CAUCHY PROBLEM FOR SOME HYPERBOLIC SYSTEMS OF NONLINEAR FIRST-ORDER PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT. The Cauchy problem for a normal weakly hyperbolic system of first-order nonlinear partial differential equations in two variables is considered. Sufficient conditions for its diagonalisation are given. The local solvability of the noncharacteristic Cauchy problem for classical weakly hyperbolic Monge-Ampère equation is proved.

1. Cauchy problem

We consider the following normal quasilinear first-order system of

1991 *Mathematics Subject Classification.* Primary 35L70; Secondary 53C21, 58C27, 58G17.
Keywords: Hyperbolic system, Diagonalised system, Weakly hyperbolic Monge-Ampère equation, Cauchy problem.
* Supported in part by the National Basic Research Program in Natural Sciences (Vietnam).

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX equations in two variables

$$\begin{cases} \dfrac{\partial X_1}{\partial \alpha_2} = (a_{12}-1)\dfrac{\partial X_1}{\partial \alpha_1} + a_{22}\dfrac{\partial X_2}{\partial \alpha_1} + \dfrac{\partial P_2}{\partial \alpha_1} \\ \dfrac{\partial X_2}{\partial \alpha_2} = -a_{11}\dfrac{\partial X_2}{\partial \alpha_1} - (a_{21}+1)\dfrac{\partial X_2}{\partial \alpha_1} - \dfrac{\partial P_1}{\partial \alpha_1} \\ \dfrac{\partial Z}{\partial \alpha_2} = (a_{12}P_1 - a_{11}P_2)\dfrac{\partial X_1}{\partial \alpha_1} + (a_{22}P_1 - a_{21}P_2)\dfrac{\partial X_2}{\partial \alpha_1} \\ \qquad\qquad - \dfrac{\partial Z}{\partial \alpha_1} - P_2\dfrac{\partial P_1}{\partial \alpha_1} + P_1\dfrac{\partial P_2}{\partial \alpha_1} \\ \dfrac{\partial P_1}{\partial \alpha_2} = (-a_{11}a_{22} + a_{12}a_{21})\dfrac{\partial X_2}{\partial \alpha_1} + (a_{12}-1)\dfrac{\partial P_1}{\partial \alpha_1} - a_{11}\dfrac{\partial P_2}{\partial \alpha_1} \\ \dfrac{\partial P_2}{\partial \alpha_2} = (a_{11}a_{22} - a_{12}a_{21})\dfrac{\partial X_1}{\partial \alpha_1} + a_{22}\dfrac{\partial P_1}{\partial \alpha_1} - (a_{21}+1)\dfrac{\partial P_2}{\partial \alpha_1} \end{cases} \tag{1}$$

Here (X_1, X_2, Z, P_1, P_2) are unknown functions of the variables α_1, α_2; a_{ij} arefunctions of (X_1, X_2, Z, P_1, P_2).
Suppose that in R^2_X there is a curve Γ that is given by equations:

$$\begin{cases} X_1 = X_1^0(\alpha_1), \\ X_2 = X_2^0(\alpha_1). \end{cases}$$

Suppose that we are given also 3 functions $Z^0(\alpha_1), P_1^0(\alpha_1), P_2^0(\alpha_1)$. The Cauchy problem for the system (1) consists in looking for $(X(\alpha_1, \alpha_2),$

$$Z(\alpha_1,\alpha_2), P(\alpha_1,\alpha_2)) \equiv$$
$$(X_1(\alpha_1,\alpha_2), X_2(\alpha_1,\alpha_2), Z(\alpha_1,\alpha_2), P_1(\alpha_1,\alpha_2), P_2(\alpha_1,\alpha_2)).$$

$\in C^2$ that is a solution of (1) such that

$$\begin{cases} X(\alpha_1,\alpha_2)\big|_{\alpha_2=0} = X^0(\alpha_1), \\ Z(\alpha_1,\alpha_2)\big|_{\alpha_2=0} = Z^0(\alpha_1), \\ P(\alpha_1,\alpha_2)\big|_{\alpha_2=0} = P^0(\alpha_1), \end{cases} \tag{2}$$

where $X^0(\alpha_1) \equiv (X_1^0(\alpha_1), X_2^0(\alpha_1)), P^0(\alpha_1) \equiv (P_1^0(\alpha_1), P_2^0(\alpha_1))$.

