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# Suppression of $\pi^0$ Condensation in Neutron Star Matter due to Neutron $^3P_2$ Superfluidity


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## Abstract

It is investigated how the occurrence of $\pi^0$ condensation in neutron star matter is affected by the neutron $^3P_2$ superfluidity. Threshold condition is derived by solving the quasi-particle RPA equation describing $\pi^0$-like collective oscillation. Although there exist five possible solutions of the $^3P_2$ gap equation, numerical calculation of the critical density is preferentially carried out for a simple case where the spin and orbital angular momenta of a bound pair are in complete alignment.

It becomes evident that the $\pi^0$ condensation is suppressed by the neutron $^3P_2$ superfluidity. The suppression, however, turns out very modest irrespective of the type of $^3P_2$ superfluid states. In conclusion the neutron $^3P_2$ superfluidity brings about no significant change in the $\pi^0$ condensation threshold predicted so far. This conclusion may be applicable to the case of $\pi^c$ condensation without serious change.

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The numerical calculations were carried out by using NEAC ACOS 900 at the Computation Center, Osaka University.

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## 1. Introduction

An understanding of nuclear matter is important both for the study of nuclear physics and astrophysics. Through the 1950's and 1960's remarkable progress has been made to understand properties of nuclear matter around the normal density ($\rho_0$=0.17fm$^{-3}$); the Brueckner theory revealed that the saturation of normal nuclear matter originates in the singularity of nuclear forces having a repulsive core which cancels its surrounding attractive forces[1]). On the other hand, the discovery of pulsars[2]) and the advent of relativistic heavy-ion accelerators have stimulated the study of nuclear matter at high densities and high temperatures. Up to now various new phenomena, such as pion condensation, neutron $^3P_2$ superfluidity, quantum solidification, abnormal isomers and quark matter, have been predicted to occur such extreme conditions[3,4]). Among them pion condensation and the neutron $^3P_2$ superfluidity have attracted special attention owing to their striking astrophysical implications in connection with neutron star phenomena.

Possibility of pion condensation in neutron stars was first suggested by Migdal[5]) and independently by Sawyer and Scalapino[6]) from theoretical grounds. This phase is expected to be realized beyond the critical density somewhat higher than but not far from $\rho_0$[7]). It is widely accepted that neutron stars cool predominantly through escaping neutrinos in their hot early period. If charged pions ($\pi^c$) are present, the β-decay processes involving pions drastically enhance the cooling rate of young neutron stars, as first pointed out by Bachall and Wolf[8]) and subsequently reexamined by many authors with explicit relevance to $\pi^c$ condensation[9-12]). This cooling mechanism (referred to as the pion cooling) gives rise to an appreciable lowering of the surface temperature of neutron stars untill later stages of their stellar evolution[13-16]. Recent progress in X-ray satellite observation on pulsars (Crab and Vela) and putative neutron stars in supernova remnants (Cas A, Kepler, Tycho, RCW103, SN1006 etc)[17] has yielded interesting upper limits to the surface temperatures of these objects, which are consistent as a whole with standard cooling scenarios without the pion cooling[14-16]. A natural interpretation is the absence of pion condensates in these neutron stars and neutron star candidates. Tatsumi[18] and Frukawa[19], however, pointed out that development of a typical $\pi^0$ condensate induces reduction of the phase space available for neutrino processes, and hence tends to retard the cooling of neutron stars. In addition, quite recently the pion cooling was shown to be less effective in a realistic situation[20]. These theoretical consequences lead to the fascinating interpretation that the observational data indicate the coexistence of $\pi^c$ and $\pi^0$ condensates. The attractive nature of the $\pi N$ p-wave interaction, which is the driving force of pion condensation, favors the coexistence of both types of condensates[21]. A definite answer to this question must wait for further investigation both on the theoretical and observational side.

It is also of astrophysical interest that pion condensation softens the equation of state for neutron star matter. The ranges of masses and moments of inertia of neutron stars, which are the most accessible mechanical parameters, are sensitive to details of the equation of state at densities where pion condensation is likely to occur. In particular, knowledge of the maximum allowable neutron star mass is an important ingredient in attempts to identify black holes from measurement of masses of compact objects. According to several model calculations undertaken by now[22-24]), the $\pi^c$ condensation, if realized, slightly decreases the maximum values of mass and moment of inertia allowable for neutron stars. This implies, although the calculations are rather preliminary due to various uncertainties, that it may be difficult by observations of masses and moments of inertia to identify the effect of the $\pi^c$ condensation. On the other hand, the $\pi^0$ condensation would have a more striking influence upon these parameters because its realization corresponds to a drastic structure change of the nucleon system characterized by remarkable localization[25-26]). Furthermore, such a localization of nucleons could produce a solid-like core in the deep interior of neutron stars, and thus could explain the Vela-pulsar glitch and the Her X-1 star high-low cycle phenomena[27]). It seems therefore premature at the present stage to say something definite about the evidence of pion condensation in neutron stars.

Possibility of another new phase, the neutron $^3P_2$ superfluid state, was noticed soon after the discovery of pulsars in 1967. In contrast to the case of pion condensation, the observation of macroscopic time scales for glitch relaxation of pulsars offers the convincing proof that some part of neutron star interior exhibits the $^3P_2$ superfluidity[28]). Besides this observational implication, the neutron $^3P_2$ superfluidity has substential relevance to the neutron star cooling mainly through reduction of the neutron specific heat [13,29]). A conclusive calculation of the $^3P_2$ energy gap indicates that this phase can exist in the density region $\rho_0<\rho<3\rho_0$[30]). Such densities significantly overlap with densities where pion condensation is predicted to occur. A question then arises: How is the occurrence of pion condensation affected by the neutron $^3P_2$ superfluidity?

Pion condensation and the neutron $^3P_2$ superfluidity characterize the phase of nuclear matter at neutron star densities. Because of their observational bearings on the cooling scenario and rotational dynamics of neutron stars, interrelation between these phenomena ( coexistence or competition) is considered to be the most promising subject of study which may provide a criterion for establishing the existence of pion condensates in neutron stars. Studies of the interrelation problem have been initiated by the Kyoto group, who investigated whether or not the neutron $^3P_2$ superfluidity persists under well-developed pion condensates[31]). Their conclusions are as follows: (i)The superfluidity can persist under the typical $\pi^0$ condensate described by the ALS model[25]), although the superfluidity in this phase becomes essentially of two-dimensional character[31a, 31b]). (ii)Its persistence is delicate under a $\pi^c$ condensate because the mixture of nucleon isospin states diminishes the pairing correlation[31c]). (iii)Such a diminishing effect is almost compensated by an enhancement due to a large nucleon effective mass if the $\pi^0$ condensation is coexistent with the $\pi^c$ one[31d]). Although observational significance of these results was extensively discussed, any positive or negative evidence of pion condensation has not been extracted yet[32]). Such a situation motivates us to study further the interrelation problem in different aspects.

The aim of this paper is to find out to what extent the occurrence of pion condensation in neutron star matter is affected by the neutron $^3P_2$ superfluidity. In the superfluid phase the nucleon single-particle spectrum is modified to have a gap, and the Fermi surface is difused. One needs to include these modifications properly. We concentrate on the critical density and threshold condition for the $\pi^0$ condensation where the condensate frequency is equal to zero. Our formulation is based on the method of normal mode within the range of the random phase approximation. There is a very important relation between stability of a Hartree-Fock type state of many-body systems and solutions of the RPA equation describing some kind of collective oscillation; if the RPA equation has an imaginary solution, the Hartree-Fock type state becomes unstable for an infinitismal deformation generated by the collective oscillation[33]). This criterion of instability has already applied to the problem of pion condensation in normal neutron star matter[34-36]). The superfluid case can be formulated in a similar line of approach; threshold condition is derived from the RPA equation describing $\pi^0$-like collective oscillation. The resulting threshold condition has a familiar form if we rewrite it in terms of the $\pi^0$ polarization operator which reflects the modifications induced by the superfluidity. This polarization operator is evaluated directly with the Feynman rules associated with nucleon Green's functions. Such an evaluation enables us to examine properties and physical meaning of the $\pi^0$ polarization operator in the superfluid phase.

There exist five types of solutions of the $^3P_2$ gap equation[37]). The $^3P_2$ gap equation is a coupled integral equation for five gap parameters $\Delta_{m_J}(q)$ $(m_J=0,\pm1,\pm2)$,

$$\Delta_{m_J}(q)= - \frac{1}{\pi}\int d^3q' V_\lambda(q,q')\sum_\mu \frac{\Delta_\mu(q')}{\sqrt{\tilde{\varepsilon}^2_{q'}+D^2(\vec{q}\,')}} \mathrm{Tr}[G_\mu(\hat{q}')G^*_{m_J}(\hat{q}')] , \tag{1.1}$$

with

$$[G_{m_J}(\hat{q})]_{\sigma\sigma'}=(\tfrac{1}{2}\ \tfrac{1}{2}\sigma\sigma'|Sm_S)(SLm_Sm_L|Jm_J)Y_{Lm_L}(\hat{q}) \ , \tag{1.2}$$

$$V_\lambda(q,q')=\sum_{spin}\int d^3rj_L(qr)\Psi^*_{\lambda m_J}(1,2)V(1,2)j_L(q'r)\Psi_{\lambda m_J}(1,2) \ . \tag{1.3}$$

Here $\tilde{\varepsilon}_q=(q^2-q_F^2)/2M^*$ with $q_F$ and $M^*$ being the Fermi momentum and nucleon effective mass, $D(\vec{q})$ is the energy gap, V(1,2) is the two-nucleon potential and $\vec{r}$ is the relative coordinate between two nucleons with momenta $(\vec{q},-\vec{q})$.

$$\Psi_{\lambda m_J}(1,2)=\sum_{m_J=m_L+m_S}(SLm_Sm_L|Jm_J)Y_{Lm_L}(\hat{r})\chi_{Sm_S}(1,2) \tag{1.4}$$

denotes the spin-orbital angular part of the wave function for a nucleon pair. The simbol $\lambda$ indecates a set of quantum numbers of spin (S=1), orbital (L=1) and total (J=2) angular momenta. The energy gap is related to the gap parameters by

$$D^2(\vec{q})=\frac{1}{2}\mathrm{Tr}[\Delta^\dagger(\vec{q})\Delta(\vec{q})] \ , \tag{1.5}$$

with

$$\Delta(\vec{q})\equiv\sum_{m_J}\Delta_{m_J}(q)G_{m_J}(\hat{q}) \ . \tag{1.6}$$

Solutions of eq. (1.1) are classified according to which of the gap parameters are dominant.

Sol. 1: the solution only with $m_J=\pm2$ gap parameters.

Sol. 2: the solution only with $m_J=0$ gap parameter.

These two simple cases were first discussed by Tamagaki (Sols. 1 and 2)[38]) and by Hoffberg et al. (Sol. 2)[39]). Their energy gaps are axially symmetric and deformations of the Fermi sphere are, respectively, oblate and prolate with respect to the axis of spin quantization, as schematically illustrated in Fig. 1.

Sol. 3: the solution where $m_J=0,\pm 2$ gap parameters are coupled.

Sol. 4: the solution where $m_J=\pm 1$ gap parameters are coupled.

Sol. 5: the solution where $m_J=0,\pm 1,\pm 2$ gap parameters are coupled.

