Effect of Residual Stress and Mechanical Heterogeneity on Brittle Fracture Strength of Welded Joints†

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Abstract

Effects of residual stress and mechanical heterogeneity caused due to welding on brittle fracture initiation in welded joints are analytically investigated by using fracture mechanics. The residual stress redistribution due to the existence of a crack is considered. The fracture strength can be approximately evaluated by the calculation neglecting the redistribution of residual stress with exception of the much lower stress level. Effects of mechanical heterogeneity on brittle fracture are equivalent in calculations to those of residual stress. The transition temperature $(T_i)_{1/2}$ *is changed by the factors such as residual stress, mechanical heterogeneity and fracture toughness. The elevation of temperature* $\Delta(T_i)_{1/2}$ *due to residual stress is predominent at lower residual stress level.*

1. Introduction

Behaviors of brittle fracture initiation in welded joints are affected with several factors which are different from factors in homogeneous materials. The main factors of these are welding residual stress and fracture toughness ahead of (welding) crack tip. In addition to them, mechanical heterogeneity caused due to welding is one of factors which play an essential role in case of brittle fracture in welded joints. Effect of residual stress on brittle fracture initiation in welded structure was already discussed in terms of fracture mechanics by Sakai et al[1]. and Kanazawa et. al.[2] However, the investigations have never been done on the effects of mechanical heterogeneity on brittle fracture or composite effects of those factors. Recently, Koshiga et. al[3]. proposed the technique of calculation in which was considered effects of residual stress and fracture toughness in order to discuss effects of stress-relieving heat treatment on brittle fracture initiation in welded structure. The basic concept employed for discussion in the present paper is that concept proposed by Koshiga, and the brittle fracture strength of welded joints having residual stress and mechanical heterogeneity was calculated. Futhermore, the authors try to take a clue in considering composite effects of those factors.

2. Basic conditions for calculating brittle fracture strength

2. 1 Distributions of residual stress

In this paper, the welded plates including a crack (or notch) under tension paralle to a weld line, so called "Wells-Kihara Test" are supposed as shown in **Fig. 1,** the residual stress distributions are approximated by step-like (rectangle) form or parabolic form in which the tensile residual stress is distributed within $2x_0$ mm across the weld, and the rest $(=W-2x_0)$ is uniformly distributed compressive residual stress with its magnitude equal to balance the tension. If the width of welded plates is infinte and the co-ordinate $x=x_0$ at which the residual stress equal to zero is unchangeable, the residual stress distribution $\sigma_r(x)$ in a infinite plate $(W\rightarrow\infty)$ is given as follows. For parabolic form

$$\left.\begin{aligned}\sigma_r(x)&=(\sigma_r)_P\left(1-\frac{x^2}{x_0^2}\right) && (|x|\leqq x_0)\\ \sigma_r(x)&\rightarrow 0 && (|x|>x_0)\end{aligned}\right\}\quad(1)$$

and for step-like (rectangular) form

$$\left.\begin{aligned}\sigma_r(x)&=\sigma_r && (|x|\leqq x_0)\\ \sigma_r(x)&\rightarrow 0 && (|x|>x_0)\end{aligned}\right\}\quad(2)$$

If the crack (or notch) exists in welded joints, the residual stress distribution becomes different from the

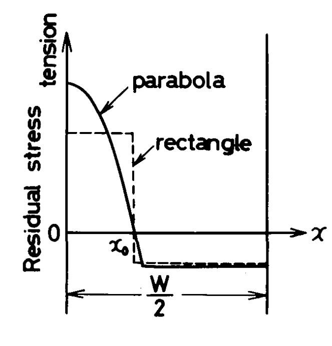

Fig. 1. Simplified representation of residual stresses.

† Received on May 31, 1972

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distributions as shown in **Fig. 1.** The residual stress redistributes when the transverse notch is made after welding. When the welding crack causes at low temperature or the fatigue crack propagates, the residual stress also redistributes. The redistribution of the residual stress in case of existence of crack can be calculated using Dugdale model, as stated afterwords.

2. 2 Mechanical heterogeneity of welded joints

In calculating the brittle fracture strength at low stress level, it is sufficient to consider only heterogeneity of yield stress σ_Y. Parts in vicinity of weld line is given thermal-cycles in welding. The relationships between the yield stress σ_Y after the thermal-cycles and the peak temperature of it were given by Suzuki's[4] experiments. If the peak temperature of parts in vicinity of weld line is known, yield stress σ_Y of that parts is estimated from these experimented data. **Figure 2** shows the relation between the peak temperature T_m (for $q/vh=6500$ cal/cm²) and σ_Y estimated from T_m using Suzuki's data (curve ①) and distance from weld line. In **Fig. 2,** the dotted line is the distribution of σ_Y assumed in the later calculation of fracture strength of welded joints.

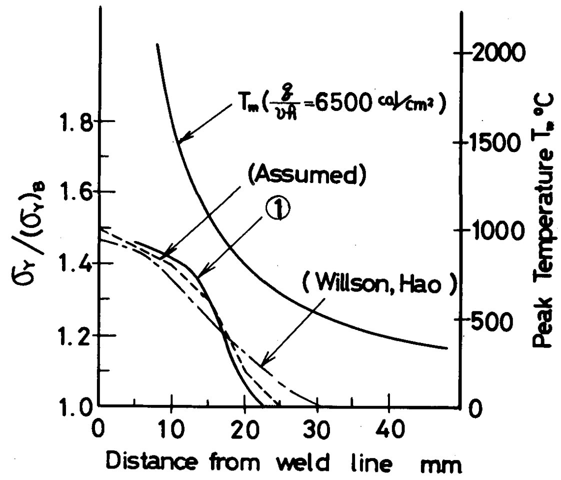

Fig. 2. Distribution of yield stress across a weld line
① : distribution evaluated from reference (4).

2. 3 Relationship between yield stress σ_Y and temperature

It is generally recognized that relation between σ_Y and temperature T(°K) is given as follows.

$$\sigma_Y=\sigma_{Y0}\exp\{D(1/T-1/273)\} \tag{3}$$

Where σ_{Y0} is yield stress at sub-zero and D is a material constant. The D-value is given in **Fig. 3** as a function of yield stress at room temperature ($\fallingdotseq\sigma_{Y0}$) using data published in EW-Committee of Japan Welding Engineering Society.

