On the cohomology of Coxeter groups and their finite parabolic subgroups

(Coxeter群とその有限位数の
放物的部分群のコホモロジーについて)

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1. Introduction

The purpose of this paper is to study the cohomology of Coxeter groups in terms of their parabolic subgroups of finite order. Given finite sets S and $\{m_{ij}\}$, $(i,j) \in S \times S$, where m_{ij} are integers or ∞ such that $m_{ii} = 1$, $2 \leq m_{ij} = m_{ji} \leq \infty$ $(i \neq j)$, the group W defined by the generators $\{r_i\}_{i \in S}$ and the fundamental relation $(r_i r_j)^{m_{ij}} = 1$, $m_{ij} \neq \infty$, is called a *Coxeter group.* We will identify the set of generators $\{r_i\}_{i \in S}$ with the set S. Also, if we wish to emphasis the set S we shall write (W, S) in place of W. (Some authors call (W, S) a *Coxeter system.*)

A subgroup of (W, S) generated by a subset $T \subseteq S$ is called a *parabolic subgroup* of W and is denoted by W_T. In particular, $W_S = W$ and $W_\emptyset$ is the trivial subgroup. A parabolic subgroup inherits a structure of a Coxeter group in an obvious way. Note that Coxeter groups of finite order are completely classified. The reader will refer [1] or [6] for a general theory of Coxeter groups.

Given a Coxeter group (W, S), let $\mathcal{F}$ be the poset of (possibly empty) subsets F of S such that W_F is a finite parabolic subgroup of W. Given a W-module A of coefficients, set

$$\mathcal{H}^*(W, A) = \operatorname*{lim.inv.}_{F \in \mathcal{F}} H^*(W_F, A),$$

where the inverse limit is taken with respect to the restriction maps $H^*(W_F, A) \to H^*(W_{F'}, A)$ (where $F \supset F'$), and define

$$\rho : H^*(W, A) \to \mathcal{H}^*(W, A) \tag{1}$$

to be the canonical homomorphism induced by the restriction maps $H^*(W, A) \to H^*(W_F, A)$. If $A = k$ is a commutative ring with unity (regarded as a W-module with the trivial W-action), then $\mathcal{H}^*(W, k)$ is a graded ring and ρ is a ring homomorphism.

Rusin [9] and Davis and Januszkiewicz [4] studied, among other things, the mod 2 cohomology ring of certain Coxeter groups. In particular, they proved that ρ yields an isomorphism $H^*(W, \mathbb{Z}/2) \cong \mathcal{H}^*(W, \mathbb{Z}/2)$ if (W, S) is a right-angled Coxeter group (i.e. $m_{ij} = 2$ or ∞ for all distinct $i, j \in S$) [4, Theorem 4.11] or a Coxeter group satisfying the hypothesis of [9, Corollary 30] (which is too complicated to repeat here). Inspired by their results, we studied the homomorphism ρ in far general context, and obtained the following results.

Theorem 1. *Let k be a commutative ring with unity. A ring homomorphism $\rho : H^*(W,k) \to \mathcal{H}^*(W,k)$ satisfies the following two properties:*

(i) *If $u \in \ker \rho$, then u is nilpotent.*

(ii) *Suppose k is a field of characteristic $p > 0$. For every $v \in \mathcal{H}^*(W,k)$, there is an integer $n \geq 0$ such that $v^{p^n} \in \operatorname{im}\rho$.*

(A homomorphism satisfying the properties (i) and (ii) in Theorem 1 is called an *F-isomorphism* in [8].) Theorem 1 provides us some understanding of the rôle of the cohomology of finite parabolic subgroups. Notice that the homomorphism ρ may have a non-trivial kernel (Remark 3), hence our result is best possible in one direction.

We do not know whether ρ may have a non-trivial cokernel at present. We give a sufficient condition for ρ to be surjective. A Coxeter group (W,S) is called *aspherical* in [7] if every three distinct elements of S generate a parabolic subgroup of infinite order.

