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# THE UNSTEADY FLOW AFTER DAM BREAKING

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## Abstract

In this paper, the following problems are presented: the unsteady flow on the river system and reservoirs, the discontinuous wave and unsteady flow after the dam breaking, numerical experiments for some tests cases and for natural Da river.

## 1. Mathematical Modelling

The equation system describing the unsteady flows is established from the laws of conservation (see [1]) and has the following form:

$$\oint_{\partial S} Q dt - \omega dx = \iint_S q dx dt, \quad (1.1)$$

$$\oint_{\partial S} \left[ P + \frac{Q^2}{\omega} \right] dt - Q dx = \iint_S \left[ g\omega \left( i - \frac{Q|Q|}{K^2} + R_x \right) \right] dx dt,$$

where:

$$P = g \int_0^h (h - \zeta) b(x, \zeta) d\zeta$$

$$R_x = g \int_0^h (h - \zeta) \frac{\partial b(x, \zeta)}{\partial x} d\zeta$$

x - the coordinate along channel.

q- lateral flow.

K- conveyance factor.

i- bottom slope.  
the bottom.

g- acceleration due to gravity.

Q- discharge.

t- time.

$\omega$ - cross-section area.

h- the depth.

b(x,  $\zeta$ )- width on the distance  $\zeta$  from

S- consideration region.

$\partial S$  - boundary of S.

## 1.1 One dimensional Saint -Venant Equation system

If the flow is continuous, from (1.1) we get the Saint -Venant equation system

$$\begin{aligned} B \frac{\partial Z}{\partial t} + \frac{\partial Q}{\partial x} &= q \\ \frac{\partial Q}{\partial t} + 2v \frac{\partial Q}{\partial x} + B(c^2 - v^2) \frac{\partial Z}{\partial x} &= \Phi \end{aligned} \quad (1.2)$$

where

$$\Phi = \left[ iB + \left( \frac{\partial \omega}{\partial x} \right)_{h=const} \right] v^2 - \frac{g\omega Q|Q|}{K^2} = \left( \frac{\partial \omega}{\partial x} - B \frac{\partial Z}{\partial x} \right) v^2 - \frac{g\omega Q|Q|}{K^2}$$

Z- level of free surface.

v- velocity.

B- width of the water surface.

c- celerity of small wave

propagation.

Equations systems (1.2) are quasilinear and of hyperbolic type, which can be rewrite in the characteristic form:

$$\frac{\partial Q}{\partial t} + (v - c) \frac{\partial Q}{\partial x} + B(-v - c) \left[ \frac{\partial Z}{\partial t} + (v - c) \frac{\partial Z}{\partial x} \right] = \Phi + (-v - c)q \quad (1.3)$$

$$\frac{\partial Q}{\partial t} + (v + c) \frac{\partial Q}{\partial x} + B(-v + c) \left[ \frac{\partial Z}{\partial t} + (v + c) \frac{\partial Z}{\partial x} \right] = \Phi + (-v + c)q \quad (1.4)$$

For solving the equation system (1.2) or (1.3) (1.4), it is necessary to give initial conditions at  $t = 0$ :  $Z(x, 0) = Z^0(x)$ ,  $Q(x, 0) = Q^0(x)$  and the boundary conditions, adjoint conditions.

### a. Boundary conditions:

For subcritical flow, one boundary condition is needed:

- At the upstream boundary :

$$Q(x_b, t) = Q_b(t). \quad (1.5)$$

- At the downstream boundary:

$$Z(x_b, t) = Z_b(t) \quad \text{or} \quad Q(x_b, t) = f(Z_b(t)). \quad (1.6)$$

For supercritical flow:

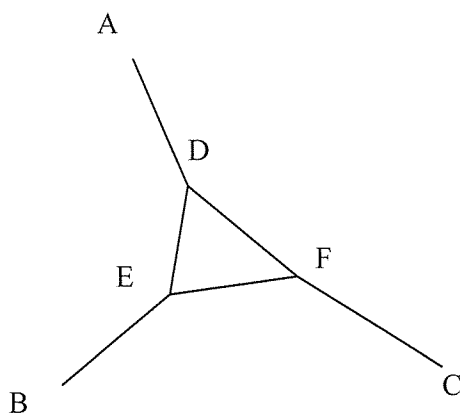
- At the upstream boundary, two boundary conditions are needed.

$$Q(x_b, t) = Q_b(t) \quad \text{and} \quad Z(x_b, t) = Z_b(t). \quad (1.7)$$

- At the downstream boundary:

No boundary condition is needed.

