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# QCD SUM RULE ON THE QUARK CONTENTS IN HADRONS

by

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**Abstract**

We study the quark contents of hadrons by the QCD sum rule method. We concentrate on the moments of the quark-parton distribution functions, which describe the quark correlation on the light-cone. One advantage of the present approach is the model independence. We can obtain results which are unambiguously comparable with experiments and with lattice calculations. We show that the QCD sum rules give results consistent with other model independent approaches. For example, we show that the QCD sum rule for the pion (kaon)-nucleon scattering lengths are consistent with the low energy theorem known as the Tomozawa-Weinberg relation. We also estimate the term beyond the low energy theorem, which are not obtainable from symmetry arguments alone.

We first study the quark momentum fractions of the pseudo-scalar mesons (the pion and kaon). We obtain only a small difference between the two in spite of the large mass difference of them, or that of the u-, d-quarks and the s-quark. The result is consistent with available experimental data and lattice calculations.

We next consider the strange scalar density in the proton. In the conventional QCD sum rules, it is expected to be small because of the strong suppression of the perturbative term and its local power corrections. We, for the first time, take the direct instantons into account, and show that they give the leading contribution to this quantity. Our calculation predicts rather small value compared with the results of other approaches.

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# 1 Introduction

Quantum chromodynamics (QCD) [1, 2] is now widely believed as the theory of the strong interactions among the quarks and the gluons. The quarks and the gluons together form the hadrons through the strong interactions. The existence of the partons (quarks and gluons) inside hadrons is evidenced by the experiments like the deep inelastic scattering (DIS) of the lepton off the hadron. An important observation in DIS experiments is the approximate Bjorken scaling [3], which implies that the quark struck by the highly virtual probes is moving almost as a free particle. This fact indicates that the effective coupling constant between the quarks and the gluons should become small at very high energy. QCD is known to possess this feature called asymptotic freedom [4]. Since asymptotic freedom is a unique property of the non-Abelian gauge theory in four dimensions [5], the observation of the Bjorken scaling gives strong support that QCD based on the color gauge group is the correct theory of the strong interactions.

Owing to asymptotic freedom, the perturbative method can be applied to the processes like DIS, which involve very large momenta. Those processes where perturbative QCD (pQCD) can be applied are called hard processes. The basis for the application of pQCD to hard processes is provided by the so-called factorization theorem [6]. For example, by using the operator product expansion (OPE) [7], the cross sections of DIS can be separated into the hard part (high-energy part) and the soft part (low-energy part). The hard part can be calculated by the perturbative method. To treat the soft part, however, one needs non-perturbative methods, since the interactions of QCD at low energies become strong and tremendously complex. The complexities of QCD at low energies are reflected in phenomena like the color confinement, the non-vanishing vacuum condensates, etc.

The soft part carries information on the parton distributions inside hadrons. In QCD, the parton distributions are defined as the light-cone Fourier transform of correlation function of the quarks and gluons in hadrons [8]. They are the function of the Bjorken variable $\xi$ ($0 \leq \xi \leq 1$) and generally depend on the renormalization scale $\mu^2$. The scale dependence of the quark distribution functions is governed by the QCD evolution equations based on the renormalization group equation (RGE). The observed scale dependence (the braking of the Bjorken scaling) of the structure functions is well-described by the QCD evolution equations.

The parton distributions represent multi-parton correlation on the light-cone. The major contribution to hard processes comes from the ones involving a few partons (low twist contributions). The higher twist contributions are suppressed by the more $\frac{1}{\mu^2}$ factor. The simplest twist-2 contribution measures the quark-quark correlation on the light-cone, which can be understood in the parton picture [9]. In the parton picture, the Bjorken variable $\xi$ stand for the probability to find the parton in the hadron.

The parton distributions can be defined for each spin state of target hadrons. The parton distribution function of the nucleon with low-twist can be classified as in table 1.1 [8]. The parton distributions which measures the correlation of quarks with opposite chirality are called chiral-odd. In table 1.1, $h(\xi, \mu^2)'$s and $e(\xi, \mu^2)$ are the chiral odd parton distributions. They can not be measured in totally inclusive DIS experiments but are accessible in, e.g., Drell-Yan processes.

The moments of the quark-parton distributions with respect to $\xi^n$ can be expressed in terms of the hadronic matrix elements of the quark bi-linear operators. Among them, the lower moments are interesting, because they have simple physical interpretation. For example, the second moment of the spin-averaged quark distri-

Table 1.1: Classification of quark distribution function

| Twist | Twist-2 | Twist-3 | Twist-4 |
|---|---|---|---|
| Target spin average | $f_1(\xi, \mu^2)$ | $e(\xi, \mu^2)$ | $f_4(\xi, \mu^2)$ |
| Longitudinal spin (Target helicity asymmetry) | $g_1(\xi, \mu^2)$ | $h_L(\xi, \mu^2)$ | $g_3(\xi, \mu^2)$ |
| Transverse spin (Target helicity flip) | $h_1(\xi, \mu^2)$ | $g_{\rm T}(\xi, \mu^2)$ | $h_3(\xi, \mu^2)$ |

bution $f_1(\xi, \mu^2)$, $\int d\xi \xi f_1(\xi, \mu^2)$, is nothing but the momentum fraction carried by the quark. The lowest moment of the longitudinally polarized quark distribution $g_1(\xi, \mu^2)$, $\int d\xi g_1(\xi, \mu^2)$, gives the spin fraction carried by the quark, and for the nucleon target it is connected with the axial vector coupling constant $g_A$. The lowest moment of the chiral-odd spin-averaged quark distribution $e(\xi, \mu^2)$, $\int d\xi e(\xi, \mu^2)$, for the nucleon is related to the sigma term.

There are several approaches to investigate the non-perturbative aspects of QCD like the parton distributions. Among them, effective models of QCD are often used. Most of these effective models share some symmetries (or approximate symmetries) with QCD. Chiral symmetry is an example of such symmetries. Studies using effective models are useful to make clear to find what kind of symmetries or which degrees of freedom are important to reproduce specific properties of QCD. Furthermore some properties directly result from symmetry arguments, like the low energy theorem (LET) [10]. One of difficulties in these approaches is the lack of direct connection with QCD.

Another approach to the non-perturbative QCD is the Monte Carlo calculation based on lattice gauge theory (LGT) [11, 12]. This approach has been successfully used to investigate the color confinement, the QCD phase transition at finite temperature, etc. LGT is powerful because it makes possible to treat non-perturbative phenomena directly from QCD. However, it has still limitations to study quantita-

tively the dynamics of the quarks and the gluons in hadrons. For example, most calculations rely on the quenched approximation.

Last of all we introduce the QCD sum rules [13] which we will employ in the present thesis. It is a useful method to compute physical quantities in a model-independent way. It also gives a lot of insights into the non-perturbative nature of QCD. For example, the essential roles of the gluon condensates in hadron physics was first recognized in the QCD sum rules.

The QCD sum rules connect physical quantities with the matrix elements of local operators. The key techniques are OPE and the spectral representation of physical amplitudes. The matrix elements of local operators are considered as parameters, which are universal to all applications. In this way, one can calculate a large number of physical quantities with a small number of matrix elements.

It is noteworthy that the QCD sum rules give the results consistent with LET, as we will show in section 3 and appendix B [13, 14]. Furthermore QCD sum rules predict the existence of the Nambu-Goldstone bosons (the pions) [13]. In the QCD sum rule method, one can also obtain the terms beyond LET, which are not obtainable from symmetry arguments alone.

In this thesis we investigate the parton distributions inside hadrons using the QCD sum rule method. Since the parton distributions are scale dependent objects, one has to specify the scale of calculations. In effective models, it is impossible to determine the scale from QCD. This may cause uncontrollable uncertainty in comparing calculation with experiments, if the results are not obtained from symmetry arguments. On the other hand, in QCD sum rules one can specify the scale and therefore unambiguously compare the results with experiments. Thus QCD sum rules provide a useful framework for the studies of the quark distributions inside

hadrons.

