Trans. JWRI, Vol. 22 (1993), No.1

Dominant Point Detection by Regularization†

Katsunori INOUE* and Wonchan SEO**

Abstract

An algorithm for detecting dominant points on a digital closed curve is presented. It uses a technique that called regularization in which polygonal approximation is achieved by minimizing a criterion function proposed in this paper. The regularized criterion function is defined as the weighted conjunction of the fitness to the given contour and the distinctness of the model. The dominant points of given contour are controlled in various degrees of approximation by changing the regularization factor, and an iterative method is presented for minimizing the regularized criterion function.

The proposed algorithm is compared with those of several other dominant point detection algorithms in terms of the approximation errors and the number of detected dominant points using a number of examples.

KEY WORDS: (Dominant Points) (Polygonal Approximation) (Regularization) (Digital Curve) (Support Region) (Chain Code) (Curvature) (Corner) (Angle)

1. Introduction

Dominant points are the high curvature points along a digital curve that have important shape attributes. This concept of dominant points has been applied in shape recognition[1)-3)], motion estimation[4)], and coding[5)].

Many algorithms[6)-16)] have been proposed for detecting dominant points. In general, there are two approaches in those algorithms. One is to find the dominant points directly through angle or corner detection schemes[6)-14)], and the other approach is to obtain a piecewise linear polygonal approximation of the digital curve subject to certain constraints on the goodness of fit[15),16)]. In polygonal approximation, dominant points are the interesting points of any two adjacent line segments. These points are also known as the vertices or break points of the closed curve(polygonal).

Most dominant point detection algorithms(either angle detection or polygonal approximation), except for that of references (6) and (11), require one or more input parameters. These parameters usually represent the support region for the measurement of local properties at each point on the curve. They are selected based on the level of detail represented by the digital curve. In general, it is difficult to find a set of parameters suitable for curve that consists of multiple size features. Too large support region will smooth out the fine features of a curve, whereas a small support region will generate a large number of redundant dominant points. This is a fundamental problem of scale because the features describing the shape of a curve vary enormously in size and extent, and there is seldom a well-defined basis for choosing an appropriate scale(or smoothing) parameter correspond to a particular feature size. This problem can be avoided by a non-parameteric algorithm in some degree. However, in the cases that the change of scales and various degrees of polygonal approximation are required, the non-parametric algorithms do not satisfy this expectation.

In this paper, a new dominant point detection algorithm by regularization[18)] is introduced. This algorithm detects the dominant points by estimating the regularized criterion function proposed here with. The criterion function is defined as the weighted conjunction of two terms which are the fitness of the approximating polygon to the given contour and the distinctness of the model. Dominant point detection is achieved by minimizing the regularized criterion function. The degree of approximation can be controlled by the regularization factor, and the semi-optimal solution is obtained by a simple iterative method.

More details are given in the following sections. The properties and problems of various dominant point detection algorithms are briefly reviewed in next section. In section 3, regularization for detecting dominant points is discussed, and a new dominant detection algorithm is

† Received on July 30,1993

* Professor
** Graduate Student of Osaka University

Transactions of JWRI is published by Welding Research Institute, Osaka University, Ibaraki, Osaka 567, Japan

proposed. In section 4, the proposed algorithm is compared with the four algorithms using four digital closed curves with respect to 1) the error introduced in approximating the closed curve by the polygon generated by joining the dominant points, and 2) the number of detected dominant points. In section 5, conclusions are presented.

2. Dominant Point Detection Algorithms

In this section, the properties and problems of various dominant point detection algorithms are briefly reviewed. Detailed literature reviews of various dominant point detection algorithms can be found in references (6) and (11).

There are two major problems with dominant point detection on digital curves. One is the precise definition of discrete curvature, and the other is the determination of the support region for the computation of the curvature. In the real Euclidean plane, curvature is defined as the rate of change of slop as a function of arc length. For the curve of $y = f(x)$, this can be expressed in terms of derivatives as

$$\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}. \qquad (1)$$

For a digital curve, if the discrete curvature is defined by simply replacing the derivatives in Eq.(1) by first differences, there is a problem that small changes in slope are impossible, since successive slope angles on the digital curve can be differ only by a multiple of 45°. This difficulty is overcome in various dominant point detection algorithms by using $k > 1$ differences in Rosenfeld-Weszka algorithm[9], rather than by simply using the first differences($k = 1$). In other words, a smoothed version of discrete curvature is measured, and k can be viewed as a smoothing parameter. Another way to overcome this problem is to use higher order chain codes[17] where the directions are quantized in more than eight steps.

A number of other authors also concentrated on techniques involving direct measurements of discrete curvature or functions of discrete curvature. These measurements are used in various dominant point detection algorithms to detect dominant points in the final steps of nonmaxima suppression. Hence, the k cosine measures in Rosenfeld-Johnstone[8] and Rosenfeld-Weszka[9], and the cornerity measure in Freeman-Davis algorithms[13] are all different types of measures of significance.

