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DETERMINATION OF THE PLANT LOCATONS FOR ENSURING SOME ENVIRONMENTAL CRITERIA

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ABSTRACT

In this paper the following problems are presented:

- Algorithms for solving the two-dimensional matter propagation and its adjoint problems,
- Stability of the difference schemes and the non-negative property of numerical solution,
- Determination of the plant locations so that some environmental criteria are satisfied,
- Numerical experiments for the test cases and for Halong Bay area.

Keywords: Partial differential equations, finite difference schemes.

1. EQUATION OF THE SUSPENDED MATTER PROPAGATION AND ITS ADJOINT EQUATION

1.1. Governing equations

The equation describing the suspended matter diffusion and transport in the horizontal 2D case has the following form (see [1]):

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\sigma C=f+\gamma\Delta C \qquad (x,y)\in G, 0<t\le T \tag{1}$$

with the initial and boundary conditions:

$$C\big|_{t=0}=C^0,\quad C\big|_{\Gamma^-}=\varphi,\quad \frac{\partial C}{\partial n}\Big|_{\Gamma^+}=0 \tag{2}$$

where: x, y, t *- space and time variables,*

(u,v) *- velocity that satisfies the condition:*

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{3}$$

C - matter concentration,

σ *- decay coefficient,*

f *- source intensity,*

γ *- diffusion coefficient;*

$\Gamma=\Gamma^+ +\Gamma^-$, Γ^+ *- boundary part, at which* $u_n\ge 0$ *;* Γ^- *- boundary part, at which* $u_n<0$, u_n *- projection of the velocity on the external normal vector* $\vec{n}$.

$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ *- Laplace operator.*

Solution of the equation (1) may be determined under the form: $C=C_1+C_2$

where, C_1 and C_2 are solutions of two following problems:

Problem 1:

$$\frac{\partial C_1}{\partial t}+u\frac{\partial C_1}{\partial x}+v\frac{\partial C_1}{\partial y}+\sigma C_1=\gamma\Delta C_1 \tag{4}$$

with the initial and boundary conditions:

$$C_1\big|_{t=0}=C^0,\quad C_1\big|_{\Gamma^-}=\varphi,\quad \frac{\partial C_1}{\partial n}\Big|_{\Gamma^+}=0 \tag{5}$$

Problem 2:

$$\frac{\partial C_2}{\partial t}+u\frac{\partial C_2}{\partial x}+v\frac{\partial C_2}{\partial y}+\sigma C_2=\gamma\Delta C_2+f \tag{6}$$

with the initial and boundary conditions:

$$C_2\,|_{t=0}=0,\qquad C_2\,|_{\Gamma^-}=0,\qquad \frac{\partial C_2}{\partial n}\,|_{\Gamma^+}=0. \tag{7}$$

It is well known that the adjoint equation of the equation (6), (7) has the following form (see [1], [2], [7]):

$$-\frac{\partial C_2^*}{\partial t}-u\frac{\partial C_2^*}{\partial x}-v\frac{\partial C_2^*}{\partial y}+\sigma C_2^*-\gamma\Delta C_2^*=p \tag{8}$$

and the initial and boundary conditions of the equation (7) be chosen as follows:

$$C_2^*\,|_{t=T}=0,\qquad C_2^*\,|_{\Gamma^-}=0,\qquad \left.\left(\gamma\frac{\partial C_2^*}{\partial n}+u_n C_2^*\right)\right|_{\Gamma^+}=0 \tag{9}$$

We have the dual form (see [1], [2], [7]) for the problem (6), (7) and adjoint problem (8), (9):

$$\int_0^T dt\int_G pC_2 dG=\int_0^T dt\int_G fC_2^* dG \tag{10}$$

From equation (8), using a variable transformation $t_1=T-t$, we obtain the another form of the adjoint equation:

$$\frac{\partial C_2^*}{\partial t_1}-u\frac{\partial C_2^*}{\partial x}-v\frac{\partial C_2^*}{\partial y}+\sigma C_2^*-\gamma\Delta C_2^*=p \tag{11}$$

$$C_2^*\,|_{t_1=0}=0,\qquad C_2^*\,|_{\Gamma^-}=0,\qquad \left.\left(\gamma\frac{\partial C_2^*}{\partial n}+u_n C_2^*\right)\right|_{\Gamma^+}=0 \tag{12}$$