In this thesis we concentrate on the moments of the parton distributions. Our first interest is in the quark momentum fraction and its dependence on the flavor structure of hadrons. Here the quark momentum fraction is defined by the second moment of the flavor-singlet $f_1(\xi, \mu^2)$ parton distribution.

For the nucleon, the structure functions have been measured extensively by experiments, and there have been many theoretical studies of them. In contrast, experimental as well as theoretical investigation of the distribution functions for other hadrons are still few, in spite of their importance in many theoretical calculations in hadron physics. For example, the parton distributions of the pion are necessary to investigate the pionic effect in the deep inelastic scattering off nuclei. The parton distributions of the pseudo-scalar mesons are needed as inputs in consistent treatment of the finite temperature QCD sum rules [15].

Furthermore, it is not yet clear how the difference in the flavor structure of hadrons affects the parton distributions. In many calculations involving the meson parton distributions, however, it has been usually assumed that the difference has little effect on the parton distributions. In section 4, we study the above problems by calculating the quark momentum fractions of the pseudo-scalar mesons, the pion and the kaon, in the framework of the QCD sum rule [16]. In view of the mass difference of the pion and the kaon, or that of the u-, d-quarks and the s-quark, it is interesting to study the role of quark contents in the parton distributions.

Next we study the strangeness content of the proton. The strangeness content of the proton is related to Okubo-Zweig-Iizuka rule (OZI rule), and its violation in the proton. A typical example of the OZI rule seen in the vector mesons: The $\omega$ meson is almost composed of u- and d-quarks, while the $\phi$ meson is almost composed of

s-quarks, corresponding to the ideal mixing. However, the realization of the OZI rule is known to be strongly channel dependent. For example, large mixing of s-quark is observed in the pseudo-scalar mesons: the $\eta'$ meson is nearly the flavor SU(3) singlet state. There have been attempts to explain the channel dependence by taking instanton effect into account [20, 21].

In section 5 we calculate the strange scalar density in the proton $\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle$ by the QCD sum rule method. The large $\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle$ corresponds to the violation of the OZI rule in the baryon sector. It is also related to the $s$-quark fraction of the proton mass and the KN sigma term relevant for the kaon condensation[17]. The latest semi-phenomenological estimation gives $y \equiv 2\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle / \langle \mathrm{p}|\bar{u}u + \bar{d}d|\mathrm{p}\rangle \simeq 0.2$ [18]. However theoretical studies are still controversial. The recent calculation in quenched lattice QCD gives rather large value $y = 0.66 \pm 0.15$ [19]. The effective models of the nucleon give the various results ranging from $y \simeq 0$ to $y \simeq 0.6$. Our calculation will clarify this controversy. In the QCD sum rule calculations for the problems related with OZI rule, the leading role is played by small size direct instantons [22]. The contributions of the conventional operator product expansion are strongly suppressed (higher order in $\alpha_s$).

This thesis is organized as follows: In section 2 we introduce the QCD sum rule. In section 3, we calculate the pion-nucleon kaon-nucleon scattering lengths, and discuss on the low energy theorem from the QCD sum rule. In section 4, we calculate the the quark momentum fractions of the pion and the kaon, and compare the result with available experimental analysis, and with result from LGT calculation, etc. In section 5 we calculate the strange scalar density in the proton, and investigate the OZI violation induced by the direct instanton.

# 2 QCD sum rule method

QCD sum rules, developed by Shifman, Vainshtein and Zakharov, provide analytic method to investigate the non-perturbative aspects of QCD[13],[23]–[29]. The two essential techniques to be used are the operator product expansion (OPE) and the spectral representation. Using the QCD sum rule method, one can connect various properties of hadrons with the matrix elements of local operators composed of the quark and gluon fields. For example, the masses of hadrons are expressed in terms of the vacuum expectation values (VEVs) of a few local operators. The matrix elements of local operators can be obtained from phenomenological information through QCD sum rules, etc. In this way, we can calculate the properties of hadrons. In this section we introduce the QCD sum rule method.

## 2.1 General formulation

In the QCD sum rule method we first consider a correlation function. Here for simplicity we consider the vacuum correlation function of $J^A$ and $J^B$:

$$\Pi(q^2) = \int \mathrm{d}^4x\, e^{iqx} \langle 0|T[J^A(x), J^B(0)]|0\rangle. \tag{2.1}$$

$J^A(x)$ and $J^B(0)$ should be chosen appropriately for each problem. For example, if we want to investigate the mass of the vector meson, we choose them to be the vector currents, which possess the same properties in the spin, parity and charge conjugation as those of the vector meson. The operators which have the same quantum numbers as those of a hadron are called the interpolating fields.

QCD sum rules for two-point correlation functions $\Pi(q^2)$ are formally given by [30]

$$\int_0^\infty \mathrm{Im}\Pi_{\mathrm{Th}}(s) W(s/M^2)\, ds = \int_0^\infty \mathrm{Im}\Pi_{\mathrm{Ph}}(s) W(s/M^2)\, ds, \tag{2.2}$$

where $\Pi_{\rm Th}(s)$ and $\Pi_{\rm Ph}(s)$ are theoretical and phenomenological estimation of correlation function $\Pi(s)$, respectively, which will be explained below.

$W(s/M^2)$ is a weight function to suppress the contributions of high $s$ region, corresponding to higher excited states. By choosing this factor appropriately, one can extract the information on the lowest lying state. Various forms are possible for $W(s/M^2)$, and for each choice of $W(s/M^2)$, we obtain different type of QCD sum rules. For example, if we choose $W(s/M^2)$ to be the exponential form, $e^{-s/M^2}$, we obtain the so-called Borel sum rules. Here $M$ sets the specific mass-scale. It should be noted that for the smaller $M$, $W(s/M^2)$ enhances the contribution of the lowest lying states more. For a special choice of $W(s/M^2)$ , $W(s/M^2) = \dfrac{1}{\pi M^2}\dfrac{1}{s/M^2+1}$, eq. (2.2) become nothing but the dispersion relation, and is called the dispersion sum rule:

$$\Pi(-M^2) = \frac{1}{\pi}\int_0^\infty \frac{{\rm Im}\Pi(s)\ ds}{s+M^2} + \text{subtraction terms.} \tag{2.3}$$

All the other types of QCD sum rules can be obtained from the dispersion sum rule eq. (2.3). For example, Borel sum rules are obtained by performing the Borel transformation to the dispersion sum rule.

By using OPE, $\Pi_{\rm Th}(s)$ is expressed as the sum of matrix elements of various composite operators multiplied by coefficients. The coefficients are called the Wilson coefficients. If $M$ is sufficiently large, we can calculate Wilson coefficients by pQCD as we describe below. For some cases, we should take the small-size instanton effects into account, since these effects can not be implicated into matrix elements of local operators.

On the other hand, $\Pi_{\rm Ph}(s)$ denotes $\Pi(s)$ parameterized by a few phenomenological parameters, e.g., the mass and the decay constant. By choosing sufficiently low $M^2$ in $W(s/M^2)$, the contributions of higher excited states can be suppressed. So

the most important contribution comes from the lowest lying state The contributions of other states are approximated as the continuum. It is usually determined to match the theoretical calculations in the asymptotic region. Moreover we usually employ the so-called narrow resonance approximation, i.e., we approximate the resonance width to be zero. Thus we obtain

$$\mathrm{Im}\Pi_{\mathrm{Ph}}(s) = f\delta(s - m^2) + \theta(s - s_0)C, \tag{2.4}$$

where $m$ and $f$ denote the mass and the decay constant of the lowest lying state, respectively. $C$ is determined to match the asymptotic behavior of OPE

By using eq. (2.2), we can connect the properties of the lowest lying state, like the masses and the decay constants, with matrix elements of operators. However eq. (2.2) holds only in the limited region of $M$ for the following reason. The calculations on the theoretical side can be performed in the large $M$ region, since in the small $M$ region undesirable non-perturbative effects come into play in calculating the Wilson coefficients. On the other hand, the simple form, eq. (2.4), for the phenomenological side can be used in the small $M$ region, where the contributions of the excited states to the integral sufficiently suppressed. The region of $M$, where these two requirements are satisfied and thus we can apply the QCD sum rule method, is often called the fiducial region of $M$. In Borel sum rule it is called the Borel window.

