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Calculation of Dose Distributions in Radiation Therapy by a Digital Computer
III. Computation of Dose Distributions in Variously Shaped Fields

By

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ディジタル型電子計算機による線量分布の計算
第3報 不規則形照射野の線量計算

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任意の形をした照射野の線量分布を計算する半実験式を，HVL 1，2，3 mm Cu X線，^{60}Co γ線， 4.3MVおよび6MVX線の6線質について開発した．その方法は深部線量を1次線と散乱線にわけて計算する方法で，散乱線の計算に必要な scatter-air ratio と1次線の分布を決める空気中線量分布の数式化を行なつた．

計算値と実測値の相違は，幾何学的照射野内では中照射野の20cm深部まではいずれの線質についても2％以内であり，照射野外の低線量域で10％程度である．

高エネルギーX線についての斜入射および不均質部照射の補正法を述べ，実測値と比較した．

ここに開発された数式を用い，婦人科領域，頭頸部および胸部の不規則形照射野による照射例の線量分布を電子計算機で計算し，その結果を直接 print-out した．

この研究は厚生省がん研究助成金により行なわれたものであり，その要旨は第12回国際放射線医学会議（1969，10，7東京）において発表した．

Introduction

Empirical formulae for calculation of the dose distributions of ^{60}Co γ-rays and 4.3 MV X-rays were reported in our previous paper[6]. The method was essentially a mathematical expression of the tissue-air ratio and decrement value. By using this expression, the three-dimensional dose distributions for multiple or moving fields can be calculated by a computer with accuracy and rapidity, provided that the shape of the field is rectangular.

However, the problem of dosage in a beam with various cross-sections is of fundamental importance in external beam radiotherapy. The method which has been universally employed for calculating depth doses in variously shaped fields is due in essence to that of Clarkson[1] who adopted a scatter function. Gupta and Cunningham[3] have recently proposed a scatter-air ratio instead of the scatter function. The use of this ratio allows a rapid calculation of dose distributions for any shaped field for a wide range of SSD.

The present paper deals with the determination of mathematical description for the scatter-air ratios and for the primary dose distributions of ^{60}Co γ-rays, diaphragm-limited beams of HVL 1, 2, and 3 mm Cu, and for 4.3 and 6 MV X-rays.

Mathematical Expression

The dose at any point P at depth d, as shown in Fig. 1, may be expressed as the sum of the

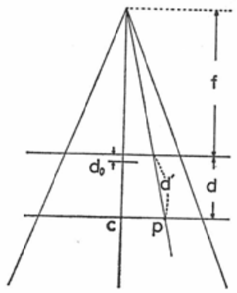

Fig. 1. Symbols used in dosage calculations

primary and scattered doses, in the form of

$$D_p(d) = D_p^a \text{ TAR } (0, d') + D_c^a \int_{2\pi} \frac{\text{SAR}(r(\theta), d)}{2\pi} d\theta \quad \cdots\cdots (1)$$

where D_p^a and D_c^a are doses in air at a point P and C, respectively,

TAR is the tissue-air ratio,

SAR is the scatter-air ratio, and

r is the radius of a circular field.

The scatter-air ratio is defined as follows:

$$\text{SAR}(r_d, d) = \text{TAR }(r_d, d) - \text{TAR }(0, d) \quad \cdots\cdots (2)$$

where TAR (r_d, d) and TAR$(0, d)$ are the tissue-air ratios for a field of radius r_d and zero area at depth d, respectively.

As shown in Fig. 2, the secondary dose S_p at any point such as P can be computed by the following expression,

$$S_p = \int \frac{\text{SAR}(r(\theta), d)}{2\pi} d\theta \fallingdotseq \frac{1}{n} \sum_{i=1}^{n} \text{SAR }(r_i, d) \quad \cdots\cdots (3)$$

where n is the number of segments in 360° and r_i is the distance from the point of calculation to the boundary of the field for the *i*th sector.

According to equations (1), (2), and (3), the depth dose may be calculated by a computer provided that the tissue-air ratios for a field of radius r and zero area can be expressed as mathematical formulae, since the dose in air, D_c^a and D_p^a, may be derived theoretically from the geometrical

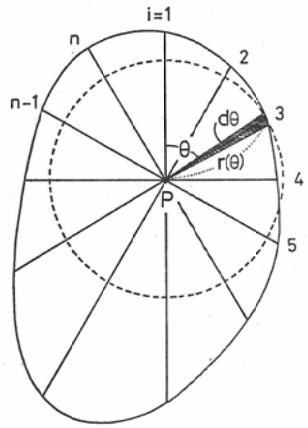

Fig. 2. Symbols used in calculation of secondary doses.

relation between the source and the diaphragm.

