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Development of Computational Evaluation Methods for Creep Deformation of SiC/SiC Composites †

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Abstract

In order to analyze crack propagation behavior in SiC/SiC composites, new computer simulation methods using time dependent interface elements were developed and applied to time-dependent crack growth in SiC/SiC composite under four-point bending tests on single-edge-notched beam bend-bars. The time-dependent crack growth in SiC/SiC composite was simulated by two methods, in which the creep property was introduced into the interface elements according to 1) the general method of FEM analysis and, 2) a new method making the best use of the potential function. In both cases, the stage-II slow crack growth of a general creep deformation was simulated. Furthermore, by using the new method, not only the stage-III crack growth but also transition phenomena from the stage-II to stage-III could be simulated. So the new method is considered to have the potential capability to demonstrate the time-dependent crack growth behavior in SiC/SiC composite.

KEY WORDS: (SiC/SiC Composites) (SiC Fibers) (Creep) (Ceramics) (Finite Element Method) (Interface Element)

1. Introduction

Silicon carbide fiber reinforced silicon carbide composites (SiC/SiC composites) are promising candidates as high flux component materials, due to their potential of low-activation, low-afterheat and their high-temperature properties1-4). From the point of view of safe design of the fusion structures, it is very important to estimate the fracture strength and behavior of materials exactly. In many previous proposed methods for evaluation of the composites, however, the formation and propagation of cracks in the composites have been microscopically and statically analyzed5,6), and the macroscopic and dynamic deformations have not been revealed sufficiently.

Recently the authors developed a new and simple computer simulation method7,8), in order to simulate the fracture phenomena that can be considered as the formation of new surfaces during the crack propagation. Based on the fact that surface energy must be supplied for the formation of a new surface, a potential function representing the density of surface energy was introduced in the proposed finite element method (FEM) using interface elements.

For SiC/SiC composites, the matrix cracks before the fibers fail. Following matrix cracking, intact fibers bridge the cracks and impose closure tractions behind the crack tip that reduce the driving force for further cracking (crack-tip shielding). The formation of bridged cracks could occur in the composites used below the known matrix fracture stress after accidental overloads or impact damage during service or in flaws resulting from processing. Thus, bridged cracks may exist in the composites even under service conditions that are constrained by accumulation of creep damage or dimensional change due to creep. Depending on the composite properties and the thermomechanical history of the composite, such cracks may be either fully or partially bridged. In either case, subsequent fracture is governed by the bridging stresses due to the fibers crossing the crack that are able to sustain loads, and it was reported that the time-dependent crack growth, in inert environments, in SiC/SiC composites is determined by the creep rate of the bridging fibers6). So, as an objective of this report, a new time-dependent interface element was developed for introducing the creep...
property of fibers into the interface element, and it was applied to analyze time-dependent crack growth, especially slow crack growth, of SiC/SiC composites.

2. Interface Potential

In the case of ordinary crack propagation problems, a method using the interface element has been proposed[7,8]. The mechanical behavior of the crack, in other words, the formation and the propagation of the crack is governed by the interface potential $\phi$ per unit area of the crack surface. The requirements of the interface potential function are

(1) It involves the surface energy $\gamma$, which is necessary to form the new surface as a material constant.

(2) It is a continuous function of opening displacement $\delta$.

Among many functions satisfying these requirements, a Lennard-Jones type potential[9] may be employed and the potential energy $\phi$ is defined by the following equation.

$$\phi(\delta) = 2\gamma \cdot \left\{ \frac{r_0}{r_0 + \delta} \right\}^{2n} - 2\cdot \left\{ \frac{r_0}{r_0 + \delta} \right\}^n$$  \hspace{1cm} (1)

Where $\delta$ is the crack opening displacement and $\gamma$, $n$ and $r_0$ are material constants. In particular, $2\gamma$ is the surface energy per unit area.