These solutions have axially asymmetric energy gaps characterized by complicated angle dependence. The energy gap of Sol. 4 has nodes, while Sols. 3 and 5 are nodeless. The most general self-consistent solution, Sol. 5, is considered to represent the ground state of neutron star matter in the density region considered here, although all the types of solutions (Sols. 1∿5) are almost degenerate.

We calculate the critical density for the $\pi^0$ condensation realized from Sol. 1 where the spin and orbital angular momenta of a bound pair are in complete alignment. In the numerical calculation we take account of two additional effects arising from the $\Delta_{33}(1236)$ isobar and NN short-range correlations.

The contents of this thesis are as follows: The next section is devoted to the derivation of the threshold condition. In sec. 3 the $\pi^0$ polarization operator is evaluated with the Feynman rules and its properties are discussed. Numerical results are shown and discussed in sec. 4. The last section is for concluding remarks.

## 2. Derivation of the threshold condition

### 2.1. HAMILTONIAN

As far as the $\pi^0$ condensation is concerned, protons admixed in neutron star matter can be disregarded because of small admixture (several percent). We start with the following Hamiltonian for the neutron + $\pi^0$ system ($\hbar$=c=pion mass=1):

$$H=H_N^0+H_\pi^0+H_{NN}^\lambda+H_{\pi N} \quad , \tag{2.1}$$

$$H_N^0=\sum_{q\alpha}\tilde{\varepsilon}_q n_{q\alpha}^\dagger n_{q\alpha} \quad , \tag{2.2}$$

$$H_\pi^0=\sum_p \omega_p a_p^\dagger a_p \quad , \tag{2.3}$$

$$H_{NN}^\lambda=\frac{(4\pi)^2}{\Omega}\sum_{qq'}V_\lambda(q,q')\sum_{m_J} b_{\lambda m_J}^\dagger(q)b_{\lambda m_J}(q') \quad , \tag{2.4}$$

$$H_{\pi N}=\sum_{pq}\sum_{\alpha\beta} f_p(\vec{\sigma}\cdot\hat{p})_{\alpha\beta} n_{q\alpha}^\dagger n_{q-p\beta}(a_p+a_{-p}^\dagger) \quad . \tag{2.5}$$

Here $\omega_p=\sqrt{p^2+1}$ , $f_p=-ifp/\sqrt{2\Omega\omega_p}$ with f and $\Omega$ being the $\pi$N p-wave coupling constant ($f^2/4\pi=0.08$) and normalization volume, $\vec{\sigma}$ are the Pauli spin matrices.

$$b_{\lambda m_J}^\dagger(q)=\frac{1}{\sqrt{2}}\sum_{\sigma\sigma'}\int d\hat{q}[G_{m_J}(\hat{q})]_{\sigma\sigma'} n_{q\sigma}^\dagger n_{-q\sigma'}^\dagger \tag{2.6}$$

is the boson operator representing a neutron pair with relative momentum q and angular momentum quantum numbers $\lambda$ and $m_J$. The

Hamiltonians $H_N^0$ and $H_\pi^0$ stand for the neutron and pion free parts, while $H_{NN}^\lambda$ and $H_{\pi N}$ correspond to the $^3P_2$ pairing and $\pi N$ p-wave interactions, respectively. We may treat these interactions separately because the most responsible for the pairing interaction is the spin-orbit force due to exchange of heavy mesons, such as "$\sigma$", $\omega$ and $\rho$[38]).

Suppose now that the pairing interaction is dominant and the system is in the $^3P_2$ superfluid state. The Hamiltonian H should be then rewritten in terms of quasi-particle operators. This is done by making use of the generalized Bogoliubov transformation introduced by Tamagaki[38]),

$$\left\{\begin{array}{l} \tilde{n}_{q\alpha}=\sum_\beta [U_{\alpha\beta}(\vec{q})n_{q\beta}-V_{\alpha\beta}(\vec{q})n^\dagger_{-q\beta}] , \\ \tilde{n}^\dagger_{q\alpha}=\sum_\beta [U^*_{\alpha\beta}(\vec{q})n^\dagger_{q\beta}-V^*_{\alpha\beta}(\vec{q})n_{-q\beta}] , \end{array}\right. \tag{2.7}$$

where $\tilde{n}_{q\alpha}$ and $\tilde{n}^\dagger_{q\alpha}$ are the quasi-particle operators, $U_{\alpha\beta}(\vec{q})$ and $V_{\alpha\beta}(\vec{q})$ are elements of the $2\times2$ transformation matrices $U(\vec{q})$ and $V(\vec{q})$. Properties of these matrices are summarized in Appendix A. The transform of H is

$$\tilde{H}=\tilde{H}_N^0+H_\pi^0+\tilde{H}_{\pi N} , \tag{2.8}$$

$$\tilde{H}_N^0=\sum_{q\alpha} E_q \tilde{n}^\dagger_{q\alpha}\tilde{n}_{q\alpha}+K_0 , \tag{2.9}$$

where $E_q=\sqrt{\tilde{\varepsilon}_q^2+D^2(\vec{q})}$ and $K_0$ denotes the ground state energy of the neutron system. Although $\tilde{H}_{\pi N}$ contains four distinct terms with respect to the quasi-particle operators, only the terms which couple pions to quasi-particle pair states are relevant to our discussion (see Fig. 2);

$$\tilde{H}_{\pi N} \sim \sum_{pq} \sum_{spin} f_p (\vec{\sigma}\cdot\hat{p})_{\alpha\gamma} [U^T_{\alpha\beta}(\vec{q}) V_{\gamma\delta}(\vec{q}-\vec{p}) \tilde{n}^\dagger_{q\beta} \tilde{n}^\dagger_{p-q\delta}$$

$$+ V^\dagger_{\alpha\beta}(\vec{q}) U_{\gamma\delta}(\vec{q}-\vec{p}) \tilde{n}_{-q\beta} \tilde{n}_{q-p\delta}] (a_p + a^\dagger_{-p}) \ , \qquad (2.10)$$

where $U^T(\vec{q})$ means the transpose of $U(\vec{q})$.

## 2.2. PIONIC INSTABILITY OF THE SYSTEM

Since the Hamiltonian $\tilde{H}_{\pi N}$ couples pions to the quasi-particle pair states with the quantum numbers of the $\pi^0$, the system may develops a $\pi^0$-like collective oscillation. In order to describe such a kind of oscillation, we introduce the following eigenmode operator:

$$S^\dagger(\vec{k})=S_\pi^\dagger(\vec{k})+S_N^\dagger(\vec{k}) \ , \tag{2.11}$$

$$S_\pi^\dagger(\vec{k})=\xi(\vec{k})a_k^\dagger+\eta(\vec{k})a_{-k} \ , \tag{2.12}$$

$$S_N^\dagger(\vec{k})=\sum_q \sum_{spin}(\vec{\sigma}\cdot\hat{k})_{\alpha\gamma}[\zeta_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k})\tilde{n}_{q\beta}^\dagger\tilde{n}_{k-q\delta}^\dagger$$
$$+\chi_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k})\tilde{n}_{-q\beta}\tilde{n}_{q-k\delta}] \ . \tag{2.13}$$

Its corresponding eigenfrequency is determined from the RPA equation

$$[\tilde{H},S^\dagger(\vec{k})]=\omega S^\dagger(\vec{k}) \ . \tag{2.14}$$

The relevant nonvanishing commutators are evaluated to be

$$[\tilde{H}_\pi^0,S_\pi^\dagger(\vec{k})]=\omega_k(\xi(\vec{k})a_k^\dagger-\eta(\vec{k})a_{-k}) \ , \tag{2.15}$$

$$[\tilde{H}_N^0,S_N^\dagger(\vec{k})]=\sum_q \sum_{spin}(\vec{\sigma}\cdot\hat{k})_{\alpha\gamma}(E_q+E_{q-k})$$
$$\times[\zeta_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k})\tilde{n}_{q\beta}^\dagger\tilde{n}_{k-q\delta}^\dagger-\chi_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k})\tilde{n}_{-q\beta}\tilde{n}_{q-k\delta}] \ , \tag{2.16}$$

$$[\tilde{H}_{\pi N}, S_{\pi}^{\dagger}(\vec{k})] = (\xi(\vec{k}) - \eta(\vec{k})) f_k \sum_q \sum_{spin} (\vec{\sigma}\cdot\hat{k})_{\alpha\gamma}$$

$$\times [U^T_{\alpha\beta}(\vec{q}) V_{\gamma\delta}(\vec{q}-\vec{k}) \tilde{n}^{\dagger}_{q\beta} \tilde{n}^{\dagger}_{k-q\delta} + V^{\dagger}_{\alpha\beta}(\vec{q}) U_{\gamma\delta}(\vec{q}-\vec{k}) \tilde{n}_{-q\beta} \tilde{n}_{q-k\delta}] \ , \qquad (2.17)$$

$$[\tilde{H}_{\pi N}, S_{N}^{\dagger}(\vec{k})] = (a_{-k} + a_k^{\dagger}) f_k \sum_q \sum_{spin} (\vec{\sigma}\cdot\hat{k})_{\alpha\gamma} (\vec{\sigma}\cdot\hat{k})_{\mu\rho}$$

$$\times [\chi_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k}) (U^T_{\mu\beta}(\vec{q}) V_{\rho\delta}(\vec{q}-\vec{k}) + U^T_{\mu\delta}(\vec{q}-\vec{k}) V_{\rho\beta}(\vec{q}))$$

$$-\zeta_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k}) (V^{\dagger}_{\mu\delta}(\vec{q}-\vec{k}) U_{\rho\beta}(\vec{q}) + V^{\dagger}_{\mu\beta}(\vec{q}) U_{\rho\delta}(\vec{q}-\vec{k}))] \ . \qquad (2.18)$$

Combining eqs. (2.11)∿(2.18), we obtain a set of equations

$$(\xi(\vec{k}) - \eta(\vec{k})) f_k U^T_{\alpha\beta}(\vec{q}) V_{\gamma\delta}(\vec{q}-\vec{k})$$

$$+ (E_q + E_{q-k} - \omega) \zeta_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k}) = 0 \ , \qquad (2.19)$$

$$(\xi(\vec{k}) - \eta(\vec{k})) f_k V^{\dagger}_{\alpha\beta}(\vec{q}) U_{\gamma\delta}(\vec{q}-\vec{k})$$

$$- (E_q + E_{q-k} + \omega) \chi_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k}) = 0 \ , \qquad (2.20)$$

$$(\omega_k - \omega) \xi(\vec{k}) + \Theta(\vec{k}) = 0 \ , \qquad (2.21)$$

$$(\omega_k+\omega)\eta(\vec{k})-\Theta(\vec{k})=0 \ , \tag{2.22}$$

with

$$\Theta(\vec{k})=f_k \sum_q \sum_{spin} (\vec{\sigma}\cdot\hat{k})_{\alpha\gamma}(\vec{\sigma}\cdot\hat{k})_{\mu\rho}[\chi_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k})$$

$$\times(U^T_{\mu\beta}(\vec{q})V_{\rho\delta}(\vec{q}-\vec{k})+U^T_{\mu\delta}(\vec{q}-\vec{k})V_{\rho\beta}(\vec{q})-\zeta_{\alpha\beta,\gamma\delta}(\vec{q};\vec{k})$$

$$\times(V^\dagger_{\mu\delta}(\vec{q}-\vec{k})U_{\rho\beta}(\vec{q})+V^\dagger_{\mu\beta}(\vec{q})U_{\rho\delta}(\vec{q}-\vec{k}))] \ . \tag{2.23}$$

