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A MATHEMATICAL MODEL FOR MANGROW FOREST DYNAMICS

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Abstract

Following the principle of self-organizations, we shall propose a mathematical model for mangrove forest dynamics by introducing trees and soils which are considered as the constituent element and the conductor of the ecosystem respectively in cooperative relations.

Keywords:*Mangrove forest dynamics, Self-organization, Mathematical model*

1 Introduction

In the study of forest growth dynamics numerical simulations on the basis of mathematical models are becoming one of indispensable methods. Observations of forest dynamics require us extremely long time and experiments cost immensely. It is a very important problem to build models which are suitable for the objectives from knowledge and information obtained by observations. This report tries to present a mathematical model describing the growth process of a mangrove forest in order to study its mechanism theoretically and numerically.

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2 Some existing models for forest dynamics

In this section we shall review some existing mathematical models which have been presented for describing the growth process of forests.

2.1 Individual-Based Model.

The JABOWA model which has been presented by Botkin *et al.* (1972) may be the first and most basic model for forest growth dynamics. Their model consider a unit area of $100m^2 \sim 300m^2$ called a plot. A certain number oft ee.s:co si?ered individually in the plot. The growth of each tree per a year is described by the formula

$$\Delta D^2 H = R.(LA)(1 - DH/D_{\max} H_{\max})$$

where D denotes the diameter of tree at the breast height, H denotes the height of tree (so D^2H corresponds to the volume of tree), LA denotes the area of leaves, $D_{\max}$ and H_{max} denote the possible maximum diameter and height respectively which the tree can attain, and R is a parameter depending on various environmental conditions. To these growth equations of trees, the effect of interaction of is also incorporated. The higher trees can absorb more light and grow more swiftly, and just one tree dominates the plot. Then the tree dies down, and the plot is again full of room called a gap.

2.2 Individual-Based Continuous Space Model

On the basis of the JABOWA model, many authors have afterward presented spatially continuous models. Pacala et al. (1996) have presented an individual-based continuous space model called the SORTIE model. They formulated a growth equation of each tree spread in a wide area by

$$\Delta D = D.G_1.(GLI)\{G_1/G_2 + (GLI)\}$$

where GLI denotes global light index and indicates the interaction with trees in its neighborhood. They considered also the seed production and dispersion.

2.3 Age-Structured Model

When we are concerned with dynamics of forest ecosystems, age-dependent tree relationship is often more interesting than the individual growth of trees. From this viewpoint many age-structured models have been presented. For example, Antonovsky *et al.* (1983) considered a very simple ecosystem of a mono-species and with only two age classes. Their growth equations are the written as

$$\begin{cases} \dot{u} = \rho v - \gamma(v)u - fu \\ \dot{v} = fu - hv \end{cases}$$

where u and v are trees densities of young and old age classes; ρ, f and h are coefficients of reproduction, ageing and mortality of old trees respectively, while $\gamma(v)$ denotes a mortality of old trees. It is assumed that there exists some optimal value of old tree density under which the recruitment of young trees is maximal. The typical form of $\gamma(v)$ is given by

$$\gamma(v) = a(v-b)^2 + c$$

with some positive constants a, band c.

2.4 Age-Structured Continuous Space Model

More recently Kuznetsov, Antonovsky, Biktashev and Aponina (1994) generalized the age-structured model to a continuous space model by taking seed dynamics into account. Their new growth equations are described by

$$\begin{cases} u_t = \delta\beta v - \gamma(v)u - fu \\ v_t = fu - hv \\ w_t = \alpha v - \beta v + d\Delta w \end{cases}$$

where w is a seed density, α γ and δ are seed production, deposition and establishment rates respectively, and d is the diffusion coefficient of seeds.

3 Results and Discussion

On the basis of the age-structured continuous space model due to Kuznetsov *et al.*, we intend to present a model describing mangrove forest growth dynamics. We take in addition soil dynamics into account. Following to the principle of the theory of selforganizations due to Haken (1983, 2000), we consider trees as the constituent elements of an ecosystem and soil as a conductor of dynamics. Soil leads trees to spread in a mangrove forest and is produced by trees. More appropriately, the roots of trees accumulate soil.

In this sense soil and trees are in cooperative relation. Precisely we assume the following conditions:

1. The ecosystem consists of a mono-species, and only two age classes are considered. They obey the growth equations due to Kuzunetsov *et ai.*
2. The establishment rate $\delta(\ell)$ depends on the height of soil ℓ.
3. The mobility of seeds consists of two factors: one is a natural diffusion and the other is the directed movement in a sense that the seawater carries them.
4. Soil is carried by the seawater and is trapped by the roots of mangrove.

Our proposed system is then written by

$$\begin{cases} u_t = \beta\delta(\ell)w - \gamma(v)u - fu, \\ v_t = fu - hv \\ w_t = \alpha v - \beta w + d_w \Delta w + \nabla.\{w \nabla_\chi (\ell)\} \\ \ell_t = \varphi(\ell)v - \psi(\ell) + d_\ell \Delta \ell \end{cases}$$

where $\ell(-\infty < \ell \leq L)$ denotes the height of soil. The level $\ell = 0$ corresponds to the level of seawater in low tide and $\ell = L$ corresponds to that in high tide. The function $\chi(\ell)$ is a potential function of the flow of seeds, $\varphi(\ell)$ is an accumulation rate of soil, $\psi(\ell)$ is an erosion rate of soil by seawater. The constant d_ℓ is the diffusion coefficient of soil.

The establishment rate $\varphi(\ell)$ is an increasing function of $-\infty < \ell \leq L$. A possible form may be $\delta(\ell) = \delta^{-(L-\ell)}$ with some constant δ. The potential function $\chi(\ell)$ is also an increasing function of $\infty < \ell \leq L$. A possible form may be $\chi(\ell) \equiv 0$ for $\ell \leq 0$ $\chi(\ell) = \lambda(1 - \ell/L)$ for $0 < \ell \leq L$ with some positive constant λ. The function $\psi(\ell)$ is a decreasing function of ℓ, possibly $\varphi(\ell) \equiv \lambda$ for $\ell \leq 0$, and $\varphi(\ell) = \lambda(1 - \ell/L)$ for $0 < \ell \leq L$ with some positive constant λ. The function $\psi(\ell)$ is an increasing function of $-\infty < \ell \leq L$ such that $\psi(L) = 0$.

We consider this system in a two-dimensional domain Ω. As the boundary conditions on $\partial\Omega$, we impose on wan w and ℓ the Neumann conditions $\frac{\partial w}{\partial n} = \frac{\partial \ell}{\partial n} = 0$.

Finally it may be very interesting to notice the similarity of this model to the chemotaxis-growth model which describes the process of aggregating pattern formation by biological individuals:

$$\begin{cases} u_t = \alpha u(1-\beta u) + d_u \Delta u - \nabla . \{u \nabla_\chi (u)\}, \\ \rho_t = fu - g\rho\Delta_\rho \end{cases}$$

see Osaki *et al.* (2002). Here, u denotes the density of biological individuals and ρ denotes the concentration of a chemical substance which attracts the amebas and which is produced by the amebas themselves. In this system the amebas are constituent elements and the chemical substance plays a role of the conductor. Aida *et al.* (2004) shows that this interaction-diffusion system possesses some pattern solutions which have very good correlation with experimental results due to Woodwart *et al.* and so on.

4 Conclusion

We reviewed some mathematical models presented for describing the forest' growth process. On the basis of the age-structured continuous space model we have presented a model for describing the growth process of a mangrove forest by being led by the principle of the theory of self-organizations.

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