We may encounter the case where the QCD sum rule calculation is not successful; there may be no fiducial region in the result. It would imply that the simple form, eq. (2.4) does not work, and therefore we should try to take the more improved form, say the first excited state, into account. It should be noted that in the framework of QCD sum rules, we can not show the mechanism how the bound states are formed. We just assume that there exist the bound states for each channel. These assumptions should be checked by investigating the stability of the sum rule with

respect to $M$.

## 2.2 Operator product expansion

The operator product expansion (OPE) is a useful tool in analyzing the hard processes, e.g., the deep inelastic scattering and the high energy $e^+e^-$ annihilation. It also plays an essential role in QCD sum rules.

OPE, first introduced by Wilson [11], is the method of expanding the products of local operators, e.g., $A(x)B(0)$, into the sum of local composite operators with coefficients:

$$A(x)B(0) = \sum_n C_n^{AB}(x) O_n, \tag{2.5}$$

where $C_n^{AB}(x)$ is c-number coefficient called the Wilson coefficient. They might be singular at $x_\mu = 0$. $O_n$ are local composite operators with the normal ordering, and thus have no singularity. In this way we can decompose the products of local operators into singular parts and non-singular parts.

At short distance, terms with the higher singularity in $x$ in the Wilson coefficients are important. This expansion is called the short distance expansion. The singularity of the Wilson coefficients at $x = 0$ is determined from the scale dimensions of the operators,

$$C_n^{AB}(x) \sim x^{-(d_A + d_B - d_{O_n})} \qquad (x^\mu \to 0), \tag{2.6}$$

where $d_A$, $d_B$ and $d_{O_n}$ are the scale dimension of $A$, $B$ and $O_n$, respectively. Thus the operators $O_n$ with the smaller scale dimension give dominant contributions to the short distance expansion. Here the scale dimension $d_A$ of an operator $A(x)$ is determined from its behavior under the scale transformation of the space-time coordinates: $x_\mu \to e^{-t} x_\mu$,

$$A(x) \to e^{t d_A} A(x), \tag{2.7}$$

where the transformation of eq. (2.7) is called the dilatation transformation. For the free field theories (or to the zeroth order in interactions in general field theories), the scale dimension is identical to the canonical dimension, which is determined from the commutation relations.

In similar way, at light-like distance, terms with the higher singularity in $x^2$ in the Wilson coefficients are important. This expansion is called the light-cone expansion. The singularity of the Wilson coefficients at $x^2 = 0$ is now determined from the twist of the operators,

$$C^{AB}_{n;\mu_1,\mu_2,\cdots\mu_s}(x) \sim (x^2)^{-(\tau_A+\tau_B-\tau_{O_n})/2} x_{\mu_1} x_{\mu_2} \cdots x_{\mu_s} \qquad (x^2 \to 0), \tag{2.8}$$

where $s$ is the Lorentz spin of $O_n$; and $\tau_A$, $\tau_B$ and $\tau_{O_n}$ are the twist of $A$, $B$ and $O_n$, respectively. The operators $O_n$ with the smaller twist give dominant contributions to the light-cone expansion. Here the twist of an operator is given by,

$$(\text{Twist}) = (\text{dimension}) - (\text{Lorentz spin}). \tag{2.9}$$

Because OPE is intended to be the operator identity, it should hold for any matrix elements. In QCD sum rule in the vacuum, we consider vacuum matrix elements. If we evaluate the vacuum matrix elements in perturbation theory, only the term with the unit operators survives, since all the operators appearing in OPE are normal ordered. However if non-perturbative effects induce non-trivial structure of the vacuum, other operators can come into play. They give the power corrections to the perturbative terms (unit operator terms). It is well known that the QCD vacuum is complex enough to allow the non-vanishing VEVs; all the spin-0 and gauge-invariant operators can have the non-vanishing VEVs. The operator with low mass-dimension, which are relevant for the QCD sum rule in the vacuum, are given

as follows:

$$
\begin{aligned}
&I(\text{unit operator}) && (d=0),\\
&\bar{\psi}M_1\psi && (d=3),\\
&F^a_{\mu\nu}F^{a\,\mu\nu} && (d=4),\\
&\bar{\psi}\sigma^{\mu\nu}gT^aF^a_{\mu\nu}M_2\psi && (d=5),\\
&\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi && (d=6),\\
&f^{abc}F^a_{\mu\nu}F^{b\,\nu\rho}F^{c\,\mu}_{\rho} && (d=6),
\end{aligned}
\tag{2.10}
$$

where $M_1$ and $M_2$ stand for some matrices in the flavor space. $\Gamma_1$ and $\Gamma_2$ stand for some matrices in the spinor, flavor and color space.

In the applications of QCD sum rules to the hadron-hadron scattering lengths and the mass-shifts in finite density/temperature medium, hadronic matrix elements of OPE are considered Then the spin non-zero operators also come into play.

The VEVs of composite operators are not given within the framework of QCD sum rules; they are regarded as parameters. Many hadronic matrix elements of the composite operators can be connected to the moments of parton distribution functions. So in favorable case, we can obtain the values from the experimental data. One may calculate the hadronic matrix elements using the QCD sum rule method. Later we will discuss on the latter approach and give some examples of the applications.

OPE is proven only in the framework of perturbation theory. Indeed it is well known that if we take the instanton contribution into account (in the dilute gas approximation), the expansion breaks down at the term corresponding to dimension-12 [13]. Later we will take the direct instanton effect into account explicitly, when it becomes relevant.

## 2.3 Calculation of the Wilson coefficients

Making use of OPE we can expand the products of local operators into the sum of local composite operators multiplied by the Wilson coefficients. In QCD the Wilson coefficients can be calculated perturbatively. We explain the two methods, the background field method [31, 32, 29] and the plane wave method [23], to calculate the Wilson coefficients.

### 2.3.1 Background field method

In the background field method, we introduce the classical background fields to automatically incorporate the non-vanishing matrix elements of local operators. We first shift the quark- and gluon-fields by the classical background fields,

$$\begin{aligned} A^s_\mu &\rightarrow B^s_\mu + C^s_\mu, \\ \psi_f &\rightarrow \chi_f + \eta_f, \\ \bar{\psi}_f &\rightarrow \bar{\chi}_f + \bar{\eta}_f, \end{aligned} \tag{2.11}$$

where f is the flavor index. $B^s_\mu$ denotes the gluon background field, while $C^s_\mu$ denotes its quantum fluctuation. $\chi_f$ and $\bar{\chi}_f$ denote the quark background field; and $\eta_f$ and $\bar{\eta}_f$ denote its quantum fluctuation. If one calculates a correlation function after these modifications, one obtains the terms depending on the background field. The Wilson coefficient of a local operator is given by the coefficient of corresponding background field.