The derivative of $\phi$ with respect to the crack opening $\delta$ as shown in the following equation gives the bonding strength per unit area of the crack surface.

$$\sigma = \frac{\partial \phi}{\partial \delta} = 4\gamma \cdot n \cdot \left\{ \frac{r_0}{r_0 + \delta} \right\}^{n+1} - 2\cdot \left\{ \frac{r_0}{r_0 + \delta} \right\}^{2n+1}$$  \hspace{1cm} (2)

3. Method of Modeling

Figure 1 shows a schematic illustration of the bridging zone. In this research, the bridging zone was analyzed macroscopically, not microscopically. One interface element was assumed to represent many fibers and matrices in the bridging zone. The properties of the interface between fiber and matrix were assumed to be invariant, with respect to time or displacement, hence they were not explicitly included in the model. In other words, effective matrix properties were used to incorporate both the true matrix and the interface. Away from the cracked region, the composite was described by ordinary elements of FEM.

In order to characterize both fibers and matrices by one interface element, the interface potential $\phi$ was defined by the following equation as the same as the simple rule of mixture.

$$\phi = \phi_f + \phi_m$$  \hspace{1cm} (3)

Where $\phi_f$ and $\phi_m$ indicate fiber and matrix, respectively. In this study, only the creep effect of fiber on the time-dependent crack growth was taken into account and the interface potential of fiber $\phi_f$ was assumed to have a time dependency. To introduce the creep effect, two methods according to: 1) the general method of FEM analysis and, 2) a new method making the best use of the potential function were applied.

4. Theoretical Formulation

4.1 General method of FEM analysis

In the general method of FEM analysis for creep deformation, total strain can be divided into elastic strain, plastic strain, creep strain and other strain, which is for example thermal strain. In the case of SiC/SiC composites, the deformation may be limited to elastic and creep deformation, because of their brittleness. The strain of ordinary elements in FEM is equal to the increment of displacement in the interface element. Then, for the interface potential of the fiber, the increment of displacement $\Delta \delta_c$ can be divided into the elastic increment $\Delta \delta^E_c$ and the creep increment $\Delta \delta^C_c$ according to following equation.

$$\Delta \delta_c = \Delta \delta^E_c + \Delta \delta^C_c$$  \hspace{1cm} (6)

Further, the creep increment $\Delta \delta_c^C$ was assumed to follow the classical Dorn's formalism[10] under the steady-state creep by the following equation.

$$\Delta \delta^C_c = A \cdot \sigma^m \cdot \Delta t$$  \hspace{1cm} (7)

Where $A$, $\sigma$, $m$ and $\Delta t$ are a constant describing the structure of the deformation, the applied stress of the interface element, the stress exponent and a time increment, respectively.

In the case of the elastic displacement $\delta^E_c$, the following equation must be satisfied at any time increment according to Eq.(4).
\[ \phi_f(\delta^e) = 2\gamma_f \left[ \left( \frac{r_{fo}}{r_{fo} + \delta^e} \right)^2 - 2 \left( \frac{r_{fo}}{r_{fo} + \delta^e} \right)^n \right] \]  

(8)

Where the elastic displacement \( \delta^e \) was obtained from the total displacement \( \delta \) by the following equation.

\[ \delta^e = \delta - \delta^e = \delta - \sum \Delta \delta^e \]  

(9)

On the other hand, for the interface potential of the matrix, since any creep deformation was not taken into account, the following equation was used.

\[ \phi_m(\delta) = 2\gamma_m \left[ \left( \frac{r_{mo}}{r_{mo} + \delta} \right)^2 - 2 \left( \frac{r_{mo}}{r_{mo} + \delta} \right)^n \right] \]  

(10)

In this general method, load vector and stiffness matrix for FEM can be obtained by the same process described in reference \(^7\), so this is omitted.

### 4.2 A new method

In Eq.(4), there are three material constants, which are \( \gamma_f, r_{fo} \) and \( n \). Among these constants, only \( r_{fo} \) has the same dimension as the total displacement \( \delta \). As written in the previous section, in general, the displacement \( \delta \) is controlled by Eq.(7) during creep. So, in this research, to make the best use of the potential function, the material constant \( r_{fo} \) was assumed to have the time dependency and the increment of \( r_{fo} \) was defined to follow the next equation.

\[ \Delta r_{fo} = B \cdot \sigma^n \cdot \Delta t \]  

(11)

Where \( B \) is a constant, and the other parameters are the same as Eq.(7). So the interface potential energy of fiber \( \phi_f \) becomes a function of both opening displacement \( \delta \) and time \( t \), and this is written as \( \phi_f(\delta, t) \). Then the total interface potential energy is also described as \( \phi(\delta, t) \), while the interface potential energy of matrix \( \phi_m \) is a function of only the opening displacement.