### b. Adjoint conditions at the internal node of river systems. (for example nodes D, E, F in the fig 1.1)



**Fig 1.1**

At the every internal node it is necessary to give the following adjoint condition.(for example, adjoint conditions at D):

$$\sum_{j \in J_D} \alpha_{Dj} Q_{Dj} = 0$$

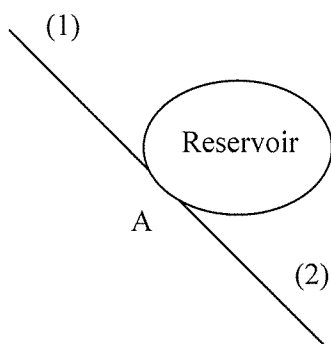
$$Z_{Dj} = Z_D, j \in J_D$$

(1.8)

where  $J_D$  is the set of the river branches having common node D.

$$\alpha_{Dj} = \begin{cases} -1 & \text{If D is left boundary of river branch j.} \\ +1 & \text{If D is right boundary of river branch j.} \end{cases}$$

**c. Adjoint conditions at the common node A of a river and a reservoir.**



**Fig 1.2**

Suppose that the reservoir has the volume  $V$  depending on the elevation  $Z_H$ :

$$V = V(Z_H).$$

The adjoint conditions are: (see fig 1.2)

$$\sum_{j=1}^2 \alpha_{Aj} Q_j + Q_3 = 0 \quad (1.9)$$

$$Z_{Aj} = Z_H \quad j=1, 2.$$

$$\text{where: } Q_3 = -\frac{dV(Z_H)}{dt}$$

## 1.2 Adjoint condition at the discontinuous front:

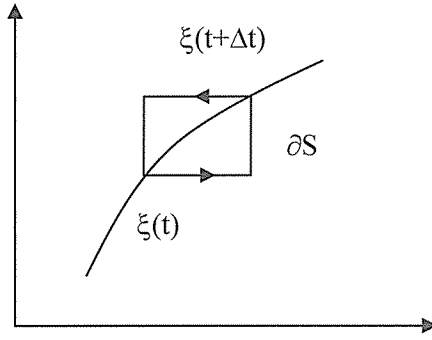


Figure 1.3

One adjoint condition at the discontinuous front is needed: (see [1],[5],[7])

$$(1.10) \quad [P] \left[ \frac{1}{\omega} \right] + [v]^2 = 0$$

where  $[f] = f^+ - f^-$ ,  
 $f^-$  is the value  $f$  at the left side of the

$\xi$   
 $f^+$  is the value  $f$  at the right side of the  $\xi$

The velocity of the discontinuous front  $\xi$  is (see fig 1.3).

$$C_* = v^+ + \sqrt{\frac{\omega^-}{\omega^+} \frac{P^+ - P^-}{\omega^+ - \omega^-}} = v^- + \sqrt{\frac{\omega^+}{\omega^-} \frac{P^+ - P^-}{\omega^+ - \omega^-}} = \frac{Q^+ - Q^-}{\omega^+ - \omega^-}$$

$$v^+ + c^+ < C_*; v^- - c^- < C_* < v^- + c^- \quad (1.11)$$

In a case when the height of discontinuous front is very small ( $\Delta h \ll l$ ), the adjoint condition and velocity  $C_*$  are:

$$Q^+ - Q^- - B^+ (v^- + c^+) (Z^+ - Z^-) = 0 \quad (1.12)$$

$$C_* = v^- + c^+ \approx v^+ + c^- \quad (1.13)$$

## 2. The Algorithms

### 2.1 Calculation of the one dimensional unsteady flows(see[2],[4]-[6]):

Equation (1.3) and (1.4) may be rewrite as follows:

$$\frac{dQ}{dt} + a_1 \frac{dZ}{dt} = b_1, \frac{dx}{dt} = c_1 \quad (2.1)$$

$$\frac{dQ}{dt} + a_2 \frac{dZ}{dt} = b_2, \frac{dx}{dt} = c_2 \quad (2.2)$$

where,

$$a_1 = B(-v - c), \quad b_1 = \Phi + (-v - c)q, \quad c_1 = v - c$$

$$a_2 = B(-v + c), \quad b_2 = \Phi + (-v + c)q, \quad c_2 = v + c$$

a. **Calculation of the values  $Z_0^{k+1}$  and  $Q_0^{k+1}$  at left boundary  $L_0$ :**

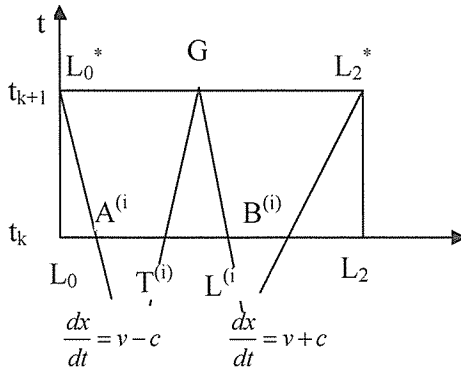


Fig 2.1

- Determine the coordination of point  $A^{(i)}$ , (i.e the intersection of a back characteristics line  $(dx/dt)=v-c$  and the line  $t=t_k$ ) at

$$x_{A^{(i)}} = x_0 + \frac{\tau}{2} \left[ (c_1)_{L_0^*}^{(i-1)} + (c_1)_{A^{(i-1)}} \right]$$

$$(c_1)_{L_0^*}^{(0)} = (c_1)_{A^{(0)}} = (c_1)^k$$

the (i) iterative step (see Fig. 2.1)

- Determine the values  $Z_{A^{(i)}}$  and  $Q_{A^{(i)}}$  by the linear interpolation.