The background fields satisfy the classical equations of the motion,

$$D_\nu G^{\nu\mu} + gT^s \sum_f \bar{q}_f T^s \gamma^\mu q_f = 0, \tag{2.12}$$

$$(i\gamma_\mu D^\mu - m_f)\chi_f = 0, \tag{2.13}$$

$$\bar{\chi}_f (i\gamma_\mu \overleftarrow{D}^\mu + m_f) = 0, \tag{2.14}$$

where $D_\nu$ is the covariant derivative composed of the classical fields,

$$D_\mu = \partial_\mu - igT^s B^s_\mu, \tag{2.15}$$

$$\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + igT^s B^s_\mu, \tag{2.16}$$

and $G^{\nu\mu}$ is the classical field strength tensor,

$$G_{\mu\nu} = T^s G^s_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]. \tag{2.17}$$

$T^s$ is the generator of the color SU(3) gauge group in the fundamental representation.

In the background field method, we take the background field gauge for the gluon fluctuation,

$$D^\mu C^s_\mu = 0. \tag{2.18}$$

$D_\nu$ is the covariant derivative composed of the classical fields given by eq. (2.15). It is known that after this gauge fixing the effective action becomes independent under gauge transformations of the background gluon field $B^s_\mu$. Thus the convenient gauge fixing condition for the background gluon field $B^s_\mu$ can be chosen, and the Fock-Schwinger gauge is usually used:

$$(x^\mu - x^\mu_0)B_\mu(x) = 0. \tag{2.19}$$

The Fock-Schwinger gauge is also known as the fixed point gauge or the radial gauge. Due to the dependence on the coordinate $x_0$, the translational invariance is broken under this gauge fixing. For example, the quark propagator, $(S(x,y))^{ab}_{ij}$ depends on both $x$ and $y$. At the final step of the calculations in the background field method, we may drop all the gauge dependent terms, because the physical matrix elements of the gauge variant operators vanish. Then the translational invariance for physical quantities is recovered. To simplify calculations, we take $x_0 = 0$ in the following.

One advantage of the Fock-Schwinger gauge is that one can straightforwardly expand the relevant amplitudes in terms of gauge invariant quantities. The field strength tensor $G^s_{\mu\nu}$ can be expressed uniquely in terms of the gluon field $B^s_\mu$:

$$B^s_\mu(x) = \int_0^1 d\alpha\ \alpha x_\mu G^{s\,\rho\mu}(\alpha x). \tag{2.20}$$

Taylor expanding the field strength tensor $G^s_{\mu\nu}(\alpha x)$, we obtain

$$\begin{aligned} B^s_\mu(x) &= \frac{1}{2}x^\rho G^s_{\rho\mu}(0) + \frac{1}{3}x^\alpha x^\rho \Big(D_\alpha G^s_{\rho\mu}(0)\Big) \\ &\quad + \frac{1}{8}x^\alpha x^\beta x^\rho \Big(D_\alpha D_\beta G^s_{\rho\mu}(0)\Big) + \cdots. \end{aligned} \tag{2.21}$$

Though the l.h.s. of the eq. (2.21) is gauge variant, the r.h.s of the eq. (2.21) is expressed as the sum of gauge covariant terms.

Similarly Taylor expanding the background quark field, we obtain

$$\chi_f(x) = \chi_f(0) + x^\alpha D_\alpha \chi_f(0) + \frac{1}{2}x^\alpha x^\beta D_\alpha D_\beta \chi_f(0) + \cdots. \tag{2.22}$$

Using eq. (2.22), we can expand $\bar\chi_f(0)\chi_f(x)$ in terms of gauge covariant terms,

$$\begin{aligned} \bar\chi^a_{f\,i}(x)\chi^b_{f\,j}(0) = \quad & \bar\chi^a_{f\,i}(0)\chi^b_{f\,j}(0) + x^\mu \Big(\bar\chi_f(0)\overleftarrow{D}_\mu(0)\Big)^a_i \chi^b_{f\,j}(0) \\ & + \frac{1}{2}x^\mu x^\nu \Big(\bar\chi_f(0)\overleftarrow{D}_\mu(0)\overleftarrow{D}_\nu(0)\Big)^a_i \chi^b_{f\,j}(0) + \cdots, \end{aligned} \tag{2.23}$$

where the color indices $a$ and $b$, and the spinor indices $i$ and $j$ are explicitly shown. Since only the spin-0 and gauge invariant operators can have the non-vanishing VEVs, we correspondingly keep only terms with the spin-0 and gauge-invariant background field. So to obtain the Wilson coefficients in the QCD sum rules in the vacuum, we perform the following replacement for $\bar\chi^a_{f\,i}(x)\chi^b_{f\,j}(0)$:

$$\begin{aligned} \bar\chi^a_{f\,i}(x)\chi^b_{f\,j}(0) \ \to\ & \frac{1}{12}\delta_{ab}\Big[\Big(\delta_{ij} + \frac{i}{4}m(\not{x})_{ji}\Big)\langle\bar\psi_f\psi_f\rangle \\ & + \frac{1}{16}gx^2\Big(\delta_{ij} + \frac{i}{6}m(\not{x})_{ji}\Big)\langle\bar\psi_f\sigma^{\mu\nu}T^s F^s_{\mu\nu}\psi\rangle \end{aligned}$$

$$+\frac{i}{288}g^2x^2(\not{x})_{ji}\langle\bar{\psi}_f\gamma^\mu T^s\psi_f\sum_f\bar{\psi}_f T^s\gamma_\mu\psi_f\rangle$$
$$+\cdots\Big]. \tag{2.24}$$

Here we used the equations of the motion, eqs. (2.13) and (2.14), and show the result up to $O(m)$ and dimension-6.

Other useful formulae can be obtained similarly:

$$B^s_\mu(x)B^t_\nu(y)\to\frac{1}{364}\delta^{st}x^\omega y^\tau(g_{\omega\tau}g_{\mu\nu}-g_{\omega\nu}g_{\mu\tau})\langle F^{\mu\nu}F_{\mu\nu}\rangle+\cdots, \tag{2.25}$$

$$\begin{aligned}&\bar{\chi}_{f\,i}(x)T^sB^s_\rho(z)\chi_{f\,j}(0)\\&\quad\to\frac{z^\mu}{96}\Big[\Big(\sigma_{\mu\rho}-\frac{m_f}{2}(x_\mu\gamma_\rho-x_\rho\gamma_\mu)+i\frac{m_f}{2}x^\nu\sigma_{\mu\rho}\gamma_\nu\Big)_{ji}\langle\bar{\psi}_f\sigma_{\omega\tau}T^sF^{s\,\omega\tau}\psi_f\rangle\\&\qquad+\Big(\frac{2}{3}(z_\rho\gamma_\mu-z_\mu\gamma_\rho)-\frac{i}{2}\not{z}\sigma_{\mu\rho}\Big)_{ji}g\langle\bar{\psi}_f\gamma^\mu T^s\psi_f\sum_f\bar{\psi}_fT^s\gamma_\mu\psi_f\rangle.\\&\qquad+\cdots\Big].\end{aligned} \tag{2.26}$$

In the QCD sum rules for the scattering lengths and for mass shift in the finite density/temperature medium, one considers the hadronic matrix elements of operators. In these cases one should also keep the terms in the spin-nonzero combination.

### 2.3.2 Plane wave method

Since OPE is the operator identity, it should, in principle, hold for any matrix elements. In the plane wave method, we take the matrix elements with respect to the plane wave state of the quarks and gluons, and calculate the amplitudes perturbatively, If we appropriately choose the state to sandwich the both sides of OPE, only specific terms in OPE can contribute. After operating appropriate projections, we can obtain the specific Wilson coefficient.

## 2.4 Vacuum expectation values of local operators

In QCD sum rules in the vacuum, VEVs of local operators are necessary to determine hadronic properties. Actually the VEVs of several low dimensional operators play roles.

