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NEWMAN'S THEOREM FOR PSEUDO-SUBMERSIONS

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1. Introduction. In 1931 M.H.A. Newman [N] proved the following result.

Theorem (Newman). *If M is a connected topological manifold with metric d , there exists a number $\varepsilon = \varepsilon(M, d) > 0$, depending only upon M and d , such that every finite group G acting effectively on M has at least one orbit of diameter at least ε .*

P.A. Smith [S] in 1941 proved a version of Newman's Theorem in terms of coverings of M and Dress [D] in 1969 gave a simplified proof of Newman's Theorem based on Newman's original approach and using a modern version of local degree.

In another direction Cernavskii [C] in 1964 generalized Newman's Theorem to the setting of finite-to-one open mappings on manifolds. His techniques were based upon those of Smith. Recently McAuley and Robinson [M-R] and Deane Montgomery [MO] have expanded upon Cernavskii's results. In fact McAuley and Robinson, using the techniques of Dress, have obtained the following version of Cernavskii's result. [M-R, Theorem 3].

Theorem (Cernavskii-McAuley-Robinson). *If M is a compact connected topological manifold with metric d , there exists a number $\varepsilon = \varepsilon(M, d) > 0$ such that if Y is a metric space and $f: M \rightarrow Y$ a continuous finite-to-one proper open surjective mapping which is not a homeomorphism, then there is at least one $y \in Y$ such that $\text{diam } f^{-1}(y) \geq \varepsilon$.*

In [H-M] we gave estimates of the ε in Newman's Theorem for Riemannian manifolds M in terms of convexity and curvature invariants of M . In this note we apply the techniques of [H-M] to obtain estimates of ε for the Cernavskii-McAuley-Robinson result for the case where M is a Riemannian manifold. In particular, if S^n is the standard unit sphere with standard metric, we show $\varepsilon > \pi/2$, i.e. if $f: S^n \rightarrow Y$ is as above, there exists $y \in Y$ with $\text{diam } f^{-1}(y) > \pi/2$. We also obtain a cohomology version of Newman's Theorem for compact orientable Riemannian manifolds which generalizes Theorem 3 of

[H-M].

We wish to thank McAuley and Robinson for sending us [M-R] prior to its publication.

2. Generalized Newman's theorem for Riemannian manifolds.

We shall call an open finite-to-one proper surjective map $f: M \rightarrow Y$, Y a metric space, which is not a homeomorphism, a *pseudo-submersion*, and $f^{-1}(f(x))$ an *orbit of f at x* and denoted by $O_f(x)$.

Now let M be a connected Riemannian manifold with a metric induced from the Riemannian metric of M . Assume that there exists at least one pseudo-submersion $f: M \rightarrow Y$. Define the *Newman's diameter* $d^T(M)$ of M by

$$d^T(M) = \sup \left\{ \varepsilon \left| \begin{array}{l} \text{for every pseudo-submersion } f: M \rightarrow Y. \\ \text{there exists } x \in M \text{ such that } \text{diam } O_f(x) \geq \varepsilon \end{array} \right. \right\}$$

Define the *cardinality* of f by $\text{Card } f = \max \{ \text{card } O_f(x) : x \in M \}$. Suppose there exists at least one pseudo-submersion $f: M \rightarrow Y$ with $\text{Card } f = p > 1$; we define the *mod p Newman's diameter* $d_p^T(M)$ as the supremum of the numbers $\varepsilon > 0$ such that for every pseudo-submersion $g: M \rightarrow Y$ with $\text{Card } g = p$, there exists an orbit of diameter at least ε .

We call a subset S of a Riemannian manifold M *convex* if for every pair of points in S there exists a unique distance measuring geodesic in S joining them. For $x \in M$, the *radius of convexity of M at x* , which we denote by r_x , is defined as the supremum of the radii of all convex embedded open balls centered at x .

The following result is based upon Lemma 3 in [D] and appears as Theorem 2 in [M-R].