Theorem 2. *If W is an aspherical Coxeter group, then ρ is surjective for any abelian group A of coefficients (with trivial W-action).*

In case $k = \mathbb{Z}/2$, we have more to say. By Theorem 1, ρ induces a homomorphism $H^*(W,k)/\sqrt{0} \to \mathcal{H}^*(W,k)/\sqrt{0}$, where $\sqrt{0}$ denotes the nilradical. Rusin proved that the mod 2 cohomology ring of any finite Coxeter group has no nilpotent elements [9, Theorem 9]. Hence the nilradical of $\mathcal{H}^*(W,\mathbb{Z}/2)$ is trivial. From this together with Theorem 1 and 2 we obtain

Corollary. *For any Coxeter group W, ρ induces a monomorphism*

$$H^*(W,\mathbb{Z}/2)/\sqrt{0} \to \mathcal{H}^*(W,\mathbb{Z}/2),$$

which is an isomprhism if W is aspherical.

Any Coxeter group has a finite virtual cohomological dimension [10] and hence its Farrell cohomology is defined. Theorem 1 and Theorem 2 hold also for the Farrell cohomology. In particular, the latter holds for all W-module A. We refer to [2] for the definition and properties of the Farrell cohomology.

The left of the paper is organized as follows. Given a Coxeter group W, Davis constructed in [3] a contractible W-complex $\mathcal{U}$, named the universal complex, such that the isotropy subgroup of each cell is a parabolic group of finite order. In §2 we will recall his construction and make a further observation on $\mathcal{U}$. Associated with the complex $\mathcal{U}$, there is a spectral sequence converging to the cohomology of W. In §3 we will investigate this spectral sequence and its differentials. In §4 we will prove Theorem 1 and 2 and their analogues for the Farrell cohomology by using the spectral sequence described in §3.

Finally, we remark that other studies concerning (co)homology of Coxeter groups can be found in [5], [7].

2. Davis' Construction of W-Complexes

Given a Coxeter group (W, S), Davis constructed a certain finite dimensional, contractible W-complex $\mathcal{U}$ on which W acts properly. We recall how this goes.

Let $\mathcal{F}$ be a poset defined in §1. Let $|\mathcal{F}|$ be the (abstract) simplicial complex defined as follows: Namely, the vertices of $|\mathcal{F}|$ are the elements of $\mathcal{F}$, and the simplices of $|\mathcal{F}|$ are the linearly ordered finite subsets $F_0 \subset \cdots \subset F_n$ of $\mathcal{F}$. $|\mathcal{F}|$ is called the nerve of $\mathcal{F}$. $|\mathcal{F}|$ is a cone with $\emptyset \in \mathcal{F}$ as a cone point. We denote the simplex $F_0 \subset \cdots \subset F_n$ by the ordered $(n+1)$-tuple $(F_0, \cdots, F_n)$, and we identify $|\mathcal{F}|$ with its geometric realization.

For $s \in S$, let $P(s)$ be the union of simplices $(F_0, \cdots, F_n)$ of $|\mathcal{F}|$ with $s \in F_0$. $P(s)$ is also a cone with a cone point $\{s\} \in \mathcal{F}$. For each $x \in |\mathcal{F}|$, set $T(x) = \{s \in S : x \in P(s)\}$. Define an equivalence relation $\sim$ in $W \times |\mathcal{F}|$, W being considered discrete, by

$$(w_1, x_1) \sim (w_2, x_2) \Leftrightarrow x_1 = x_2 \text{ and } w_1 w_2^{-1} \in W_{T(x_1)}.$$

Let $\mathcal{U} = W \times |\mathcal{F}|/\sim$ be the quotient space. $\mathcal{U}$ is a simplicial complex, whose simplex is the image of $\{w\} \times (F_0, \cdots, F_n) \subset W \times |\mathcal{F}|$ in $\mathcal{U}$ ($w \in W$, $(F_0, \cdots, F_n) \in \mathcal{F}$). One of the main result in [3] is that $\mathcal{U}$ is contractible.

