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Innovation, Technology, and Economic Growth
(技術革新、技術と経済成長)

Tatsuro Iwaisako

Graduate School of Economics, Osaka University

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Contents

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Chapter 1

Introduction

Levels of per capita income vary enormously across countries. In particular, there is a considerable gap of per capita income between developed countries and developing countries. Jones(2002) emphasizes some empirical evidences about economic growth and one of them is the fact that the poorest countries have per capita income that are less than 5 percent of per capita incomes the richest countries. Furthermore, as he emphasizes as the second fact, not only levels of per capita income but also growth rates of per capita income vary substantially across countries. For example, the economy of the United States has grown at a rate of 1.8 percent per year for more than one hundred years, on the other hand the economies of some countries in Africa and Latin America have never grown from 1960 to 1997.

What causes the difference in levels and growth rates of per capita income? What determines the levels and growth rates of per capita income? Many economic growth researchers have tried to answer this question in different ways. First, they consider investment rates as the most important determinant of levels of per capita income. That is, they consider that economies with higher investment rate can sustain higher level of per capita income. This fact is confirmed by the Solow model(Solow, 1956). Second, some studies consider that the determinant of the levels of per capita income is investment rate of human capital, that is, educational attainment as well as the investment rate of physical capital(Uzawa, 1965 and Lucas, 1988). They consider that economies with not only the higher investment rate in physical capital but also higher educational attainment can sustain higher levels or growth rates of per capita income. Finally, some studies, especially recent studies, consider that the most important determinant of levels or growth rates of per capita income is technological progress. Constructing the growth models where the research effort is endogenous, Romer(1990), Grossman and Helpman(1991), Aghion and Howitt(1992), and many other studies examine how R&D efforts and growth rates are determined.

Of course, all of these three factors are necessary for sustained growth. However, technological progress that improves the factor productivity is the most important to the developed countries that have already accumulated physical capital and human capital sufficiently. In fact, Young(1995) provides the following important empirical fact. Using growth accounting, he analyzes rapid economic growth of the East Asian countries since 1960 and shows that a large part of the growth of output relies on factor accumulation, such as investment in physical capital and increases in labor inputs. From this empirical finding, Krugman(1994) predicted that these East Asian economies would not maintain this high growth rate. This Krugman's prediction is based on the fact that improvements of technology are essential for sustained growth. [1]

For these reasons, this doctoral dissertation also consider that technological progress is essential for sustained growth and the present paper analyzes the growth model including endogenous technological progress and examines what determines levels or growth rates of per capita income.

First, what yields technological progress? That is the research and development(R&D) activity by private firms. Then, on what do the R&D activities depend? They depend on the laws and institutions concerning intellectual property right protection. In particular, patent policies affect the incentive of R&D. Hence, in Chapters 2 and 3, developing growth models in which R&D activities are determined endogenously, I examine how patent policies affect economic growth and the welfare level.

In Chapter 2, we investigate how extending patent length affects economic growth and the social welfare based on an endogenous growth model with R&D activities. The first study of optimal patent length in a dynamic general equilibrium model, Judd(1985) has concluded that the patent length that maximizes the social welfare is infinite. In contrast to this result, we show that the patent length that maximizes the social welfare is finite. Moreover, we analyze not only patent length policy but also patent breadth policy. In order to introducing patent breadth policy into the model, we assume that all patented goods are subject to compulsory licensing and that patent authorities can control the royalty rate that licensees must pay to licensors. We can interpret this royalty rate as patent breadth. In this extension, we show that the patent length that maximizes the social welfare is not infinite even if the royalty rate can be controlled. This result is contrast to the result obtained by Gilbert and Shapiro(1990), that the optimal patent policy involves infinite patent length. In addition, we also show that the patent breadth that maximizes the social welfare is not maximum one.

In contrast to Chapter 2, Chapter 3 develops an endogenous growth model in which not only innovation but also capital accumulation is a driving force of economic growth.

[1] Using the growth model where both physical capital accumulation and innovation are driving forces of economic growt, Matsuyama(1999) shows that innovations can occur, once capital is accumulated sufficiently.

That is, the model has two engines of economic growth, innovation and capital accumulation. We investigate how the patent policy affects economic growth in this more general endogenous growth model. In the endogenous growth models with only innovation, tightening patent protection necessarily raises the return of innovation and accelerates innovation, and enhances economic growth. We can find this result in the models of Chapter 1, Kwan and Lai(2003), and O'Donoghue and Zweimuller(2004). In contrast to the results of these models, stronger patent protection accelerates innovation but discourages capital accumulation in the model of this chapter. As a result, strengthening patent protection may reduce the growth rate of output and the growth-maximizing degree of the patent protection is lower than the maximum degree of the patent protection. We also investigate how the patent protection affects social welfare and show that the welfare-maximizing degree of the patent protection is lower than the growth-maximizing degree of the patent protection.

Chapters 2 and 3 analyze the endogenous growth models that have no transitional dynamics. Consequently, per capita income of economies is always growing at a constant rate in these models. However, as Jones(2002) argues as the third fact, growth rates of individual countries are not generally constant over time. Furthermore, patterns of growth are quite different among countries. In the latter half of the doctorial dissertation, I construct growth models with endogenous technological progress that explain these empirical facts.

Dissimilar to the other chapters, Chapter 4 analyzes issues of technology choice. Chapter 4 investigates the equilibrium dynamics of an economy where two production technologies are available: one is a primitive production technology which realizes constant returns to scale, and the other is an advanced technology, which exhibits increasing returns to scale due to specialization of intermediate goods production. It is shown that if investors' choice of technology is introduced, then even a quite simple two-period overlapping generations model can generate endogenous cycles, poverty traps, or permanent growth. Consequently, this growth model can explain the observed differences of patterns of growth among countries. The paper also discusses how underdeveloped countries can achieve production that puts them on a permanent growth path.

Chapter 5 explores dynamic properties of a Schumpeterian growth model in the environment where innovations are imitated costlessly after one period. In this environment, the sector that obtains an innovation becomes monopolistic and the sector that does not obtain an innovation becomes competitive. If more sectors become monopolistic, this decreases the labor demand and lowers the wage rate, and raises the return of innovation. Because of this pecuniary externality, the rate of return of innovation rises as the aggregate investment in R&D increases. As a result, multiple balanced growth paths can be generated. In the balanced growth path with the higher growth rate, many sectors obtain innovations and become monopolistic and consequently the average markup of this economy is high. In the one with the lower growth rate, only a small number of sectors obtain innovations and the many other sectors become competitive and consequently the average markup is low. Furthermore, the model in this chapter can generate indeterminacy of equilibrium paths and growth cycles. Hence the present model can explain the observed differences of growth rates among countries and the observed fluctuations of growth rates.

Finally, Chapter 6 concludes this doctorial dissertation.

Chapter 2

Patent Policy in an Endogenous Growth Model

2.1 Introduction

Research and development activity by private firms is one of the most important factors as the engine of economic growth. There are many studies of economic growth that view R&D as the engine of economic growth (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). What has a great impact on the private firms' R&D activity is the patent policy. This paper investigates how the patent policy affects economic growth and social welfare.[1]

There are two welfare effects of extending the patent length. One is the growth enhancing effect: extending the patent length enhances economic growth by raising the rate of return of R&D. The other is the static inefficiency effect: extending the patent length reduces the amount of output by increasing the proportion of monopolistic sector, and thus the amount of consumption. Considering these two opposite effects, the patent length that maximizes the social welfare can be finite. On the contrary to the argument above, the famous study of optimal patent length in a dynamic general equilibrium model, Judd (1985) has concluded that the patent length that maximizes the social welfare is infinite based on an exogenous growth model. The reason for this conclusion is that under the infinite patent length all goods are equally priced and there is no distortion due to monopoly. On the contrary, this paper shows that the patent length that maximizes the social welfare can be finite based on an endogenous growth model. In the endogenous growth model, the long-run growth rate depends on the patent length. Because the growth rate is determined endogenously in the present model, reducing the patent length that is infinitely long raises the ratio of competitive

[1]The first analysis based on a partial equilibrium model was conducted by Nordhaus (1969).

sectors sharply. This also increases output and consumption sharply, thus can increase the utility level. This results in that infinitely-lived patents do not maximize the social welfare.[2]

In addition to the analysis of patent length, we investigate how the patent policy including patent breadth affects the social welfare. Under the circumstances that the patent breadth, namely, the licensing royalty rate can be controlled, we show also that the patent length that maximizes the social welfare is not infinite. The seminal paper that examines the patent policy based on a partial equilibrium model (Gilbert and Shapiro (1990)) suggests that the optimal patent policy involves an infinite patent length and therefore this paper's result that infinite patent length is not optimal is quite important. Additionally, the optimal patent policy including patent breadth has not been examined in a dynamic general equilibrium model. Hence this paper makes a significant contribution also in this respect.[3]

The paper is organized as follows. Section 2 sets up the model and solves the social welfare function on the balanced growth path. Section 3 shows that the patent length that maximizes the social welfare is finite. Moreover, introducing compulsory licensing in the same way as Tandon (1982), section 4 examines the patent policy that includes patent length and patent breadth. Section 5 concludes the paper.

2.2 The Model

We extend the endogenous growth model developed by Rivera-Batiz and Romer (1991) and Barro and Sala-i-Martin (1995) in order to examine how the patent policy affects economic growth and social welfare.

2.2.1 Firms

The final good sector produces by use of intermediate goods and labor inputs. Production technology of the final good sector is given by

$$Y = AL^{1-\alpha}D^{\alpha}, \quad 0 < \alpha < 1 \tag{2.1}$$

where Y is an amount of the final goods, L is labor input, D is a composite of intermediate goods and A denotes a given factor productivity. The composite of intermediate

[2]Chou and Shy (1991) argue that the patent length that maximizes the social welfare can be finite also. However, their analysis uses a partial equilibrium model where the interest rate is constant. In addition, their model is an exogenous growth model. Chou and Shy (1993) investigate how a long duration of patents affects investment in R&D activities by using an overlapping generations model.

[3]Li (2001) examines the patent policy focusing only on patent breadth in an extension of Grossman and Helpman (1991).

goods is specified as a CES production function:[4]

$$D = \left[\int_0^N X(i)^{\alpha} di \right]^{\frac{1}{\alpha}}, \tag{2.2}$$

where N is the number of intermediate goods, $X(i)$ is the quantity of the intermediate input $i \in [0, N]$, and $1/(1-\alpha)(> 1)$ represents the elasticity of substitution among the intermediate inputs.

We assume that perfect competition prevails in the final goods market. Let the final good be the numeraire. The first-order conditions for the profit maximization of the final good sector are given by the following:

$$(1-\alpha)AL^{-\alpha}D^{\alpha} = w, \tag{2.3}$$

$$\alpha AL^{1-\alpha}X(i)^{\alpha-1} = P(i), \tag{2.4}$$

where w denotes the real wage rate and $P(i)$ denotes the price of intermediate good i.

Every intermediate good is produced by using one unit of final goods. Then the profit of the firm producing intermediate good i is

$$\pi(i) = [P(i) - 1]X(i)$$

There are patented intermediate goods and nonpatented goods at each point in time. In the patented intermediate good sectors firms behave monopolistically and then the price charged by the firms is

$$P^M = \frac{1}{\alpha},$$

where P^M is the price of patented intermediate good. The output level and the profit of patented intermediate good, X^M and $\bar{\pi}$, respectively become

$$X^M = \left(\alpha^2 AL^{1-\alpha}\right)^{\frac{1}{1-\alpha}}, \tag{2.5}$$

$$\bar{\pi} = \frac{1-\alpha}{\alpha}\left(\alpha^2 AL^{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{2.6}$$

On the other hand, perfect competition prevails in the nonpatented intermediate goods sectors and the price P^C is equal to the marginal cost.

$$P^C = 1.$$

The output level of nonpatented intermediate good X^C is given by

$$X^C = \left(\alpha AL^{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{2.7}$$

[4]Ethier (1982) first used the CES function as a production function.

2.2.2 Households

We consider an economy populated by L households, who supply one unit of labor inelastically. The household seeks to maximize the lifetime utility

$$\max_{\{c_t\}_{t=0}^{\infty}} U = \int_0^{\infty} e^{-\rho t} \log c_t dt, \tag{2.8}$$

where c_t is consumption per household and ρ is the discount rate.

The households can save by investing in R&D activities and thus having the property right of the firms producing the patented intermediate goods. R&D sector can invent one unit of intermediate good by using η unit of the final good. The household that has the property right of one firm can get the profit $\bar{\pi}$ during the patent length, T. Let e_t and a_t denote the investment in the R&D activity and the number of shares of firms producing the patented goods respectively. Then the intertemporal budget constraint of the household is given by

$$e_t = \bar{\pi} a_t + w_t - c_t, \tag{2.9}$$

$$e_t \geq 0. \tag{2.10}$$

The number of shares of firms producing the patented goods at time t increases by the investment at time t, and decreases by the number of expiring patents that are invented at time $t - T$ and thus we get

$$\dot{a}_t = \frac{e_t}{\eta} - \frac{e_{t-T}}{\eta}. \tag{2.11}$$

The optimality condition for e_t is that the marginal cost of R&D investment is equal to the marginal benefit of R&D investment. The marginal cost of R&D investment is the marginal loss of utility due to the reduction of current consumption, $e^{-\rho t}(1/c_t)$. The marginal benefit of R&D investment is the discounted sum of the marginal utility due to the profit flow protected by the patent. Therefore the optimality condition becomes [5]

$$e^{-\rho t} \frac{1}{c_t} = \int_t^{t+T} e^{-\rho \tau} \frac{1}{c_\tau} \frac{\bar{\pi}}{\eta} d\tau. \tag{2.12}$$

Differentiation of this equation with respect to time, t yields the following differential-difference equation in c_t:

$$\frac{\dot{c}_t}{c_t} = \frac{\bar{\pi}}{\eta} \left(1 - e^{-\rho T} \frac{c_t}{c_{t+T}}\right) - \rho. \tag{2.13}$$

[5] See Judd(1985).

2.2.3 The Dynamics of Variety of The Intermediate Goods

In this subsection, we describe the dynamics of the number of goods N_t. Let N_t^C, N_t^M be the number of nonpatented and patented goods respectively. The output Y_t becomes

$$\begin{aligned} Y_t &= AL^{1-\alpha}\left[\int_0^{N_t} X_t(i)^\alpha di\right] \\ &= AL^{1-\alpha}\left[N_t^C(X^C)^\alpha + N_t^M(X^M)^\alpha\right]. \end{aligned} \tag{2.14}$$

Let Q_t be GDP.[6] Then we obtain

$$\begin{aligned} Q_t &= Y_t - \int_0^{N_t} X_t(i)di \\ &= AL^{1-\alpha}\left[N_t^C(X^C)^\alpha + N_t^M(X^M)^\alpha\right] - \left(N_t^C X^C + N_t^M X^M\right). \end{aligned} \tag{2.15}$$

Letting $C_t(= c_t L)$ the aggregate consumption, the equilibrium condition for the final goods market is

$$AL^{1-\alpha}\left[N_t^C(X^C)^\alpha + N_t^M(X^M)^\alpha\right] - \left(N_t^C X^C + N_t^M X^M\right) = C_t + \eta\dot{N}_t. \tag{2.16}$$

The number of competitive intermediate goods sectors at time t, N_t^C is the number of all intermediate goods sectors at time $t-T$, that is, $N_t^C = N_{t-T}$. Therefore the number of monopolistic sectors at time t, N_t^M, is given by

$$N_t^M = N_t - N_t^C = N_t - N_{t-T}. \tag{2.17}$$

From (2.16) and (2.17), the dynamics of the number of intermediate goods N_t is characterized by the following differential-difference equation:

$$\eta\dot{N}_t = q_M N_t + (q_C - q_M)N_{t-T} - C_t, \tag{2.18}$$

where

$$q_C = AL^{1-\alpha}(X^M)^\alpha - X^M = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L, \tag{2.19}$$

$$q_M = AL^{1-\alpha}(X^C)^\alpha - X^C = (1-\alpha)(1+\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L. \tag{2.20}$$

q_M represents the added value generated by production of a good that one monopolistic firm supplies and q_C represents the added value generated by production of a good that one competitive firm supplies. Because the value of X that maximizes the added value that one sector yields, $q(X) = AL^{1-\alpha}X^\alpha - X$ is X^C, q_C and q_M satisfy the inequality $q_C > q_M$. The dynamics of this economy is characterized by equations (3.10) and (2.18). The complete analysis of the dynamics is beyond this paper and we limit the attention to the characteristic of the balanced growth path.

[6]Note that GDP is given by the sum of the added value of the final goods sector and that of the intermediate goods sector.

2.2.4 Balanced Growth Path

Consumption C_t, the number of intermediate goods N_t and output Y_t grow at the same rate g along the balanced growth path. From equation (13), the growth rate, g satisfies

$$g + \rho = \frac{\bar{\pi}}{\eta}\left[1 - e^{-(g+\rho)T}\right]. \tag{2.21}$$

This equation defines the function that determines the growth rate corresponding to the patent length. Let's denote this function $g(T)$. This function $g(T)$ is an increasing function with respect to T. That is, extending the patent length raises the growth rate of the economy. $g(T)$ takes its maximum value, $\bar{\pi}/\eta - \rho$ when the patent length is infinite.

Let us denote the proportion of the competitive intermediate sectors by μ. Along the balanced growth path, μ becomes

$$\mu(T) \equiv \frac{N_t^C}{N_t} = \frac{N_{t-T}}{N_t} = e^{-g(T)T}. \tag{2.22}$$

From (2.18) and (2.22), on the balanced growth path the following equations must be satisfied,

$$\begin{aligned} \eta\frac{\dot{N}_t}{N_t} &= q_M + (q_C - q_M)\frac{N_t^C}{N_t} - \frac{C_t}{N_t} \\ \eta g(T) &= q_M + (q_C - q_M)\mu(T) - \chi, \end{aligned} \tag{2.23}$$

where $\chi \equiv C/N$. Suppose that the economy is on the balanced growth path, then the consumption path is given by

$$c_t L = C_0(T)e^{g(T)t}. \tag{2.24}$$

From equation (2.23), the consumption at time 0 becomes

$$C_0(T) = N_0\left[q_M + (q_C - q_M)\mu(T) - \eta g(T)\right]. \tag{2.25}$$

We get the representative consumer's life time utility, $U(T)$ as the following function of T

$$\begin{aligned} U(T) &= \int_0^\infty e^{-\rho t}\log c_t dt \\ &= \frac{1}{\rho^2}\left[\rho\log C_0(T) + g(T)\right] - \frac{1}{\rho}\log L \end{aligned} \tag{2.26}$$

As is stated in the introduction, there are two welfare effects of extending the patent length. One is the growth enhancing effect: extending the patent length enhances economic growth by raising the rate of return of R&D. The other is the static inefficiency effect: extending the patent length reduces the amount of output by increasing the proportion of monopolistic sector, and thus the amount of consumption (see equation (2.25)).

2.3 Patent Length

In this section, by deriving the representative household's lifetime utility with respect to the patent length, we examine how the patent length affects the social welfare and show that the patent length that maximizes the social welfare is finite. We can get the following proposition.

Proposition 1. *The patent length that maximizes the social welfare is finite. Moreover, if the following inequality*

$$(A) \qquad \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L-\rho\eta}{\left[1-(1+\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\right](1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L} > -\log\left[1-\frac{\rho\eta}{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L}\right]$$

is satisfied, the patent length that maximizes the social welfare obtains the positive growth rate. [7]

Proof. See Appendix 1. □

First, the intuition of the first half of this proposition can be understood as follows: There are two opposite effects of reducing the patent length. Reducing the patent length decreases the monopolistic profit and the growth rate, but increases the proportion of the competitive sectors. When the patent length is sufficiently long, an increase of the welfare level due to the rising of the competitive sectors dominates a decrease of the welfare level due to the falling of the growth rate.

Second, we state about the latter half of proposition 1. The left hand side of inequality (A) is an increasing function of A and L and the right hand side of inequality (A) is a decreasing function of A and L. We can draw the graph of the left hand side and that of the right hand side of inequality (A) as functions of $A^{\frac{1}{1-\alpha}}L$ (see Figure 1). When $\rho\eta$ is constant, the values of $A^{\frac{1}{1-\alpha}}L$ more than Ω satisfy the inequality (A) as depicted in Figure 1. Moreover lowering $\rho\eta$ moves both the LHS curve and the RHS curves to the left as illustrated by the dotted curves in Figure 1. Thus the lower $\rho\eta$ is, the lower Ω is. These results are understood as follows: the patent length that maximizes the social welfare obtains the sustainable growth in the economy with higher productivity, larger labor resource, lower cost of R&D, η, and lower discount rates, ρ.

[7] Because $q_C > q_M > \bar{\pi} > \rho\eta$, the left and right hand sides are always positive respectively.

2.4 Patent Length and Patent Breadth

In the literature on optimal patent design, not only patent length but also patent breadth is important. The patent breadth generally means the scope of protection offered by a patent over its lifetime. Many researchers have examined optimal patent length and breadth by using partial equilibrium analyses.

In order to examine the optimal mix between patent length and patent breadth in the present model, we consider the following patent policy: all patented goods are subject to compulsory licensing and the owners of the patents get the royalty fee that firms producing the patented goods pay over the lifetime of the patents.[8] [9] In this extension, we can interpret that increasing the royalty rate is equivalent to increasing the patent breadth. [10]

Let $\beta(< 1/\alpha - 1)$ denote the royalty rate, which specifies the licensing fee per unit of output. The marginal cost of the firms producing the patented goods is 1 plus the royalty rate. Thus the price charged by firms producing the patented goods is given by

$$P^M = 1 + \beta. \tag{2.27}$$

Using the intermediate goods demand (2.4), the output level and the profit flow of patented intermediate good, $X^M(\beta)$ and $\bar{\pi}(\beta)$, are respectively given by

$$X^M(\beta) = \left(\frac{1}{1+\beta}\alpha^2 AL^{1-\alpha}\right)^{\frac{1}{1-\alpha}}, \tag{2.28}$$

$$\bar{\pi}(\beta) = \beta\left(\frac{1}{1+\beta}\alpha AL^{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{2.29}$$

We limit the attention to the balanced growth path with a non-negative growth rate, and thus we assume that $\beta \in [\underline{\beta}, 1/\alpha - 1]$, where $\underline{\beta}$ satisfies that $\bar{\pi}(\underline{\beta})/\eta = \rho$.[11]

From (2.28), we get the added value generated by production of a good that one patented good sector supplies.

$$q_M(\beta) = AL^{1-\alpha}[X^M(\beta)]^\alpha - X^M(\beta) = \frac{1+\beta-\alpha}{(1+\beta)^{\frac{1}{1-\alpha}}}\alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L. \tag{2.30}$$

By examining the representative consumer's lifetime utility with respect to the patent length for different values of β, we can get the following proposition.

[8] The analysis of this section is suggested by a referee of this journal.

[9] Tandon (1982) examines the use of compulsory licensing in the partial equilibrium model.

[10] Gilbert and Shapiro (1990) also identify the patent breadth with the flow rate of profit available to the patentee over the lifetime of patent and examine the optimal mix between patent length and patent breadth.

[11] $\underline{\beta}$ denotes the lowest royalty rate that brings non-negative growth when the patent length is infinity. In other words, the royalty rate lower than $\underline{\beta}$ cannot obtain non-negative growth even if the patent length is infinite.

Proposition 2. *For all values of patent breadth $\beta \in [\underline{\beta}, 1/\alpha - 1]$, the patent length that maximizes the social welfare is finite. Moreover, if the following inequality*

$$(B) \quad \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L - \rho\eta}{\left[(1-\alpha) - \frac{1+\beta-\alpha}{(1+\beta)^{\frac{1}{1-\alpha}}}\right] \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L} > -\log\left[1 - \frac{\rho\eta}{\frac{\beta}{(1+\beta)^{\frac{1}{1-\alpha}}} \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L}\right]$$

is satisfied, the patent length that maximizes the social welfare obtains the positive growth rate.

Proof. See Appendix 2. □

The studies of optimal patent policy in partial equilibrium models have suggested that optimal patent policy involves infinite patent length if the patent breadth is increasing costly in terms of deadweight loss (see Gilbert and Shapiro (1990)). On the contrary to the results of these researches, we can show that the patent length that maximizes the social welfare is finite even in the situation that the patent breadth can be controlled. [12]

Next, we examine how the patent breadth affects the social welfare when the patent length is constant. By deriving the representative household's lifetime utility with respect to the patent length and examining how the patent breadth affects the social welfare, we can get the following proposition.

Proposition 3. *For all values of patent length $T \in [\underline{T}, \infty]$, the patent breadth that maximizes the social welfare level is less than the maximum patent breadth.*

Proof. See Appendix 3. □

Propositions 2 and 3 state that the finite patent length with a less-than-maximum patent breadth maximizes the social welfare level. This result is different from the results of the existing studies based on the partial equilibrium model (see Gilbert and Shapiro (1990) and Tandon (1982)).

2.5 Conclusion

The famous study of optimal patent length in the general equilibrium model, Judd(1985) has concluded that the patent length that maximizes the social welfare is infinite.

In this paper, by developing the model with deadweight loss due to monopoly protected by the patent policy, we have shown that the patent length that maximizes the social welfare can be finite.

[12] Klemperer (1990) shows that finitely-lived patents can be optimal in a static model of spatial product differentiation.