- Substituting these values into equation (2.1) we get:

$$Q_0^{(i)} + a_0^{(i)} Z_0^{(i)} = d_0^{(i)} \quad (2.3)$$

- From the equation (2.3) and boundary conditions (1.5) one deduces  $Z_0^{(i)}$ .

The iterative process is stopped if  $|Z_0^{(i)} - Z_0^{(i-1)}| < \varepsilon |Z_0^{(i-1)}|$ ,  $\varepsilon \leq 0.01$

b. **Calculation of the values  $Z_N^{k+1}$  and  $Q_N^{k+1}$  at right boundary  $L_2$ :**

By the analogous argument from the equation (2.2) we have the following equation at the iterative step (i) (see fig 2.1).

$$Q_N^{(i)} + a_N^{(i)} Z_N^{(i)} = d_N^{(i)} \quad (2.4)$$

Solving this equation (2.4) and the boundary condition (1.6)  $Z_N^{k+1} = Z_b(t_{k+1})$  or linearized boundary condition  $Q_N^{(i)} + \alpha_N^{(i)} Z_N^{(i)} = \beta_N^{(i)}$ , where

$$\alpha_N^{(i)} = - \left. \frac{\partial f}{\partial z} \right|_N^{(i-1)}; \quad \beta_N^{(i)} = Q_N^{(i-1)} - \left. \frac{\partial f}{\partial z} \right|_N^{(i-1)} \cdot Z_N^{(i-1)} \quad \text{we get } Q_N^{(i)},$$

$Z_N^{(i)}$ .

The iterative process is stopped if

$$|Z_N^{(i)} - Z_N^{(i-1)}| < \varepsilon |Z_N^{(i-1)}|, \quad |Q_N^{(i)} - Q_N^{(i-1)}| < \varepsilon |Q_N^{(i-1)}|,$$

c. **Calculation of the values  $Z^{k+1}$  and  $Q^{k+1}$  at the internal node of river system:** (for example D on the fig.1.1)

For each river branch  $j$  ( $j=1, 2, \dots, J_D$ ) having common internal node D we have one linear equation:

$$Q_{Dj}^{(i)} = a_{Dj}^{(i)} Z_{Dj}^{(i)} + d_{Dj}^{(i)} \quad (2.5)$$

where  $a_{Dj}^{(i)} = -(a_0^{(i)})_j$  and  $d_{Dj}^{(i)} = (d_0^{(i)})_j$  if D is left boundary of the branch  $j$

$a_{Dj}^{(i)} = -(a_N^{(i)})_j$  and  $d_{Dj}^{(i)} = (d_N^{(i)})_j$  if D is right boundary of the branch j

From adjoint condition (1.8) and (2.5) we obtain:

$$Z_{Dj}^{(i)} = Z_D^{(i)} = \frac{-\sum_{j=1}^{J_D} \alpha_{Dj} d_{Dj}^{(i)}}{\sum_{j=1}^{J_D} \alpha_{Dj} a_{Dj}^{(i)}} \quad (2.6)$$

The iterative process is stopped if  $|Z_D^{(i)} - Z_D^{(i-1)}| < \varepsilon |Z_D^{(i-1)}|$ .

**d. Calculation of the values  $Z^{k+1}$  and  $Q^{k+1}$  at the common node of a river and a reservoir:** (for example node A on the fig 1.2)

Linearizing the equation  $Q_3 = -\frac{dV(Z)}{dt}$ , we have

$$Q_3^{k+1} \approx -\frac{V(Z^{k+1}) - V(Z^K)}{\tau} \approx -\frac{1}{\tau} \left[ V(Z^K) + \frac{dV}{dZ} (Z^{k+1} - Z^K) - V(Z^K) \right]$$

$$Q_3^{(i)} \approx \beta^{(i)} (Z^{(i)} - Z^K)$$

$$\text{where } \beta^{(i)} = -\frac{1}{2\tau} \left[ \left( \frac{dV}{dZ} \right)^{(i-1)} + \left( \frac{dV}{dZ} \right)^K \right]$$

From the equation (2.5) for each river branch, adjoint conditions (1.9) and equation (2.7) we get:

$$Z_H^{(i)} = \frac{\beta^{(i)} Z_H^K - \sum_{j=1}^2 \alpha_j d_{Aj}^{(i)}}{\beta^{(i)} + \sum_{j=1}^2 \alpha_j a_{Aj}^{(i)}} \quad (2.8)$$

$$|Z_H^{(i)} - Z_H^{(i-1)}| < \varepsilon |Z_H^{(i-1)}| \quad \text{Iterative process is stopped if}$$

The discharge are calculated from (2.5) and (2.7).

**e. Calculation of Z and Q at interior nodes of river branch:** (For example, node G on the fig. 2.1)

From the equations (2.1) and (2.2), by method of characteristic we get the following equations for determining the values Z and Q at the (i) iterative step.

$$Q_G^{(i)} + a_L^{(i)} Z_G^{(i)} = d_L^{(i)}$$

$$Q_G^{(i)} + a_T^{(i)} Z_G^{(i)} = d_T^{(i)}$$

Solving this equation system we obtain  $Z_G^{(i)}$  and  $Q_G^{(i)}$ .