For $(w, x) \in W \times |\mathcal{F}|$, denote its image in $\mathcal{U}$ by $[w, x]$. By the correspondence $x \leftrightarrow [1, x]$, a complex $|\mathcal{F}|$ can be regarded as a subcomplex of $\mathcal{U}$. W acts on $\mathcal{U}$ by $w \cdot [w', x] = [ww', x]$, and the action is simplicial and properly discontinuous. Moreover, a subcomplex $|\mathcal{F}|$ is a fundamental domain, in the sense that every W-orbit intersects $|\mathcal{F}|$ in exactly one point. Notice that the isotropy subgroup of a simplex $(F_0, \cdots, F_n)$ of $|\mathcal{F}| \subset \mathcal{U}$ is a finite parabolic subgroup W_{F_0}. See [3] for the detail of the construction. ($\mathcal{U}$ is called the *universal complex* in [3].)

We give a further property of $\mathcal{U}$ which will be used in the next section. Recall that an ordered (simplicial) complex is a simplicial complex together with a partial ordering on its vertices, such that the vertices of any simplex are linearly ordered. We have:

Lemma 1. *$\mathcal{U}$ is an ordered simplicial complex. An action of W on $\mathcal{U}$ preserves the ordering on vertices.*

Proof. Since a complex $|\mathcal{F}|$ is the nerve of a poset $\mathcal{F}$, $|\mathcal{F}|$ is an ordered complex. Any vertex of $\mathcal{U}$ is of the form $[w, F] \in \mathcal{U}$ for some $w \in W$ and $F \in \mathcal{F}$. Define a partial ordering on vertices of $\mathcal{U}$ by $[w, F_1] < [w, F_2] \Leftrightarrow F_1 \subset F_2$. If $[w, F_1] < [w, F_2]$ then $[w'w, F_1] < [w'w, F_2]$ for any $w' \in W$. Hence the action of W preserves the ordering on vertices. Using the fact that $|\mathcal{F}|$ is a fundamental domain of the W-action, we see that any simplex σ of $\mathcal{U}$ is expressed as $\sigma = w\tau$ for some $w \in W$ and some simplex τ of $|\mathcal{F}|$. If $\tau = (F_0, \cdots, F_n)$ is such a simplex of $|\mathcal{F}|$, then vertices of σ are $[w, F_0], \cdots, [w, F_n]$, which are linearly ordered. $\square$

3. Some Spectral Sequence

Arguments of this section can be applied with no change to the case of the ordinary as well as the Farrell cohomology (cf. Remark 1). We deal with the both two cases at once.

Let $|\mathcal{F}|_p$ (*resp.* $\mathcal{U}_p$) denote the set of p-simplices of $|\mathcal{F}|$ (*resp.* $\mathcal{U}$). Since $|\mathcal{F}|$ is a fundamental domain of the action of W, a set $|\mathcal{F}|_p$ can be regarded as a set of representatives of elements of $\mathcal{U}_p$ mod W. Since $\mathcal{U}$ is a finite dimensional, contractible, ordered simplicial complex with an order preserving W-action, there is a spectral sequence of the form

$$E_1^{pq} = \prod_{\sigma \in |\mathcal{F}|_p} H^q(W_\sigma, A) \Rightarrow H^{p+q}(W, A), \tag{2}$$

where W_σ is the isotropy subgroup at σ [2, p.282]. Since $W_\sigma = W_{F_0}$ for $\sigma = (F_0, \cdots, F_n)$, the E_1-term consists of the cohomology of finite parabolic subgroups of W.