In addition to the analysis of patent length, we investigate how the patent policy including patent breadth affects the social welfare. Gilbert and Shapiro (1990) examine the patent policy based on a partial equilibrium model and suggest that the optimal patent policy involves an infinite patent length. On the contrary to the results, we have shown also that the finite patent length maximizes the social welfare. Our results may rationalize why finite patent length policies are taken in an actual world.

2.6 Appendix 1

We prove that the patent length that maximizes the welfare level of the representative household, T^* is finite.

Because $g(T)$ is one to one function from T to g, we can define the inverse function of $g(T)$. By defining a new variable as $r = g + \rho$ for simplicity and we denote this inverse function as $T = T(r)$. By deriving the value of r that maximizes the welfare of the representative consumer $V(r)$, we show that the patent length that maximizes the social welfare, T^* is finite.

By taking the logarithm of the both side of (2.21) and using $r = g + \rho$, we obtain

$$T(r) = \frac{1}{r}\log\frac{\bar{\pi}}{\bar{\pi} - \eta r}. \tag{2.31}$$

Substituting this into (2.22) results in

$$\mu(r) = \left(1 - \frac{\eta}{\bar{\pi}}r\right)^{\frac{r-\rho}{r}}. \tag{2.32}$$

We limit the attention to the balanced growth path with a non-negative growth rate, and thus the domain of r is given by

$$\rho \leq r \leq \frac{\bar{\pi}}{\eta}. \tag{2.33}$$

From (2.31), the domain of T that can sustain a non-negative growth rate is given by $[\underline{T}, \infty]$, where $\underline{T} = 1/\rho \log\left[\bar{\pi}/\left(\bar{\pi} - \eta\rho\right)\right]$. We can denote the utility of the representative household as a function of r; that is, $V(r)$ is

$$V(r) = \frac{1}{\rho^2}\left\{\rho \log N_0\left[q_M + (q_C - q_M)\mu(r) - \eta(r - \rho)\right] + r\right\} - \frac{1}{\rho}(\log L + 1). \tag{2.34}$$

In order to examine the properties of the function $V(r)$, it is enough to examine

$$v(r) = \rho \log\left[q_M + (q_C - q_M)\mu(r) - \eta(r - \rho)\right] + r, \tag{2.35}$$

Because $v(r)$ depends on $\mu(r)$, we have to examine $\mu(r)$ first.

Properties of $\mu(r)$

Values of $\mu(r)$ at the boundaries of the domain of r are

$$\mu(\rho) = \left(\frac{\bar{\pi} - \eta\rho}{\bar{\pi}}\right)^0 = 1, \tag{2.36}$$

$$\mu(\frac{\bar{\pi}}{\eta}) = 0^{\frac{\bar{\pi} - \eta\rho}{\bar{\pi}}} = 0. \tag{2.37}$$

From $1-\frac{\eta}{\bar{\pi}}r<1$, $\log\left(1-\frac{\eta}{\bar{\pi}}r\right)<0$. Therefore, we obtain

$$\mu'(r)=\left[\frac{\rho}{r^2}\log\left(1-\frac{\eta}{\bar{\pi}}r\right)-\frac{r-\rho}{r}\frac{\frac{\eta}{\bar{\pi}}}{1-\frac{\eta}{\bar{\pi}}r}\right]\mu(r)<0. \tag{2.38}$$

Thus, $\mu(r)$ is a decreasing function of r.

Values of $\mu'(r)$ at the boundaries of the domain of r are

$$\mu'(\rho)=\frac{1}{\rho}\log\left(1-\frac{\eta}{\bar{\pi}}\rho\right)<0, \tag{2.39}$$

$$\mu'(\frac{\bar{\pi}}{\eta})=\lim_{r\to\frac{\bar{\pi}}{\eta}}\left[\frac{\rho}{r^2}\log\left(1-\frac{\eta}{\bar{\pi}}r\right)\mu(r)-\frac{r-\rho}{r}\frac{\eta}{\bar{\pi}}\left(1-\frac{\eta}{\bar{\pi}}r\right)^{-\frac{\rho}{r}}\right]=-\infty. \tag{2.40}$$

Note that the first term of (2.40) always takes a negative value and the second term of (2.40) becomes negative infinite when r is approaching to $\bar{\pi}/\eta$.

Properties of $v(r)$

Values of $v(r)$ at the boundaries of the domain of r are

$$v(\rho)=\rho\left(\log q_C+1\right)>0, \tag{2.41}$$

$$v(\frac{\bar{\pi}}{\eta})=\rho\log\left(q_M-\bar{\pi}+\eta\rho\right)+\frac{\bar{\pi}}{\eta}>0. \tag{2.42}$$

$v'(r)$ becomes as follows:

$$v'(r)=\rho\frac{(q_C-q_M)\mu'(r)-\eta}{q_M+(q_C-q_M)\mu(r)-\eta(r-\rho)}+1. \tag{2.43}$$

By showing that $v'(\rho)>0$ and $v'(\bar{\pi}/\eta)<0$, we show that there exists r^* such that $v'(r)=0$ in the domain of r. First, the value of $v'(\rho)$ is given by

$$v'(\rho)=\frac{(q_C-q_M)\log\left(1-\frac{\eta}{\bar{\pi}}\rho\right)-\rho\eta}{q_C}+1. \tag{2.44}$$

If the following inequality is satisfied, $v'(\rho)>0$ and therefore $r=\rho$ is not optimal, that is, the patent length that generates no growth is not optimal.

$$q_C>\rho\eta-(q_C-q_M)\log\left(1-\frac{\eta}{\bar{\pi}}\rho\right). \tag{2.45}$$

Substituting (3.13) and (2.20) into this inequality, we get the following inequality.

$$(A)\qquad \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L-\rho\eta}{\left[1-(1+\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\right](1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L}>-\log\left[1-\frac{\rho\eta}{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L}\right].$$

Second, the value of $v'(r)$ at the opposite boundary is

$$v'(\frac{\bar{\pi}}{\eta}) = \rho\frac{(q_C - q_M)}{q_M - \bar{\pi} + \eta\rho}\left[\lim_{r\to\frac{\bar{\pi}}{\eta}} \mu'(r)\right] - \frac{\rho\eta}{q_M - \bar{\pi} + \eta\rho} + 1 = -\infty. \tag{2.46}$$

From (2.44) and (2.46), the graph of $v'(r)$ intersects with the horizontal axis at least once as depicted in Figure 2. Therefore $v(r)$ possesses the unique maximum value in the domain of r and there uniquely exists a value, r^* that maximizes $V(r)$. Consequently we have shown that the finite patent length T^* corresponding to r^* maximizes the lifetime utility of the representative household.

2.7 Appendix 2

Similarly to Appendix 1, we construct the social welfare function with respect to r for different values of β and prove that the patent length that maximizes the welfare level of the representative household, T^* is finite for any values of the royalty rate, $\beta \in [\underline{\beta}, 1/\alpha - 1]$.

We limit the attention to the balanced growth path with a non-negative growth rate, and thus the domain of r is given by

$$\rho \leq r \leq \frac{\bar{\pi}(\beta)}{\eta}. \tag{2.47}$$

From (2.31), the domain of T that can sustain a non-negative growth rate is given by

$$\frac{1}{\rho}\log\frac{\bar{\pi}(\beta)}{\bar{\pi}(\beta) - \eta\rho} \leq T \leq \infty. \tag{2.48}$$

We get the lifetime utility of the representative household as a function of r and β as follows:

$$V(r;\beta) = \tfrac{1}{\rho^2}\left[\rho\log N_0 + v(r;\beta)\right] - \tfrac{1}{\rho}(\log L + 1), \tag{2.49}$$

$$v(r;\beta) = \rho\log\left\{q_M(\beta) + [q_C - q_M(\beta)]\mu(r;\beta) - \eta(r-\rho)\right\} + r, \tag{2.50}$$

where

$$\mu(r;\beta) = \left(1 - \frac{\eta}{\bar{\pi}(\beta)}r\right)^{\frac{r-\rho}{r}}. \tag{2.51}$$

We can easily show that $\mu(\rho;\beta) = 1$ and $\mu(\bar{\pi}(\beta)/\eta;\beta) = 0$. With respect to the first derivative of $\mu(r;\beta)$, we can obtain

$$\mu'(\rho;\beta) = \frac{1}{\rho}\log\left(1 - \frac{\eta}{\bar{\pi}(\beta)}\rho\right) < 0, \tag{2.52}$$

$$\begin{aligned}\mu'(\frac{\bar{\pi}}{\eta};\beta) &= \lim_{r \to \frac{\bar{\pi}(\beta)}{\eta}} \left[\frac{\rho}{r^2}\log\left(1 - \frac{\eta}{\bar{\pi}(\beta)}r\right)\mu(r;\beta) - \frac{r-\rho}{r}\frac{\eta}{\bar{\pi}(\beta)}\left(1 - \frac{\eta}{\bar{\pi}(\beta)}r\right)^{-\frac{\rho}{r}}\right] \\ &= -\infty. \end{aligned}\tag{2.53}$$

Next, the values of $v(r;\beta)$ at boundaries of the domain of r are given by

$$v(\rho;\beta) = \rho\,(\log q_C + 1) > 0, \tag{2.54}$$

$$v(\frac{\bar{\pi}(\beta)}{\eta};\beta) = \rho\log\left[q_M(\beta) - \bar{\pi}(\beta) + \eta\rho\right] + \frac{\bar{\pi}(\beta)}{\eta} > 0. \tag{2.55}$$

By showing that $v'(\rho;\beta) > 0$ and $v'(\bar{\pi}(\beta)/\eta;\beta) < 0$, we show that there exists r^* such that $v'(r;\beta) = 0$ in the domain of r. First, the value of $v'(\rho;\beta)$ is given by

$$v'(\rho;\beta) = \frac{[q_C - q_M(\beta)]\log\left(1 - \frac{\eta}{\bar{\pi}(\beta)}\rho\right) - \rho\eta}{q_C} + 1. \tag{2.56}$$

If the following inequality is satisfied, $v'(\rho;\beta) > 0$ and therefore $r = \rho$ is not optimal, that is, the patent length that generates 0-growth is not optimal.

$$q_C > \rho\eta - [q_C - q_M(\beta)]\log\left(1 - \frac{\eta}{\bar{\pi}(\beta)}\rho\right). \tag{2.57}$$

Substituting (2.29) and (2.30) into this inequality, we get the following inequality.

$$(B) \quad \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L - \rho\eta}{\left[(1-\alpha) - \frac{1+\beta-\alpha}{(1+\beta)^{\frac{1}{1-\alpha}}}\right]\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}L} > -\log\left[1 - \frac{\rho\eta}{\frac{\beta}{(1+\beta)^{\frac{1}{1-\alpha}}}\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L}\right]$$

Second, the value of $v'(r;\beta)$ at the opposite boundary is

$$\begin{aligned} v'(\frac{\bar{\pi}(\beta)}{\eta};\beta) &= \rho\frac{[q_C - q_M(\beta)]}{q_M(\beta) - \bar{\pi}(\beta) + \eta\rho}\left[\lim_{r \to \frac{\bar{\pi}(\beta)}{\eta}}\mu'(r;\beta)\right] - \frac{\rho\eta}{q_M(\beta) - \bar{\pi}(\beta) + \eta\rho} + 1 \\ &= -\infty. \end{aligned}\tag{2.58}$$

From (2.56) and (2.58), $v(r;\beta)$ has the unique maximum value in the domain of r for any values of β. Therefore we have shown that the finite patent length $T^*(\beta)$ corresponding to $r^*(\beta)$ maximizes the lifetime utility of the representative household.

2.8 Appendix 3

We prove that the patent breadth that maximizes the welfare level of the representative household, β^* is less than the maximum patent breadth for any duration of patent, $T \in [\underline{T}, \infty]$.

From (2.29), (2.21), and $r = g + \rho$, β and r satisfy the following equality:

$$\bar{\pi}(\beta) = \eta \frac{r}{1 - e^{-rT}}. \tag{2.59}$$

The functions $\bar{\pi} = \bar{\pi}(\beta)$ and $\bar{\pi} = \eta \frac{r}{1-e^{-rT}}$ are one to one functions respectively, and thus we can define the function from β to r as $r(\beta)$. Therefore, we can derive the social welfare function of β.

Because we limit the attention to non-negative growth paths, $r(\beta)$ satisfies that $r(\beta) \geq \rho$. Thus, from (2.59), the domain of β is given by $[\underline{\beta}_T, 1/\alpha - 1]$, where $\underline{\beta}_T$ satisfies that $\bar{\pi}(\underline{\beta}_T) = \eta \frac{\rho}{1-e^{-\rho T}}$.

We get the lifetime utility of the representative household with respect to β as follows:

$$\tilde{V}(\beta) = \tfrac{1}{\rho^2} \left(\rho \log N_0 + \tilde{v}(\beta) \right) - \tfrac{1}{\rho} (\log L + 1), \tag{2.60}$$

$$\tilde{v}(\beta) = \rho \log \left[q_M(\beta) + (q_C - q_M(\beta)) e^{-(r(\beta) - \rho)T} - \eta (r(\beta) - \rho) \right] + r(\beta). \tag{2.61}$$

$\tilde{v}'(\beta)$ becomes as follows:

$$\begin{aligned} \tilde{v}'(\beta) &= \rho \frac{\frac{dq_M(\beta)}{d\beta} \left[1 - e^{-(r(\beta)-\rho)T} \right] + (q_C - q_M(\beta)) \left[-T \frac{dr(\beta)}{d\beta} e^{-(r(\beta)-\rho)T} \right] - \eta \frac{dr}{d\beta}}{q_M(\beta) + (q_C - q_M(\beta)) e^{-(r(\beta)-\rho)T} - \eta (r(\beta) - \rho)} \\ &\quad + \frac{dr(\beta)}{d\beta}. \end{aligned} \tag{2.62}$$

By showing that $\tilde{v}'(\frac{1-\alpha}{\alpha}) < 0$, we show that the patent breadth that maximizes the social welfare is less than the maximum patent breadth, $(1-\alpha)/\alpha$.

Differentiating (2.59) with respect to β and r, we get

$$\frac{dr(\beta)}{d\beta} = \frac{\bar{\pi}'(\beta)}{\eta} \frac{\left(1 - e^{-rT}\right)^2}{1 - e^{-rT}(1 + rT)}. \tag{2.63}$$

From (2.29), we get

$$\frac{d\bar{\pi}(\beta)}{d\beta} = \left(1 - \frac{\alpha}{1-\alpha}\beta\right) \left(\frac{1}{1+\beta} \alpha A L^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \geq 0, \quad \text{for} \quad \forall \beta \in [\underline{\beta}_T, \frac{1-\alpha}{\alpha}]. \tag{2.64}$$

From this, we get $\frac{dr(\frac{1-\alpha}{\alpha})}{d\beta} = 0$. Therefore, the value of $\tilde{v}'(\frac{1-\alpha}{\alpha})$ is given by

$$\tilde{v}'(\frac{1-\alpha}{\alpha}) = \rho \frac{\frac{dq_M(\frac{1-\alpha}{\alpha})}{d\beta}\left[1 - e^{-\left(r(\frac{1-\alpha}{\alpha})-\rho\right)T}\right]}{q_M + (q_C - q_M)e^{-(r(\frac{1-\alpha}{\alpha})-\rho)T} - \eta(r(\frac{1-\alpha}{\alpha}) - \rho)}. \tag{2.65}$$

Using the facts that $q_C > q_M > \bar{\pi}$ and $r(\frac{1-\alpha}{\alpha}) < \bar{\pi}/\eta$,

$$q_M + (q_C - q_M)e^{-(r(\frac{1-\alpha}{\alpha})-\rho)T} - \eta(r(\frac{1-\alpha}{\alpha}) - \rho) > q_M - \bar{\pi} + \rho\eta > 0. \tag{2.66}$$

Therefore, we have shown that the denominator of the left hand side of (2.65) is positive. From (2.30), we get

$$\frac{dq_M(\beta)}{d\beta} = -\beta(1+\beta)^{-\frac{1}{1-\alpha}-1}(1-\alpha)^{-1}\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L < 0. \tag{2.67}$$

From (2.66) and (2.67), we show that $\tilde{v}'(\frac{1-\alpha}{\alpha}) < 0$. Therefore we show that the maximum patent breadth, $\beta = 1/\alpha - 1$ does not maximize the social welfare.

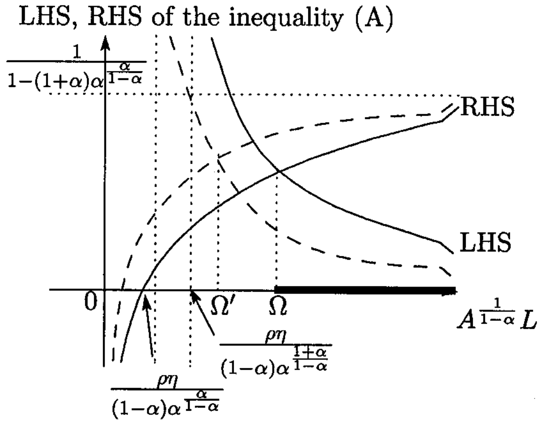


Figure 1. The parameters that satisfy the inequality (A)

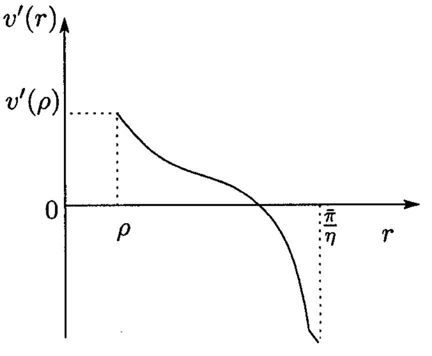


Figure 2. The property of $v'(r)$

Chapter 3

Patent Enforcement, Capital Accumulation, and Economic Growth

3.1 Introduction

For the last two decades, the United States, Japan, and other many countries have amended patent laws so that patent protection is strengthened. One of the purposes of strengthening patent protection is to stimulate innovation and to enhance economic growth. Because innovation improves productivities and thus raises the growth rate of output, strengthening patent protection is growth-enhancing policy in this sense. However, what induces the growth of output is not only innovation but also physical and human capital accumulation and growth of labor. Of course, all these factors are important for economic growth; however, we concentrate especially on capital accumulation, because innovation is an intentional activity that advances productivity and accumulation of physical capital unintentionally advance productivity through learning by doing. This latter point was pointed out by a seminal paper of Romer(1986). Therefore, to examine whether strengthening patent protection enhances economic growth, we must examine how strengthening patent protection affects the capital accumulation as well as innovation. This is the main purpose of the present paper.

A number of papers have examined how strengthening patent protection affects economic growth in endogenous growth models.[1] Introducing patent length into a variety-expansion growth model of Grossman and Helpman(1990), Michel and Nyssen(1998)

[1] Judd(1985) and Chou and Shy(1991) also examine how patent length affects market equilibrium paths. However, their models are exogenous growth models and thus they cannot examine how strengthening patent protection affects economic growth.

examine how extending patent length affects economic growth and welfare level. Goh and Olivier(2002) develop a variety-expansion growth model with a downstream and an upstream sector and introduce patent breadth as a policy instrument into their model. They examine how tightening patent protection in the downstream and upstream sectors affects economic growth and welfare. Furthermore, Iwaisako and Futagami(2003) introduce not only patent length but also patent breadth 'into a variety-expansion growth model based on Romer(1990) and examine how patent length and breadth affect economic growth and welfare. Kwan and Lai(2003) incorporate an exogenous imitation rate into a variety-expansion growth model and investigate how strengthening patent protection, that is, a decrease in the imitation rate affects economic growth and welfare. On the other hand, some papers have examined how strengthening patent protection affects economic growth in a vertical innovation growth model(Futagami et al., 1999 and O'Donoghue and Zweimuller, 2004). However, the models of these papers do not include capital accumulation, and innovation is the unique driving force of economic growth in the models. Therefore all of these papers except Michel and Nyssen(1998) and Goh and Olivier(2002) conclude that strengthening the patent protection necessarily enhances economic growth.[2]

In this paper, we construct an endogenous growth model that includes not only innovation but also capital accumulation as driving force of economic growth. That is, we integrate a variety expansion model of Grossman and Helpman (1991) with learning-by-doing model of Romer (1986). In such a general model including capital accumulation, we investigate how tightening patent protection affects innovative activities and economic growth. In this setting, we show that stronger patent protection accelerates innovation but discourages capital accumulation. The reason for this result is as follows: strengthening the patent protection increases the profit flow of the patent holders but reduces the distribution to the product inputs such as labor and capital. In particular, a decrease in the rent of capital reduces the market value of capital and discourages production of capital, that is, capital accumulation. Due to the negative effect on capital accumulation, strengthening patent protection may reduce the growth rate of output. This result is in contrast to the results of the early studies that do not include capital accumulation.[3] Moreover, we investigate how strengthening patent

[2]Michel and Nyssen(1998) and Goh and Olivier(2002) show that tightening patent protection may impede innovation in different ways. Michel and Nyssen(1998) assume that extending the patent length impedes knowledge spillovers that contribute to economic growth and show that when the diffusion of knowledge is low, a finite patent length maximizes the growth rate of output. Goh and Olivier(2002) show that tightening patent protection in the downstream sector discourages innovation in the upstream sector in their model.

[3]Using cross-country data, Gould and Gruben(1996) examine the relationship between level of patent protection and growth rate of output. Their cross-country data does not show a positive correlation between patent protection and growth rate of output clearly without controlling for other determinates of growth. The relation is rather not monotone. The result of the present paper may explain this

protection affects social welfare. We show that the welfare-maximizing degree of the patent protection is lower than the growth-maximizing degree of the patent protection. This result suggests that the patent authority must be cautious about strengthening patent protection. Too strong protection can hurt the social welfare.

The paper is organized as follows. Section 2 sets up the model and Section 3 solves the balanced growth path and examines how tightening patent protection affects the growth rates of innovation, capital, and output. Section 4 examines how patent protection affects social welfare. Section 5 concludes the paper.

3.2 The Model

Introducing capital accumulation into an R&D-based growth model with an expansion of product variety (Grossman and Helpman, 1991), we develop the model where both R&D activities and capital accumulation are the engines of economic growth.

3.2.1 Households

Each economy has a measure one of households. The representative household maximizes his or her lifetime utility over an infinite horizon. The lifetime utility is given by:

$$\int_0^\infty e^{-\rho t} \log D_t dt, \tag{3.1}$$

where D_t denotes a composite of final goods and ρ denotes the subjective discount rate. The composite of goods D_t is specified as Dixit and Stiglitz (1977) type utility function:

$$D_t = \left[\int_0^{N_t} x_t(i)^\alpha di \right]^{\frac{1}{\alpha}}, \quad 0 < \alpha < 1, \tag{3.2}$$

where N_t is the number of final goods available, $x(i)$ is the quantity of the good $i \in [0, N_t]$, and $1/(1-\alpha)(> 1)$ represents the elasticity of substitution among the intermediate inputs. The representative households maximize (3.1) subject to the following lifetime budget constraint:

$$\int_0^\infty e^{-\int_0^\tau r_v dv} E_\tau d\tau = \int_0^\infty e^{-\int_0^\tau r_v dv} w_\tau d\tau + W_0, \tag{3.3}$$

where E_τ denotes the instantaneous expenditure and w_τ and W_0 are the wage rate and the initial asset holding. As is well-known, the solution of the above dynamic

observed relationship.

maximization problem is characterized by the following demand function of $x(i)$ and the Euler equation:

$$x_t(i) = E_t(P_t^D)^{\frac{\alpha}{1-\alpha}} p_t(i)^{-\frac{1}{1-\alpha}}, \tag{3.4}$$

and

$$\frac{\dot{E}_t}{E_t} = r_t - \rho, \tag{3.5}$$

where

$$P_t^D = \left[\int_0^{N_t} p_t(i)^{-\frac{\alpha}{1-\alpha}} di \right]^{-\frac{1-\alpha}{\alpha}}. \tag{3.6}$$

Following Grossman and Helpman (1991), we normalize prices so that the instantaneous expenditure E_t is equal to 1 for all t. Because E_t satisfies (3.5) for all t, this normalization implies that $r_t = \rho$ for all t.

3.2.2 Final goods producing firms

Each final good is produced by using labor and capital, and the production function of good i is given by:

$$x_t(i) = A[\ell_t(i)]^{\gamma}[k_t(i)]^{1-\gamma}, \tag{3.7}$$

where $x_t(i)$ is output of good i and $\ell_t(i)$ and $k_t(i)$ denote the amounts of labor and capital devoted to production of good i. We can split the firm's maximization problem into two steps. In the first step, solving the firm's cost minimization, we obtain the labor and capital demands and the unit cost function as follows:

$$\ell_t(i) = \frac{1}{A}\left(\frac{\gamma}{1-\gamma}\right)^{1-\gamma}\left(\frac{q_t}{w_t}\right)^{1-\gamma} x_t(i), \tag{3.8}$$

$$k_t(i) = \frac{1}{A}\left(\frac{1-\gamma}{\gamma}\right)^{\gamma}\left(\frac{w_t}{q_t}\right)^{\gamma} x_t(i), \tag{3.9}$$

$$c(w_t, q_t) = \frac{1}{A}\gamma^{-\gamma}(1-\gamma)^{\gamma-1} {w_t}^{\gamma} {q_t}^{1-\gamma}, \tag{3.10}$$

where $c(w, q)$ denotes the unit cost function and q and w denote the rental rate of capital and the wage rate respectively.