Iterative process is stopped if

$$\left| Z_G^{(i)} - Z_G^{(i-1)} \right| < \varepsilon \left| Z_G^{(i-1)} \right| \quad \text{and} \quad \left| Q_G^{(i)} - Q_G^{(i-1)} \right| < \varepsilon \left| Q_G^{(i-1)} \right|$$

## 2.2 Discontinuous wave on a river

Suppose that the dam is totally and instantaneously broken. The computational process includes (see [1],[6]-[8])

### a. Calculation of $Z^-$ , $Q^-$ at the moment of dam breaking:

According to references [3],[5]-[7] these values can be calculated by an iterative method using formula:

$$V_i = v^+ + \sqrt{\frac{g}{2} \left[ (h^{(i)})^2 - (h^+)^2 \right] \cdot \left( \frac{1}{h^+} - \frac{1}{h^{(i)}} \right)} \quad \text{and} \quad h_s = \frac{1}{4g} \left( v_1 + 2\sqrt{gh_1} - V_i \right)^2$$

where

$$\begin{aligned} v_1 &= v(L_1 - 0, 0), & h_1 &= h(L_1 - 0, 0) \\ h^{(i)} &= h^{(i-1)} + 0,01h^+, & h^{(0)} &= h^+ = h(L_1 + 0, 0) \end{aligned}$$

Iterative process is stopped if  $h_s \leq h^{(i)}$ .

### b. Determine the position of the discontinuous front $\xi$ .

$$\xi^{K+1} = \xi(t_{K+1}) = \xi^K + C_*^K \tau$$

where,  $C_* = v^+ + \sqrt{\frac{\omega^-}{\omega^+} \cdot \frac{P^+ - P^-}{\omega^+ - \omega^-}}$

### c. Determine the values $(Z^+)^{K+1}$ , $(Q^+)^{K+1}$ at the right side of the discontinuous front $\xi$ :

From the Saint -Venant equation system in the characteristic form (2.1), (2.2) one deduces the equations at the (i) iterative step

$$(Q^+)^{(i)} + a_L^{(i)} (Z^+)^{(i)} = d_L^{(i)}$$

$$(Q^+)^{(i)} + a_T^{(i)} (Z^+)^{(i)} = d_T^{(i)}$$

Solving this equation system we get  $(Z^+)^{(i)}$  and  $(Q^+)^{(i)}$ .

$$\text{We take } (Z^+)^{(K+1)} = (Z^+)^{(i)}, (Q^+)^{(K+1)} = (Q^+)^{(i)}$$

$$\text{if } \left| (Z^+)^{(i)} - (Z^+)^{(i-1)} \right| < \varepsilon \left| (Z^+)^{(i-1)} \right| \quad \text{and} \quad \left| (Q^+)^{(i)} - (Q^+)^{(i-1)} \right| < \varepsilon \left| (Q^+)^{(i-1)} \right|$$

### d. Determine the values $(Z^-)^{K+1}$ , $(Q^-)^{K+1}$ at the left side of the discontinuous front $\xi$ :

From the equation (2.2) it yields

$$(Q^-)^{(i)} + a_T^{(i)} (Z^-)^{(i)} = d_T^{(i)}$$

Linearising adjoint condition (1.10) one deduces

$$\gamma^{(i)} (Q^-)^{(i)} + \mu^{(i)} (Z^-)^{(i)} = \theta^{(i)}$$

where  $\gamma^{(i)}$ ,  $\mu^{(i)}$ ,  $\theta^{(i)}$  are known coefficients.

Solving this equation system we obtain  $(Z^-)^{(i)}$  and  $(Q^-)^{(i)}$ .

$$\text{If } \left| (Z^-)^{(i)} - (Z^-)^{(i-1)} \right| < \varepsilon \left| (Z^-)^{(i-1)} \right|, \left| (Q^-)^{(i)} - (Q^-)^{(i-1)} \right| < \varepsilon \left| (Q^-)^{(i-1)} \right|$$



we take  $(Z^-)^{(K+1)} = (Z^-)^{(i)}$ ,  $(Q^-)^{(K+1)} = (Q^-)^{(i)}$ .

e. The values  $Z^{K+1}$  and  $Q^{K+1}$  at the boundary nodes, internal nodes of river system, common nodes of a river and a reservoir or interior nodes of each river branch are calculated by the method of characteristic as in the point 1.

### 2.3 Unsteady flow after the dam breaking on river

Suppose that the dam breaking is gradual. The condition at dam is the function:

$$Q = f(Z_T, Z_D) \quad (2.9)$$

$$\text{and} \quad Q_T = Q_D = Q \quad (2.10)$$

Linearising the equation (2.9) we get

$$Q_T^{(i)} = \alpha^{(i)} Z_T^{(i)} + \beta^{(i)} Z_D^{(i)} + \gamma^{(i)} \quad (2.11)$$

#### a. For the supercritical flow

Analogously, from the equation (2.1), (2.2) one deduces two following equations at the left side of the dam.

