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# Validity and Efficiency of Simple Ranking Algorithm for Optimal Portfolio Selection under Limited Diversification<sup>\*</sup>

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## Abstract

In this paper, we analyze the problem of selecting portfolios which maximize the ratio of the average excess return to the standard deviation (equivalently to the Sharpe Ratio), among all those portfolios including the optimal portfolio with the optimal number  $k$  of securities. Under the assumptions of constant pairwise correlations and no short-selling, by using Matlab' programming, we present the simple ranking algorithm (SRA) to reform the simple ranking procedure of Elton, Gruber, and Padberg (1995) effectively solving the problem for all values of  $k$ . The validity and efficiency of the simple ranking algorithm (SRA) will be proved by comparing portfolio investment performance with that by the basic Markowitz (1952)'s nonconstant correlation model .

JEL Classification: G11; G12; D81.

Keyword: Optimal Portfolio Selection; the Simple Ranking Algorithm; Marginal Benefits from Diversification; Nonconstant Correlation Model ; the Sharpe Ratio; the Type of Industry; Constant Pairwise Correlation; No Short-Selling; Limited Diversification.

## 1 Introduction

Mean–variance model, which is nonconstant correlation model , being the foundation of modern portfolio theory, was presented as early as 1952 in Markowitz's pioneering article. In his model, variance is a risk measure to measure risk on risky investment, and risk management will be conducted by measuring the variance of expect return. Before Markowitz presented his theory, the investors found the stocks whose returns were large, and used to put their money choose on these stocks. But at that time, these investors did not pay attention to dispersion of stockkeeper return. Markowitz presented that variance, as a risk measure, can measure risk on risky investment. In Markowitz' model, one should choose the securities whose variance were small even if they had the same expect return.

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However, when a portfolio which includes a large number securities is made, the burden of calculating the security's variance and the variance–covariance matrix of returns is very large, with the shortcoming of the nonconstant correlation model, Elton, Gruber, and Padberg presented the simple ranking procedure solving effectively the problem for all values of  $k$ . Sankaran and Patil (1999) then presented the algorithm of the Elton, Gruber, and Padberg's simple ranking procedure based on the mean–variance model. Using the simple ranking procedure of Elton, Gruber, and Padberg, we can get the optimal portfolio whose expect return is the biggest, at the same time, the optimal number  $k$  of securities is also decided. We model the simple ranking procedure of Elton, Gruber, and Padberg by Matlab, through Matlab programming. By the algorithm, the optimal portfolio will be got, and the optimal number of securities will be decided. We make the problem of selecting the optimal portfolio is more simply and perfectly, the method will be beneficial to the investor or risk management, and so on.

One basic implication of modern portfolio theory is that investors hold well–diversified portfolios. However, there is empirical evidence that individual investors typically hold only a small number of securities.<sup>1</sup>

There exist several practical reasons why a small investor failed to make this compromise in the best possible manner. Besides saving on transaction, market imperfections such as fixed transaction costs provide one explanation for the prevalence of undiversified portfolios. A small investor who chooses to invest in only a limited number of securities can devote more attention to the individual behavior of those securities and their mean–variance characteristics. Thirdly, the recent empirical evidence on the relation between risk and return on stocks, which suggests that diversification beyond 8 – 10 securities may not be worthwhile. Also, the existing empirical evidence on the benefits of diversification as a function of the number of securities held in the portfolio has been based invariably on the principle of random selection of securities, which tends to bias the comparison of actual alternatives in favor of mutual fund selection. The third reason also own to Szego (1980) who emphasizes the point that the variance–covariance matrix of returns of a large size portfolio tends to conceal significant singularities or near–singularities, so that enlarging the portfolio beyond the limited diversification size may be superfluous.<sup>2</sup>

With the reason of not being well–diversified and the complex of calculating the variance–covariance matrix of returns of a large size portfolio, we should find an efficiency and validity algorithm to replace the nonconstant correlation model to deal with the problem of selecting optimal portfolio and determining the optimal weights. If we know the number of the securities and the characteristic of these securities, how can we choose the securities to compose the portfolio that makes us to get the maximum return, simultaneously, how can we find the optimal portfolio?

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<sup>1</sup> See Jacob (1974).

<sup>2</sup> Some of researchers, such that Sengupta and Sfeir (1995), Szego (1980), who also observe that the variance–covariance matrix of the returns on the securities in a portfolio that has a large number of securities tends to conceal significant singularities or near–singularities. They also suggest that it may therefore be superfluous to enlarge the number of securities in a portfolio beyond a limited.

lio investment weight. Some of investors select the optimal portfolio by using the Sharpe Ratio.<sup>3</sup> and effectively determine the optimal weights of a optimal portfolio by using the simple ranking procedure of Elton, Gruber, and Padberg (1995). In this paper, under the assumptions of constant pairwise correlations and no short-selling, by using Matlab' programming, we present the simple ranking algorithm (SRA) to reform the simple ranking procedure of Elton, Gruber, and Padberg (1995) effectively solve the optimal portfolio selection problem.

It is easy to solve the problem of determining the optimal weights in a portfolio that comprises a given subset of securities in the universe at a variety of situations by simple ranking algorithm (EGP).<sup>4</sup>

We reform the simple ranking algorithm by Matlab, The reformation of the simple ranking algorithm (SRA) can deal with the problem of determining the optimal weight in a portfolio with massive dates and securities. The simple ranking algorithm can also solve the problem of determining an optimal portfolio that comprises at most a given number of securities from the universe. There is no restriction on the input date, not only the number of the input dates, but also the style of the securities. It is the only one condition that the Sharpe ratios should be positive. If the efficiency and adequacy of the simple ranking algorithm (SRA) can be proved, we can say SRA can be used efficiently to select the optimal portfolio and determine the optimal weights, and the time of calculation and error coming from the calculation of the large scale of securities' the variance-covariance matrix of returns.

With the purpose, we will prove the validity and efficiency of the simple ranking algorithm (SRA) by an empirical analysis of comparing the investment performance to that of the nonconstant correlation model .

The note is organized as follows. In Section 2, we model the problem formally. In section 3, we present the algorithm in detail. Section 4 illustrate the result on empirical analysis. The examination and the conclusion are described in Section 5.

## 2 Notations and Model

At first, we will introduce the notation before we present the model:

- $n \in \mathbb{Z}_{++} := \{1, 2, \dots\}$ : the number of securities in the universe;
- $N$ : the set of securities in the universe, i.e.,  $N := \{1, \dots, n\}$ ;

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<sup>3</sup> See Sharpe (1963).

<sup>4</sup> Elton, Gruber, and Padberg (1995) address the problem of selecting portfolios which maximize the ratio of the average excess return to the standard deviation, equivalently to the Sharpe Ratio, among all those portfolios which comprise at most a pre-specified number,  $k$ , of securities from among the  $n$  securities that comprise the universe. A  $k$ -optimal portfolio as one that maximizes the ratio of the average excess return to the standard deviation over all portfolios that comprise at most  $k$  securities ( $1 \leq k \leq n$ ). Under the assumptions of constant pairwise correlations and no short-selling, the simple ranking procedure of Elton, Gruber, and Padberg (1995) effectively solving the problem for all values of  $k$ , and that as a function of  $k$ , the optimal ratio increases at a decreasing rate.

- $k$ : the pre-specified upper limit on the number of securities in the portfolio ( $1 \leq k \leq n$ );
- $x_i$  the weight of security  $i \in N$  (it is assumed that  $x_i \geq 0$  for all  $i$ );
- $r_f$ : the rate of return on the riskless asset;
- $\bar{r}_i$ : the expected rate of return on security  $i \in N$ ;
- $\sigma_i (> 0)$ : the standard deviation of the rate of return on security  $i \in N$ ;
- $b_i := (\bar{r}_i - r_f)/\sigma_i$ : the Sharpe ratio of security  $i \in N$  defined as the ratio of the average excess return to the standard deviation of the rate of return on security  $i$ ;
- $\rho$ : an estimate of the (average) correlation coefficient of any pair of security returns (it is assumed that  $\rho \geq 0$ );
- $C_t$ : the cut-off value of securities.