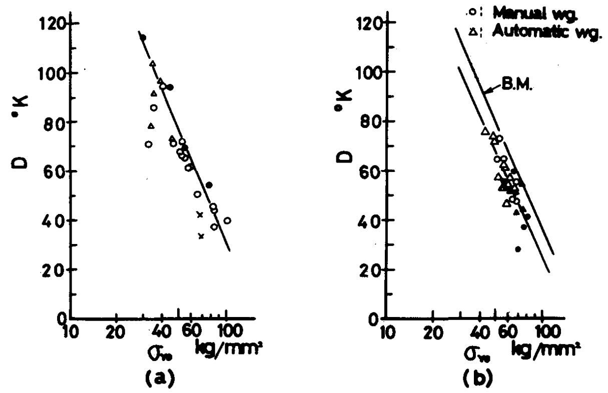

Fig. 3. The D-value representing temperature dependency of yield stress for several kinds of steel and weld metal.

2. 4 Criterion for brittle fracture initiation and relationship between fracture toughness and temperature

The current engineering fracture criteria K_c, Φ_c and ρ_c^+ for brittle fracture initiation are critically reviewed analytically and experimentally by Kanazawa et. al.[7], and it is concluded that the fracture initiation in existence of initial stress ahead of the crack tip due to welding, pre-loading etc. is explained equally well by Φ_c or ρ_c^+ criterion, but not by K_c criterion. In this paper, the basic concept employed expediently for discussion is that brittle fracture should initiate when tensile yield zone ρ^+ formed ahead of a pre-existing crack attains a critical size ρ_c^+ depending on the material, because the authors adopt the experimental equations showed by Koshiga et. al. as the relationship between fracture toughness and temperature.

The relationship between the ρ_c^+-value and temperature was given by Koshiga et. al. as follows

$$\rho_c^+=\alpha(T/100)^5 \tag{4}$$

where α is a material constant.

For the case when the residual stress is equal to zero, the tensile yield zone ρ^+ in a infinite plate subjected to uniform tensile stress σ is determined by

$$\left.\begin{aligned} &\sigma_Y\cos^{-1}\left(\frac{c}{a}\right)=\frac{\pi}{2}\sigma \\ &\text{where} \quad a=c+\rho^+ \end{aligned}\right\} \tag{5}$$

Denoting the temperature at which the fracture stress $\sigma_f=1/2\ \sigma_{Y0}$ by $(T_{i0})_{1/2}$, from eqs, (3), (4), (5), it follows that

$$\frac{\alpha}{c}\left\{\frac{(T_{i0})_{1/2}}{100}\right\}^5\left[\sec\left\{\frac{\pi}{4}\exp\left(\frac{D}{273}-\frac{D}{(T_{i0})_{1/2}}\right)\right\}-1\right]^{-1}=1 \tag{6-1}$$

Figure 4 illustrates the relationships between $(T_{i0})_{1/2}$

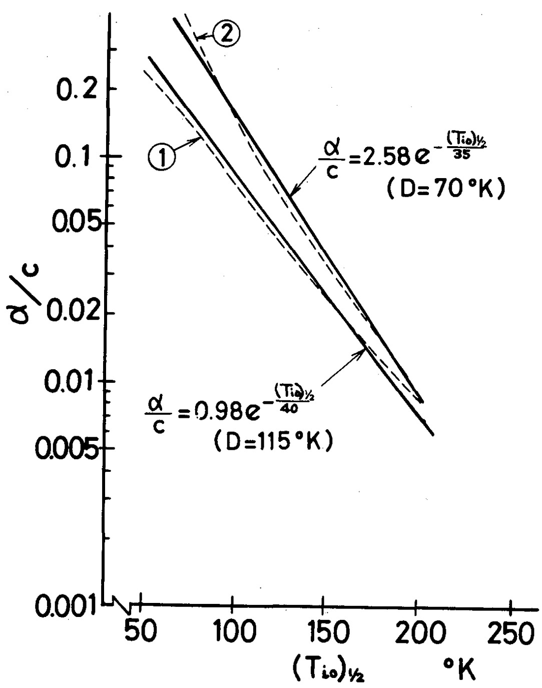

Fig. 4. Relation between α/c and $(T_{i0})_{½}$.

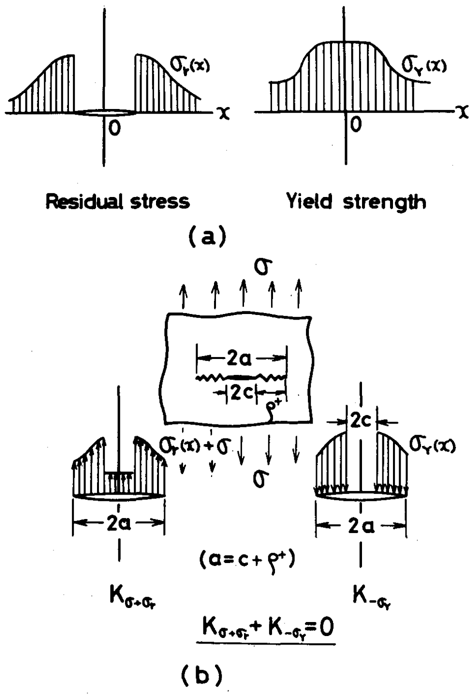

Fig. 5.

and α/c. The dotted curves ①, ② in **Fig. 4** represent the relations when $D=115$ (°K) and $D=70$ (°K) respectively. These relationships approximate to the relationships shown by the solid lines in **Fig. 4** which are given as follows.

$$\left.\begin{aligned}\frac{\alpha}{c}&=0.98\exp\left\{-\frac{(T_{io})_{½}(^\circ\mathrm{K})}{40}\right\}\\&=1.06\times10^{-3}\exp\left\{-\frac{(T_{io})_{½}(^\circ\mathrm{C})}{40}\right\}\ (D=115^\circ\mathrm{K})\\\frac{\alpha}{c}&=2.58\exp\left\{-\frac{(T_{io})_{½}(^\circ\mathrm{K})}{35}\right\}\\&=1.06\times10^{-3}\exp\left\{-\frac{(T_{io})_{½}(^\circ\mathrm{C})}{35}\right\}\ (D=70^\circ\mathrm{K})\end{aligned}\right\}\quad(6\text{-}2)$$

On the following calculations for brittle fracture strength of welded joints having residual stress and machanical heterogeneity, the authors employ the $(T_{io})_{½}$-value as the parameter representing the fracture toughness.

