Title	On weak convergence of diffusion processes generated by energy forms
Author(s)	Uemura, Toshihiro
Citation	Osaka Journal of Mathematics. 32(4) p861-p.868
Issue Date	1995
oaire:version	VoR
URL	https://doi.org/10.18910/12736
DOI	
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Uemura, T.
Osaka J. Math.
32 (1995), 861–868

ON WEAK CONVERGENCE OF DIFFUSION PROCESSES GENERATED BY ENERGY FORMS

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(Received February 10, 1994)

0. Introduction

Convergences of closed forms, energy forms or energy functions have been studied by many authors (see e.g. [1]-[3], [5]-[8]). It is important to know, given an energy form, if it can be approximated by "nice" ones, or, given a sequence of energy forms, what their "limit" is.

In this paper we consider a sequence of forms $\mathcal{E}^n(u,v)=\int_{\boldsymbol{R}^d}(A_n(x)\nabla u(x), \nabla v(x))_d\phi_n^2(x)dx$ with certain domains on $L^2(\boldsymbol{R}^d;\phi_n^2dx)$, where ϕ_n are locally bounded functions on $\boldsymbol{R}^d$, A_n are $(d\times d)$ symmetric matrix valued functions on $\boldsymbol{R}^d$, $(\cdot,\cdot)_d$ means the inner product on $\boldsymbol{R}^d$ and $\nabla u=(\nabla_1 u,\nabla_2 u,\cdots,\nabla_d u)$ is the distributional (weak) derivative of u. Take strictly positive, bounded functions f_n with $\int_{\boldsymbol{R}^d}f_n\phi_n^2dx=1$ and denote by $\{X_t,P_x^n,x\in\boldsymbol{R}^d\}$ the diffusion processes associated with the forms $\mathcal{E}^n$. We study the weak convergence of the probability measures $\{P_{m_n}^n,n=1,2,\cdots\}$ with $dm_n=f_n\phi_n^2dx$, when the date A_n, ϕ_n, and f_n converge *a.e.* on $\boldsymbol{R}^d$, as $n\to\infty$.

Although our main result (see section 1) is similar to that of T.J. Lyons and T.S. Zhang [5], we assume only a certain local boundedness of ϕ_n, while a uniform boundedness on the whole space is assumed in [5]. In order to obtain the result in [5], they generalized the theorem of Kato and Simon on monotone sequence of closed forms (see M. Reed and B. Simon [7]) used by S. Albeverio, R. Høegh-Krohn and L. Streit [1]. We will instead adopt the Mosco-convergence of closed forms(see U. Mosco [6]) to prove our theorem.

S. Albeverio, S. Kusuoka and L. Streit [2] obtained a semigroup convergence by imposing the regularity conditions that there exist $R>0$ and $C>0$ such that, the restrictions of ϕ_n to $\boldsymbol{R}^d-B_R$ is of class C^2 and the growth order of $x\cdot\nabla\phi_n/\phi_n$ is not greater than $C|x|^2$ on $\boldsymbol{R}^d-B_R$. No smoothness on A_n, ϕ_n is required in the present approach.

1. Statement of Theorem

Let $\phi_n(x)$, $\phi(x)$ be measurable functions on $\boldsymbol{R}^d$ and $A_n(x)$, $A(x)$ be $(d\times d)$ symmetric matrix valued functions on $\boldsymbol{R}^d$. Consider the following conditions:

(A.1) (i) there exists a constant $\delta>0$ such that

$$0\leq\frac{1}{\delta}|\xi|^2\leq(A_n(x)\xi,\xi)_d\leq\delta|\xi|^2, \qquad \text{for } dx\text{-}a.e.\, x\in \boldsymbol{R}^d, \xi\in \boldsymbol{R}^d, n\in \boldsymbol{N}.$$

(ii) for any relatively compact open set G of $\boldsymbol{R}^d$, there exist constants $\lambda(G)$, $\Lambda(G)>0$ such that,

$$0<\lambda(G)\leq\phi_n(x)\leq\Lambda(G), \qquad \text{for } dx\text{-}a.e.\, x\in G, n\in \boldsymbol{N},$$

(iii) $\phi_n(x)\to\phi(x)$, dx-$a.e.$ on $\boldsymbol{R}^d$,
(iv) $A_n(x)\to A(x)$ in matrix norm, dx-$a.e.$ on $\boldsymbol{R}^d$.