These equations lead to the eigenvalue equation for $\omega$

$$1=\frac{2\omega_k f_k^2}{\omega^2-\omega_k^2}\sum_q \sum_{spin} (\vec{\sigma}\cdot\hat{k})_{\alpha\gamma}(\vec{\sigma}\cdot\hat{k})_{\mu\rho}[\frac{V^\dagger_{\alpha\beta}(\vec{q})U_{\gamma\delta}(\vec{q}-\vec{k})}{E_q+E_{q-k}+\omega}$$

$$\times(U^T_{\mu\beta}(\vec{q})V_{\rho\delta}(\vec{q}-\vec{k})+U^T_{\mu\delta}(\vec{q}-\vec{k})V_{\rho\beta}(\vec{q}))+\frac{U^T_{\alpha\beta}(\vec{q})V_{\gamma\delta}(\vec{q}-\vec{k})}{E_q+E_{q-k}-\omega}$$

$$\times(V^\dagger_{\mu\delta}(\vec{q}-\vec{k})U_{\rho\beta}(\vec{q})+V^\dagger_{\mu\beta}(\vec{q})U_{\rho\delta}(\vec{q}-\vec{k}))] \ . \tag{2.24}$$

After some manipulations, eq. (2.24) is reduced to the compact form

$$1=\frac{f^2k^2}{(\omega^2-\omega_k^2)\Omega}\sum_q \frac{(E_q+E_{q-k})Tr[W^\dagger(\vec{q};\vec{k})W(\vec{q};\vec{k})]}{\omega^2-(E_q+E_{q-k})^2} \ , \tag{2.25}$$

with

$$W(\vec{q};\vec{k})=U(\vec{q}-\vec{k})(\vec{\sigma}\cdot\hat{k})V(\vec{q})+[U(\vec{q})(\vec{\sigma}\cdot\hat{k})V(\vec{q}-\vec{k})]^T . \qquad (2.26)$$

A straightforward calculation yields

$$\mathrm{Tr}[W^\dagger(\vec{q};\vec{k})W(\vec{q};\vec{k})]=(1-\tilde{\varepsilon}_q\tilde{\varepsilon}_{q-k}/E_qE_{q-k})$$
$$+\mathrm{Tr}[(\vec{\sigma}\cdot\hat{k})^T\Delta^\dagger(\vec{q}-\vec{k})(\vec{\sigma}\cdot\hat{k})\Delta(\vec{q})]/2E_qE_{q-k} , \qquad (2.27)$$

where $\Delta(\vec{q})$ is the 2×2 gap matrix defined by eq. (1.6).

Eq. (2.25) indicates that the $\pi^0$ polarization operator in the superfluid phase should be defined as follows:

$$\tilde{\Pi}(\vec{k},\omega)\equiv \frac{f^2k^2}{\Omega}\sum_q \frac{(E_q+E_{q-k})\mathrm{Tr}[W^\dagger(\vec{q};\vec{k})W(\vec{q};\vec{k})]}{\omega^2-(E_q+E_{q-k})^2} . \qquad (2.28)$$

Eq. (2.25) is then expressed as

$$1=\frac{\tilde{\Pi}(\vec{k},\omega)}{\omega^2-\omega_k^2} . \qquad (2.29)$$

Validity of this definition will be verified in the next section.

Owing to $\mathrm{Tr}[W^\dagger(\vec{q};\vec{k})W(\vec{q};\vec{k})]\geqq 0$, eq. (2.29) may have an imaginary solution when the following condition is satisfied:

$$1\leqq -\tilde{\Pi}(\vec{k},0)/\omega_k^2 . \qquad (2.30)$$

In this case the system is no longer stable for the $\pi^0$-like collective oscillation; the $\pi^0$ condensation develops. It should be noted that the eigenvalue equation (2.29) and threshold condition (2.30) are very similar to those in the normal phase (Fermi gas phase),

$$1 = \frac{\Pi(\vec{k},\omega)}{\omega^2-\omega_k^2} , \tag{2.31}$$

$$1 \leqq -\Pi(\vec{k},0)/\omega_k^2 , \tag{2.32}$$

with

$$\Pi(\vec{k},\omega) = \frac{2(fk)^2}{\Omega} \sum_q \frac{\Theta(q_F-|\vec{q}-\vec{k}|)-\Theta(q_F-q)}{\omega+\tilde{\varepsilon}_{q-k}-\tilde{\varepsilon}_q} , \tag{2.33}$$

where $\Theta(q_F-q)$ is the neutron occupation function corresponding to the sharp Fermi sea. This means that the $\pi^0$ polarization operator given by eq. (2.28) includes all the modifications induced by the superfluidity.

## 2.3. INCLUSION OF THE $\Delta_{33}(1236)$ ISOBAR AND NN SHORT-RANGE CORRELATIONS

In the above treatment two important contributions are left out; the one coming from the $\Delta_{33}(1236)$ isobar and the other coming from the NN short-range correlations.

Fig. 3 illustrates pion self-energy processes in the normal phase via the isobar, which greatly reduce the energy of pions in matter, and hence encourage pion condensation. The contribution of these processes is evaluated to be

$$\Pi_{res}(\vec{k},\omega)= \frac{16(fk)^2}{9\Omega} \sum_q \left[\frac{\Theta(q_F-q)}{\omega-(2\omega+\omega_\Delta+(\vec{q}-\vec{k})^2/2M_\Delta-q^2/2M^*)} + \frac{\Theta(q_F-q)}{\omega-(\omega_\Delta+(\vec{q}+\vec{k})^2/2M_\Delta-q^2/2M^*)}\right] , \qquad (2.34)$$

where $\omega_\Delta$(=296MeV) is the mass difference between the isobar and neutron. For $|\omega\pm\omega_\Delta|>>|(\vec{q}\mp\vec{k})^2/2M_\Delta-q^2/2M^*|$ we may neglect the isobar and neutron kinetic energies. Then we have

$$\Pi_{res}(\vec{k},\omega)= \frac{32}{9}(fk)^2\rho\frac{\omega_\Delta}{\omega^2-\omega_k^2} , \qquad (2.35)$$

where $\rho$ is the neutron density. Under the same condition eq. (2.35) turns out to represent the isobar contribution in the superfluid phase without any modification because $\omega_\Delta$ is much larger than the energy gap existing in the quasi-particle spectrum $E_q$.

The NN short-range correlations in the one-pion channel arise mainly from the repulsive core of nuclear forces and exchange of ρ-mesons. Their effects are usually simulated by a zero-range effective interaction of the form[4,7]),

$$H_{NN}=\frac{1}{2}g'(\psi^{\dagger}\sigma_i\tau_i\psi)(\psi^{\dagger}\sigma_i\tau_i\psi) \quad . \tag{2.36}$$

Here g' is the density- and momentum-independent coupling constant (g'>0). Due to this repulsive interaction, the quasi-particle pair exitations undergo repeated scatterings, which raise the energies of the pair states, and hence prevent pion condensation. Such scattering processes induce renormalization of the $\pi^0$ polarization operator, as illustrated in Fig. 4. The matrix element of eq. (2.36) coupling the pair states with momentum $\vec{k}$ is

$$\langle\vec{q}',\vec{k}-\vec{q}'|H_{NN}|\vec{q},\vec{k}-\vec{q}\rangle=g' \quad . \tag{2.37}$$

Thus, the renormalized $\pi^0$ polarization operator is given by

$$\tilde{\Pi}^R(\vec{k},\omega)=\tilde{\Pi}(\vec{k},\omega)+\tilde{\Pi}(\vec{k},\omega)[g'/(fk)^2]\tilde{\Pi}^R(\vec{k},\omega) \quad , \tag{2.38}$$

or

$$\tilde{\Pi}^R(\vec{k},\omega)=\frac{\tilde{\Pi}(\vec{k},\omega)}{1-g'\tilde{\Pi}(\vec{k},\omega)/(fk)^2} \quad . \tag{2.39}$$

We apply the same renormalization to the isobar contribution.

Combining these calculations, we find that the original threshold condition (2.30) becomes

$$1 \leqq -\tilde{\Pi}^R_{tot}(\vec{k},0)/\omega_k^2 \ , \tag{2.40}$$

with

$$\tilde{\Pi}^R_{tot}(\vec{k},\omega) = \frac{\tilde{\Pi}(\vec{k},\omega)+\Pi_{res}(\vec{k},\omega)}{1-g'[\tilde{\Pi}(\vec{k},\omega)+\Pi_{res}(\vec{k},\omega)]/(fk)^2} \ , \tag{2.41}$$

where $\Pi_{res}(\vec{k},\omega)$ is given by eq. (2.35).

## 3. The $\pi^0$ polarization operator in the superfluid phase

### 3.1. EVALUATION OF THE $\pi^0$ POLARIZATION OPERATOR BASED ON THE FEYNMAN RULES

The $\pi^0$ polarization operator $\tilde{\Pi}(\vec{k},\omega)$ defined by eq. (2.28) consists of two pieces $\tilde{\Pi}_n(\vec{k},\omega)$ and $\tilde{\Pi}_a(\vec{k},\omega)$;

$$\tilde{\Pi}(\vec{k},\omega)=\tilde{\Pi}_n(\vec{k},\omega)+\tilde{\Pi}_a(\vec{k},\omega) \ , \tag{3.1}$$

with

$$\tilde{\Pi}_n(\vec{k},\omega)= \frac{f^2k^2}{\Omega} \sum_q \frac{(E_q+E_{q-k})(1-\tilde{\varepsilon}_q\tilde{\varepsilon}_{q-k}/E_qE_{q-k})}{\omega^2-(E_q+E_{q-k})^2} \ , \tag{3.2}$$

$$\tilde{\Pi}_a(\vec{k},\omega)= \frac{f^2k^2}{\Omega} \sum_q \frac{(E_q+E_{q-k})\mathrm{Tr}[(\vec{\sigma}\cdot\hat{k})^T\Delta^\dagger(\vec{q}-\vec{k})(\vec{\sigma}\cdot\hat{k})\Delta(\vec{q})]}{2E_qE_{q-k}[\omega^2-(E_q+E_{q-k})^2]} \ . \tag{3.3}$$

We first verify that $\tilde{\Pi}_n(\vec{k},\omega)$ and $\tilde{\Pi}_a(\vec{k},\omega)$ represent the proper self-energies indicated in Fig. 5. The indicated diagrams involve three distinct particle-lines, which are, respectively, associated with the nucleon Green's functions $F^\dagger(q)$, $F(q)$ and $G(q)$, as shown in Fig. 6. These Green's functions are derived from generalized Gor'kov equations for a nonzero angular momentum pairing[40,41]. Details of the derivation are presented in Appendix B. The Feynman rules yield the expressions

$$\tilde{\Pi}_n(k)= -i \frac{f^2|\vec{k}|^2}{\Omega} \sum_q \int \frac{dq_0}{2\pi} \mathrm{Tr}[(\vec{\sigma}\cdot\hat{k})G(q)(\vec{\sigma}\cdot\hat{k})G(q)] \ , \tag{3.4}$$

$$\tilde{\Pi}_a(k) = -i \frac{f^2|\vec{k}|^2}{\Omega} \sum_q \int \frac{dq_0}{2\pi} \mathrm{Tr}[(\vec{\sigma}\cdot\hat{k})^T F^\dagger(k-q)(\vec{\sigma}\cdot\hat{k})F^T(q)] \ , \quad (3.5)$$

where k and q stand for the four momenta $(\vec{k},k_0)$ and $(\vec{q},q_0)$. In writing down eq. (3.5), we have used the relation

$$(\vec{\sigma}\cdot\hat{k})_{\gamma\beta}F^\dagger_{\gamma\delta}(k-q)(\vec{\sigma}\cdot\hat{k})_{\delta\alpha}F_{\beta\alpha}(q)$$
$$= \mathrm{Tr}[(\vec{\sigma}\cdot\hat{k})^T F^\dagger(k-q)(\vec{\sigma}\cdot\hat{k})F^T(q)] \ .$$

Inserting eqs. (B.17)~(B.19) into eqs. (3.4) and (3.5), and performing the integrals with respect to $q_0$, we readily obtain eqs. (3.2) and (3.3). It is reasonable to refer to $\tilde{\Pi}_n(\vec{k},\omega)$ and $\tilde{\Pi}_a(\vec{k},\omega)$ as the normal and abnormal terms, respectively.