Expending the Dugdale's idea, the ρ^+-value in a infinite plate having residual stress and mechanical heterogeneity can be easily determined. The residual stress $\sigma_r(x)$ and yield stress $\sigma_Y(x)$ (as shown schematically in **Fig. 5 (a)**) are distributed on crack line. In this case, using the Dugdale model (as shown in **Fig. 5 (b)**), the ρ^+-value in a infinite plate subject to uniform tensile stress σ is determined by

$$\begin{aligned}&-\left[\int_{-a}^{-c}+\int_{c}^{a}\right]\frac{\sigma_Y(x)}{\sqrt{a^2-x^2}}dx\\&+\int_{-a}^{a}\frac{\sigma+\sigma_r(x)}{\sqrt{a^2-x^2}}dx=0,\\&a=c+\rho^+\end{aligned}\qquad(7)$$

3. Effects of residual stress on brittle fracture strength

3.1 Effect of form of residual stress distribution

Brittle fracture strength was calculated in the two cases of residual stress distribution, or rectangular type and parabolic type as shown in **Fig. 1.** In this case, the redistribution of residual stress due to the existence of the crack was neglected and it is assumed that materials are homogeneous and isotropic. The residual stress distribution $\sigma_r(x)$ was already given in eqs. (1) and (2), and the ρ^+-value in this case is given by following equations from eq. (7)

$$\left.\begin{aligned}&\sigma_Y\cos^{-1}\left(\frac{c}{a}\right)=\int_0^a\frac{T_0(x)}{\sqrt{a^2-x^2}}dx\\&a=c+\rho^+\\&T_0(x)=\sigma+\sigma_r(x)\end{aligned}\right\}\qquad(8)$$

The relations between ρ^+-value and tensile stress σ in this two type is given as follows.

(1) Parabolic type

(i) $a(=c+\rho^{+}) \leqq x, \sigma+(\sigma_r)_P \leqq \sigma_Y$:

(Where, $(\sigma_r)_P$ is the peak value of tensile residual stress.)

$$\sigma_Y \cos^{-1}\left(\frac{c}{a}\right)=\frac{\pi}{2}\left\{\sigma+(\sigma_r)_P\left(1-\frac{a^2}{2x_0^2}\right)\right\} \quad (9)$$

(ii) $a \leqq x_0,\ \sigma+(\sigma_r)_P > \sigma_Y$

$$\sigma_Y \cos^{-1}\left(\frac{c}{a}\right)=\left\{\sigma_Y-\sigma-(\sigma_r)_P\left(1-\frac{a^2}{2x_0^2}\right)\right\}$$

$$\times \sin^{-1}\left(\frac{x_Y}{a}\right)+\frac{\pi}{2}\left\{\sigma+(\sigma_r)_P\left(1-\frac{a^2}{2x_0^2}\right)\right\}$$

$$-\frac{(\sigma_r)_P}{2}\frac{x_Y}{x_0}\sqrt{\left(\frac{a}{x_0}\right)^2-\left(\frac{x_Y}{x_0}\right)^2} \quad (10)$$

where, $x_Y=x_0\sqrt{1-\frac{\sigma_Y-\sigma}{(\sigma_r)_P}}$

(iii) $a > x_0,\ \sigma+(\sigma_r)_P \leqq \sigma_Y$

$$\sigma_Y \cos^{-1}\left(\frac{c}{a}\right)=\frac{\pi}{2}\ \sigma+(\sigma_r)_P\left(1-\frac{a^2}{2x_0^2}\right)$$

$$\times \sin^{-1}\left(\frac{x_0}{a}\right)-\frac{(\sigma_r)_P}{2}\sqrt{\left(\frac{a}{x_0}\right)^2-1} \quad (11)$$

(iv) $a > x_0,\ \sigma+(\sigma_r)_P > \sigma_Y$

$$\sigma_Y \cos^{-1}\left(\frac{c}{a}\right)=\left\{\sigma_Y-\sigma-(\sigma_r)_P\left(1-\frac{a^2}{2x_0^2}\right)\right\}$$

$$\times \sin^{-1}\left(\frac{x_Y}{a}\right)-\frac{\pi}{4}(\sigma_r)_P\left(\frac{a}{x_0}\right)^2$$

$$+(\sigma+(\sigma_r)_P)\sin^{-1}\left(\frac{x_0}{a}\right)-\frac{(\sigma_r)_P}{2}\frac{x_Y}{x_0}$$

$$\times\sqrt{\left(\frac{a}{x_0}\right)^2-\left(\frac{x_Y}{x_0}\right)^2} \quad (12)$$

(2) Rectangular type

(i) $a \leqq x_0$

$$\sigma=\frac{2}{\pi}\sigma_Y\cos^{-1}\left(\frac{c}{a}\right)-\sigma_r \quad (13)$$

(ii) $a > x_0,\ \sigma+\sigma_r \leqq \sigma_Y$

$$\sigma=\frac{2}{\pi}\left\{\sigma_Y\cos^{-1}\left(\frac{c}{a}\right)+\sigma_r \times \cos^{-1}\left(\frac{x_0}{a}\right)\right\}-\sigma_r \quad (14)$$

(iii) $a > x_0,\ \sigma+\sigma_r > \sigma_Y$

$$\sigma=\frac{\sigma_Y\left\{\cos^{-1}\left(\frac{c}{a}\right)+\cos^{-1}\left(\frac{x_0}{a}\right)-\frac{\pi}{2}\right\}}{\cos^{-1}\left(\frac{x_0}{a}\right)} \quad (15)$$

If the relationships between σ_Y and ρ_c^{+}-value and temperature are given as shown in eq. (3) and eq. (4) respectively, the fracture strength of plates having residual stress can be calculated. **Figure 6** shows the relationships between fracture strength σ_f/σ_{Y0} and temperature when the point at which the sign of the residual stress change is fixed and the sum of magnitude of residual stress is equal each other in two types. For an example, when $c=20$ mm, $x_0=40$ mm and $(T_{i0})_{1/2}=-170$°C. In **Fig.6,** the fracture strengths in the rectangular type and parabolic type of residual stress distribution are compared at $\sigma_r/\sigma_{Y0}=0.25$ and 0.5, where dotted lines show for rectangular type. The fracture strength is not so much affected by the form of residual stress distribution. Therefore, in the following calculation, the rectangular type of residual stress distibution is adopted as the calculation of strength can be done simply.