Before the second step, we must consider patent policy. We assume that there are many potential imitators and that patented products can be imitated without any costs by the imitating firms. The patent authority conducts the following patent enforcement policy. If imitation occurred, then the patent authority would punish the firms that imitate patented products with a fine to protect the patent holders from imitation. That is, if a firm produced the patented good without patent, this illegal production would be exposed and fined as long as the imitator produces the patented product. The expected value of the fine corresponds to this legal marginal cost. Therefore it costs imitating firms a constant legal cost in addition to the technical marginal cost to produce one unit of patented goods without the patent.[4] On this assumption, the patentee will charge the price equal to the marginal cost of the imitators that includes the fine so that they cannot enter the market of the goods. Therefore a higher legal cost for imitation, that is, stronger patent protection allows the patentee to charge a higher price and make higher profit flow. Moreover, in order to make the analysis as simple as possible, we assume that the patent protection lasts forever, and that the controllable instrument of patent policies is only the fine.

Let $b = \beta c(w, q)$ denote the legal cost per unit of output for producing the patented goods illegally. The higher β imlies the stronger patent protection. The total marginal cost of the firms producing the patented goods without the patent becomes $(1+\beta)c(w, q)$. The patentee charges the price so that the imitators cannot make positive profit and thus the price of the good must be:

$$p_t(i) = (1+\beta)c(w_t, q_t). \tag{3.11}$$

From Eqs.(3.4), (3.11), and $E = 1$, the output level of each good is given by:

$$x_t = [(1+\beta)c(w_t, q_t)]^{-\frac{1}{1-\alpha}} (P_t^D)^{\frac{\alpha}{1-\alpha}}. \tag{3.12}$$

The profit flow of patented good is given by:

$$\pi_t = \beta c(w_t, q_t)x_t = \beta\left[(1+\beta)c(w_t, q_t)\right]^{-\frac{1}{1-\alpha}} (P_t^D)^{\frac{\alpha}{1-\alpha}}. \tag{3.13}$$

Substituting (3.11) into (3.6), we obtain:

$$P_t^D = (1+\beta)c(w_t, q_t)N_t^{-\frac{1-\alpha}{\alpha}}. \tag{3.14}$$

[4]Goh and Olivier (2002) interpret this legal marginal cost for producing the patented goods as patent breadth and assume that the patent authority can control this legal cost indirectly. In the literature, Gilbert and Shapiro(1990) identify the patent breadth with the flow rate of profit available to the patentee and assume that the patent authority can control this profit flow. On the other hand, Tandon (1982) assumes that all patented goods are subject to compulsory licensing and that the owners of the patents get the royalty fee that firms producing the patented goods pay over the lifetime of the patents. In this model, a higher royalty fee implies a higher cost for imitation and a higher profit flow of the patentee. O'Donoghue(1998) and Takalo(2001) discuss how the earlier studies have used patent breadth in the literature of patent policy in detail.

Substituting this into (3.12) and (3.13), we obtain the following quantity of each good and the profit flow:

$$x_t = \frac{1}{(1+\beta)c(w_t, q_t)N_t}. \tag{3.15}$$

$$\pi_t = \frac{\beta}{(1+\beta)N_t}. \tag{3.16}$$

3.2.3 R&D firms

We let v_t denote the value of the patent at time t, which is equal to the discounted present value of the profit flow subsequent to t. Therefore we obtain:

$$v_t = \int_t^{\infty} e^{-\int_t^{\tau} r_v dv} \pi_\tau d\tau, . \tag{3.17}$$

Differentiation of this equation with respect to time t yields the following no-arbitrage condition:

$$\dot{v_t} = r_t v_t - \pi_t. \tag{3.18}$$

If a firm engaging in R&D uses ℓ_N units of labor during the time interval dt, the firm can produce $dN = N(\ell_N/a_N)dt$ new products. Because the free entry condition requires that the revenue of R&D is equal to the cost of R&D, then we obtain:

$$v_t = \frac{a_N}{N_t} w_t. \tag{3.19}$$

3.2.4 Capital goods producing firms

We let $v_{K,t}$ denote the value of one unit of capital stock at time t, which is equal to the present discounted value of the stream of rent of capital subsequent to t. Therefore we obtain:

$$v_{K,t} = \int_t^{\infty} e^{-\int_t^{\tau} r_v dv} q_\tau d\tau, . \tag{3.20}$$

Differentiation of this equation with respect to t yields the following no-arbitrage condition with respect to capital:

$$\dot{v_{K,t}} = r_t v_{K,t} - q_t. \tag{3.21}$$

Producing capital goods requires labor input and capital input. The production function of capital goods is given by:

$$\dot{K}_t = {a_K}^{-1}(\bar{K}_t \ell_{K,t})^{\delta}(k_{K,t})^{1-\delta}, \tag{3.22}$$

where $\ell_{K,t}$ and $k_{K,t}$ denote the amounts of labor and capital devoted to production of capital goods and $\bar{K}_t$ represents effectiveness of labor in production of capital goods. We assume that this effectiveness is accumulated through the production of capital goods, and thus $\bar{K}_t = K_t$.[5] We assume that perfect competition prevails in the capital goods market. We can split the capital goods producing firm's maximization problem into two steps. In the first step, solving the firm's cost minimization, we obtain the labor and capital demands and the unit cost function as follows:

$$\ell_{K,t} = a_K \left(\frac{\delta}{1-\delta}\right)^{1-\delta} \left(\frac{q_t}{w_t}\right)^{1-\delta} \frac{\dot{K}_t}{{K_t}^{\delta}}, \tag{3.23}$$

$$k_{K,t} = a_K \left(\frac{1-\delta}{\delta}\right)^{\delta} \left(\frac{w_t}{q_t}\right)^{\delta} \frac{\dot{K}_t}{{K_t}^{\delta}}, \tag{3.24}$$

$$c_K(w_t, q_t) = a_K \delta^{-\delta} (1-\delta)^{\delta-1} {w_t}^{\delta} {q_t}^{1-\delta} {K_t}^{-\delta}, \tag{3.25}$$

where $c_K(w, q)$ denotes the unit cost function of capital goods production. Because the free entry condition requires that the revenue of capital goods production is equal to its cost, then we obtain:

$$v_{K,t} = c_K(w_t, q_t). \tag{3.26}$$

3.2.5 Market equilibrium

The households supply L units of labor flow inelastically at each time. The equilibrium condition of the labor market is given by:

$$\int_0^{N_t} \ell_t(i)di + \ell_{K,t} + \ell_{N,t} = L.$$

Using (3.8) and (3.15), we obtain the labor demand of one final good producing firm:

$$\ell_t = \frac{\gamma}{(1+\beta)w_t N_t}.$$

Substituting this, the R&D technology, and (3.23) into the labor market clearing condition, we obtain:

$$\frac{\gamma}{(1+\beta)w_t} + a_K \left(\frac{\delta}{1-\delta}\right)^{1-\delta} \left(\frac{q_t}{w_t}\right)^{1-\delta} \frac{\dot{K}_t}{{K_t}^{\delta}} + a_N \frac{\dot{N}_t}{N_t} = L. \tag{3.27}$$

[5]See Romer (1986).

On the other hand, the equilibrium condition of the capital market becomes:

$$\int_0^{N_t} k_t(i)di + k_{K,t} = K_t.$$

Substituting (3.15) into (3.9), we obtain the capital demand of one final good producing firm:

$$k_t = \frac{1-\gamma}{(1+\beta)q_t N_t}.$$

Substituting this and (3.24) into the market equilibrium condition of the capital goods, we obtain:

$$K_t = \frac{1-\gamma}{(1+\beta)q_t} + a_K \left(\frac{1-\delta}{\delta}\right)^{\delta} \left(\frac{w_t}{q_t}\right)^{\delta} \frac{\dot{K}_t}{{K_t}^{\delta}}. \tag{3.28}$$

3.3 Equilibrium Path

In this section, we derive the equilibrium path of the economy.

The wage rate and the rent of capital are determined by (3.19) and (3.26). From (3.19), (3.25), and (3.26), we obtain:

$$w_t = \frac{v_t N_t}{a_N}, \tag{3.29}$$

$$q_t = {K_t}^{-1} \delta^{\frac{\delta}{1-\delta}} (1-\delta) \left(\frac{v_{K,t} K_t}{a_K}\right)^{\frac{1}{1-\delta}} \left(\frac{v_t N_t}{a_N}\right)^{-\frac{\delta}{1-\delta}}. \tag{3.30}$$

Substituting (3.19) and (3.30) into (3.27) and (3.28), we obtain the growth rate of N_t and K_t as follows:

$$\frac{\dot{N}_t}{N_t} = \frac{L}{a_N} - \frac{\gamma-\delta}{(1-\delta)(1+\beta)v_t N_t} - {a_N}^{-1} \left(\delta \frac{a_N v_{K,t} K_t}{a_K v_t N_t}\right)^{\frac{1}{1-\delta}}. \tag{3.31}$$

$$\frac{\dot{K}_t}{K_t} = {a_K}^{-1} \left(\delta \frac{a_N v_{K,t} K_t}{a_K v_t N_t}\right)^{\frac{\delta}{1-\delta}} - \frac{(1-\gamma)}{(1-\delta)(1+\beta)v_{K,t} K_t}. \tag{3.32}$$

On the other hand, substituting the profit flow, (3.16) into the non-arbitrage condition, (3.18) yields:

$$\frac{\dot{v}_t}{v_t} = \rho - \frac{\beta}{(1+\beta)v_t N_t}. \tag{3.33}$$

Substituting the rent of capital, (3.30) into the non-arbitrage condition, (3.21) yields:

$$\frac{\dot{v_{K,t}}}{v_{K,t}} = \rho - a_K{}^{-1}(1-\delta)\left(\delta\frac{a_N v_{K,t} K_t}{a_K v_t N_t}\right)^{\frac{\delta}{1-\delta}}. \tag{3.34}$$

Consequently, the equilibrium paths of the economy are characterized by the market equilibrium conditions of labor and capital, (3.31) and (3.32), and the non-arbitrage conditions, (3.33) and (3.34). Here, we define that $y_t \equiv 1/(v_t N_t)$, $z_t \equiv 1/(v_{K,t} K_t)$. Combining (3.31), (3.32), (3.33), and (3.34), the equilibrium paths are characterized by the following differential equations with respect to y_t and z_t:

$$\frac{\dot{y}_t}{y_t} = \left[1 - \frac{1-\gamma}{(1-\delta)(1+\beta)}\right] y_t + a_N{}^{-1}\left(\delta\frac{a_N y_t}{a_K z_t}\right)^{\frac{1}{1-\delta}} - \left(\frac{L}{a_N} + \rho\right), \tag{3.35}$$

$$\frac{\dot{z}_t}{z_t} = \frac{1-\gamma}{(1-\delta)(1+\beta)} z_t - a_K{}^{-1}\delta^{\frac{1}{1-\delta}}\left(\frac{a_N y_t}{a_K z_t}\right)^{\frac{\delta}{1-\delta}} - \rho. \tag{3.36}$$

The balanced growth paths of this economy are determined so that $\dot{y} = \dot{z} = 0$. Letting y^* and z^* denote the values of y_t and z_t on the BGP, these values satisfy the following equations:

$$\left[1 - \frac{1-\gamma}{(1-\delta)(1+\beta)}\right] y^* + a_N{}^{-1}\left(\delta\frac{a_N y^*}{a_K z^*}\right)^{\frac{1}{1-\delta}} = \left(\frac{L}{a_N} + \rho\right). \tag{3.37}$$

$$\frac{1-\gamma}{(1-\delta)(1+\beta)} z^* - a_K{}^{-1}\delta^{\frac{1}{1-\delta}}\left(\frac{a_N y^*}{a_K z^*}\right)^{\frac{\delta}{1-\delta}} = \rho. \tag{3.38}$$

Using (3.35) and (3.36), we obtain the phase diagram of the present model as depicted in Fig. 1. We find that the unique steady state (y^*, z^*) is unstable. Because both y_t and z_t are jumpable variables, y_t and z_t jump to this steady state values when the patent policy changes. In other words, the present model has no transitional dynamics and the equilibrium path jumps to the BGP instantaneously.[6]

Next, we derive the growth rate of N_t and K_t on the BGP, g_N and g_K. Substituting y^* and z^* into (3.31) and (3.32) yields the growth rate of N and K. Because y and

[6]There exist two stock variables in this model, the number of consumption goods and capital stock. As is well known, the Grossman-Helpman model has no transitional dynamics. Moreover, because the production function of capital goods is an AK-type technology, there is no transitional dynamics in this model.

z must be constant on the BGP, $g_N + \dot{v}_t/v_t = 0$ and $g_K + \dot{v_{K,t}}/v_{K,t} = 0$. Using this, (3.33), and (3.34), we obtain:

$$g_N = -\frac{\dot{v}_t}{v_t} = \frac{\beta}{1+\beta}y^* - \rho, \tag{3.39}$$

$$g_K = -\frac{\dot{v_{K,t}}}{v_{K,t}} = a_K{}^{-1}(1-\delta)\left(\delta\frac{a_N y^*}{a_K z^*}\right)^{\frac{\delta}{1-\delta}} - \rho. \tag{3.40}$$

From (3.31) and (3.32), g_N and g_K satisfy the following equations:

$$g_N = \frac{L}{a_N} + \frac{(\delta-\gamma)y^*}{(1-\delta)(1+\beta)} - a_N{}^{-1}\left(\delta\frac{a_N y^*}{a_K z^*}\right)^{\frac{1}{1-\delta}}. \tag{3.41}$$

$$g_K = a_K{}^{-1}\left(\delta\frac{a_N y^*}{a_K z^*}\right)^{\frac{\delta}{1-\delta}} - \frac{(1-\gamma)}{(1-\delta)(1+\beta)}z^*. \tag{3.42}$$

Defining that $G_N = \left[a_N(g_N+\rho)\right]/(1-\delta)$ and $G_K = \left[a_K(g_K+\rho)\right]/(1-\delta)$ and eliminating y^* and z^* from these equations, we obtain the following equations of G_K and G_N:

$$\left(1-\delta+\frac{\gamma-\delta}{\beta}\right)G_N = L + \rho a_N - G_K{}^{\frac{1}{\delta}}. \tag{3.43}$$

$$\frac{\delta(1-\gamma)}{\beta}G_N = \delta G_K{}^{\frac{1}{\delta}} + \rho a_K G_K{}^{\frac{1-\delta}{\delta}}. \tag{3.44}$$

This system determines the values of g_N and g_K.

First, we examine how strengthening the patent protection affects capital accumulation. Eliminating G_N from (3.43) and (3.44) yields:

$$\frac{(1-\delta)\beta+\gamma-\delta}{\delta(1-\gamma)} = \frac{L+\rho a_N - G_K{}^{\frac{1}{\delta}}}{\delta G_K{}^{\frac{1}{\delta}} + \rho a_K G_K{}^{\frac{1-\delta}{\delta}}}. \tag{3.45}$$

This equation characterizes the relation between β and G_K. Totally differentiating (3.45), we obtain:

$$\frac{d\beta}{dG_K} = -\frac{\delta(1+\beta)+(1-\gamma)(L+\rho a_K)G_K{}^{-\frac{1}{\delta}}}{\delta G_K + \rho a_K} < 0. \tag{3.46}$$

This shows a negative correlation between the extent of patent protection and the growth rate of capital. Hence we obtain the following proposition:

Proposition 4. *Strengthening the patent protection impedes the growth of capital stock.*

The intuition for this result is as follows: strengthening the patent protection increases the profit flow of the patent holders but reduces the distribution to the product inputs such as labor income and capital income. In particular, a decrease in the capital income reduces the market value of capital and discourages production of capital, that is, capital accumulation.

Next, we examine how strengthening the patent protection affects innovation. Eliminating β from (3.43) and (3.44), we obtain G_N as a function of solely G_K as follows:

$$G_N = \frac{L + \rho a_N}{1-\delta} - \frac{{G_K}^{\frac{1}{\delta}}}{1-\gamma} - \frac{\gamma - \delta}{\delta(1-\gamma)(1-\delta)} \rho a_K {G_K}^{\frac{1-\delta}{\delta}}. \tag{3.47}$$

Differentiating (3.47) with respect to G_K yields:

$$\frac{dG_N}{dG_K} = -\frac{{G_K}^{\frac{1}{\delta}-2}}{\delta(1-\gamma)} \left(G_K + \frac{\gamma - \delta}{\delta} \rho a_K \right). \tag{3.48}$$

This show that if the capital goods sector is relatively intensive in capital, that is, $\gamma > \delta$, then $dG_N/dG_K > 0$. From Proposition, 4 $dG_K/d\beta < 0$. Therefore we obtain the following proposition:

Proposition 5. *If the consumption goods is more labor-intensive than the capital goods ($\gamma \geq \delta$) or If the consumption goods is more capital-intensive than the capital goods ($\gamma < \delta$) and $G_K > [(\delta - \gamma)\rho a_K]/\delta$, strengthening the patent protection enhances innovation. Otherwise, strengthening the patent protection impedes innovation.*

For simplicity, we assume that the capital goods sector is relatively intensive in capital, that is, $\gamma > \delta$ in the rest of the paper.

Finally we examine how the patent policy affects the growth rate of output. We define the output of the economy as follows:

$$Y_t = \left[\int_0^{N_t} x_t(i)^\alpha di \right]^{\frac{1}{\alpha}}. \tag{3.49}$$

Because the good sectors are symmetric, $Y_t = {N_t}^{\frac{1}{\alpha}} x_t$. From (3.15), we obtain: $Y_t = {N_t}^{\frac{1-\alpha}{\alpha}} [(1+\beta)c(w_t, q_t)]^{-1}$. Substituting (3.19) and (3.30) into this yields:

$$Y_t = A\gamma^\gamma (1-\gamma)^{1-\gamma} [\delta^{\frac{\delta}{1-\delta}} (1-\delta)]^{\gamma-1} (1+\beta)^{-1} (a_N y^*)^{1-\frac{1-\gamma}{1-\delta}} (a_K z^*)^{\frac{1-\gamma}{1-\delta}} {N_t}^{\frac{1-\alpha}{\alpha}} {K_t}^{1-\gamma} \tag{3.50}$$

From (3.50), the growth rate of output becomes the weighted sum of the rate of innovation and the rate of capital accumulation:

$$g_Y = \frac{1-\alpha}{\alpha} g_N + (1-\gamma) g_K. \tag{3.51}$$

To examine whether strengthening the patent protection enhances economic growth, we differentiate (3.51) with respect to G_K. Using the definitions of G_N and G_K and (3.48), we obtain:

$$\begin{aligned}\frac{dg_Y}{dG_K} &= \frac{1-\alpha}{\alpha}\frac{1-\delta}{a_N}\frac{dG_N}{dG_K} + (1-\gamma)\frac{1-\delta}{a_K} \\ &= \frac{1-\delta}{\delta(1-\gamma)a_N}\left[-\frac{1-\alpha}{\alpha}G_K{}^{\frac{1}{\delta}-2}\left(G_K + \frac{\gamma-\delta}{\delta}\rho a_K\right) + \delta(1-\gamma)^2\frac{a_N}{a_K}\right]\end{aligned} \tag{3.52}$$

If dg_Y/dG_K is positive at G_K corresponding to the maximum patent protection, that is, $\beta = (1-\alpha)/\alpha$, the maximum patent protection does not maximize the growth rate of output. Thus we have the following proposition:

Proposition 6. *The maximum patent protection ($\beta = (1-\alpha)/\alpha$) does not maximize the growth rate of output, if the values of the parameters satisfy the following inequality:*

$$\frac{1-\alpha}{\alpha}\frac{G_K{}^{\frac{1}{\delta}-2}}{a_N}\left[G_K + \frac{\gamma-\delta}{\delta}\rho a_K\right] < \frac{\delta(1-\gamma)^2}{a_K}, \tag{3.53}$$

where

$$\frac{(1-\delta)\frac{1-\alpha}{\alpha} + \gamma - \delta}{\delta(1-\gamma)} = \frac{L + \rho a_N - G_K{}^{\frac{1}{\delta}}}{\delta G_K{}^{\frac{1}{\delta}} + \rho a_K G_K{}^{\frac{1-\delta}{\delta}}}.$$

Using (3.52), we can explore the function $g_Y(G_K)$ and find what values of parameters satisfy the above inequality. According to Appendix, the maximum patent protection does not maximize the growth rate of output when the cost of capital goods production is relatively lower and when the labor resource is relatively smaller. In other words, Proposition 6 shows that the perfect enforcement of patent does not maximize the growth rate of output and that reducing the patent protection level (decreasing β) raises the growth rate of output in the economy with higher productivity of capital goods production.

Finally, using a numerical example, we show that the perfect enforcement of patent($\beta = (1-\alpha)/\alpha$) does not maximize the growth rate of output in the economy with specific values of parameters. In the numerical example of Fig. 3, the perfect enforcement of patent($\beta = 1$) actually does not maximize the growth rate of output and rather the milder enforcement of patent maximizes the growth rate of output.[7]

[7]Parameter values are $L = 1$, $a_K = 1$, $a_N = 15$, $\gamma = 0.7$, $\delta = 0.6$, $\rho = 0.02$, and $\alpha = 0.5$. Because $\alpha = 0.5$, the range of the degree of patent enforcement, β, is $[0, 1]$.

3.4 Welfare Analysis

In this section, we examine how strengthening the patent protection affects social welfare. First, we replace y^* and z^* with G_N and G_K in (3.50). From (3.39), we obtain

$$\frac{a_N y^*}{1+\beta} = (1-\delta)\frac{G_N}{\beta}.$$

Moreover, using (3.40), we obtain

$$\left(\delta \frac{a_N y^*}{a_K z^*}\right)^{-\frac{1-\gamma}{1-\delta}} = G_K{}^{-\frac{1-\gamma}{\delta}}.$$

Substituting them into (3.50) yields the output path on the balanced growth path as follows:

$$\begin{aligned} Y_t &= A\left[\gamma(1-\delta)\right]^{\gamma}\left[(1-\gamma)\delta\right]^{1-\gamma}\beta^{-1}G_N G_K{}^{-\frac{1-\gamma}{\delta}} N_0{}^{\frac{1-\alpha}{\alpha}} K_0{}^{1-\gamma} e^{g_Y t} \\ &\equiv Y_0 e^{g_Y t}. \end{aligned} \tag{3.54}$$

Thus we get the representative households' lifetime utility as follows:

$$\begin{aligned} U(G_K) &= \int_0^{\infty} e^{-\rho t} \ln Y_t dt \\ &= \frac{1}{\rho^2}\left[\rho \ln Y_0(G_K) + g_Y(G_K)\right]. \end{aligned} \tag{3.55}$$

To examine how strengthening patent protection affects the social welfare, we differentiate the social welfare with respect to G_K. Then we obtain:

$$\frac{dU(G_K)}{dG_K} = \frac{1}{\rho}\frac{1}{Y_0(G_K)}\frac{dY_0(G_K)}{dG_K} + \frac{1}{\rho^2}\frac{dg_Y(G_K)}{dG_K}. \tag{3.56}$$

Because we have already examined the relation between the growth rate of output, g_Y and G_K in the last section, next we examine the relation between the initial value of output, Y_0 and G_K. Differentiating $\ln Y_0(G_K)$, we obtain:

$$\begin{aligned} \frac{d\ln Y_0(G_K)}{dG_K} &= -\frac{1}{\beta}\frac{d\beta}{dG_K} + \frac{1}{G_N}\frac{dG_N}{dG_K} - \frac{1-\gamma}{\delta}\frac{1}{G_K} \\ &= \frac{(1+\beta)G_K + \left[(1-\delta)\beta + \gamma - \delta\right]\frac{\rho a_K}{\delta}}{\delta G_K \beta\left(G_K + \frac{\rho a_K}{\delta}\right)} + \frac{G_K + (\gamma-\delta)\frac{\rho a_K}{\delta}}{\delta G_K \beta\left(G_K + \frac{\rho a_K}{\delta}\right)} - \frac{1-\gamma}{\delta}\frac{1}{G_K} \\ &= \frac{\gamma G_K + (\gamma-\delta)\frac{\rho a_K}{\delta}}{\delta G_K\left(G_K + \frac{\rho a_K}{\delta}\right)} > 0, \end{aligned} \tag{3.57}$$

where we use (3.44), (3.45), (3.48), and $\gamma > \delta$. According to (3.57), strengthening the patent protection reduces the initial value of output.

From Appendix, we obtain the value of G_K that maximizes the growth rate of output, that is, the value of G_K that satisfies $dg_Y/dG_K = 0$ as depicted in Fig. 2-1 and 2-2. Moreover, the sign of $d\ln Y_0/dG_K$ is necessarily positive. Hence the value of G_K that satisfies $dU(G_K)/dG_K = 0$ is higher than the value of G_K that satisfies $dg_Y/dG_K = 0$. In other words, the value of G_K that maximizes the social welfare, $U(G_K)$ is higher than the value of G_K that maximizes the growth rate of output. Because G_K is a decreasing function of β from (3.46), we have the following proposition:

Proposition 7. *The welfare-maximizing patent protection level is lower than the growth-maximizing patent protection level.*

Strengthening patent protection has three effects on welfare. The first effect is the innovation-enhancing effect: increasing patent protection level raises the rate of return of innovation and economic growth. This improves the social welfare. The second effect is the capital-accumulation-reducing effect: increasing patent protection level reduces the rent of capital and thus hampers capital accumulation. This lowers the social welfare. The third effect is the initial value effect: from (3.57), this necessarily reduces the social welfare. If the capital-accumulation-reducing effect and the initial value effect dominate the innovation-enhancing effect, strengthening the patent protection reduces the social welfare as is mentioned in Proposition 7.