1. The tissue-air ratio for a field of zero area

The tissue-air ratio for a field of zero area can be expressed as an exponential function of depth as

$$\mathrm{TAR}(0,\ d) = \exp\ \{-\mu(d-d_o)\} \quad \cdots\cdots (4)$$

where d_o is the depth of the peak absorbed dose.

The effective absorption coefficients of water μ in equation (4) were determined by using the tissue-air ratio data for zero area in B.J.R. Supplement 10[2] for ^{60}Co γ-rays and for diaphragm-limited beams of HVL 1, 2, and 3 mm Cu. For 4.3 and 6 MV X-rays, the values measured in this center and in the National Cancer Center Hospital, respectively, were used.

The values of μ for six radiation qualities obtained are shown in the second column of Table 1. Fig. 3 shows a comparison between the tissue-air ratios for zero area calculated from equation (4) and the experimental values. The two values are in good agreement.

Table 1. Effective absorption coefficients of water, μ, and empirical equations for K(d) and m(d)

Radiation	μ, cm^{-1}		K(d) and m(d)
HVL 1 mm Cu X-rays	0.182	K	$1.1323 \exp(-0.2114d)$
		m	$0.072146 + 0.030989d - 0.00083473d^2 + 0.000012103d^3$
2 mm Cu	0.163	K	$1.0743 \exp(-0.1926d)$
		m	$0.06677 + 0.028506d - 0.00071275d^2 + 0.0000087466d^3$
3 mm Cu	0.150	K	$1.0667 \exp(-0.1806d)$
		m	$0.054378 + 0.027423d - 0.00071585d^2 + 0.0000081221d^3$
^{60}Co γ-rays	0.0657	K	$1.00778 - 0.063527d + 0.0014216d^3 - 0.00001d^3$
		m	$0.01113 + 0.0070903d - 0.0000593d^2$
4.3 MV X-rays	0.0570	K	$1.04555 - 0.063207d + 0.0015608d^2 - 0.00001474d^3$
		m	$0.00081 + 0.0083966d - 0.00023696d^2 + 0.0000035224d^3$
6 MV X-rays	0.0471	K	$1.0738 - 0.052254d + 0.00091420d^2 - 0.0000036591d^3$
		m	$-0.002041 + 0.0061734d - 0.00011073d^2 + 0.0000065750d^3$

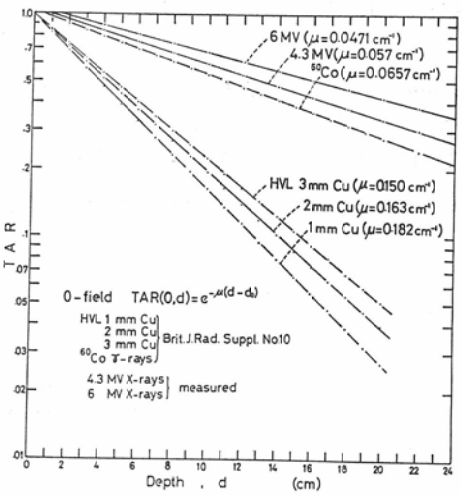

Fig. 3. Comparison of calculated and experimental values of tissue-air ratios for field of zero area for six radiation qualities.
Solid lines were calculated from equation (4), using μ in Table 1, and dots are experimental values.

2. The scatter-air ratio

From equations (2) and (4), the scatter-air ratio may be expressed as

$$SAR\ (r_d,\ d) = TAR\ (r_d,\ d) - \exp\ \{-\mu\ (d - d_o)\} \quad \cdots\cdots(5)$$

Pfalzner[9] has pointed out that the tissue-air ratio for ^{60}Co γ-rays may be expressed as a power function. We proved that this approximation would be applicable for other radiation qualities, in the form of

$$TAR\ (r_d,\ d) = K(d)\ (\pi r_d^2)^{m(d)} \quad \cdots\cdots(6)$$

where K and m are constants for a given depth and a given radiation quality.

