \( \sigma_f / \sigma_m \) and temperature when the point at which the sign of the residual stress change is fixed and the sum of magnitude of residual stress is equal each other in two types. For an example, when \( c = 20 \text{ mm}, \ x_x = 40 \text{ mm} \) and \( (T_{\text{in}})_1 = -170^\circ \text{C} \). In Fig. 6, the fracture strengths in the rectangular type and parabolic type of residual stress distribution are compared at \( \sigma_r / \sigma_m = 0.25 \) and 0.5, where dotted lines show for rectangular type. The fracture strength is not so much affected by the form of residual stress distribution. Therefore, in the following calculation, the rectangular type of residual stress distribution is adopted as the calculation of strength can be done simply.

![Fig. 6. Comparison of fracture strength for two types of residual stress distribution. (Solid line: Parabolic type, Dotted line: Rectangular type.)](image)

3.2 Brittle fracture strength considering the redistribution of residual stress due to the existence of a crack

When

\[
\sigma_r \cos^{-1} \left( \frac{c}{c + \rho^*_r} \right) < \frac{\pi}{2} \sigma_r (16)
\]

in eq. (13), the fracture strength \( \sigma_f \) becomes smaller zero. This means that brittle fracture will initiate even if the external load is not given. This inconsistency is caused as it is supposed that the residual stress acts over the crack surface. It is that the redistribution of residual stress due to the existence of a notch (or crack) is considered. The method of calculation
considered the redistributed residual stress was also attempted by Kanazawa et al. The
When the transverse notch (crack length is equal to 2c) is made by saw cut at room temperature (≈0°C) in
the infinite plate having residual stress of rectangular type as shown in Fig. 1. The plastic zone size p,
formed ahead of crack tip by redistribution of residual stress can be calculated from eq. (8) as follows.

\[
\text{when } a_1 \leq x_0 \quad \frac{c}{a_1} = \cos \left( \frac{\pi}{2}, \frac{\sigma_r}{\sigma_{\infty}} \right) \\
\text{when } a_1 > x_0 \quad \frac{c}{a_1} = \cos \left( \frac{\sigma_r}{\sigma_{\infty}} \sin^{-1} \frac{x_0}{a_1} \right)
\]

The residual stress distributions the crack line in this
(17)
case are also obtained with the application of Dugdale model.

(a) When \( a_1 \equiv x_0 \);

The results required are obtained by superposition of three states, within linear elasticity theory, or states (I), (II) and (III) as shown in Fig. 7 (a). The solution
of stress distribution when a force distribution \( T_e(x) \)
(18)
is applied to crack surfaces can be taken in several forms from several sources. A form particularly
convenient here, taken from Kerner, gives the stress \( \sigma_r \), on
the prolongation of the crack as
\[
\sigma_r = \frac{2}{\pi} |x| (x^2 - a_1^2)^{-1/2} \int_0^{\pi} T_0(t)(a_1^2 - t^2)^{-1/2} dt \quad (x \geq a_1)
\]

If \( T_0(t) \) is constant regardless of x-coordinate, using
the following relation
\[
\int \frac{a_1^2 - t^2}{x^2 - t^2} dt = \frac{1}{2x} \sqrt{x^2 - a_1^2} \left\{ \sin^{-1} \frac{a_1^2 - xt}{a_1 (x - t)} - \sin^{-1} \frac{a_1^2 + xt}{a_1 (x + t)} \right\} + \sin^{-1} \frac{t}{a_1}
\]

(b) When \( a_1 > x_0 \);