Proposition 2.1 (Dress-McAuley-Robinson). *Let U be an open, connected, relatively compact subset of R^n and $f: \bar{U} \rightarrow Y$ a pseudo-submersion. Then*

$$\begin{aligned} D &= \text{Max} \{ \text{Min} \{ \|x - y\| : y \in \partial \bar{U} \} : x \in U \} \\ &\leq C = \text{Max} \{ \text{diam } O_f(x) : x \in \partial \bar{U} \} . \end{aligned}$$

Here $\|x - y\|$ is the euclidean norm.

It is well-known that the exponential map locally stretches distances for manifolds of nonpositive curvature. In [H-M] the following analogous result was obtained for manifolds of bounded curvature.

Proposition 2.2. *Suppose $K \leq b^2$, $b > 0$, (respectively $K \leq 0$) on a Riemannian manifold M with distance function d . Let $B_r(z) = \{y: d(y, z) < r\}$ be a convex embedded ball centered at z in M . Suppose further that $r < \pi b^{-1}/2$ (respectively $0 < r < \infty$ when $K \leq 0$). For any $x, y \in B_r(z)$, if $\hat{x} = \exp_z^{-1}x$ and $\hat{y} = \exp_z^{-1}y$, then*

$d(x, y) \geq (2/\pi) \|\hat{x} - \hat{y}\|$ (respectively $d(x, y) \geq \|\hat{x} - \hat{y}\|$ when $K \leq 0$). Here $\|\hat{x} - \hat{y}\|$ is the euclidean norm in the tangent space M_z .

Using Propositions 2.1 and 2.2 and the techniques of [H-M] we are able to prove the main result of this section.

Theorem 2.3. *Let*

$$\bar{r} = \sup_{z \in M} r_z.$$

(1) *If $K \leq 0$, $d^T(M) \geq \bar{r}/2$. In particular if $\bar{r} = +\infty$, there exist point inverses of arbitrarily large diameters.*

(2) *If $K \leq b^2$, and $a = \text{Min}\{\pi/2b, \bar{r}\}$, $d^T(M) \geq 2a/(\pi+2)$.*

Proof. Fix any $z \in M$ and let r_z = the radius of convexity at z . For any $r > 0$ satisfying

$$r < \begin{cases} r_z & \text{if } K \leq 0 \\ \text{Min}\{r_z, \pi b^{-1}/2\} & \text{if } K \leq b^2, \end{cases}$$

and any α , $1/2 \leq \alpha < 1$, suppose that

$$(H) \text{ diam } O_f(x) < (1-\alpha)r, \text{ all } x \in M.$$

Define $U = f^{-1}[f(B_{\alpha r}(z))]$. Clearly U is open. We claim U is connected. Let V be a component of U . Now it is known [C], [MO] that V maps onto $f(U) = f(B_{\alpha r}(z))$. Hence, V intersects $O_f(z)$. But since

$$\begin{aligned} \text{diam } O_f(z) &< (1-\alpha)r \leq \alpha r, \\ O_f(z) &\subset B_{\alpha r}(z). \text{ Furthermore by (H),} \\ B_{\alpha r}(z) &\subset U \subset B_r(z). \end{aligned}$$

Let $U_\wedge = \exp_z^{-1}U$. Then U_\wedge is an open and connected subset of $R^n = M_z$. It can be verified that

$$\bar{U}_\wedge = \exp_z^{-1} \circ f^{-1}[f(\bar{B}_{\alpha r}(z))].$$

Consequently we can apply Proposition 2.1 to $f_\wedge = f \circ \exp_z: \bar{U}_\wedge \rightarrow Y$. Now

$$\begin{aligned} \{\hat{x} \in M_z \mid \|\hat{x}\| \leq \alpha r\} &= \exp_z^{-1} \bar{B}_{\alpha r}(z) \subset \bar{U}_\wedge \\ &\subset \exp_z^{-1} \bar{B}_r(z) = \{\hat{x} \in M_z \mid \|\hat{x}\| \leq r\} \end{aligned}$$

The left-hand inclusion implies

$$D = \text{Max}\{\text{Min}\{\|\hat{x} - \hat{y}\| \mid \hat{y} \in \partial \bar{U}_\wedge \mid \hat{x} \in U_\wedge\} \geq \alpha r \quad (\text{Simply let } \hat{x} = 0)$$