On the other hand, although balanced equations of the new method are the same as the equations of the general method \(^7\), load vector and stiffness matrix are different. So these should be introduced. The time dependent interface potential energy \( \phi(\delta, t) \) is given by Eq.(12).

\[ U^e_5[u_0^e, t] = \int \phi(\delta, t) \cdot dS^e \]  

(12)

Where \( U^e_5 \) and \( u_0^e \) show interface potential energy of one element and nodal displacement. In Eq.(13), \( U^e_5[u_0^e + \Delta u_0^e, t + \Delta t] \) is transformed with Taylor’s expansion about \( \Delta u_0^e \) and \( \Delta t \).

\[ U^e_5[u_0^e + \Delta u_0^e, t + \Delta t] = \int \phi(\delta + \Delta \delta, t + \Delta t) \cdot dS^e = \int \phi \cdot dS^e \]  

\[ + \int \left\{ \frac{\partial \phi}{\partial \delta} \cdot \left[ \frac{\partial \delta}{\partial u_0^e} \right]^T \cdot \left[ \frac{\partial \delta}{\partial u_0^e} \right] \cdot \Delta t \right\} \cdot \left\{ \Delta u_0^e \right\} \cdot dS^e \]  

\[ + \frac{1}{2} \int \left\{ \frac{\partial^2 \phi}{\partial \delta^2} \cdot \Delta \delta \right\} \cdot \left\{ \Delta u_0^e \right\} \cdot \left\{ \frac{\partial \delta}{\partial u_0^e} \right\} \cdot dS^e \]  

\[ + H.O.T. \]  

(13)

First order term and second order term about increment of nodal displacement \( \Delta u_0^e \) are arranged to these equations.

\[ \int \left\{ \frac{\partial \phi}{\partial \delta} \cdot \left[ \frac{\partial \delta}{\partial u_0^e} \right]^T \cdot \Delta t \right\} \cdot \left\{ \Delta u_0^e \right\} \cdot dS^e = - \int \{ f^T \cdot \{ K \} \} \cdot \{ \Delta u_0^e \} \]  

(14)

\[ \int \left\{ \frac{\partial^2 \phi}{\partial \delta^2} \cdot \Delta \delta \right\} \cdot \left\{ \Delta u_0^e \right\} \cdot \left\{ \frac{\partial \delta}{\partial u_0^e} \right\} \cdot dS^e = - \frac{1}{2} \int \{ \Delta u_0^e \} \cdot \{ K \} \cdot \{ \Delta u_0^e \} \]  

(15)

Load vector \( \{ f \} \) and stiffness matrix \( \{ K \} \) in time dependent interface element are as coefficients of first order term and second order term. That is,

\[ \{ f \} = - \int \left\{ \frac{\partial \phi}{\partial \delta} \cdot \left[ \frac{\partial \delta}{\partial u_0^e} \right]^T \right\} \cdot \left\{ \Delta u_0^e \right\} \cdot dS^e \]  

(16)

\[ \{ K \} = \int \left\{ \frac{\partial^2 \phi}{\partial \delta^2} \cdot \Delta \delta \right\} \cdot \left\{ \frac{\partial \delta}{\partial u_0^e} \right\} \cdot \left\{ \frac{\partial \delta}{\partial u_0^e} \right\} \cdot dS^e \]  

(17)

### 5. Model for Analysis

In this report, as an example of time-dependent crack growth in SiC/SiC composites, slow crack growth, which was obtained by loading single-stage-notched beam bend-bar specimens in four-point bending (Fig.2), was examined. In this test, the crack propagated from the tip of the notch in the direction, which is parallel to the applied load. Experimental details are published elsewhere \(^7\). The reinforcements of the composites were 2-dimensional, plain-weave Hi-Nicalon fiber mats, which were stacked in the direction of thickness, and the matrices were deposited by chemical vapor infiltration.

