A MATHEMATICAL MODEL FOR MANGROVE GEO-ECOSYSTEM FOCUSING ON INTERACTIONS BETWEEN TREES AND SOILS

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ABSTRACT

We revise the mangrove geo-ecosystem introduced by Yagi, Ho and Duc [1] in the Annual Report of FY 2003. In modelling we will pay more attention on the effects of interactions between trees and soils which are in a cooperative relation. We will present also a simplified model focusing on the effects.

KEYWORD

Mangrove geo-ecosystem, Mathematical model, Reciprocal actions, Self-organization

INTRODUCTION

In the study of forestry kinetics, the numerical simulations on the basis of suitable mathematical models are becoming one of indispensable methods. Observations of forest dynamics require us extremely long time and experiments cost immensely. In the report Yagi, Ho and Duc [1], they have reviewed existing forestry kinematic models constructed at several spatial scales such as, Individual-Based Model, Individual-Based Continuous Space Model, Age-Structured Model, Age-Structured Continuous Space Model etc. In addition, they have presented a mangrove kinematic model on the basis of Age-Structured Continuous Model.

In this report we intend to revise their model. In the new model announced below, we will pay more attention on the effects of reciprocal actions between mangroves and soils. See [2,3] and [4].

We assume that mangroves and soils are in a cooperative relation by the following facts:

(1) Soils give the grounds on which seeds are established and trees grow.
(2) Mangroves trap soils by their roots to facilitate sedimentation of soils transported by the water flow.

We assume also that soils facilitate the dispersion of mangroves via the following process:

(1) The flow of sea water from the center of forest to the open sea is caused by ebb tide and by the inhomogeneity of soil level.
(2) Seeds produced by mangroves are transported by the flow from the center of forest to the boundary region.

By these assumptions we find the mangrove ecosystem as a self-organization system of trees and soils with active reciprocal actions which are cooperative and with facilitation of seed dispersion by soils. According to the satellite photographs (see [4,Fig. 6.1] and [5,Fig. 3] etc.), the mangroves form a remarkable fractal pattern in their growing process. The authors believe that such a pattern formation might be understood theoretically by these isotropic feedback laws, namely, by the principle of self-organization.

RESULTS AND DISCUSSION

Our full system of equations governing the mangrove geo-ecosystem is given by

$$
(1)\qquad \begin{cases}
\dfrac{\partial u}{\partial t} = \beta\delta(\ell)w - \gamma(v)u - fu & \text{in} \quad \Omega \times (0,\infty), \\
\dfrac{\partial v}{\partial t} = fu - hv & \text{in} \quad \Omega \times (0,\infty), \\
\dfrac{\partial w}{\partial t} = d_w \Delta w + \mu\nabla\cdot\{w\chi\} - \beta w + \alpha v & \text{in} \quad \Omega \times (0,\infty), \\
\dfrac{\partial \ell}{\partial t} = d_\ell \Delta \ell + \nu\nabla\cdot\{\ell\chi\} + \varphi(\ell)(pu+qv) & \text{in} \quad \Omega \times (0,\infty), \\
\dfrac{\partial w}{\partial n} = \dfrac{\partial \ell}{\partial n} = 0 & \text{on} \quad \Gamma_N \times (0,\infty), \\
w = \ell = 0 & \text{on} \quad \Gamma_D \times (0,\infty), \\
u(x,0) = u_0(x),\ v(x,0) = v_0(x), & \\
\qquad w(x,0) = w_0(x),\ \ell(x,0) = \ell_0(x) & \text{in} \quad \Omega,
\end{cases}
$$

where $\chi = \{(L-\ell)/(1+pu+qv)\}\nabla\ell$.
Here, Ω is a two-dimensional bounded domain which denotes a region where the mangroves spread out, say, a swamp, and its boundary $\partial\Omega$ is divided into two parts Γ_N and Γ_D, $\partial\Omega = \Gamma_N \cup \Gamma_D$, Γ_N being a part of the boundary faced to the land and Γ_D being another part of the boundary faced to the open sea.
We have four unknown functions u, v, w and ℓ. More precisely, $u = u(x,t)$ and $v = v(x,t)$ denote the mangrove densities of young age class and old age class, respectively, at a position $x \in \Omega$ and time $t \in [0,\infty)$; $w = w(x,t)$ denotes the seed density at $x \in \Omega$ and $t \in [0,\infty)$ on the surface; and $\ell = \ell(x,t)$ the soil level at $x \in \Omega$ and $t \in [0,\infty)$. We set $\ell = 0$ to denote the lowest level of tide and similarly set $\ell = L$ (which is fixed) to denote the highest level of tide. The mangroves are expected to grow on a ground whose soil level is in the range of $0 \le \ell \le L$.
We consider the natural diffusion for seeds and soils. The constants $d_w > 0$ and $d_\ell > 0$ are diffusion constants of seeds and soils, respectively. We consider also the water flow caused by the ebb tide. The fluid is described fundamentally by the vector $-(L-\ell)\nabla\ell$, that is, the water flows from a higher soil level position to a lower soil level with rate $L-\ell$. Friction against the water flow by the trees or their roots is also considered. Its rate is given by $1/(1+pu+qv)$, where $p > 0$ and $q > 0$ are the friction coefficients by young trees and old trees, respectively. So, the fluid of water is ultimately given by the vector $\chi = \{(L-\ell)/(1+pu+qv)\}\nabla\ell$. The seeds on the surface and the soils are transported by the water flow. The terms $\mu\nabla\cdot\{w\chi\}$ and $\nu\nabla\cdot\{\ell\chi\}$ denote the transporting of seeds and soils, respectively, the constants $\mu > 0$ and $\nu > 0$ are their transporting rates. The transport of soils toward the open sea means erosion of soils.
The first two equations of (1) describe growth of mangrove trees for young age class and for old age class. The function $\delta(\ell)$ denotes an establishment rate of seeds which is an