Since an approximate smoothed version of discrete curvature is measured in various algorithms, an appropriate smoothing factor(for example, m in the case of the Rosenfeld-Johnston algorithm[8]) has to be selected based on the level of detail represented in the digital curve in order to measure the discrete curvature to a certain degree of accuracy. This smoothing factor is in fact a function of the support region which is used to compute the measure of significance. In general, the higher the level of detail, the smaller the smoothing factor to be selected. A major difficulty arises when a digital curve has features at various levels of detail.

It has been remarked in (8) that the user of these procedures has to select a smoothing factor appropriate to the class of curves to be processed. The difficulty of selecting a suitable smoothing factor was avoided in some degree by the Teh-Chin algorithm[6]. However, the non-parametric dominant point detection algorithm has a problem, that is only one solution on the given contour can be obtained. It does not satisfy the expectation that the various degrees of approximating polygon or different number of dominant points are demanded. At these points, a new dominant point detection algorithm is presented in next section.

3. Dominant Point Detection by Regularization

Regularization[18] proposes to solve ill-posed problems by restricting the space of acceptable solutions by imposing additional constrains. One of the formulation with regularization is as follows. The regularization of finding a solution z from the input data y such that $Az = y$ requires the choice of norms $\|\cdot\|$ and a stabilizing functional $\|Pz\|$ One method that can be applied is to find z that minimizes the cost functional of

$$\|Az - y\|^2 + \lambda \|Pz\|^2 \qquad (2)$$

, where λ is a so-called regularization parameter. The first term expresses the closeness of the solution to the input data, the second expresses the degree of regularization, or the additional constraints, and the factor λ controls the compromise between these two terms.

In the dominant point detection problem, the approximating polygon model should be fitted closely without redundant dominant points to a given digital curve. The following criterion function is proposed for the dominant point detection as a concrete form considering these matters.

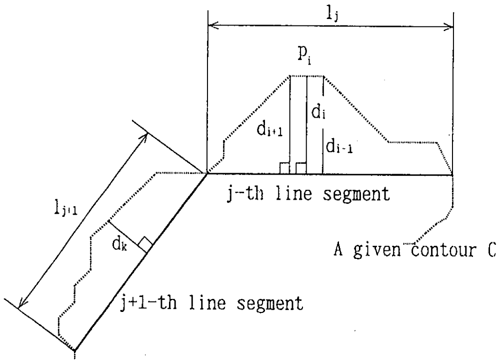

Fig. 1 Measures constructing the criterion function

$$Fc = \sum_{i=1}^{n} d_i^2 + \lambda \frac{\left(\sum_{j=1}^{n_d} l_j\right)^2}{\sum_{j=1}^{n_d} l_j^2} \qquad (3)$$

, where d_i is the perpendicular distance between a point p_i on a given contour C and the line segment of approximating polygon corresponding to the point p_i (See **Fig. 1**). l_j is the length of each line segment of approximating polygon. n and n_d are the total number of given contour points and the number of detected dominant points, respectively.

The regularization factor λ controls the weight of conjunction between two terms. The first term expresses the degree of fitness of the approximating polygon to the given contour. The second term is to obtain the simple and adaptable model. The approximating polygon which has the smaller number of dominant points and the wide variety in the length of the line segments in case the same number of dominant points is estimated as the excellent model. It is also sufficient to describe the fine features of given contour at various levels of detail.

These two terms of fitness to the given contour and distinctness of the model have a confronted property with each other. That is, the approximating polygon with a high fitness becomes to be low distinctness of the model itself. The first term is weighted more significantly with the smaller value of λ, and the fitness of the approximating polygon to the given contour becomes to be larger, but the shape of the obtained polygon model becomes to be bad. When the large value of λ, on the contrary, the second term is weighted, and the approximating polygon becomes a good shape. By changing the regularization factor appropriate to the class of contours to be processed, the degree of polygonal approximation on the given contour is controlled, and the suitable number of dominant points is obtained by minimizing the criterion function.

Iterative Method to Minimize criterion Function

An iterative method is implemented, it allows an attention point to converge on the given contour with low cost functional. This method does not guarantee an optimum solution but the semi-optimum solution may be obtained in the limited calculation time. The method works as follows.

Step 1. Put the candidate points on the given contour with an interval of one point.

Step 2. Compute the criterion function using the current location of the candidate points.

Step 3. Evaluate the criterion function sequentially at the location of the attention point obtained by eliminating or moving the attention point from the former to the latter.

Step 4. Fix the attention point on that position if the value of criterion function is lower than the calculated value of Step 2.