Therefore, regarding eq. (17), the stress distributions
(21)
(\( \sigma_y \)) in this case is given as
\[
(\sigma_y)_2 = (\sigma_y)_1 + (\sigma_y)_2 + (\sigma_y)_3
\]
\[
\begin{align*}
\sigma_r + \sigma_n & \quad \sin^{-1} \frac{a_t^2 - xc}{a_1 (x + c)} \\
- \sin^{-1} \frac{a_t^2 + xc}{a_1 (x + c)} & \quad (a_1 \leq |x| \leq x_0)\\
\sigma_r & \quad + \sigma_n \quad \sin^{-1} \frac{a_t^2 - xc}{a_1 (x - c)} \\
- \sin^{-1} \frac{a_t^2 + xc}{a_1 (x - c)} & \quad (x > x_0)
\end{align*}
\]

In this case, the residual stress distribution is also
obtained by superposition of states (II) and (III) as
shown in Fig. 7 (b).

(22)
(\( \sigma_y \)) _2 = \sigma_r + \sigma_n \quad \left\{ \sin^{-1} \frac{a_t^2 - xx_0}{a_1 (x - x_0)} - \sin^{-1} \frac{a_t^2 + xx_0}{a_1 (x + x_0)} \right\}
\[
+ \frac{2x}{\sqrt{x^2 - a_1^2}} \sin^{-1} \frac{x_0}{a_1}
\]

(\( \sigma_y \)) _r = \sigma_r \quad \left\{ \sin^{-1} \frac{a_t^2 - xx_0}{a_1 (x - x_0)} - \sin^{-1} \frac{a_t^2 + xx_0}{a_1 (x + x_0)} \right\}
\[
+ \sigma_r + \sigma_n \quad \left\{ \sin^{-1} \frac{a_t^2 - xc}{a_1 (x - c)} - \sin^{-1} \frac{a_t^2 + xc}{a_1 (x + c)} \right\}
\]

(\( x \geq a_1 \))

(23)

Figure 8 shows the redistributed residual stress on the
prolongation of the crack. For examples, when c = 30 mm, \( x_0 = 60 \) mm and \( \sigma_r / \sigma_{\infty} = 0.4 \) or 0.8.

Figure 9 shows one example of the relationships between the fracture strength \( \sigma_f \) of plate having residual stress and testing temperature \( T \) for various value of magnitude of residual stress, when c = 30 mm, \( x_0 = 60 \) mm, \( (T_{\infty})_{1/2} = -125°C \) and \( D = 115°K \). The solid
The temperature \((T_r)_{\frac{1}{2}}\) at which \(\sigma_f/\sigma_{ne} = 0.5\) is adopted in order to consider effects of initial magnitude of residual stress \(\sigma_0\) on brittle fracture strength. Figure 10 shows relationship between \((T_r)_{\frac{1}{2}}\) and \(\sigma_f/\sigma_{ne}\) obtained from Fig. 9. The change of \((T_r)_{\frac{1}{2}}\)-value with residual stress is considerable in the range of small value of residual stress \(\sigma_f/\sigma_{ne}\). The \(\Delta(T_r)_{\frac{1}{2}}\) (=\((T_r)_{\frac{1}{2}} - (T_{ne})_{\frac{1}{2}}\))-value becomes nearly equal to 60°C when \(\sigma_f/\sigma_{ne} = 0.3\).

When the temperature is sufficiently low or high, the fracture strength \(\sigma_f\) is not so much affected by the magnitude of residual stress (as shown in Fig. 9).

Figure 11 shows the relationships between \(\sigma_f/\sigma_{ne}\) and \(\sigma_f/\sigma_{ne}\) for various testing temperatures. The results in Fig. 11 was calculated in the case of \((T_{ne})_{\frac{1}{2}} = -125°C \pm 150°K\). It is considered that effect of residual stress on brittle fracture appears remarkably in the temperature range of \((T_{ne})_{\frac{1}{2}} +80°C\).

4. Effect of mechanical heterogeneity on brittle fracture strength

It is well known by the previous reports that mechanical heterogeneity arised in the welded joints have effects upon the behaviors of plastic deformation of them subjected to tensile stress. As the yield stress of weld metal and heat-affected zones are different from
it of base metals and changes continuously in them as shown in Fig. 2, the behaviors of plastic deformation in the weld will be different it of homogeneous and isotropic materials. This suggests that the mechanical heterogeneity have effects upon the brittle fracture strength when the basic concept of brittle fracture initiation is employed by COD- or \( \rho^* \)-concept.