The quark condensate (the VEV of the quark bilinear operators) is known to be relevant for the spontaneous breakdown of the chiral symmetry. Its effect has been incorporated into many effective models of QCD. The non-vanishing quark condensate is ensured by the following low energy theorem (LET),

$$2m_\pi^2 f_\pi^2 = -(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle, \tag{2.27}$$

known as the Gell-Mann-Oakes-Renner relation [1] [34]. $m_\pi$ and $f_\pi$ are the mass and the decay constant of the pion: $m_\pi = 140\text{MeV}$ and $f_\pi = 93\text{MeV}$. Assuming isospin symmetry, we get

$$\hat{m}\langle \bar{q}q \rangle \simeq -(0.096\text{GeV})^4 \tag{2.28}$$

where $q$ denote u- or d-quark field, and $\hat{m} = (m_u + m_d)/2$. If we use standard estimate of the quark mass $\hat{m} \simeq 7 - 8\text{MeV}$ [2, 35, 38], we have,

$$\langle \bar{q}q \rangle \simeq -(0.225\text{GeV})^3. \tag{2.29}$$

Here all the renormalization scale dependent quantities, like quark masses are given at the renormalization scale 1GeV, unless it is explicitly mentioned. The strange quark condensate is estimated from QCD sum rules for the strange baryons [65, 37],

$$\langle \bar{s}s \rangle \simeq 0.8\langle \bar{q}q \rangle, \tag{2.30}$$

We will use $m_s \simeq 0.2\text{GeV}$ for the strange quark mass [2, 35, 38].

[1] One can also derive this relation using the QCD sum rule method. We will discuss more on LET from QCD sum rules in section 3 and appendix B

The importance of the non-vanishing gluon condensate, which indicate the breaking of dilatation symmetry, was first emphasized by Vainshtein, Zakharov and Shifman [33]. It is estimated from the charmonium sum rules [13]:

$$\langle \frac{\alpha_s}{\pi} FF \rangle \simeq (0.33\text{GeV})^4. \tag{2.31}$$

There have been attempts to calculate the gluon condensate in the lattice gauge theory [39]. They give lager value than eq. (2.31), though they contain large uncertainty. Instantons is known to contribute the the gluon condensate, though the result is rather sensitive to the instanton density.

The mixed condensate is usually estimated by connecting it with the quark condensate,

$$\langle \bar{q}\sigma_{\mu\nu} g T^a F^{a\,\mu\nu} q \rangle = 2\langle \bar{q} D^\mu D_\mu q \rangle = 2M^2 \langle \bar{q} q \rangle = m_0^2 \langle \bar{q} q \rangle, \tag{2.32}$$

where $M$ is the average virtual momentum of vacuum quarks. QCD sum rule estimation of $m_0$ is given by $m_0^2 \simeq 0.8\text{GeV}^2$ [37]. The mixed condensate for the strange quark is determined from the QCD sum rule for the strange baryon, and is given by [37]

$$\langle \bar{s}\sigma_{\mu\nu} F^{\mu\nu} s \rangle \simeq 0.8 \langle \bar{q}\sigma_{\mu\nu} F^{\mu\nu} q \rangle. \tag{2.33}$$

The four-quark condensate is often estimated with the help of the vacuum saturation hypothesis [13],

$$\langle \bar{\psi}\Gamma_1 \psi \bar{\psi}\Gamma_2 \psi \rangle \longrightarrow \frac{1}{122} \left( \text{Tr}[\Gamma_1]\text{Tr}[\Gamma_2] - \text{Tr}[\Gamma_1\Gamma_2] \right) \langle \bar{q} q \rangle^2, \tag{2.34}$$

where $\Gamma_1$ and $\Gamma_2$ stand for some matrices in the spinor, flavor and color space. The vacuum saturation hypothesis is correct in large $N_c$ limit. In the vector and axial-vector channel the above hypothesis is shown to be phenomenologically good. In the scalar and pseudo-scalar channel, however, there has been discussion on the

validity of the hypothesis, because of large mixing with the vacuum gluon in these channels [40].

## 2.5 Borel sum rule

There are variety types of QCD sum rules depending on the choice of the weight function $W(s/M^2)$, In QCD sum rules for light quark systems, the Borel sum rule is shown to have advantages over the dispersion sum rule eq. (2.3) [13].

The Borel sum rule is obtained by operating the Borel transformation [2]

$$\hat{L}_M \equiv \lim_{\substack{Q^2,n\to\infty \\ \frac{Q^2}{n}=M^2}} \frac{1}{(n-1)!}(Q^2)^n(-\frac{d}{dQ^2})^n, \tag{2.35}$$

on both sides of the dispersion sum rule eq.( 2.3). The Borel sum rule for a two-point correlation function is formally given by the following equation,

$$\hat{L}_M \left[\Pi_{\rm Th}(-Q^2)\right] = \int_0^\infty {\rm Im}\Pi_{\rm Ph}(s)e^{-s/M^2}\, ds, \tag{2.36}$$

where $M$ is called the Borel mass.

Advantages of Borel sum rule over the dispersion sum rule are summarized as follows:

- *OPE series on the theoretical side converges faster.* By applying the Borel transformation to OPE series, we obtain better convergent series. This can be easily seen from the Borel transformation of $(1/Q^2)^k$,

$$\hat{L}_M \left[\frac{1}{(Q^2)^k}\right] = \frac{1}{(k-1)!}\frac{1}{(M^2)^k}. \tag{2.37}$$

  Thus the convergence improves by the factorial factor $1/(k-1)!$. The terms with higher dimensional operators become unimportant in the Borel sum rule.

[2] See appendix A for detail.

- *Contribution of the lowest lying state enhances on the phenomenological side.* In the dispersion sum rule excited states contributions are suppressed by the $1/(s+Q^{2)}$ factor. On the other hand, in the Borel sum rule they are suppressed by the exponential factor $e^{-s/M^2}$, and accordingly the contribution of the lowest lying state enhances.

- *No subtraction terms on the phenomenological side.* In the dispersion sum rule, subtraction terms may be needed on the phenomenological side. However in the Borel sum rule, there are no subtraction term, since all the analytic terms in $Q^2$ disappear after the Borel transformation,

$$\hat{L}_M \left[ (Q^2)^k \right] = 0. \tag{2.38}$$

From the next section, we apply the QCD sum rule method to investigate the properties of hadrons.

# 3 QCD sum rules and low energy theorems

## 3.1 Introductory remarks

In this section we study the pion-nucleon and kaon-nucleon scattering lengths in QCD sum rule, and derive the low energy theorems (LETs) from the QCD sum rules. It is well known that the interactions of the Goldstone bosons with the nucleon at low energies are determined by LETs [10]. Namely, the $T$-matrices for the pion-nucleon (kaon-nucleon) scattering can be calculated by the PCAC and current algebra up to corrections by higher order terms in the pion (kaon) mass.

For the pion-nucleon system, the low energy theorem is quite successful because of the small mass of the pion, and therefore we can check the validity of the QCD sum rules in this application. In fact, we will re-derive the Tomozawa-Weinberg relation [43, 44] as the leading result of the QCD sum rules. The situation is similar to the QCD sum rules for the pion in the vacuum, where Shifman, Vainshtein and Zakharov [13] re-derived the Gell-Mann-Oakes-Renner relation [34].

For the kaon-nucleon system, however, the low energy theorems are not so reliable due to the large kaon mass. Moreover, the experimental results themselves are contradictory: the $K^-$-atom experiment [45] and the $K^-p$ scattering experiment [46] give the scattering lengths which differ from each other in sign. In the QCD sum rule, we do not assume at least formally that the kaon mass is small. Therefore, there is a hope that the QCD sum rule gives a better prediction than the low energy theorem for the kaon-nucleon system and provides some guide to the resolution of the problem.