We consider the forms

$$\begin{aligned}\mathcal{E}^n(u,v)&=\int_{\boldsymbol{R}^d}(A_n(x)\nabla u(x),\nabla v(x))_d\phi_n^2(x)dx,\\ \mathcal{F}^n&=\{u\in L^2(\boldsymbol{R}^d;\phi_n^2dx):\nabla_i u\in L^2(\boldsymbol{R}^d;\phi_n^2dx), i=1,2,\cdots,d\},\end{aligned}\tag{1.1}$$

for $n=1,2,3,\cdots$,

$$\begin{aligned}\mathcal{E}(u,v)&=\int_{\boldsymbol{R}^d}(A(x)\nabla u(x),\nabla v(x))_d\phi^2(x)dx,\\ \mathcal{F}&=\{u\in L^2(\boldsymbol{R}^d;\phi^2dx):\nabla_i u\in L^2(\boldsymbol{R}^d;\phi^2dx), i=1,2,\cdots,d\},\end{aligned}\tag{1.2}$$

Our assumption (A.1) implies that the forms (1.1) and (1.2) are regular local Dirichlet forms on $L^2(\boldsymbol{R}^d;\phi_n^2dx)$ and $L^2(\boldsymbol{R}^d;\phi^2dx)$ (called "energy forms") respectively. It follows from M. Fukushima, Y. Oshima and M. Takeda [4] that there exist diffusion processes $M^n=\{X_t,P_x^n,x\in\boldsymbol{R}^d\}$ and $M^n=\{X_t,P_x,x\in\boldsymbol{R}^d\}$ associated with $\mathcal{E}^n$ and $\mathcal{E}$ respectively. Further, we consider the following condition:

(A.2) there exists a constant $c>0$ such that $\sup_n\int_{B_r}\phi_n^2dx\leq e^{cr^2}$, for all $r>0$.

Then by condition (A.2) and Theorem 2.2 in M. Takeda [8], these processes are conservative. For every relatively compact open set G of $\boldsymbol{R}^d$, we consider the Dirichlet forms of part on G associated with (1.1) and (1.2):

$$\begin{aligned}\mathcal{E}^{n,G}(u,v)&=\int_G(A_n(x)\nabla u(x),\nabla v(x))_d\phi_n^2(x)dx,\\ \mathcal{F}_G^n&=H_0^1(G)\quad \text{on } L^2(G;\phi_n^2dx),\end{aligned}\tag{1.3}$$

for $n=1,2,3,\cdots$,

$$\mathscr{E}^G(u,v)=\int_G (A(x)\nabla u(x),\nabla v(x))_d\phi^2(x)dx,$$
$$\mathscr{F}_G=H_0^1(G) \quad \text{on } L^2(G;\phi^2 dx), \tag{1.4}$$

Now take strictly positive functions f_n of $L^1(\boldsymbol{R}^d;\phi_n^2 dx)$ and f of $L^1(\boldsymbol{R}^d;\phi^2 dx)$ and assume the conditions below:

(A.3) (i) $\int_{\boldsymbol{R}^d} dm_n=\int_{\boldsymbol{R}^d} dm=1$, where $dm_n=f_n\phi_n^2 dx$ and $dm=f\phi^2 dx$,

(ii) for any compact set K, $\sup_n \|f_n\|_{L^\infty(K;\phi_n^2 dx)}<\infty$,

(iii) $f_n(x)\to f(x)$, dx-*a.e.* on $\boldsymbol{R}^d$.

It follows from conditions (A.2), (A.3) and Theorem 3.1 in M. Takeda [8] that the sequence of probability measures $\{P^n_{m_n}, n=1,2,\cdots\}$ is tight on $C([0,\infty)\to\boldsymbol{R}^d)$. Moreover we can assert as follows:

Theorem. *Assume the conditions* (A.1)-(A.3). *Then* $\{P^n_{m_n}, n=1,2,\cdots\}$ *converges weakly to* P_m *on* $C([0,\infty)\to\boldsymbol{R}^d)$.