Step 5. Go to Step 6 when the value of criterion function is not changed. Otherwise go to Step 2.

Step 6. Retain all of the candidate points as the dominant points.

4. Experimental Results

In this section, the proposed algorithm is compared with the four algorithms. These include 1) the angle detection procedure by Rosenfeld and Johnston[8], 2) the improved angle detection procedure by Rosenfeld and Weszka[9], 3) the corner finding algorithm by Freeman and Davis[13], and 4) the determination of support region by Teh-Chin algorithm[6].

Four closed curves were chosen to compare the algorithms with respect to 1) the approximation errors, and 2) the number of detected dominant points. According to the chain codes provided in reference (6), the original contours are shown in **Figs.2(a)**, **3(a)**, **4(a)** and **5(a)**, namely CHROMOSOME, LEAF, FIGURE-8 and SEMICIR, respectively. Chain codes of above contours are provided in the appendix.

The results obtained by applying the five algorithms to Fig. 2(a) are shown in **Figs.2(b)-(i)**, which are also summarized in **Table 1**. For the comparison, the results by the regularized algorithm used four sets of regularization factors are shown. The processing of, Figs.2(a)-4(a) requires one set of input parameter each, these are listed in their corresponding tables. The processing of Fig.5(a) uses two sets of input parameters to demonstrate the problem of varying feature size. **Table 1-4** show the results corresponding to Figs.2-5. Several observations are drawn from the results.