## 3.2 Borel sum rule for pion (kaon)-nucleon scattering lengths

Recently, it was pointed out that the framework of the QCD sum rule can be extended to the description of hadronic interactions [41, 42]. In ref. [41] it was shown that the nucleon-nucleon scattering lengths are related to the expectation values of the operators such as $q^\dagger q$ and $\bar{q}q$ with respect to the nucleon. The calculated scattering lengths are in qualitative agreement with the experimental values. The same formalism is applied to the vector meson-nucleon scattering lengths in ref. [42]. Here we apply the formalism to the calculation of the pion-nucleon and kaon-nucleon scattering lengths [14].

We first summarize the derivation of the Borel sum rule for the correlation function of the axial-vector current. Later we will show that the obtained Borel sum rule with taking the expectation value with respect to the nucleon give the Tomozawa-Weinberg relation for scattering lengths.

Let us consider the correlation function of axial-vector currents,

$$\Pi_{\mu\nu}(k) = -i \int d^4x e^{ikx} \langle T[A_\mu(x), A_\nu^\dagger(0)] \rangle, \tag{3.1}$$

where the expectation value can be taken with respect to any state. In this section, for the vacuum expectation value we explicitly wrote as $\langle \cdots \rangle_0$. $A_\mu$ denotes the axial vector current.

$$A_\mu(x) = \bar{q}_1(x) \gamma_\mu \gamma_5 q_2(x). \tag{3.2}$$

The spectral representation of the correlation function is given by

$$\Pi_{\mu\nu}(\omega, \mathbf{k}) = \int d\omega' \frac{\rho_{\mu\nu}(\omega', \mathbf{k})}{\omega - \omega'}, \tag{3.3}$$

where $\rho^{\mu\nu}(\omega', \mathbf{k})$ is the spectral density:

$$\begin{aligned} \rho_{\mu\nu}(\omega, \mathbf{k}) &= \frac{i}{2\pi} \{\Pi_{\mu\nu}(\omega + i\eta, \mathbf{k}) - \Pi_{\mu\nu}(\omega - i\eta, \mathbf{k})\} \\ &= -\frac{1}{\pi} \mathrm{Im}\, \Pi_{\mu\nu}(\omega + i\eta, \mathbf{k}). \end{aligned} \tag{3.4}$$

Hereafter we simply write, e.g., $\mathrm{Im}\,\Pi_{\mu\nu}(\omega,\mathbf{k})$ instead of $\mathrm{Im}\,\Pi_{\mu\nu}(\omega+i\eta,\mathbf{k})$. We split $\Pi_{\mu\nu}(\omega,\mathbf{k})$ into even and odd parts in $\omega$:

$$\Pi_{\mu\nu}(\omega,\mathbf{k}) = \Pi_{\mu\nu\mathrm{even}}(\omega^2,\mathbf{k}) + \omega\Pi_{\mu\nu\mathrm{odd}}(\omega^2,\mathbf{k}). \tag{3.5}$$

Then, eq. (3.3) can be rewritten in terms of even and odd parts as

$$\begin{aligned}
\Pi_{\mu\nu\mathrm{even}}(\omega^2,\mathbf{k}) &= -\int_{-\infty}^{\infty} \frac{\rho_{\mu\nu}(\omega',\mathbf{k})}{\omega'^2-\omega^2}\omega' d\omega', \\
\Pi_{\mu\nu\mathrm{odd}}(\omega^2,\mathbf{k}) &= -\int_{-\infty}^{\infty} \frac{\rho_{\mu\nu}(\omega',\mathbf{k})}{\omega'^2-\omega^2} d\omega'.
\end{aligned} \tag{3.6}$$

By applying the Borel transformation,

$$\hat{L}_M \equiv \lim_{\substack{n\to\infty \\ -\omega^2\to\infty \\ -\omega^2/n=M^2}} \frac{(\omega^2)^n}{(n-1)!}\left(-\frac{d}{d\omega^2}\right)^n, \tag{3.7}$$

to eq. (3.6) we get

$$\begin{aligned}
\hat{L}_M[\Pi_{\mu\nu\mathrm{even}}(\omega^2,\mathbf{k})] &= -\int_{-\infty}^{\infty} \rho_{\mu\nu}(\omega',\mathbf{k})\frac{\omega'}{M^2}\exp\left(-\frac{\omega'^2}{M^2}\right)d\omega', \\
\hat{L}_M[\Pi_{\mu\nu\mathrm{odd}}(\omega^2,\mathbf{k})] &= -\int_{-\infty}^{\infty} \rho_{\mu\nu}(\omega',\mathbf{k})\frac{1}{M^2}\exp\left(-\frac{\omega'^2}{M^2}\right)d\omega',
\end{aligned} \tag{3.8}$$

where $M$ is the Borel mass. Subtraction terms may be needed in the spectral representation, eq. (3.3), but will disappear after the Borel transformation in eq. (3.8). These equations, eq. (3.8), with the correlation functions on the l.h.s. evaluated by the operator product expansion (OPE) are the Borel sum rules.

Let us next consider the physical content of the spectral function. The following Ward-Takahashi identity is useful for this purpose:

$$\begin{aligned}
&-i\int d^4x e^{ikx}k^\mu k^\nu \langle T[A_\mu(x),A_\nu^\dagger(0)]\rangle \\
&= -i\int d^4x e^{ikx}\Big\{\langle T[\partial^\mu A_\mu(x),\partial^\nu A_\nu^\dagger(0)]\rangle \\
&\qquad + ik_\mu\langle\delta(x_0)[A_\mu(x),A_0^\dagger(0)]\rangle \\
&\qquad + \langle\delta(x_0)[A_0(x),\partial^\nu A_\nu^\dagger(0)]\rangle\Big\}.
\end{aligned} \tag{3.9}$$

Since the second and the third terms of the r.h.s are real, we get the following relation for the imaginary part of eq. (3.9):

$$k^\mu k^\nu \mathrm{Im}\,\Pi_{\mu\nu}(k) = \mathrm{Im}\left\{-i\int d^4x e^{ikx}\langle T[\partial^\mu A_\mu(x), \partial^\nu A_\nu^\dagger(0)]\rangle\right\}. \tag{3.10}$$

Hereafter we take $\mathbf{k} = 0$. Then only the $\mu = \nu = 0$ component of $\Pi_{\mu\nu}$ is relevant. Therefore, we simplify our notation as follows: $\Pi(\omega) = \Pi_{00}(\omega, \mathbf{k} = 0)$, $\rho(\omega) = \rho_{00}(\omega, \mathbf{k} = 0)$. Thus,

$$\rho(\omega) = -\frac{1}{\pi\omega^2}\mathrm{Im}\left\{-i\int d^4x e^{i\omega t}\langle T[\partial^\mu A_\mu(x), \partial^\nu A_\nu^\dagger(0)]\rangle\right\}. \tag{3.11}$$

We assume that the spectral density in the vacuum, $\rho_0$, is saturated by the pion (kaon) pole terms:

$$\rho_0(\omega) = m_\varphi f_\varphi^2\{\delta(\omega - m_\varphi) - \delta(\omega + m_\varphi)\}. \tag{3.12}$$

It should be noted that $\rho_0$ does not have the pole term due to the axial-vector meson since $\mathbf{k} = 0$.