3.5 Conclusion

Based on endogenous growth models that include only innovation as the engine of economic growth, many studies show that stronger patent protection enhances economic growth. However, in this paper, we show that stronger patent protection accelerates innovation but discourages capital accumulation in an endogenous growth model with both innovation and capital accumulation. Consequently, we show that tightening patent protection may reduce the growth rate of output. We also investigate how the patent protection affects social welfare and show that the welfare-maximizing degree of the patent protection is lower than the growth-maximizing degree of the patent protection.

3.6 Appendix

In this appendix, we examine the relation between g_Y and G_K. Differentiating (3.52) with respect to G_K, we obtain:

$$\frac{d^2 g_Y}{d{G_K}^2} = \frac{1-\delta}{\delta(1-\gamma)a_N}\left[-\frac{1-\alpha}{\alpha}{G_K}^{\frac{1}{\delta}-3}\left(\frac{1-\delta}{\delta}G_K + \frac{(1-2\delta)(\gamma-\delta)}{\delta^2}\rho a_K\right)\right]. \quad (3.58)$$

If $\delta < 1/2$, the sign of (3.58) is necessarily negative. Therefore the relation between G_K and the growth rate of output is as depicted in Fig. 2-1. We let $\widetilde{G}_K$ denote the value of G_K that satisfies $dg_Y/dG_K = 0$. From (3.52), $\widetilde{G}_K$ is determined by:

$$\frac{1-\alpha}{\alpha}{G_K}^{\frac{1}{\delta}-2}\left(G_K + \frac{\gamma-\delta}{\delta}\rho a_K\right) = \delta(1-\gamma)^2\frac{a_N}{a_K}. \quad (3.59)$$

From (3.59) and Fig. 2-1, we can show that if a_N/a_K is higher, $\widetilde{G}_K$ becomes higher. On the other hand, we let ${G_K}^{min}$ denote the value of G_K that corresponds to the maximum patent protection level, $\beta = (1-\alpha)/\alpha$. From (3.44), ${G_K}^{min}$ satisfies the following equation:

$$\frac{1-\delta}{\alpha}{G_K}^{\frac{1}{\delta}} + \frac{(1-\delta)\frac{1-\alpha}{\alpha} + \gamma - \delta}{\delta}\rho a_K {G_K}^{\frac{1-\delta}{\delta}} = (1-\gamma)(L + \rho a_K). \quad (3.60)$$

This equation implies that if the value of L is lower, G_K^{min} becomes lower. Consequently, in the economy with higher a_N/a_K and smaller L, G_K^{min} is lower than $\widetilde{G}_K$ as depicted in Fig. 2-1. That is, the maximum patent protection level does not maximize the growth rate of output.

If $\delta > 1/2$, the sign of (3.58) is necessarily not negative. The relation between G_K and dg_Y/dG_K is not monotone as depicted in Fig. 2-2. However, (3.59) shows that if a_N/a_K is higher, the range of G_K that satisfies $dg_Y/dG_K > 0$ becomes broader. Therefore the maximum patent protection does not maximize the growth rate of output in the economy with higher a_N/a_K and smaller L also in the case when $\delta > 1/2$.

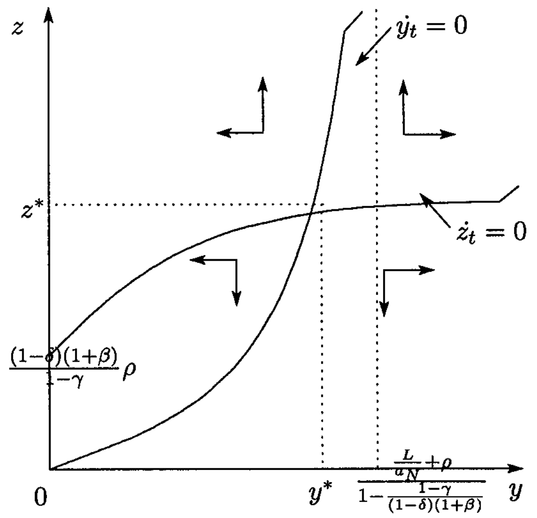


Fig. 1. The phase diagram

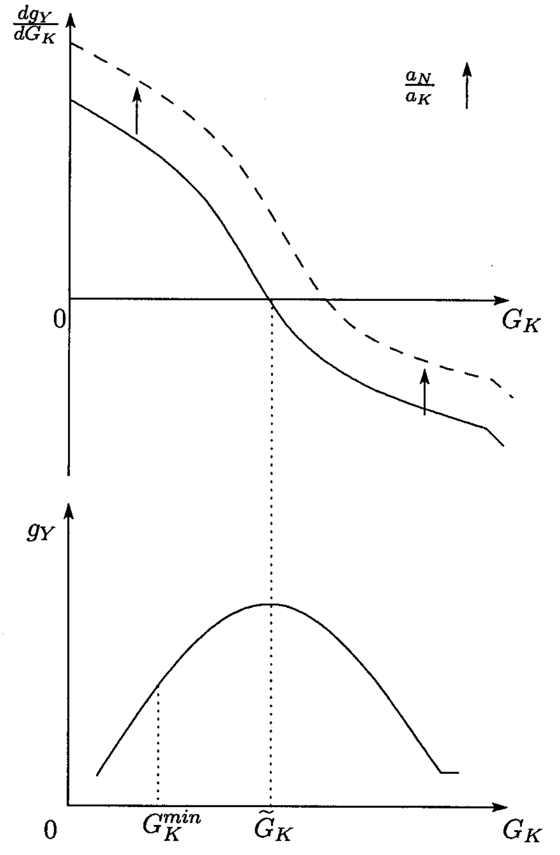


Fig. 2-1. The growth rate of output when $\delta < \frac{1}{2}$

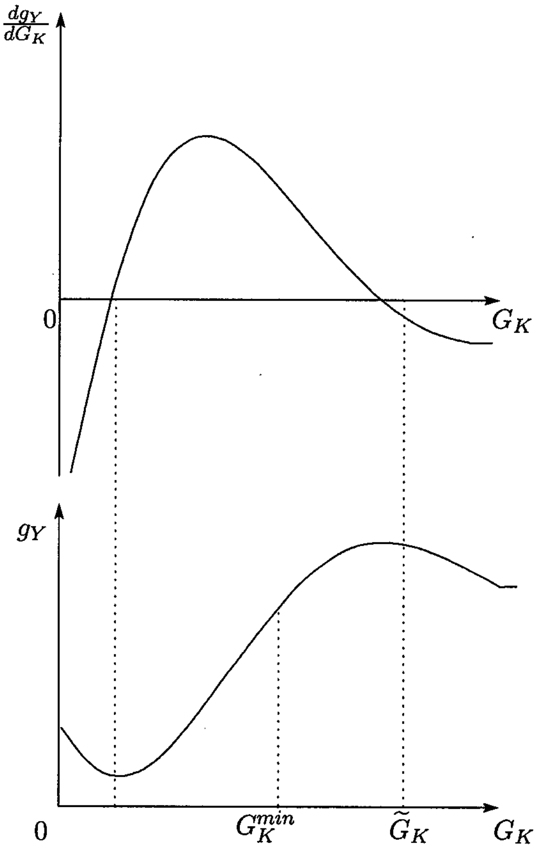


Fig. 2-2. The growth rate of output when $\delta > \frac{1}{2}$

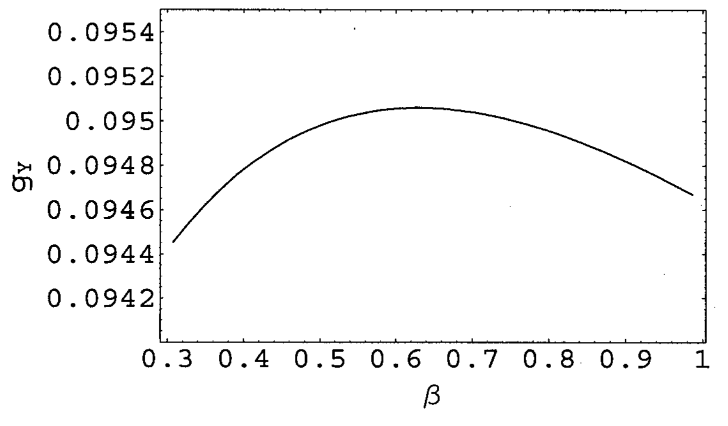


Fig.3. Patent enforcement and growth rate of output–a numerical example.

Chapter 4
Technology Choice and Patterns of Growth in an Overlapping Generations Model

4.1 Introduction

It is well recognized that, in the standard one-sector growth model, economies with access to similar technologies will converge to a common balanced growth path. Some empirical papers, Baumol (1986) and Barro and Sala-i-Martin (1992), have argued that income levels show a tendency to converge to a common growth path. In recent empirical research (Quah 1993; Quah 1996), however, the opinion that economies will not necessarily converge to a common steady state has become quite popular. Why does the divergence of income levels happen? Why cannot every country sustain permanent growth? The purpose of this paper is to answer these questions.

Some papers have attempted to explain the empirically observed divergence of income. Introducing human capital accumulation into the Diamond (1965) overlapping generations model, Azariadis and Drazen (1990) showed that multiple, locally stable steady states can be generated due to increasing returns to scale in the accumulation of human capital. Galor and Tsiddon (1991) introduced threshold externalities in production into an overlapping generations model. They assumed that once a country reaches a threshold level of the capital-effective labor ratio, other countries can utilize the more advanced technology even if their own capital-effective labor ratios are below the threshold level. Then they showed that countries other than the leading country may not reach the high productivity steady state and that they converge to a low development trap.

In infinitely-lived agent models many studies have shown that not only multiple steady states but also limit cycles can emerge. Along similar lines as Azariadis and Drazen (1990), Futagami and Mino (1995) showed that multiple equilibria can be generated if there are threshold externalities in public capital accumulation. Greiner and Semmler (1996) built the two-sector growth model in which human capital is acquired only through learning by doing, and they showed that multiple steady states, indeterminacy of the equilibrium and limit cycles can be generated in their model.[1] In a more general model, Greiner (1996) showed the necessary conditions for endogenous growth cycles.

This paper shows that, in an overlapping generations model, a variety of patterns of growth including permanent cyclical fluctuations can be generated if investors' choice of technology is introduced. Such phenomena are not analyzed by most of the above mentioned studies. Because this paper's model generates not only sustained growth but also underdevelopment traps, this paper can analyze both the underdevelopment traps and the take-off to sustained growth. This paper's model predicts that some countries can obtain sustained growth and others cannot and converge to the underdeveloped traps. This prediction agrees with the empirical fact that the world income distribution is polarized into twin peaks of rich and poor (found by Quah 1993; Quah 1996). In addition this paper shows that the dynamics that will be generated are dependent on income distribution among investors and workers.

One technology is primitive and the other is advanced. The primitive production technology yields constant returns to scale. Under the advanced technology, one kind of final good is produced using a variety of intermediate goods. The final goods production function exhibits a constant elasticity of substitution, and the production technology of the intermediate goods exhibits increasing returns to scale due to specialization of production. The intermediate goods market is assumed to be monopolistically competitive.

This model exhibits various patterns of equilibrium dynamics, permanent cyclical fluctuations and poverty traps. As well, it demonstrates perpetual growth and convergence to a steady state, depending on the degree of competitiveness of the intermediate goods market, which in turn is determined by the elasticity of substitution in final goods production under the advanced technology. The result shows that even if the size of the effective labor supply is the same in two countries, differing parameters within the two economies will result in different paths. If intermediate goods tend to be more complementary, that is, the intermediate goods market tends toward more monopoly, then the economy is likely to experience permanent cyclical fluctuations. If the intermediate goods are more substitutive and the market is more competitive, then the economy is likely to have poverty traps. Because this paper assumes that the old people (investors) get all of the monopoly rents, increasing the monopoly power of the intermediate goods market reduces the proportion of total output that is distributed to workers. Therefore the above results can be interpreted as follows: If the distribution between workers and investors is too biased and uneven, the economy is likely to have underdevelopment traps (permanent cyclical fluctuations and poverty traps).

These results differ from those of previous papers with respect to the relationship between the distribution of total output and the growth rate.[2] Uhlig and Yanagawa (1996) showed that in a two-period overlapping generations model with endogenous growth, decreasing the labor income tax (increasing the income share of workers can) hikes the growth rate. In addition to this growth enhancing effect, in this paper's model increasing the income share of workers (decreasing the income share of investors) has a negative effect on obtaining permanent growth; decreasing the income share of investors prevents investors from adopting the advanced technology that puts economies on permanent growth path. Due to these two opposite effects, this paper shows that the economy cannot obtain permanent growth when the distribution between workers and investors is too biased and uneven. There is no research that finds the above mechanism and concludes that the economy that has too low or too high a share of labor cannot sustain permanent growth.

The organization of this paper is as follows. Section 2 establishes the basic structure of the model and solves the maximization problems of consumers and firms. Section 3 shows that several patterns of dynamics can be achieved in this model. Section 4 clarifies the reason why underdevelopment traps are generated in the case where either one of two classes possesses most of the output in the economy. Section 5 shows that a tax or a subsidy can obtain permanent growth. Section 6 gives some concluding remarks.

4.2 The Model

4.2.1 Consumers

Consider a simple two-period overlapping generations model. In this economy L (a constant) individuals are born in every period, and each individual lives for two periods, one period called "young" and the other called "old". In the first period, each individual supplies one unit of labor inelastically and divides the resulting labor income between first-period consumption and saving. In the second period, the individual simply consumes the saving and any interest he or she obtains. The utility function of individuals born at time t is given by

$$\log c_{1t} + \frac{1}{1+\rho} \log c_{2t+1}, \tag{1}$$

where c and ρ stand for consumption and the subjective discount rate, respectively. The budget constraint in each period is given by $c_{1t} + s_t = w_t$ and $c_{2t+1} = (1 + r_{t+1})s_t$, where s, w, and r represent saving, labor income, and the interest rate, respectively. Given labor income and the rationally anticipated interest rate, each consumer maximizes utility (1) under the budget constraints above. Because this is a typical textbook problem, the optimal saving function can

$$s_t = \frac{1}{2+\rho} w_t . \tag{2}$$

Note that the two-period overlapping generations model can be interpreted in a different way: a younger economic agent, worker, gets income only by supplying labor inelastically; and an older economic agent, investor, gets income only by lending his or her capital. This naming emphasizes the fact that there are heterogeneous agents in this economy similar to those in two-class models.

4.2.2 Technologies

In this economy there are two kinds of available production technologies. One is primitive and underdeveloped and can be described as a constant returns to scale Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{3}$$

where Y_t, K_t and L_t are output, capital, and the number of workers employed, respectively.

The other is an advanced and developed product technology, which exhibits increasing returns to scale. This is the same process of production as Benhabib and Farmer (1994) assumed in order to analyze increasing returns economies.[3] The process of production is as follows. At the first stage, each intermediate goods producing firm produces a kind of intermediate good, and the production of the intermediate good exhibits increasing returns to scale. At the next stage, by using these intermediate goods, the final goods producing firms produce a kind of final good. The production functions of final goods and intermediate goods are described as follows:

$$Y_t = \left[\int_0^1 X_t(i)^{\lambda} di \right]^{\frac{1}{\lambda}}, \quad 0 < \lambda < 1, \tag{4}$$

and

$$X_t(i) = K_t(i)^a L_t(i)^b, \qquad a \geq 1, \ b > 0 \tag{5}$$

where Y_t is the same output as in the primitive production case, $X_t(i)$, $K_t(i)$, and $L_t(i)$ represent intermediate goods input, capital, and labor inputs for sector i, respectively. $\lambda = 1 - \frac{1}{\sigma}$, where σ is the elasticity of substitution across goods. The intermediate goods are complementary when λ is close to 0 and substitutes when λ is close to 1. We assume that the productivity of capital in the production of the intermediate goods is high.

4.2.3 Production Process

Next, the firm's profit maximization problems are solved in this subsection.

4.2.3.1 The Case of Primitive Technology

Under the primitive production function, the profit of firms is (here we normalize all prices using the final good as numeraire) $\Pi_t^P = K_t^{\alpha} L_t^{1-\alpha} - w_t^P L_t - r_t^P K_t$, where w_t^P and r_t^P represent the wage and the interest rate under the primitive technology respectively. The first-order conditions of profit maximization are easily rewritten by using capital stock per capita, k_t:

$$r_t^P = \alpha k_t^{\alpha - 1}, \tag{6}$$

and

$$w_t^P = (1-\alpha)k_t^\alpha .$$

(7)

4.2.3.2 The Case of Advanced Technology

Consider the behavior of firms under the advanced production technology. Assume that the market for final goods is perfectly competitive. The profit of firms is

$$\Pi_t^A = Y_t - \int_0^1 p_t(i)X_t(i)di, \tag{8}$$

where $p_t(i)$ represents the price of the i-th intermediate goods firm. Applying the first-order conditions of profit maximization allows us to derive the inverse demand function of each intermediate good as follows:[4]

$$p_t(i) = \left[\frac{X_t(i)}{Y_t}\right]^{\lambda-1} .$$

(9)

The profit of firms producing intermediate goods is

$$\Pi_t(i) = p_t(i)X_t(i) - w_t^A L_t(i) - r_t^A K_t(i), \tag{10}$$

where w_t^A and r_t^A represent the wage and the interest rate under the advanced technology, respectively. As long as $\lambda \neq 1$, intermediate goods are differentiated and firms have some degree of monopolistic power. Because it is assumed that the intermediate goods market is monopolistically competitive, firms choose the production quantities (the prices) so as to equalize marginal revenue and marginal cost. Substituting the inverse demand function for intermediate goods and the production function into (10), we have

$\Pi_t(i) = K_t(i)^{\lambda a} L_t(i)^{\lambda b} Y_t^{1-\lambda} - w_t^A L_t(i) - r_t^A K_t(i)$. Assume that the profit function is concave, that is, $\lambda(a+b) \leq 1$ in order to assume the second-order conditions of the maximization problem. The first-order conditions of profit maximization are

$$r_t^A = \lambda a \frac{p_t(i) X_t(i)}{K_t(i)}, \tag{11}$$

and

$$w_t^A = \lambda b \frac{p_t(i) X_t(i)}{L_t(i)}. \tag{12}$$

From the assumption that all firms producing intermediate goods are symmetric, in equilibrium, the following relationships are derived: $K_t(i) = K_t$, $L_t(i) = L$, $X_t(i) = X_t$. Because the profit is 0 in (8), $Y_t = p_t X_t$. From this relationship and (9), the intermediate goods price, we find that $p_t = 1$ and $Y_t = X_t = K_t^a L^b$. Hence (11) and (12) reduce to

$$r_t^A = \lambda a \frac{Y_t}{K_t}, \tag{13}$$

and

$$w_t^A = \lambda b \frac{Y_t}{L}. \tag{14}$$

If the production function for the firm is of constant returns to scale ($a + b = 1/\lambda$), the profit of the firm producing intermediate goods is zero. If it is of decreasing returns to scale, the profit is $\Pi_t^A = [1 - \lambda(a+b)] Y_t > 0$. This paper assumes that each of investors that invest in the intermediate goods production owns one intermediate goods producing firms. Hence the profit is distributed among investors. Under this assumption, the gross rate of return on capital $\hat{r}_t$ is given by the sum of the rental price of capital and the dividend from the profit per unit of capital, that is,

$$\hat{r}_t^A = r_t^A + \frac{\Pi_t^{\ A}}{K_t} = (1-\lambda b)\frac{Y_t}{K_t}. \tag{15}$$

Using the capital-labor ratio $k_t(= K_t/L_t)$ and the fact that $Y_t = K_t^{\ a} L^b$, (15) and (14) can be rewritten as $\hat{r}_t^A = (1-\lambda b)k_t^{a-1}L^{a+b-1}$ and $w_t^A = \lambda b k_t^a L^{a+b-1}$.

For simplicity, we assume that a=1 in the rest of the paper;[5] therefore these equations are easily rewritten;

$$\hat{r}_t^A = (1-\lambda b)L^b, \tag{16}$$

and

$$w_t^A = \lambda b L^b k_t. \tag{17}$$

4.2.4 The Choice of Technology

In this subsection behavior of the investors, in particular, the choice of technology is established. It is assumed that investors can coordinate their choice of technology, primitive or advanced, so that they get the highest return.[6]

Under these assumptions, the choice of all investors is as follows:[7]

All investors invest in the primitive technology if $r^P(k) > \hat{r}^A(k)$.

All investors invest in the advanced technology if $r^P(k) \le \hat{r}^A(k)$.

The interest rate function of k under primitive production intersects the interest rate function under advanced production from above at unique point. This intersection is the critical value of k for investors, the point at which investors switch to the other product technology. Next calculate this critical value, $\tilde{k}_r$. The following relationships are easily obtained:

$$r^P = \hat{r}^A$$

$$\alpha \tilde{k}_r^{\alpha-1} = (1-\lambda b)L^b .$$

From this, the critical value is given by

$$\tilde{k}_r = \left[\frac{\alpha}{(1-\lambda b)L^b}\right]^{\frac{1}{1-\alpha}} . \tag{18}$$

When k reaches this value, investors choose the advanced production technology.

4.3 Market Equilibrium and Dynamics

4.3.1 Market Equilibrium

The capital market equilibrium requires that investment be equal to net saving, which is the saving of the young minus the dissaving of the old. Therefore the market equilibrium condition is:

$$K_{t+1} = s_t L \tag{19}$$

Substituting (2) and (7) into this condition gives the following capital accumulation equation under the primitive technology:

$$k_{t+1} = \frac{1-\alpha}{2+\rho} k_t^{\alpha} . \tag{20}$$

Similarly substituting (2) and (17) into (19) gives the following capital accumulation equation under the advanced technology:

$$k_{t+1} = \frac{\lambda b L^b}{2+\rho} k_t . \tag{21}$$

4.3.2. Equilibrium Dynamics

From (18), (20) and (21), the dynamics of this system can be characterized by the equations

$$k_{t+1} = \begin{cases} \dfrac{1-\alpha}{2+\rho} k_t^{\alpha} & if \quad k_t < \left[\dfrac{\alpha}{(1-\lambda b)L^b}\right]^{\frac{1}{1-\alpha}} \\ \dfrac{\lambda b L^b}{2+\rho} k_t & if \quad k_t \geq \left[\dfrac{\alpha}{(1-\lambda b)L^b}\right]^{\frac{1}{1-\alpha}} \end{cases} . \tag{22}$$

In order to analyze how the values of the parameters determine the dynamic properties of the model, we can use some conditions to divide the λ-L space into regions. It can be proved that one growth pattern – permanent cyclical fluctuations, poverty traps, perpetual growth, or convergence to a unique steady state – can emerge in each region of the λ - L space. [8]

First, find the conditions of λ and L under which permanent growth can be generated in the advanced technology production scenario. The condition under which the growth rate is strictly positive is $k_{t+1}/k_t = \lambda b L^b/(2+\rho) > 1$. We get the condition as follows:

$$L > \left(\frac{2+\rho}{\lambda b}\right)^{\frac{1}{b}} . \tag{23}$$

Call this boundary line the *Permanent Growth Boundary* line (hereafter the PGB line). In the region located above this line, a permanent growth path exists. In the region located below this line, permanent growth is not possible. This line is sloping downward, such as in Figure 1-1. Under the assumption that investors get all of the firms' profits, λ represents not only the degree of competitiveness of the intermediate goods market but also the proportion of goods that is distributed to workers. From (16) and (17),

$$\hat{r}_t^A K_t = (1-\lambda b) Y_t$$

and

(24)

$$w_t^A L = \lambda b Y_t .$$

Increasing λ makes labor income higher. The increase in the labor income raises the saving from (2), and the increase in the saving raises capital for the next period as shown by the capital market equilibrium condition (19). Therefore a high λ enables an economy with even low labor force participation L to grow permanently. This is why the PGB line is sloping downward.

Second, we can find conditions of λ and L under which steady states cannot exist in the primitive production case. Define the steady state under the primitive technology as k_{OLG}. Substituting $k_{t+1} = k_t = k_{OLG}$ into (20), we get $k_{OLG} = [(1-\alpha)/(2+\rho)]^{1/1-\alpha}$. The condition requiring that the advanced technology be adopted before the economy can converge to the steady state under the primitive technology is $\widetilde{k}_r < k_{OLG}$. Substituting for the above terms yields

$$L > \left[\frac{\alpha(2+\rho)}{(1-\alpha)(1-\lambda b)} \right]^{\frac{1}{b}} . \tag{25}$$

Call this boundary line the *Neoclassical Boundary* line (hereafter the NB line). This line divides the space of λ and L into two regions. In the upper region the economy does not converge to a steady state under the primitive production technology. In the lower region the economy may converge to a steady state depending on the initial capital stock. Low λ and high L means that the investors' income share is large under the advanced technology, and thus investors want to switch to the advanced technology when k is lower. This is why the NB line is sloping upward.