Since $\bar{B}_r(z)$ is a convex, embedded ball with $r < \pi b^{-1}/2$ when $K \leq b^2$ ($r < \infty$ when $K \leq 0$), we may apply Proposition 2.2. So

$$\begin{aligned}
C &= \text{Max} \{ \text{diam } O_f(\hat{x}) \mid \hat{x} \in \partial \bar{U}_\wedge \} \\
&\leq \begin{cases} \text{Max} \{ \text{diam } O_f(x) \mid x \in \partial \bar{U} \} & \text{if } K \leq 0 \\ \pi/2 \text{ Max} \{ \text{diam } O_f(x) \mid x \in \partial \bar{U} \} & \text{if } K \leq b^2 \end{cases} \\
&< \begin{cases} (1-\alpha)r & \text{if } K \leq 0 \\ (1-\alpha)\pi r/2 & \text{if } K \leq b^2 \end{cases}
\end{aligned}$$

by (H).

By Proposition 2.1, $D \leq C$. Consequently

$$\alpha r < \begin{cases} (1-\alpha)r & \text{if } K \leq 0 \\ (1-\alpha)\pi r/2 & \text{if } K \leq b^2 \end{cases}$$

or

$$\alpha < \begin{cases} 1/2 & \text{if } K \leq 0 \\ \pi/(\pi+2) & \text{if } K \leq b^2 \end{cases}$$

Consequently, (H) is false for

$$\alpha = \begin{cases} 1/2 & \text{if } K \leq 0 \\ \pi/(\pi+2) & \text{if } K \leq b^2 \end{cases}$$

So there exists an $x \in M$ with $\text{diam } O_f(x) \geq r/2$ if $K \leq 0$; $2r/(\pi+2)$ if $K \leq b^2$.

It is possible to obtain a version of Theorem 2.3 in terms of *injectivity radius*. For a complete connected Riemannian manifold M define the *injectivity radius* $i(M)$ by

$$i(M) = \sup \{ d(x, C(x)) : x \in M \}$$

where $C(x)$ denotes the cut locus of x .

Theorem 2.4.

- (1) If $K \leq 0$, $d^T(M) \geq i(M)/2$.
- (2) If $K \leq b^2$, M is compact and $a = \text{Min} \{ \pi/2b, i(M)/2 \}$, $d^T(M) \geq 2a/\pi$.

3. Estimate of Newman's diameter $d^T(S^n)$ and related topics. We use the notion of *degree of a map* defined by Dress [D].

Let $f: M^n \rightarrow Y$ be a pseudo-submersion. The *branch set* B_f of f is defined as $B_f = \{x \in M: f \text{ is not a local homeomorphism at } x\}$. By [C] or [M-R], $M - f^{-1}(f(B_f))$ is a dense open subset of M^n .

Lemma 3.1: Newman's Lemma (Dress [D], McAuley-Robinson [M-R]). Let $f: M \rightarrow Y$ be a pseudo-submersion, X a locally compact metric space, $g: M \rightarrow X$ and $j: Y \rightarrow X$ be a proper map such that $g = j \circ f$. Let $x \in X$ be such that

$$g^{-1}(x) \cap f^{-1}(f(B_f)) = \emptyset,$$

and $y \in j^{-1}(x)$. If $\text{Card } f^{-1}(y) = p$, then g is inessential at x for Z_p ; that is, the degree of g at x , $d(g, x)$, is zero (with Z_p as coefficients).

Theorem 3.2. Let M be a compact connected oriented topological n -manifold and $f: M^n \rightarrow Y$ be a pseudo-submersion with $\text{Card } O_f(x_0) = p > 1$ for some $x_0 \in M - f^{-1}(f(B_f))$. Suppose $\varphi: M \rightarrow S^n$ is a map such that the $\deg \varphi \not\equiv 0 \pmod{p}$. If we denote $\varphi(z)$ by \bar{z} , then there exists $x \in M$ such that the following holds:

- (1) $\sum_{z \in O_f(x)} \bar{z} = c\bar{x}$ in R^{n+1} for some $c \leq 0$.
- (2) $\angle \bar{x} o \bar{z} \begin{cases} = \pi & \text{if } \text{Card } O_f(x) = 2 \\ \geq 2\pi/3, \text{ and } \angle \bar{x} o \bar{z} = \angle \bar{x} o \bar{y}, & \text{if } \text{Card } O_f(x) = 3 \text{ and} \\ & O_f(x) = \{x, y, z\}. \\ \geq \pi - \cos^{-1}(1/(p-1)) > \pi/2 & \text{if } \text{Card } O_f(x) \geq 4 \end{cases}$

for some $z \in O_f(x)$, where $\angle \bar{x} o \bar{z}$ denotes the angle between $o\bar{x}$ and $o\bar{z}$, $o \in R^{n+1}$ the origin, and S^n the standard unit sphere in R^{n+1} .