2. Proof of Theorem

In order to carry out the proof of Theorem, we need some lemmas and notations.

Henceforth, for a form $(\mathscr{E},\mathscr{D}(\mathscr{E}))$ on a Hilbert space $\mathscr{H}$, we let $\mathscr{E}(u,u)=\infty$ for every $u\in\mathscr{H}-\mathscr{D}(\mathscr{E})$. Here a form means a non-negative definite symmetric form on $\mathscr{H}$, not necessarily densely defined. As was mentioned in the introduction, we use the notion of the Mosco-convergence of forms, which is defined as follows:

DEFINITION. A sequence of forms $\mathscr{E}^n$ on a Hilbert space $\mathscr{H}$ is said to be Mosco-convergent to a form $\mathscr{E}$ on $\mathscr{H}$ if the following conditions are satisfied;

(M.1) for every sequence u_n weakly convergent to u in $\mathscr{H}$,

$$\liminf_{n\to\infty}\mathscr{E}^n(u_n,u_n)\geq\mathscr{E}^n(u,u).$$

(M.2) for every u in $\mathscr{H}$, there exists u_n converging to u in $\mathscr{H}$, such that

$$\limsup_{n\to\infty}\mathscr{E}^n(u_n,u_n)\geq\mathscr{E}(u,u).$$

In [6], U. Mosco showed that a sequence of closed forms $\mathscr{E}^n$ on a Hilbert space

$\mathscr{H}$ is Mosco-convergent to a closed form $\mathscr{E}$ on $\mathscr{H}$ if and only if the resolvents associated with $\mathscr{E}^n$ converges to the resolvent associated with $\mathscr{E}$ strongly on $\mathscr{H}$.

In order to use Mosco's theorem, we introduce related forms:

$$\begin{aligned}&\mathscr{A}^{n,G}(u,v)=\mathscr{E}^{n,G}\left(\frac{u}{\phi_n},\frac{v}{\phi_n}\right)=\int_G\left(A_n(x)\nabla\frac{u(x)}{\phi_n(x)},\nabla\frac{v(x)}{\phi_n(x)}\right)_d\phi_n^2(x)dx,\\&\mathscr{D}(\mathscr{A}^{n,G})=\{u\in L^2(G;dx):u/\phi_n\in H_0^1(G)\}\end{aligned}\tag{2.1}$$

for $n=1,2,3,\cdots$,

$$\begin{aligned}&\mathscr{A}^{G}(u,v)=\mathscr{E}^{G}\left(\frac{u}{\phi},\frac{v}{\phi}\right)=\int_G\left(A(x)\nabla\frac{u(x)}{\phi(x)},\nabla\frac{v(x)}{\phi(x)}\right)_d\phi^2(x)dx,\\&\mathscr{D}(\mathscr{A}^{G})=\{u\in L^2(G;dx):u/\phi\in H_0^1(G)\}.\end{aligned}\tag{2.2}$$

By the unitary map $f\mapsto\phi_n^{-1}f$ between $L^2(G;dx)$ and $L^2(G;\phi_n^2dx)$ and by the condition (A.1), the forms $\mathscr{A}^{n,G}$ and $\mathscr{A}^G$ are closed on $L^2(G;dx)$.

Lemma. *Assume the condition* (A.1). *Then the forms $\mathscr{A}^{n,G}$ is Mosco-convergent to the form $\mathscr{A}^G$ on $L^2(G;dx)$.*

Proof. We have to check the conditions (M.1) and (M.2).

First we note that, from the condition (A.1), there exist $(d\times d)$ symmetric matrix valued functions $\sqrt{A_n(x)}=(\sigma_{ij}^n(x))$ and $\sqrt{A(x)}=(\sigma_{ij}(x))$ defined on $\boldsymbol{R}^d$ with the following properties:

(i) $A_n(x)=(\sqrt{A_n(x)})^2,\ A(x)=(\sqrt{A(x)})^2$,

(ii) $\sqrt{A_n(x)}\to\sqrt{A(x)}$ in matrix norm, dx-*a.e.* on $\boldsymbol{R}^d$.