From (3) we have the following necessary condition for the initial Cauchy data

$$\frac{\partial Z^0(\alpha_1)}{\partial \alpha_1} = P_1^0(\alpha_1)\frac{\partial X_1^0(\alpha_1)}{\partial \alpha_1} + P_2^0(\alpha_1)\frac{\partial X_2^0(\alpha_1)}{\partial \alpha_1}, \qquad (3)$$

which is assumed to be fulfilled.
We introduce now the following condition for the system (1)

$(\mathcal{C}_1)$: $a_{ij}(X, Z, P)$

$X_1^0(\alpha_1), X_2^0(\alpha_1),$

$Z^0(\alpha_1), P_1^0(\alpha_1), P_2^0(\alpha_1)$ ( )

$$\left|\frac{D(a_{11},a_{12})}{D(P_1,P_2)}\right|(X_1^0(\alpha_1))^2 + \left|\frac{D(a_{21},a_{22})}{D(P_1,P_2)}\right|(X_2^0(\alpha_1))^2 +$$

$$[\left|\frac{D(a_{11},a_{22})}{D(P_1,P_2)}\right| + \left|\frac{D(a_{21},a_{12})}{D(P_1,P_2)}\right|]X_1^0(\alpha_1)X_2^0(\alpha_1) +$$

$$+(\frac{\partial a_{11}}{\partial P_1} + \frac{\partial a_{12}}{\partial P_2})X_1^0(\alpha_1) + (\frac{\partial a_{21}}{\partial P_1} + \frac{\partial a_{22}}{\partial P_2})X_2^0(\alpha_1) + 1 \neq 0, \qquad (4)$$

a_{ij} $(X_1^0(\alpha_1), X_2^0(\alpha_1),$

$Z^0(\alpha_1), P_1^0(\alpha_1), P_2^0(\alpha_1))$.

2. Hyperbolicity

We set

$$U = (X_1, X_2, Z, P_1, P_2)^T,$$

A(U)=

$$\begin{bmatrix} a_{12}-1 & a_{22} & 0 & 0 & 1 \\ -a_{11} & -a_{21}-1 & 0 & -1 & 0 \\ a_{12}P_1 - a_{11}P_2 & a_{22}P_1 - a_{21}P_2 & -1 & -P_2 & P_1 \\ 0 & -a_{11}a_{22}+a_{12}a_{21} & 0 & a_{12}-1 & -a_{11} \\ a_{11}a_{22}-a_{12}a_{21} & 0 & 0 & a_{22} & -a_{21}-1 \end{bmatrix}. \qquad (5)$$

We write the system (1) in the matrix form

$$\frac{\partial U}{\partial \alpha_2} = A(U)\frac{\partial U}{\partial \alpha_1}, \tag{6}$$

Now we recall some definitions and results on hyperbolic systems. To do this we may consider following more genéral normal system in two variables

$$\frac{\partial V}{\partial \alpha_2} = H(V)\frac{\partial V}{\partial \alpha_1} + G(V), \tag{7}$$

where $V, G(V)$ are columm-vectors of size $m \times 1$ and $H(V)$ is matrix of size $m \times m$.

The Cauchy problem for system (7) consists in looking for $V(\alpha_1, \alpha_2) \in C^1$ such that

$$V(\alpha_1, \alpha_2)\big|_{\alpha_2=0} = V^0(\alpha_1), \tag{8}$$

where $V^0(\alpha_1)$ is a given vector function.