From equations (5) and (6), the scatter-air ratio may be expressed as

$$SAR\ (r_d,\ d) = K(d)\ (\pi r_d^2)^{m(d)} - \exp\ \{-\mu\ (d - d_o)\} \quad \cdots\cdots(7)$$

According to this equation, the scatter-air ratio may be calculated by a computer provided that the parameters K and m are expressed as a mathematical formula. We have already presented the mathematical expressions for K and m for ^{60}Co γ-rays and 4.3 MV X-rays, and that the errors are less than 3%.

In the present work, the mathematical expressions for K and m of other four radiation qualities have been developed by using the tissue-air ratio data in B.J.R. Supplement 10 for X-rays of HVL 1–3 mm Cu. For 6 MV X-ryas, the data measured in the National Cancer Center Hospital are used.

The empirical expressions for K and m of six radiation qualities are shown in the right-hand column of Table 1. The equations for X-rays of HVL 1–3 mm Cu are applicable to the depths in the range of 1.0 to 20 cm. Back-scatter factors are given by the following expressions.

$BSF(r) = 0.96590(\pi r^2)^{0.073171}$ for HVL 1 mm Cu,

$BSF(r) = 0.94202(\pi r^2)^{0.068884}$ for HVL 2 mm Cu, and

$BSF(r) = 0.95165(\pi r^2)^{0.056999}$ for HVL 3 mm Cu.

The maximum error of estimate of these equations at a field of 20 × 20 cm is 6.6% for 1 mm Cu, 5.6% for 2 mm Cu, and 5.3% for 3 mm Cu and, if the field is less than 10 × 10 cm, the errors are less than 2%, as shown in Fig. 4. For 6 MV X-rays, the error is less than 2% up to 20 cm depth as shown in Fig. 5, and the maximum error is 3.8% at 30 cm depth.

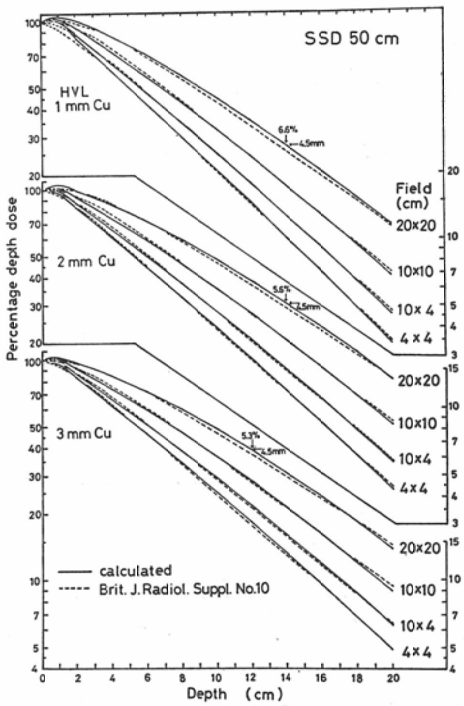

Fig. 4. Comparison of calculated and experimental values of percentage depth doses for X-rays of HVL 1,2, and 3mm Cu

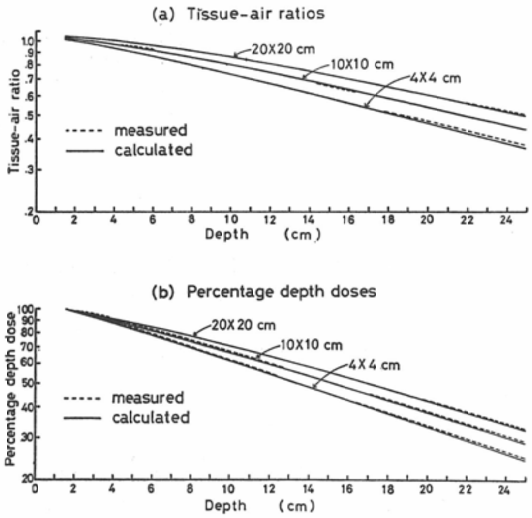

Fig. 5. Comparison of calculated and measured values of (a) tissue-air ratios and (b) percentage depth doses for 6MV X-rays.