Under the assumption of constant coefficient of correlation and no short-selling, the investor's problem can be formulated as follows:

$$\text{Maximize} \quad \frac{\sum_{i=1}^n (\bar{r}_i - r_f) x_i}{\sqrt{\sum_{i=1}^n \sigma_i^2 x_i^2 + \rho \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_i x_j}} \quad (1)$$

$$\text{subject to} \quad x_i \geq 0, \quad i = 1, \dots, n; \quad (2)$$

$$\text{at most } k \text{ of } \{x_i | i = 1, \dots, n\} \text{ are strictly positive.} \quad (3)$$

An optimal solution of the above problem is called as a  $k$ -optimal portfolio.

Let  $F$  be an arbitrary subset of  $N$ , and  $w(F)$  denote the maximum value of Sharpe ratio of portfolios which are composed of only securities in  $F$ . Formally,  $w(F)$  is defined as the maximum value of the following portfolio selection problem:

$$\text{Maximize} \quad \frac{\sum_{i \in F} (\bar{r}_i - r_f) x_i}{\sqrt{\sum_{i \in F} \sigma_i^2 x_i^2 + \rho \sum_{i \in F} \sum_{j \in F, j \neq i} \sigma_i \sigma_j x_i x_j}} \quad (4)$$

$$\text{subject to} \quad x_i \geq 0, \quad i \in F. \quad (5)$$

For a subset  $F$  of  $N$ , let  $|F|$  denote the cardinality of  $F$ . Then, our problem (1) – (3) could be expressed as follows:

$$\text{Maximize } w(F) \text{ subject to } F \subset N \text{ and } |F| \leq k. \quad (6)$$

### 3 Algorithm and Programming

Without any loss of generality, we first assume that the securities in the universe are numbered in a descending order of  $b_i$ ,  $i = 1, \dots, n$ , so that  $b_1 \geq b_2 \geq \dots \geq b_n$ . For an arbitrary subset  $F$  of  $N$  and for  $t = 1, \dots, |F|$ , let  $i(t; F)$  denote the (or a) security with the  $t$ -th largest value of  $b$  among the securities in  $F$ ;

$$\begin{aligned} F &= \{i(t; F) \mid t = 1, \dots, |F|\}; \\ i(1; F) &< i(2; F) < \dots < i(|F|; F); \\ b_{i(1; F)} &\geq b_{i(2; F)} \geq \dots \geq b_{i(|F|; F)}. \end{aligned}$$

Sankaran and Patil (1999) proposed an algorithm for solving the maximization problem (1) – (3) based on the following Simple Ranking Algorithm (SRA) proposed by Elton, Gruber, and Padberg (1976, 1977, 1978). For an arbitrary subset  $F$  of  $N$  as an input, it computes a portfolio composed of securities in a subset  $S_F$  of securities from  $F$ .

**Algorithm 1** (Simple Ranking Algorithm (SRA)).

**Input:** an arbitrary nonempty subset  $F$  of  $N = \{1, \dots, n\}$ ;

**Output:** a portfolio composed of securities in a subset from  $F$ , namely,  $S_F$ .

**Step 1:** If  $b_{i(1; F)} \leq 0$ , then set  $t := 0$  and go to **Step 4**; else, initialize as  $t := 1$ .

**Step 2:** If  $t \geq |F|$  or

$$b_{i(t+1; F)} \leq \rho \frac{\sum_{u=1}^t b_{i(u; F)}}{(t-1)\rho + 1}, \quad (7)$$

then go to **Step 4**; else,  $t := t + 1$ .

**Step 3:** Go to **Step 2**.

**Step 4:** Set

$$S_F := \{i(u; F) \mid u = 1, \dots, t\}, \quad (8)$$

and construct the portfolio weights  $\{x_i \mid i \in F\}$  as follows:

$$x_{i(u; F)} \propto \frac{1}{\sigma_{i(u; F)}} \left( b_{i(u; F)} - \rho \frac{\sum_{u=1}^t b_{i(u; F)}}{(t-1)\rho + 1} \right), \quad i = 1, \dots, t; \quad (9)$$

$$x_{i(u; F)} := 0, \quad i = t+1, \dots, |F|. \quad (10)$$

□

Step 2 in SRA represents the search for the optimal cut-off value for the Sharpe ratio to be included in the portfolio. Thus, those securities in  $F$  with Sharpe ratios that are greater than the cut-off have the positive weights, while others in  $F$  with Sharpe ratios that are not greater than the cut-off have zero weight.<sup>5</sup>

For the validity of SRA, Sankaran and Patil (1999) proved the following propositions and corollary.

**Proposition 1.** Let  $F$  denotes an arbitrary nonempty subset of  $N$ , then

$$w(F) = \sqrt{\frac{1}{1-\rho} \left( \sum_{i \in S_F} b_i^2 - \frac{\rho \left( \sum_{i \in S_F} b_i \right)^2}{\rho(|S_F| - 1) + 1} \right)}. \quad (11)$$

Further, the portfolio that attains  $w(F)$  is given by Equ. (9) and (10). □

**Proposition 2.** Let  $F$  denotes an arbitrary subset of  $N$  containing  $m$  ( $2 \leq m \leq n$ ) securities such that  $S_F = F$ , and let  $\ell$  denote the largest-numbered security in  $F$ . (Thus,  $\ell$  has the smallest value of Sharpe ratio  $b_i$  among all the securities in  $F$ .) If  $j$  is the a security which is not in  $F$  such that  $j < \ell$ , then we have

$$w((F \cup \{j\}) \setminus \{\ell\}) \geq w(F). \quad (12)$$

□

**Corollary 1.** There is a  $k$ -optimal portfolio which is composed of securities  $\{1, \dots, t\}$  for some  $t \leq k$ . Further, the simple ranking algorithm SRA finds such a portfolio when  $F$  is defined as  $\{1, \dots, k\}$ . □

Corollary 1 implies that the following algorithm finds a  $k$ -optimal portfolio for all values of  $k \leq n$ , which is proposed by Sankaran and Patil (1999) as an extension of the simple ranking algorithm SRA. It will be beneficial to calculate the optimal weights of portfolio selection problem (1) – (3) under limited diversification.

**Algorithm 2.**

**Step 0:** Renumber the securities so that the Sharpe ratios  $b_i, i = 1, \dots, n$  are ordered in a descending order. The 1-optimal portfolio comprises only security 1.

**Step 1:** Initialize as  $k = 2$ .

**Step 2:** If

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<sup>5</sup> See Elton and Gruber (1995).

$$b_k \leq \rho \frac{\sum_{j=1}^{k-1} b_j}{(k-2)\rho + 1}, \quad (13)$$

then go to Step 4;

**Step 3:** The  $k$ -optimal portfolio comprises securities 1 to  $k$ , and the optimal weight of security  $i$  ( $= 1, \dots, k$ ) is proportional to

$$\frac{1}{\sigma_i} \left( b_i - \rho \frac{\sum_{j=1}^k b_j}{(k-1)\rho + 1} \right). \quad (14)$$

Make an increment as  $k := k + 1$ . If  $k \leq n$  then go to Step 2.

**Step 4:** Set  $K := k - 1$  and stop; for all  $k > K$ , the  $k$ -optimal portfolio is identical to the  $K$ -optimal portfolio.  $\square$

Using Matlab, the above algorithm can be written as follows:

- a. input  $\rho$  and index ( $n$ ) at random, we can choose the pairwise correlation  $\rho$  and  $n$  as we want.  
↓
- b. input  $b_i, i = 1, \dots, n$ , here user can input  $b_i, i = 1, \dots, n$  of all kinds of securities.  
↓
- c. arranging  $b_i, i = 1, \dots, n$  in descending order, the programming can arrange  $b_i$  in descending order automatically. It is beneficial to users who need input many  $b_i$ .  
↓
- d. calculating  $C_t, t = 1, \dots, n$ .  
↓
- e. finding the optimal number  $t$  of securities among  $n$ .

## 4 Empirical Analysis

We use part of NIKKEI needs index of Tokyo securities's type of industry average stock monthly price date to calculate the performance to compare the performance of Nonconstant Correlation Model and the simple produce by Elton, Gruber. We also use LIBOR yearly interest rate date as the rate of return on the riskless asset. The in-the-sample date is from 1981.1 to 1985.12; the out-of-sample date is from 1986.1 to 1990.12. We use the date that was not the current dates, because the

finance market in Japan was very stable, before the Bubble economy happened to be broken, the return of stocks were positive.<sup>6</sup> In the period of in-the-sample, the optimal weight of the optimal portfolio will be calculated, in the period of out-of-sample, we used the outcome of the optimal weight to construct portfolio, and then estimate the performance of the two models. We should pay attention to the period of in-the-sample, the style of the period of in-the-sample is rolling, so the beginning monthly date will be replaced by the first monthly date of the out-of-sample. We calculate each of 60 monthly dates by the way of rolling.