1.2. Algorithm (see [2]-[6])

The equation (6) and the adjoint equation (11) may be rewritten in a common form:

$$\frac{\partial C}{\partial t}+\Lambda C=f \tag{13}$$

where, $\Lambda=\Lambda_1+\Lambda_2$, $\Lambda_1=\pm u\frac{\partial}{\partial x}-\gamma\frac{\partial^2}{\partial x^2}+\frac{\sigma}{2}$, $\Lambda_2=\pm v\frac{\partial}{\partial y}-\gamma\frac{\partial^2}{\partial y^2}+\frac{\sigma}{2}$

Equation (13) may be solved by one of two methods of the directional decomposition (splitting method):

1.2.1. First method:

$$\frac{C^{k+1}-C^k}{dt}+\Lambda\left[\theta C^{k+1}+(1-\theta)C^k\right]=f^{k+1}$$

or $(I+dt\theta\Lambda)C^{k+1}=\left[I-dt(1-\theta)\Lambda\right]C^k+dt\,f^{k+1}$ (14)

where $0\le\theta\le1$, I is the unique operator. Using approximation:

$$\left[I+dt\theta(\Lambda_1+\Lambda_2)\right]=(I+dt\theta\Lambda_1)(I+dt\theta\Lambda_1)+O(dt^2)$$

from (14), one deduces:

$$(I+dt\theta\Lambda_1)(I+dt\theta\Lambda_2)C^{k+1}=\left[I-dt(1-\theta)\Lambda\right]C^k+dtf^{k+1}$$

The computational process includes two steps:

$$(I+dt\theta\Lambda_1)C^{k+1/2}=\left[I-dt(1-\theta)\Lambda\right]C^k+dt\,f^{k+1} \tag{15}$$

$$(I+dt\theta\Lambda_2)C^{k+1}=C^{k+1/2} \tag{16}$$

1.2.2 Second method:

$$\frac{C^{k+1/2}-C^{k}}{dt}+\overline{\Lambda}_1\left[\theta C^{k+1/2}+(1-\theta)C^{k}\right]+a\sigma C^{k+1/2}=af^{k+1/2} \quad (17)$$

$$\frac{C^{k+1}-C^{k+1/2}}{dt}+\overline{\Lambda}_2\left[\theta C^{k}+(1-\theta)C^{k+1/2}\right]+(1-a)\sigma C^{k+1}=(1-a)f^{k+1} \quad (18)$$

where, $0\le a\le 1;\ \overline{\Lambda}_1=\Lambda_1-\frac{\sigma}{2};\ \overline{\Lambda}_2=\Lambda_2-\frac{\sigma}{2}$

1.2.3. Discretizing the equations (15) and (17) by an implicit finite difference scheme in the x - direction:

$$\left(u\frac{\partial C}{\partial x}\right)_{m,n}^{k+1/2}=\frac{(u+|u|)_{m,n}^{k+1}}{2}\frac{C_{m,n}^{k+1/2}-C_{m-1,n}^{k+1}}{dx}+\frac{(u-|u|)_{m,n}^{k+1/2}}{2}\frac{C_{m+1,n}^{k+1/2}-C_{m,n}^{k+1/2}}{dx}$$

$$\left(\frac{\partial^2 C}{\partial x^2}\right)_{m,n}^{k+1/2}=\frac{C_{m+1,n}^{k+1/2}-2C_{m,n}^{k+1/2}+C_{m-1,n}^{k+1/2}}{dx^2}$$

we obtain: $a_m C_{m+1,n}^{k+1/2}+b_m C_{m,n}^{k+1/2}+c_m C_{m-1,n}^{k+1/2}=d_m$ (19)

where, a_m, b_m, c_m are known values satisfying the following conditions :

$b_m>0,\ a_m<0,\ c_m<0$ and $|b_m|\ge|a_m|+|c_m|+\delta,\ \delta>1$ (20)

So, the linear equation system (19) has the unique solution and the computational error of the following double sweep method:

$$C_{m,n}^{k+1}=L_m C_{m+1,n}^{k+1}+K_m \quad (21)$$

where, $L_m=\frac{-a_m}{b_m+c_m L_{m-1}}$, $K_m=\frac{d_m-c_m K_{m-1}}{b_m+c_m L_{m-1}}$,

is not accumulated (see [7]).