Definition 1. ([9]) ( )

$V \in R^m$

) $H(V)$

) R^m,

Theorem 1. ([9]) ( )

$H(V)$

( ) ( )

Theorem 2. ([9]) ( )

$2m$

$2m$

Theorem 3. $a_{12} \neq a_{21}$ $(X_1, X_2, Z, P_1, P_2) \in R^5$,

( )

For the system (6) we do not assume that $a_{12} \neq a_{21}$. In this case only condition 1) in Definition 1 is valid. We show below (Theorem 5) that **under some restrictions on coefficients** $a_{ij}(X, Z, P)$, the system (6) can be reduced to a diagonal one of 7 quasilinear equations with respect to 7 unknowns. From the Theorem 1 it follows that there exists locally unique smooth solution for the Cauchy problem (6), (2). In this case the system (6) could be said to be **weakly hyperbolic.**

3. Reduced system

Set

$$\mathcal{C}(X_1, X_2, Z, P_1, P_2) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -a_{11} & -a_{21} & 0 & 1 & 0 \\ -a_{12} & -a_{22} & 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

Then

$$\mathcal{C}^{-1}(X_1, X_2, Z, P_1, P_2) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ a_{11} & a_{21} & 0 & 1 & 0 \\ a_{12} & a_{22} & 0 & 0 & 1 \end{bmatrix}.$$

Set

$$V \equiv (\tilde{X}_1, \tilde{X}_2, \tilde{Z}, Q_1, Q_2)^T = C^{-1}U.$$

That means

$$\begin{cases} \tilde{X}_1 = X_1, \\ \tilde{X}_2 = X_2, \\ \tilde{Z} = Z, \\ Q_1 = P_1 + a_{11}(X, Z, P)X_1 + a_{21}(X, Z, P)X_2, \\ Q_2 = P_2 + a_{12}(X, Z, P)X_1 + a_{22}(X, Z, P)X_2. \end{cases} \tag{10}$$

Proposition 1. (C_1)
(X, Z, Q)

$\in R^5$, ﬃ $(X^0(\alpha_1), Z^0(\alpha_1), Q^0(\alpha_1))$,
P_1, P_2

$$\begin{cases} P_1 + a_{11}(X,Z,P)X_1 + a_{21}(X,Z,P)X_2 = Q_1, \\ P_2 + a_{12}(X,Z,P)X_1 + a_{22}(X,Z,P)X_2 = Q_2 \end{cases} \tag{11}$$

$$P_1 = f(X_1, X_2, Z, Q_1, Q_2), P_2 = g(X_1, X_2, Z, Q_1, Q_2), \tag{12}$$

$$Q^0(\alpha_1) = P^0(\alpha_1) + X^0(\alpha_1)H(X^0(\alpha_1), Z^0(\alpha_1), P^0(\alpha_1))$$

$$H(X,Z,P) = \begin{bmatrix} a_{11}(X,Z,P) & a_{12}(X,Z,P) \\ a_{21}(X,Z,P) & a_{22}(X,Z,P) \\ & \end{bmatrix}. \tag{13}$$

Theorem 4. V

$$\frac{\partial V}{\partial \alpha_2} = \mathcal{A}(V)\frac{\partial V}{\partial \alpha_1} + \mathcal{B}(V)V, \tag{14}$$

$$\mathcal{A} = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & -P_2 & P_1 \\ 0 & 0 & 0 & a_{12}-a_{21}-1 & 0 \\ 0 & 0 & 0 & 0 & a_{12}-a_{21}-1 \end{bmatrix}, \tag{15}$$

$$\mathcal{B} = \mathcal{C}^{-1}(A\frac{\partial C}{\partial \alpha_1} - \frac{\partial C}{\partial \alpha_2}) =$$

$$\begin{bmatrix} \frac{\partial a_{12}}{\partial \alpha_1} & \frac{\partial a_{22}}{\partial \alpha_1} & 0 & 0 & 0 \\ -\frac{\partial a_{11}}{\partial \alpha_1} & -\frac{\partial a_{21}}{\partial \alpha_1} & 0 & 0 & 0 \\ -\frac{\partial a_{11}}{\partial \alpha_1}P_2 + \frac{\partial a_{12}}{\partial \alpha_1}P_1 & -\frac{\partial a_{21}}{\partial \alpha_1}P_2 + \frac{\partial a_{22}}{\partial \alpha_1}P_1 & 0 & 0 & 0 \\ (a_{12}-a_{21}-1)\frac{\partial a_{11}}{\partial \alpha_1} - \frac{\partial a_{11}}{\partial \alpha_2} & (a_{12}-a_{21}-1)\frac{\partial a_{21}}{\partial \alpha_1} - \frac{\partial a_{21}}{\partial \alpha_2} & 0 & 0 & 0 \\ (a_{12}-a_{21}-1)\frac{\partial a_{12}}{\partial \alpha_1} - \frac{\partial a_{12}}{\partial \alpha_2} & (a_{12}-a_{21}-1)\frac{\partial a_{22}}{\partial \alpha_1} - \frac{\partial a_{22}}{\partial \alpha_2} & 0 & 0 & 0 \end{bmatrix}, \tag{16}$$