In particular, $|\sqrt{A_n(x)}\xi|\le\sqrt{\delta}|\xi|, dx-a.e. x\in\boldsymbol{R}^d,\xi\in\boldsymbol{R}^d,n\in\boldsymbol{N}$. Hence $\sigma_{ij}^n(x)$ is uniformly bounded on $\boldsymbol{R}^d$ and converges to $\sigma_{ij}(x)dx$-*a.e.* as $n\to\infty$ for each i,j.

Proof of (M.1). Suppose $u_n\to u$ weakly in $L^2(G;dx)$. We may assume

$$\liminf\mathscr{A}^{n,G}(u_n,u_n)<\infty.$$

Then we have

$$\begin{aligned}+\infty&>\liminf_{n\to\infty}\mathscr{A}^{n,G}(u_n,u_n)\\&=\liminf_{n\to\infty}\int_G\left(A_n\nabla\left(\frac{u_n}{\phi_n}\right),\nabla\left(\frac{u_n}{\phi_n}\right)\right)_d\phi_n^2dx\\&\ge\frac{\lambda(G)^2}{\delta}\liminf_{n\to\infty}\int_G\left|\nabla\left(\frac{u_n}{\phi_n}\right)\right|^2dx,\end{aligned}$$

and we can take a subsequence $\{n_k\}$ such that $\nabla_i\left(\frac{u_{n_k}}{\phi_{n_k}}\right)$ is weakly convergent to an element $h_i \in L^2(G;dx)$ for each $i=1,2,\cdots,d$ and $\liminf_{n\to\infty} \mathscr{A}^{n,G}(u_n,u_n)=\lim_{k\to\infty} A^{n_k,G}(u_{n_k},u_{n_k})$.

On the other hand, for all $\eta \in C_0^\infty(G)$, $\int_G \nabla_i\left(\frac{u_{n_k}}{\phi_{n_k}}\right)\eta dx = -\int_G \frac{u_{n_k}}{\phi_{n_k}}\nabla_i \eta dx$, and u_n/ϕ_n converges to u/ϕ weakly in $L^2(G;dx)$, because ϕ_n^{-1} is uniformly bounded and converges to ϕ^{-1} *dx-a.e.* on G. This shows that

$$\int_G h_i \eta dx = -\int_G \frac{u}{\phi}\nabla_i \eta dx, \qquad \text{for all } \eta \in C_0^\infty(G).$$

Thus we have $h_i=\nabla_i\left(\frac{u}{\phi}\right)$, $i=1,2,\cdots,d$, and in particular $u\in\mathscr{D}(\mathscr{A}^G)$.

Furthermore $\Sigma_{j=1}^d \sigma_{ij}^{n_k}\nabla_j\left(\frac{u_{n_k}}{\phi_{n_k}}\right)\phi_{n_k}$ converges to $\Sigma_{j=1}^d \sigma_{ij}\nabla_j\left(\frac{u}{\phi}\right)\phi$ weakly in $L^2(G;dx)$, since $\sigma_{ij}^n\phi_n$ is uniformly bounded and converges to $\sigma_{ij}\phi$ *dx-a.e.* on G as $n\to\infty$ for each i, j.
Consequently,

$$\begin{aligned}
\liminf_{n\to\infty} \mathscr{A}^{n,G}(u_n,u_n) &= \lim_{n_k\to\infty} \mathscr{A}^{n_k,G}(u_{n_k},u_{n_k}) \\
&= \lim_{n_k\to\infty} \int_G \left(A_{n_k}\nabla\left(\frac{u_{n_k}}{\phi_{n_k}}\right),\nabla\left(\frac{u_{n_k}}{\phi_{n_k}}\right)\right)_d \phi_{n_k}^2 dx \\
&= \lim_{n_k\to\infty} \int_G \left|\sqrt{A_{n_k}}\nabla\left(\frac{u_{n_k}}{\phi_{n_k}}\right)\right|^2 \phi_{n_k}^2 dx \\
&= \lim_{n_k\to\infty} \sum_{i=1}^{d} \left\| \sum_{j=1}^{d} \sigma_{ij}^{n_k}\nabla_j\left(\frac{u_{n_k}}{\phi_{n_k}}\right)\phi_{n_k}\right\|_{L^2(G;dx)}^2 \\
&\geq \sum_{i=1}^{d} \left\| \sum_{j=1}^{d} \sigma_{ij}\nabla_j\left(\frac{u}{\phi}\right)\phi\right\|_{L^2(G;dx)}^2 = \mathscr{A}^G(u,u).
\end{aligned}$$