$$Q_T^{(i)} + a_T^{(i)} Z_T = d_T^{(i)} \quad (2.12)$$

$$Q_T^{(i)} + a_L^{(i)} Z_T = d_L^{(i)} \quad (2.13)$$

Solving the equations (2.10)-(2.13) we obtain the values  $Z_T^{(i)}, Z_D^{(i)}, Q_T^{(i)}, Q_D^{(i)}$ . The iterative process is stopped if the error is small enough and we take

$$Z_T^{K+1} = Z_T^{(i)}, Z_D^{K+1} = Z_D^{(i)}, Q_T^{K+1} = Q_T^{(i)}$$

#### b. For the subcritical flow.

From the equation (2.2) at the right side and (2.1) at the left side of dam we have

$$Q_T^{(i)} + a_T^{(i)} Z_T = d_T^{(i)} \quad (2.14)$$

$$Q_D^{(i)} + a_L^{(i)} Z_D = d_L^{(i)} \quad (2.15)$$

Solving the equations (2.10), (2.11), (2.14), (2.15) we obtain the values  $Z_T^{(i)}, Z_D^{(i)}, Q_T^{(i)}, Q_D^{(i)}$

The iterative is stopped if the error is small enough.

c. The values  $Z^{K+1}$  and  $Q^{K+1}$  at the boundary, internal, common nodes or interior nodes of river branch are calculated by the same method as in the point 1.

## 3 Numerical Experiments

The method of characteristic is applied to solve some test problems and natural DA river-system problem(see[8]).

### 1. Test case 1:

Channel of 1.5 km long in which every section is rectangular. Its geometry is described in Fig. 3.1 and 3.2. The bed slop is about 10% with reverse gradients. One can notice the important contrating section at  $x= 800$  m which creat an acceleration of the flow.

This test enables to check that these source terms are correctly evaluaed, in the case of flat water at rest.

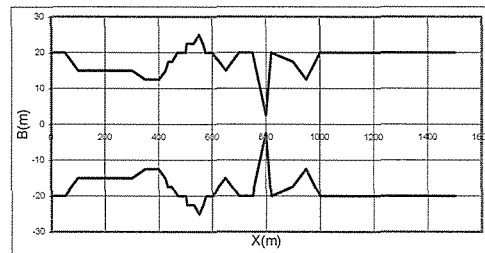
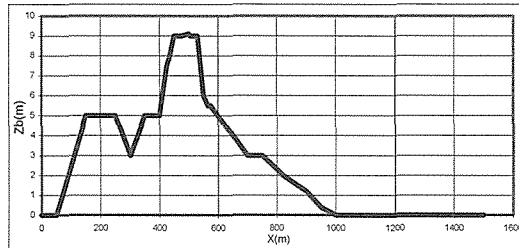


Fig 3.1: Channel geometry – Profile view

Fig 3.2: Channel geometry – Top view

The complete description of the geometry is given in the following table.

Cross-sec	X(m)	Z <sub>b</sub> (m)	B(m)	Cross-sec	X(m)	Z <sub>b</sub> (m)	B(m)
1	0	0	40	16	530	9	45
2	50	0	40	17	550	6	50
3	100	2.5	30	18	565	5.5	45
4	150	5	30	19	575	5.5	40
5	250	5	30	20	600	5	40
6	300	3	30	21	650	4	30
7	350	5	25	22	700	3	40
8	400	5	25	23	750	3	40
9	425	7.5	30	24	800	2.3	5
10	435	8	35	25	820	2	40
11	450	9	35	26	900	1.2	35
12	470	9	40	27	950	0.4	25
13	475	9	40	28	1000	0	40
14	500	9.1	40	29	1500	0	40
15	505	9	45				

- \* In each configuration the boundary and initial conditions are as follows:
  - Downstream boundary and initial condition: level impose equal to 12 m.
  - Upstream boundary condition: no discharge.
  - Initial condition: water at rest at the level 12m.

\* The analytical solution is very simple in this test case.

- Water at rest: discharge and flow velocity must be equal to zero.
  - Flat free surface water level stays at the initial level of 12 m.
- \* The numerical solution(see fig 3.3):
- Discharge flow is  $0 \text{ m}^3/\text{s}$
  - Water surface level is 12 m.

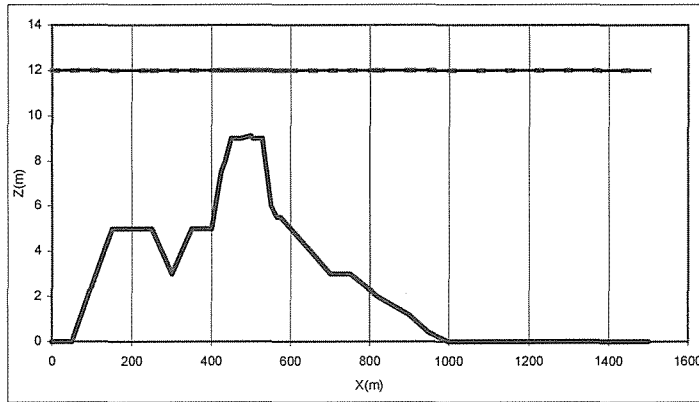


Fig 3.3 :  
The numerical  
solution and the  
analytical  
solution.

..... Numerical  
—— Analytical

## 2. Test case 2:

The steady flow over a bump in a rectangular channel with a constant width. According to the boundary and initial condition, the flow may be subcritical, transcritical with a steady shock, supercritical or at rest.