$a_{ij}(X_1, X_2, Z, P_1, P_2)$ P_1, P_2

$P_1 = f(X_1, X_2, Z, Q_1, Q_2), P_2 = g(X_1, X_2, Z, Q_1, Q_2)$

4. Diagonalization

It is clear from (15) that the system (14) is **not diagonal**. We give now some **sufficient conditions** under which the system (14) **can be reduced to a diagonal** quasilinear one.

We introduce now other condition for the system (1)

(C_2) : $a_{ij}(X, Z, P)$ fi

$$\begin{cases} \dfrac{\partial a_{ij}}{\partial Z} = 0, \\ \dfrac{\partial a_{ij}}{\partial X_1} - a_{11}\dfrac{\partial a_{ij}}{\partial P_1} - a_{12}\dfrac{\partial a_{ij}}{\partial P_2} = 0, \\ \dfrac{\partial a_{ij}}{\partial X_2} - a_{21}\dfrac{\partial a_{ij}}{\partial P_1} - a_{22}\dfrac{\partial a_{ij}}{\partial P_2} = 0. \end{cases} \tag{17}$$

We set

$$S_1 = \frac{\partial Q_1}{\partial \alpha_1}, S_2 = \frac{\partial Q_2}{\partial \alpha_1}.$$

Proposition 2. (C_1) (C_2)

)

$$(a_{12} - a_{21} - 1)\frac{\partial a_{ij}}{\partial \alpha_1} - \frac{\partial a_{ij}}{\partial \alpha_2} = 0, \tag{18}$$

) $b_{ij}(X, Q)$ $c_{ij}(X, Q)$

$$\frac{\partial a_{ij}}{\partial \alpha_1} = b_{ij}(X, Q)S_1 + c_{ij}(X, Q)S_2, \forall i, j = 1, 2. \tag{19}$$

We introduce **new dependent variables**

$$W = (X_1, X_2, Z, Q_1, Q_2, S_1, S_2)^T.$$

From Proposition 2 it follows

Theorem 5. Assume the conditions (C_1) and (C_2). Then the system (14) can be diagonalized, i.e. it may be reduced to following diagonal one:

$$\frac{\partial W}{\partial \alpha_2} = \tilde{A}(W)\frac{\partial W}{\partial \alpha_1} + F(W), \tag{20}$$

where

$$\tilde{A} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t \end{bmatrix}, \tag{21}$$

where $t = a_{12} - a_{21} - 1$ and

$$F(W) = F_1(W) + F_2(W),$$

where

$$F_1(W) =$$

$$\begin{bmatrix} S_2 + (b_{12}S_1 + c_{12}S_2)X_1 + (b_{22}S_1 + c_{22}S_2)X_2 \\ -S_1 + (b_{11}S_1 + c_{11}S_2)X_1 - (b_{21}S_1 + c_{21}S_2)X_2 \\ -P_2S_1 + P_1S_2 + [-(b_{11}S_1 + c_{11}S_2)P_2 + (b_{12}S_1 + c_{12}S_2)P_1)]X_1 \\ 0 \\ 0 \\ [(b_{12} - b_{21})S_1 + (c_{12} - c_{21})S_2)]S_1 \\ [(b_{12} - b_{21})S_1 + (c_{12} - c_{21})S_2)]S_2 \end{bmatrix},$$

$$F_2(W) = \begin{bmatrix} 0 \\ 0 \\ [-(b_{21}S_1 + c_{21}S_2)P_2 + (b_{22}S_1 + c_{22}S_2)P_1]X_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where in $F_1(W), F_2(W)$ the variables P_1, P_2 must be replaced by $f(X_1, X_2, Z, Q_1, Q_2)$ and $g(X_1, X_2, Z, Q_1, Q_2)$ respectively.