Next, we investigate the differential $d_1^{pq} : E_1^{p,q} \longrightarrow E_1^{p+1,q}$ explicitly. For simplicity, write $H^*(W_\sigma) = H^*(W_\sigma, A)$. For a simplex σ of $\mathcal{U}$ and $w \in W$, let $c(w^{-1})^* : H^*(W_\sigma) \longrightarrow H^*(W_{w\sigma})$ be the isomorphism induced by a conjugation. Write $c(w^{-1})^*(u) = wu$. Then E_1^{pq} can be identified with the subgroup of $\prod_{\sigma\in\mathcal{U}_p} H^q(W_\sigma)$ consisting of $\prod_{\sigma\in\mathcal{U}_p} u_\sigma$ satisfying $wu_\sigma = u_{w\sigma}$ for all $w \in W$, $\sigma \in \mathcal{U}_p$. The differential d_1^{pq} is the restriction to this subgroup of the map

$$d : \prod_{\sigma\in\mathcal{U}_p} H^q(W_\sigma) \to \prod_{\tau\in\mathcal{U}_{p+1}} H^q(W_\tau).$$

defined as follows. For all $\tau = (F_0, \cdots, F_{p+1}) \in \mathcal{U}_{p+1}$ let $\tau_i = (F_0, \cdots \hat{F_i}, \cdots, F_{p+1})$, $0 \le i \le p+1$, and let $\rho_i : H^q(W_{\tau_i}) \longrightarrow H^q(W_\tau)$ be the restriction maps. (If $i \ne 0$, then $W_{\tau_i} = W_\tau$ and ρ_i is the identity map.) Then d is given by

$$d\left(\prod_{\sigma\in\mathcal{U}_p} u_\sigma\right) = \prod_{\tau\in\mathcal{U}_{p+1}} \left(\sum_{i=0}^{p+1} (-1)^i \rho_i u_{\tau_i}\right) \tag{3}$$

[2, Lemma X.4.2]. Moreover the edge homomorphism $H^*(W) \longrightarrow E_2^{0,*} \subset \prod_{\sigma\in\mathcal{U}_p} H^*(W_\sigma)$ is identified with the map induced by restriction maps (cf. proof of [2, Proposition X.4.6]). Now we have the description of the differentials of the spectral sequence (3).

Lemma 2. *The differential of the spectral sequence* (2) *is given by the formula* (3) *with* $\mathcal{U}_p$ *replaced by* $|\mathcal{F}|_p$.

Proof. Let $\mathcal{W}_\sigma$ be a set of representatives of left cosets of W_σ in W. Then

$$\prod_{\tau\in\mathcal{U}_p} H^q(W_\tau) = \prod_{\sigma\in|\mathcal{F}|_p} \prod_{w\in\mathcal{W}_\sigma} H^q(W_{w\sigma}).$$

We assume $1 \in W_\sigma$. Regarding E_1^{pq} as a subgroup of $\prod_{\tau\in\mathcal{U}_p} H^q(W_\tau)$, define $\alpha : \prod_{\sigma\in|\mathcal{F}|_p} H^*(W_\sigma) \longrightarrow E_1^{pq}$ by

$$\prod_{\sigma\in|\mathcal{F}|_p} u_\sigma \mapsto \prod_{\sigma\in|\mathcal{F}|_p} \prod_{w\in W_\sigma} wu_\sigma.$$

Then α is a map which gives the prescribed identification of E_1^{pq} with a subgroup of $\prod_{\tau\in\mathcal{U}_p} H^q(W_\tau)$ (cf. proof of [2, Lemma X.4.2]). A map $\beta : E_1^{pq} \longrightarrow \prod_{\sigma\in|\mathcal{F}|_p} H^*(W_\sigma)$ defined by

$$\prod_{\sigma\in|\mathcal{F}|_p} \prod_{w\in W_\sigma} u_{w\sigma} \mapsto \prod_{\sigma\in|\mathcal{F}|_p} u_\sigma$$

is the inverse of α. Both maps are well-defined since any element $\prod_{\sigma\in\mathcal{U}_p} u_\sigma$ of E_1^{pq} satisfies $wu_\sigma = u_{w\sigma}$.