#### 4.1 Calculation by The Simple Ranking Algorithm (SRA) of Elton and Gruber

Full historical model is one of useful models. Using the model, we calculate each pairwise correlation coefficient over a historical period and use this value as an estimate of the future. No assumptions are made as to how or why any pair of securities might move together. Instead, the amount of their co-movement is estimated directly. The most aggregate type of averaging that can be done is to use the average of all pairwise correlation coefficient for the future. this is equivalent to the assumption that the past correlation matrix contains information about what the average correlation will be in the future but no information about individual differences from this average. The average correlation models can be thought of as a naive model against which more elaborate models should be judged.

The methods of selecting optimal portfolios that are appropriate when the single-index model and the constant-correlation model are accepted as descriptions of the covariance structure between securities. Here, there is an assumption that the programming is made based on the average correlation models, so the correlation is constant.<sup>7</sup>

We calculated the optimal weights by the Simple Ranking Algorithm (SRA) by using constant  $\rho$ . we will calculate singularly with different  $\rho$ ,  $\rho = 1/2$ , and  $\rho = 1/3$  and  $\rho = 2/3$ .

#### 4.2 Calculation by Nonconstant Correlation Model

The Nonconstant Correlation Model are formulated as below:

$$\text{Maximize} \quad \frac{\sum_{i=1}^n (\bar{r}_i - r_f) x_i}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_j x_i}} \quad (15)$$

$$\text{subject to} \quad \sum_{i=1}^n x_i = 1; \quad (16)$$

$$x_i \geq 0, \quad i = 1, \dots, n. \quad (17)$$

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<sup>6</sup> The model (SRA) we made should use the positive Sharpe Ratio.

<sup>7</sup> The theory of computational complexity implies that the problem of finding the  $k$ -optimal portfolios for all the values of  $k (k : 1 \rightarrow n)$  is impossible to be efficiently solvable under the single-index model of stock returns (Blog et al. (1983)).

The problem is a quadratic programming problem. In order to deal with the above optimization problem, we should get the variance–covariance matrix at first. The different between the two models is just pairwise correlations because pairwise correlations is not constant in tradition Markowitz model. When we calculate the variance–covariance in the period of out–of–sample, we choose the period just like the period of calculating the simple ranking algorithm (EGP) of Elton and Gruber. The results of the optimal weights in a portfolio by using nonconstant correlation model are presented at Table 6 – 16.<sup>8</sup>

## 5 Examination and Conclusion

In this section, by using the optimal weights by nonconstant correlation model and the simple ranking algorithm (SRA), monthly portfolio's returns are calculated at the period of out–of–sample (1986.1 – 1990.12), and then based on the monthly portfolio's returns, we get yearly return and calculate the mean and variance of the yearly returns, finally, we compare the investment performance of two models by using the mean and variance of the yearly returns.

In Table 17, from monthly portfolio return, the mean and standard deviation of yearly portfolio return are be showed at the period of out–of–sample, the transition of the ratio (mean/standard deviation) are revealed at Figure 1, and with the different  $\rho$ , from 1986 to 1990, the ratios are showed by the two model at Table 18.

In figure 1, NCM is nonconstant correlation model, which is traditional mean-variance model, also. From Figure 1, we can clearly know that portfolio performance based on the simple ranking algorithm (SRA) is not worse than that of the nonconstant correlation model by using the dates that we choose, and with the assumption of the constant pairwise correlation, the conclusion can be got. Though the constant correlation ( $\rho$ ) are set by  $\rho = 1/2$ ,  $\rho = 1/3$  and  $\rho = 2/3$ , the outcomes are the same. Different  $\rho$  cause different portfolios and different yearly returns in the period of out–of–sample. But we can make a conclusion that the simple ranking algorithm can be widely used, because the method is easier more to calculate than the traditional nonconstant correlation model. It is very difficult to estimate the variance–covariance matrix when faced large–scale portfolio selection problem. Even faced the large–scale portfolio selection problem, we still do not need spent much time to calculate the variance–covariance matrix, and avoid computational errors.

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<sup>8</sup> The industry of agriculture, forestry and fisheries (aff).

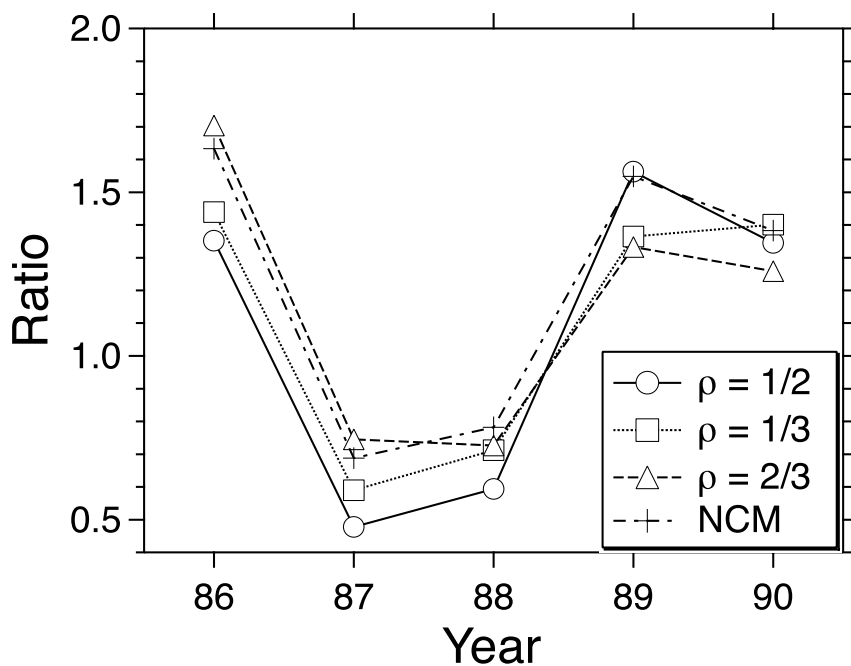


Figure 1:

Table 1: Each industry's Sharpe ratio from 1981.1 to 1985.12

| type of industry      | Sharpe Ratio | type of industry     | Sharpe Ratio |
|-----------------------|--------------|----------------------|--------------|
| aff                   | 0.541379     | mining               | 0.965454     |
| building              | 0.999038     | grocery              | 0.95056      |
| fiber manufacture     | 0.885878     | valve.paper          | 0.963169     |
| medicament            | 0.788276     | oil.coal             | 0.751068     |
| rubble                | 1.050848     | glass.soil.stone     | 1.085672     |
| steel                 | 0.717836     | hardware             | 1.059506     |
| machinery             | 1.074802     | electric manufacture | 0.790076     |
| transport application | 1.172873     | electricity gas      | 0.192622     |
| transport             | 0.788329     | shipping             | 0.667343     |
| airlift               | 0.741498     | IT                   | 0.794956     |
| other instrument      | 1.074064     | precision instrument | 0.810726     |
| commerce              | 0.534361     | real estate          | 0.954462     |
| service               | 1.209829     | finance.insurance    | 0.570315     |
| nonferrous metal      | 0.183212     | warehouse            | 0.173514     |