3.2. PROPERTIES OF THE NORMAL AND ABNORMAL TERMS

Anisotropy is characteristic of the $^3P_2$ energy gap. Concerning the normal term, such an anisotropy is smoothed out by the summation with respect to $\vec{q}$. Consequently the normal term depends only on an angular average of the energy gap,

$$\bar{D}^2(q) \equiv \int d\hat{q}\ D^2(q,\hat{q}) \ , \tag{3.6}$$

at $q \simeq q_F$. Here it should be noted that the averaged energy gaps $\bar{D}^2(q_F)$ are almost equal for five possible solutions. For example,

$$\begin{aligned}
&\text{Sol. 1:} \qquad \bar{D}^2(q_F)=3.37\ (\text{MeV})^2,\\
&\text{Sol. 2:} \qquad \bar{D}^2(q_F)=3.37\ (\text{MeV})^2,\\
&\text{Sol. 3:} \qquad \bar{D}^2(q_F)=3.48\ (\text{MeV})^2,\\
&\text{Sol. 4:} \qquad \bar{D}^2(q_F)=3.48\ (\text{MeV})^2,\\
&\text{Sol. 5:} \qquad \bar{D}^2(q_F)=3.48\ (\text{MeV})^2,
\end{aligned} \tag{3.7}$$

at $E_F$=100MeV[37]). This implies that $\tilde{\Pi}_n(\vec{k},\omega)$ is independent of $\hat{k}$, and moreover its magnitude is almost independent of the type of solutions for the $^3P_2$ gap equation. Setting the energy gaps equal to zero in eq. (3.2), we can easily see that the normal term is reduced to the ordinary $\pi^0$ polarization operator in the normal phase given by eq. (2.33).

The abnormal term is a consequence of the nonconservation of neutrons, and thus vanishes in the normal phase. Since $\tilde{\Pi}_a(\vec{k},\omega)$ reflects the anisotropic feature of the $^3P_2$ superfluid state through the factor $Tr[(\vec{\sigma}\cdot\hat{k})^T\Delta^\dagger(\vec{q}-\vec{k})(\vec{\sigma}\cdot\hat{k})\Delta(\vec{q})]$, it explicitly depends on the direction of $\vec{k}$.

It is worthwhile to estimate the contribution of this novel term to the $\pi^0$ condensation threshold. Roughly speaking, eqs. (3.2) and (3.3) yield

$$\tilde{\Pi}_n(\vec{k},0)\propto\rho \quad , \tag{3.8}$$

$$\tilde{\Pi}_a(\vec{k},0)\propto\rho[D(q_F)/E_F]^2 \quad , \tag{3.9}$$

where $E_F$ is the Fermi energy. These relations of proportionality imply that the normal term is associated with the behavior of all neutrons in the system, whereas the abnormal term only with that of paired neutrons near the Fermi surface. Fig. 7 sketches such situations in an intutive manner. From eqs. (3.8) and (3.9) we have

$$\kappa\equiv\tilde{\Pi}_a(\vec{k},0)/\tilde{\Pi}_n(\vec{k},0)$$

$$\simeq[D(q_F)/E_F]^2 \quad . \tag{3.10}$$

Considering $E_F\simeq 100\text{MeV}$ and $D(q_F)\simeq 1\text{MeV}$ at the density of interest, we find

$$\kappa\simeq 10^{-4} \quad . \tag{3.11}$$

The contribution of the abnormal term is therefore expected to be negligible.

## 4. Numerical results and discussion

### 4.1. THE $^3P_2$ ENERGY GAP

In order to calculate the critical density for $\pi^0$ condensation realized from the $^3P_2$ superfluid state, we must in advance solve the gap equation (1.1) and obtain a set of gap parameters. Here we deal with Sol. 1 referred to as the maximum $|m_J|$ coupling case. This is sufficient to get general conclusions which are valid for all the types of solutions, as we shall see later. In this case eq. (1.1) becomes[38]

$$\Delta_2(q) = -\frac{1}{\pi}\int dq' q'^2 V_\lambda(q,q') \times \int d\hat{q}' \frac{(3/8\pi)\Delta_2(q')\sin^2\theta_{q'}}{\sqrt{\tilde{\varepsilon}^2_{q'} + (3/8\pi)\Delta_2^2(q')\sin^2\theta_{q'}}} , \tag{4.1}$$

and $\Delta_2(q)=\Delta_{-2}(q)$. The gap matrix has a diagonal form;

$$\Delta(\vec{q}) = \begin{pmatrix} \Delta_2(q)Y_{11}(\hat{q}) & 0 \\ 0 & \Delta_2(q)Y_{1-1}(\hat{q}) \end{pmatrix} . \tag{4.2}$$

The energy gap is given by

$$D(\vec{q}) = \sqrt{3/8\pi}\,\Delta_2(q)\sin\theta_q . \tag{4.3}$$

Eq. (4.1) is solved by using one of Mongan's nonlocal separable potentials[42],

$$\begin{cases} V_\lambda(q,q') = -(\pi/2)h_l(q)h_l(q') \ , \\ h_l(q) = C_A q^l/(q^2+a_A^2)^{(l+1)} \ , \end{cases} \tag{4.4}$$

where $l=1$, $a_A=2.72\mathrm{fm}^{-1}$ and $C_A=122.5[\mathrm{MeVfm}^{-3}]^{1/2}$. This potential is adjusted to NN scattering phase shifts up to $E_{NN}<400\mathrm{MeV}$ in the lab. system. The gap parameter is very sensitive to the neutron effective mass M*. To maximize the gap parameter and thus its effect on the $\pi^0$ condensation threshold, we take M*=M=940MeV, although its realistic value at neutron star densities is believed to be $M^*\simeq 0.8M$.

Calculated results are displayed in Figs. 8 and 9. In the momentum and density regions neighboring $q=q_F$ and $E_F=100\mathrm{MeV}$, the displayed curves reproduce fairly well both momentum and density dependences of the gap parameters calculated from more realistic two-nucleon potentials[37]. On the other hand, the magnitude of $\Delta_2(q_F)$ and its existing density region are considerably larger than those obtained from the conclusive calculation[30].

Since the Mongan's potential (4.4) has no repulsive core, the resulting gap parameter $\Delta_2(q)$ is still positive for $q\gtrsim 2q_F$, where the realistic potentials with repulsive cores yield $\Delta_2(q)$ with negative values. We need not, however, take this discrepancy seriously because the $\pi^0$ condensation threshold is insensitive to details of $\Delta_2(q)$ in such a high momentum region.

## 4.2. CRITICAL DENSITY

The precise value of g' is of crucial importance in determining the threshold for pion condensation. Up to now considerable efforts have been made to achieve a reliable estimate of g'. Recent investigations[43-45]) yield values in the region of $g'=(0.5\sim0.7)f^2$, but no conclusive value has not been reported yet. Here we treat g' as a parameter because our purpose is not to accomplish a precise estimate of the critical density. The direction of the pion momentum $\vec{k}$ is taken to be parallel to the axis of spin quantization. Then we have

$$\mathrm{Tr}[(\vec{\sigma}\cdot\hat{k})^T\Delta^\dagger(\vec{q}-\vec{k})(\vec{\sigma}\cdot\hat{k})\Delta(\vec{q})]=\frac{3q\Delta_2(q)\Delta_2(|\vec{q}-\vec{k}|)\sin^2\theta_q}{4\pi|\vec{q}-\vec{k}|}, \qquad (4.5)$$

$$D^2(\vec{q}-\vec{k})=\frac{3q^2\Delta_2^2(|\vec{q}-\vec{k}|)\sin^2\theta_q}{8\pi|\vec{q}-\vec{k}|}, \qquad (4.6)$$

where $|\vec{q}-\vec{k}|=\sqrt{q^2+k^2-2qk\cos\theta_q}$. With these expressions we can solve the threshold condition (2.40).

Numerical calculations of the critical density $\rho_c$ have been carried out for two cases with and without the isobar contribution. Calculated results are plotted in Fig. 10 as a function of g'. For $\rho\gtrsim0.5\rho_0$ the neutron $^3P_2$ superfluidity comes into existence and increases the critical density from its values in the normal phase. Roughly speaking, the increase in $\rho_c$ defined by

$$\Delta\rho_c\equiv(\rho_{c,super}-\rho_{c,normal})/\rho_{c,normal}, \qquad (4.7)$$

is proportional to the magnitude of $\Delta_2(q_F)$.

Since in the superfluid phase excited states are separated from the ground state by a finite gap, some excess of energy corresponding to this gap is required to break a neutron bound pair and excite a $\pi^0$-like quasi-particle pair. The $\pi^0$ condensation cannot take place until the energy gain due to the $\pi N$ p-wave interaction exceeds further this extra energy. Such a suppression mechanism is operative only for the neutron bound pairs near the Fermi surface, while the $\pi^0$ condensation is followed by the change of all neutron states. Thus, the order of magnitude of $\Delta\rho_c$ is expected to be comparable with that of $\Delta_2(q_F)/E_F$ which measures the extent of diffusion of the Fermi surface. The various quantities calculated from $g'=0.6f^2$ are listed in table 1. In this case $\Delta\rho_c$ is 0.13, which is about an order of magnitude larger than the value of $\Delta_2(q_F)/E_F$. This means that the neutron $^3P_2$ superfluidity affects the $\pi^0$ condensation threshold beyond the expectation. Nevertheless, the superfluidity induces only a small increase in $\rho_c$ compared with the one resulting from the NN short-range correlations.

We have confirmed that the contribution of the abnormal term of the $\pi^0$ polarization operator is completely negligible compared with that of the normal term. The ratio $\kappa$ defined by eq. (3.10) is extremely small ($\kappa=1.6\times10^{-3}$ for $g'=0.6f^2$), which is consistent with the rough estimation (3.11). Relating this fact with the properties of the normal term, we are led to the following conclusions: (i)We need not specify the direction of the pion momentum $\vec{k}$; the $\pi^0$ condensation favors no particular direction even in the case realized from the anisotropic $^3P_2$ superfluid state. (ii)Critical densities are very close for all the types of solutions (Sols. 1∿5).