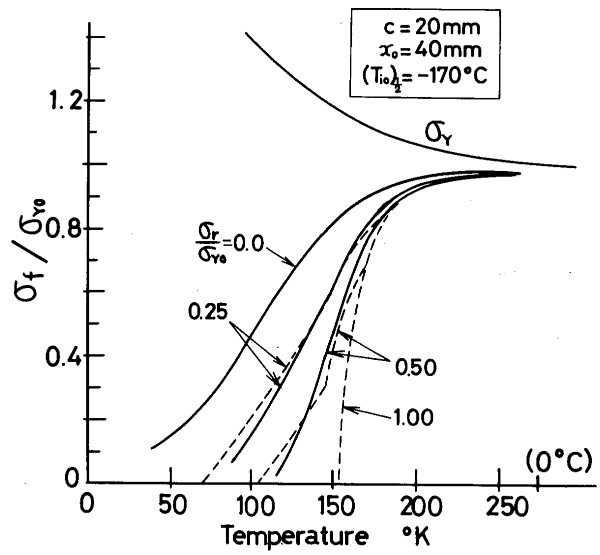

Fig. 6. Comparison of fracture strength for two types of residual stress distribution. (Solid line: Parabolic type, Dotted line; Rectangular type).

3. 2 Brittle fracture strength considering the redistribution of residual stress due to the existence of a crack

When

$$\sigma_Y \cos^{-1}\left(\frac{c}{c+\rho_c^{+}}\right) < \frac{\pi}{2}\sigma_r \quad (16)$$

in eq. (13), the fracture strength σ_f becomes smaller zero. This means that brittle fracture will initiate even if the external load is not given. This inconsistency is caused as it is supposed that the residual stress acts over the crack surface. It is that the redistribution of residual stress due to the existence of a notch (or crack) is considered. The method of calculation considered the redistributed residual stress was also attempted by Kanazawa et. al.[7]

When the transverse notch (crack length is equal to $2c$) is made by saw cut at room temperature ($\approx$0°C) in the infinite plate having residual stress of rectangular type as shown in **Fig. 1.** The plastic zone size ρ_1 formed ahead of crack tip by redistribution of residual stress can be calculated from eq. (8) as follows.

$$\left.\begin{array}{ll} \text{when } a_1 \leqq x_0 & \dfrac{c}{a_1} = \cos\left(\dfrac{\pi}{2}\cdot\dfrac{\sigma_r}{\sigma_{Y0}}\right) \\ \text{when } a_1 > x_0 & \dfrac{c}{a_1} = \cos\left(\dfrac{\sigma_r}{\sigma_{Y0}}\sin^{-1}\dfrac{x_0}{a_1}\right) \end{array}\right\} \quad (17)$$

The residual stress distributions the crack line in this case are also obtained with the application of Dugdale model.

(a) When $a_1 \leqq x_0$;

The results required are obtained by superposition of three states, within linear elasticity theory, or states (I), (II) and (III) as shown in **Fig. 7 (a).** The solution of stress distribution when a force distribution $T_0(x)$ is applied to crack surfaces can be taken in several forms from several sources. A form particularly convenient here, taken from Keer,[8] gives the stress σ_y on the prolongation of the crack as

$$\sigma_y = \frac{2}{\pi}|x|(x^2-a_1^2)^{-1/2}\int_0^{a_1} T_0(t)(a_1^2-t^2)^{1/2} \times (x^2-t^2)^{-1}\,dt \quad (x \geqq a_1) \qquad (18)$$

If $T_0(t)$ is constant regardless of x-coordinate, using the following relation

$$\int\frac{\sqrt{a_1^2-t^2}}{x^2-t^2}dt = \frac{1}{2x}\sqrt{x^2-a_1^2}\left\{\sin^{-1}\frac{a_1^2-xt}{a_1(x-t)} - \sin^{-1}\frac{a_1^2+xt}{a_1(x+t)}\right\} + \sin^{-1}\frac{t}{a_1} \qquad (19)$$

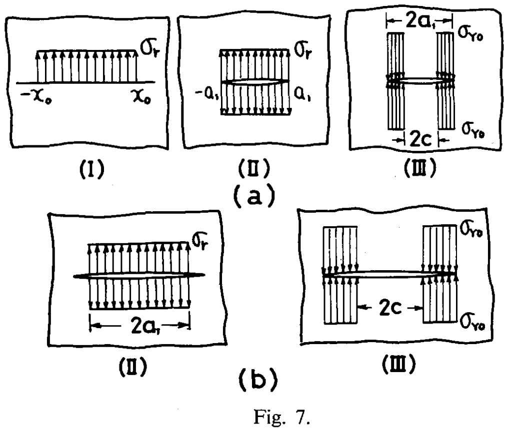

Fig. 7.

the stress distribution in states (II) and (III) are given as follows.

$$(\sigma_y)_2 = \frac{\sigma_r x}{\sqrt{x^2-a_1^2}} - \sigma_r$$

$$(\sigma_y)_3 = -\frac{\sigma_{Y0}}{\pi}\left[\left(\cos^{-1}\frac{c}{a_1}\right)\frac{2x}{\sqrt{x^2-a_1^2}} - \pi - \sin^{-1}\frac{a_1^2-xc}{a_1(x-c)} + \sin^{-1}\frac{a_1^2+xc}{a_1(x+c)}\right] \qquad (20)$$

Therefore, regarding eq. (17), the stress distributions $(\sigma_y)_r$ in this case is given as

$$(\sigma_y)_r = (\sigma_y)_1 + (\sigma_y)_2 + (\sigma_y)_3$$

$$= \begin{cases} \sigma_{Y0} + \dfrac{\sigma_{Y0}}{\pi}\left\{\sin^{-1}\dfrac{a_1^2-xc}{a_1(x-c)} - \sin^{-1}\dfrac{a_1^2+xc}{a_1(x+c)}\right\} & (a_1 \leqq |x| \leqq x_0) \\ \sigma_{Y0} - \sigma_r + \dfrac{\sigma_{Y0}}{\pi}\left\{\sin^{-1}\dfrac{a_1^2-xc}{a_1(x-c)} - \sin^{-1}\dfrac{a_1^2+xc}{a_1(x+c)}\right\} & (x > x_0) \end{cases} \qquad (21)$$

(b) When $a_1 > x_0$;

In this case, the residual stress distribution is also obtained by superposition of states (II) and (III) as shown in **Fig. 7 (b).**

$$(\sigma_y)_2 = \frac{\sigma_r}{\pi}\left[\left\{\sin^{-1}\frac{a_1^2-xx_0}{a_1(x-x_0)} - \sin^{-1}\frac{a_1^2+xx_0}{a_1(x+x_0)}\right\} + \frac{2x}{\sqrt{x^2-a_1^2}}\sin^{-1}\frac{x_0}{a_1}\right] \qquad (22)$$

$$(\sigma_y)_r = \frac{\sigma_r}{\pi}\left\{\sin^{-1}\frac{a_1^2-xx_0}{a_1(x-x_0)} - \sin^{-1}\frac{a_1^2+xx_0}{a_1(x+x_0)}\right\} + \sigma_{Y0} + \frac{\sigma_{Y0}}{\pi}\left\{\sin^{-1}\frac{a_1^2-xc}{a_1(x-c)} - \sin^{-1}\frac{a_1^2+xc}{a_1(x+c)}\right\} \quad (x \geqq a_1) \qquad (23)$$

Figure 8 shows the redistributed residual stress on the prolongation of the crack. For examples, when c=30 mm, x_0=60 mm and σ_r/σ_{Y0}=0.4 or 0.8.