1.2.4. Discretizing the equations (16) and (18) by a difference scheme in the y direction:

$$\left(v\frac{\partial C}{\partial y}\right)_{m,n}^{k+1}=\frac{(v+|v|)_{m,n}^{k+1}}{2}\frac{C_{m,n}^{k+1}-C_{m,n-1}^{k+1}}{dy}+\frac{(v-|v|)_{m,n}^{k+1}}{2}\frac{C_{m,n+1}^{k+1}-C_{m,n}^{k+1}}{dy}$$

$$\left(\frac{\partial^2 C}{\partial y^2}\right)_{m,n}^{k+1}=\frac{C_{m,n+1}^{k+1}-2C_{m,n}^{k+1}+C_{m,n-1}^{k+1}}{dy^2}$$

we also get:

$$\tilde{a}_n C_{m,n+1}^{k+1}+\tilde{b}_n C_{m,n}^{k+1}+\tilde{c}_n C_{m,n-1}^{k+1}=\tilde{d}_n \quad (22)$$

where, $\tilde{a}_m, \tilde{b}_m, \tilde{c}_m$ are known values satisfying the following conditions :

$\tilde{b}_m>0,\ \tilde{a}_m<0,\ \tilde{c}_m<0$ and $|\tilde{b}_m|\ge|\tilde{a}_m|+|\tilde{c}_m|+\delta,\ \delta>1$ (23)

Also, the equation system (22) has the unique solution and the double sweep method (21) does not produce an accumulated computational error.

1.3. Stability of the finite-difference schemes and non-negative property of the numerical solutions

1.3.1. Stability of the difference scheme

Suppose that $u = const, v = const, \delta = const, \gamma = const, \theta = 1$, then $a_m = const, b_m = const$, $c_m = const$ and $d_m = C^k_{m,n} + dt\, f^{k+1}_{m,n}$. Difference schemes satisfy the necessary and sufficient conditions of the stability.

a) Necessary condition

Let $f = 0$ and the solution C is defined by the form:

$$C^k_{m,n} = \lambda^k_x C^0_n e^{im\varphi},\ i^2 = -1,\ \varphi \in [0, 2\pi) \quad (24)$$

then

$$C^{k+1/2}_{m,n} = \frac{\lambda^{k+1/2}_x}{\lambda^k_x} C^k_{m,n} \quad (25)$$

Putting (24) into (19) it yields

$$\lambda^{k+1/2}_x [b_m + (a_m + c_m)\cos\varphi + i(a_m - c_m)\sin\varphi] = \lambda^k_x \quad (26)$$

From (20) we have:

$$b_m + (a_m + c_m)\cos\varphi \geq \delta + |a_m| + |c_m| + (a_m + c_m)\cos\varphi > 1$$

Therefore

$$\left|\frac{\lambda^{k+1/2}_x}{\lambda^k_x}\right| = \frac{1}{\sqrt{[b_m + (a_m + c_m)\cos\varphi]^2 + (a_m - c_m)^2 \sin^2\varphi}} < 1$$

Similarly, from equation (22), (23) one deduces

$$C^{k+1}_{m,n} = \frac{\lambda^{k+1}_y}{\lambda^{k+1/2}_y} C^{k+1/2}_{m,n} \quad (27)$$

and

$$\left|\frac{\lambda^{k+1}_y}{\lambda^{k+1/2}_y}\right| < 1.$$

From (25) and (27), we obtain:

$$C^{k+1}_{m,n} = \lambda^k C^k_{m,n}$$

where

$$|\lambda^k| = \left|\frac{\lambda^{k+1}_y}{\lambda^{k+1/2}_y}\right| \left|\frac{\lambda^{k+1/2}_x}{\lambda^k_x}\right| < 1.$$

b) Sufficient condition

Let $\left|C^{k+1/2}_{m_0,n_0}\right| = \sup\limits_{1\le m\le M-1} \sup\limits_{1\le n\le N-1} \left|C^{k+1/2}_{m,n}\right|$

From equation (19), (20) we get:

$$\left|b_m C^{k+1/2}_{m_0,n_0} - |a_m| C^{k+1/2}_{m_0+1,n_0} - |c_m| C^{k+1/2}_{m_0-1,n_0}\right| = \left|C^k_{m_0,n_0} + dt\, f^k_{m_0,n_0}\right|,$$

$$\sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C^{k+1/2}_{m,n}\right| \le \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C^k_{m,n}\right| + dt \sup_k \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|f^k_{m,n}\right|$$

Similarly, from equation (22), (23) we get:

$$\sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C^{k+1}_{m,n}\right| \le \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C^{k+1/2}_{m,n}\right|$$