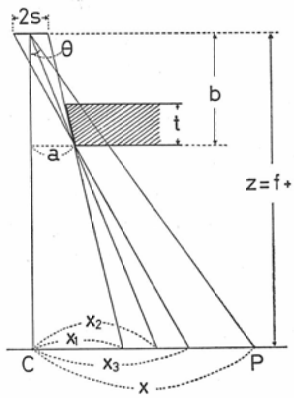

Fig. 6. Symbols used in dosage calculation in air

3. The dose distribution in air

The ratio of the dose in air at P to the dose in air at C of Fig. 6 is denoted by the function P_1. The air dose function P_1 may be approximated by the following formula

$$P_1 = \frac{D_p^a}{D_c^a} = P_2 \{P_3 + (T_x + S_x)(1-P_3)\} \qquad (8)$$

the terms of which are defined below.

The function P_2 is the correction for inverse-square law effects across the field and is given by

$$P_2 = z^2/(x^2 + z^2) \qquad (9)$$

When the beam flattening filter is used, P_2 takes the value of unity or the actual dose distribution in air.

The function P_3 is the correction for the geometrical penumbral regions. With reference to Fig. 6, it may be expressed as follows[10]:

when $x \leqq x_1$ (i.e. in the full-illumination regions), $P_3 = 1$,

when $x \geqq x_3$ (i.e. in the umbral regions), $P_3 = 0$, and

when $x_1 < x < x_3$ (i.e. in the penumbral regions)

$$P_3 = \frac{1}{2} + \frac{1}{\pi}(Y\sqrt{1-Y^2} + \sin^{-1} Y) \qquad (10)$$

where Y is given by

$$Y = \frac{az - bx}{s(z-b)} \qquad (11)$$

The function $(T_x + S_x)(1-P_3)$ is the correction for the transmission penumbral regions.

T_x is the correction for transmission through a diaphragm material and is expressed as

$$T_x = \exp(-\mu' t \sec\theta) \qquad (12)$$

where μ' is the effective absorption coefficient of a diaphragm material, t is the thickness of the diaphragm, and $\sec\theta$ is expressed by $\sqrt{x^2 + z^2}/z$, which is approximately a unity when SSD is large.

S_x is the correction for radiation scattered from the materials near the source and others, and may be given by an empirical formula.

For 4.3 MV X-rays, S_x may be given by the following empirical formula,

$$S_x = 0.15 \exp\{-1.54(x-x_2)\} \qquad (13)$$

where x_2 is a distance from the central axis to the edge of the geometrical field, and $\mu' = 0.513\ \mathrm{cm^{-1}}$ is used in equation (12).

Examples of dose profiles in air and water for 4.3 MV X-rays calculated from these equations are shown in Figs. 7, 8, 9, and 10, in comparison with the experimental values, which were measured with Baldwin Ionex 0.2-cc chamber and with Fujilith contact film. Solid and broken lines are calculated from equation (8) using equation (13) and $S_x = 0$, respectively, and circles are measured. When S_x is zero, a difference is observed between the measured and calculated values of dose in low-dose regions. Although these differences may be ignored because of the discrepancies in the low-dose regions, the results calculated from equation (13) show that the differences can be reduced to approximately 10%.

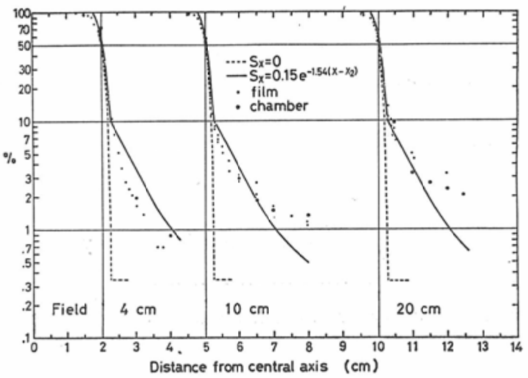

Fig. 7. Comparison of calculated and measured values of dose distributions in air for 4.3 MV X-rays with SDD 51 cm, diaphragm thickness of 11 cm Pb, and SCD 100 cm.

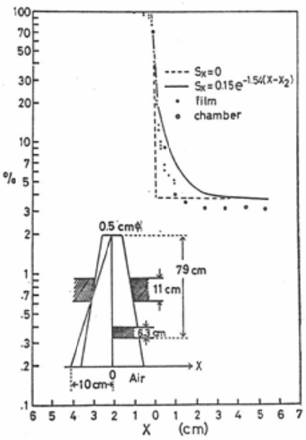

Fig. 9. Comparison of calculated and measured values of dose distributions in air for 4.3 MV X-rays with SDD 79cm, diaphragm thickness of 6.3cm Pb, and SCD100cm.