Following ref. [44] we define the off-shell $T$-matrix by

$$\begin{aligned} &T(\nu, t, k^2, k'^2) = \\ &-i\frac{(k^2 - m_\varphi^2)(k'^2 - m_\varphi^2)}{2f_\varphi^2 m_\varphi^4}\int d^4x e^{ikx}\langle \mathrm{N}(p)|T[\partial^\mu A_\mu(x), \partial^\nu A_\nu^\dagger(0)]|\mathrm{N}(p')\rangle, \end{aligned} \tag{3.13}$$

where $\nu = \omega + t/4M_\mathrm{N}$, $t = (k - k')^2$ and $k + p = k' + p'$. In this and later equations $\langle \mathrm{N}|\mathcal{O}|\mathrm{N}\rangle$ means the matrix element with the disconnected part, $\langle 0|\mathcal{O}|0\rangle\langle \mathrm{N}|\mathrm{N}\rangle$, subtracted. $f_\varphi$ is defined in the standard way:

$$\langle 0|A_\mu(0)|\varphi(k)\rangle = i\sqrt{2}f_\varphi k_\mu. \tag{3.14}$$

In this definition the $T$-matrix is related to the $S$-matrix by $S_{fi} = \delta_{fi} - i(2\pi)^4\delta^4(k' + p' - k - p)T_{fi}$.

From eqs. (3.11) and (3.13) the spectral density, in which the expectation value is taken with respect to the nucleon, becomes

$$\begin{aligned}\rho_{\rm N}(\omega) &= \frac{-1}{\pi\omega^2}{\rm Im}\left\{\frac{2f_\varphi^2 m_\varphi^4}{(\omega^2-m_\varphi^2)^2}T(\omega,0,\omega^2,\omega^2)\right\} \\ &= -\frac{1}{2}f_\varphi^2\left[\delta'(\omega-m_\varphi){\rm Re}T_+ - \delta(\omega-m_\varphi){\rm Re}\left(T'_+ - \frac{3}{m_\varphi}T_+\right)\right. \\ &\quad +\delta'(\omega+m_\varphi){\rm Re}T_- + \delta(\omega+m_\varphi){\rm Re}\left(T'_- - \frac{3}{m_\varphi}T_-\right) \\ &\quad \left. + \frac{4m_\varphi^4}{\omega^2}{\rm Re}\frac{1}{(\omega^2-m_\varphi^2)^2}\frac{1}{\pi}{\rm Im}\,T(\omega,0,\omega^2,\omega^2)\right], \end{aligned} \tag{3.15}$$

where $T_\pm = T(\pm m_\varphi, 0, m_\varphi^2, m_\varphi^2)$, $T'_\pm = \pm\frac{\partial}{\partial\omega}T(\omega,0,\omega^2,\omega^2)|_{\omega=\pm m_\varphi}$. For the time being we ignore the last term in eq. (3.15). Then the spectral function becomes

$$\begin{aligned}\rho_{\rm N}(\omega) &= -\frac{1}{2}f_\varphi^2\left[\delta'(\omega-m_\varphi){\rm Re}T_+ - \delta(\omega-m_\varphi){\rm Re}\left(T'_+ - \frac{3}{m_\varphi}T_+\right)\right. \\ &\quad \left. +\delta'(\omega+m_\varphi){\rm Re}T_- + \delta(\omega+m_\varphi){\rm Re}\left(T'_- - \frac{3}{m_\varphi}T_-\right)\right]. \end{aligned} \tag{3.16}$$

Hereafter we simply write $T$ instead of ${\rm Re}T$ for notational simplicity.

By substituting eqs. (3.12) and (3.16) into eq. (3.8) we get

$$-2\frac{m_\varphi^2}{M^2}f_\varphi^2\exp\left(-\frac{m_\varphi^2}{M^2}\right) = \hat{L}_M[\Pi_{\rm 0even}(\omega^2)], \tag{3.17}$$

$$-2\frac{m_\varphi^2}{M^6}f_\varphi^2 T_\pm\exp\left(-\frac{m_\varphi^2}{M^2}\right) = \hat{L}'_M[\Pi_{\rm Neven}(\omega^2)] \pm m_\varphi\hat{L}'_M[\Pi_{\rm Nodd}(\omega^2)], \tag{3.18}$$

$$-2\frac{m_\varphi^3}{M^6}f_\varphi^2 T'_\pm\exp\left(-\frac{m_\varphi^2}{M^2}\right) = 2\hat{L}''_M[\Pi_{\rm Neven}(\omega^2)] \pm 3m_\varphi\hat{L}'''_M[\Pi_{\rm Nodd}(\omega^2)], \tag{3.19}$$

where

$$\begin{aligned}\hat{L}'_M[\Pi_{\rm N}(\omega^2)] &= \frac{d}{dM^2}\hat{L}_M[\Pi_{\rm N}(\omega^2)] \\ &\quad +\left(1-\frac{m_\varphi^2}{M^2}\right)\frac{1}{M^2}\hat{L}_M[\Pi_{\rm N}(\omega^2)],\end{aligned}$$

$$
\begin{aligned}
\hat{L}''_M[\Pi_{\rm N}(\omega^2)] &= \left(1+\frac{m_\varphi^2}{M^2}\right)\frac{d}{dM^2}\hat{L}_M[\Pi_{\rm N}(\omega^2)] \\
&\quad + \left(1+\frac{m_\varphi^2}{M^2}-\frac{m_\varphi^4}{M^4}\right)\frac{1}{M^2}\hat{L}_M[\Pi_{\rm N}(\omega^2)], \\
\hat{L}'''_M[\Pi_{\rm N}(\omega^2)] &= \left(1+\frac{2}{3}\frac{m_\varphi^2}{M^2}\right)\frac{d}{dM^2}\hat{L}_M[\Pi_{\rm N}(\omega^2)] \\
&\quad + \left(1+\frac{1}{3}\frac{m_\varphi^2}{M^2}-\frac{2}{3}\frac{m_\varphi^4}{M^4}\right)\frac{1}{M^2}\hat{L}_M[\Pi_{\rm N}(\omega^2)]. \qquad (3.20)
\end{aligned}
$$

From eqs. (3.17), (3.18) and (3.19) $T_\pm$ and $T'_\pm$ are given by

$$
T_\pm = M^4\frac{\hat{L}'_M[\Pi_{\rm N\,even}(\omega^2)] \pm m_\varphi \hat{L}'_M[\Pi_{\rm N\,odd}(\omega^2)]}{\hat{L}_M[\Pi_{0\,\rm even}(\omega^2)]}, \qquad (3.21)
$$

$$
T'_\pm = \frac{M^4}{m_\varphi}\frac{2\hat{L}''_M[\Pi_{\rm N\,even}(\omega^2)] \pm 3m_\varphi \hat{L}'''_M[\Pi_{\rm N\,odd}(\omega^2)]}{\hat{L}_M[\Pi_{0\,\rm even}(\omega^2)]}. \qquad (3.22)
$$

The scattering lengths can be calculated from $T_\pm$ using the following relation.

$$
a_{\varphi^\pm {\rm N}} = -\frac{1}{4\pi m_{\varphi^\pm}}\frac{m_{\varphi^\pm} M_{\rm N}}{m_{\varphi^\pm}+M_{\rm N}}T_\pm \qquad (3.23)
$$

Therefore, we can calculate the r.h.s of eq. (3.21) by the OPE and compare it with experimentally observed scattering length.

Here, we would like to make a brief comment on the physical significance of $T'$, the derivative of the $T$-matrix with respect to the pion energy, $\omega$. In the sum rule approach, only on-shell quantities appear in the spectral function. Therefore, off-shell $T$-matrices, which are important when one discusses the pion (kaon) mass in nuclear matter [47], cannot be directly obtained. We can, however, obtain $T'$, the slope of the $T$-matrix as a function of the pion energy, at on-shell points, with which we can linearly extrapolate the $T$-matrices from the on-shell to off-shell points.