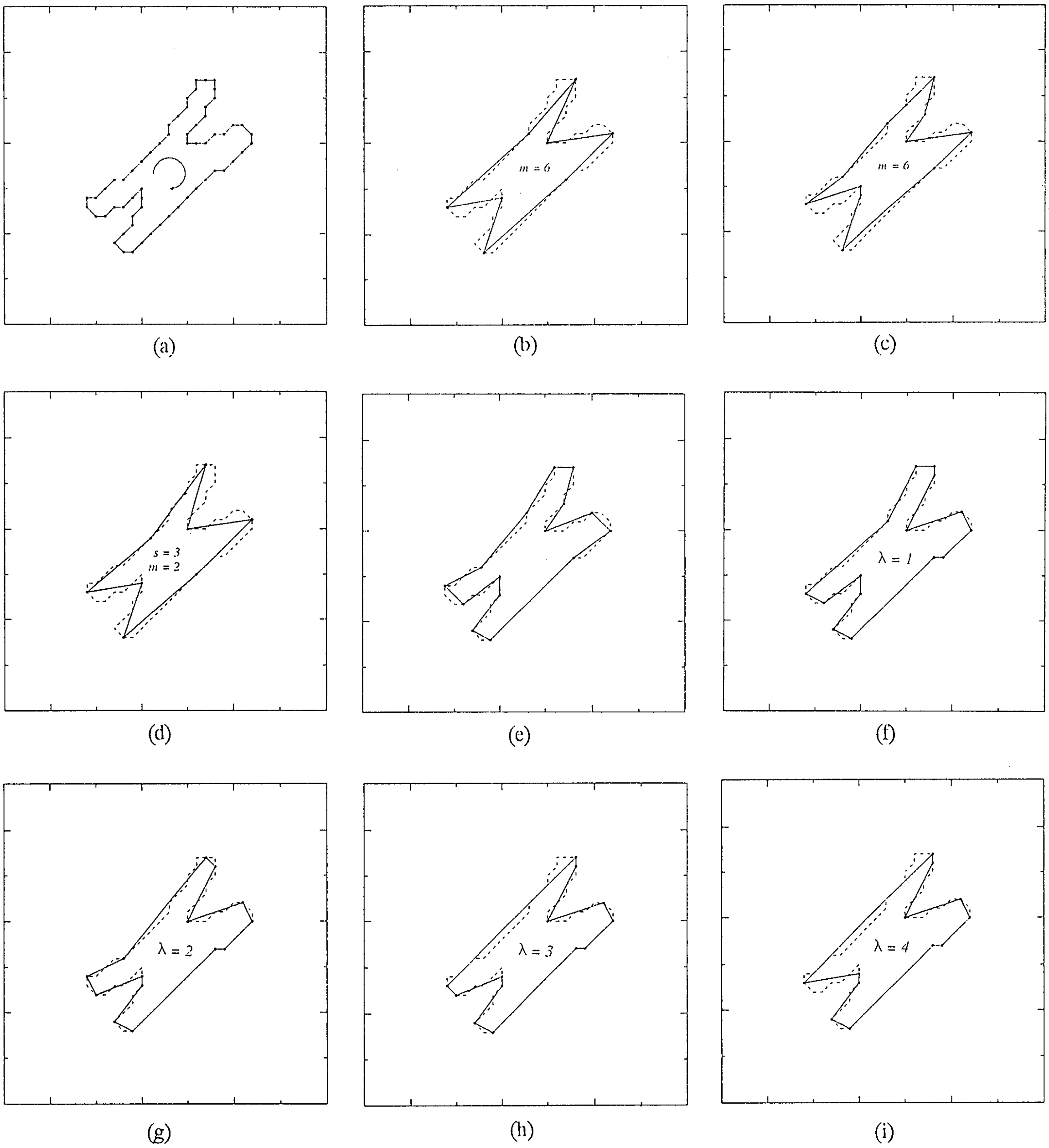

Fig. 2 Results of obtaining dominant points of the CHROMOSOME contour: (a) original contour, (b) Rosenfeld-Johnston algorithm, (c) Rosenfeld-Weszka algorithm, (d) Freeman-Davis algorithm, (e) Teh-Chin algorithm, (f) proposed regularization algorithm($\lambda = 1$), (g) $\lambda = 2$, (h) $\lambda = 3$ and (i) $\lambda = 4$.

The polygon drawn by joining the adjacent dominant point is used to approximate the shape of the object. A quantitative measure of the quality of the detected dominant points is used, defined as the piecewise error between the digital curve and the approximating polygon. The error between a point p_i of a given contour C and the approximating polygon is defined as the perpendicular distance of the point to the approximating line segment. This error is denoted by E_i. Two error norms between C and its approximating polygon defined below are used:

(1) Integral square error,

$$E_2 = \sum_{i=1}^{n_d} d_i^2 , \tag{4}$$

(2) Maximum error,

$$E_{\max} = \max d_i . \tag{5}$$

1) Approximation Errors: The regularized algorithm consistently outperforms the rest of the algorithms in terms of both approximation error measures. The Teh-Chin algorithm results in very small errors. The approximation errors obviously depend on the number of detected dominant points, n_d. The results of the proposed