These conditions form two lines in the λ - L space of Figure 1-1. The PGB line and the NB line make four regions. See what happens in each region. In region 1, the economy starting from any initial capital stock can ride on a permanent growth path (Figure 2-1). In region 2, every economy converges to the steady state under the primitive technology (Figure 2-2). The dynamics of economies with parameters in regions 3 and 4 are very important and interesting. First, in region 3, every economy exhibits permanent cyclical fluctuations (Figure 2-3).[9] In this region λ is low. Low λ means that the investors' share is high under the advanced technology, and thus investors switch to the advanced technology with lower k. Low λ, however, means that the workers' share is low under the advanced technology and the growth rate is negative (that is, capital contracts). Due to this contraction the economy will return to the primitive technology sooner or later. This process is repeated, and permanent cyclical fluctuations emerge in the economies with parameters in region 3. Second, in the region 4, every economy has a poverty trap depending on initial capital stock (Figure 2-4). Because λ is high in this region, the workers' share is high under the advanced technology. Hence permanent growth can be obtained. High λ, however, means that the share of the investors is low under the advanced technology, and thus they do not switch to the advanced technology when initial k is lower than $\tilde{k}_r$. This process generates poverty traps. Let us summarize these results in the following proposition.

Proposition 1. *This OLG economy has a variety of possible patterns of growth depending on the values of λ and L as follows (See Figures 1-1, 2-1, 2-2, 2-3, and 2-4):*

(i) if $L > \left(\dfrac{2+\rho}{\lambda b}\right)^{\frac{1}{b}}$ *and* $L > \left[\dfrac{\alpha(2+\rho)}{(1-\alpha)(1-\lambda b)}\right]^{\frac{1}{b}}$, *permanent growth can be sustained (Figure 2-1).*

(ii) if $L < \left(\dfrac{2+\rho}{\lambda b}\right)^{\frac{1}{b}}$ *and* $L < \left[\dfrac{\alpha(2+\rho)}{(1-\alpha)(1-\lambda b)}\right]^{\frac{1}{b}}$, *this economy will converge to a unique*

steady state under the primitive technology (Figure 2-2).

(iii) *if* $L < \left(\frac{2+\rho}{\lambda b}\right)^{\frac{1}{b}}$ *and* $L < \left[\frac{\alpha(2+\rho)}{(1-\alpha)(1-\lambda b)}\right]^{\frac{1}{b}}$, *permanent cyclical fluctuations emerge (Figure 2-3).*

(iv) *if* $L > \left(\frac{2+\rho}{\lambda b}\right)^{\frac{1}{b}}$ *and* $L > \left[\frac{\alpha(2+\rho)}{(1-\alpha)(1-\lambda b)}\right]^{\frac{1}{b}}$, *a poverty trap exists and this economy will converge to the trap when the initial level of* k *is lower than* $\tilde{k}_r$ *(Figure 2-4).*

One of the important findings that this proposition makes is that having λ too high or too low λ leads to underdevelopment traps (permanent cyclical fluctuation or poverty traps) as depicted in Figure 1-1.

Both permanent cyclical fluctuations and poverty traps cannot always be generated, because the range of λ that can be analyzed must be $\lambda \in [0,\ 1/1+b]$ by the assumption that the profit function is concave. When the degree of increasing returns to scale of the advanced technology $a+b=1+b$ is higher than $1/\alpha$, this paper's model predicts that both permanent cyclical fluctuations and poverty traps can be generated as depicted in Figure 1-1. When the degree of increasing returns to scale of the advanced technology $1+b$ is lower than $1/\alpha$, this paper's model predicts that permanent cyclical fluctuations can be generated and poverty traps cannot be generated as depicted in Figure 1-2. Recent researches on economic growth, especially on *indeterminacy* (e.g., Benhabib, Meng and Nishimura 2000) quote the empirical facts which are concerned with the degree of increasing returns to scale. A number of recent empirical studies show that the degree of increasing returns to scale in U.S. data is relatively low (e.g., Basu and Fernard 1997). When the degree of increasing returns to scale of the advanced technology is relatively low, the region of poverty traps are likely to vanish as depicted in Figure 1-2.

In this economy the market equilibrium is not necessarily desirable in the sense that perpetual growth is not necessarily obtained. Whether this undesirability occurs is dependent on λ. In next section, how λ can affect the equilibrium dynamics of the economy is analyzed.

4.4 The Role of Distribution

As is shown in Figure 1-1, the economy with a too-high or too-low λ cannot exhibit perpetual growth. Why does such an undesirable phenomenon occur when λ is too high or too low? λ represents not only the degree of competitiveness of the intermediate goods market but also the proportion that is distributed to labor. Therefore, we will call λ the parameter of distribution hereafter.

Increasing λ has two opposite effects: the *growth enhancing effects* and the *incentive weakening effects* on investors. The former means that increasing λ raises the growth rate and the latter means that increasing λ prevents investors from switching to the advanced technology.

The growth effect is generated as follows. If each individual has a homothetic utility function, his/her optimal saving function; $S_t(w_t,\ r_{t+1})$ is homogeneous of degree one in labor income w_t; therefore the optimal saving function can be written as follows: $S_t(w_t, r_{t+1}) = s(r_{t+1})w_t$. Using (24) and the reduced production function under the advanced technology, the capital market equilibrium equation (19) can be rewritten as $K_{t+1} = s(r_{t+1})w_t L = s(r_{t+1})\lambda b L^b K_t$, and the growth rate γ_K can be written as

$$\gamma_K = \frac{K_{t+1}}{K_t} - 1 = s(r_{t+1})\lambda b L^b - 1. \tag{26}$$

Because λ represents the proportion that is distributed to labor, in the OLG model increasing labor income means increasing saving. And because saving becomes capital in the next period, the growth rate rises such as in (26). This mechanism is the same as that of Uhlig and Yanagawa (1996). Using this property of the OLG model, they show that increasing the income tax on capital necessarily enhances the growth rate. That is why the economy that has too low a λ cannot sustain permanent growth.

In addition to the growth effect, in this paper's model, increasing the parameter has a negative effect: the *incentive effect*. This is generated through the mechanism as follows. $(1-\lambda)$ is the proportion that is distributed to investors under the advanced technology. Therefore increasing λ makes the capital income under the advanced technology lower. At the same time, the critical level of capital that motives investors to switch to the advanced technology, $\tilde{k}_r = \{\alpha/[(1-\lambda b)L^b]\}^{1/(1-\alpha)}$, rises and generates poverty traps (depicted as Figure 2-4). That is why the economy that has a too-high λ cannot sustain permanent growth.

Due to these two opposite effects, the economy in this model cannot ride on a permanent growth path when the proportion that is distributed to workers, that is, λ, is either too low or too high.

In this paper, we assume that the investors own the intermediate goods producing firm and get all of the profit. We examine how the results will change if we assume that the workers get most of the profit. Increasing λ reduces the profit. Under the new assumption, the decrease of the profit raises the rate of return to capital and reduces the labor income under the advanced technology. Therefore, the higher λ is, the lower the critical level of capital that motives investors to switch to the advanced technology is and the lower the growth rate under the advanced technology is. The results are reverse to the previous results, and poverty traps are likely to be generated when λ is low and permanent cyclical fluctuations are likely to be generated when λ is high. However, the conclusion that the economy that has too low or too high λ cannot sustain permanent growth is the same as before.

Next, the mechanism that generates permanent cyclical fluctuations and poverty traps is clarified. Define as $\tilde{k}_w$ the critical value of capital stock by which the labor income under the advanced technology exceeds the value of labor income under the primitive technology. Calculate the critical value of labor income $\tilde{k}_w$ using the following steps. Substituting (7) and (17) into $w_t^P = \hat{w}_t^A$, we get

$$\tilde{k}_w = \left(\frac{1-\alpha}{\lambda b L^b}\right)^{\frac{1}{1-\alpha}}. \tag{27}$$

Figure 3 shows how these critical values of the interest rate and labor income are determined. The relationship between per capita capital and the interest rate is depicted in Figure 3. This unique intersection is the critical value of k for investors, $\tilde{k}_r$. The relationships between per capita capital stock and labor income in each production situation are depicted in Figure 3. This unique intersection determines the critical value of k for workers, $\tilde{k}_w$.

If $\tilde{k}_r < \tilde{k}_w$, that is, $\lambda b < 1-\alpha$, by switching to the advanced technology, investors cause labor income $w_t^P(k_t)$ to decrease to $w_t^A(k_t)$. Hence $k_{t+1}(k_t)$ jumps downward at $\tilde{k}_r$ (for an example, see Figure 2-3). In this case (when λ is relatively low) the incentive for investors is strong enough, but the growth rate under the advanced technology is low, and permanent cyclical fluctuations are likely to be generated. On the contrary, if $\widetilde{k}_r > \widetilde{k}_w$, that is, $\lambda b > 1-\alpha$, a switch to the advanced technology causes labor income $w_t^P(k_t)$ to increase to $w_t^A(k_t)$ and therefore $k_{t+1}(k_t)$ jumps upward at $\widetilde{k}_r$ (for an example, see Figure 2-4). In this case (where λ is relatively high) the growth rate is high enough to sustain perpetual growth under the advanced technology, but the incentive for investors is weak, and poverty traps are likely to be generated.

These findings show that changing the income distribution is necessary to lead an economy that has fallen into underdevelopment traps out of such traps to perpetual growth. In the next section redistribution policies that lead economies to a permanent growth path are considered.

4.5 Economic Policies for Escaping from Underdevelopment Traps

In this section the effects of tax and subsidy policies are examined. Because there have already been many studies on the effects of capital income tax on the growth rate in an overlapping generations model (such as Jones and Manuelli 1992; Uhlig and Yanagawa 1996), this paper restricts its attention to analyzing how capital tax (or subsidy) affects the patterns of growth of the economy.

Are there any policies that can make economies recover from permanent cyclical fluctuations, poverty traps, or convergence to a nongrowing steady state? In order to answer this question, the following redistribution policy is introduced. Under the primitive technology the real interest rate that investors face and the amount of labor income realized by workers are

$$r_t^P = \alpha k_t^{\alpha-1} - \tau_P \tag{28}$$

and

$$w_t^P = (1-\alpha)k_t^\alpha + \tau_P k_t . \tag{29}$$

Under the advanced technology the gross rate of return on capital stock for investors and labor income for workers are

$$\hat{r}_t^A = (1-\lambda b)L^b - \tau_A \tag{30}$$

and

$$w_t^A = \lambda b L^b k_t + \tau_A k_t = (\lambda b L^b + \tau_A)k_t . \tag{31}$$

Examine the case where the government imposes different tax rates on capital stock holdings in each production scenario, τ_P and τ_A, and distributes the tax revenue to workers in the same time period. If $\tau_P < 0$ and $\tau_A < 0$, capital is being subsidized on and workers' income is being taxed. Under this redistribution policy, the permanent growth boundary line and the neoclassical boundary line are as follows.

<The PGB line> $\qquad \dfrac{k_{t+1}}{k_t} = 1$

$$L = \left(\frac{2+\rho-\tau_A}{\lambda b} \right)^{\frac{1}{b}} . \tag{32}$$

<The NB line> $\qquad \tilde{k}_r(\tau_P, \tau_A) = k_{OLG}(\tau_P)$

$$L = \left[\frac{\alpha(2+\rho)-\tau_P+(1-\alpha)\tau_A}{(1-\alpha)(1-\lambda b)} \right]^{\frac{1}{b}} , \tag{33}$$

where $\tilde{k}_r(\tau_P,\ \tau_A)$ and $k_{OLG}(\tau_P)$ represent the critical values where investors switch the production technologies and the steady state under the redistribution policies, respectively.

Using these equations, we analyze the effects of some redistribution policies on growth patterns.

4.5.1 Tax on the Primitive Production Technology ($\tau_P > 0$, $\tau_A = 0$)

Consider the effect of imposing a tax on the capital input used in the primitive production method and redistributing this tax revenue to workers. From (33) increasing τ_P shifts the NB line downward. Figure 4-1 indicates the new NB line. This shows that increasing τ_P makes region 1 (where permanent growth exists) broader and region 4 (poverty traps) narrower. Let us summarize this result as the following proposition.

Proposition 2-1. *Imposing a tax on capital input in the primitive production (and redistributing this tax revenue to workers) can make some of the economies that have fallen into poverty traps able to achieve a permanent growth path.*

This policy gives investors incentive to adopt the advanced production technology and gives workers under the primitive production method incentive to save because of their receipt of redistributed tax revenues. Such effects keep the economy from convergence to poverty traps.

4.5.2 Subsidies for the Advanced Production Technology ($\tau_P = 0$, $\tau_A < 0$)

Next, we examine the effect of the following redistribution policy: The government gives subsidies to capital used in the advanced sector and finances the subsidies through an income tax on workers. From (32) and (33) this redistribution policy shifts the PGB line upward and the NB line downward. Figure 4-2 shows these two effects: decreasing τ_P narrows region 4 (poverty traps) and extends region 3 (permanent cyclical fluctuations). Let us summarize these results as the following proposition.

Proposition 2-2. *Giving subsidies to the capital used in the advanced sector (by financing the subsidies through an income tax on workers) enables some of the economies that have fallen into poverty traps to achieve permanent growth, however, such a scheme also causes some of the growing economies to experience permanent cyclical fluctuations.*

This policy makes the critical value at which investors switch to the advanced technology ($\tilde{k}_r$) lower and the workers' income lower. The decrease of the workers' income lowers the growth rate through the growth effect referred in Section 4. This effect is not desirable. In the next subsection we will consider the policy that enables the economy that experiences permanent cyclical fluctuations to achieve permanent growth.

4.5.3 Taxing Both Technologies ($\tau_P > 0$, $\tau_A > 0$)

As is shown by Uhlig and Yanagawa (1996), in the standard OLG model with endogenous growth, increasing the capital income tax is necessary to obtain permanent growth. In our model this is not necessarily true; increasing the capital income tax has another effect, that of lowering the investors' income under the advanced technology by the incentive effect that was referred to in Section 4. This policy makes $\tilde{k}_r$ lower and thus generates poverty traps. How can this negative effect be eliminated? The answer is by imposing a tax also on the returns to capital used in the primitive technology.

Hence we examine the effect of taxing not only the capital used in the advanced technology but also the capital used in the primitive technology and redistributing this tax revenue among workers. Increasing the tax on capital stock for the advanced technology makes the PGB line shift down, but also makes the NB line shift up and broadens region 4 (poverty traps). In order to eliminate the second effect on the NB line, the government must impose a tax on capital stock for the primitive technology so as to satisfy the following inequality:

$$\tau_P > (1-\alpha)\tau_A . \tag{34}$$

This redistribution policy can make the NB line shift down without raising the growth line as is depicted in Figures 4-3. Let us summarize these results in the following proposition.

Proposition 2-3. *Taxing the returns of capital used in both the primitive and advanced technologies so as to satisfy (34) (and redistributing this tax revenue among workers) can lead some of the economies that have fallen into permanent cyclical fluctuations or poverty traps to attain permanent growth.*

We emphasize that that the policy described in proposition 2-3 is effective for both the economy experiencing permanent cyclical fluctuations and the economy that finds itself in poverty traps.

4.6 Concluding Remarks

This paper has shown that the endogenous determination of product technologies by investors may generate not only convergence to steady states and permanent growth but also permanent cyclical fluctuations and poverty traps in a very simple overlapping generations model. Because this paper's model has a variety of patterns of growth, this paper can analyze both underdevelopment traps and the take-off to sustained growth.

Whether underdevelopment traps are generated is dependent on the parameter that represents how total output is distributed between workers and investors. Underdevelopment traps are likely to emerge when the share that workers get is too large or too small. In the standard overlapping generations model with endogenous growth, the larger the share that workers get, the higher the growth rate will be. In this paper, there is a negative effect on obtaining a permanent growth path: the higher the proportion of labor income (and the lower the proportion of capital income) under the advanced technology, the weaker the incentive of investors to adopt it. In this case poverty traps are likely to emerge.

It has been also shown that redistribution policies among workers and investors can change the patterns of growth. This fact shows that these policies can make underdeveloped countries "take-off" from underdevelopment traps and launch themselves into permanent growth.

In this paper, we have discussed only the redistribution policies. Extending this paper's model, however, we can investigate how other public policies affect patterns of growth. For example, introducing public infrastructure that enhances the productivity of the advanced technology to the model studied here, we can examine how the public capital accumulation affects patterns of growth. Under this extension, the governments must finance the public spending with labor income taxes and capital income taxes. Underdeveloped countries cannot ride on a permanent growth path when their governments fail to adopt the adequate proportion of labor income taxes to capital income taxes, because higher labor income taxes reduce the growth rate and lower labor income taxes (higher capital income taxes) prevent investors from adopting the advanced technology. It is worth discussing what kind of tax policies can make the underdeveloped countries take off from underdevelopment traps.

Appendix 1

In this appendix it is shown that permanent cyclical fluctuations necessarily emerge in region 3, in the case where $k_{t+1}/k_t > 1$, $\tilde{k}_r < k_{OLG}$. To show this, it is necessary and sufficient to show the following three facts. i) There exists an interval such that k_t remains in the interval once k_t enters the interval. ii) There exists no steady state in this interval. iii) Every k_t that begins outside the interval enters the interval.

Proof) First, define the function (22) from k_t to k_{t+1} $f;[0,\infty) \to [0,\infty)$. In particular, economies with combinations of λ and L located in region 3 follow dynamics such as those shown in Figure 2-3.

i) Define an interval that satisfies the following two conditions on $\underline{k}$ and $\overline{k}$.

$$f^n(k) \geq \underline{k} \quad \text{for } \forall k \geq \underline{k} \quad \text{for } \forall n = 1,2,\cdots.$$

$$f^n(k) \leq \overline{k} \quad \text{for } \forall k \leq \overline{k} \quad \text{for } \forall n = 1,2,\cdots.$$

By the definitions of $\underline{k}$ and $\overline{k}$,

$$f^n(k) \in [\underline{k},\overline{k}] \quad \text{for } \forall k \in [\underline{k},\overline{k}] \quad \text{for } \forall n = 1,2,\cdots. \tag{A.1}$$

From Figure 2-3, the existence of this interval is proved if $k_{t+1}/k_t < 1$, $\tilde{k}_r < k_{OLG}$.

ii) Figure 2-3 shows that $f(k)$ cannot intersect the 45° line under the condition, $\tilde{k}_r < k_{OLG}$ and therefore no steady state exists in the interval.

iii) Because $k_{t+1} = [(1-\alpha)/(2+\rho)]k_t^{\alpha}$ is above the 45° line and (A.1) holds,

$$\exists \underline{n} \quad f^n(k) \in [\underline{k}, \bar{k}] \quad \text{for } \forall n \geq \underline{n} \quad \text{for } \forall k \in (0, \underline{k}). \tag{A.2}$$

Because $k_{t+1}/k_t = \lambda b L^b/(2+\rho) < 1$ and (A.1) holds,

$$\exists \bar{n} \quad f^n(k) \in [\underline{k}, \bar{k}] \quad \text{for } \forall n \geq \bar{n} \quad \text{for } \forall k \in (\bar{k}, \infty). \tag{A.3}$$

From (A.2) and (A.3),

$$f^n(k) \in [\underline{k}, \bar{k}] \quad \text{for } \forall n \geq \max\{\underline{n}, \bar{n}\} \quad \text{for } \forall k \in (0, \infty).$$

From i), ii), and iii), it has been shown that permanent cyclical fluctuations necessarily emerge in region 3.

Q.E.D.

---

[1] Their model is the same as Romer's (1986) model, where the quantity of human capital is equal to the quantity of physical capital.

[2] Bertola (1996) investigated the effects of a change in the share of labor or capital stock in disposable income on the growth rate. He used a continuous-time overlapping generations model with endogenous growth similar to Saint-Paul's (1992) model. The paper shows that the share of labor is positively related to the growth rate only when the declining rate of labor income is high.

[3] Because a profit-maximizing firm cannot attain an interior optimum in the presence of a nonconvex technology, increasing returns cannot be introduced directly.

[4] Because the final goods market is assumed to be perfectly competitive, profit is zero.

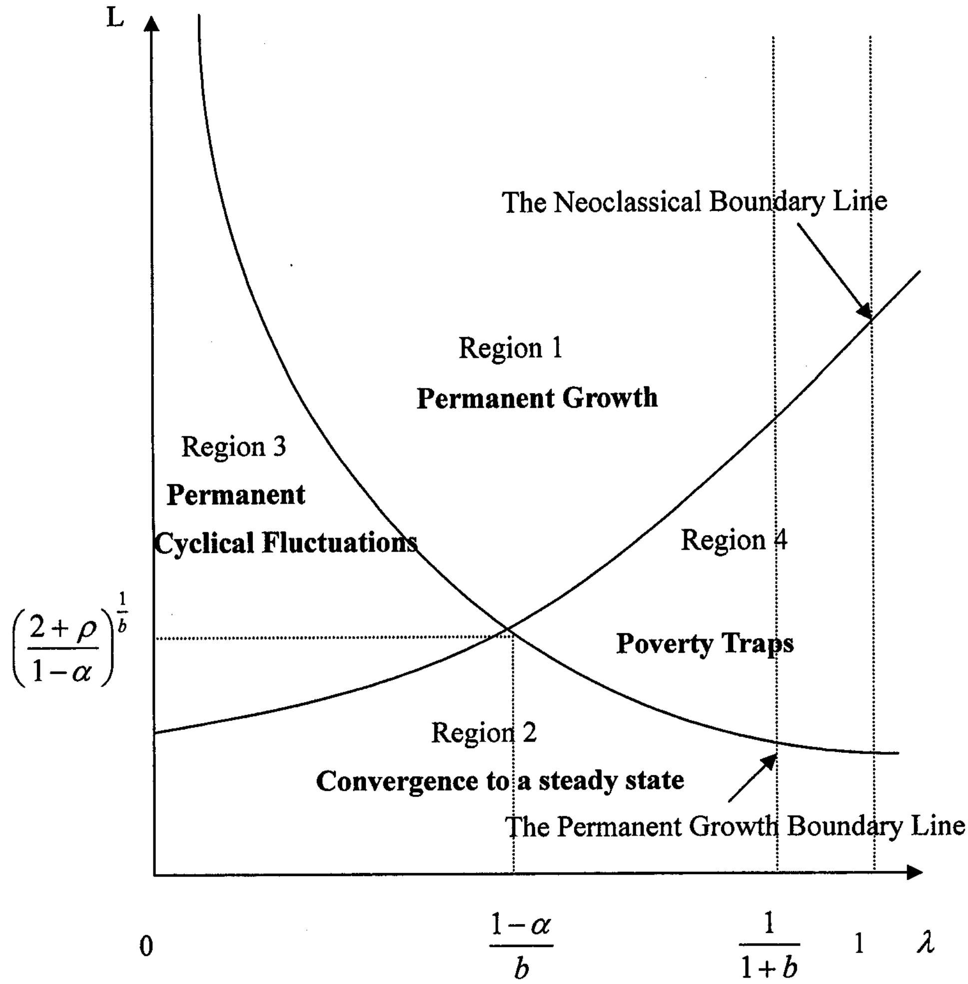
[5] This assumption gives us clearer results about the patterns of growth. If $a > 1$, this model results in more complex divergences of the patterns of growth.

[6] There are some organizations that coordinate investors' or entrepreneurs' choice, such as Keidanren (Japan Federation of Economic Organizations) in Japan.

[7] The interest rate when all of capital is invested in the advanced technology is always higher than the interest rate when capital is divided with two technologies, because the advanced production technology exhibits increasing returns to capital. Therefore, the investors need to compare only the two rates of return: the rate of return when investing all of capital to the primitive technology and the rate of return when investing all of capital to the advanced technology.

[8] The assumption that a=1 enables economies adopting the advanced technology to obtain perpetual growth.

[9] This fact is proved in the Appendix.