Table 2: the selection of risky securities in the optimal portfolio in the period of in-the-sample by SRA ( $\rho = 1/2$ )

| type of industry      | $\rho/(1 - \rho + t\rho)$ | $\sum b_i$ | $C_t$     | Sharpe Ratio |
|-----------------------|---------------------------|------------|-----------|--------------|
| service               | 50%                       | 1.209829   | 0.604915  | 1.2098294    |
| transport application | 33%                       | 2.382702   | 0.794155  | 1.172873     |
| glass.soil.stone      | 25%                       | 3.468374   | 0.867094  | 1.085672     |
| machinery             | 20%                       | 4.543177   | 0.908635  | 1.074802     |
| other instrument      | 16.67%                    | 5.61724    | 0.936394  | 1.074064     |
| hardware              | 14.29%                    | 6.676747   | 0.954107  | 1.059506     |
| rubble                | 12.5%                     | 7.727594   | 0.965949  | 1.050848     |
| building              | 11.11%                    | 8.726632   | 0.969626  | 0.999038     |
| mining                | 10%                       | 9.692086   | 0.969209  | 0.965454     |
| valve.paper           | 9.0909%                   | 10.65526   | 0.968659  | 0.963169     |
| real estate           | 8.333%                    | 11.60972   | 0.967473  | 0.954462     |
| grocery               | 7.6923%                   | 12.56028   | 0.966174  | 0.95056      |
| fiber manufacture     | 7.14285%                  | 13.44615   | 0.960439  | 0.885878     |
| precision instrument  | 6.66667%                  | 14.25688   | 0.950458  | 0.810726     |
| IT                    | 6.25%                     | 15.05184   | 0.94074   | 0.794956     |
| electric manufacture  | 5.88235%                  | 15.84191   | 0.931877  | 0.790076     |
| transport             | 5.55556%                  | 16.63024   | 0.923902  | 0.788329     |
| medicament            | 5.26315%                  | 17.41852   | 0.916763  | 0.788276     |
| oil.coal              | 5%                        | 18.16959   | 0.908479  | 0.751068     |
| airlift               | 4.7619%                   | 18.91108   | 0.900527  | 0.741498     |
| steel                 | 4.54545%                  | 19.62892   | 0.892223  | 0.717836     |
| shipping              | 4.34782%                  | 20.29626   | 0.882445  | 0.667343     |
| finance.insurance     | 4.16667%                  | 20.86658   | 0.869439  | 0.570315     |
| aff                   | 4%                        | 21.40796   | 0.856318  | 0.541379     |
| commerce              | 3.84615%                  | 21.94232   | 0.843934  | 0.534361     |
| electricity gas       | 3.7037%                   | 22.13494   | 0.819812  | 0.192622     |
| nonferrous metal      | 0.035714                  | 22.318152  | 0.797075  | 0.183212     |
| warehouse             | 0.034483                  | 22.491666  | 0.7755726 | 0.173514     |

At the Table 2,  $t$  is the number of securities in the portfolio.

Table 3: the selection of risky securities in the optimal portfolio in the period of in-the-sample by SRA ( $\rho = 1/3$ )

| type of industry      | $\rho/(1 - \rho + t\rho)$ | $\sum b_i$ | $C_t$     | Sharpe Ratio |
|-----------------------|---------------------------|------------|-----------|--------------|
| service               | 33%                       | 1.209829   | 0.403236  | 1.2098294    |
| transport application | 25%                       | 2.382702   | 0.595675  | 1.172873     |
| glass.soil.stone      | 20%                       | 3.468374   | 0.693675  | 1.085672     |
| machinery             | 16.67%                    | 4.543177   | 0.757348  | 1.074802     |
| other instrument      | 14.29%                    | 5.61724    | 0.802704  | 1.074064     |
| hardware              | 12.5%                     | 6.676747   | 0.834593  | 1.059506     |
| rubble                | 11.11%                    | 7.727594   | 0.858622  | 1.050848     |
| building              | 10%                       | 8.726632   | 0.872663  | 0.999038     |
| mining                | 9.090909%                 | 9.692086   | 0.881098  | 0.965454     |
| vavl.paper            | 8.333%                    | 10.65526   | 0.887934  | 0.963169     |
| real estate           | 7.6923%                   | 11.60972   | 0.893054  | 0.954462     |
| grocery               | 7.14285%                  | 12.56028   | 0.897162  | 0.95056      |
| fiber manufacture     | 6.6667%                   | 13.44615   | 0.896409  | 0.885878     |
| precision instrument  | 6.25%                     | 14.25688   | 0.891055  | 0.810726     |
| IT                    | 5.88235%                  | 15.05184   | 0.885402  | 0.794956     |
| electric manufacture  | 5.55556%                  | 15.84191   | 0.880106  | 0.790076     |
| transport             | 5.26315%                  | 16.63024   | 0.875275  | 0.788329     |
| medicament            | 5%                        | 17.41852   | 0.870926  | 0.788276     |
| oil.coal              | 4.7619%                   | 18.16959   | 0.865217  | 0.751068     |
| airlift               | 4.54545%                  | 18.91108   | 0.859594  | 0.741498     |
| steel                 | 4.34782%                  | 19.62892   | 0.85343   | 0.717836     |
| shipping              | 4.16667%                  | 20.29626   | 0.845676  | 0.667343     |
| finance.insurance     | 4%                        | 20.86658   | 0.834663  | 0.570315     |
| aff                   | 3.84615%                  | 21.40796   | 0.823382  | 0.541379     |
| commerce              | 3.7037%                   | 21.94232   | 0.812678  | 0.534361     |
| electricity gas       | 3.57%                     | 22.13494   | 0.790217  | 0.192622     |
| nonferrous metal      | 0.034483                  | 22.318152  | 0.7695901 | 0.183212     |
| warehouse             | 0.03333                   | 22.491666  | 0.7496472 | 0.173514     |

Table 4: the selection of risky securities in the optimal portfolio in the period of in-the-sample by SRA ( $\rho = 2/3$ )

| type of industry      | $\rho/(1 - \rho + t\rho)$ | $\sum b_i$ | $C_t$     | Sharpe Ratio |
|-----------------------|---------------------------|------------|-----------|--------------|
| service               | 67%                       | 1.209829   | 0.810586  | 1.2098294    |
| transport application | 40%                       | 2.382702   | 0.953081  | 1.172873     |
| glass.soil.stone      | 28.57%                    | 3.468374   | 0.990915  | 1.085672     |
| machinery             | 22.22%                    | 4.543177   | 1.009494  | 1.074802     |
| other instrument      | 18.20%                    | 5.61724    | 1.022338  | 1.074064     |
| hardware              | 15.40%                    | 6.676747   | 1.028219  | 1.059506     |
| rubble                | 13.30%                    | 7.727594   | 1.02777   | 1.050848     |
| building              | 11.80%                    | 8.726632   | 1.029743  | 0.999038     |
| mining                | 10.50%                    | 9.692086   | 1.017669  | 0.965454     |
| vavle.paper           | 9.50%                     | 10.65526   | 1.012249  | 0.963169     |
| real estate           | 8.70%                     | 11.60972   | 1.010045  | 0.954462     |
| grocery               | 8.0%                      | 12.56028   | 1.004822  | 0.95056      |
| fiber manufacture     | 7.40%                     | 13.44615   | 0.995015  | 0.885878     |
| precision instrument  | 6.90%                     | 14.25688   | 0.983725  | 0.810726     |
| IT                    | 6.50%                     | 15.05184   | 0.978369  | 0.794956     |
| electric manufacture  | 6.10%                     | 15.84191   | 0.966357  | 0.790076     |
| transport             | 5.70%                     | 16.63024   | 0.947924  | 0.788329     |
| medicament            | 5.40%                     | 17.41852   | 0.9406    | 0.788276     |
| oil.coal              | 5.10%                     | 18.16959   | 0.926649  | 0.751068     |
| airlift               | 4.90%                     | 18.91108   | 0.926643  | 0.741498     |
| steel                 | 4.70%                     | 19.62892   | 0.922559  | 0.717836     |
| shipping              | 4.40%                     | 20.29626   | 0.893035  | 0.667343     |
| finance.insurance     | 4.30%                     | 20.86658   | 0.897263  | 0.570315     |
| aff                   | 4.1%                      | 21.40796   | 0.877726  | 0.541379     |
| commerce              | 3.90%                     | 21.94232   | 0.855575  | 0.534361     |
| electricity gas       | 3.80%                     | 22.13494   | 0.841128  | 0.192622     |
| nonferrous metal      | 0.0363636                 | 22.318152  | 0.8115683 | 0.183212     |
| warehouse             | 0.03508772                | 22.491666  | 0.7891808 | 0.173514     |