To examine the validity of the proposed method
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Fig. 2 Schematic illustration of four-point bending test on single-edge-notched beam bend-bar.

using the time dependent interface element for the analysis of slow crack growth, the model shown in Fig. 3 was analyzed as the plain strain problem. Because of the symmetry of the problem, only the half of the specimen was used. The time dependent interface model was arranged along the crack propagation path. In this analysis, only the mode-I type crack propagation parallel to y-axis in Fig. 3 was taken into account according to the experimental results. The parameter \( n \) in Eq.(4) and (5) was assumed to be 6.

6. Results and Discussion

Figure 4 shows an experimental result of the time-dependent crack growth in SiC/SiC composite reported in reference\(^6\), where the temperature was 1200 °C, atmosphere was gettered Ar (<20 ppm O\(_2\)), and applied load was 556 N. In this paper, the proposed method with the time dependent interface element was only applied to demonstrate the stage-II type slow crack growth. Therefore the area in the time-displacement curve, where the displacement was more than 0.06 mm, was compared with the calculation results since the rate of change of the displacement, with respect to time, was minimal. Accurate modeling of all stages of crack growth behavior must include a time-dependent term. The parameter \( m \) was assumed to be 1 according to reference\(^1\) since the test was performed at high-temperature.

Figure 5 shows the experimental result and the calculation results using both the general method and new method. Although both calculation results simulated the stage-II slow crack growth, the result using the new method represented not only the stage-II slow crack growth but also the stage-III crack growth. Although stage-III crack growth is not often observed under conditions relevant to fusion devices, it may occur under other conditions. So after this, the calculations using the new method under the different applied load were examined and discussed.

The calculation results under various applied load using the new method were shown in Fig. 6. The ends of curves indicate the fracture point of the specimen. The transition time from stage-II to stage-III decreases with increasing applied load, similar to the general creep

Fig. 4 Displacement-time curve during time-dependent crack growth in SiC/SiC composite\(^6\).

Fig. 5 Comparison between experiment and computation in time-dependent crack growth.

Fig. 6 Calculated displacement-time curves under various applied loads.
deformation. Therefore the new method is considered to be useful for analyzing such creep behavior. **Figure 7** shows the deformation and stress distribution in the time-dependent crack growth of SiC/SiC composite at 556 N applied load. The crack propagation behavior was clearly demonstrated and the stress relaxation behavior at the crack tip was also calculated. From the definition of the interface potential, the stresses of fiber and matrix in the interface element can be obtained. The stress changes of fiber and matrix along the crack path are shown in **Fig.8**. After the crack propagated, both the stress of fiber and matrix decreased, and the stress of matrix became almost zero. On the other hand, the stress of fiber maintained around 200 MPa after the crack propagated. In this research, as the first calculation using the new method, the fiber was assumed not to fracture but to continue to creep. Although the calculation result does not exactly represent the experimental behavior, the new method is considered to have the potential capability to demonstrate the time-dependent crack growth behavior in SiC/SiC composite. As future work, the effect of fiber fracture and the effects of other parameters in Eq.(4) and (5) will be examined.

**7. Conclusions**

In order to analyze crack propagation behavior in SiC/SiC composite, a new computer simulation method using time dependent interface elements was developed and applied to time-dependent crack growth in SiC/SiC composites. The conclusions can be summarized as follows.

1) The time-dependent crack growth in SiC/SiC composite was simulated by two methods, in which the creep property was introduced into the interface elements according to : 1) the general method of FEM analysis and, 2) a new method making the best use of the potential function. In both cases, the stage-II slow crack growth of a general creep deformation was simulated.

2) By using the new method, not only the stage-III crack growth but also transition phenomena from stage-II to stage-III could be simulated. So the new method is considered to have the potential capability to model all stages of time-dependent crack growth behavior in SiC/SiC composites.

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**References**

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