Proof of (M.2). Let u be in $\mathscr{D}(\mathscr{A}^G)$, that is, $u\in L^2(G;dx)$ and $u/\phi\in H_0^1(G)$. Accordingly there exists a sequence $\{\eta_n\}$ in $C_0^\infty(G)$ such that $\|u/\phi-\eta_n\|_{H^1(G)}$ converges to 0 as $n\to\infty$. Put $u_n=\phi_n\eta_n$. Then we can see that $u_n\to u$ in $L^2(G;dx)$. Further, using again the property of the sequence $\sigma_{ij}^n\phi_n$ observed above, we get that

$$\sum_{j=1}^{d} \sigma_{ij}^n(\nabla_j\eta_n)\phi_n \to \sum_{j=1}^{d}\sigma_{ij}\nabla_j\left(\frac{u}{\phi}\right)\phi \text{ in } L^2(G;dx), \quad \text{for } i=1,2,\cdots,d.$$

Therefore we have

$$\mathscr{A}^{n,G}(u_n,u_n)=\int_G (A_n\nabla\left(\frac{u_n}{\phi_n}\right),\nabla\left(\frac{u_n}{\phi_n}\right))_d\phi_n^2dx$$
$$=\int_G |\sqrt{A_n}\nabla\eta_n|^2\phi_n^2dx=\sum_{i=1}^d \|\sum_{j=1}^d \sigma_{ij}^n(\nabla_j\eta_n)\phi_n\|^2_{L^2(G;dx)}$$
$$\to \sum_{i=1}^d \|\sum_{j=1}^d \sigma_{ij}\nabla_j\left(\frac{u}{\phi}\right)\phi\|^2_{L^2(G;dx)}=\mathscr{A}^G(u,u), \quad n\to\infty.$$

q.e.d.

This lemma shows that, if we let $H^{n,G}$ and H^G be the selfadjoint operators associated with the forms $\mathscr{A}^{n,G}$ and $\mathscr{A}^G$ respectively, then $H^{n,G}$ converges to H^G in the strong resolvent sense, hence, in the semigroup sense on $L^2(G;dx)$ by Mosco's theorem.

Let $H^{n,G}_{\phi_n}$ and H^G_ϕ also denote the selfadjoint operators associated with the forms $\mathscr{E}^{n,G}$ and $\mathscr{E}^G$ respectively. Then by the unitary map $f\mapsto\phi_n^{-1}f$ between $L^2(G;dx)$ and $L^2(G;\phi_n^2dx)$, $H^{n,G}=\phi_nH^{n,G}_{\phi_n}\phi_n^{-1}$.

On the other hand, let $M^{n,G}=\{X_t,P^{n,G}_x,x\in G\}$ and $M^G=\{X_t,P^G_x,x\in G\}$ be the diffusion processes associated with the forms $\mathscr{E}^{n,G}$ and $\mathscr{E}^G$ respectively. Because $\mathscr{E}^{n,G}$ is the part of $\mathscr{E}^n$ on G as we have already noted, the behaviour of the process $\{X_t,P^n_x,x\in \boldsymbol{R}^d\}$ is the same as that of $\{X_t,P^{n,G}_x,x\in G\}$ before it leaves G for each n.

Now we can give the proof of Theorem:

Proof of Theorem. By Lemma and the argument following it, we see that $\phi_ne^{-tH^{n,G}_{\phi_n}}\phi_n^{-1}$converges to $\phi e^{-tH^G_\phi}\phi^{-1}$ strongly on $L^2(G;dx)$. Here $e^{-tH^{n,G}_{\phi_n}}$ and $e^{-tH^G_\phi}$ denotes the semigroups associated with $\mathscr{E}^{n,G}$ and $\mathscr{E}^G$ respectively. Therefore, by virtue of Theorem 7 in [1], $P^{n,G}_{m_n}$ converges to P^G_m in the finite dimensional distribution sense.