Table 5: monthly return of portfolio in the period of out-of-sample by SRA

| year/month | $\rho = 1/2$ | $\rho = 1/3$ | $\rho = 2/3$ | year/month | $\rho = 1/2$ | $\rho = 1/3$ | $\rho = 2/3$ |
|------------|--------------|--------------|--------------|------------|--------------|--------------|--------------|
| 1986/1     | 0.24928      | 0.191        | 0.364811     | 1988/7     | 0.2654       | 0.5212       | 0.3212       |
| 1986/2     | 0.417051     | 0.2945       | 2.2201       | 1988/8     | 0.1121       | 0.4321       | 0.3251       |
| 1986/3     | 0.64889      | 0.441        | 0.93362      | 1988/9     | 0.2119       | 0.2412       | 0.4321       |
| 1986/4     | 0.2087424    | 0.80255      | 0.3726814    | 1988/10    | 0.2316       | 0.2908       | 0.3213       |
| 1986/5     | 1.018573     | 0.3429       | 1.226148     | 1988/11    | 0.7871       | 0.6652       | 0.4241       |
| 1986/6     | 0.5234       | 0.1177       | 0.34736      | 1988/12    | 1.068        | 1.121        | 1.3212       |
| 1986/7     | 2.37522      | 0.681        | 2.45401      | 1989/1     | 1.2101       | 1.4021       | 1.3943       |
| 1986/8     | 0.525774     | 0.153        | 0.524119     | 1989/2     | 2.627        | 2.3617       | 2.1627       |
| 1986/9     | 0.0334       | 0.02         | 0.014399     | 1989/3     | 1.5284       | 1.284        | 1.354        |
| 1986/10    | 1.8111       | 1.4.491      | 1.01556      | 1989/4     | 2.01         | 2.411        | 2.322        |
| 1986/11    | 0.1431       | 0.122        | 0.19927      | 1989/5     | 0.76         | 0.7423       | 0.5243       |
| 1986/12    | 0.383518     | 0.277        | 0.38953      | 1989/6     | 0.685022     | 0.257989     | 0.8112       |
| 1987/1     | 0.85929      | 0.2946       | 0.803258     | 1989/7     | 0.3788       | 0.1265       | 0.1818       |
| 1987/2     | 0.381836     | 0.1334       | 0.781673     | 1989/8     | 0.50513      | 0.51182      | 0.4082       |
| 1987/3     | 0.136932     | 0.337        | 0.4588       | 1989/9     | 4.109667     | 5.41945      | 1.167503     |
| 1987/4     | 1.258157     | 1.22841      | 1.255757     | 1989/10    | 2.1029       | 2.3112       | 2.9721       |
| 1987/5     | 0.633348     | 0.36801      | 0.6371483    | 1989/11    | 0.1219       | 0.6211       | 0.3421       |
| 1987/6     | 1.42788      | 0.85         | 1.50127      | 1989/12    | 1.616        | 0.6754       | 0.6545       |
| 1987/7     | 1.202923     | 1.529        | 1.296955     | 1990/1     | 1.2668       | 2.3212       | 2.5241       |
| 1987/8     | 1.590413     | 1.2547       | 1.58845      | 1990/2     | 0.3128       | 0.2383       | 0.3128       |
| 1987/9     | 0.19         | 0.5137       | 0.883456     | 1990/3     | 0.2424       | 0.1271       | 0.3217       |
| 1987/10    | 1.89422      | 1.894        | 1.14885      | 1990/4     | 0.4412       | 0.4114       | 0.3871       |
| 1987/11    | 1.223        | 1.979        | 1.443        | 1990/5     | 0.1759       | 0.68654      | 0.81306      |
| 1987/12    | 1.669        | 1.5604       | 1.669        | 1990/6     | 0.131059     | 0.1415       | 0.27         |
| 1988/1     | 0.113        | 0.2121       | 0.2421       | 1990/7     | 1.63894      | 1.18569      | 1.29505      |
| 1988/2     | 0.413        | 0.5112       | 0.32212      | 1990/8     | 1.80086      | 1.79         | 1.81917      |
| 1988/3     | 0.6721       | 0.5721       | 0.4542       | 1990/9     | 1.08547      | 1.26117      | 0.78835      |
| 1988/4     | 0.2323       | 0.3452       | 0.6101       | 1990/10    | 0.7462       | 0.35693      | 0.477856     |
| 1988/5     | 0.2121       | 0.4249       | 0.3332       | 1990/11    | 0.302        | 0.322944     | 0.310864     |
| 1988/6     | 0.5212       | 0.4241       | 0.4323       | 1990/12    | 0.183        | 0.165        | 0.197        |

Table 6: the optimal weights of the optimal portfolio in the period of out-of-sample (1986.1 – 6) by nonconstant correlation model

| type of industry      | 1986/1   | 1986/2   | 1986/3   | 1986/4    | 1986/5   | 1986/6   |
|-----------------------|----------|----------|----------|-----------|----------|----------|
| aff                   | 0.042513 | 0.051214 | 0.054401 | 0.048752  | 0.033423 | 0.037414 |
| mining                | 0        | 0        | 0        | 0         | 0        | 0        |
| building              | 0        | 0        | 0        | 0         | 0        | 0        |
| grocery               | 0.462206 | 0.47437  | 0.508294 | 0.421675  | 0.374468 | 0.38449  |
| fiber manufacture     | 0        | 0        | 0        | 0         | 0        | 0        |
| valv.paper            | 0        | 0        | 0        | 0         | 0        | 0        |
| medicament            | 0        | 0        | 0        | 0         | 0        | 0        |
| oil.coal              | 0        | 0        | 0        | 0         | 0        | 0        |
| rubble                | 0        | 0        | 0        | 0         | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0         | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0         | 0        | 0        |
| hardware              | 0        | 0        | 0.010708 | 0         | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0         | 0        | 0        |
| electric manufacture  | 0        | 0        | 0        | 0         | 0        | 0        |
| transport application | 0        | 0.136473 | 0.177653 | 0.1888373 | 0.211719 | 0.199635 |
| finance.insurance     | 0        | 0        | 0        | 0         | 0        | 0        |
| electricity gas       | 0        | 0.03661  | 0.057794 | 0.100686  | 0.111957 | 0.141276 |
| transport             | 0        | 0        | 0        | 0         | 0        | 0        |
| shipping              | 0        | 0        | 0        | 0         | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0         | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0         | 0        | 0        |
| other instrument      | 0        | 0        | 0        | 0         | 0        | 0        |
| commerce              | 0        | 0        | 0        | 0         | 0        | 0        |
| precision instrument  | 0.118258 | 0.08704  | 0.05112  | 0.240516  | 0.268434 | 0.237186 |
| real estate           | 0        | 0        | 0        | 0         | 0        | 0        |
| service               | 0.377023 | 0.217294 | 0.140032 | 0         | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0         | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0         | 0        | 0        |

Table 7: the optimal weights of the optimal portfolio in the period of out-of-sample (1986.7 – 12) by nonconstant correlation model

| type of industry      | 1986/7   | 1986/8   | 1986/9   | 1986/10  | 1986/11  | 1986/12  |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0.312054 | 0.005877 | 0.082245 | 0        | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0.474446 | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0.399105 | 0.235312 | 0.474489 | 0        |
| fiber manufacture     | 0        | 0        | 0        | 0        | 0        | 0        |
| valve.paper           | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0        | 0        | 0        | 0.09001  | 0        | 0.421878 |
| oil.coal              | 0        | 0        | 0        | 0        | 0        | 0        |
| rubble                | 0        | 0        | 0        | 0.036728 | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0.034939 | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0.14833  | 0        | 0        | 0.016826 | 0        | 0        |
| transport application | 0        | 0        | 0.023737 | 0.090476 | 0        | 0        |
| finance.insurance     | 0.11997  | 0        | 0        | 0.000938 | 0        | 0        |
| electricity gas       | 0        | 0.049054 | 0.16444  | 0.13879  | 0.150041 | 0.078011 |
| transport             | 0        | 0        | 0        | 0.011978 | 0        | 0        |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0.0263   | 0.03703  | 0        |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0.240772 | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0.638893 | 0.406843 | 0.217288 | 0.338441 | 0.500112 |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0        | 0        | 0.018171 | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 8: the optimal weights of the optimal portfolio in the period of out-of-sample (1987.1 – 6) by nonconstant correlation model