5. Concluding remarks

We have found that the $\pi^0$ condensation in neutron star matter is suppressed by the neutron $^3P_2$ superfluidity. The increase in the critical density is, however, not more than 30% of its values in the normal phase, although we have deliberately overestimated the effect of the superfluidity by employing the maximal gap parameter. This means that the suppression of the $\pi^0$ condensation is very modest in the realistic situation, i.e., $\Delta\rho_c \lesssim$ several percent in accordance with $\Delta_2(q_F) \lesssim 0.6/\sqrt{2}$ MeV[30]). Thus we may conclude that the neutron $^3P_2$ superfluidity brings about no significant change in the $\pi^0$ condensation threshold predicted so far.

This conclusion applies to the case of $\pi^c$ condensation as well. A straightforward extension of the formulation presented in sec. 2 leads to the $\pi^c$ polarization operator of the form

$$\tilde{\Pi}^c(\vec{k},\omega) = \frac{f^2k^2}{\Omega}\sum_q\Big[\frac{(1+\tilde{\varepsilon}_q^{(n)}/E_q^{(n)})(1-\tilde{\varepsilon}_{q-k}^{(p)}/E_{q-k}^{(p)})}{\omega-E_q^{(n)}-E_{q-k}^{(p)}} - \frac{(1-\tilde{\varepsilon}_q^{(n)}/E_q^{(n)})(1+\tilde{\varepsilon}_{q-k}^{(p)}/E_{q-k}^{(p)})}{\omega+E_q^{(n)}+E_{q-k}^{(p)}}\Big], \qquad (5.1)$$

where $\tilde{\varepsilon}_q^{(i)} = (q^2-q_F^{(i)2})/2M^{(i)*}$ and $E_q^{(i)} = \sqrt{\tilde{\varepsilon}_q^{(i)2}+D^{(i)2}(\vec{q})}$ with i=n,p. In the derivation of eq. (5.1) we have taken account of the fact that the proton $^1S_0$ superfluid coexists with the neutron $^3P_2$ superfluid at the density of interest. Because of the charge conservation law, $\tilde{\Pi}^c(\vec{k},\omega)$ does not have a term corresponding to the abnormal term.

The energy gaps existing in the nucleon single-particle spectra are masked by a large condensate frequency ($\omega_c^{(-)}\simeq$pion mass>>energy gaps). Therefore, as far as the threshold condition and critical density are concerned, it seems unlikely that the neutron $^3P_2$ superfluidity, as well as the proton $^1S_0$ superfluidity, has a notable influence upon the $\pi^c$ condensation. This observation is in sharp contrast to the well-developed case where the $\pi^c$ condensation strongly attenuates the neutron $^3P_2$ superfluidity[31c]. In this respect, it is quite interesting to see whether or not the same situation is realized in our line of approach which starts from the superfluid phase side.

In this paper we have left another interesting problem untouched: How does the neutron $^3P_2$ superfluidity behave after the $\pi^0$ condensation develops? Studies of these remaining problems will lead to a deeper understanding of the interrelation between pion condensation and the neutron $^3P_2$ superfluidity.

Appendix A

*Properties of $U(\vec{q})$ and $V(\vec{q})$*

The transformation matrices $U(\vec{q})$ and $V(\vec{q})$ are, respectively, proportional to the unit matrix and gap matrix;

$$\begin{cases} U(\vec{q})=u_q\times 1 \ , \\ V(\vec{q})=\dfrac{v_q}{D(\vec{q})}\times\Delta(\vec{q}) \ , \end{cases} \tag{A.1}$$

where $u_q$ and $v_q$ are given by

$$\begin{cases} u_q^2=\dfrac{1}{2}(1+\dfrac{\tilde{\varepsilon}_q}{E_q}) \ , \\ v_q^2=\dfrac{1}{2}(1-\dfrac{\tilde{\varepsilon}_q}{E_q}) \ , \\ u_q v_q=\dfrac{D(\vec{q})}{2E_q} \ . \end{cases} \tag{A.2}$$

Since the gap parameter satisfies $\Delta^T(\vec{q})=-\Delta(-\vec{q})=\Delta(\vec{q})$, we have

$$\begin{cases} U^\dagger(\vec{q})=U^*(\vec{q})=U^T(\vec{q})=U(-\vec{q})=U(\vec{q}) \ , \\ V^T(\vec{q})=-V(-\vec{q})=V(\vec{q}) \ . \end{cases} \tag{A.3}$$

Appendix B

*Derivation of the nucleon Green's functions*

We start with the Hamiltonian

$$H_B = H_N^0 + H_{pair} \ , \tag{B.1}$$

with

$$H_{pair} = \frac{1}{2} \sum_{qq'} \sum_{spin} \langle \vec{q}'\mu', -\vec{q}'\nu' | V(1,2) | \vec{q}\mu, -\vec{q}\nu \rangle$$

$$\times n^{\dagger}_{q'\mu'} n^{\dagger}_{-q'\nu'} n_{-q\nu} n_{q\mu} \ . \tag{B.2}$$

Applying the Hartree-Fock-Gor'kov approximation[46]) to $H_B$, we have

$$H_B = H_N^0 + N_{pair})_{HF}$$

$$+ \frac{1}{2} \sum_{q} \sum_{\mu\nu} [\Delta^{\dagger}_{\nu\mu}(\vec{q}) n_{-q\nu} n_{q\mu} + \Delta_{\mu\nu}(\vec{q}) n^{\dagger}_{q\mu} n^{\dagger}_{-q\nu}]$$

$$+\text{correction term} \ . \tag{B.3}$$

Here $H_{pair})_{HF}$ correspond to the Hartree-Fock contributions, which are taken into account by using an effective mass approximation for the nucleon kinetic energy. The order parameters for a nonzero angular momentum pairing are defined by

$$\left( \begin{array}{l} \Delta^{\dagger}_{\nu\mu}(\vec{q}) \equiv \sum_{q'} \sum_{\alpha\beta} \langle \vec{q}'\alpha, -\vec{q}'\beta | V(1,2) | \vec{q}\mu, -\vec{q}\nu \rangle F^{\dagger}_{\alpha\beta}(\vec{q}', 0) \ , \\ \\ \Delta_{\mu\nu}(\vec{q}) \equiv \sum_{q'} \sum_{\alpha\beta} \langle \vec{q}\mu, -\vec{q}\nu | V(1,2) | \vec{q}'\alpha, -\vec{q}'\beta \rangle F_{\beta\alpha}(\vec{q}', 0) \ , \end{array} \right. \tag{B.4}$$

where $F^{\dagger}_{\alpha\beta}(\vec{q}',0)$ and $F_{\beta\alpha}(\vec{q}',0)$ denote the thermal Green's functions. These definitions guarantee that $\Delta_{\mu\nu}(\vec{q})$ and $\Delta^{\dagger}_{\mu\nu}(\vec{q})$ satisfy

$$\Delta_{\mu\nu}(\vec{q})=-\Delta_{\nu\mu}(-\vec{q}) \ , \quad \Delta^{\dagger}_{\mu\nu}(\vec{q})=\Delta^{*}_{\nu\mu}(\vec{q}) \ . \tag{B.5}$$

The order parameters $\Delta_{\mu\nu}(\vec{q})$ form the gap matrix $\Delta(\vec{q})$. The thermal Green's functions are defined by

$$\left\{\begin{aligned} &F^{\dagger}_{\alpha\beta}(\vec{q},\tau-\tau')\equiv\langle T_{\tau}(n^{\dagger}_{q\alpha}(\tau)n^{\dagger}_{-q\beta}(\tau'))\rangle \ , \\ &F_{\alpha\beta}(\vec{q},\tau-\tau')\equiv\langle T_{\tau}(n_{-q\alpha}(\tau)n_{q\beta}(\tau'))\rangle \ , \\ &G_{\alpha\beta}(\vec{q},\tau-\tau')\equiv-\langle T_{\tau}(n_{q\alpha}(\tau)n^{\dagger}_{q\beta}(\tau'))\rangle \ , \end{aligned}\right. \tag{B.6}$$

where $T_{\tau}$ is the time-ordering operator and the angular brackets mean an ensemble average in the grand canonical ensemble.

The Hamiltonian (B.3) yields the equations of motion

$$\frac{\partial}{\partial\tau}n_{q\alpha}(\tau)=-\tilde{\varepsilon}_{q}n_{q\alpha}(\tau)-\sum_{\mu}\Delta_{\alpha\mu}(\vec{q})n^{\dagger}_{-q\mu}(\tau) \ , \tag{B.7}$$

$$\frac{\partial}{\partial\tau}n^{\dagger}_{q\alpha}(\tau)=-\tilde{\varepsilon}_{q}n^{\dagger}_{q\alpha}(\tau)-\sum_{\mu}\Delta^{\dagger}_{\alpha\mu}(-\vec{q})n_{-q\mu}(\tau) \ . \tag{B.8}$$

Combining eqs. (B.7) and (B.8) with eqs. (B.6), we obtain the equations of motion for the thermal Green's functions

$$\left(\frac{\partial}{\partial\tau}+\tilde{\varepsilon}_q\right)G_{\alpha\beta}(\vec{q},\tau-\tau')-\sum_{\mu}\Delta_{\alpha\mu}(\vec{q})F^{\dagger}_{\mu\beta}(-\vec{q},\tau-\tau')$$

$$=-\delta_{\alpha\beta}\delta(\tau-\tau')\ , \qquad \text{(B.9)}$$

$$\left(\frac{\partial}{\partial\tau}-\tilde{\varepsilon}_q\right)F^{\dagger}_{\alpha\beta}(\vec{q},\tau-\tau')-\sum_{\mu}\Delta^{\dagger}_{\alpha\mu}(-\vec{q})G_{\mu\beta}(-\vec{q},\tau-\tau')=0\ , \qquad \text{(B.10)}$$

It is advantageous to employ the Fourier representation in the variable $\tau$[47]. The Fourier transforms of eqs. (B.9) and (B.10) are evaluated to be

$$(i\omega_n-\tilde{\varepsilon}_q)G_{\alpha\beta}(\vec{q},\omega_n)+\sum_{\mu}\Delta_{\alpha\mu}(\vec{q})F^{\dagger}_{\mu\beta}(-\vec{q},\omega_n)=\delta_{\alpha\beta}\ , \qquad \text{(B.11)}$$

$$(i\omega_n+\tilde{\varepsilon}_q)F^{\dagger}_{\alpha\beta}(\vec{q},\omega_n)+\sum_{\mu}\Delta^{\dagger}_{\alpha\mu}(-\vec{q})G_{\mu\beta}(-\vec{q},\omega_n)=0\ . \qquad \text{(B.12)}$$

Eqs. (B.11) and (B.12) are generalized Gor'kov equations.