\* Geometry data:

- The channel width  $B=1 \text{ m}$
- The channel length  $L=25 \text{ m}$
- Bottom  $Z_b$  equation
  - $x < 8 \text{ m}$  and  $x > 12 \text{ m} : Z_f = 0.$
  - $8 \text{ m} < x < 12 \text{ m} : Z_f = 0.2 - 0.05(x-10)^2.$

\* Transcritical flow without shock:

- Downstream: level imposed equal to 0.66 m, no level imposed when the flow becomes supercritical.
- Upstream : discharge imposed equal to  $1.53 \text{ m}^3/\text{s}$ .
- Analytic and numerical solution. (See fig 3.4)

\* Transcritical flow with shock:

- Downstream: level imposed equal to 0.33 m.
- Upstream : discharge imposed equal to  $0.18 \text{ m}^3/\text{s}$ .
- Analytic and numerical solution. (See fig 3.5)

\* Subcritical flow

- Downstream: level imposed equal to 2 m.
- Upstream : discharge imposed equal to  $4.42 \text{ m}^3/\text{s}$ .

- Analytic and numerical solution. (See fig 3.6).

\* Initial conditions

- Constant level equal to the level imposed downstream. Discharge equal to zero.
- Friction term equal to zero.

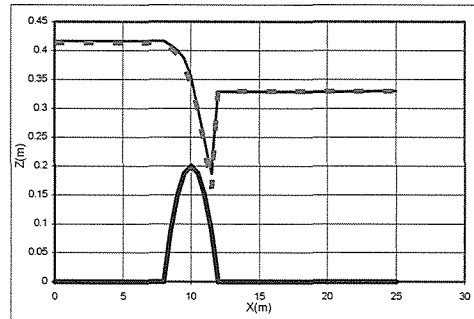
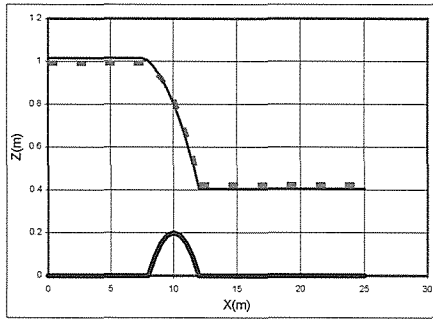


Fig 3.4 Fig 3.5

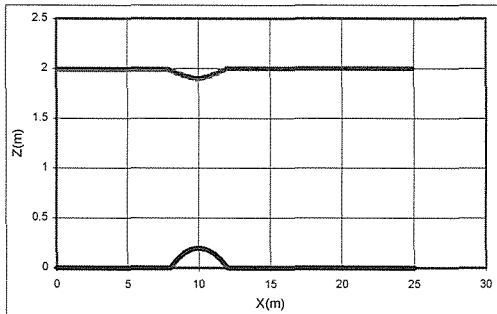


Fig 3.6 :

The numerical solution and the analytical solution.

..... Numerical  
 — Analytical

### 3. Test case 3

Our purpose is to calculate the unsteady flow resulting from an instantaneous dam breaking in a rectangular channel with constant width.

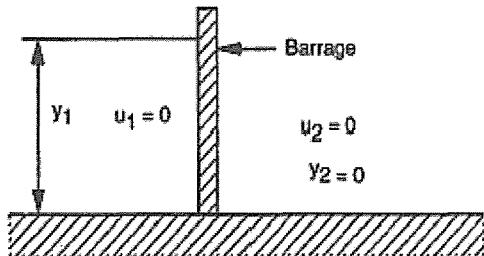


Fig 3.7:

Dam break on dry bed, initial state.

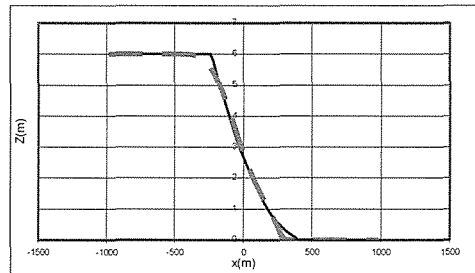


Fig 3.8

The numerical solution and the analytical solution at t=30s

\* Geometrical data(see fig 3.7):

- Channel length 2000 m.
- Dam position  $x=0$  m.
- Channel width  $L=1$  m.

\* Physical parameters

- No friction.
- Boundary conditions.

Downstream: level imposed equal to  $y_2$ .

Upstream : no discharge.

\* Initial conditions

$$y = y_1 = 6 \text{ if } x < 0.$$

$$y = y_2 = 0 \text{ m if } x > 0.$$

\* Analytic and numerical solution. (See fig 3.8).

#### 4. Test case 4:

Our purpose is to calculate the unsteady flow of an instantaneous dambreak on an already wet bed.

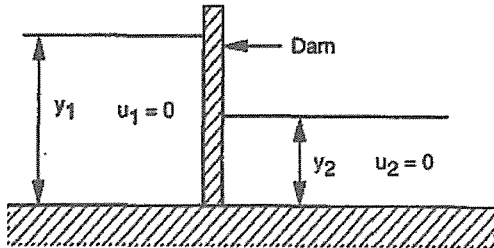


Fig 3.9:

Dam break on wet bed, initial state.