Figure 9 shows one example of the relationships between the fracture strength σ_f of plate having residual stress and testing temperature T for various value of magnitude of residual stress, when c=30 mm, x_0=60 mm, $(T_{i0})_{1/2}$=−125°C and D=115°K. The solid

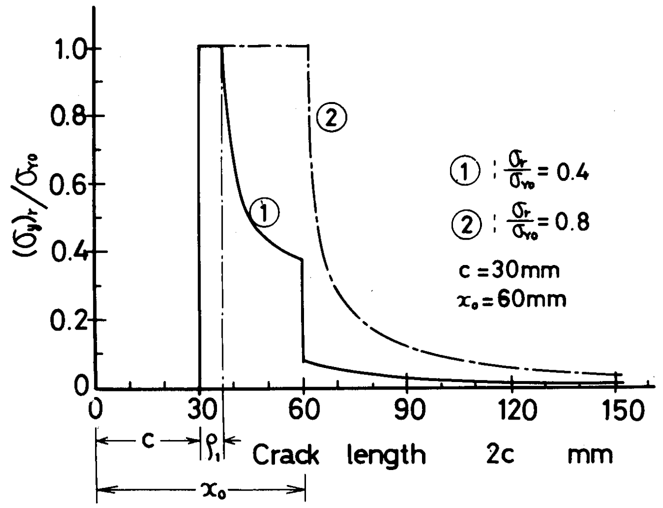

Fig. 8. Redistributed residual stress ahead of crack tip.

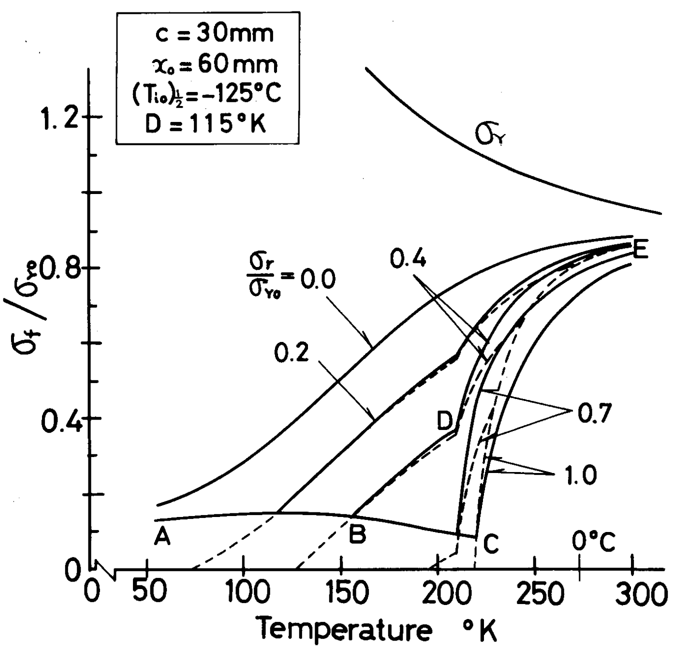

Fig. 9. Effect of redistribution of residual stress on brittle fracture strength.

lines in **Fig. 9** are fracture strength σ_f considering the redistribution of residual stress and the dotted lines in **Fig. 9** are σ_f not considering the redistribution as mentioned in 3. 1. Considering the redistribution, the inconsistency which σ_f becomes smaller than zero in the range of low temperature does not cause, and as temperature become lower, the fracture strength comes close to the strength of plates free of residual stress. The curve ABC in **Fig. 9** shows the brittle fracture strength in case of residual stress on the prolongation of notch is equal to σ_{Y0}. No matter what the magnitude of the residual stress is, the fracture strength σ_f does not become smaller than the value given by the curve ABC. What is obvious on comparing the two curves in **Fig. 9**, or solid curves and dotted curves, is that the two curves almost agree at temperature above it at which the fracture toughness ρ_c^+ is equal to ρ_1, for example temperature at the point B in **Fig. 9** for σ_r/σ_{Y0}=0.4. Effect of redistribution of residual stress due to the existence of crack appears only at low temperature in which $\rho_c^+<\rho_1$. The calculations of the curve ABC and the dotted curves are simple comparatively. For practical usage, the brittle fracture strength of welded plates having residual stress can be obtained easily by the curve ABC and the fracture curve not considering the redistribution.

The temperature $(T_i)_{1/2}$ at which σ_f/σ_{Y0} =0.5 is adopted in order to consider effects of initial magnitude of residual stress σ_r on brittle fracture strength. **Figure 10** shows relationship between $(T_i)_{1/2}$ and σ_r/σ_{Y0} obtained from **Fig. 9.** The change of $(T_i)_{1/2}$-value with residual stress is considerable in the range of small value of residual stress σ_r/σ_{Y0}. The $\Delta(T_i)_{1/2}$ $(=(T_i)_{1/2}-(T_{i0})_{1/2})$-value becomes nearly equal to 60°C when σ_r/σ_{Y0} =0.3.

When the temperature is sufficiently low or high, the fracture strength σ_f is not so much affected by the magnitude of residual stress (as shown in **Fig. 9**). **Figure 11** shows the relationships between σ_f/σ_{Y0} and σ_r/σ_{Y0} for various testing temperatures. The results in **Fig. 11** was calculated in the case of $(T_{i0})_{1/2}$ = −125°C ≒ 150°K. It is considered that effect of residual stress on brittle fracture appears remarkablely in the temperature range of $(T_{i0})_{1/2}$ +80°C.