Proof. (1) Suppose on the contrary, then $\sum_{z \in O_f(x)} \bar{z} \neq 0$ for all x in M . Define a map $g: M^n \rightarrow S^n$ by

$$g(x) = \sum_{z \in O_f(x)} \bar{z} / \left| \sum_{z \in O_f(x)} \bar{z} \right|.$$

Then for any $z \in O_f(x)$, $g(z) = g(x)$. Hence g induces a map $j: Y \rightarrow S^n$ such that $g = j \circ f$. It follows from Lemma 3.1 that g is inessential at $g(x)$ for Z_p .

On the other hand, by hypothesis there is a well defined homotopy $H: M \times [0, 1] \rightarrow S^n$ between φ and g defined by

$$H(x, t) = \{t\varphi(x) + (1-t)g(x)\} / |t\varphi(x) + (1-t)g(x)|.$$

Hence, $\deg \varphi = \deg g = d(g, g(x)) = 0 \pmod{p}$. This is a contradiction.

(2) For any $y, z \in O_f(x)$, set $\theta_{yz} = \angle \bar{y} o \bar{z}$. Let \langle, \rangle be the standard inner product in R^{n+1} . From (1) there exists an element x in M such that

$$\langle \bar{x}, \bar{x} \rangle + \sum_{z \neq x, z \in O_f(x)} \langle \bar{x}, \bar{z} \rangle = c \langle \bar{x}, \bar{x} \rangle$$

for some $c \leq 0$; that is,

$$(**) \sum_{z \neq x, z \in O_f(x)} \cos \theta_{xz} = c - 1 \leq -1$$

If $\text{Card } f = 2$, it is easy to see from (**) that $c = 0$, and $\theta_{xz} = \pi$.

If $\text{Card } f = 3$, then $\cos \theta_{xy} + \cos \theta_{xz} = c - 1$. From (1) we have

$$|(1-c)\bar{x} + \bar{z}|^2 = |-\bar{y}|^2.$$

Hence $\cos \theta_{xy} = \cos \theta_{xz} = (c-1)/2$. That is, $\theta_{xy} = \theta_{xz} \geq 2\pi/3$. If $\text{Card } f = p \geq 4$, there exists at least one $z \in O_f(x)$ such that $\cos \theta_{xz} \leq -1/(p-1)$; that is, $\theta_{xz} \geq \pi$.

$$-\cos^{-1}(1/(p-1)) > \pi/2.$$

Theorem 3.2 implies the following:

Corollary 3.3. (1) $d_2^T(S^n) = \pi$, i.e., for any pseudo-submersion $f: S^n \rightarrow Y$ with $\text{Card } f = 2$, there exists $x \in S^n$ such that $f^{-1}(f(x)) = \{x, -x\}$.

$$(2) \quad d_3^T(S^n) = 2\pi/3.$$

$$(3) \quad (p-1)\pi/p \geq d_p^T(S^n) \geq \pi - \cos^{-1}(1/(p-1)) > \pi/2 \text{ if } p \geq 4.$$

$$(4) \quad 2\pi/3 \geq d^T(S^n) > \pi/2.$$

Proof. In [K], the equivariant diameter $D(M)$ and modulo p equivariant diameter $D_p(M)$ have been defined. They are precisely defined by the pseudo-submersions $\pi: M \rightarrow M/G$ which are orbit maps of isometric actions of compact Lie groups G or $G = Z_p$ on M respectively. Hence $D(M) \geq d^T(M)$ and $D_p(M) \geq d_p^T(M)$ for some p . But $D(S^n) = 2\pi/3$ and $D_p(S^n) = (p-1)\pi/p$ if $p \geq 3$ by [K]. Hence, the result follows from Theorem 3.2 by applying it to the identity map $S^n \rightarrow S^n$.