Therefore

$$\begin{aligned}\sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C^{k+1}_{m,n}\right| &\le \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C^k_{m,n}\right| + dt \sup_k \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|f^k_{m,n}\right| \\ &\le \ldots \le \sup_{1\le m\le M} \sup_{1\le n\le N} \left|C^0_{m,n}\right| + dt(k+1) \sup_k \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|f^k_{m,n}\right|\end{aligned} \quad (28)$$

For second difference scheme (17), (18) we also get inequality (28)

If the boundary condition is an function φ, then

$$C_{0,n}^k = \varphi_{0,n}^k,\ C_{M,n}^k = \varphi_{M,n}^k,\ C_{m,0}^k = \varphi_{m,0}^k,\ C_{m,N}^k = \varphi_{m,N}^k$$

If the boundary condition is $\dfrac{\partial C}{\partial n} = 0$, $C_{0,0}^k = C_{1,1}^k,\ C_{0,N}^k = C_{1,N-1}^k$, $C_{M,0}^k = C_{M-1,1}^k$, and $C_{M,N}^k = C_{M-1,N-1}^k$, we obtain

$$\sup_n \left|C_{0,n}^k\right| \le \sup_{1\le m\le M-1}\ \sup_{1\le n\le N-1} \left|C_{m,n}^k\right| = A$$

$$\sup \left|C_{M,n}^k\right| \le A,\quad \sup \left|C_{m,0}^k\right| \le A,\quad \sup \left|C_{m,N}^k\right| \le A \tag{29}$$

Let the norms of the functions are defined as follow:

$$\|C\| = \sup_k \sup_{0\le m\le M} \sup_{0\le n\le N} \left|C_{m,n}^k\right| = \sup_k \sup_m \sup_n \left|C_{m,n}^k\right|$$

$$\left\|C^0\right\| = \sup_m \sup_n \left|C_{m,n}^0\right|$$

$$\|f\| = \sup_k \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|f_{m,n}^k\right| \tag{30}$$

$$\|\varphi\| = \max\left\{\sup_k \sup_n \left|\varphi_{0,n}^k\right|, \sup_k \sup_n \left|\varphi_{M,n}^k\right|, \sup_k \sup_m \left|\varphi_{m,0}^k\right|, \sup_k \sup_m \left|\varphi_{m,N}^k\right|\right\}$$

Then, from the inequality (28), (29) and (30), we obtain:

$$\sup_k \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C_{m,n}^k\right| \le \left\|C^0\right\| + T\|f\| \tag{31}$$

$$\max\left\{\sup_k \sup_n \left|C_{0,n}^k\right|, \sup_k \sup_n \left|C_{M,n}^k\right|, \sup_k \sup_m \left|C_{m,0}^k\right|, \sup_k \sup_m \left|C_{m,N}^k\right|\right\} \le \max\left\{\|\varphi\|, \left\|C^0\right\| + T\|f\|\right\} \tag{32}$$

From (30), (31) and (32) we get:

$$\|C\| = \max\left\{\sup_k \sup_{1\le m\le M-1} \sup_{1\le n\le N-1} \left|C_{m,n}^k\right|, \sup_k \sup_n \left|C_{0,n}^k\right|, \sup_k \sup_n \left|C_{M,n}^k\right|, \sup_k \sup_m \left|C_{m,0}^k\right|, \sup_k \sup_m \left|C_{m,N}^k\right|\right\}$$

$$\le \max\left\{\left\|C^0\right\| + T\|f\|, \|\varphi\|\right\}$$

Therefore

$$\|C\| \le \left\|C^0\right\| + T\|f\| + \|\varphi\|$$

and the stability of the difference schemes is proved.

1.3.2. Non-negative property of the numerical solution

The equation (19) can be solved by the double sweep method

$$C_{m,n}^{k+1/2} = L_m C_{m+1,n}^{k+1/2} + K_m \tag{33}$$

where $\quad L_m = \dfrac{-a_m}{b_m + c_m L_{m-1}},\ K_m = \dfrac{d_m - c_m K_{m-1}}{b_m + c_m L_{m-1}};\ L_0 = 0, K_0 = C_{0,n}^{k+1/2}$

Using the inductive method, we can prove

$0 \le L_m < 1$ and $K_m \ge 0$. ($m=1,\ldots,M-1$)

Indeed, assumed that $0 \le L_{m-1} \le 1$ and $K_{m-1} \ge 0$. Let $\theta=1$, we obtain $d_m \ge 0$. From equation (20) and $f(x, y, t) \ge 0$, we get:

$$\begin{aligned} b_m + c_m L_{m-1} &= b_m - |a_m| - |c_m| + |a_m| + |c_m| - |c_m| L_{m-1} = \\ &= \delta + |a_m| + (1 - L_{m-1})|c_m| > |a_m| \end{aligned} \tag{34}$$