Regard $\prod_{\sigma\in|\mathcal{F}|_p} H^q(W_\sigma)$ as a subgroup of $\prod_{\sigma\in\mathcal{U}_p} H^q(W_\sigma)$ consisting of those $\prod_{\sigma\in\mathcal{U}_p} u_\sigma$ with $u_\sigma = 0$ whenever $\sigma \notin |\mathcal{F}|_p$. Since $|\mathcal{F}|$ is a subcomplex of $\mathcal{U}$, we see that $\tau_i \in |\mathcal{F}|_p$ for each $\tau \in |\mathcal{F}|_{p+1}$. Hence

$$d\left(\prod_{\sigma\in|\mathcal{F}|_p} H^q(W_\sigma)\right) \subseteq \prod_{\tau\in|\mathcal{F}|_{p+1}} H^q(W_\tau),$$

where d is the map given by the formula (3). Define $D^{pq} : \prod_{\sigma\in|\mathcal{F}|_p} H^q(W_\sigma) \longrightarrow \prod_{\tau\in|\mathcal{F}|_{p+1}} H^q(W_\tau)$ by the restriction of the map d to the subgroup $\prod_{\sigma\in|\mathcal{F}|_p} H^q(W_\sigma)$. Then D^{pq} is the map satisfying the formula (3) with $\mathcal{U}_p$ replaced by $|\mathcal{F}|_p$.

Now the lemma follows from the commutative diagram

$$\begin{tikzcd} E_1^{p,q} \arrow[r, "d_1^{pq}"] \arrow[d, shift left, "\beta"] & E_1^{p+1,q} \arrow[d, shift left, "\beta"] \\ \prod_{\sigma\in|\mathcal{F}|_p} H^q(W_\sigma) \arrow[r, "D^{pq}"] \arrow[u, shift left, "\alpha"] & \prod_{\tau\in|\mathcal{F}|_{p+1}} H^q(W_\tau). \arrow[u, shift left, "\alpha"] \end{tikzcd}$$

□

Remark 1. The description of the spectral sequences in this section quoted from [2] is written for the Farrell cohomology. However, the same argument holds for the ordinary cohomology.

4. Proof of Theorems

Thanks to Lemma 2, we can compute $E_2^{0,q}$-terms of the spectral sequence (2) as follows. Recall that $|\mathcal{F}|_0$ is identified with $\mathcal{F}$, while $|\mathcal{F}|_1$ is identified with $F_0 \subset F_1$ in $\mathcal{F}$. By Lemma 2, $E_2^{0,q} = \ker d_1^{0,q}$ can be identified with the subgroup of $\prod_{F\in\mathcal{F}} H^q(W_F)$ consisting of those families $\prod_{F\in\mathcal{F}} u_F$ satisfying the following condition: if $F_0 \subset F_1$ in $\mathcal{F}$, then u_{F_1} restricts to u_{F_0} via the restriction map $H^*(W_{F_1}) \longrightarrow H^*(W_{F_0})$. That is,

$$E_2^{0,q} = \mathcal{H}^q(W) = \operatorname*{lim.inv.}_{F\in\mathcal{F}} H^q(W_F).$$

Compare [2, Lemma X.4.3]. An edge homomorphism $H^*(W) \to E_2^{0,*}$ can be identified with ρ. This follows from the commutative diagram

$$\begin{tikzcd} H^q(W) \arrow[r] \arrow[dr] & E_1^{0,q} \arrow[d, shift left, "\beta"] \\ & \prod_{\sigma\in|\mathcal{F}|_0} H^q(W_\sigma) \arrow[u, shift left, "\alpha"] \end{tikzcd}$$

where arrows starting at $H^q(W)$ indicate maps induced by the restrictions, and α, β are maps defined in the proof of Lemma 2.