We now assume that the system does not have spin polarization. This assumption requires that $G_{\alpha\beta}(\vec{q},\omega_n)$ should be diagonal with respect to the spin indices, which is equivalent to the requirement

$$\sum_{\mu}\Delta_{\alpha\mu}(\vec{q})\Delta^{\dagger}_{\mu\beta}(\vec{q})=D^2(\vec{q})\delta_{\alpha\beta}\ . \qquad \text{(B.13)}$$

In this case solutions of eqs. (B.11) and (B.12) are easily found to be

$$F^{\dagger}_{\alpha\beta}(\vec{q},\omega_n)=\frac{\Delta^{\dagger}_{\alpha\beta}(-\vec{q})}{\omega_n^2+\tilde{\varepsilon}_q+D^2(\vec{q})}\ , \qquad \text{(B.14)}$$

$$F_{\alpha\beta}(\vec{q},\omega_n)= \frac{\Delta_{\alpha\beta}(-\vec{q})}{\omega_n^2+\tilde{\varepsilon}_q+D^2(\vec{q})} , \tag{B.15}$$

$$G_{\alpha\beta}(\vec{q},\omega_n)= - \frac{i\omega_n+\tilde{\varepsilon}_q}{\omega_n^2+\tilde{\varepsilon}_q+D^2(\vec{q})} . \tag{B.16}$$

Utilizing analytic continuation[47]), we can show that eqs. (B.14) ∿(B.16) lead to the following expressions for the nucleon Green's functions at zero temperature:

$$F^{\dagger}_{\alpha\beta}(q)= - \frac{\Delta^{\dagger}_{\alpha\beta}(-\vec{q})}{2E_q} [\frac{1}{q_0-E_q+i\varepsilon} - \frac{1}{q_0+E_q-i\varepsilon}] , \tag{B.17}$$

$$F_{\alpha\beta}(q)= - \frac{\Delta_{\alpha\beta}(-\vec{q})}{2E_q} [\frac{1}{q_0-E_q+i\varepsilon} - \frac{1}{q_0+E_q-i\varepsilon}] , \tag{B.18}$$

$$G_{\alpha\beta}(q)= \frac{\delta_{\alpha\beta}}{2E_q} [\frac{E_q+\tilde{\varepsilon}_q}{q_0-E_q+i\varepsilon} + \frac{E_q-\tilde{\varepsilon}_q}{q_0+E_q-i\varepsilon}] , \tag{B.19}$$

where q stands for the four momenta $(\vec{q},q_0)$.

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Figure captions

Fig. 1. Deformed Fermi spheres; (a) the case of Sol. 1 and (b) the case of Sol. 2. Arrows indicate the direction of spin quantization.

Fig. 2. Processes contained in $\tilde{H}_{\pi N}$ given by eq. (2.10).

Fig. 3. Pion self-energy processes via the $\Delta_{33}(1236)$ isobar.

Fig. 4. Renormalization of $\tilde{\Pi}(\vec{k},\omega)$ due to the NN short-range correlations. Shaded triangles are renormalized vertices.

Fig. 5. Proper self-energy diagrams in the superfluid phase; (a) the normal one corresponding to $\tilde{\Pi}_n(\vec{k},\omega)$ and (b) the abnormal one corresponding to $\tilde{\Pi}_a(\vec{k},\omega)$.

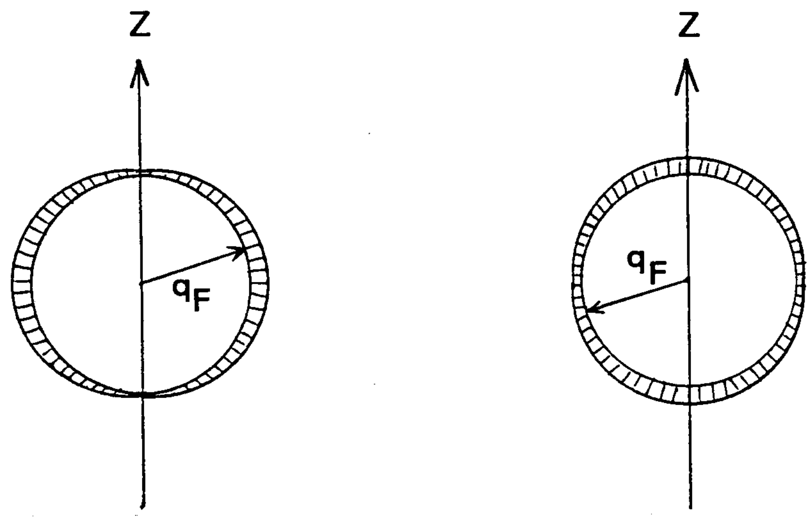
Fig. 6. The nucleon Green's functions in the superfluid phase.

Fig. 7. Pion self-energy processes producing the normal term (A) and the abnormal term (B). Double circles stand for the Fermi sphere with diffused surface. Solid lines connecting two nucleons with opposite momenta indicate that these nucleons form a bound pair.

Fig. 8. Momentum dependence of the $^3P_2$ gap parameter at $E_F$=100MeV.

Fig. 9. Density dependence of the $^3P_2$ gap parameter at the Fermi momentum. The $^1S_0$ energy gap is also shown for comparsion.

Fig. 10. Critical densities for the $\pi^0$ condensation realized from the $^3P_2$ superfluid state (Sol. 1). Critical densities in the normal phase are also shown with dashed lines for comparsion. Lower and upper curves represent the cases with and without the isobar contribution, respectively.




(a) (b)