On the other hand, one has from condition (A.2) and Lemma 2.1 in [8] that

$$\lim_{r\to\infty}\sup_n P^n_{I_{B_R}\phi_n^2dx}\left(\sup_{0\le t\le T}(|X_t|-|X_0|)\ge r\right)=0, \quad \text{for all } R>0,T>0.$$

Then, for any $0<t_1<t_2\cdots<t_p$, $A_i\in\mathscr{B}(\boldsymbol{R}^d)$, $i=1,2,\cdots,p$ and $\varepsilon>0$, there exists an $r>0$ such that $\sup_nP^n_{m_n}(t_p\ge\tau_r)<\varepsilon/2$. Moreover, we can see that $P_m(t_p\ge\tau_r)<\varepsilon/2$. Here τ_r denotes the exit time for the open ball B_r with radius r and center O.

Let $\Lambda=\{X_{t_1}\in A_1,X_{t_2}\in A_2,\cdots,X_{t_p}\in A_p\}$. Then we see that

$$\begin{aligned}|P^n_{m_n}(\Lambda)-P_m(\Lambda)|\le&|P^n_{m_n}(\Lambda)-P^n_{m_n}(\Lambda\cap\{t_p<\tau_r\})|\\&+|P^n_{m_n}(\Lambda\cap\{t_p<\tau_r\})-P_m(\Lambda\cap\{t_p<\tau_r\})|\\&+|P_m(\Lambda\cap\{t_p<\tau_r\})-P_m(\Lambda)|\end{aligned}$$

$$\leq P^n_{m_n}(t_p \geq \tau_r) + P_m(t_p \geq \tau_r)$$
$$+ |P^n_{m_n}(\Lambda \cap \{t_p < \tau_r\}) - P_m(\Lambda \cap \{t_p < \tau_r\})|.$$

The first and second term of the right hand side are less than ε. Since the last term is the finite dimensional distribution of $\boldsymbol{M}^{n,B_r}$ and $\boldsymbol{M}^{B_r}$, we conclude that $P^n_{m_n}$ converges to P_m in the finite dimensional distribution sense.

We have already noted the tightness of $\{P^n_{m_n}\}$ on $C([0,\infty) \to \boldsymbol{R}^d)$. Thus the proof of Theorem is completed. q.e.d.

Example. Let f be a locally bounded measurable function on $\boldsymbol{R}^d$, and consider a mollifier, e.g., $j(x) = \gamma \exp(-1/1-|x|^2)$ for $|x|<1$, $j(x)=0$ for $|x| \geq 1$, where γ is a constant to make $\int_{\boldsymbol{R}^d} j(x)dx = 1$. We put $j_\varepsilon(x) = j(x/\varepsilon)/\varepsilon^d, f_\varepsilon(x) = \int_{\boldsymbol{R}^d} j_\varepsilon(x-y)f(y)dy$, for any $\varepsilon > 0$. Since f_ε converges to f in $L^2(G;dx)$ for each relatively compact open set G, we can take a sequence ε_n converging to 0 such that f_{ε_n} converges to f, dx-$a.e.$ on $\boldsymbol{R}^d$. Thus if we set $\phi_n(x) = \exp f_{\varepsilon_n}(x), \phi(x) = \exp f(x)$, and assume that there exists a constant $c>0$ with $\int_{B_r} e^{2f(x)}dx \leq e^{cr^2}$, for $r>0$, then ϕ_n, ϕ satisfies the conditions (A.1) and (A.2). Therefore we have the weak convergence statement for the processes associated with ϕ_n, ϕ and $A_n = A =$ identity matrix.

ACKNOWLEDGEMENT. The author would like to express his gratitude to Professor M. Fukushima for helpful advice. The author thanks Professor M. Takeda, who encourages him to study this subject and gives him valuable comments. The author is also grateful to Professors N. Jacob, I. Shigekawa and K. Kuwae for helpful suggestions.

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