Denoting the yield stress by \( \sigma_r (|x|) \), as shown in Fig. 2, the \( \rho^* \)-value ahead of a crack in an infinite plate subjected to uniform tensile stress \( \sigma \) is given as follows from eq. (7).

\[
\frac{\pi}{2} \frac{\sigma}{a} = \int_0^a \frac{\sigma_r (x)}{a^2 - x^2} \frac{dx}{a^2 - x^2}
\]

When the yield stress at the crack tip is denoted by \((\sigma_r)_c\), eq. (27) follows that

\[
(\sigma_r)_c \cos^{-1} \left( \frac{c}{a} \right) = \frac{\pi}{2} - \int_0^c \frac{\sigma_r (x) - (\sigma_r)_c}{a^2 - x^2} \frac{dx}{a^2 - x^2}
\]  

(25)

What is obvious on comparing eq. (25) with eq. (8) is that the two equations agree precisely when

\[
\sigma_r (x) = -|\sigma_r (x) - (\sigma_r)_c|
\]  

(26)

This fact shows that the existence of mechanical heterogeneity is equivalent to it of residual stress in calculation of \( \rho^* \)-value.

Let consider the fracture strength when the interlayers in which the yield stress is different from it of other parts exist across a crack in a infinite plate, as shown in Fig. 12(a). But it is assumed that the fracture toughness is equal in the all region and the effect of mechanical constraint on fracture toughness is neglected. Denoting the ratio of yield stress of material (II) to material (I) by \( \lambda (= \sigma_{r2}/\sigma_{r1}) \), the relationships of external stress \( \sigma \) and the \( \rho^* \)-value are given as follows. eq. (24).

\[
(i) \quad \alpha = c + \rho^* < l_1 \quad \Rightarrow \frac{\pi}{2} \frac{\sigma}{\sigma_{r1}} = \cos^{-1} \frac{c}{a}
\]  

(27)

\[
(ii) \quad l_1 \leq a < l_2 \quad \Rightarrow \frac{\pi}{2} \frac{\sigma}{\sigma_{r1}} = \cos^{-1} \frac{c}{a} + (\lambda - 1)
\]

\[\times \cos^{-1} \frac{l_1}{a}
\]  

(28)

Fig. 12. Effect of mechanical heterogeneity on fracture strength. (b) Effect of hard regions, (c) Effect of soft regions.

(95)
Figure 12 (b) and (c) show the examples of calculation using these equations. ( for examples when \(2c=60 \text{ mm}, \quad l_2-l_1=20 \text{ mm}, \) in Fig. 12 (b) \((\sigma_{\gamma 1}/\sigma_{\gamma 0}) = -3/2, \quad (T_{\alpha})_{\Delta} = -155^\circ \text{C}, \quad D_1 = 115^\circ \text{K}, \quad D_2 = 70^\circ \text{K};\) in Fig. 12 (c) \((\sigma_{\gamma 1}/\sigma_{\gamma 0}) = 2/3, \quad (T_{\alpha})_{\Delta} = -140^\circ \text{C}, \quad D_1 = 70^\circ \text{K}, \quad D_2 = 115^\circ \text{K}). \text{ Figure 12 (b) shows when the hard interlayers in which yield stress is larger than it of base metals exist in the neighbourhood of crack tips, on the other hand Fig. 12 (c) shows when the soft interlayers exist.}\) The fracture strength is affected by the distance \(d_0\) which is the distance from the crack tip to the heterogeneous interlayer. When the hard interlayer exists on the prolongation of the crack, the fracture strength becomes larger than it of homogeneous base metal. Contrary to this, when the soft interlayer exists, the fracture strength becomes smaller.