Fig. 1

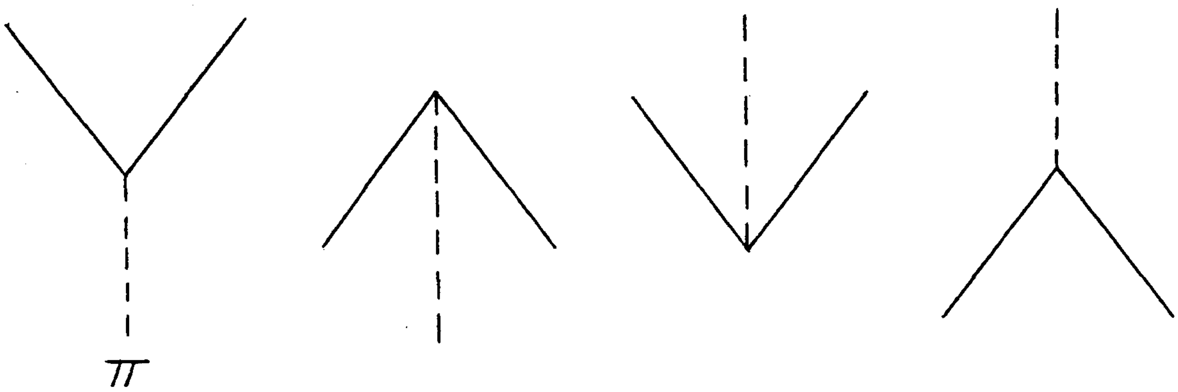



Fig. 2

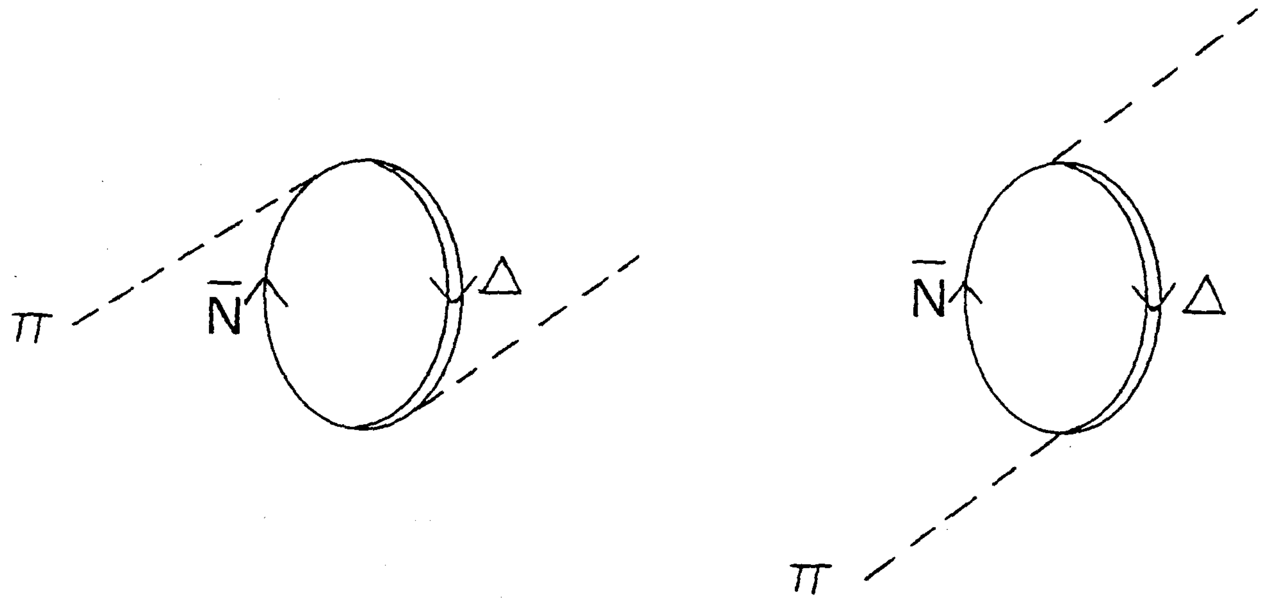



Fig. 3

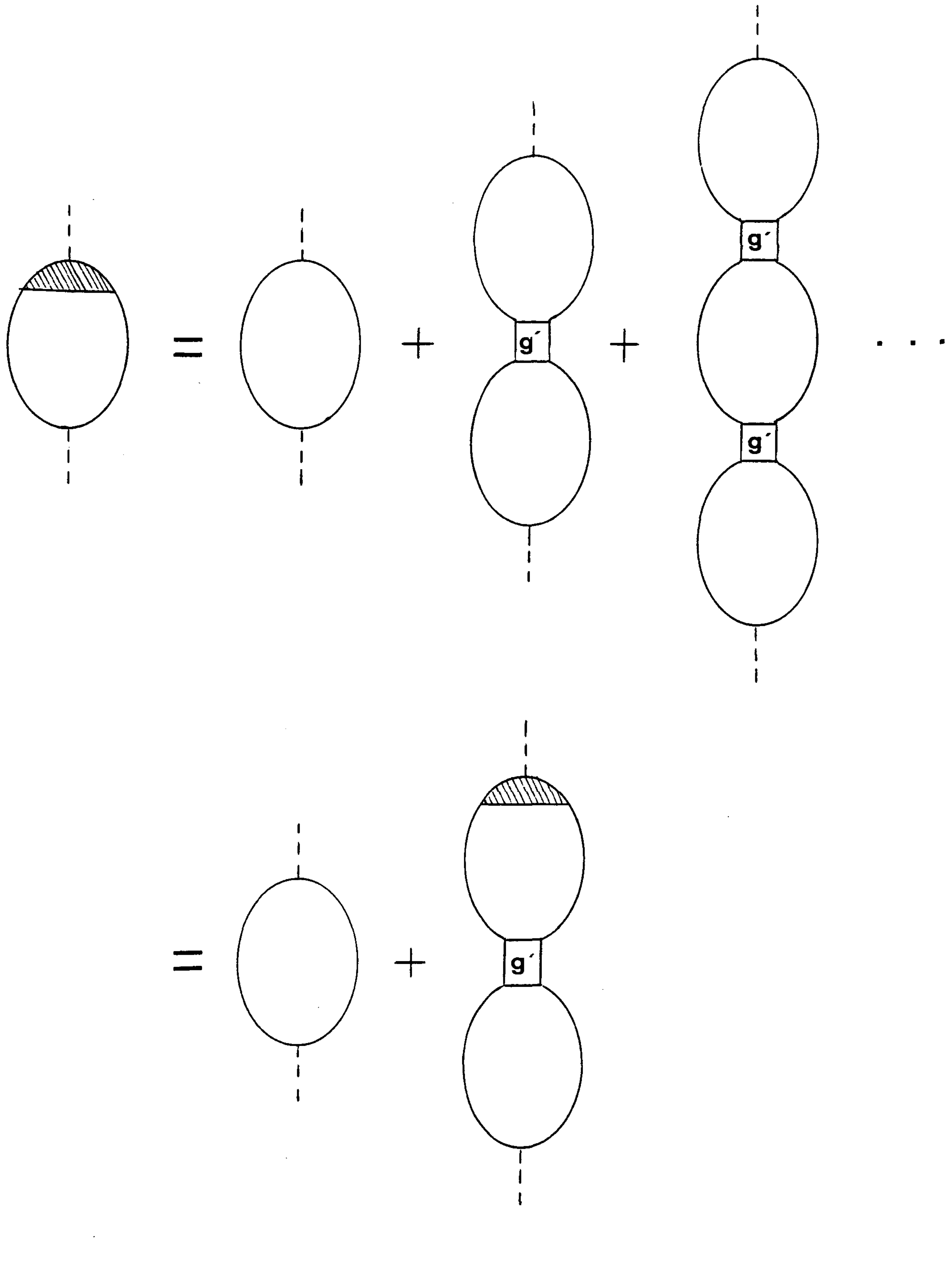



Fig. 4

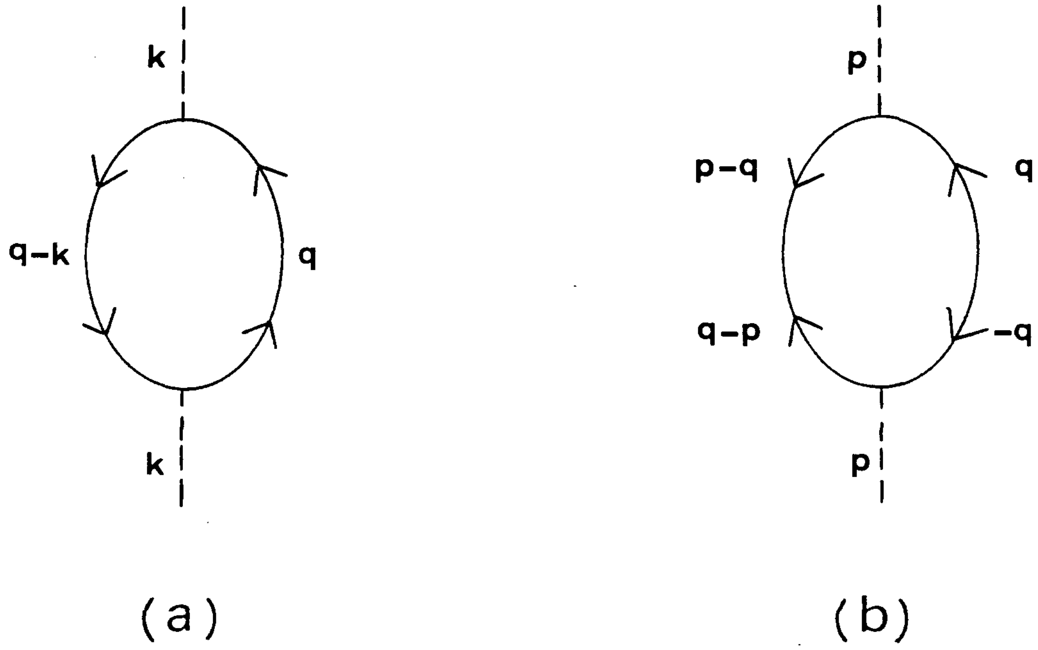



Fig. 5

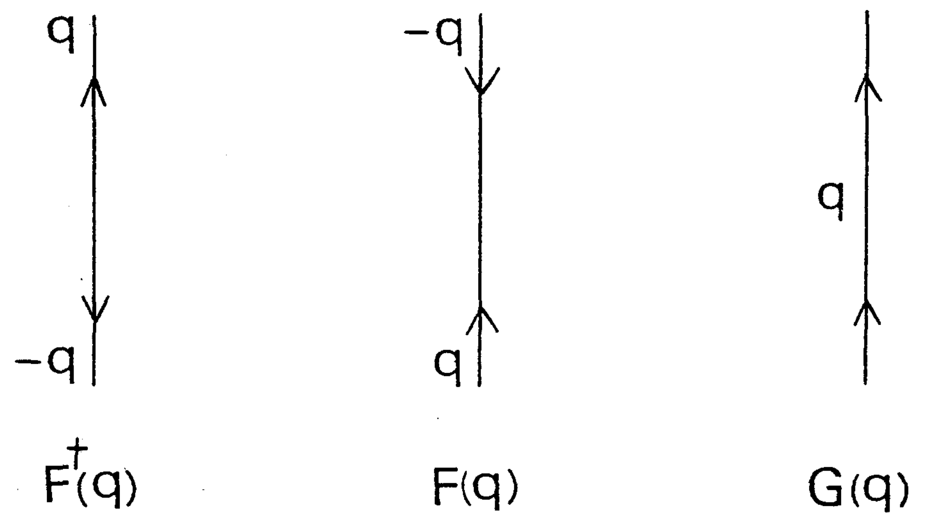
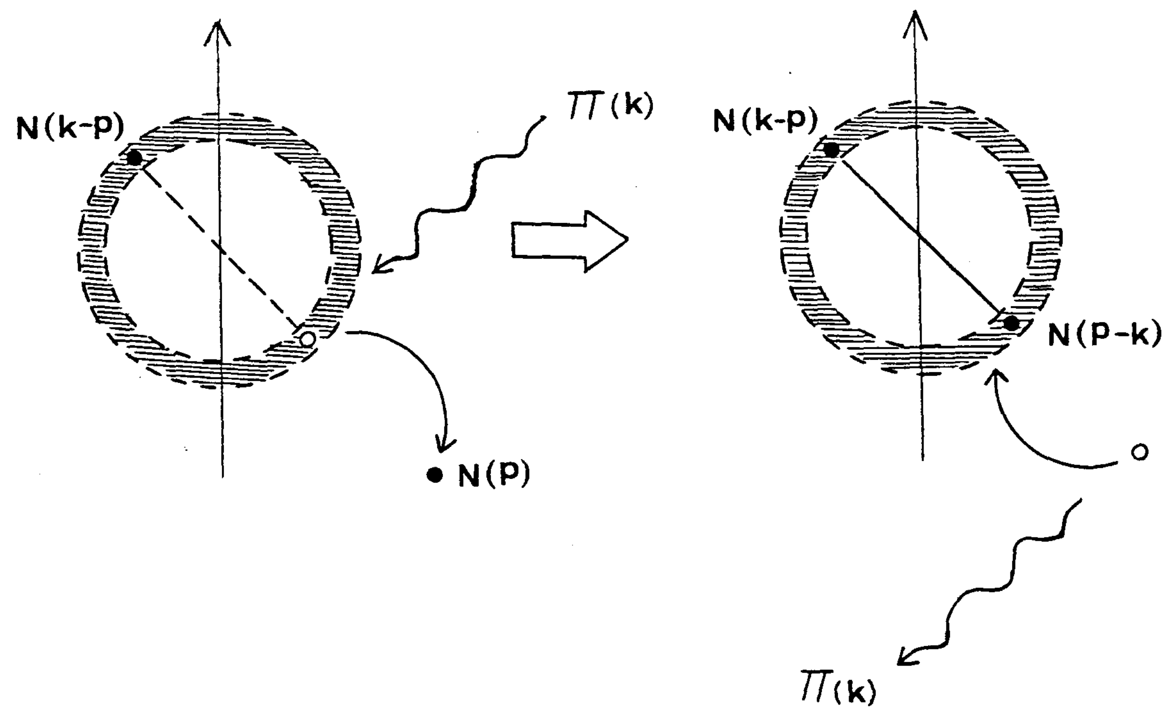



Fig. 6

(A)




(B)

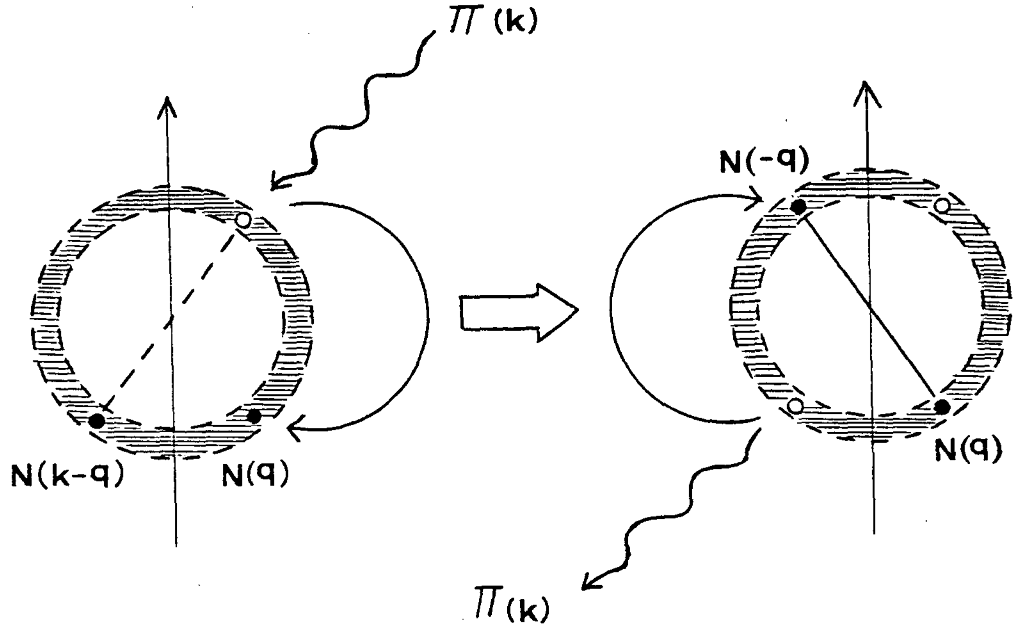
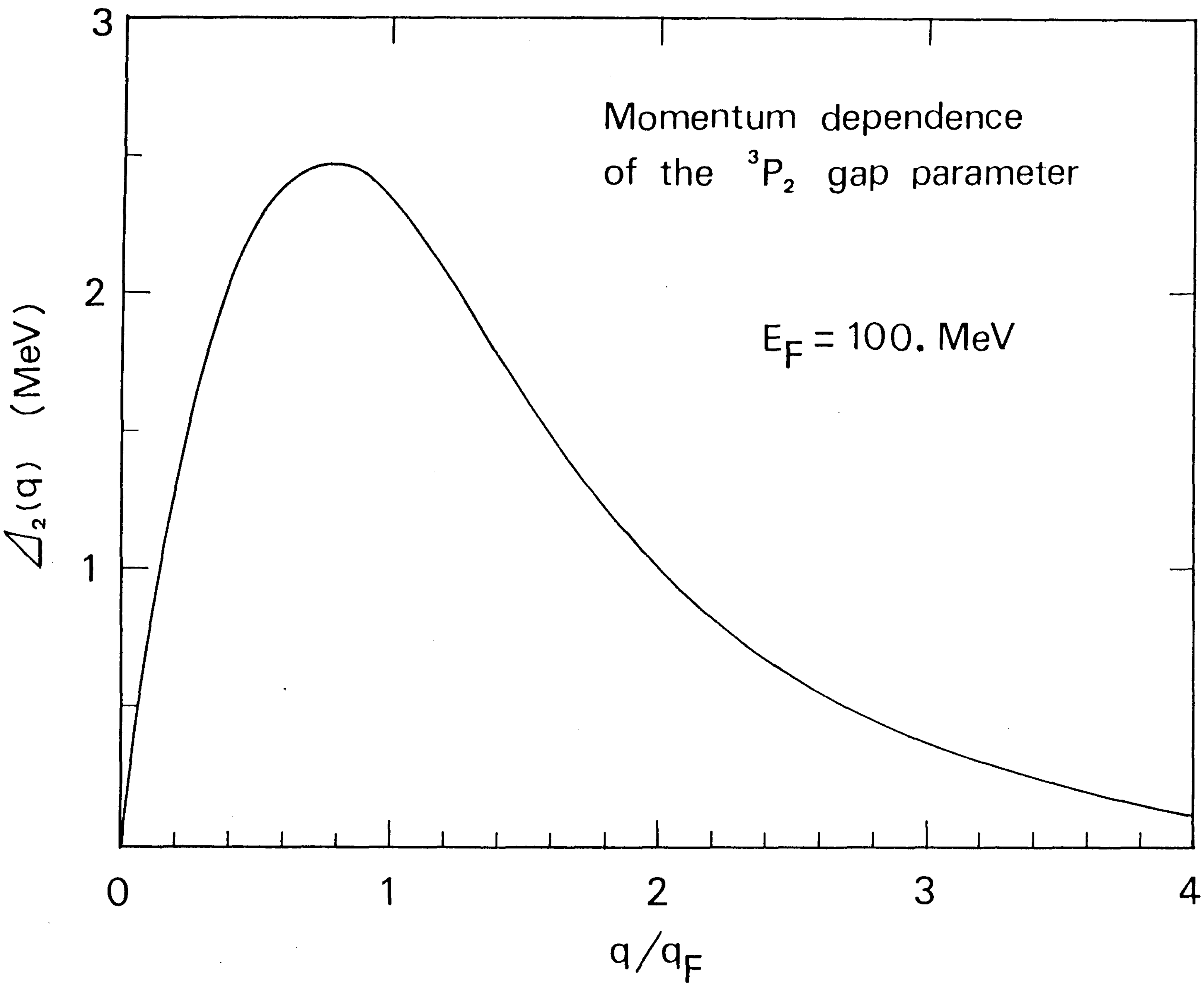