Therefore,

$$0 \le L_m = \frac{-a_m}{b_m + c_m L_{m-1}} = \frac{|a_m|}{b_m + c_m L_{m-1}} < \frac{|a_m|}{|a_m|} = 1 \tag{35}$$

$$K_m = \frac{d_m - c_m K_{m-1}}{b_m + c_m L_{m-1}} = \frac{d_m + |c_m| K_{m-1}}{b_m + c_m L_{m-1}} \ge 0. \tag{36}$$

From inequalities (35), (36) and the non-negative boundary conditions $C_b \geq 0$, we have:

$C_{m,n}^{k+1/2} = L_m C_{m+1,n}^{k+1/2} + K_m \geq 0$

Similarly, we also get $C_{m,n}^{k+1} \geq 0$. So we obtain the non-negative property of the numerical solution.

1.4. Comparison with the analytical solution

The matter propagation problem:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \sigma C - \gamma \Delta C = Q\delta(r - r_0)\delta(t - t_0), \qquad (37)$$

$$C = 0 \text{ for } t = 0$$

$$C \to 0 \text{ for } |r| \to \infty$$

with assumption: $u = const \geq 0$ and $v = const \geq 0$, has the following analytical solution (see [1] [2]):

$$C(x,y,t) = \begin{cases} \dfrac{Q}{4\pi\gamma(t - t_0)} \exp\{-\alpha(r - r_0, t - t_0)\}, & t \in (t_0, T], \\ 0, & t \in [0, t_0] \end{cases}$$

where

$$\alpha(r,t) = \sigma t + \frac{(x - ut)^2 + (y - vt)^2}{4\gamma t}$$

Let the computational region *G*, that containing source point r_0, is large enough so that *C=0* at the boundaries. The algorithm is applied for calculating the matter propagation problem of two test cases with and without the advection term:

1.4.1. Without the advection term (u=v=0)

The input parameters are as follows: $\gamma = 0.5\ m^2/s$, $\sigma = 0.01$ 1/s, $Q = 100$ mg/l/s, $t_0 = 10.$ s, $r_0 = (100,100)$, $dx = dy = 1$m, dt = 1. s. We obtain very good agreement between the computed and analytical solutions (see Fig. 1).

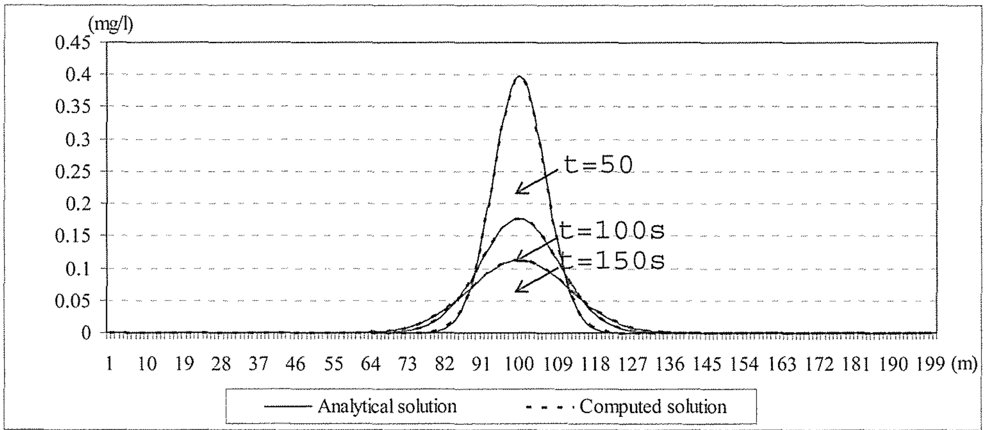

Figure 1. Concentration distribution along a ray passing the source point and parallel with the Ox axis, at t = 50 s, t = 100 s and t = 150 s

1.4.2. With the advection term

The input parameters are as follows: u = 0.5 m/s, v = 0. m/s, $\gamma = 0.5\ m^2/s$, $\sigma = 0.01$ 1/s, $Q = 100$ mg/l/s, $t_0 = 10.$ s, $r_0 = (30,100)$, $dx = dy = 1$m, dt = 1.2 s. Figure 2 shows an agreement between the computed and analytical solutions.