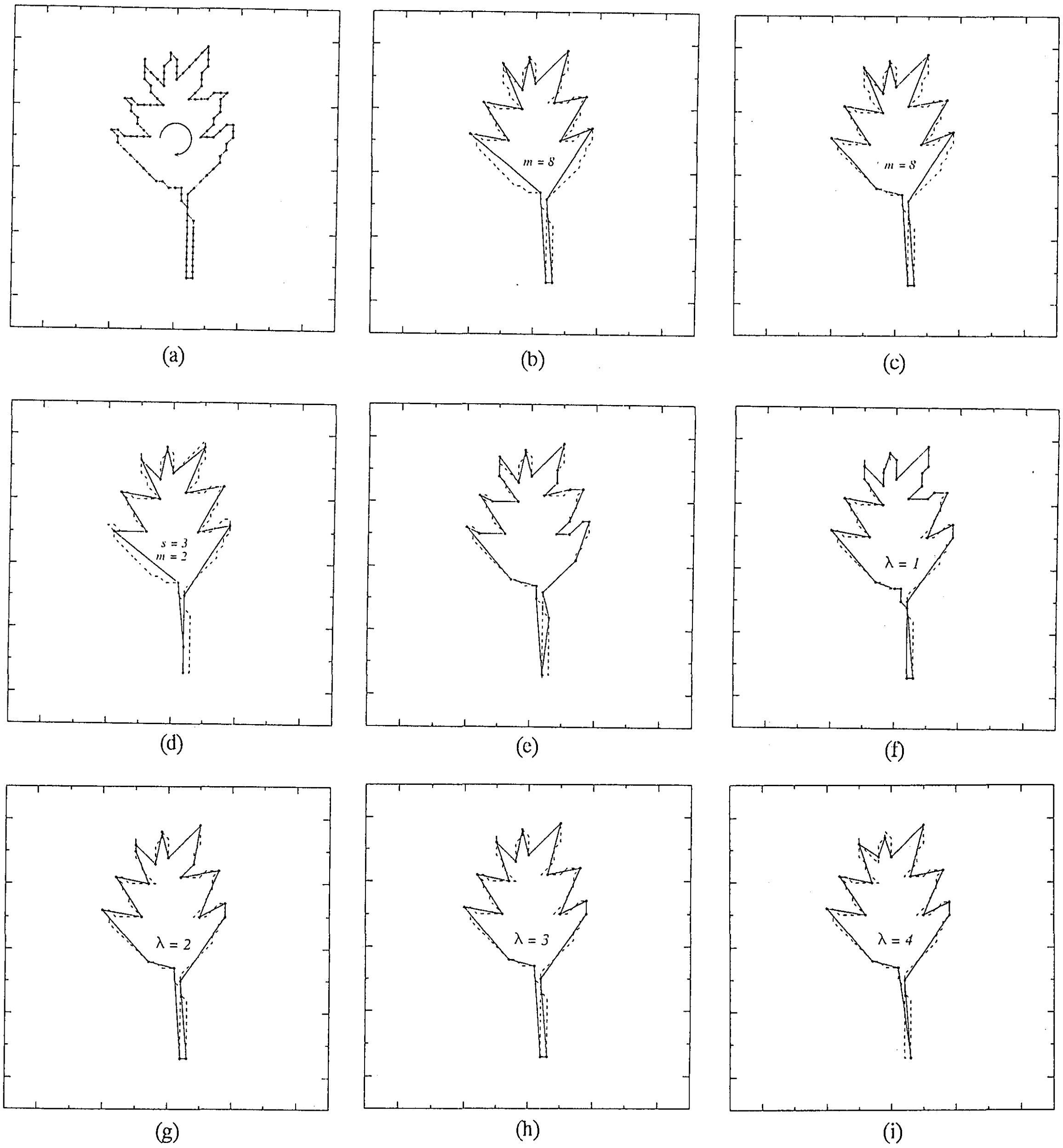

Fig. 3 Results of obtaining dominant points of the LEAF contour: (a) original contour, (b) Rosenfeld-Johnston algorithm, (c) Rosenfeld-Weszka algorithm, (d)Freeman-Davis algorithm, (e) Teh-Chin algorithm, (f) proposed regularization algorithm(λ = 1), (g) λ = 2, (h) λ = 3 and (i) λ = 4.

algorithm show the smallest errors at the similar number of dominant points, and it is confirmed that various degrees of polygonal approximation on the given contour is controlled accurately by changing the regularization factor. The data compression ratios of the total number of input contour points to the number of detected dominant points, n/n_d, are tabulated in the tables.

2) Number of Detected Dominant Points: The number of dominant points detected by the four algorithms except Teh-Chin algorithm depends on the input parameters. In general, the larger the support region introduced by the input parameters, the lower the number of dominant points detected, and the errors became larger. The regularized algorithm proposed in this paper has the smallest number of dominant points at the same errors. The regularized algorithm outperforms the rest of algorithms in terms of both approximation error measures and the number of detected dominant points.

5. Conclusion

In two-dimensional shape representation, dominant points are an important attribute of shape. The locations of the detected dominant points must be accurate and the number of dominant points must provide a good