\* Geometrical data(see 3.9):

- Channel length 2000 m.
- Dam position  $x=0$  m.
- Channel width  $L=1$  m.

\* Physical parameters

- No friction.
- Boundary conditions.

Downstream: level imposed equal to  $y_2$ .

Upstream : no discharge.

\* Initial conditions

$$y = y_1 = 6 \text{ if } x < 0.$$

$$y = y_2 = 2 \text{ m if } x > 0.$$

\* Analytic and numerical solution. (See fig 3.10).

#### 5. Discontinuous wave on the Da- river of 555 km long: (see fig 3.11)

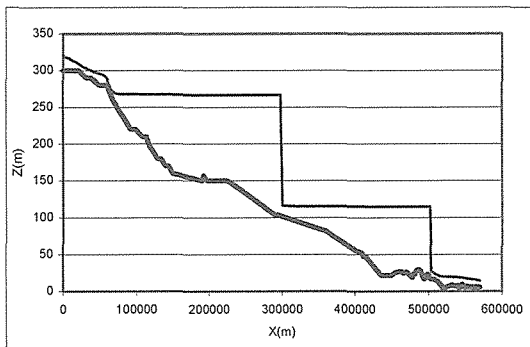


Fig 3.11a

SonLa dam on the Da river, initial state.

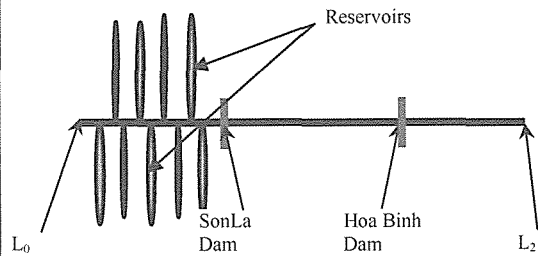


Fig 3.11b

Da river and the reservoirs.

The SonLa dam is situated at the distance of 300 km from the upstream boundary  $L_0$ . The water level on upstream side of dam is 265 m and on the other one is 116 m. The cross-section area  $\omega(x,h)$ , top with  $B(x,h)$  and the river bed  $Z_0(x)$  are given in a table of field measurements. The water volume of main river at upstream side of SonLa dam is  $14.10^9$

$\text{m}^3$ . Suppose that dam is totally and instantly broken. The numerical solution is presented in the fig 3.12, fig 3.13 and fig 3.14a, fig 3.14b.

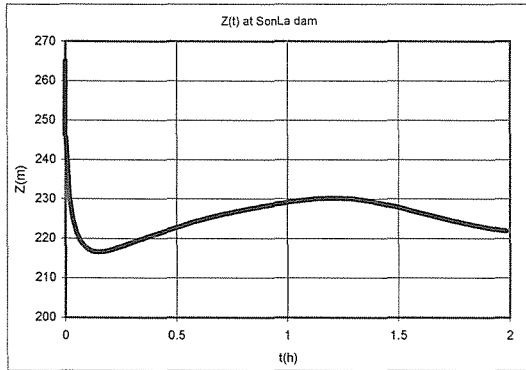


Fig 3.12

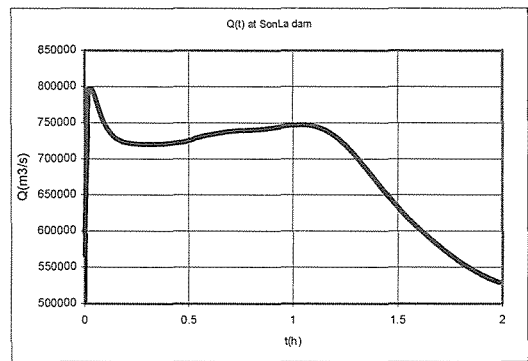


Fig 3.13

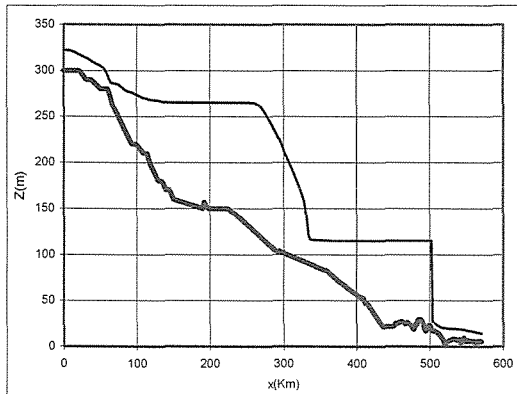


Fig 3.14a:  
Water surface elevation at  $t=0.25$  h.