Now we can prove Theorem 1, for both the ordinary and the Farrell cohomology at once. Let $H^{n,*}(W,k)$ be the filtration of $H^*(W,k)$ associated with the spectral sequence (2). Since $E_2^{p,q}$ is concentrated at $0 \le p \le \dim|\mathcal{F}|$, $H^{n,*}(W,k) = 0$ for $n > \dim|\mathcal{F}|$. Since $\ker\rho = H^{1,*}(W,k)$ and the spectral sequence has a multiplicative structure compatible with the cup product of $H^*(W,k)$ [2, pp.284–285], for any $u \in \ker\rho$, $u^n \in H^{n,*}(W,k) = 0$ whenever $n > \dim|\mathcal{F}|$. So u is nilpotent. Next, suppose k is a field of characteristic p, p prime, and let $v \in E_2^{0,*} = \mathcal{H}^*(W,k)$. Then $d_2(v^p) = 0$, since

$$d_2(v^p) = pv^{p-1}d_2(v) = 0.$$

Hence $v^p \in E_3^{0,*}$. Iterating the argument we obtain $v^{p^n} \in E_\infty^{0,*}$ whenever $n+2 > \dim|\mathcal{F}|$. Thus Theorem 1 is proved.

Now we turn to the proof of Theorem 2. It suffices to show that $E_2^{*,*} = E_\infty^{*,*}$. If a Coxeter group (W,S) is aspherical, then $|\mathcal{F}|$ is at most two dimensional and hence the $E_1^{p,*}$-terms of the spectral sequence (2) are concentrated at $0 \le p \le 2$. Any 2-simplex of $|\mathcal{F}|$ is of the form $(\emptyset, F_0, F_1)$, and the isotropy subgroup of such simplex is trivial. The Farrell cohomology group of the trivial group vanishes for any coefficients, and hence $E_1^{2,*} = 0$ in this case, which proves Theorem 2 for the Farrell cohomology.

The case of the ordinary cohomology needs more work, since $E_1^{2,0} \neq 0$. We claim that $E_2^{2,0} = 0$, which proves the spectral sequence collapses at E_2-page. Observe that for every 1-simplex (F_0, F_1) of $|\mathcal{F}|$ with $F_0 \neq \emptyset$, a 2-simplex $(\emptyset, F_0, F_1)$ is the only one having (F_0, F_1) as its face. For a 2-simplex $(\emptyset, F_0, F_1)$ and $a \in A$, define $u = \prod_{\sigma \in |\mathcal{F}|_2} u_\sigma \in E_1^{2,0}$ and $v = \prod_{\tau \in |\mathcal{F}|_1} v_\tau \in E_1^{1,0}$ by

$$u_\sigma = \begin{cases} a & \sigma = (\emptyset, F_0, F_1), \\ 0 & \text{otherwise}, \end{cases}$$
$$v_\tau = \begin{cases} a & \tau = (F_0, F_1), \\ 0 & \text{otherwise}. \end{cases}$$

This is possible since $H_0(\{1\}, A) \cong H_0(W_F, A) \cong A$. The isomorphims follow from that W-action on A is assumed to be trivial for the case of ordinary cohomology. Applying the defining formula (3) of differentials, we have $d_1^{1,0}(v) = u$. Since any element of $E_1^{2,0}$ is a sum of such u's, it follows that $d_1^{1,0}$ is surjective. Thus $E_2^{2,0} = 0$ and this completes the proof.

Remark 2. The hypothesis of Theorem 2 for the ordinary cohomology that the action of W on A is trivial cannot be removed. For if the action is non-trivial, then the restriction map $H^0(W_F, A) \to H^0(\{1\}, A)$ is not an isomorphism but only an injection.

Remark 3. There is an aspherical Coxeter group for which ρ is not an isomorphism. Let W be an aspherical Coxeter group defined by

$$W =< s_1, s_2, s_3 : s_i^2 = 1, (s_i s_j)^3 = 1 \text{ if } i \neq j > .$$

Then the mod 2 ordinary cohomology of W is given by $H^*(W, \mathbb{Z}/2) = \mathbb{Z}/2[u, v]/(u^2)$, where $\deg u = 2$, $\deg v = 1$, and (u^2) is the ideal generated by u^2. The calculation is due to Rusin [9, p.52]. As we have mentioned in §1, $\mathcal{H}^*(W, \mathbb{Z}/2)$ has no nilpotent elements. Hence $\rho(u) = 0$.

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