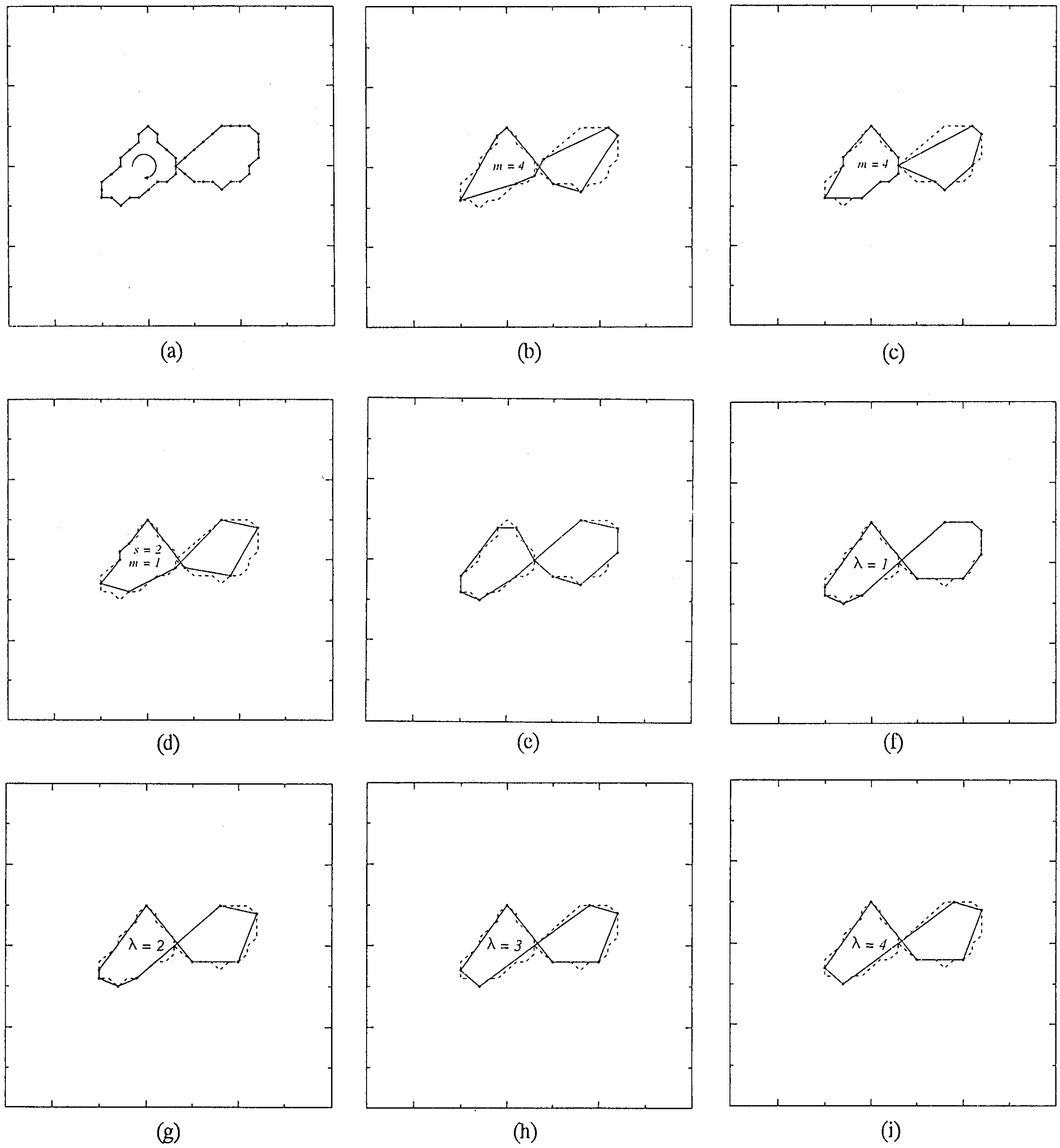

Fig. 4 Results of obtaining dominant points of the FIGURE-8 contour: (a) original contour, (b) Rosenfeld-Johnston algorithm, (c) Rosenfeld-Weszka algorithm, (d) Freeman-Davis algorithm, (e) Teh-Chin algorithm, (f) proposed regularization algorithm(λ = 1), (g) λ = 2, (h) λ = 3 and (i) λ = 4.

representation of the shape without redundancy.

A new algorithm for detecting dominant points is presented. Outperformed dominant point detection is realized by minimizing the regularized criterion function defined as the weighted conjunction of fitness of approximating polygon to the input contour and the distinctness of the model.

It is confirmed that the proposed algorithm is reasonably efficient, and the degree of polygonal approximation on the given contour and the suitable number of dominant points can be controlled by changing the regularization factor.

References

1) N. Ansari and E. J. Delp : Partial Recognition: A Landmark-based Approach, IEEE Trans. Pattern Anal. Machine Intell., Vol. PAMI-12(1990), pp. 470-483.
2) N. Ayache and O. D. Fangeras : HYPER: A New Approach for the Recognition and Positioning of Two-dimensional Objects, IEEE Trans. Pattern Anal. Machine Intell., Vol. PAMI-8(1986), pp. 44-54.
3) E. E. Milos : Shape Matching Using Curvature Process, Comput. Vision, Graphics, Image Processing, Vol. 47(1989), pp. 203-226.