4. Effect of mechanical heterogeneity on brillte fracture strength

It is well known by the previous reports[9] that mechanical heterogeneity arised in the welded joints have effects upon the behaviors of plastic deformation of them subjected to tensile stress. As the yield stress of weld metal and heat-affected zones are different from

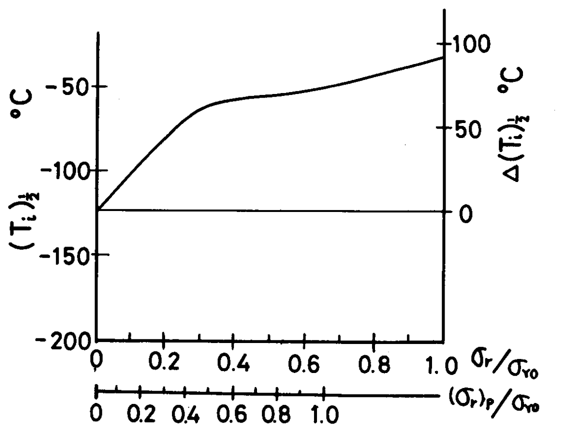

Fig. 10. Relation between resididual stress and $(T_i)_{1/2}$-value.

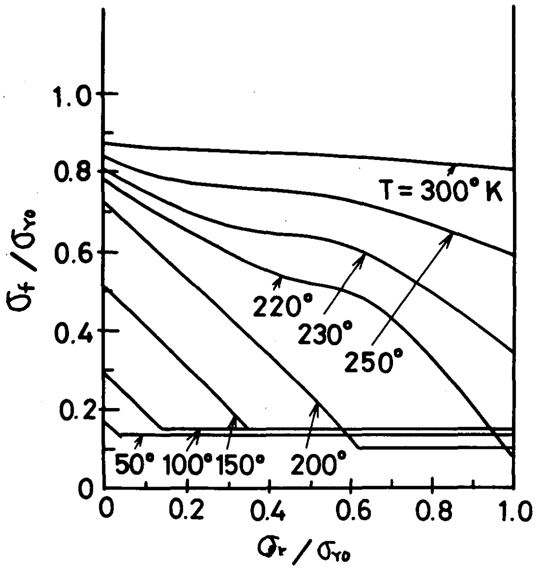

Fig. 11. Relation between fracture strength and residual stress at constant testing temperature.

it of base metals and changes continuously in them as shown in **Fig. 2,** the behaviors of plastic deformation in the weld will be different it of homogeneous and isotropic materials. This suggests that the mechanical heterogeneity have effects upon the brittle fracture strength when the basic concept of brittle fracture initiation is employed by COD- or ρ^+-concept.

Denoting the yield stress by $\sigma_Y(|x|)$, as shown in **Fig. 2,** the ρ^+-value ahead of a crack in a infinite plate subjected to uniform tensile stress σ is given as follows from eq. (7).

$$\frac{\pi}{2}\sigma = \int_c^a \sigma_Y(x)\frac{dx}{\sqrt{a^2-x^2}} \qquad (24)$$

$$a = c + \rho^+$$

When the yield stress at the crack tip is denoted by $(\sigma_Y)_c$, eq. (27) follows that

$$(\sigma_Y)_c \cos^{-1}\left(\frac{c}{a}\right) = \frac{\pi}{2} - \int_c^a \frac{\{\sigma_Y(x)-(\sigma_Y)_c\}}{\sqrt{a^2-x^2}}dx \qquad (25)$$

What is obvious on comparing eq. (25) with eq. (8) is that the two equations agree precisely when

$$\sigma_r(x) = -\{\sigma_Y(x)-(\sigma_Y)_c\} \qquad (26)$$

This fact shows that the existence of mechanical heterogeneity is equivalent to it of residual stress in calculation of ρ^+-value.

Let consider the fracture strength when the interlayers in which the yield stress is different from it of other parts exist across a crack in a infinite plate, as shown in **Fig. 12(a).** But it is assumed that the fracture toughness is equal in the all region and the effect

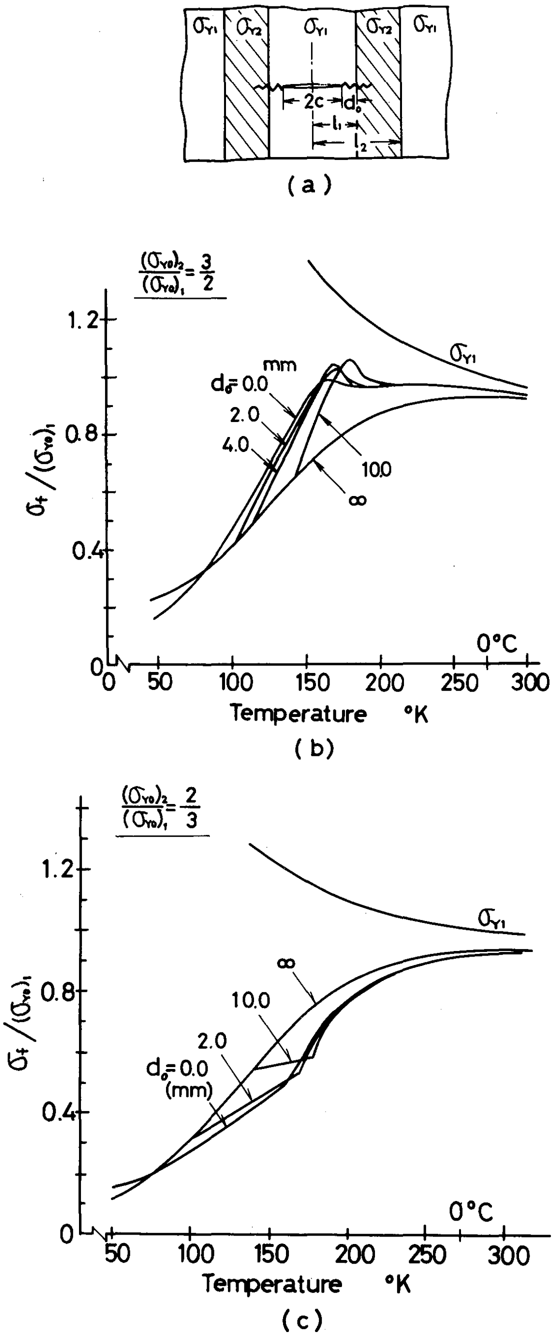

Fig. 12. Effect of mechanical heterogeneity on fracture strength. (b); Effect of hard regions, (c); Effect of soft regions.

of mechanical constraint on fracture toughness is neglected. Denoting the ratio of yield stress of material (II) to material (I) by $\lambda(=\sigma_{Y2}/\sigma_{Y1})$, the relationships of of external stress σ and the ρ^+-value are given as follows. eq. (24).