Fig. 7



3
2
1
$\Delta_2(q)$ (MeV)
Momentum dependence
of the $^3P_2$ gap parameter
$E_F = 100.$ MeV
0
1
2
3
4
$q/q_F$


Fig. 8

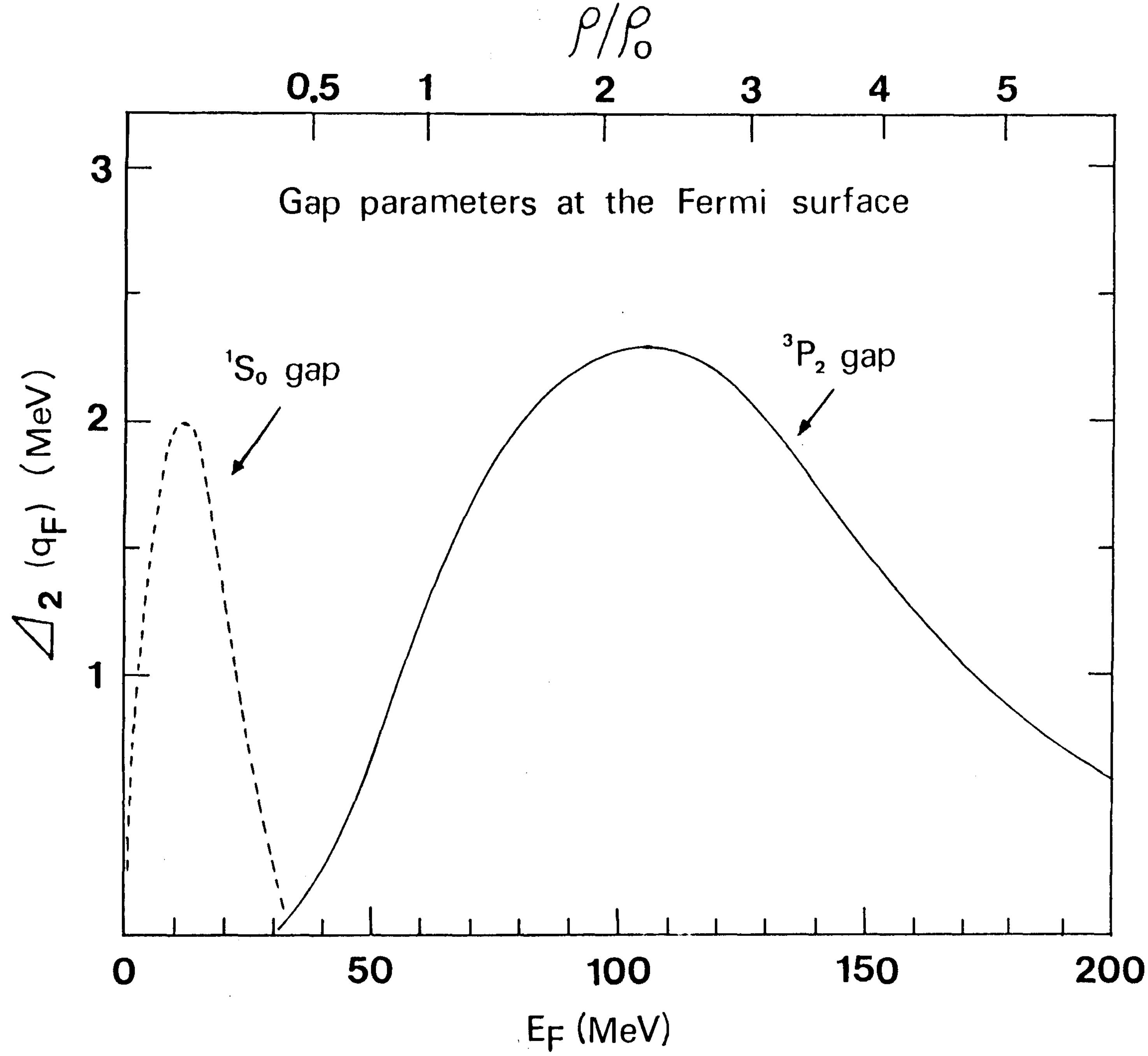
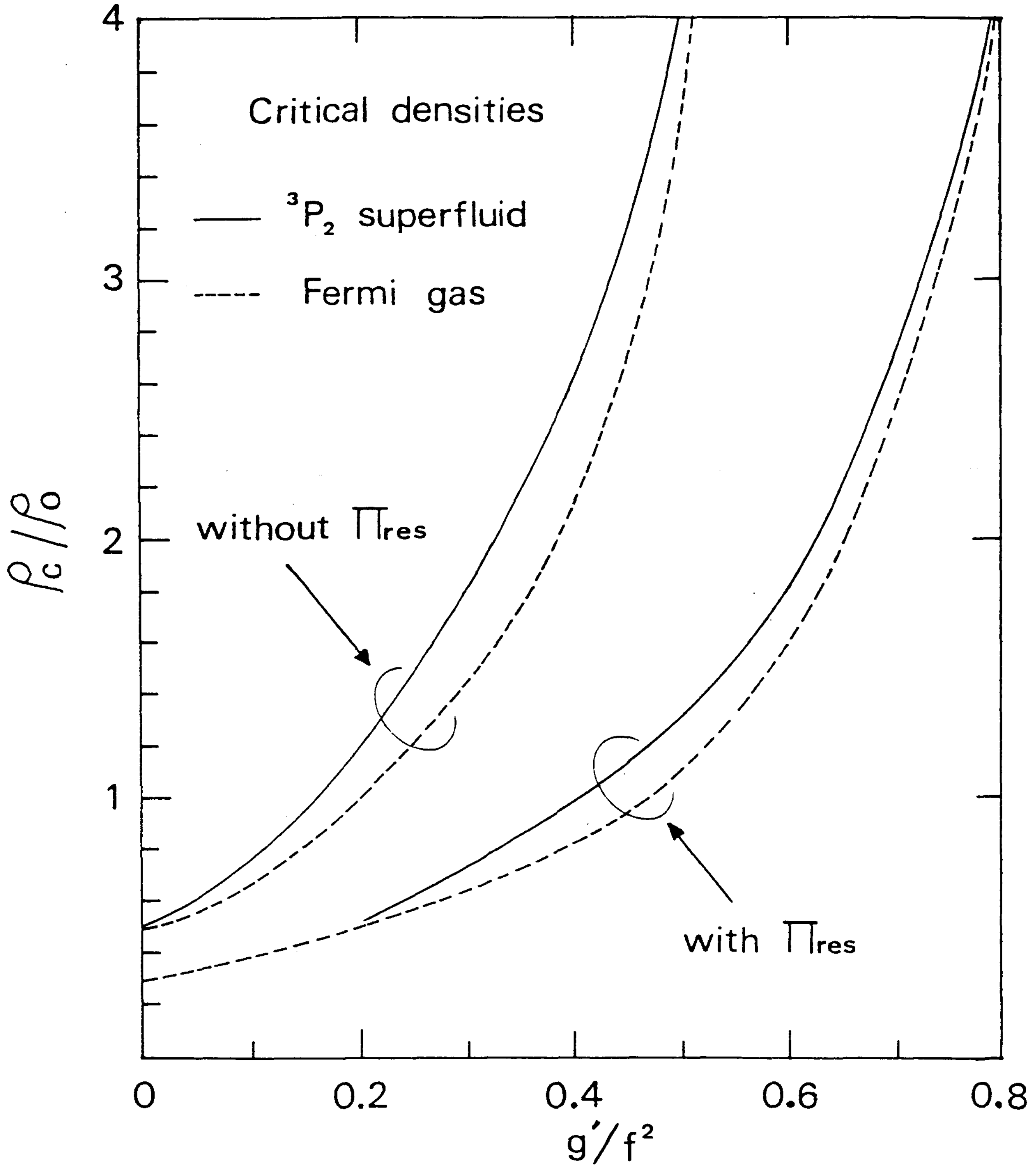

ρ/ρ0
0,5
1
2
3
4
5
3
2
1
Gap parameters at the Fermi surface
1S0 gap
3P2 gap
Δ2 (qF) (MeV)
0
50
100
150
200
EF (MeV)

Fig. 9



Critical densities
$^3P_2$ superfluid
Fermi gas
without $\Pi_{res}$
with $\Pi_{res}$
$\rho_c/\rho_o$
$g'/f^2$
0
0.2
0.4
0.6
0.8
1
2
3
4


Fig. 10

| | |
|---|---|
| $k_{c,super}$ | 2.8 $fm^{-1}$ |
| $\rho_{c,super}/\rho_0$ | 1.8 |
| $\rho_{c,normal}/\rho_0$ | 1.6 |
| $\Delta\rho_c$ | $1.3\times10^{-1}$ |
| $\tilde{\Pi}_a(k_c,0)/\tilde{\Pi}_n(k_c,0)$ at $\rho=\rho_{c,super}$ | $1.6\times10^{-3}$ |
| $\Delta_2(q_F)/E_F$ at $\rho=\rho_{c,normal}$ | $2.5\times10^{-2}$ |

Table 1. Various quantities calculated from $g'=0.6f^2$. The isobar contribution is included.