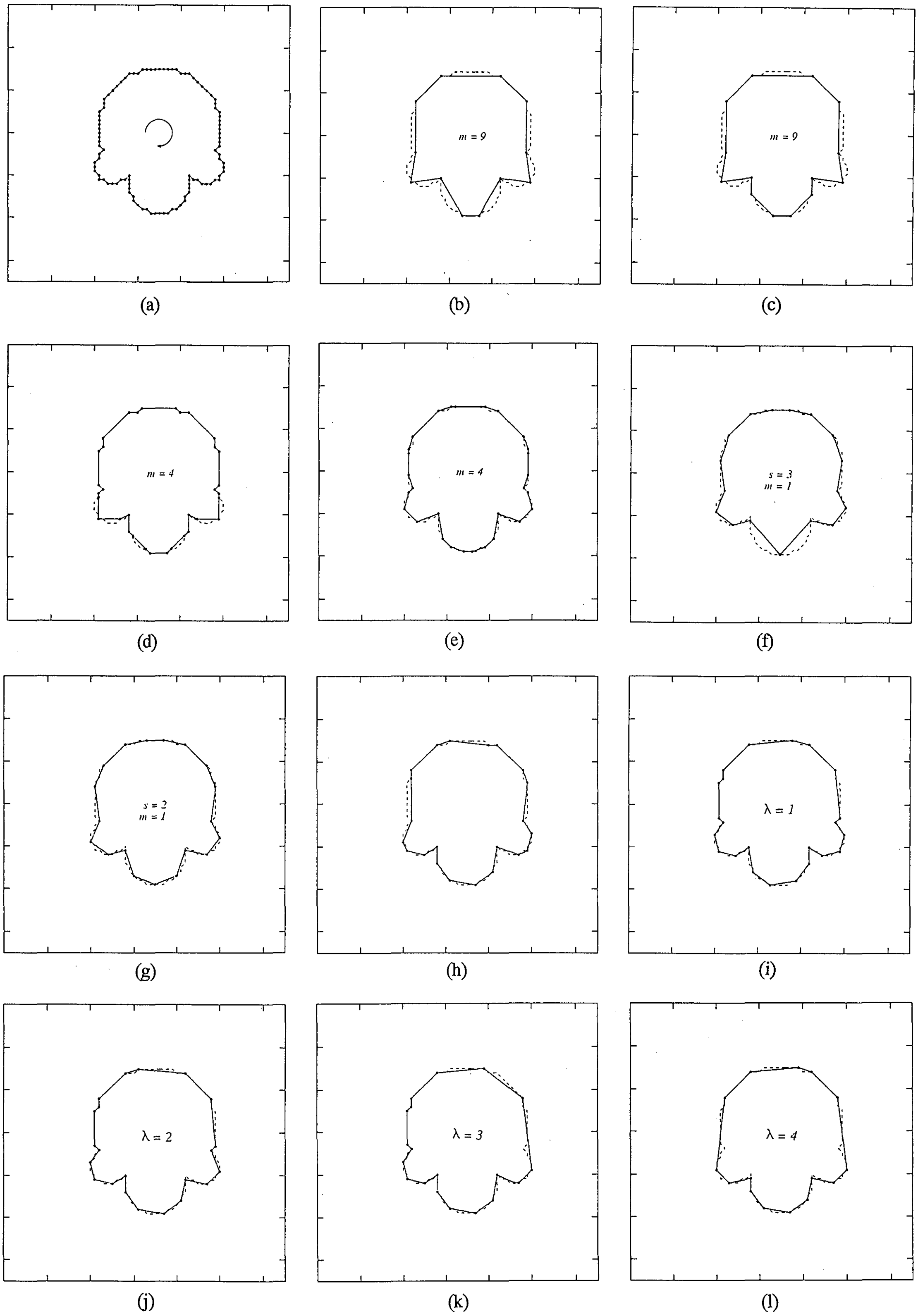

Fig. 5 Results of obtaining dominant points of the SEMICIR contour: (a) original contour, (b) and (c) Rosenfeld-Johnston algorithm, (d) and (e) Rosenfeld-Weszka algorithm ,(f) and (g) Freeman-Davis algorithm, (h) Teh-Chin algorithm, (i) proposed regularization algorithm(λ = 1), (j) λ = 2, (k) λ = 3 and (l) λ = 4.

4) J. K. Aggarwal and N. Nandhakumr : On the Computation of Motion from Sequence of Images-a review, Proc. IEEE, Vol. 76(1988), pp. 917-935.

5) S. N. Biswas, S. K. Pal and D. B. Majumder : Binary Contour Coding Using Beizer Approximation, Pattern Recognition Lett., Vol. 8(1988), pp. 237-249.

Table 1 Results of the CHROMOSOME contour

Number of Input Contour Points, $n = 60$					
Algorithm	Input Parameter	Number of Dominant Pts n_d	Compression Ratio n/n_d	Max Error E_{max}	Integral Sq Error E_2
Rosenfeld-Johnston	$m = 6$	8	7.5	1.54	21.94
Rosenfeld-Weszka	$m = 6$	12	5.0	1.58	22.61
Freeman-Davis	$s = 3$, $m = 2$	8	7.5	1.51	22.56
Teh-Chin(k-cosine)	None	15	4.0	0.74	7.20
Proposed Algorithm	$\lambda = 1$	15	4.0	0.75	4.01
	$\lambda = 2$	14	4.3	0.93	6.21
	$\lambda = 3$	13	4.6	1.41	10.15
	$\lambda = 4$	12	5.0	1.41	13.67

Table 2 Results of the LEAF contour

Number of Input Contour Points, $n = 120$					
Algorithm	Input Parameter	Number of Dominant Pts n_d	Compression Ratio n/n_d	Max Error E_{max}	Integral Sq Error E_2
Rosenfeld-Johnston	$m = 8$	17	7.1	1.76	43.42
Rosenfeld-Weszka	$m = 8$	18	6.7	1.53	30.57
Freeman-Davis	$s = 3$, $m = 2$	17	7.1	1.72	45.27
Teh-Chin(k-cosine)	None	29	4.1	0.99	14.96
Proposed Algorithm	$\lambda = 1$	30	4.0	0.92	8.76
	$\lambda = 2$	20	6.0	0.95	14.93
	$\lambda = 3$	19	6.3	0.97	16.41
	$\lambda = 4$	18	6.7	0.99	18.99