$$\text{(i)}\quad a(=c+\rho^+) < l_1,\quad \frac{\pi}{2}\frac{\sigma}{\sigma_{Y1}} = \cos^{-1}\frac{c}{a} \qquad (27)$$

$$\text{(ii)}\quad l_1 \leqq a < l_2,\quad \frac{\pi}{2}\frac{\sigma}{\sigma_{Y1}} = \cos^{-1}\frac{c}{a} + (\lambda-1) \times \cos^{-1}\frac{l_1}{a} \qquad (28)$$

(ii) $l_2 \leqq a$, $$\frac{\pi}{2}\frac{\sigma}{\sigma_{Y1}} = \cos^{-1}\frac{c}{a} + (\lambda - 1) \times \left\{ \cos^{-1}\frac{l_1}{a} - \cos^{-1}\frac{l_2}{a} \right\} \qquad (29)$$

Figure 12 (b) and **(c)** show the examples of calculation using these equations. (for examples when $2c=60$ mm, $l_2-l_1=20$ mm, in **Fig. 12 (b)** $(\sigma_{Y0})_2/(\sigma_{Y0})_1=3/2$, $(T_{i0})_{1/2}=-155°C$, $D_1=115°K$, $D_2=70°K$; in **Fig. 12 (c)** $(\sigma_{Y0})_2/(\sigma_{Y0})_1=2/3$, $(T_{i0})_{1/2}=-140°C$, $D_1=70°K$, $D_2=115°K$). **Figure 12 (b)** shows when the hard interlayers in which yield stress is larger than it of base metals exist in the neighbourhood of crack tips, on the other hand **Fig. 12 (c)** shows when the soft interlayers exist. The fracture strength is affected by the distance d_0 which is the distance from the crack tip to the heterogeneous interlayer. When the hard interlayer exists on the prolongation of the crack, the fracture strength becomes larger than it of homogeneous base metal. Contrary to this, when the soft interlayer exists, the fracture strength becomes smaller.

Subsequently let consider the effect on fracture strength when both residual stress and mechanical heterogeneity exist. Kihara et. al. made an experiment on the effect of crack length of the fracture strength of welded joints which recived tension load parallel to a weld line. In order to compare with the data of that experiments and discuss, we calculated in the following condition.

(1) The residual stress distribution is approximated by step-like form as shown in **Fig. 1** and $\sigma_r/\sigma_{Y0}=0.8$, $x_0=70$ mm.
(2) The redistribution of residual stress due to the existence of a crack is regarded.
(3) The yield stress distribution is approximated by the dotted line in **Fig. 2** and $D=130°K$.
(4) $(T_{i0})_{1/2}=-120°C$.
(5) The plate is infinite.

Figure 13 shows the relationship between the fracture strength σ_f and crack length $2c$. The curve ① in **Fig. 13** shows the calculated fracture strength when existence of both residual stress and mechanical heterogeneity are regarded, the curve ① shows it when the existence of residual stress is only regarded and the curve ② shows it of homogeneous base metal. The marks ○ and □ in **Fig. 13** show the Kihara's data of welded joints and base metal respectively. The calculated curves ① and ② show a similar tendency of experimental data.

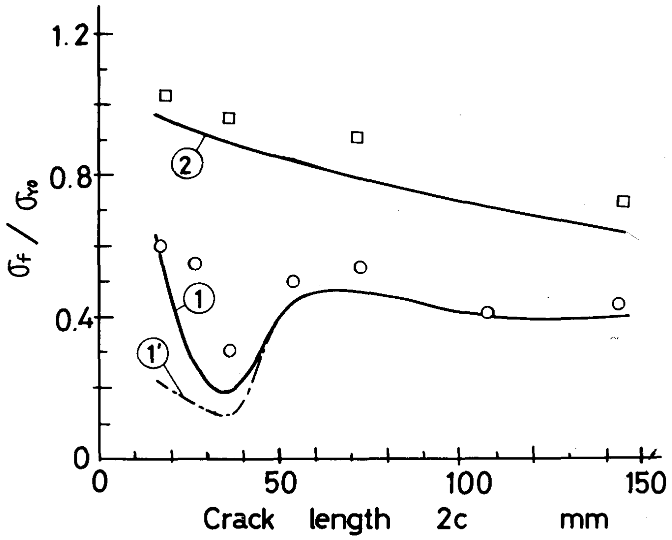

Fig. 13. Effect of crack length on fracture strength.

5. Synthetic consideration on the brittle fracture strength of welded joints.

The brittle fracture strength of welded joints is affected synthetically by the residual stress ahead of crack tip, the mechanical heterogeneity and the fracture toughness at which fracture will initiate. In this parapraph, the effects of residual stress and fracture toughness on the brittle fracture strength are considered synthetically.

In the foregoing parapraph, we adopted the $(T_{i0})_{1/2}$-value as the parameter which was a typical example of fracture toughness. It is very convenient that transition temperature obtained from the small scale testing, for example V-notch Charpy impact test, can be adopted as that parameter. It is shown by Koshiga et. al. that the relatively good correlation between the fracture toughness ρ_c^+-value and the fracture transition temperature ${}_vT_s$ may exist. The relation between the α-value showed in eq. (4) and ${}_vT_s$-value is given as follows by Koshiga's report.

$$\alpha = 1.53 \exp\left\{ -\frac{{}_vT_s\,(°C)}{40} \right\} \qquad (30)$$

From eq. (6-2) and eq. (30), the relationships between ${}_vT_s$ (°C) and $(T_{i0})_{1/2}$(°C) are given as follows.

$$\left.\begin{aligned} (T_{i0})_{1/2}(°C) &= {}_vT_s(°C) - 92.1(3.21 - \log_{10} c) \\ &\qquad : (D=115°K) \\ (T_{i0})_{1/2}(°C) &= 0.875\ {}_vT_s(°C) - 80.6(3.21 - \log_{10} c) \\ &\qquad : (D=70°K) \end{aligned}\right\} \qquad (31)$$

When $c=30$ mm:

$$\left.\begin{array}{ll} (T_{io})_{½} \doteqdot {}_vT_S - 160 & (D=115°\mathrm{K}) \\ (T_{io})_{½} \doteqdot 0.875\ {}_vT_S - 140 & (D=70°\mathrm{K}) \end{array}\right\} \quad (32)$$

where $-220°\mathrm{C} < (T_{i0})_{½} < -70°\mathrm{C}$.