L
The Neoclassical Boundary Line
Region 1
Permanent Growth
Region 3
Permanent
Cyclical Fluctuations
Region 4
$\left(\frac{2+\rho}{1-\alpha}\right)^{\frac{1}{b}}$
Poverty Traps
Region 2
Convergence to a steady state
The Permanent Growth Boundary Line
0
$\frac{1-\alpha}{b}$
$\frac{1}{1+b}$
1
λ

Figure 1-1

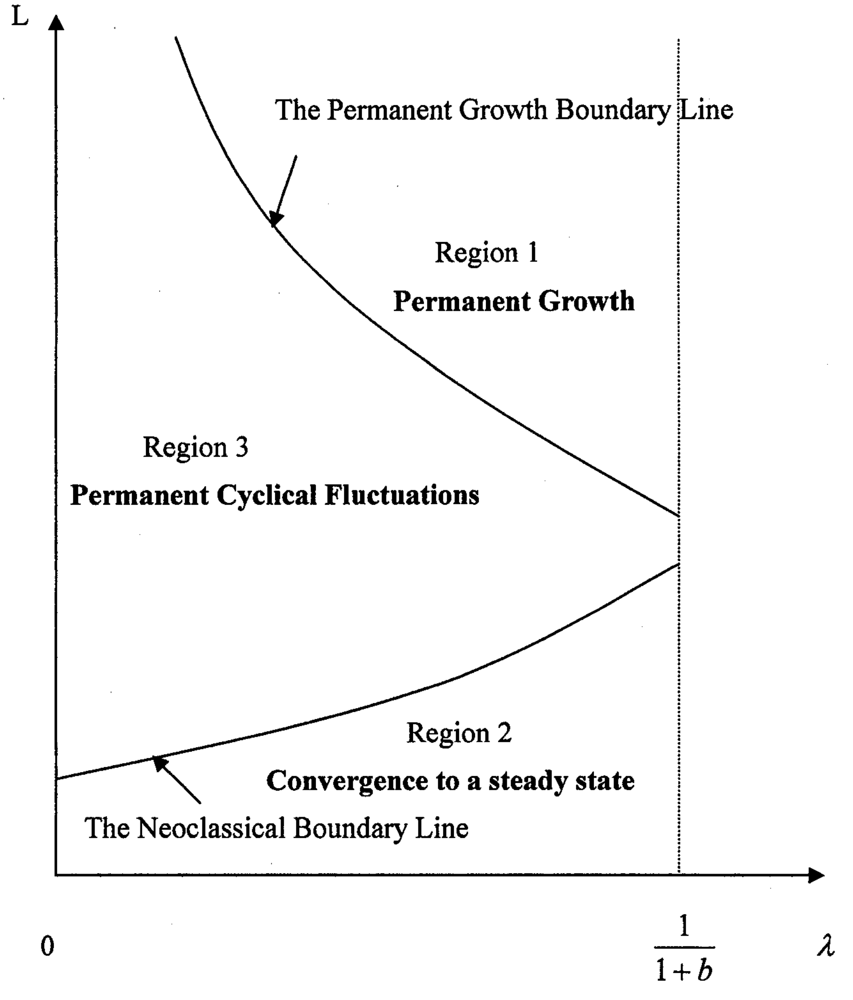

L
The Permanent Growth Boundary Line
Region 1
Permanent Growth
Region 3
Permanent Cyclical Fluctuations
Region 2
Convergence to a steady state
The Neoclassical Boundary Line
0
$\frac{1}{1+b}$
λ

Figure 1-2

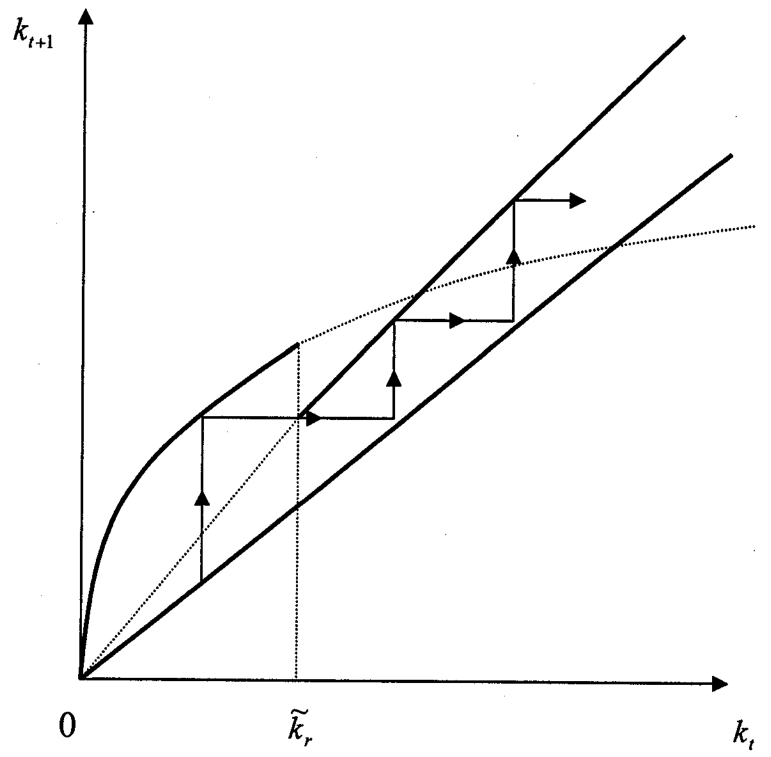


Figure 2-1. Permanent Growth

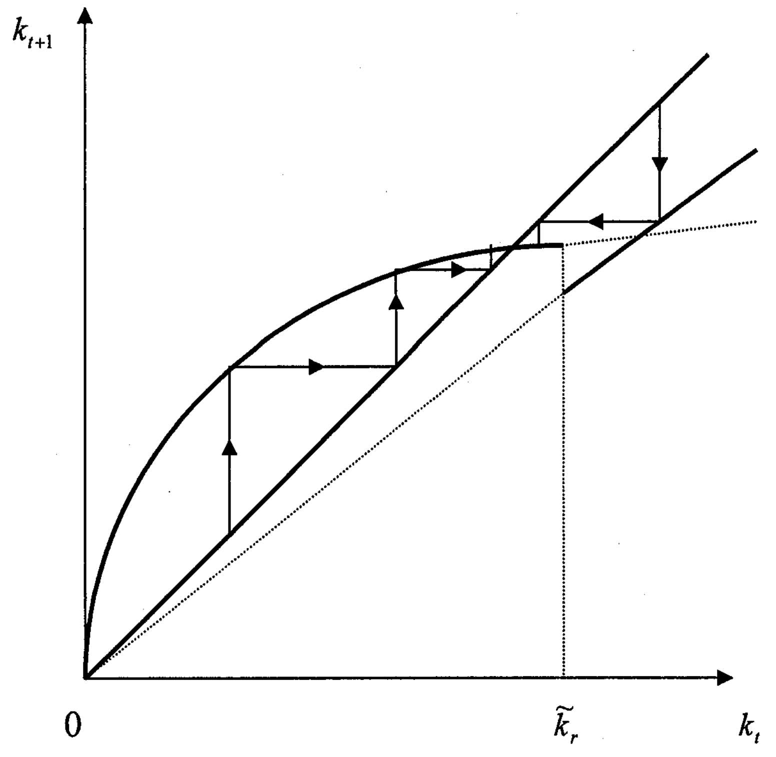


Figure 2-2. Convergence to a steady state

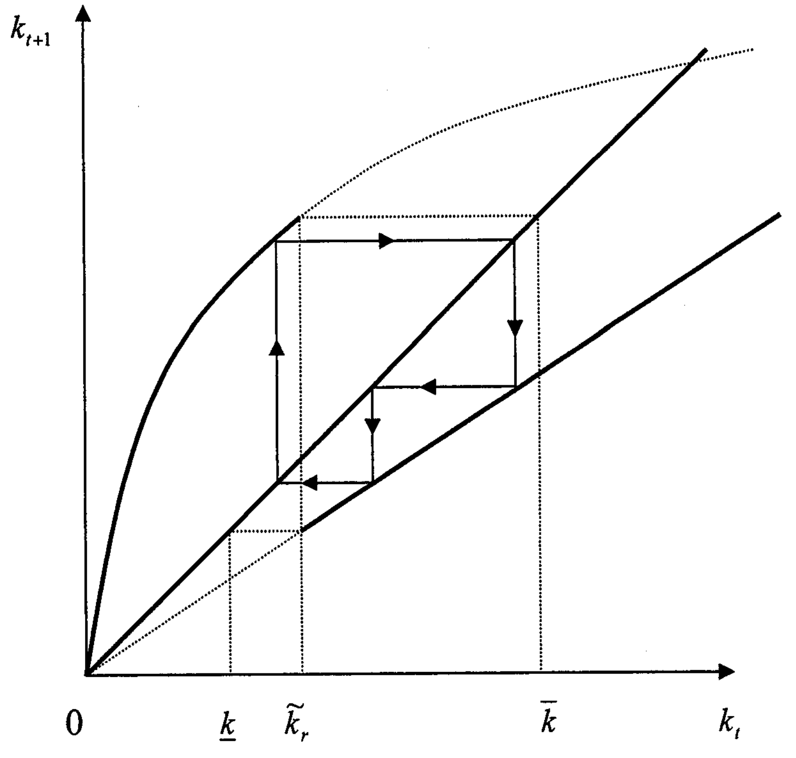


Figure 2-3. Permanent Cyclical Fluctuations

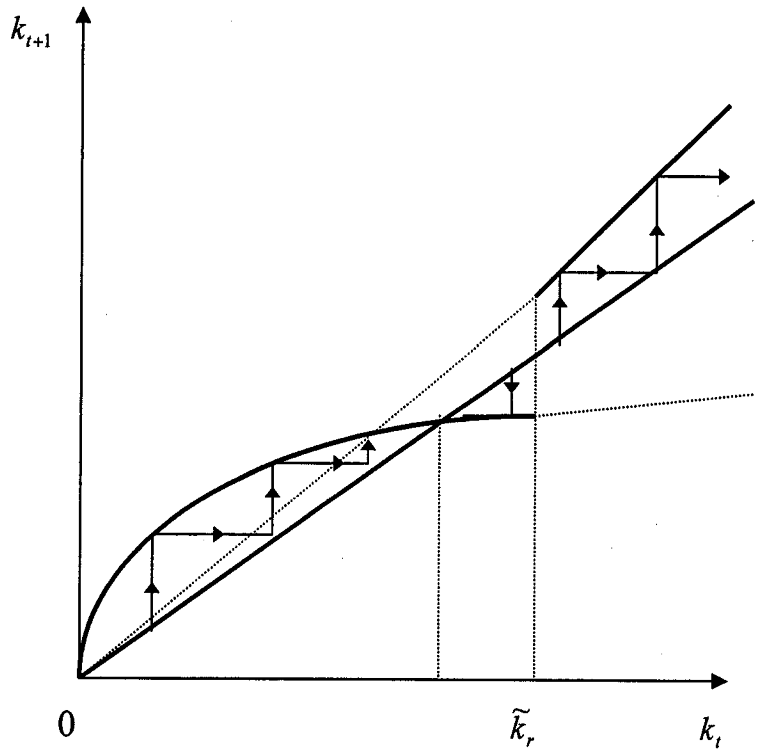
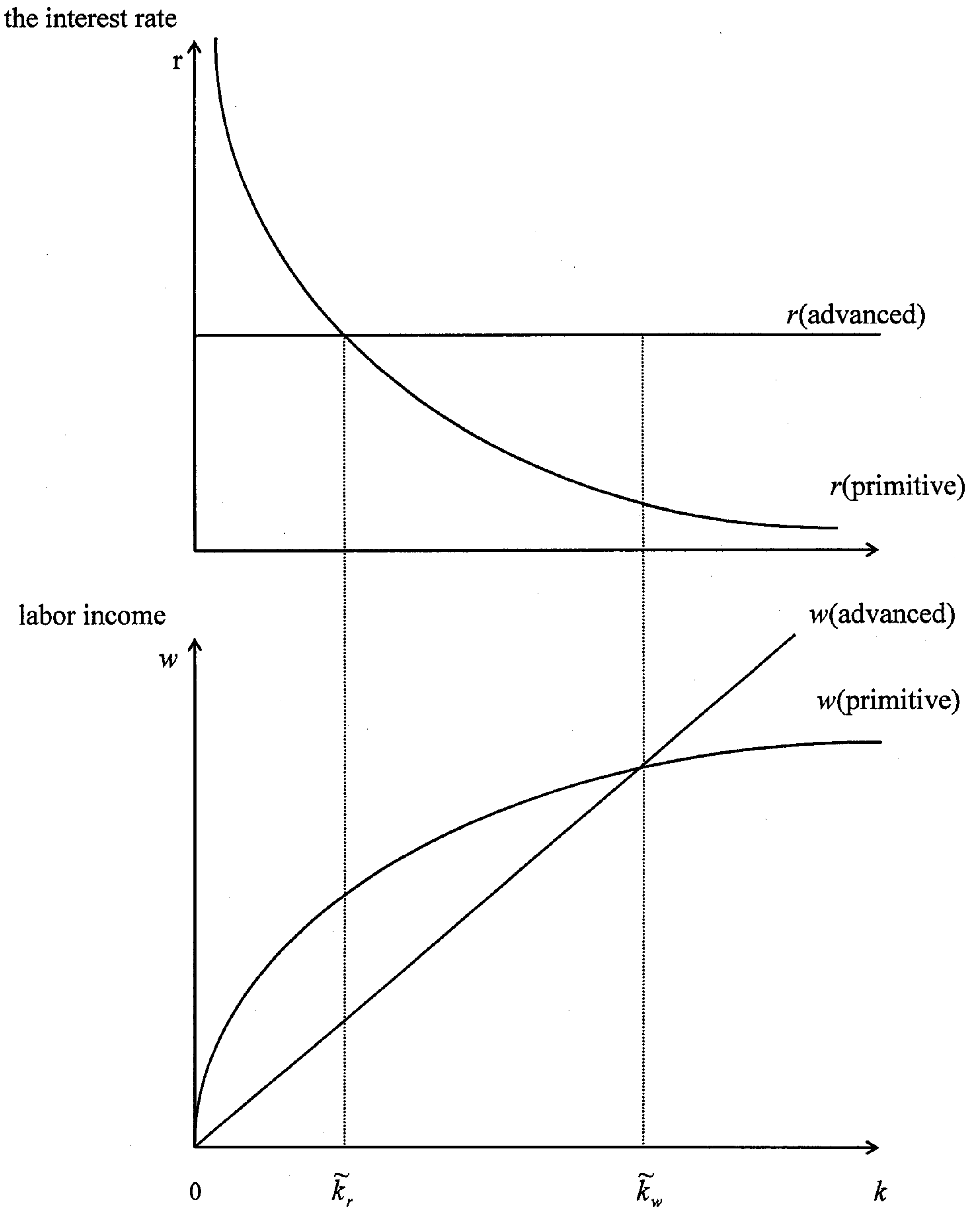


Figure 2-4. Poverty Traps



the interest rate
r
r(advanced)
r(primitive)
labor income
w
w(advanced)
w(primitive)
0
$\tilde{k}_r$
$\tilde{k}_w$
k

Figure 3

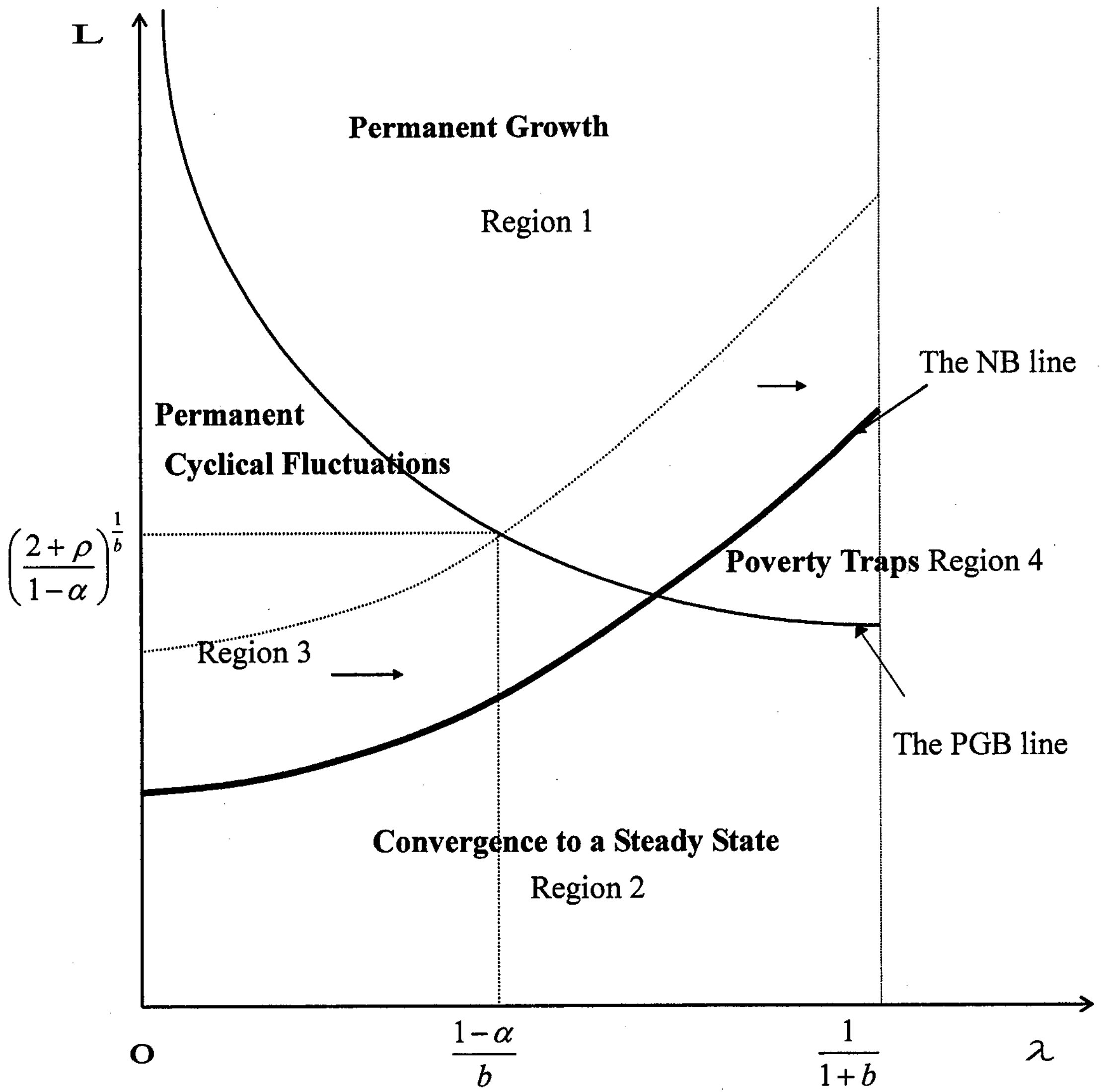


Figure 4-1. Tax on the primitive production technology

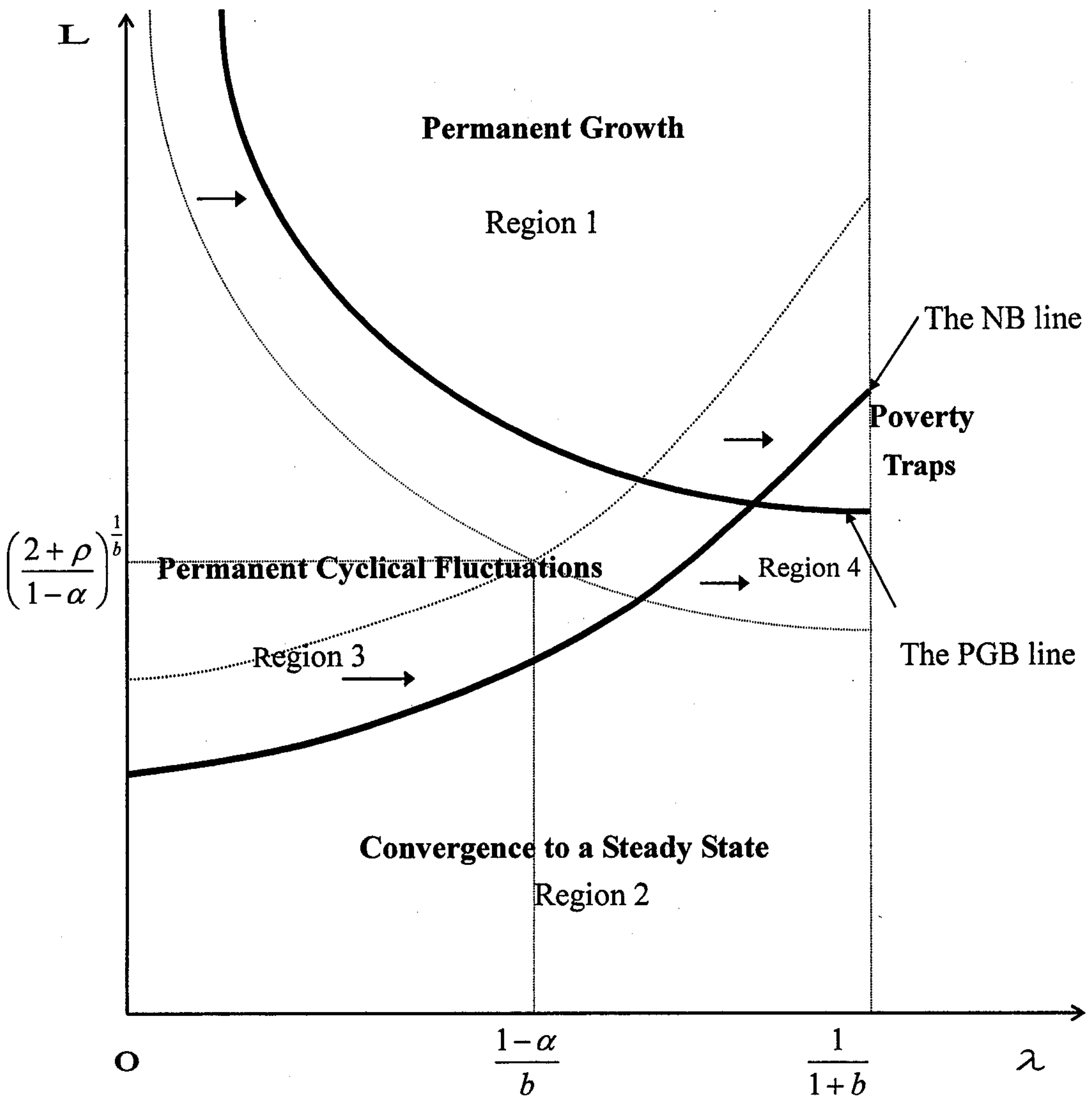


Figure 4-2. Subsidies for the advanced technology

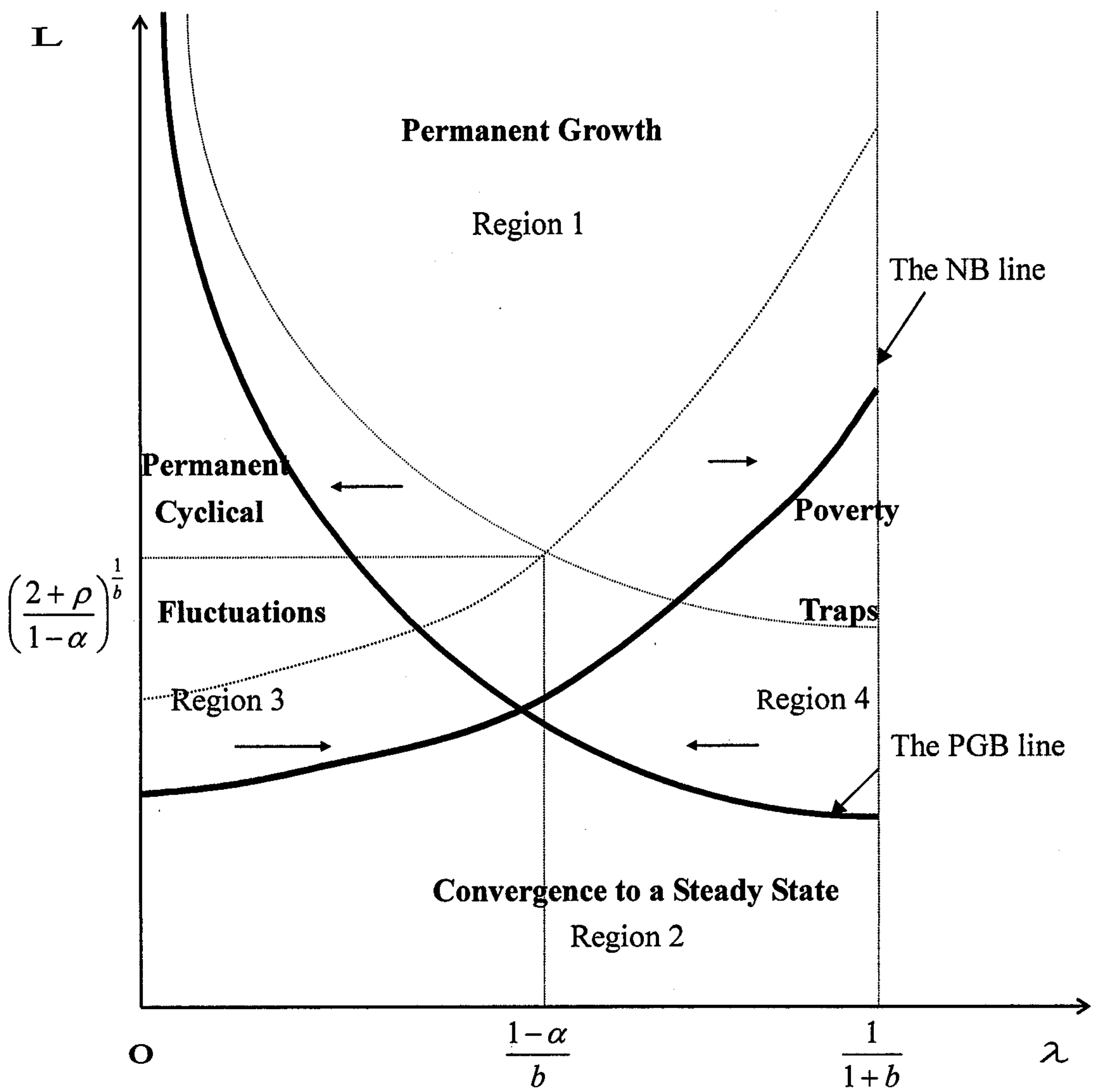


Figure 4-3. Taxing both technologies

Chapter 5

Multiple Balanced Growth Paths in a Schumpeterian Growth Model

5.1 Introduction

Research and development activity by private firms is one of the most important factors as the engine of economic growth. Many studies of economic growth view R&D as the engine of economic growth(Aghion and Howitt 1992; Romer 1990; Grossman and Helpman 1991). However almost all the existing R&D-based endogenous growth model cannot explain the differences of growth rate among countries and the fluctuations of growth rate that are observed empirically.[1] The purpose of this paper is to construct the R&D-based model of endogenous growth that can explain these phenomena.