Table 3 Results of the FIGURE-8 contour

Number of Input Contour Points, $n = 45$					
Algorithm	Input Parameter	Number of Dominant Pts n_d	Compression Ratio n/n_d	Max Error E_{max}	Integral Sq Error E_2
Rosenfeld-Johnston	$m = 4$	10	4.5	1.61	22.83
Rosenfeld-Weszka	$m = 4$	16	2.8	1.59	12.67
Freeman-Davis	$s = 2$, $m = 1$	11	4.1	1.34	14.61
Teh-Chin(k-cosine)	None	13	3.5	1.00	5.93
Proposed Algorithm	$\lambda = 1$	11	4.1	1.00	3.70
	$\lambda = 2$	9	5.0	1.00	6.27
	$\lambda = 3$	7	6.4	1.00	7.83
	$\lambda = 4$	7	6.4	1.00	7.83

Table 4 Results of the SEMICIR contour

Number of Input Contour Points, $n = 102$					
Algorithm	Input Parameter	Number of Dominant Pts n_d	Compression Ratio n/n_d	Max Error E_{max}	Integral Sq Error E_2
Rosenfeld-Johnston	$m = 9$	12	8.5	2.04	92.37
	$m = 4$	30	3.4	0.74	8.85
Rosenfeld-Weszka	$m = 9$	14	7.3	1.56	59.12
	$m = 4$	34	3.0	1.00	15.40
Freeman-Davis	$s = 3$, $m = 1$	17	6.0	2.54	79.53
	$s = 2$, $m = 1$	19	5.4	1.41	23.31
Teh-Chin(k-cosine)	None	22	4.6	1.00	20.61
Proposed Algorithm	$\lambda = 1$	23	4.4	0.72	7.68
	$\lambda = 2$	22	4.6	0.98	9.72
	$\lambda = 3$	20	5.1	1.40	12.69
	$\lambda = 4$	15	6.8	1.40	15.58

6) C. H. Teh and R. T. Chin : On the Detection of Dominant Points on Digital Curves, IEEE Trans. Pattern Anal. Machine Intell., Vol. PAMI-11(1989), pp. 859-872.
7) I. M. Anderson and J. C. Bezedek : Curvature and Tangential Deflection of Discrete Arcs: A Theory based on the Commutator of Scatter Matrix Pairs and its Application to Vertex Detection in Planar Shape Data, IEEE Trans. Pattern Anal. Machine Intell., Vol. PAMI-6(1984), pp. 27-41.
8) A. Rosenfeld and E. Johnston : Angle Detection on Digital Curves, IEEE Trans. Comput., Vol. C-22(1973), pp. 875-878.
9) A. Rosenfeld and J.S. Weszka : An Improved Method of Angle Detection on Digital Curves, IEEE Trans. Comput., Vol. C-24(1975), pp. 940-941.
10) J. Roberge : A Data Reduction Algorithm for Planar Curves, Comput. Vision, Graphics, Image Processing, Vol. 29(1985), pp. 168-195.
11) N. Ansari and K. W. Hwang : Non-Parametric Dominant Point Detection, Pattern Recognition, Vol. 24(1991), No.9, pp. 849-862.
12) N. Ansari and E. J. Delp : On Detecting Dominant Points, Pattern Recognition, Vol. 24(1991), pp. 441-451.
13) H. Freeman and L. S. Davis : A Corner-Finding Algorithm for Chain Coded Curves, IEEE Trans. Comput., Vol. C-26(1977), pp. 297-303.
14) L. S. Davis : Understanding Shape: Angle and Sides, IEEE Trans. Comput., Vol. C-26(1977), No.3, pp. 236-242.
15) J. G. Dunham : Optimum Uniform Piecewise Linear Approximation of Planar Curves, IEEE Trans. Pattern Anal. Machine Intell., Vol. PAMI-8(1986), pp. 67-75.
16) K. Wall and P. E. Danieisson : A Fast Sequential Method for Polygonal Approximation of Digitized Curves, Comput. Vision, Graphics, Image Processing, Vol. 28(1984), pp. 220-227.
17) J. A. Saghr and H. Freeman : Analysis of Precision of Generalized Chain Codes for the Representation of Planar Curves, IEEE Trans. Pattern Anal. Machine Intell., Vol. PAMI-3(1981), pp. 533-539.
18) T. Poggio, V. Torre and C. Koch : Computational Vision and Regularization Theory, Nature, Vol. 26(1985), pp. 314-319.

Appendix

Chain Code of the CHROMOSOME Contour					
55454	32011	01111	12112	12006	65655
60010	10765	55455	55555	55431	12122

Chain Code of the LEAF Contour					
33332	30700	00332	32307	00003	32322
26777	22212	76661	11116	66566	55000
10056	65655	00110	66565	65555	56667
66666	66664	22222	22222	23224	43433

Chain Code of the FIGURE-8 Contour					
76776	77007	10121	22234	44555	55654
55453	42211	21121			

Chain Code of the SEMICIR Contour					
00007	00777	77766	76666	66665	76766
56454	43436	66656	55454	44434	33232
22254	54434	23221	21322	22222	21221
11111	00100	00			