| type of industry      | 1987/1   | 1987/2   | 1987/3   | 1987/4   | 1987/5   | 1987/6   |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0        | 0        | 0        |          | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber manufacture     | 0        | 0        | 0        | 0        | 0        | 0        |
| valve.paper           | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0.499266 | 0.659316 | 0.68091  | 0.219793 | 0.280282 | 0.198325 |
| oil.coal              | 0        | 0        | 0        | 0        | 0        | 0        |
| rubber                | 0        | 0        | 0        | 0        | 0        | 0.000552 |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0        | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0        | 0        | 0        | 0        | 0        | 0        |
| transport application | 0        | 0        | 0        | 0        | 0        | 0        |
| finance.insurance     | 0        | 0        | 0        | 0        | 0        | 0        |
| electricity gas       | 0.090349 | 0.184173 | 0.182466 | 0.780208 | 0.719719 | 0.801124 |
| transport             | 0        | 0        | 0        | 0        | 0        | 0        |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0        | 0        | 0        |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0.410386 | 0.156512 | 0.136625 | 0        | 0        | 0        |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0        | 0        | 0        | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 9: the optimal weights of the optimal portfolio in the period of out-of-sample (1987.7 – 12) by nonconstant correlation model

| type of industry      | 1987/7   | 1987/8   | 1987/9   | 1987/10  | 1987/11  | 1987/12  |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0        | 0        | 0        | 0        | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber manufacture     | 0        | 0        | 0        | 0        | 0        | 0        |
| valve.paper           | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0.17575  | 0.22379  | 0.188157 | 0.187578 | 0.195247 | 0.176338 |
| oil.coal              | 0        | 0        | 0        | 0        | 0        | 0        |
| rubber                | 0.022289 | 0.021006 | 0        | 0        | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0        | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0        | 0        | 0.044198 | 0.051924 | 0.07261  | 0.073004 |
| transport application | 0        | 0        | 0        | 0        | 0        | 0        |
| finance.insurance     | 0        | 0        | 0        | 0        | 0        | 0        |
| electricity gas       | 0.801962 | 0.755205 | 0.767646 | 0.760499 | 0.732144 | 0.750659 |
| transport             | 0        | 0        | 0        | 0        | 0        | 0        |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0        | 0        | 0        |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0        | 0        | 0        | 0        | 0        | 0        |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0        | 0        | 0        | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 10: the optimal weights of the optimal portfolio in the period of out-of-sample (1988.1 – 6) by nonconstant correlation model

| type of industry      | 1988/1   | 1988/2   | 1988/3   | 1988/4   | 1988/5   | 1988/6   |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0        | 0        | 0        | 0        | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber manufacture     | 0        | 0        | 0        | 0        | 0        | 0        |
| valve.paper           | 0.138734 | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0        | 0.14698  | 0.010229 | 0.028755 | 0.02583  | 0.071186 |
| oil.coal              | 0        | 0        | 0        | 0        | 0        | 0        |
| rubber                | 0        | 0        | 0.007221 | 0        | 0.031963 | 0.080081 |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| machinery             | 0.049641 | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0        | 0.050933 | 0.150255 | 0.155893 | 0.145038 | 0.071522 |
| transport application | 0        | 0        | 0        | 0        | 0        | 0        |
| finance.insurance     | 0.811625 | 0        | 0        | 0        | 0        | 0.019784 |
| electricity gas       | 0        | 0.802088 | 0.832295 | 0.812858 | 0.79717  | 0.757429 |
| transport             | 0        | 0        | 0        | 0        | 0        | 0        |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0.002495 | 0        | 0        |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0        | 0        | 0        | 0        | 0        | 0        |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0        | 0        | 0        | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 11: the optimal weights of the optimal portfolio in the period of out-of-sample (1988.7 – 12) by nonconstant correlation model

| type of industry      | 1988/7   | 1988/8   | 1988/9   | 1988/10  | 1988/11  | 1988/12  |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0        | 0        | 0        | 0        | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber manufacture     | 0        | 0        | 0        | 0        | 0        | 0        |
| valve.paper           | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0.130876 | 0.124446 | 0.088886 | 0.069712 | 0.093344 | 0.085479 |
| oi.coal               | 0        | 0        | 0        | 0        | 0        | 0        |
| rubble                | 0.063119 | 0        | 0        | 0        | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0        | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0.085812 | 0.124454 | 0.166955 | 0.17312  | 0.139502 | 0.108814 |
| transport application | 0        | 0        | 0        | 0        | 0        | 0        |
| finance.insurance     | 0.020766 | 0        | 0        | 0        | 0        | 0        |
| electricity gas       | 0.699428 | 0.751102 | 0.686394 | 0.711785 | 0.715496 | 0.74991  |
| transport             | 0        | 0        | 0.057766 | 0.045383 | 0.045541 | 0.055798 |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0        | 0.006119 | 0        |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0        | 0        | 0        | 0        | 0        | 0        |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0        | 0        | 0        | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 12: the optimal weights of the optimal portfolio in the period of out-of-sample (1989.1 – 6) by nonconstant correlation model

| type of industry      | 1989/1   | 1989/2   | 1989/3   | 1989/4   | 1989/5   | 1989/6   |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0        | 0        | 0        |          | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber manufacture     | 0        | 0        | 0        | 0        | 0.065966 | 0        |
| valve.paper           | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0.100086 | 0.134924 | 0.131722 | 0.142953 | 0        | 0.041494 |
| oil.coal              | 0        | 0        | 0        | 0        | 0        | 0        |
| rubble                | 0        | 0        | 0        | 0.001812 | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0        | 0.069581 | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0.104799 | 0.093497 | 0.098713 | 0.074438 | 0        | 0.049192 |
| transport application | 0        | 0        | 0        | 0        | 0.786016 | 0        |
| finance.insurance     | 0        | 0        | 0        | 0        | 0.078438 | 0        |
| electricity gas       | 0.742612 | 0.723435 | 0.721184 | 0.761473 | 0        | 0.753561 |
| transport             | 0.052504 | 0.048146 | 0.047306 | 0.019326 | 0        | 0.155754 |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0        | 0        | 0        |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0        | 0        | 0        | 0        | 0        | 0        |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0        | 0.001075 | 0        | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 13: the optimal weights of the optimal portfolio in the period of out-of-sample (1989.7 – 12) by nonconstant correlation model

| type of industry      | 1989/7   | 1989/8   | 1989/9   | 1989/10  | 1989/11  | 1989/12  |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0        | 0        | 0        | 0        | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber manufacture     | 0        | 0        | 0        | 0        | 0        | 0        |
| valv.paper            | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0        | 0.003796 | 0.016028 | 0        | 0        | 0        |
| oil.coal              | 0        | 0        | 0        | 0        | 0        | 0        |
| rubble                | 0        | 0        | 0        | 0        | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0        | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0.038073 | 0.053901 | 0.01508  | 0        | 0        | 0        |
| transport application | 0        | 0        | 0        | 0        | 0        | 0        |
| finance.insurance     | 0        | 0        | 0        | 0        | 0        | 0        |
| electricity gas       | 0.809222 | 0.82039  | 0.822    | 0.715773 | 0.662859 | 0.662148 |
| transport             | 0.020106 | 0.006408 | 0        | 0        | 0        | 0        |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0        | 0        | 0        | 0        | 0        | 0        |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0.1326   | 0.115505 | 0.146892 | 0.264553 | 0.337142 | 0.337852 |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0        | 0        | 0.019675 | 0        | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 14: the optimal weights of the optimal portfolio in the period of out-of-sample (1990.1 – 6) by nonconstant correlation model

| type of industry      | 1990/1   | 1990/2   | 1990/3   | 1990/4   | 1990/5   | 1990/6   |
|-----------------------|----------|----------|----------|----------|----------|----------|
| type of industry      | 0        | 0        | 0        | 0        |          | 0        |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber application     | 0        | 0        | 0        | 0        | 0        | 0        |
| valve.paper           | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0        | 0        | 0.029727 | 0.023325 | 0.028996 | 0.047692 |
| oi.coal               | 0        | 0        | 0        | 0        | 0        | 0        |
| rubber                | 0        | 0        | 0        | 0        | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel                 | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0        | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0.041845 | 0.047455 | 0.032317 | 0.028999 | 0.028651 | 0.027988 |
| transport application | 0        | 0        | 0        | 0        | 0        | 0        |
| finance. insurance    | 0        | 0        | 0        | 0        | 0        | 0        |
| electricity gas       | 0.674003 | 0.666052 | 0.481087 | 0.479064 | 0.47699  | 0.374722 |
| transport             | 0.051836 | 0.082519 | 0.341532 | 0.368058 | 0.357224 | 0.446419 |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0.004612 | 0.006017 | 0.016339 | 0.016185 | 0.020183 | 0.024714 |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0.2226   | 0.197958 | 0.099    | 0.084369 | 0.074419 | 0.078466 |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0.005105 | 0        | 0        | 0        | 0.01354  | 0        |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 15: the optimal weights of the optimal portfolio in the period of out-of-sample (1990.7 – 12) by nonconstant correlation model