REMARKS. (i) The statement (1) extends the following well known result: For any non-trivial involution g of S^n , there exists $x \in S^n$ such that $gx = -x$.

(ii) By using the arguments of Milnor in [MI] we can also show the following: Let $f: M^n \rightarrow Y$ and $\tilde{f}: \tilde{M} \rightarrow \tilde{Y}$ be pseudo-submersions with $\text{Card } f = \text{Card } \tilde{f} = 2$, $B_f = B_{\tilde{f}} = \phi$, where M is a compact connected oriented n -manifold and \tilde{M} a mod 2 homology n -sphere. Suppose there exists a map $\varphi: M \rightarrow \tilde{M}$ of odd degree. Then there exists x in M such that $\varphi O_f(x) = O_{\tilde{f}}(\varphi x)$.

Theorem 3.4. Let M be a compact connected n -dimensional submanifold of R^{n+1} , $n \geq 2$, and let $y \in R^{n+1} - M$ be in a bounded component. Suppose $f: M \rightarrow Y$ is a pseudo-submersion. Then there exists $x \in M$ such that

$$(1) \quad \text{If } \text{Card } f = 2, \{O_f(x), y\} \text{ lies on a line in } R^{n+1}.$$

$$(2) \quad \text{If } \text{Card } f = 3, \text{ and } O_f(x) = \{x, u, v\}, \text{ then}$$

$$\angle xyu = \angle uyv = \angle vyx = 2\pi/3.$$

In particular $\{O_f(x), y\}$ lies in a 2-plane in R^{n+1} .

$$(3) \quad \text{If } \text{Card } f = p \geq 4, \text{ then } \angle uyv \geq \pi - \cos^{-1}(1/(p-1)) > \pi/2 \text{ for some } u, v \in O_f(x), \text{ and } \{O_f(x), y\} \subset R^{p-1} \cap M, \text{ for some } (p-1)\text{-plane } R^{p-1} \text{ of } R^{n+1} \text{ (if } n \geq p-2) \text{ passing through the origin.}$$

Proof. Apply Theorem 3.2 to the map $\varphi: M \rightarrow S^n$ defined by $\varphi(x) = (y-x)/\|y-x\|$ because $\deg \varphi = \pm 1$. The equality in (2) follows from Corollary 3.3 (2).

4. Cohomology version of Newman's theorem for pseudo-submersions

Let $f: M \rightarrow Y$ be a pseudo-submersion. A subset A of M is called satur-

ated if $A = O_f(A)$, where $O_f(A) = \bigcup \{O_f(x) : x \in A\}$, or equivalently $A = f^{-1}(f(A))$. Let $x \in M - f^{-1}(f(B_f))$. Then there exists an open neighborhood V of x which is homeomorphic to R^n and $f|V: V \rightarrow f(V)$ is a homeomorphism. Hence by excision we have

$$H_n(Y, Y - f(x); Z_p) \approx H_n(f(V), f(V) - f(x); Z_p) \approx Z_p,$$

where $p = \text{Card } O_f(x)$.

We shall say a pseudo-submersion $f: M \rightarrow Y$ satisfies the (LOA) (*local orientable condition* for A) if A is a closed saturated subset of M , $B = f(A)$ is closed in Y and such that the inclusion $i_B: (Y, B) \rightarrow (Y, Y - x)$ induces an isomorphism

$$i_{B*}: H_n(Y, B; Z_p) \rightarrow H_n(Y, Y - f(x); Z_p)$$

for some $x \in M - f^{-1}(f(B_f))$, $\text{Card } O_f(x) = p$.

The following result extends the cohomology version of Newman's Theorem for group actions [B], [S] due to Smith.

Theorem 4.1. *Let A be a closed subspace of a compact oriented n -manifold M such that $H_n(M, A; Z_p) \approx Z_p$. Let \mathcal{U} be any open covering of M such that*

$$H^n(K(\mathcal{U}), K(\mathcal{U}|A); Z_p) \rightarrow H^n(M, A; Z_p)$$

is surjective, where $K(\mathcal{U})$ denotes the nerve of the covering \mathcal{U} . Then there does not exist a pseudo-submersion $f: M \rightarrow Y$ satisfying (LOA) and such that each orbit of f is contained in some open set in \mathcal{U} .