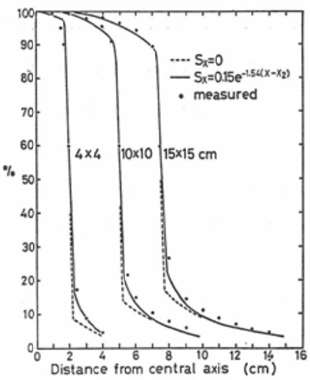

Fig. 8. Comparison of calculated and measured values of dose distributions in water for 4.3 MV X-rays with SDD 51 cm, diaphragm thickness of 11 cm Pb, SSD 85 cm, and depth of 15 cm.

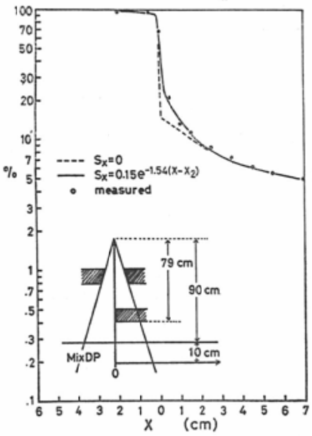

Fig. 10. Comparison of calculated and measured values of dose distributions in MixDP[5] (water-equivalent material) for 4.3 MV X-rays with SDD 79 cm, diaphragm thickness of 6.3 cm Pb, SSD 90 cm and depth of 10 cm.

For telecobalt units which have large geometrical penumbral regions and for X-rays of HVL 1–3 mm Cu, in which a large scattering takes place in water, the difference between the measured and calculated values of dose in water may be small, even if S_x is zero.

4. The correction for oblique incidence and lung for megavoltage radiation

In the case of an irregular field, the theoretical solution for the problem of inhomogeneity and oblique incidence corrections is not easy. Even if one can solve this problem, extensive computer time may be required for the calculation of dose distributions in a real patient situation. We have,

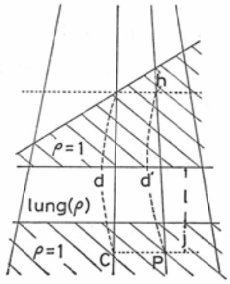

Fig. 11. Symbols used in dosage calculation for oblique incidence and inhomogeneity.

therefore, employed a formulation which is theoretically correct for the primary beam, but not for the scattered radiation, since the scattered dose for megavoltage radiation is small.

In the case of oblique incidence, as shown in Fig. 11, the dose D_p at a point P becomes

$$D_p = D_c^a\left[\mathrm{TAR}\ (0, d' + h)\ P_1 + \frac{1}{n}\sum_{i=1}^{n}\left\{\mathrm{TAR}\ (r_i, d) - \mathrm{TAR}\ (0, d)\right\}\right] \quad \cdots\cdots(14)$$

and the dose D'_p in an inhomogeneous phantom may be expressed as

$$D'_p = S_c D_c^a\left[\mathrm{TAR}\left\{0, (d + h) - l\,(1 - \rho)\right\}\ P_1 + \frac{1}{n}\sum_{i=1}^{n}\left\{\mathrm{TAR}\ (r_i, d) - \mathrm{TAR}\ (0, d)\right\}\right] \quad \cdots\cdots(15)$$

where h is the tissue deficit and/or excess in the case of oblique field, and l and ρ are thickness and density of the lung, respectively. The position of the boundaries and density values for the lung in an individual patient, that is, l and ρ, can be obtained by the transverse axial tomography and the radiographic transit dose measurement. These methods have already been reported[8].