increasing function of ℓ such that $\delta(0) = 0$. The function $\gamma(v)$ is a death rate of young age trees which is a function for v having its minimum at a certain value; a prototype of $\gamma(v)$ is a quadratic function $\gamma(v) = a(v-b)^2 + c$ (where a, b, $c > 0$) which takes its minimum c at $v = b$. The constant $f > 0$ is an aging rate of young age trees to old age. The constant $h > 0$ is a death rate of old age trees. The constant $\alpha > 0$ is a production rate of seeds by the old age trees. The constant $\beta > 0$ is a deposition rate of seeds on the ground. The three equations concerning u, v and w then describe the forestry kinematics of mangroves. These are constructed on the basis of the work due to Kuznetsov et. al [6]. The fourth equation of (1) describes the kinematics of soils. The function $\varphi(\ell)$ denotes a sedimentation rate of soils which are transported by sea water and are trapped by the trees or their roots; $\varphi(\ell)$ is a decreasing function of ℓ such that $\varphi(L) = 0$.
On Γ_N (the boundary of Ω faced to the land) we impose the Neumann boundary conditions for w and ℓ. On the other hand, on Γ_D (the boundary of Ω faced to the open sea) we impose the Dirichlet boundary conditions.
Finally, initial functions for u, v, w and ℓ are given by $u_0(x)$, $v_0(x)$, $w_0(x)$ and $\ell_0(x)$, respectively.

In the view point of mathematical analysis, the model (1) is a parabolic-ordinary system of differential equations. It is very hard in general to investigate the mathematical structure of solutions for such a model. So, it is meaningful to introduce a simplified model focusing the reciprocal actions. For this purpose we try to integrate the kinetic equations of mangroves, namely, the three equations for u, v and w, into a single equation. We assume that u, v and w have similar asymptotic behavior as $t \to \infty$. As a matter of fact, such a nature is verified in the case when the soil level is kept homogeneously and is independent of time variable, namely, when $\ell \equiv \ell_0$ (constant) for $x \in \Omega$ and $t \in [0, \infty)$, see Chuan and Yagi [7,8,9]. Then we arrive at the following model

$$(2) \quad \begin{cases} \dfrac{\partial u}{\partial t} = d_u \Delta u + \mu \nabla \cdot \left\{ \dfrac{(L-\ell)u}{1+pu} \nabla \ell \right\} + \delta(\ell) u - \gamma(u) u & \text{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial \ell}{\partial t} = d_\ell \Delta \ell + \nu \nabla \cdot \left\{ \dfrac{(L-\ell)\ell}{1+pu} \nabla \ell \right\} + \varphi(\ell) u & \text{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial \ell}{\partial n} = 0 & \text{on} \quad \Gamma_N \times (0,\infty), \\ u = \ell = 0 & \text{on} \quad \Gamma_D \times (0,\infty), \\ u(x,0) = u_0(x), \ \ell(x,0) = \ell_0(x) & \text{in} \quad \Omega, \end{cases}$$

in a two-dimensional region Ω as before.
Here, the unknown function $u = u(x,t)$ denotes the mangrove density, not only the tree density of young age class but also that of old age class and even the density of seeds on the surface produced by mangroves. In this way, $u = u(x,t)$ denotes some integrated density of mangroves. The transport of mangroves and soils is now described by $\mu \nabla \cdot \{u\chi\}$ and $\nu \nabla \cdot \{\ell \chi\}$, respectively, where χ denotes the fluid of sea water caused by the ebb tide and is given by $\chi = \frac{L-\ell}{1+pu} \nabla \ell$ with friction coefficient $\mu > 0$ of mangroves.
The function $\delta(\ell)$ denotes a growth rate of mangrove and is an increasing function of ℓ with $\varphi(0) = 0$. The function $\gamma(u)$ denotes a death rate of mangroves, as before, the quadratic function $\gamma(u) = a(u-b)^2 + c$ may be a prototype. The function $\varphi(\ell)$ is a

sedimentation rate of soils trapped by the mangroves and is a decreasing function of ℓ with $\varphi(L) = 0$.

CONCLUSIONS

Finding the mangrove geo-ecosystem as a cooperative self-organization system by trees and soils, we have presented a mathematical model, a full model and a simplified model. It may be very interesting to compare the model with other self-organization models, say, a chemotaxis-growth model

$$(3) \qquad \begin{cases} \dfrac{\partial u}{\partial t} = a\Delta u - \mu\nabla\cdot\{u\nabla\rho\} + f(u) & \text{in} \quad \Omega\times(0,\infty), \\ \dfrac{\partial \rho}{\partial t} = b\Delta\rho - c\rho + du & \text{in} \quad \Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial \rho}{\partial n} = 0 & \text{on} \quad \partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x), \quad \rho(x,0) = \rho_0(x) & \text{in} \quad \Omega. \end{cases}$$

Here, $u = u(x,t)$ denotes the density of biological individuals and $\rho = \rho(x,t)$ the concentration of chemical substances produced by the biological individuals. This model describes the process of aggregating pattern formation by chemotactic bacteria. See [10]. We find significant similarity between two models (2) and (3). The model (3) also shows that bacteria and chemical substances are cooperative and they are mutually attractive (this means the sign before ∇ is negative). In the meantime, (2) shows that trees and soils are cooperative but they are repulsive (this means the sign before ∇ is positive).

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