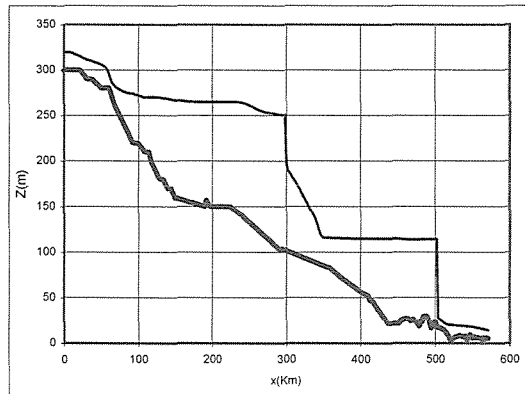
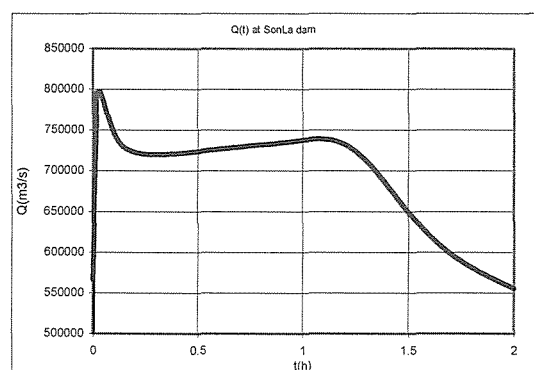
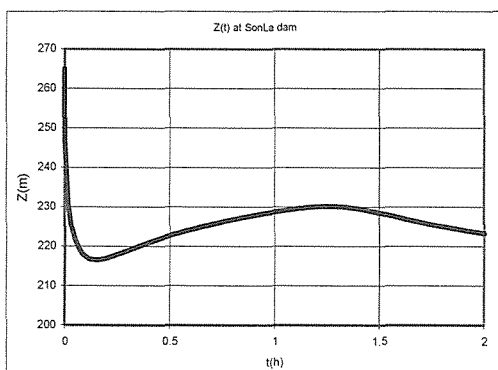


Fig 3.14b:  
Water surface elevation at  $t=0.5$  h

## 6. Discontinuous wave on the Da- river connecting with some reservoir:(see fig 3.11b)

The data is given as in the problem 5 and the water volume of the main river and the reservoirs at the upstream side of SonLa dam is  $25.10^9 \text{ m}^3$ . Suppose that the dam is totally and instantly broken . The numerical solution is presented in the fig 3.15 and fig 3.16.



### 7. The unsteady flow on the Da- river with the mentioned above data (of the problem 5)

In the case the main river is not connecting with the reservoirs, we suppose that dam is gradual failure and rectangular breach size is 135m x 155m (width x depth). Maximum of breach size at  $t = 0.25$  h. The numerical solution is presented in the fig 3.17, fig 3.18 and fig 3.14b.

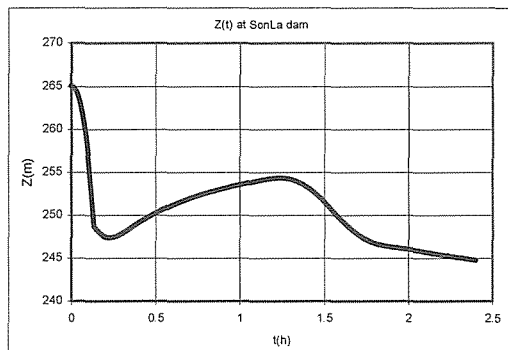


Fig 3.17

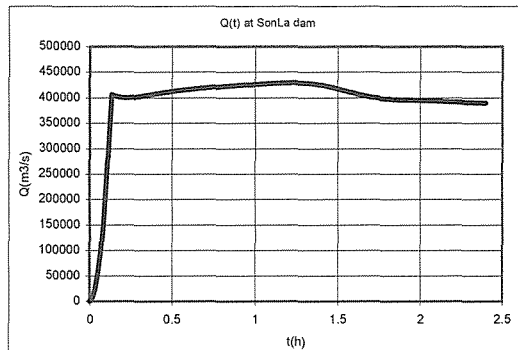


Fig 3.18

### 8. The unsteady flow on the Da- river connecting with some reservoir: (see Fig 3.11b)

The data are given as in the problem 5, we suppose that dam is gradual failure and rectangular breach 135m x 155m (width x depth). Maximum of breach size at  $t = 0.25$  h. The numerical solution is presented in the fig 3.19. and fig 3.20.

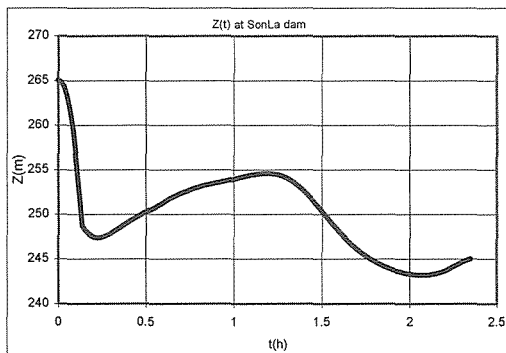


Fig 3.19

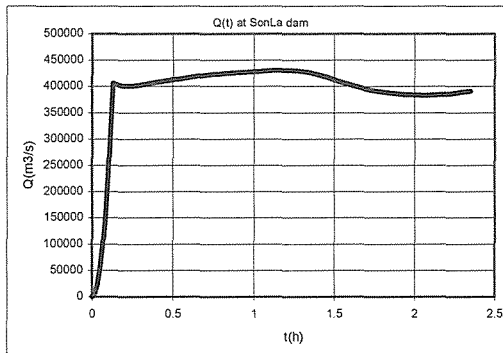


Fig 3.20

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