Theorem 6. (C_1) (C_2).

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5. Application to the classical weakly hyperbolic Monge-Ampère equation

The classical hyperbolic Monge-Ampère equation with two variables is that of the form

$$F(x_1, x_2, z, p_1, p_2, r, s, t) = Ar + Bs + Ct + (rt - s^2) - E = 0, \quad (22)$$

where $z = z(x_1, x_2)$ is an unknown function defined for $(x_1, x_2) \in R^2, p_1 = \frac{\partial z}{\partial x_1}, p_2 = \frac{\partial z}{\partial x_2}, r = \frac{\partial^2 z}{\partial x_1^2}, s = \frac{\partial^2 z}{\partial x_1 \partial x_2}$ and $t = \frac{\partial^2 z}{\partial x_2^2}$. The coefficients A, B, C and E are real smooth functions of (x_1, x_2, z, p_1, p_2) and satisfy the condition of hyperbolicity:

$$\Delta := B^2 - 4(AC + E) > 0.$$

In this case the characteristic equation

$$\lambda^2 + B\lambda + (AC + E) = 0 \quad (23)$$

has two different real roots $\lambda_1 = \lambda_1(x_1, x_2, z, p_1, p_2)$, $\lambda_2 = \lambda_2(x_1, x_2, z, p_1, p_2)$. In the case, where the equation (1) is hyperbolic, it can be written in the following equivalent form

$$\begin{vmatrix} z_{x_1x_1} + C & z_{x_1x_2} + \lambda_1 \\ z_{x_2x_1} + \lambda_2 & z_{x_2x_2} + A \end{vmatrix} = 0. \quad (24)$$

Equation (22) was investigated in [1], [2] by G. Darboux and E. Goursat under the assumptions that $\Delta > 0$ and **there are two independent first integrals**for the equation (22). In this case the equation (22) had been also considered in [3], [4], [6], [7] by reducing it to a hyperbolic quasilinear system of first-order partial differential equations with two variables. For the case $\Delta \geq 0$ in [5] M.Tsuji proved loal solvability of Cauchy problem (22), (25) provided that there exist two independent first integrals. In [10] D. V. Tuniski considered the case $\Delta \geq 0$ and proved solvability of the Cauchy problem in class of multivalue functions, but under rather strong assumptions on coefficients A, B, C, E.

In [8] we have proposed a solving method for the equation (24) that reduces it to the system (1) with $a_{11} = C, a_{12} = \lambda_1, a_{21} = \lambda_2, a_{22} = A$. Applying Theorem 6 stated above to the last system **we can consider the case $\Delta \geq 0$ and we do not assume existence of two independent first integrals.**

Suppose the functions $X_1^0(\alpha_1), X_2^0(\alpha_1), Z^0(\alpha_1), P_1^0(\alpha_1), P_2^0(\alpha_1)$ are given as§in 1, that satisfy the condition (3).

Cauchy problem: The Cauchy problem for the equation (22) consists in looking for $z(x) \in C^2$ that is a solution of (22) such that

$$\begin{cases} z(x)\big|_{x=X^0(\alpha_1)} = Z^0(\alpha_1), \\ z_{x_j}(x)\big|_{x=X^0(\alpha_1)} = P_j^0(\alpha_1), j = 1, 2, \end{cases} \tag{25}$$

where $X^0(\alpha_1) \equiv (X_1^0(\alpha_1), X_2^0(\alpha_1))$.

We assume that the Cauchy problem (22), (25) is not charateristic, i.e.

$$C\big(X_1^{0'}(\alpha_1)\big)^2 + A\big(X_2^{0'}(\alpha_1)\big)^2 - BX_1^{0'}(\alpha_1)X_2^{0'}(\alpha_1) +$$

$$\big(X_1^{0'}(\alpha_1)P_1^{0'}(\alpha_1) + X_2^{0'}(\alpha_1)P_2^{0'}(\alpha_1)\big) \neq 0, \tag{26}$$

where the coefficients A, B, C are calculated at $(X_1^0(\alpha_1), X_2^0(\alpha_1), Z^0(\alpha_1), P_1^0(\alpha_1), P_2^0(\alpha_1))$.