S_c in equation (15) is the scatter correction factor, and it may be expressed as[7]

$$S_c = k\left[1 - \frac{B}{C}\left\{\exp\ (-0.28j) - \exp\ (-0.28\ (l + j))\right\}\right] \quad \cdots\cdots(16)$$

where j is the depth in water-equivalent tissue beyond the lung. When $j = 0$, k is C, and when $j > 0$, k is unity. B and C are given by

$$B = 0.138 - 0.243\rho + 0.105\rho^2 \quad \cdots\cdots(17)$$

$$C = 0.975 + 0.016\rho - 0.086\rho^2 \quad \cdots\cdots(18)$$

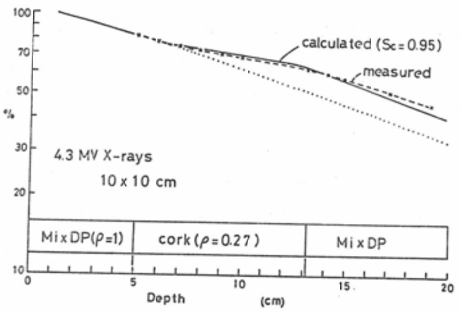

Fig. 12. Comparison of calculated and measured values of central axis depth doses in inhomogeneous phantom for 4.3 MV X-rays

In clinical practice, however, it may be sufficient to use 0.95 for the scatter correction factor S_c.

Fig. 12 shows the central axis depth doses for a 10 × 10 cm field of 4.3 MV X-rays in an inhomogeneous phantom. The discrepancy between calculated and measured values is of the order of 5%, even if the values of 0.95 is used for S_c.

Application

1. Standard isodose curves.

Figs. 13 and 14 show examples of computer printout of the standard isodose curves for HVL 1 mm of Cu X-rays for a field of closed applicator and 4.3 MV X-rays, respectively. In the case of X-rays with HVL 1 mm of Cu, the resultant distribution was corrected by the method of displacement from the data for a diaphragm-limited field, and was calculated as $T_x + S_x = 0$ in equation (8). For 4.3 MV X-rays, equation (13) is used.

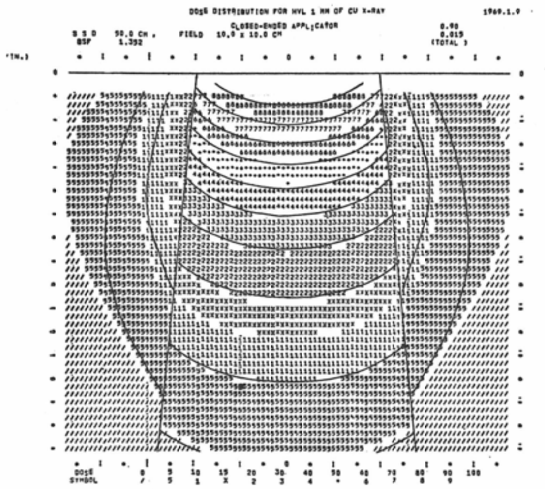

Fig. 13. Comparison of isodose distributions plotted by computer and from values taken from Isodose Chart published by IAEA 1962 for HVL 1 mm of Cu X-rays, with SSD 50cm and 10×10cm field of closed applicator

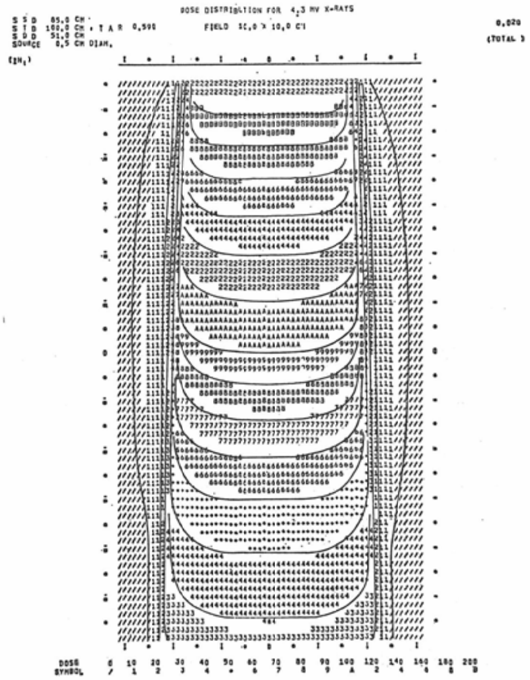

Fig. 14. Comparison of isodose distributions plotted by computer and by measurements for 4.3 MV X-rays, with SSD 85cm and 10 ×10 cm field at STD 100 cm

Printout points are computed dose values and lines are the actual isodose curves, which are measured values for 4.3 MV X-rays and are taken from Isodose Chart published by IAEA in 1962[4] for HVL 1 mm Cu X-rays. The computed dose values agree closely with the experimental values for any point inside the geometrical beam.