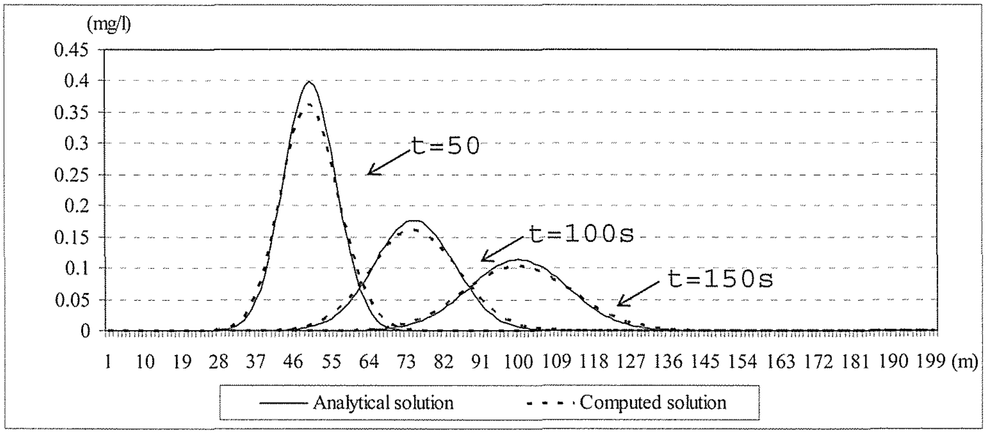

Figure 2. Concentration distribution along a ray passing the source point and parallel with the flow direction, at t = 50 s, t = 100 s and t = 150 s .

2. OPTIMIZATION PROBLEM OF PLANT LOCATION (SEE [1])

Assume that the suspended matter concentration C is calculated from the equation (1). We consider the following generalized functional called the pollution-level reflecting functional (see [1]):

$$Y_k = \int_0^T dt \int_G p_k\, C\, dG \quad \text{, where } p_k = \begin{cases} \frac{1}{T} + a_k & ,(x,y) \in G_k \\ 0 & ,(x,y) \notin G_k \end{cases}$$

and p_k is a function referring to the economic, sanitary, ecological, health standards and so on, a_k is a settling coefficient.

Let $G_k(k=1,2,...,m)$ be considered areas, recreation zones or other environmentally sensitive areas on the region G. Our problem is to determine the domain $\Omega_k \subset G$ so that the pollution matter from a plant located in this domain Ω_k satisfies the following condition for the sensitive area G_k:

$$Y_k \le c_k \qquad (38)$$

where, c_k is a given value.

Assume that on the region G there are m sensitive areas G_k (k=1, ..., m) and the source of matter emission is located at a point $r_o = (x_o, y_o)$. Then, the source intensity can be described by the function:

$$f(x,y) = Q\delta(r - r_0), \qquad Q = const$$

where, $\delta(r) = \begin{cases} \infty & , r = r_0 \\ 0 & , r \ne r_0 \end{cases}$ is Dirac function,

For the purpose of determination of the domain Ω, in which the plant can be located so that in all sensitive areas G_k , the generalized functional Y_k satisfies the condition (38), we take the following steps:

2.1. Step 1

Calculation of concentration C from the problem (4) and (5) and generalized functional:

$$Y_k = \int_0^T dt \int_{G_k} p\,C\,dG = \widetilde{c}_k$$

2.2. Step 2

Solving *m* adjoint equations (11):

$$\frac{\partial C_k^*}{\partial t_1} - u\frac{\partial C_k^*}{\partial x} - v\frac{\partial C_k^*}{\partial y} + \sigma C_k^* - \gamma \Delta C_k^* = p_k$$

where, $$p_k = \begin{cases} \frac{1}{T} + a_k & , (x,y) \in G_k \\ 0 & , (x,y) \notin G_k \end{cases}$$

with the conditions:

$$C_k^*\big|_{t_1=0} = 0, \quad C_k^*\big|_{\Gamma^-} = 0, \quad (\nu\frac{\partial C_k^*}{\partial n} + u_n C_k^*)\big|_{\Gamma^+} = 0$$

we obtain the solutions $C_k^*(k = 1,2,...,m)$. From the dual form (10), we get:

$$Y_k^* = \int_0^T dt \int_G p_k\,C\,dG = \int_0^T dt \int_G Q\delta(r - r_0)C_k^* dG = \int_0^T QC_k^*(r_0,t)dt = \int_0^T QC_k^*(r_0, T - t_1)dt_1$$

which must satisfy the condition: $Y_k^* \le c_k - \widetilde{c}_k = c_k^*$

Now we consider the function: $Y_k^*(r) = Q\int_0^T C_k^*(r,t)dt$ and draw the iso-grams of $Y_k^*(r) = const$.