There are some existing R&D-based endogenous growth models that attempt to account for the differences of the growth rate among countries. Constructing the R&D-based growth model with in-house R&D and free entry, Peretto (1999) shows that there are a 0-growth equilibrium and a positive-growth path and that to which economies converge depends on the expectations of economic agents. However his model cannot account for the permanent fluctuations of growth rate. Introducing the complementarity among differentiated capital goods, Evans, Honkapohja and Romer (1998) shows that there can exist multiple growth paths and, however their basic model without adjustment cost has no transitional dynamics.

[1] Greiner and Semmler (1996) build the two-sector growth model in which human capital is acquired only through learning by doing, and they show that multiple steady states, indeterminacy of the equilibrium and limit cycles can be generated in their model. However, their model has no intended R&D.

Moreover, there are some existing R&D-based endogenous growth models that attempt to account for fluctuations of the growth rate. Introducing the assumption of one period monopoly into Rivera-Batiz and Romer (1991)'s discrete-time model, Matsuyama (1999) shows that not only poverty traps but also nonconstant periodic paths can be generated. Though this model can account for permanent fluctuations of growth rates, this model cannot generate the balanced growth paths with different positive growth rates.

In this paper, we construct an endogenous growth model with quality improvements under the assumption that an innovation is imitated costlessly after one period and show that multiple balanced growth paths, indeterminacy, and growth cycles can appear.

Why does the present model generate such a rich array of dynamics including indeterminacy and growth cycles? Under the circumstance that every innovation is imitated after one period, there are different sectors: monopolistic and competitive sectors. The proportion of monopolistic sectors, that is, the sectors that obtain innovations influences the allocation of labor among the intermediate sectors through the labor demand and affects the profit of innovations. Because of this pecuniary externality, the rate of return of innovations rises as the aggregate investment in R&D increases. As a result, the present paper's model generates multiple balanced growth paths: one is the balanced growth path with the higher innovation and growth rate, the other is that with the lower innovation and growth rate. [2] Moreover there is transitional dynamics in the present model and around the higher-growth path indeterminacy of the competitive equilibrium path and growth cycles can generate. In particular, these endogenous cycles can account for the fluctuations of the real economies.

In contrast to the existing papers that generate indeterminacy and growth cycle, this paper's results concerning the dynamic properties do not rely on the assumption of increasing returns to scale of production technology. Non-convexities in the reduced form production function due to the pecuniary externalities mentioned above yield such dynamic properties as indeterminacy and growth cycles. Gali(1996) also shows that multiple steady states and indeterminacy without the assumption of increasing returns. However his model cannot generate endogenous growth path. As far as I know, there is no research that the R&D-based endogenous growth model can generate multiple growth paths, indeterminacy and growth cycles.

In addition to this paper's contribution mentioned above, we get interesting results with respect to the R&D subsidy policy; whether the R&D subsidy policy enhances economic growth or reduces depends on whether the economy exists on the balanced growth path with the lower growth or on the balanced growth path with the higher

[2]Laussel and Nyssen (1999) construct the R&D-based endogenous growth model similar to the present model. However their model can generate only multiple growth paths and has no transitional dynamics, and therefore their model cannot account for the fluctuations of growth rate.

growth. If the economy is on the balanced growth path with the higher growth rate, the R&D subsidy reduces the growth rate. Moreover, the present paper can show that a negative scale effect can be generated.

The structure of the paper is as follows. Section 2 sets up the model. Section 3 solves the general equilibrium and Section 4 explores the local stability of the balanced growth paths and shows that multiple growth paths, indeterminacy, and growth cycles can appear. Section 5 examines the effects of R&D subsidy and scale effect. Section 6 concludes the paper.

5.2 The model

We construct a discrete-time model of endogenous growth with quality improvements based on Grossman and Helpman (1991). We investigate the dynamics of the economy under the assumption that an innovation is imitated costlessly after one period.

5.2.1 Production

Final good

One final good is produced competitively using a continuum of intermediate goods distributed along a unite interval. $q(j,\omega)$ denotes quality level of product ω after the quality is improved j times. We assume that the quality of every good at time $t=0$ equals one and that the size of the quality improvement is $\lambda > 1$, therefore we get $q(j,\omega) = \lambda^j$. Production technology of the final good sector is given by

$$Y_t = \exp\left\{ \int_0^1 \log\left[\sum_j q_t(j,\omega) x_t(j,\omega) \right] d\omega \right\}, \tag{5.1}$$

where Y_t denotes the quantity of the final good and $x_t(j,\omega)$ denotes the quantity of quality $q(j,\omega)$ of product ω. Production technology of final good sector is homogeneous of degree 1. We take the final good as the numeraire.

Within a given sector ω, intermediate goods are perfectly substitutable. Hence the final good producers use the single quality that has the lowest quality-adjusted price, $p_t(j,\omega)/q_t(j,\omega)$ for a given sector $\omega \in [0,1]$. Letting $J_t(\omega)$ denote the quality that has the lowest quality-adjusted price for a sector ω at time t, demand functions are given by

$$x_t(j,\omega) = \begin{cases} \dfrac{Y_t}{p_t(j,\omega)} & j = J_t(\omega), \\ 0 & \text{otherwise.} \end{cases} \tag{5.2}$$

Intermediate good

Every intermediate good is produced by using one unit of labor. Being different from Grossman and Helpman (1991), we assume that every quality is imitated costlessly after one period.[3] Due to this assumption, there are monopolistic sectors and competitive sectors at every period.

First, in the sector where the quality is improved at time t due to innovation, the firm that manages to invent the new quality behaves monopolistically. Because other firms in the sector can produce the intermediate good with the leading-edge quality at time $t-1$, the innovative firm charges the price so that it can get all demand in product ω. The monopolistic firms are symmetric, then the price charged by the firms, p^m is given by

$$p_t^m = \lambda w_t. \tag{5.3}$$

From the demand function (5.2), the labor demand of the monopolistic sector, x_t^m is given by

$$x_t^m = \frac{Y_t}{\lambda w_t}. \tag{5.4}$$

From (5.2) and (5.4), the profit of the monopolistic firm, π_t is

$$\pi_t = \frac{\lambda - 1}{\lambda} Y_t. \tag{5.5}$$

Second, in the sector where the quality is not improved at time t, every firm can produce the highest quality at time $t-1$ due to costless imitation, therefore the intermediate good is produced competitively. The competitive sectors are symmetric, then the price of the intermediate good in the competitive sector, p_t^c is given by

$$p_t^c = w_t. \tag{5.6}$$

From the demand function (5.2), x_t^c is

$$x_t^c = \frac{Y_t}{w_t}. \tag{5.7}$$

Research and Development

Intermediate good producing firm can invent the intermediate good with the state-of-the-art quality and produce it at time $t+1$ by investing ηD_t units of final good at time

[3] Shleifer(1986) makes the assumption similar to this one.

t, where D_t represents the parameter of the productivity at time t. This means that the higher the productivity is, the more difficult inventing the next quality is.[4] Since monopoly profits always exceed individual firm profits under duopoly, no two firms will choose to engage in R&D for the same product.

The firm engaging in the quality improvement R&D at time t can get the profit by monopoly, π_{t+1} for only one period, because the innovation is imitated costlessly after one period. Hence the free entry condition is given by

$$\frac{\pi_{t+1}}{1+r_{t+1}} = \eta D_t, \tag{5.8}$$

where r_t denotes the interest rate. Letting μ_t be the number of the innovative sectors, that is, monopolistic sectors at time t, the aggregate investment in R&D at time $t-1$, Z_{t-1} is given by

$$Z_{t-1} = \eta D_{t-1}\mu_t. \tag{5.9}$$

The firms engaging in R&D raise funds for this cost of R&D by the issue of shares.

5.2.2 Households

We consider an economy populated by L households, who supply one unit of labor inelastically. The household's lifetime utility is given by

$$\max_{\{c_t\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \beta^t \log c_t, \tag{5.10}$$

where c_t denotes consumption per household and β denotes the discount factor. The household's intertemporal budget constraint is

$$c_t + a_{t+1} = (1+r_t)a_t + w_t, \tag{5.11}$$

$$a_0 \geq 0, \tag{5.12}$$

$$\lim_{T=\infty} \prod_{s=0}^{T} \frac{1}{1+r_s} a_{T+1} \geq 0, \tag{5.13}$$

where a_t denotes asset per household. Dynamic optimization of the utility function, (5.10), subject to the intertemporal constraint, (5.11), (5.12), and (5.13), yields the Euler equation and a transversality condition

$$\frac{c_{t+1}}{c_t} = \beta(1+r_{t+1}), \tag{5.14}$$

$$\lim_{T=\infty} \beta^T \frac{a_{T+1}}{c_T} = 0. \tag{5.15}$$

[4]Segerstrom(1998) makes the assumption similar to this one.

5.3 Market Equilibrium

5.3.1 Labor Market

Labor market equilibrium requires that the total labor used in intermediate goods production equals the labor supply, L. That is,

$$\mu x_t^m + (1-\mu)x_t^c = L,$$

where μ_t denotes the proportion of the monopolistic sector at time t. Substituting (5.4) and (5.7) into this labor market equilibrium condition, we get

$$\left(1 - \frac{\lambda-1}{\lambda}\mu_t\right) Y_t = w_t L. \tag{5.16}$$

From (5.16), the quantity of labor used in the monopolistic sectors and the quantity of labor used in the competitive sectors are

$$x_t^m = \frac{L}{\lambda\left(1 - \frac{\lambda-1}{\lambda}\mu_t\right)}, \tag{5.17}$$

$$x_t^c = \frac{L}{\left(1 - \frac{\lambda-1}{\lambda}\mu_t\right)}, \tag{5.18}$$

Next, we derive an expression for GDP, Y_t. From (5.1),

$$\begin{aligned}\log Y_t &= \int_0^1 \log q_t(\omega)x_t(\omega)d\omega \\ &= \int_0^1 \log q_t(\omega)d\omega + \int_0^1 \log x_t(\omega)d\omega. \end{aligned} \tag{5.19}$$

where $q_t(\omega)$ denotes the highest quality of intermediate good ω at time t. We define that $\log Q_t = \int_0^1 \log q_t(\omega)d\omega$ and that $\log X_t = \int_0^1 \log x_t(\omega)d\omega$. Because μ_t represents the proportion of the sectors that obtain the quality improvement, $\log Q_t$ is given by

$$\log Q_t = \mu_t \log\lambda + \log Q_{t-1}. \tag{5.20}$$

Substituting the quantity of labor used in every sector into $\int_0^1 \log x_t(\omega)d\omega$, we get the average volume of the intermediate product across industries,

$$\log X_t = \log L - \log\left(1 - \frac{\lambda-1}{\lambda}\mu_t\right) - \mu_t\log\lambda. \tag{5.21}$$

From (5.20) and (5.21), Y_t depends on the proportion of the monopolistic sector at time t as follows,

$$Y_t = Q_t L\phi(\mu_t), \tag{5.22}$$

where

$$\phi(\mu_t) = \left(\frac{\lambda^{-\mu_t}}{1 - \frac{\lambda-1}{\lambda}\mu_t}\right), \qquad 0 \leq \mu \leq 1. \tag{5.23}$$

The graph of the function $\phi(\cdot)$ is always U-shaped as depicted in Figure 1.[5] There are two effects of an increase in the proportion of the sectors that obtain innovation, μ_t on X_t. One is a negative effect: the increase in the proportion of the monopolistic sectors reduces the average volume of product, X_t, because the volume of product is relatively small in the monopolistic sectors. This effect is indicated by the term, $\lambda^{-\mu_t}$ in (5.23). The other is a positive effect: the increase in the proportion of the monopolistic sectors raises the volume of labor input in every sectors and consequently raises X_t, because the labor input is relatively small in the monopolistic sectors. This effect is indicated by the term, $1/[1 - (\lambda - 1)\mu_t/\lambda]$ in (5.23). When the proportion of the monopolistic sectors, μ_t is low (resp. high), the negative (resp. positive) effect dominates the other effect. This is why the function $\phi(\cdot)$ is U-shaped.

5.3.2 Asset Market

Letting A_t denote the aggregate asset, the equilibrium condition in the asset market implies $A_t = Z_{t-1}$. Here we define the parameter of productivity as $D_t = Q_t$. From this equation and (5.9), we get

$$\mu_t = \frac{A_t}{\eta Q_{t-1}}. \tag{5.24}$$

Substituting (5.20), (5.22) and (5.5) into the free entry condition for R&D (5.8), the interest rate is given by

$$1 + r_t = \frac{L}{\eta}\frac{\lambda - 1}{\lambda}\left(\frac{1}{1 - \frac{\lambda-1}{\lambda}\mu_t}\right). \tag{5.25}$$

Since the positive effect by the quality improvement on Y_t cancels out the negative effect on Y_t, only the positive effect remains. This equation shows that the rate of return of innovations is the increasing function of the proportion of the sectors that obtain innovation, μ_t. In other words, an increase in aggregate R&D investment increases the rate of return of innovations. Because of the pecuniary externality, increasing returns to scale with respect to R&D is generated in the present model.

[5]This fact is proved in Appendix A.

5.4 The Dynamics

Combining equations (5.14), (5.25), the equilibrium condition of final goods market, $Y_t = C_t + \eta Q_t \mu_{t+1}$, (5.22), and (5.20), the dynamics of this economy can be summarized by three difference equations of the first order.

$$
\begin{aligned}
\frac{C_{t+1}}{C_t} &= \beta \frac{L}{\eta} \frac{\lambda-1}{\lambda} \frac{1}{1-\frac{\lambda-1}{\lambda}\mu_{t+1}}, && (5.26)\\
\eta Q_t \mu_{t+1} &= Q_t \phi(\mu_t) L - C_t, && (5.27)\\
Q_{t+1} &= \lambda^{\mu_{t+1}} Q_t. && (5.28)
\end{aligned}
$$

From the market equilibrium condition (5.27), μ_t is constant and C_t and Q_t grow at the same rate on the balanced growth paths. Thus we rewrite the above difference equations by using $\chi_t = C_t/Q_t$.

The competitive equilibrium is defined as a sequence $\{\mu_t, \chi_t; t > 0\}$ which satisfies

$$
\begin{aligned}
\frac{\chi_{t+1}}{\chi_t} &= \beta \frac{L}{\eta} \frac{\lambda-1}{\lambda} \phi(\mu_{t+1}), && (5.29)\\
\eta \mu_{t+1} &= \phi(\mu_t) L - \chi_t. && (5.30)
\end{aligned}
$$

and the transversality condition

$$\lim_{T=\infty} \beta^T \eta \frac{\mu_{T+1}}{\chi_T} = 0. \tag{5.31}$$

On the balanced growth paths, μ and χ are constant. Letting (μ^*, χ^*) be the balanced-growth-path values of (μ, χ), they satisfy the following equations:

$$1 = \beta \frac{L}{\eta} \frac{\lambda-1}{\lambda} \phi(\mu^*), \tag{5.32}$$

$$\chi^* = \phi(\mu^*) L - \eta \mu^*. \tag{5.33}$$

From (5.32) or Figure 2, we find that two balanced growth paths E_L and E_H are generated if $\frac{\lambda}{\lambda-1} < \beta \frac{L}{\eta} < \frac{\lambda^{\frac{\lambda}{\lambda-1}}}{e \log \lambda}$. The dynamics of this economy is depicted as Figure 3. Letting (μ_L, χ_L) and (μ_H, χ_H) denote the balanced-growth-path values of (μ, χ) with the lower growth rate and the higher growth rate respectively, we find the following:

Proposition 1. *If the following inequality*

$$\frac{\lambda}{\lambda-1} < \beta \frac{L}{\eta} < \frac{\lambda^{\frac{\lambda}{\lambda-1}}}{e \log \lambda} \tag{5.34}$$

is satisfied, multiple growth paths can be generated. One is the balanced growth path with the higher growth rate, λ^{μ_H} and the other is the balanced growth path with the lower growth rate, λ^{μ_L}.

Proof. See Figure 2. □

In the rest of the paper, we focus on the case that the multiple balanced growth paths can be generated.

We must examine the local stability of these balanced growth paths E_L and E_H. Linearization of the dynamic system (5.29) and (5.30) around the balanced growth path (μ^*, χ^*) gives the following Jacobian matrix

$$\begin{bmatrix} \mu_{t+1} - \mu^* \\ \chi_{t+1} - \chi^* \end{bmatrix} = \begin{bmatrix} \frac{L}{\eta}\phi'(\mu^*) & -\frac{1}{\eta} \\ \frac{L}{\eta}\phi'(\mu^*)\beta\frac{L}{\eta}\frac{\lambda-1}{\lambda}\phi(\mu^*)\chi^* & 1 - \frac{1}{\eta}\beta\frac{L}{\eta}\frac{\lambda-1}{\lambda}\phi'(\mu^*)\chi^* \end{bmatrix} \begin{bmatrix} \mu_t - \mu^* \\ \chi_t - \chi^* \end{bmatrix} \quad (5.35)$$

Letting J denote Jacobian matrix, the trace and the determinant of the Jacobian matrix are

$$\mathrm{tr}J = \beta\frac{L}{\eta}\frac{\lambda-1}{\lambda}\phi'(\mu^*)\mu^* + 1 \quad (5.36)$$

$$\mathrm{det}J = \frac{L}{\eta}\phi'(\mu^*) \quad (5.37)$$

We make α denote $\frac{L}{\eta}$ for simplicity in the rest of the paper. [6] Let us summarize the results about the stability of the balanced growth path with the lower growth rate E_L in the following proposition.[7]

Proposition 2. *If μ_L that satisfies (5.32) and $\phi'(\mu_L) < 0$, and the parameters, α and β satisfy the following inequality:*

$$2 + \alpha\beta\frac{\lambda-1}{\lambda}\phi'(\mu_L)\mu_L + \alpha\phi'(\mu_L) > 0, \quad (5.38)$$

The balanced growth path E_L is a saddle point. If the above inequality is not satisfied, the balanced growth path E_L is a source.

Proof. $p(\nu)$ denotes the characteristic equation, that is, $p(\nu) = \nu^2 - trJ\nu + detJ$. Inspecting Figure 2, we find that the inequality $\phi'(\mu_L) < 0$ is always satisfied. Therefore, using the inequality $1 < \beta(\lambda-1)/\lambda\mu$, we get

$$p(1) = 1 - trJ + detJ = \frac{L}{\eta}\phi'(\mu_L)\left(1 - \beta\frac{\lambda-1}{\lambda}\mu_L\right) < 0 \quad (5.39)$$

[6]We can understand that $\alpha = \frac{L}{\eta}$ represents the potential productivity of economies.

[7]Since the region of (α, β) where the inequality (5.38) is too complicated, we do not find this region analytically.

In addition to the above condition, if the following inequality

$$p(-1) = 1 + trJ + detJ > 0 \tag{5.40}$$

is satisfied, the balanced growth path E_L is a saddle point. The second condition is rewritten as follows:

$$2 + \alpha\beta\frac{\lambda-1}{\lambda}\phi'(\mu_L)\mu_L + \alpha\phi'(\mu_L) > 0$$

□

Next, we examine the stability of the balanced growth path with the higher growth rate. First, we get the inequality $\phi'(\mu_H) > 0$ from Figure 2. Therefore, the following inequalities are satisfied.

$$\begin{aligned} p(1) &= 1 - trJ + detJ = \frac{L}{\eta}\phi'(\mu_H)\left(1 - \beta\frac{\lambda-1}{\lambda}\mu_H\right) > 0, && (5.41) \\ p(-1) &= 1 + trJ + detJ = 2 + \alpha\phi'(\mu_H)\left(\beta\frac{\lambda-1}{\lambda}\mu_H + 1\right) > 0. && (5.42) \end{aligned}$$

Let us summarize the results about the stability of the balanced growth path with the higher growth rate in the following proposition.

Proposition 3. *If the values of the parameters, α and β satisfy (5.34) and the following inequality:*

$$\alpha > \frac{\lambda^{\frac{\lambda}{\lambda-1} - \frac{1}{\log\lambda + \beta\frac{\lambda-1}{\lambda}}}}{\beta\left(\log\lambda + \beta\frac{\lambda-1}{\lambda}\right)}, \tag{5.43}$$

the balanced growth path E_H is a sink. In other words, the competitive equilibrium path is indeterminate. If the above inequality is not satisfied, the balanced growth path E_H is a source.

Proof. Since the inequalities (5.41) and (5.42) are satisfied, the last thing that we have to do is to find the range of parameters that satisfy $detJ < 1$. First, we find the range of μ that satisfies $detJ < 1$ and $\phi'(\mu) > 0$, and then we find the range of parameters α and β satisfy (5.32) and the above range of μ. On the balanced growth path, $\phi'(\mu_H)$ is written as follows:

$$\phi'(\mu_H) = \left(-\log\lambda + \frac{\frac{\lambda-1}{\lambda}}{1 - \frac{\lambda-1}{\lambda}\mu_H}\right)\phi(\mu_H) = \left(-\log\lambda + \frac{\frac{\lambda-1}{\lambda}}{1 - \frac{\lambda-1}{\lambda}\mu_H}\right)\frac{1}{\alpha\beta}\frac{\lambda}{\lambda-1}. \tag{5.44}$$

Substituting this equality into $detJ < 0$, we get

$$\mu < \frac{\lambda}{\lambda - 1} - \frac{1}{\log \lambda + \frac{\lambda}{\lambda-1}\beta}. \tag{5.45}$$

From this inequality and (5.32), the range of (α, β) satisfies the following inequality:

$$1 < \alpha\beta \left(\log \lambda + \beta \frac{\lambda - 1}{\lambda} \right) \lambda^{\frac{\lambda}{\lambda-1} - \frac{1}{\log \lambda + \beta\frac{\lambda-1}{\lambda}}}, \tag{5.46}$$

and thus we get the inequality (5.43). □

From the inequality (5.43), we get the region of (α, β) where the balanced growth path E_H is a sink as depicted in Figure 4-1.[8]

If the balanced growth path E_L is a saddle point and the balanced growth path E_H is a sink, not only the local indeterminacy of the competitive equilibrium path but also the indeterminacy of which balanced growth path economies converge to can be generated. We can show that global indeterminacy can be generated in the present model. For example, suppose that the parameter of innovation size, λ satisfies that $\frac{\lambda}{\lambda-1}\log\lambda < 1 + \sqrt{2}$. The region of (α, β) where the balanced growth path E_L is a saddle point is $\beta > \frac{\frac{\lambda}{\lambda-1}\log\lambda}{2}$.[9] In this case there exists the region of (α, β) where the balanced growth path E_L is a saddle point and the balanced growth path E_H is a sink as depicted in Figure 4-2.

It is well known that bifurcations may take place when the stability of steady states undergoes changes. In the present paper, Hopf bifurcations can take place. Let us summarize the results about the bifurcation in the following proposition.

Proposition 4. *Suppose that*

$$\alpha^* = \frac{\lambda^{\frac{\lambda}{\lambda-1} - \frac{1}{\log \lambda + \beta\frac{\lambda-1}{\lambda}}}}{\beta\left(\log\lambda + \beta\frac{\lambda-1}{\lambda}\right)}, \tag{5.47}$$

then α^* is a Hopf bifurcation value and there is an invariant closed curve bifurcating from α^*. In other words, there exist some nonconstant periodic growth paths at some parameter values α which are sufficiently close to α^*.

Proof. We prove this proposition by using the Hopf bifurcation theorem for discrete time systems.

[8] The proof with respect to the characters of the region is in the Appendix.

[9] In appendix B, we prove that the balanced growth path E_L is saddle point if this inequality is satisfied.

First, since the inequalities (5.41) and (5.42) are satisfied, the eigenvalues ν_1 and ν_2 are the complex conjugate when $detJ = 1$.

Second, letting $\mu_H(\alpha^*)$ denote the value of μ_H on the balanced growth path with the higher growth when $\alpha = \alpha^*$, α^* and $\mu(\alpha^*)$satisfies the following equalities and inequality:

$$1 = \alpha^* \beta \frac{\lambda - 1}{\lambda} \phi(\mu_H(\alpha^*)), \tag{5.48}$$

$$\alpha^* \phi'(\mu_H) = 1, \tag{5.49}$$

$$\mu_H(\alpha^*) < 1 \tag{5.50}$$

By using $\mu_H(\alpha)$, we show that $\frac{d}{d\delta}|\nu(\alpha)|_{\alpha=\alpha^*} \neq 0$.

$$\left.\frac{d\sqrt{detJ}}{d\alpha}\right|_{\alpha=\alpha^*} = \frac{1}{2}(detJ)^{-\frac{1}{2}} \left[\phi'(\mu_H(\alpha^*)) + \alpha^* \phi''(\mu_H(\alpha^*))\frac{d\mu_H(\alpha^*)}{d\alpha}\right] \tag{5.51}$$

Using (5.48),(5.49), and the following equalities:

$$\frac{d\mu_H(\alpha)}{d\alpha} = -\frac{\phi}{\alpha\phi'}, \tag{5.52}$$

$$\phi'(\mu) = \left(-\log\lambda + \frac{\frac{\lambda-1}{\lambda}}{1 - \frac{\lambda-1}{\lambda}\mu}\right)\phi(\mu), \tag{5.53}$$

$$\phi''(\mu) = \left(\frac{\frac{\lambda-1}{\lambda}}{1 - \frac{\lambda-1}{\lambda}\mu}\right)^2 \phi(\mu) + \frac{\phi'(\mu)^2}{\phi(\mu)} \tag{5.54}$$

we get the following inequality:

$$\left.\frac{d\sqrt{detJ}}{d\alpha}\right|_{\alpha=\alpha^*} = -\frac{1}{2}\alpha^* \left(\frac{\frac{\lambda-1}{\lambda}}{1 - \frac{\lambda-1}{\lambda}\mu_H(\alpha^*)}\right)^2 \phi(\mu_H(\alpha^*))^2 < 0 \tag{5.55}$$

Finally, we can perturb the other parameters slightly so that the eigenvalues are not roots of unity, and therefore we get $\nu^j(\alpha^*) \neq 1 \quad for\ j = 1, 2, 3, 4$.