| type of industry      | 1990/7   | 1990/8   | 1990/9   | 1990/10  | 1990/11  | 1990/12  |
|-----------------------|----------|----------|----------|----------|----------|----------|
| aff                   | 0        | 0        | 0        | 0        | 0        | 0.007308 |
| mining                | 0        | 0        | 0        | 0        | 0        | 0        |
| building              | 0        | 0        | 0        | 0        | 0        | 0        |
| grocery               | 0        | 0        | 0        | 0        | 0        | 0        |
| fiber application     | 0        | 0        | 0        | 0        | 0        | 0        |
| valve.paper           | 0        | 0        | 0        | 0        | 0        | 0        |
| medicament            | 0.060413 | 0.064337 | 0.053459 | 0.049724 | 0.05296  | 0.050488 |
| oil.coal              | 0        | 0        | 0        | 0        | 0        | 0        |
| rubber                | 0        | 0        | 0        | 0        | 0        | 0        |
| glass.soil.stone      | 0        | 0        | 0        | 0        | 0        | 0        |
| steel industry        | 0        | 0        | 0        | 0        | 0        | 0        |
| hardware              | 0        | 0        | 0        | 0        | 0        | 0        |
| machinery             | 0        | 0        | 0        | 0        | 0        | 0        |
| electric manufacture  | 0.029534 | 0.02886  | 0.003317 | 0        | 0.005316 | 0.001356 |
| transport application | 0        | 0        | 0        | 0        | 0        | 0        |
| finance. insurance    | 0        | 0        | 0        | 0        | 0        | 0        |
| electricity gas       | 0.360663 | 0.387708 | 0.332843 | 0.3203   | 0.367628 | 0.286488 |
| transport             | 0.42484  | 0.333586 | 0.397714 | 0.411387 | 0.34608  | 0.456602 |
| shipping              | 0        | 0        | 0        | 0        | 0        | 0        |
| airlift               | 0        | 0        | 0        | 0        | 0        | 0        |
| IT                    | 0.024626 | 0.020727 | 0.018107 | 0.016393 | 0.016864 | 0.015199 |
| other instrument      | 0        | 0        | 0        | 0        | 0        | 0        |
| precision instrument  | 0        | 0        | 0        | 0        | 0        | 0        |
| commerce              | 0.099926 | 0.150289 | 0.17737  | 0.181738 | 0.187065 | 0.170734 |
| real estate           | 0        | 0        | 0        | 0        | 0        | 0        |
| service               | 0        | 0.014494 | 0.017191 | 0.02046  | 0.024088 | 0.011827 |
| nonferrous metal      | 0        | 0        | 0        | 0        | 0        | 0        |
| warehouse             | 0        | 0        | 0        | 0        | 0        | 0        |

Table 16: monthly the rate of portfolio 's return at the period of out-of-sample by nonconstant correlation model

| year/month | return rate | year/month | return rate |
|------------|-------------|------------|-------------|
| 1986/1     | 3.162914    | 1988/7     | 2.548837    |
| 1986/2     | 1.212163    | 1988/8     | 1.15546     |
| 1986/3     | 2.89646     | 1988/9     | 1.05828     |
| 1986/4     | 4.6934334   | 1988/10    | 1.291299    |
| 1986/5     | 3.572424    | 1988/11    | 2.874649    |
| 1986/6     | 3.2169      | 1988/12    | 1.74909     |
| 1986/7     | 5.170508    | 1989/1     | 3.29555     |
| 1986/8     | 3.98227     | 1989/2     | 1.23371     |
| 1986/9     | 1.48358     | 1989/3     | 2.30697     |
| 1986/10    | 1.51577     | 1989/4     | 2.651107    |
| 1986/11    | 2.0724      | 1989/5     | 1.107406    |
| 1986/12    | 0.373181    | 1989/6     | 0.961786    |
| 1987/1     | 0.726508    | 1989/7     | 1.65245     |
| 1987/2     | 0.3322      | 1989/8     | 1.85977     |
| 1987/3     | 1.52292     | 1989/9     | 1.51368     |
| 1987/4     | 0.419728    | 1989/10    | 1.39499     |
| 1987/5     | 3.549474    | 1989/11    | 1.82194     |
| 1987/6     | 0.342587    | 1989/12    | 1.50233     |
| 1987/7     | 2.37709     | 1990/1     | 2.55946     |
| 1987/8     | 0.411661    | 1990/2     | 0.735974    |
| 1987/9     | 0.77563     | 1990/3     | 0.139044    |
| 1987/10    | 0.669728    | 1990/4     | 0.711013    |
| 1987/11    | 1.1224      | 1990/5     | 2.52991     |
| 1987/12    | 2.77857     | 1990/6     | 0.316343    |
| 1988/1     | 1.81662     | 1990/7     | 1.332647    |
| 1988/2     | 1.08357     | 1990/8     | 1.64275     |
| 1988/3     | 0.78031     | 1990/9     | 1.45726     |
| 1988/4     | 0.36943     | 1990/10    | 0.97078     |
| 1988/5     | 0.518672    | 1990/11    | 1.67635     |
| 1988/6     | 2.884589    | 1990/12    | 0.273693    |

Table 17: comparison of the two models 'performance

| item                                      | 86year  | 87year  | 878year  | 89 year  | 90 year  |
|-------------------------------------------|---------|---------|----------|----------|----------|
| mean (nonconstant correlation model)      | 5.3128  | 1.9896  | 1.579    | 3.962    | 3.2996   |
| sd (nonconstant correlation model)        | 3.254   | 2.894   | 2.0186   | 2.5642   | 2.3856   |
| the ratio                                 | 1.63271 | 0.6875  | 0.782341 | 1.548495 | 1.383128 |
| mean (SRA ( $\rho = 1/2$ ))               | 3.661   | 0.9902  | 1.8157   | 1.658    | 3.3829   |
| standard deviation (SRA ( $\rho = 1/2$ )) | 2.7072  | 2.0715  | 3.056    | 1.061    | 2.5146   |
| the ratio                                 | 1.35232 | 0.478   | 0.5941   | 1.5627   | 1.3449   |
| mean (SRA ( $\rho = 1/3$ ))               | 3.8929  | 2.045   | 1.56     | 2.5489   | 3.6265   |
| standard deviation (SRA ( $\rho = 1/3$ )) | 2.7042  | 3.4661  | 2.1906   | 1.868    | 2.5894   |
| the ratio                                 | 1.43957 | 0.59    | 0.7121   | 1.3645   | 1.4005   |
| mean (SRA ( $\rho = 2/3$ ))               | 5.011   | 1.63013 | 1.56     | 4.1023   | 1.61404  |
| standard deviation (SRA ( $\rho = 2/3$ )) | 2.9416  | 2.1881  | 2.1491   | 3.078    | 1.282    |
| the ratio                                 | 1.7034  | 0.745   | 0.7259   | 1.3328   | 1.259    |

Table 18: the dates of figure.1

| item                          | 1986 year | 1987 year | 1988 year | 1989 year | 1990 year |
|-------------------------------|-----------|-----------|-----------|-----------|-----------|
| $\rho = 1/2$                  | 1.35232   | 0.47778   | 0.594089  | 1.562695  | 1.3449    |
| $\rho = 1/3$                  | 1.43957   | 0.58988   | 0.712127  | 1.364499  | 1.4005    |
| $\rho = 2/3$                  | 1.7034    | 0.745     | 0.7259    | 1.3328    | 1.259     |
| nonconstant correlation model | 1.63271   | 0.6875    | 0.782341  | 1.548495  | 1.383128  |

## Appendix

The original maximization problem of the Sharpe ratio of the portfolio with no short sales constraint is formulated as the following mathematical programming problem:

$$\text{Maximize} \quad f(\mathbf{x}) := \frac{\sum_{i=1}^n (\bar{r}_i - r_f) x_i}{\sqrt{\sum_{i=1}^n \sigma_i^2 x_i^2 + \rho \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_i x_j}} \quad (18)$$

$$\text{subject to} \quad \sum_{i=1}^n x_i = 1; \quad (19)$$

$$x_i \geq 0, \quad i = 1, \dots, n. \quad (20)$$

Since both of the denominator and numerator of the objective function are positively homogeneous, we have

$$f(\alpha \mathbf{x}) = f(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}_+^n \setminus \{\mathbf{0}\}; \quad \alpha > 0. \quad (21)$$