Theorem 7. ( )

( ) ( )

) ﬃ A, B, C, E z;

) $\Delta \geq 0$;

)

$$\left|\frac{D(C,\lambda_1)}{D(p_1,p_2)}\right|(X_1^0(\alpha_1))^2 + \left|\frac{D(\lambda_2,A)}{D(p_1,p_2)}\right|(X_2^0(\alpha_1))^2 +$$

$$[\left|\frac{D(C,A)}{D(p_1,p_2)}\right| + \left|\frac{D(\lambda_2,\lambda_1)}{D(p_1,p_2)}\right|]X_1^0(\alpha_1)X_2^0(\alpha_1) +$$

$$+(\frac{\partial C}{\partial p_1} + \frac{\partial \lambda_1}{\partial p_2})X_1^0(\alpha_1) + (\frac{\partial \lambda_2}{\partial p_1} + \frac{\partial A}{\partial p_2})X_2^0(\alpha_1) + 1 \neq 0, \qquad (27)$$

$A, C, \lambda_1, \lambda_2$

$(X_1^0(\alpha_1), X_2^0(\alpha_1),$
$Z^0(\alpha_1), P_1^0(\alpha_1); P_2^0(\alpha_1))$;

) $A(x,z,p), C(x,z,p), \lambda_1(x,z,p), \lambda_2(x,z,p)$

fi

$$\begin{cases} \dfrac{\partial \varphi}{\partial x_1} - C\dfrac{\partial \varphi}{\partial p_1} - \lambda_1 \dfrac{\partial \varphi}{\partial p_2} = 0, \\ \dfrac{\partial \varphi}{\partial x_2} - \lambda_2 \dfrac{\partial \varphi}{\partial p_1} - A\dfrac{\partial \varphi}{\partial p_2} = 0. \end{cases} \qquad (28)$$

( ) ( )

The Monge-Ampère equations, satisfying conditions 1) - 4) of Theorem 7 are said to be **weakly hyperbolic** ones.

Exemples.

1) ([5], [10]) The coeffiients A, B, C, E are constants with $\Delta \geq 0$. It is easy to see that all the assumptions of the Therem 7 are satisfied.

2) Suppose $v(y,t)$ is a solution of the **Burger equation**

$$v_t + vv_y = 0, \tag{29}$$

which satisfies the following condition

$$v_y(P_1^0(\alpha_1) - P_2^0(\alpha_1), X_1^0(\alpha_1) + X_2^0(\alpha_1))(X_1^0(\alpha_1) + X_2^0(\alpha_1)) + 1 \neq 0. \tag{27'}$$

Then the Monge-Ampère equation

$$rt - s^2 + v^2(z_{x_1} - z_{x_2}, x_1 + x_2) = 0$$

with $A = B = C = 0, E = -v^2(z_{x_1} - z_{x_2}, x_1 + x_2), \Delta = 4v^2(p_1 - p_2, x_1 + x_2), \lambda_1 = -\lambda_2 = v(p_1 - p_2, x_1 + x_2)$ satisfies all conditions of the Theorem 7.

3) Suppose $v(y,t), w(y,t)$ are some solutions of the equation (29) that satisfy the condition:

$$v_y(-P_1^0(\alpha_1), X_1^0(\alpha_1))w_y(-P_2^0(\alpha_1), X_2^0(\alpha_1))X_1^0(\alpha_1)X_2^0(\alpha_1) -$$

$$v_y(-P_1^0(\alpha_1), X_1^0(\alpha_1))X_1^0(\alpha_1) - w_y(-P_2^0(\alpha_1), X_2^0(\alpha_1))X_2^0(\alpha_1) + 1 \neq 0. \tag{27''}$$

Then the equation

$$w(-z_{x_2}, x_2)r + v(-z_{x_1}, x_1)t + (rt - s^2) - v(-z_{x_1}, x_1)w(-z_{x_2}, x_2) = 0$$

with $A = w(-p_2, x_2), B = 0, C = v(-p_1, x_1), E = v(-p_1, x_1)w(-p_2, x_2), \Delta =\equiv 0, \lambda_1 = \lambda_2 = 0$ satisfies all conditions of the Theorem 7.

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