We obtain the domains Ω_k in which if the plant is located, then the functional $Y_k^*(r) \le c_k^*$ in the area G_k. If there is perchance no area Ω_k inside *G*, it may be re-established anyway by reducing the discharge intensity *Q*.

Overlaying all the areas $\Omega_k(k = 1,...,m)$, we obtain the domain Ω, $(\Omega = \bigcap^m \Omega_k)$. Ω will be the domain in which the plant can be located so that pollution standards will be met in all the areas $G_k \subset G$, (k = 1, 2, ...*m*).

3. NUMERICAL EXPERIMENTS

The first mentioned-above method is applied to solve the following two optimization problems of plant location:

3.1. Test case 1

The computed rectangular region *G* = 1000m x 1000m is covered by a uniform grid 51x51 with spacing steps: *dx* = 20m, *dy* = 20m. A constant velocity field (*u*,*v*): *u* = 0.5 m/s, *v* = -0.5 m/s. Diffusion coefficient : $\gamma = 0.5m^2/s$. Decay coefficient : $\sigma = 0.0005s^{-1}$. Time step: *dt* = 5 s. Time simulation : *T* = 20000 s. 3 considered sensitive rectangular areas G_k inside *G* (*k*=1,2,3) with the left-bottom corner coordinates and the right-top corner coordinates are as follows:

G_1 = [(24.5,8.5),(25.5,9.5)],
G_2 = [(37.5,12.5),(39.5,14.5)],
G_3 = [(29.5,33.5),(30.5,34.5)].

And standard concentration: $c_k^* = 10mg/l$ (k=1,2,3).

The numerical results are illustrated in Fig. 3. In this figure, the number on the contour lines indicates value of the pollution-level reflecting functionals Y_k^*. As a result, the domain Ω where the plant can be located so that the sanitary condition in the all areas G_k are satisfied (that means $Y_k^* \le c_k^*$) is in white.

3.2. Test case 2

The computed area, Ha Long Bay, is covered by a uniform grid 69x45 with spacing steps: dx = 1000m, dy = 1000m. Diffusion coefficient: $\gamma = 10m^2/s$. The decay coefficient: $\sigma = 0.001$ 1/s. Time step: dt = 10s. Simulation time: T = 24 h. The current is determined by solving the Navier-Stockes equation for the incompressible water as the follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = \eta\,\Delta u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = \eta\,\Delta v$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

with η is the viscosity of water.

3 considered sensitive rectangular areas G_k inside G (k=1,2,3) with the left-bottom corner coordinates and the right-top corner coordinates are as follows:

G_1 = [(13.5,13.5),(16.5,16.5)]- DoSon beach area,
G_2 = [(25.5,34.5),(26.5,35.5)]- HaLong beach area,
G_3 = [(33.5,20.5),(35.5,22.5)]- a some area.

And standard concentration: $c_k^* = 10 mg/l$ (k=1,2,3).

The numerical results are illustrated in Fig. 4. Also, in this figure, the number on the contour lines indicates value of the pollution level-reflecting functionals Y_k^*. Consequently, the domain Ω where the plant can be located so that the sanitary standards in the all areas G_K are satisfied (that means $Y_k^* \le c_k^*$) is also in white.