When the residual stress exists ahead of crack tip, the temperature $(T_i)_{½}$ at which $\sigma_f/\sigma_{Y0}=0.5$ rises. This elevation of $(T\ )_{½}$-value, or $\Delta(T_i\ )_{½}\,(=(T_i)_{½}-(T_{i0})_{½})$-value, is affected by the magnitude of residual stress σ_r/σ_{Y0} as shown in **Fig. 10.** **Figure 14** shows the relationships between $\Delta(T_i)_{½}$-value and σ_r/σ_{Y0}-value for various value of ${}_vT_S$. The relationship between $\Delta(T_i)_{½}$ and σ_r/σ_{Y0} is not affected very much by ${}_vT_S$-value in $-40°\mathrm{C} \leqq {}_vT_S \leqq 40°\mathrm{C}$, is approximated by the following equations when $c=30$, $x_0=60$ mm,

$$\left.\begin{array}{ll} 0 \leqq \sigma_r/\sigma_{Y0} \leqq 0.3 & ;\ \Delta(T_i)_{½} = 200 \cdot \dfrac{\sigma_r}{\sigma_{Y0}} \\ 0.3 < \sigma_r/\sigma_{Y0} \leqq 1.0 & ;\ \Delta(T_i)_{½} = 45 \cdot \dfrac{\sigma_r}{\sigma_{Y0}} + 46.5 \end{array}\right\} \quad (33)$$

From eqs. (32), (33), the $(T_i)_{½}$-value in welded joints is generally given as follows.

$$(T_i)_{½} = A \cdot ({}_vT_S) + B \cdot \frac{\sigma_r}{\sigma_{Y0}} + C \quad (34)$$

Where the constants A, B, C are values which are determined by the D-value, the crack length $2c$, the magnitude of residual stress etc..

When $D=115°\mathrm{K}$, $c=30$ mm, the constants are given as follows.

$$\left.\begin{array}{ll} 0 \leqq \sigma_r/\sigma_{Y0} \leqq 0.3 & ;\ A=1.0,\ B=200°\mathrm{C},\ C=-160°\mathrm{C} \\ 0.3 < \sigma_r/\sigma_{Y0} < 1.0 & ;\ A=1.0,\ B=45°\mathrm{C},\ C=-113.5°\mathrm{C} \end{array}\right\} \quad (35)$$

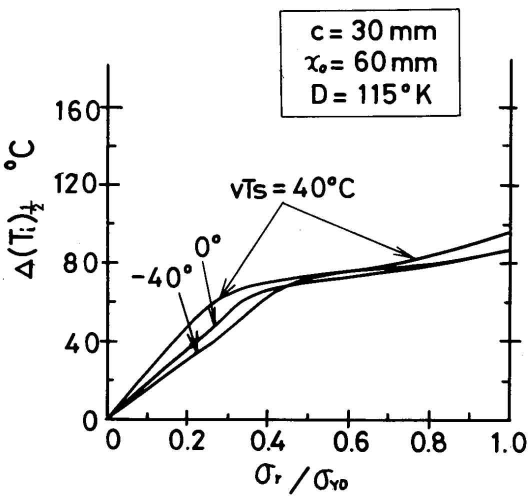

Fig. 14. Relation between residual stress and $\Delta(T_i)_{½}$-value.

It is seen from eq. (34) that there are many combinations of ${}_vT_S$ and σ_r/σ_{Y0} in order to obtaine the same $(T_i)_{½}$-value in welded joints.

Figure 15 shows the relationships between ${}_vT_S$ and σ_r/σ_{Y0} obtained using the constants shown in eq. (35) for various values of $(T_i)_{½}$. Because **Fig. 15** is obtained by the calculations based on many assumptions, there are many problems to be solved to understand this relation quantitatively. It is convenient that many quantiative informations on that subject can be obtained. The calculated results which the $(T_i)_{½}$-value is affected significantly by residual stress in the range of $\sigma_r/\sigma_{Y0} \lesssim 0.3$ and the $(T_i)_{½}$-value is not affected very much in $\sigma_r/\sigma_{Y0} \gtrsim 0.3$ are very interesting.

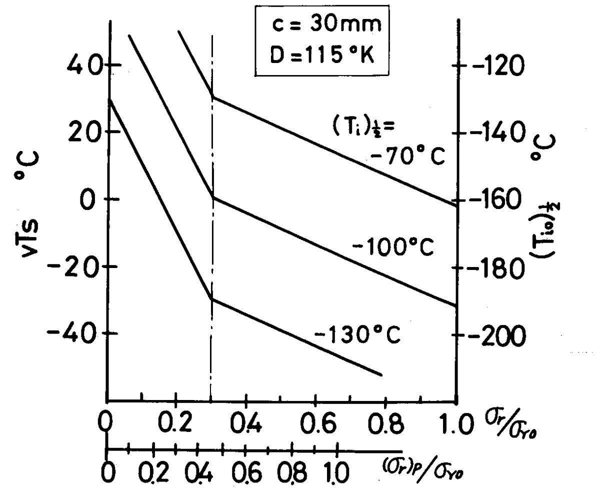

Fig. 15. Relation between ${}_vT_S$ and residual stress for different values of $(T_i)_{½}$

6. Conclusion

The results obtained in this report are summarized as follows:

(1) The brittle fracture strength of welded joints is intensely affected by magnitude of residual stress rather than from of its distribution (See **Fig. 6**). The fracture strength can be approximately evaluated by the calculation neglecting the redistribution of residual stress in the vicinity of notch with exception of the much lower stress level (See **Fig. 9**).

(2) Effects of mechanical heterogeneity on brittle fracture strength are equivalent in calculations to those of residual stress. The brittle fracture strength is elevated by the hard region surrounding the notch and is lowered by the soft region (See **Fig. 12**).

(3) The transition temperature $(T_i)_{½}$, at which fracture strength becomes equal to half the yield stress σ_{Y0} at room temperature, is changed by the factors such as residual stress, mechanical heterogeneity and fracture toughness near the tip of the notch. The

elevation of the temperature $\Delta(T_i)_{1/2}$ due to residual stress is predominent at lower residual stress level (See **Figs. 10, 14**).

Acknowledgment

The authors gratefully acknowledge precious advice given by Prof. Dr. Engi., H. Kihara, Director of Welding Recearch Institute of Osaka University and Prof. Dr. Engi., T. Kanazawa of Tokyo University. Thanks are due also to the member of the Welding Research Committee of the Society of Naval Architects of Japan, for their discussion and remarks given in the course of this study.

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