Subsequently let consider the effect on fracture strength when both residual stress and mechanical heterogeneity exist. Kihara et al. made an experiment on the effect of crack length of the fracture strength of welded joints which received tension load parallel to a weld line. In order to compare with the data of that experiments and discuss, we calculated in the following condition.

1. The residual stress distribution is approximated by step-like form as shown in Fig. 1 and \(\sigma_1/\sigma_{\gamma 0} = 0.8, \quad x_0 = 70 \text{ mm}.\)
2. The redistribution of residual stress due to the existence of a crack is regarded.
3. The yield stress distribution is approximated by the dotted line in Fig. 2 and \(D = 130^\circ \text{K}.\)
4. \((T_{\alpha})_{\Delta} = -120^\circ \text{C}.\)
5. The plate is infinite.

Figure 13 shows the relationship between the fracture strength \(\sigma_1\) and crack length \(2c\). The curve ① in Fig. 13 shows the calculated fracture strength when existence of both residual stress and mechanical heterogeneity are regarded, the curve ② shows it when the existence of residual stress is only regarded and the curve ③ shows it of homogeneous base metal. The marks ① and ② in Fig. 13 show the Kihara’s data of welded joints and base metal respectively. The calculated curves ① and ② show a similar tendency of experimental data.

5. Synthetic consideration on the brittle fracture strength of welded joints.

The brittle fracture strength of welded joints is affected synthetically by the residual stress ahead of crack tip, the mechanical heterogeneity and the fracture toughness at which fracture will initiate. In this paragraph, the effects of residual stress and fracture toughness on the brittle fracture strength are considered synthetically.

In the foregoing paragraph, we adopted the \((T_{\alpha})_{\Delta}\) -value as the parameter which was a typical example of fracture toughness. It is very convenient that transition temperature obtained from the small scale testing, for example V-notch Charpy impact test, can be adopted as that parameter. It is shown by Koshiga et al. that rather well good correlation between the fracture toughness \(\sigma_1\) -value and the fracture transition temperature \(\gamma T_{\alpha}\) may exist. The relation between the \(\alpha\)-value showed in eq. (4) and \(\gamma T_{\alpha}\) -value is given as follows by Koshiga’s report.

\[
\alpha = 1.53 \exp \left[ -\frac{\gamma T_{\alpha} (^\circ \text{C})}{40} \right]
\]  

(30)

From eq. (6-2) and eq. (30), the relationships between \(\gamma T_{\alpha} (^\circ \text{C})\) and \((T_{\alpha})_{\Delta}(^\circ \text{C})\) are given as follows.

\[
(T_{\alpha})_{\Delta}(^\circ \text{C}) = \gamma T_{\alpha}(^\circ \text{C}) - 92.1(3.21 - \log_{10} c)
\]

\[
: (D = 115^\circ \text{K})
\]

\[
(T_{\alpha})_{\Delta}(^\circ \text{C}) = 0.875 \gamma T_{\alpha}(^\circ \text{C}) - 80.6(3.21 - \log_{10} c)
\]

\[
: (D = 70^\circ \text{K})
\]
When \( c = 30 \text{ mm} \):

\[
\begin{align*}
(T_{1o})_H & = qT - 160 \quad (D = 115^\circ\text{K}) \\
(T_{1o})_H & = 0.875 qT - 140 \quad (D = 70^\circ\text{K})
\end{align*}
\] (32)

where \(-220^\circ\text{C} < (T_{1o})_H < -70^\circ\text{C}\).