Because we proved that these conditions are satisfied, the Hopf bifurcation theorem(e.g. Guckenheimer and Holmes 1986) can be applied. □

Proposition 4 shows that the growth rate can fluctuate permanently in the present model. This result shows that the present model can account for the fluctuations of the growth rates.

5.5 The effect of the subsidy policy on economic growth and scale effect

5.5.1 R&D subsidy policy

In this subsection, we examine how the R&D subsidy affects the growth rate on the balanced growth path. Letting s_R denote the subsidy rate, the free entry condition under this subsidy policy is given by

$$\frac{\pi_t}{1+r_t} = \frac{\eta Q_{t-1}}{1+s_R}. \tag{5.56}$$

Letting $\mu^*(s_R)$ be the balanced-growth-path values of μ under the subsidy policy, they satisfy the following equations:

$$1 = (1+s_R)\beta\frac{L}{\eta}\frac{\lambda-1}{\lambda}\phi(\mu^*(s_R)). \tag{5.57}$$

The R&D subsidy affects μ_L and μ_H as depicted in Figure 5. The subsidy policy raises the growth rate of the economy on the balanced growth path with the lower growth rate. However, this policy affects the growth rate on the balanced growth path with the higher growth rate inversely; it lowers the growth rate.

5.5.2 Scale effect

In this subsection, we examine how an increase in the population size, L affects the growth rate on the balanced growth paths. Let us examine these effects by using equation (5.57) or Figure 5. Then we find that whether the effects of an increase in the population size on economic growth are positive or negative depends on whether the economy exists on the balanced growth path with the lower growth or on the balanced growth path with the higher growth; an increase in the population size raises (resp. lowers) the growth rate if the economy is on the balanced growth path with the lower (resp. higher) growth rate.

With respect to scale effect, almost all R&D-based endogenous growth models generate positive scale effects (Romer 1990; Grossman and Helpman 1991). In contrast to the existing papers, this paper shows that negative scale effects can be generated. [10]

5.6 Conclusion

As far as I know, there is no R&D-based endogenous growth model that can account for both the differences of growth rates among countries and the fluctuations of growth

[10] Peretto (1996) shows that negative scale effects can be generated in the model with multiple growth paths.

rate that are observed empirically.

We have constructed an endogenous growth model with quality improvements under the assumption that an innovation is imitated costlessly after finite period. Under the circumstance, due to the pecuniary externalities, the rate of return of innovations rises as the aggregate investment in R&D increases. As a result, we have shown that multiple growth paths, indeterminacy, and growth cycles can appear in the R&D-based endogenous growth model.

Appendix A

In the Appendix, let us show that the graph of $\phi(\mu)$ is U-shaped.
The first derivative and second derivative are given by

$$\phi'(\mu) = \left(-\log\lambda + \frac{\frac{\lambda-1}{\lambda}}{1-\frac{\lambda-1}{\lambda}\mu}\right)\phi(\mu), \tag{5.58}$$

$$\phi''(\mu) = \left(\frac{\frac{\lambda-1}{\lambda}}{1-\frac{\lambda-1}{\lambda}\mu}\right)^2 \phi(\mu) + \frac{\phi'(\mu)^2}{\phi(\mu)} > 0. \tag{5.59}$$

Since the second derivative is always positive, $\phi(\mu)$ obtains the minimum value at the $\tilde{\mu} = \frac{\lambda}{\lambda-1} - \frac{1}{\log\lambda}$.

Appendix B

From proposition 2, the condition that the balanced growth path E_L is saddle point is (5.38). Using $\phi'(\cdot)$ in Appendix A and (5.32), we get

$$\phi'(\mu) = \left(-\log\lambda + \frac{\frac{\lambda-1}{\lambda}}{1-\frac{\lambda-1}{\lambda}\mu}\right)\left(\mu + \frac{1}{\beta}\frac{\lambda}{\lambda-1}\right) > -2. \tag{5.60}$$

By some calculation, we get

$$\frac{1}{\beta}\frac{\lambda}{\lambda-1} < \frac{2}{\log\lambda} + \frac{\frac{2}{(\log\lambda)^2}}{\frac{\lambda}{\lambda-1} - \frac{1}{\log\lambda} - \mu} - \mu. \tag{5.61}$$

Suppose that $\frac{\lambda}{\lambda-1}\log\lambda < 1+\sqrt{2}$. In this case the right hand side of (5.61) obtains the minimum value at $\mu = 0$. Therefore if the minimum value satisfies the inequality (5.61), the balanced growth path E_L is a saddle point. We can reduce this inequality to

$$\beta > \frac{\frac{\lambda}{\lambda-1}\log\lambda - 1}{2}. \tag{5.62}$$

Appendix C

Letting $F(\beta)$ denote the RHS of (5.43), we show that $F(\beta)$ is a decreasing function of β. Taking the logarithm of $F(\beta)$, total differentiation gives

$$-\frac{1}{\alpha}d\alpha = \left[\frac{1}{\beta} + \left(\frac{\frac{\lambda-1}{\lambda}}{\log\lambda + \frac{\lambda-1}{\lambda}\beta}\right)^2 \beta\right] d\beta > 0. \tag{5.63}$$

The values of the boundary are given by

$$F(0) = \infty \tag{5.64}$$

$$F(1) = \frac{\lambda^{\frac{\lambda}{\lambda-1} - \frac{1}{\log\lambda + \frac{\lambda-1}{\lambda}}}}{\left(\log\lambda + \frac{\lambda-1}{\lambda}\right)} > 0 \tag{5.65}$$

From these facts, we prove that the region of (α, β) is depicted as Figure 4-1.

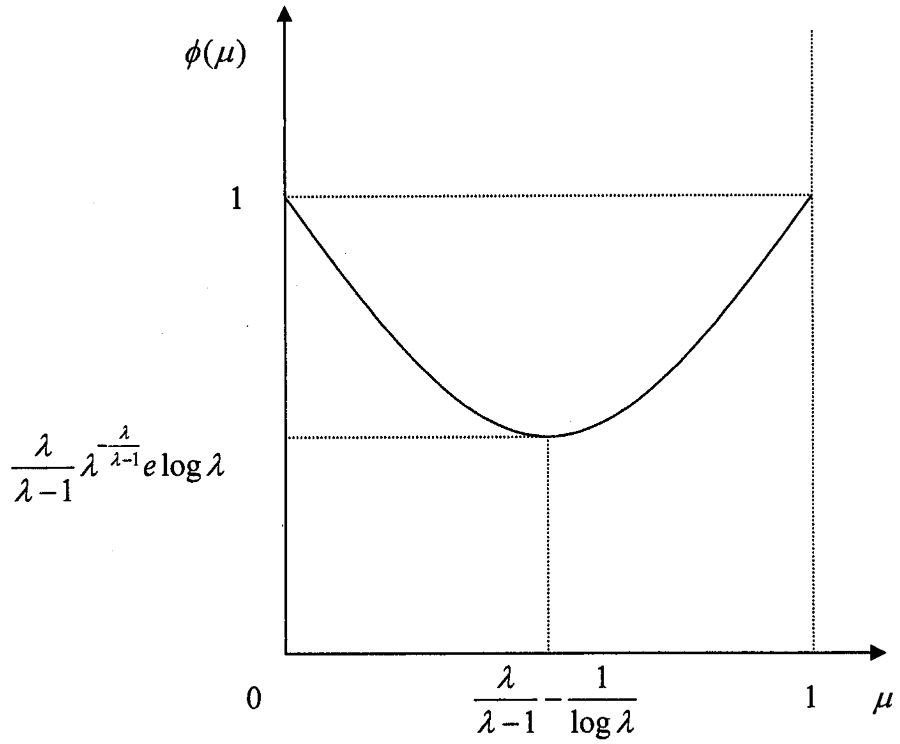

$\phi(\mu)$
1
$\frac{\lambda}{\lambda-1}\lambda^{-\frac{\lambda}{\lambda-1}}e\log\lambda$
0
$\frac{\lambda}{\lambda-1}-\frac{1}{\log\lambda}$
1
μ

Figure 1

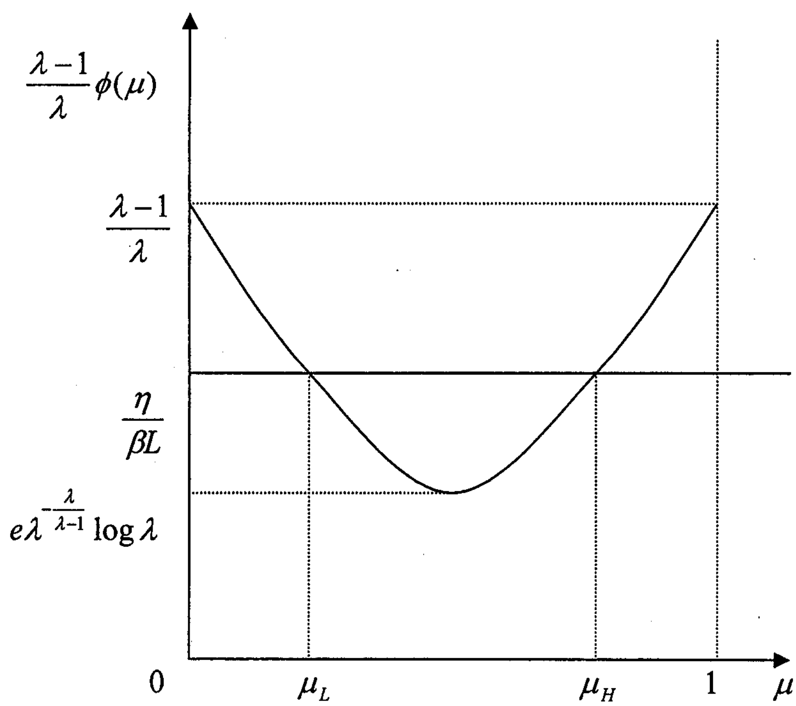

$\frac{\lambda-1}{\lambda}\phi(\mu)$
$\frac{\lambda-1}{\lambda}$
$\frac{\eta}{\beta L}$
$e\lambda^{-\frac{\lambda}{\lambda-1}}\log\lambda$
0
μ_L
μ_H
1
μ

Figure 2

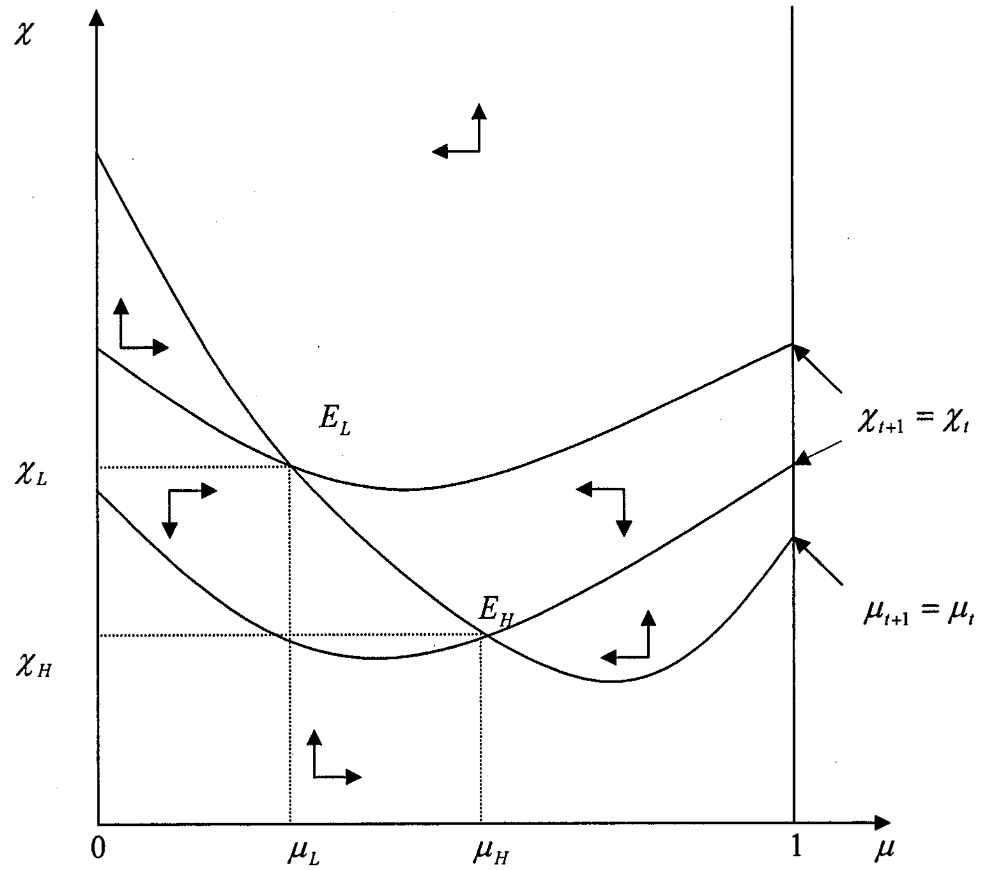


Figure 3

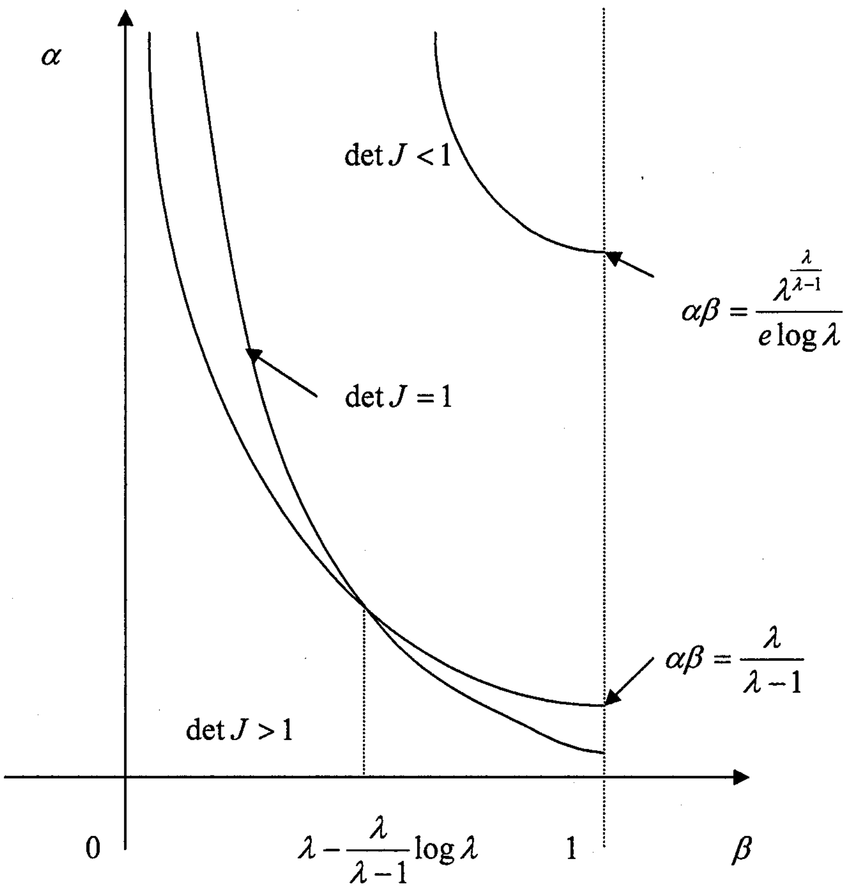

α
$\det J < 1$
$\alpha\beta = \frac{\lambda^{\frac{\lambda}{\lambda-1}}}{e \log \lambda}$
$\det J = 1$
$\alpha\beta = \frac{\lambda}{\lambda-1}$
$\det J > 1$
0
$\lambda - \frac{\lambda}{\lambda-1}\log\lambda$
1
β

Figure 4-1

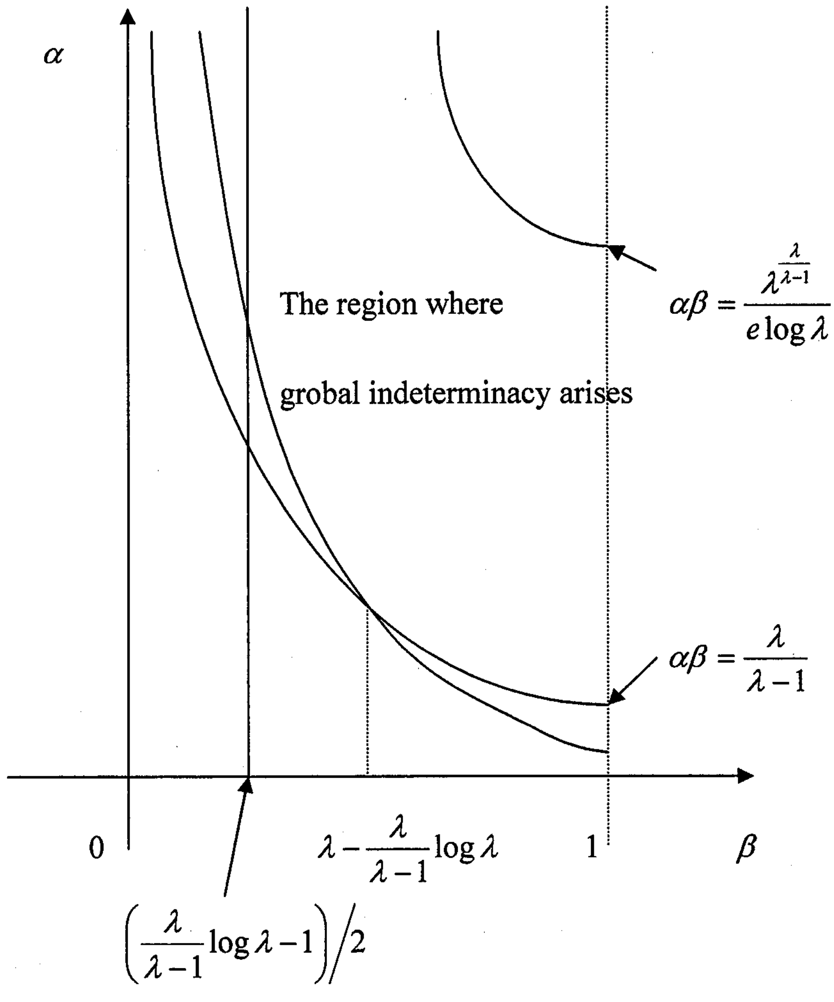

α
The region where
grobal indeterminacy arises
$\alpha\beta = \frac{\lambda^{\frac{\lambda}{\lambda-1}}}{e\log\lambda}$
$\alpha\beta = \frac{\lambda}{\lambda-1}$
0
$\lambda - \frac{\lambda}{\lambda-1}\log\lambda$
1
β
$\left(\frac{\lambda}{\lambda-1}\log\lambda - 1\right)\Big/2$

Figure 4-2

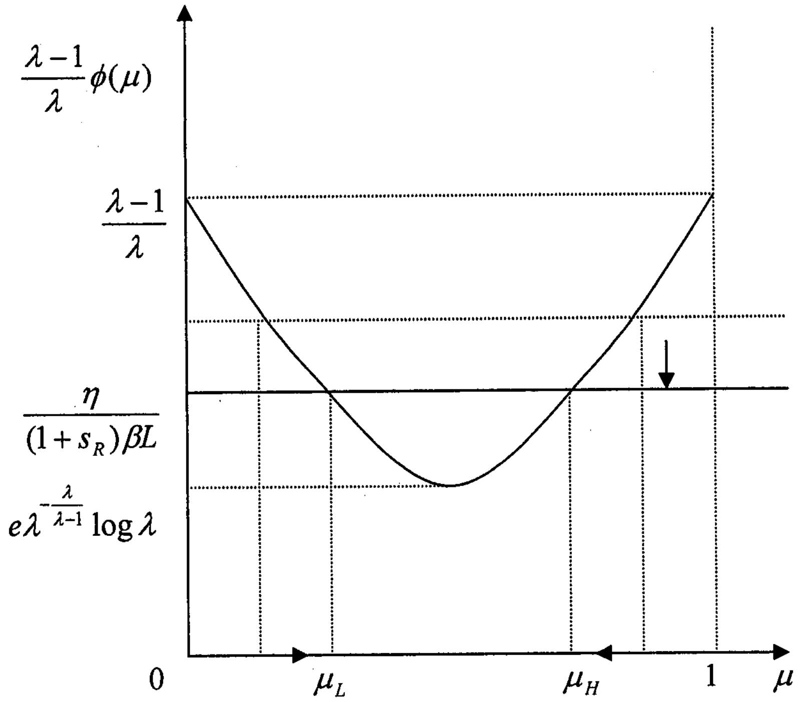

$\frac{\lambda-1}{\lambda}\phi(\mu)$
$\frac{\lambda-1}{\lambda}$
$\frac{\eta}{(1+s_R)\beta L}$
$e\lambda^{-\frac{\lambda}{\lambda-1}}\log\lambda$
0
μ_L
μ_H
1
μ

Figure 5

Chapter 6

Conclusion

This dissertation analyzes innovation, technology choice, and economic growth by using various growth models where technological progress is endogenously determined.

In Chapters 2 and 3, we examine how patent policy affects economic growth and welfare level. Chapter 2 investigates how extending patent length affects economic growth and the welfare level in an endogenous growth model with R&D activities. In contrast to the first study of optimal patent length in a dynamic general equilibrium model, Judd(1985), we show that the patent length that maximizes the social welfare is finite. Moreover, we analyze not only patent length policy but also patent breadth policy. Extending the partial equilibrium analysis of Gilbert and Shapiro(1990) and Tandon(1982) into dynamic general equilibrium analysis, we show that the patent length that maximizes the social welfare is not infinite even if the royalty rate can be controlled. Because Gilbert and Shapiro(1990) show that the optimal patent policy involves infinite patent length, our analysis provides policy implications that are different from theirs.

Chapter 3 develops an endogenous growth model that has two engines of economic growth, innovation and capital accumulation. We investigate how the patent policy affects economic growth in this more general endogenous growth model. In contrast to the growth models with only innovation, stronger patent protection accelerates innovation but discourages capital accumulation in the model of this chapter. Consequently, we show that strengthening patent protection may reduce the growth rate of output and that the growth-maximizing degree of the patent protection is lower than the maximum degree of the patent protection. We also investigate how the patent protection affects social welfare and show that the welfare-maximizing degree of the patent protection is lower than the growth-maximizing degree of the patent protection. As mentioned in this chapter, some papers point out the possibility that tighter intellectual property rights protection may reduce the growth rate of output. In addition to these papers, using the Schumpeterian model of Grossman and Helpman(1991), Horii and Iwaisako(2004) analyze the effect of intellectual property rights protection on economic growth. By the mechanism different from this chapter, we show that tighter intellectual property rights protection may impede economic growth.

Chapter 4 analyzes issues of technology choice. Chapter 4 investigates the equilibrium dynamics of an economy with two technologies: one involves decreasing marginal productivity of capital, and the other involves non-decreasing marginal productivity of capital. This chapter shows that this simple two-period overlapping generations model can generate endogenous cycles, poverty traps, or permanent growth. Consequently, this growth model can explain the observed differences of patterns of growth among countries.

Chapter 5 explores dynamic properties of a Schumpeterian growth model in the environment where innovations are imitated costlessly after one period. This chapter shows that the rate of return of innovation rises as the aggregate investment in R&D increases because of pecuniary externality. Consequently, multiple balanced growth paths can be generated. Furthermore, the model in this chapter can generate indeterminacy of equilibrium paths and growth cycles. Hence the present model can explain the observed differences of growth rates among countries and the observed fluctuations of growth rates.

Some chapters of this dissertation show that we need to analyze the effect of industrial policies such as patent policy by using not the partial equilibrium analysis but the general equilibrium analysis. Actually, some results that we obtain in Chapter 2 are opposite to the results in the partial equilibrium analysis. Why can be two analyses different from each other? It is because patent policies affect not only the incentives of R&D but also market structures, for example, the proportion of monopolistic sectors. If we use the partial equilibrium analysis, changing the market structure does not affect the factor prices. Therefore we cannot analyze the policy effect on R&D activities through the factor prices. On the other hand, as mentioned in Chapter 5, we can analyze this effect if we use the general equilibrium analysis. This dissertation analyzes only patent policies mainly, however, we must analyze the other policies, i.e. regulation of entry, subsidy of R&D, etc.. by using the general equilibrium analysis after this.[1]

[1]Futagami et al.(2004) examine how regulation of entry affects welfare in dynamic general equilibrium environments. In this paper, we show that insufficient entry may occur in the free entry equiribrium. This result is opposite to the results in the partial equilibrium analysis.

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