2. Irregular field.

Figs. 15 and 16 show an example of the dose distribution for a uterine cervix treatment by

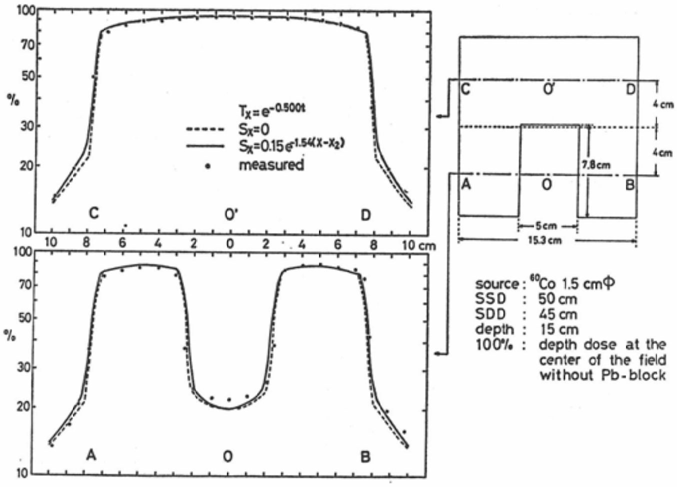

Fig. 15. Shape of field and calculated values of dose profiles for uterine cervix treatment by telecobalt in comparison with measured values

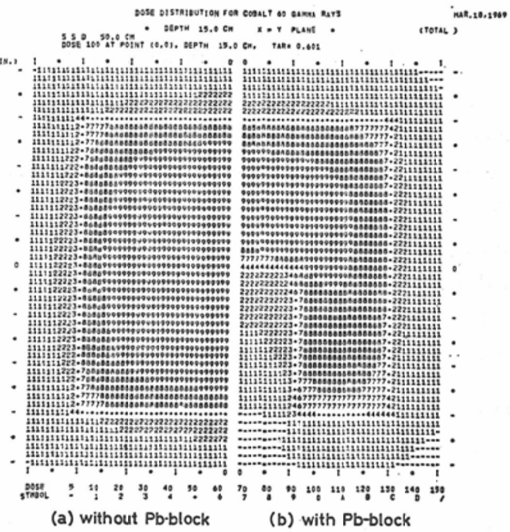

Fig. 16. Computer printout of dose distributions for cross-sectional plane of beam for cervix treatment by telecobalt with field as shown in Fig. 13. (a) Left: without Pb-block (b) Right: with Pb-block

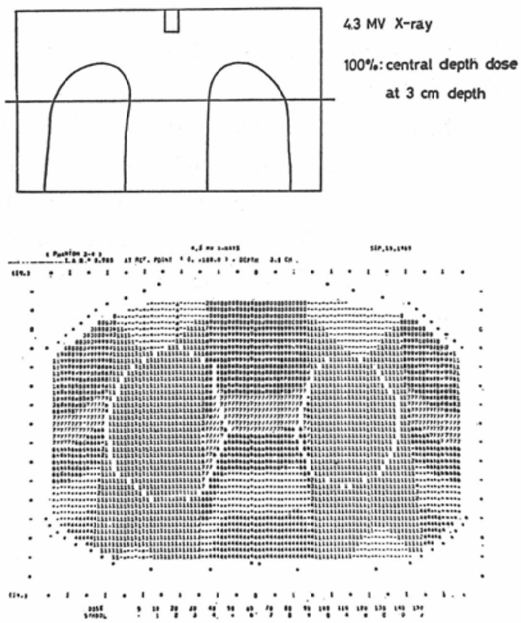

Fig. 17. Shape of field and computer printout of isodose distribution of "mantle technique" irradiation for malignant lymphoma by 4.3 MV X-rays

telecobalt. Fig. 15 shows the shape of the field and calculated values of the dose profile in comparison with the experimental values. Computer printout of the isodose curves for the cross-sectional plane of the beam is shown in Fig. 16. Fig. 16–(a) is the case without a shelter filter in the