Accordingly, first, we could solve the following mathematical programming problem without the equality condition:

$$\text{Maximize} \quad f(\mathbf{x}) := \frac{\sum_{i=1}^n (\bar{r}_i - r_f) x_i}{\sqrt{\sum_{i=1}^n \sigma_i^2 x_i^2 + \rho \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_i x_j}} \quad (22)$$

$$\text{subject to} \quad x_i \geq 0, \quad i = 1, \dots, n, \quad (23)$$

and then we could derive the optimal solution of the original mathematical programming by a normalization:

$$x_i^* := \frac{x_i}{\sum_{i=1}^n x_i}, \quad i = 1, \dots, n. \quad (24)$$

Let

$$\begin{aligned} L(\mathbf{x}; \boldsymbol{\lambda}) &:= f(\mathbf{x}) + \boldsymbol{\lambda}^\top \mathbf{x} \\ &= \left( \sum_{i=1}^n (\bar{r}_i - r_f) x_i \right) \left( \sum_{i=1}^n \sigma_i^2 x_i^2 + \rho \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_i x_j \right)^{-1/2} + \sum_{i=1}^n \lambda_i x_i, \\ &\text{for } \mathbf{x} \in \mathbb{R}_+^n; \quad \boldsymbol{\lambda} \in \mathbb{R}_+^n. \end{aligned} \quad (25)$$

$$\frac{\partial L}{\partial x_i}(\mathbf{x}; \boldsymbol{\lambda}) = \left[ \sum_{i=1}^n (\bar{r}_i - r_f) x_i \right] \left[ \left( -\frac{1}{2} \right) v(\mathbf{x})^{-3/2} \left( 2\sigma_i^2 x_i + 2\rho \sum_{j=1, j \neq i}^n x_j \sigma_i \sigma_j \right) \right] \quad (26)$$

$$+ v(\mathbf{x})^{-1/2} (\bar{r}_i - r_f) + \lambda_i \quad (27)$$

$$= 0. \quad (28)$$

where

$$v(\mathbf{x}) := \sum_{i=1}^n \sigma_i^2 x_i^2 + \rho \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_i x_j \quad (29)$$

Multiplying the above derivative by

$$v(\mathbf{x})^{1/2} = \left( \sum_{i=1}^n \sigma_i^2 x_i^2 + \rho \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_i x_j \right)^{1/2} \quad (30)$$

and rearranging yields

$$-\left( \sum_{i=1}^n (\bar{r}_i - r_f) x_i \right) \left( \sigma_i^2 x_i + \rho \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_j \right) v(\mathbf{x})^{-1} + (\bar{r}_i - r_f) + \lambda_i v(\mathbf{x})^{1/2} = 0. \quad (31)$$

Further, if we let

$$u(\mathbf{x}) := \left( \sum_{i=1}^n (\bar{r}_i - r_f) x_i \right) v(\mathbf{x})^{-1} = \frac{\sum_{i=1}^n (\bar{r}_i - r_f) x_i}{\sum_{i=1}^n \sigma_i^2 x_i^2 + \rho \sum_{i=1}^n \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_i x_j} \quad (32)$$

then we have

$$\begin{aligned} 0 &= -\left( \sigma_i^2 x_i + \rho \sum_{j=1, j \neq i}^n \sigma_i \sigma_j x_j \right) u(\mathbf{x}) + (\bar{r}_i - r_f) + \lambda_i v(\mathbf{x})^{1/2} \\ &= -\sigma_i \left( \sigma_i \{u(\mathbf{x}) x_i\} + \rho \sum_{j=1, j \neq i}^n \sigma_j \{u(\mathbf{x}) x_j\} \right) + (\bar{r}_i - r_f) + \lambda_i v(\mathbf{x})^{1/2}, \end{aligned} \quad (33)$$

that is,

$$(\bar{r}_i - r_f) = -\lambda_i v(\mathbf{x})^{1/2} + \sigma_i \left( \sigma_i \{u(\mathbf{x}) x_i\} + \rho \sum_{j=1, j \neq i}^n \sigma_j \{u(\mathbf{x}) x_j\} \right). \quad (34)$$

If we define

$$z_i = u(\mathbf{x}) x_i, \quad i = 1, \dots, n \quad (35)$$

then

$$\begin{aligned} \bar{r}_i - r_f &= -\lambda_i v(\mathbf{x})^{1/2} + \sigma_i \left( \sigma_i z_i + \rho \sum_{j=1, j \neq i}^n \sigma_j z_j \right) \\ &= -\lambda_i v(\mathbf{x})^{1/2} + \sigma_i \left( (1 - \rho) \sigma_i z_i + \rho \sum_{j=1}^n \sigma_j z_j \right), \quad i = 1, \dots, n. \end{aligned} \quad (36)$$

The complementarity condition yields

$$z_i \geq 0; \lambda_i \geq 0; z_i \lambda_i = 0, \quad i = 1, \dots, n. \quad (37)$$

If we define

$$x_i := \frac{z_i}{\sum_{i=1}^n z_i}, \quad i = 1, \dots, n \quad (38)$$

then we have

$$\sum_{i=1}^n x_i = 1. \quad (39)$$

If  $z_i > 0$  then, from the above complementarity condition, we have  $\lambda_i = 0$ . Therefore, it holds that

$$\bar{r}_i - r_f = \sigma_i \left( (1 - \rho) \sigma_i z_i + \rho \sum_{j=1}^n \sigma_j z_j \right).$$

Rearranging and solving for  $z_i$ , we have

$$z_i = \frac{1}{(1 - \rho) \sigma_i} \left( \frac{\bar{r}_i - r_f}{\sigma_i} - \rho \sum_{j=1}^n \sigma_j z_j \right). \quad (40)$$

In order eliminate the term

$$\sum_{j=1}^n \sigma_j z_j$$

in the right hand side of the above expression, for

$$i \in M := \left\{ j \in N = \{1, \dots, n\} \mid z_j > 0 \right\} \quad (41)$$

by multiplying each equation by  $\sigma_i$ , and then adding together all such  $i$ . This yields

$$\begin{aligned}
\sum_{j \in M} \sigma_j z_j &= \frac{1}{1-\rho} \left( \sum_{j \in M} \frac{\bar{r}_j - r_f}{\sigma_j} - |M|\rho \sum_{j=1}^n \sigma_j z_j \right) \\
&= \frac{1}{1-\rho} \left( \sum_{j \in M} \frac{\bar{r}_j - r_f}{\sigma_j} - |M|\rho \sum_{j \in M} \sigma_j z_j \right)
\end{aligned} \tag{42}$$

where  $|M|$  denotes the cardinality of the set  $M$  (i.e., the number of elements in the set  $M$ ).

By rearranging, we have

$$\sum_{j \in M} \sigma_j z_j = \left( \frac{1}{1-\rho + |M|\rho} \right) \sum_{j \in M} \frac{\bar{r}_j - r_f}{\sigma_j}. \tag{43}$$

Thus,

$$z_i = \frac{1}{(1-\rho)\sigma_i} \left( \frac{\bar{r}_i - r_f}{\sigma_i} - C \right), \quad i \in M, \tag{44}$$

where

$$C := \left( \frac{\rho}{1-\rho + |M|\rho} \right) \sum_{j \in M} \frac{\bar{r}_j - r_f}{\sigma_j}. \tag{45}$$

Furthermore, if the securities are numbered so that their Sharpe ratios:

$$\frac{\bar{r}_j - r_f}{\sigma_j} \tag{46}$$

are decreasing in  $j$  then, for some  $k$  ( $\in N = \{1, \dots, n\}$ ), we have

$$M = \{1, \dots, k\} \tag{47}$$

so that

$$z_i = \begin{cases} \frac{1}{(1-\rho)\sigma_i} \left( \frac{\bar{r}_i - r_f}{\sigma_i} - C_k \right), & i = 1, \dots, k; \\ 0, & i = k+1, \dots, n, \end{cases} \tag{48}$$

where

$$C_k := \left( \frac{\rho}{1-\rho + k\rho} \right) \sum_{j=1}^k \frac{\bar{r}_j - r_f}{\sigma_j}. \tag{49}$$

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