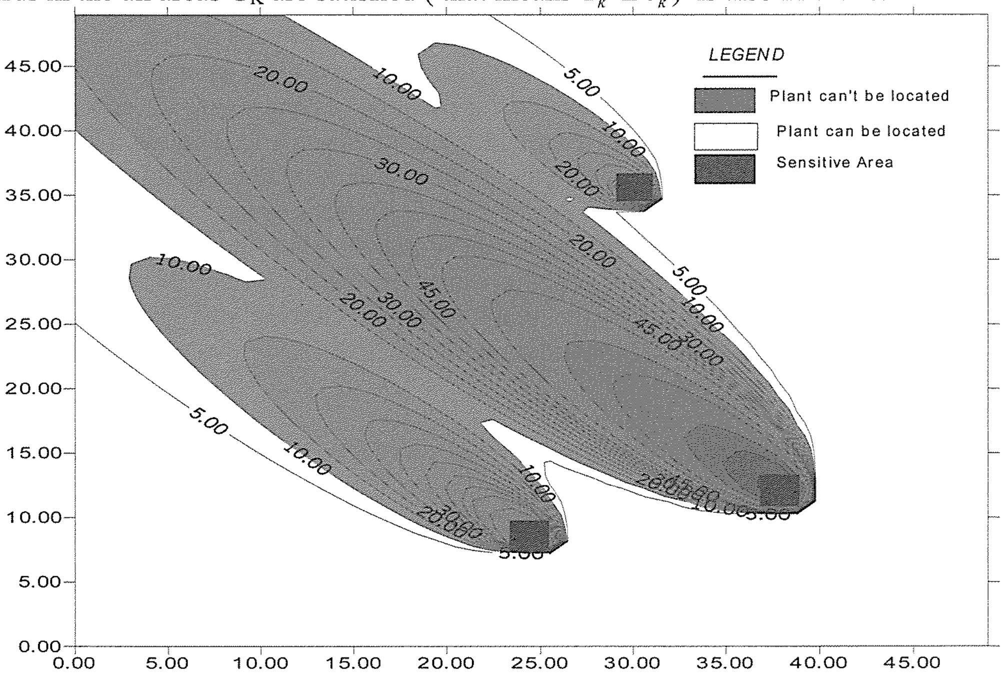

Figure 3: Distribution of value of the pollution level-reflecting functionals Y_k^* for test case 1

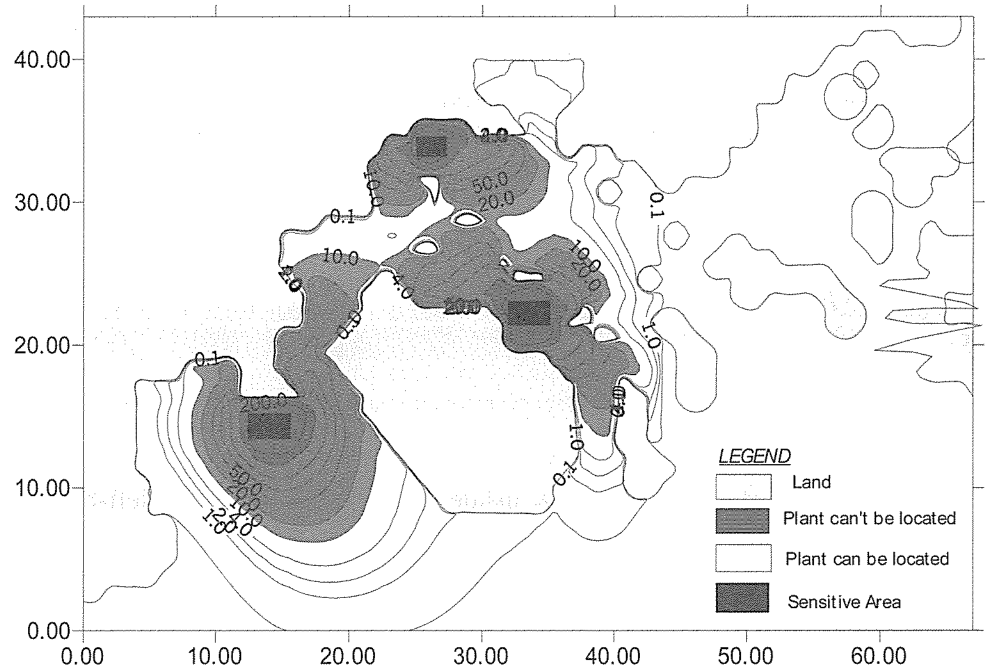

Figure 4: Distribution of value of the pollution level-reflecting functionals Y_k^* for test case 2

4. CONCLUSIONS

The algorithms for solving the matter propagation and its adjoint problems are stable. The numerical solution is non-negative and agreement with the analytical solution.

For determination of the plant location satisfying the condition (38), we suppose that the plant locates at the point $r_0 = (x_0, y_0)$, then we solve the equation (1) and verify the condition (38). If the condition (38) is satisfied, the requisite plant location at point r_0 is found. Conversely, we must suppose new plant location at the other point r_1 and recur the previous process of the above computation and verification. This process might be recur several times. However, if we use the adjoint equation (11), then we solve the equation (11) and (4) only one time for determining the region, in which plant pollution satisfied the condition (38). As a result, it is very convenient for determination of the plant location ensuring the given environmental criteria in the sensitive areas if the adjoint equation is applied.

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