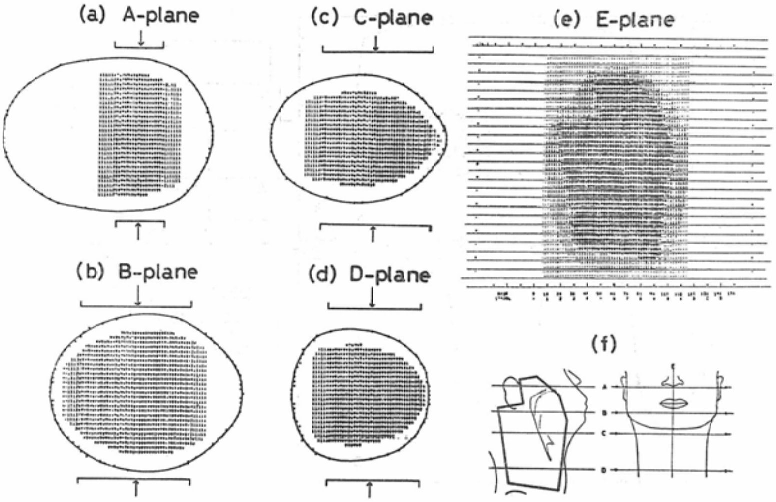

Fig. 18. Dose distributions for pharynx treatment by 6 MV X-rays. (a)-(e) Computer printout of dose distributions for planes as shown in (f). (f) Shape of field and planes chosen for calculation of dose distributions

inner part and Fig. 16–(b) is the case with the shelter filter and shows that the region of the 70% dose exists inside the field.

Fig. 17 is a computer printout of the isodose curves for the parallel plane of the beam for treatment of malignant lymphoma of the breast by 4.3 MV X-rays and the shape of the field. The resultant distribution is corrected for oblique incidence and the lung using equation (15).

Examples of dose distributions for the pharynx treatment by the opposing pairs of 6 MV X-rays are shown in Fig. 18. Fig. 18–(f) shows the shape of the field and the planes chosen for the calculation of the dose distributions, and Figs. 18(a)–(e) are the dose distributions on these planes.

The present programme was written in FORTRAN IV for a GE 635 digital computer.

Summary

A digital computer method is presented for calculating the dose distributions in a variously shaped field for ^{60}Co γ-rays, HVL 1, 2, and 3 mm Cu, and 4.3 and 6 MV X-rays. This method is essentially a mathematical expression of the scatter-air ratio and the dose distribution in air. Correction for oblique incidence and an inhomogeneity is also described. Examples of the computer printout of the dose distributions for a single square field and irregular fields for the treatment of the uterine cervix, malignant lymphoma, and the pharynx are shown.

Acknowledgments

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References

1) Clarkson, J.R.: A note on depth doses in fields of irregular shape, Brit. J. Radiol. 14, (1941) 265–268.
2) Depth dose tables for use in radiotherapy, Brit. J. Radiol. Suppl. No. 10, 1961.
3) Gupta, S.K. and Cunningham, J.R.: Measurement of tissue-air ratios and scatter functions for large field sizes, for cobalt 60 gamma radiation, Brit. J. Radiol. 39, (1966) 7–11.
4) Isodose charts and depth dose tables for medium energy X-rays, IAEA, 1962.
5) Onai, Y. and Kusumoto, G.: Trial production of a water-equivalent solid phantom material, Nipp. Act. Radiol. 19, (1959) 1012–1015.
6) Onai, Y., Irifune, T., Tomaru, T. and Konishi, K.: Calculation of dose distributions in radiation therapy by a digital computer. I. The computation of dose distributions in a homogeneous body for cobalt 60 γ-rays and 4.3 MV X-rays, Nipp. Act. Radiol. 27, (1967) 653–666.
7) Onai, Y., Tomaru, T. and Irifune, T.: Consideration on methods of constructing isodose curves from minimum experimental data. III. Dose distribution in the thorax in 4.3 MV X-ray therapy, Nipp. Act. Radiol. 27, (1968) 1480–1493.
8) Onai, Y., Tomaru, T., Irifune, T. and Uchida, I.: Estimation of average density of lung by a radiographic transit dose method and correction factors for a lesion in the midthoracic oesophagus treated with rotation of 4.3 MV X-rays and cobalt-60 γ-rays, Nipp. Act. Radiol. 29, (1970) 1397–1404.
9) Pfalzner, P.M.: A general formula for axial depth dose derived from an empirical power law for tumour-air ratios, Radiology 75, (1960) 438–445.
10) Worthley, B.W. and Cooper, R.E.M.: Computer-based external beam radiotherapy planning I. Empirical formulae for calculation of depth-doses, Phys. Med. Biol. 12, (1967) 229–240.