When the residual stress exists ahead of crack tip, the temperature \((T_{1o})_H\) at which \(\sigma_r/\sigma_{rn} = 0.5\) rises. This elevation of \((T_{1})_H\)-value, or \(\Delta(T_{1})_H = (T_{1o})_H - (T_{1o})_H\) value, is affected by the magnitude of residual stress \(\sigma_r/\sigma_{rn}\) as shown in Fig. 10. Figure 14 shows the relationships between \(\Delta(T_{1})_H\)-value and \(\sigma_r/\sigma_{rn}\)-value for various value of \(\nu T_s\). The relationship between \(\Delta(T_{1})_H\) and \(\sigma_r/\sigma_{rn}\) is not affected very much by \(\nu T_s\)-value in \(-40^\circ\text{C} \leq \nu T_s \leq 40^\circ\text{C}\), is approximated by the following equations when \(c = 30\), \(x_0 = 60\) mm,

\[
\begin{align*}
0 \leq \sigma_r/\sigma_{rn} \leq 0.3 & \quad ; \Delta(T_{1})_H = 200 - \frac{\sigma_r}{\sigma_{rn}} \\
0.3 < \sigma_r/\sigma_{rn} \leq 1.0 & \quad ; \Delta(T_{1})_H = 45 \cdot \frac{\sigma_r}{\sigma_{rn}} + 46.5
\end{align*}
\] (33)

From eqs. (32), (33), the \((T_{1})_H\)-value in welded joints is generally given as follows.

\[
(T_{1})_H = A \cdot (\nu T_s) + B \cdot \frac{\sigma_r}{\sigma_{rn}} + C
\] (34)

Where the constants \(A, B, C\) are values which are determined by the \(D\)-value, the crack length \(2c\), the magnitude of residual stress etc.

When \(D = 115^\circ\text{K}, c = 30\) mm, the constants are given as follows.

\[
\begin{align*}
0 \leq \sigma_r/\sigma_{rn} \leq 0.3 & \quad ; A = 1.0, \quad B = 200^\circ\text{C}, \quad C = -160^\circ\text{C} \\
0.3 < \sigma_r/\sigma_{rn} < 1.0 & \quad ; A = 1.0, \quad B = 45^\circ\text{C}, \quad C = -113.5^\circ\text{C}
\end{align*}
\] (35)

It is seen from eq. (34) that there are many combinations of \(\nu T_s\) and \(\sigma_r/\sigma_{rn}\) in order to obtain the same \((T_{1})_H\)-value in welded joints.

Figure 15 shows the relationships between \(\nu T_s\) and \(\sigma_r/\sigma_{rn}\) obtained using the constants shown in eq. (35) for various values of \((T_{1})_H\). Because Fig. 15 is obtained by the calculations based on many assumptions, there are many problems to be solved to understand this relation quantitatively. It is convenient that many quantitative informations on that subject can be obtained. The calculated results which the \((T_{1})_H\)-value is affected significantly by residual stress in the range of \(\sigma_r/\sigma_{rn} \leq 0.3\) and the \((T_{1})_H\)-value is not affected very much in \(\sigma_r/\sigma_{rn} \geq 0.3\) are very interesting.

![Fig. 14. Relation between residual stress and \(\Delta(T_{1})_H\)-value.](image)

6. Conclusion

The results obtained in this report are summarized as follows:

1. The brittle fracture strength of welded joints is intensely affected by magnitude of residual stress rather than from of its distribution (See Fig. 6). The fracture strength can be approximately evaluated by the calculation neglecting the redistribution of residual stress in the vicinity of notch with exception of the much lower stress level (See Fig. 9).

2. Effects of mechanical heterogeneity on brittle fracture strength are equivalent in calculations to those of residual stress. The brittle fracture strength is elevated by the hard region surrounding the notch and is lowered by the soft region (See Fig. 12).

3. The transition temperature \((T_{1})_H\), at which fracture strength becomes equal to half the yield stress \(\sigma_{rn}\) at room temperature, is changed by the factors such as residual stress, mechanical heterogeneity and fracture toughness near the tip of the notch.
elevation of the temperature $\Delta T_i$ due to residual stress is predominant at lower residual stress level (See Figs. 10, 14).

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