Proof. Suppose the conclusion is false. Then there exists a pseudo-submersion $f: M \rightarrow Y$ satisfying (LOA) and each orbit $O_f(x)$ is contained in a saturated open set V_x which is contained in some member of \mathcal{U} . Let $\mathcal{C}\mathcal{V} = \{f(V_x) : x \in V\}$. Then $f^{-1}\mathcal{C}\mathcal{V}$ is a refinement of \mathcal{U} . By [B, p. 154], $f^*: H^n(Y, B; Z_p) \rightarrow H^n(M, A; Z_p)$ is an epimorphism. But the Kronecker product induces a canonical epimorphism [G, p. 132]

$$\alpha: H^n(M, A; Z_p) \rightarrow H_n(M, A; Z_p)^* = \text{Hom}(H_n(M, A; Z_p); Z_p);$$

hence we have an isomorphism $f_*: H_n(M, A; Z_p) \rightarrow H_n(Y, B; Z_p)$.

Let $K = O_f(x)$, and $O_K \in H_n(M, M - K; Z_p)$ be the fundamental class which is the element such that for any $z \in K$, the inclusion $i_z: (M, M - K) \rightarrow (M, M - z)$ satisfies $i_{z*}(O_K) = 1_z$, the identity element of $H_n(M, M - z; Z_p) \approx Z_p$ (cf. [D]). We have the following commutative diagram

$$\begin{array}{ccc} Z_p = H_n(M; Z_p) & \xrightarrow{\quad} & H_n(M, A; Z_p) \\ \downarrow & \searrow i_* & \downarrow \\ H_n(M, M - K; Z_p) & \xleftarrow{k_{z*}} & H_n(M, M - z; Z_p) = Z_p \\ & \nearrow i_{z*} & \end{array}$$

where all homomorphisms are induced by inclusions. Since k_{z*} is an isomorphism for all z in K , there exists an element a in $H_n(M, A; Z_p)$ such that $i_*(a) = O_K$. Now we consider the following commutative diagram

$$\begin{array}{ccc} H_n(M, A; Z_p) & \xrightarrow[\approx]{f_*} & H_n(Y, B; Z_p) \\ i_* \downarrow & & \approx \downarrow i_{B*} \\ H_n(M, M-K; Z_p) & \xrightarrow{f_*} & H_n(Y, Y-f(x); Z_p) \end{array}$$

By definition, $d(f, f(x)) = f_*(O_K)$ (cf. [D]). It follows that

$$d(f, f(x)) = f_* i_*(a) = i_{B*} f_*(a) \neq 0.$$

On the other hand, we can apply Lemma 3.1 to the map f , with $f = j \circ f$, to obtain $d(f, f(x)) = 0$, where j is the identity map. This is an obvious contradiction and the proof of the theorem is complete.

Corollary 4.2. *Let M be a compact connected oriented n -manifold, and \mathcal{U} an open covering of M such that*

$$(*) \quad H^q(|\sigma|; Z_p) = 0 \text{ for any } \sigma \in K(\mathcal{U}) \text{ and any } q \geq 1.$$

Then there does not exist a pseudo-submersion $f: M \rightarrow Y$ such that

- (1) $i_{x*}: H_n(Y; Z_p) \xrightarrow{\cong} H_n(Y, Y-x; Z_p)$, where $i_x: Y \rightarrow (Y, Y-x)$ is inclusion, $x \in M - f^{-1}(f(B_f))$, $\text{Card } O_f(x) = p$, and
- (2) Each orbit of f is contained in some member of \mathcal{U} .

Proof. The hypothesis $(*)$ implies that

$$H^q(K(\mathcal{U}); Z_p) \xrightarrow{\cong} H^q(M; Z_p)$$

for all $q \geq 0$ by Leray's Theorem [G-R, p. 189].

As an example, if M is a compact connected oriented Riemannian manifold, and \mathcal{U} consists of all open convex proper subsets of M , then the condition $(*)$ of Corollary 4.2 is satisfied.

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