Let us now consider the last term in eq. (3.15). In the pion-nucleon channel this term is mainly due to the pion-nucleon continuum states above the pion-nucleon threshold. In the kaon-nucleon channel in addition to the kaon-nucleon continuum states the pion-sigma continuum states also contribute even below the kaon-nucleon

threshold. In particular, there exists a resonance $\Lambda(1405)$ which is phenomenologically known to have the most of the strength below the kaon-nucleon threshold. If these contributions are not small, we have to add them to the observed scattering lengths and the results should be compared with the calculations by the OPE. The importance of these contributions cannot be directly checked by experiments, because the spectral functions, or equivalently the off-shell pion-nucleon and kaon-nucleon $T$-matrices, are not direct observables. In sections 3.3 and 3.4 we show the results without this contribution for the pion-nucleon channel and those with the contribution of $\Lambda(1405)$ taken into account by using the effective coupling strength determined by the analysis of the scattering data. The contribution of the continuum states above the threshold will be estimated for the pion-nucleon case by employing the non-linear sigma model in section 3.5. We should emphasize again here that what can be calculated by the OPE is just the l.h.s. of eq. (3.8) and the estimation of the contributions of $\Lambda(1405)$ and continuum states above the threshold in the following sections are meant to be indirect experimental information for the l.h.s. of eq. (3.8).

## 3.3 Results with leading and next-to-leading terms of OPE

Let us now turn to the OPE. The leading and next-to-leading order terms of the correlation function in the OPE can be read off from the Ward-Takahashi identity, eq. (3.9), without explicitly performing the OPE. We rewrite eq. (3.9) as

$$\begin{aligned} & -i\int d^4x e^{ikx} k^\mu k^\nu \langle T[A_\mu(x), A_\nu^\dagger(0)]\rangle \\ &= \; k^\mu \langle \bar{q}_1\gamma_\mu q_1 - \bar{q}_2\gamma_\mu q_2\rangle - (m_1+m_2)\langle \bar{q}_1 q_1 + \bar{q}_2 q_2\rangle \\ &\quad -(m_1+m_2)^2 i\int d^4x e^{ikx}\langle T\left[\varphi(x), \varphi^\dagger(0)\right]\rangle, \end{aligned} \tag{3.24}$$

where

$$\varphi(x) = i\bar{q}_1(x)\gamma_5 q_2(x). \tag{3.25}$$

On the r.h.s. of eq. (3.24), the dimensions of the operators in the first, second and third terms are three, four and five or higher, respectively. (The OPE for the correlation function of $\varphi$ has at least dimension three.) Moreover, their quark mass dependence is constant, linear and quadratic, respectively. Therefore, the first term is the most important, the second is next and the third is the least important not only in the sense of the OPE but also in the sense of the chiral symmetry breaking expansion. Thus, we first concentrate on the first two terms in eq. (3.24). The effect of the higher dimension operators in the last term will be discussed later.

In ref. [13], Shifman, Vainshtein and Zakharov showed that from the sum rules in the vacuum, essentially eqs. (3.17) and (3.24), the Gell-Mann-Oakes-Renner relation [34] is re-derived

$$2m_\varphi^2 f_\varphi^2 = -\left(m_1 + m_2\right)\langle \bar{q}_1 q_1 + \bar{q}_2 q_2\rangle_0, \tag{3.26}$$

where higher order terms of $m_\varphi^2/M^2$ are neglected.

Similarly, we can show that from the sum rules in the nucleon, eqs. (3.21), (3.22) and (3.24), the following relations for the $T$-matrix and its derivative are derived:

$$T_{\varphi \mathrm{N}} = -\frac{m_\varphi \langle q_1^\dagger q_1 - q_2^\dagger q_2\rangle_{\mathrm{N}} - (m_1+m_2)\langle \bar{q}_1 q_1 + \bar{q}_2 q_2\rangle_{\mathrm{N}}}{2f_\varphi^2}, \tag{3.27}$$

$$T'_{\varphi \mathrm{N}} = -\frac{\langle q_1^\dagger q_1 - q_2^\dagger q_2\rangle_{\mathrm{N}}}{2f_\varphi^2}. \tag{3.28}$$

The matrix elements of the operators in eqs. (3.27) and (3.28), $\langle q^\dagger q\rangle_{\mathrm{N}}$ and $m_q\langle \bar{q}q\rangle_{\mathrm{N}}$, are given by the quark number in the nucleon and the nucleon sigma term, respectively:

$$\langle u^\dagger u\rangle_{\mathrm{p}} \;=\; \langle d^\dagger d\rangle_{\mathrm{n}} = 2,$$

$$\begin{aligned}\langle d^\dagger d\rangle_{\rm p} &= \langle u^\dagger u\rangle_{\rm n} = 1,\\ \langle s^\dagger s\rangle_{\rm p} &= \langle s^\dagger s\rangle_{\rm n} = 0,\end{aligned} \tag{3.29}$$

and

$$\begin{aligned}\frac{m_u+m_d}{2}\langle\bar{u}u+\bar{d}d\rangle_{\rm p} &= \frac{m_u+m_d}{2}\langle\bar{u}u+\bar{d}d\rangle_{\rm n} = \sigma_{\pi{\rm N}},\\ \frac{m_u+m_s}{2}\langle\bar{u}u+\bar{s}s\rangle_{\rm p} &= \frac{m_s+m_d}{2}\langle\bar{s}s+\bar{d}d\rangle_{\rm n} = \sigma_{\rm Kp},\\ \frac{m_u+m_s}{2}\langle\bar{u}u+\bar{s}s\rangle_{\rm n} &= \frac{m_s+m_d}{2}\langle\bar{s}s+\bar{d}d\rangle_{\rm p} = \sigma_{\rm Kn}.\end{aligned} \tag{3.30}$$

We use the following values for the sigma terms in later calculations: $\sigma_{\pi{\rm N}} = 45{\rm MeV}$, $\sigma_{\rm Kp} = 374{\rm MeV}$ and $\sigma_{\rm Kn} = 330{\rm MeV}$. The pion-nucleon sigma term is taken from ref. [18] and the kaon-nucleon sigma terms are calculated from the above pion-nucleon sigma term, the quark masses and the $y = 0.2$ [18] [3] Here $y$ parameter is given by

$$y = \frac{2\langle\bar{s}s\rangle_{\rm p}}{\langle\bar{u}u+\bar{d}d\rangle_{\rm p}}. \tag{3.31}$$

### 3.3.1 Pion-nucleon scattering lengths

Let us first look at the pion-nucleon channel. The $T$-matrices and their derivatives are given by

$$\begin{aligned}T^{(+)}_{\pi{\rm N}} &= \frac{(m_u+m_d)\langle\bar{u}u+\bar{d}d\rangle_{\rm N}}{2f_\pi^2} = \frac{\sigma_{\pi{\rm N}}}{f_\pi^2},\\ T^{(-)}_{\pi{\rm N}} &= -\frac{m_\pi\langle u^\dagger u - d^\dagger d\rangle_{\rm p}}{2f_\pi^2} = -\frac{m_\pi}{2f_\pi^2},\\ T'^{(+)}_{\pi{\rm N}} &= 0,\\ T'^{(-)}_{\pi{\rm N}} &= -\frac{\langle u^\dagger u - d^\dagger d\rangle_{\rm p}}{2f_\pi^2} = -\frac{1}{2f_\pi^2},\end{aligned} \tag{3.32}$$

where $T^{(\pm)}_{\pi{\rm N}} = \frac{1}{2}(T_{\pi^-{\rm p}} \pm T_{\pi^+{\rm p}}) = \frac{1}{2}(T_{\pi^+{\rm n}} \pm T_{\pi^-{\rm n}})$.

[3] In section 5, we will estimate the $y$ parameter by the QCD sum rule method. The calculated value is much smaller than the value appearing here.

The leading order term, the dimension-three operator, contributes to the isospin-odd component of the $T$-matrix and gives the Tomozawa-Weinberg term [43, 44]. The next-to-leading order term, the dimension-four operator, contributes to the isospin-even component of the $T$-matrix and gives the sigma term, which is the same as that obtained by using the PCAC and current algebra at the Weinberg point [48] [4].

It is interesting that the quark number in the nucleon determines the leading-order form of the $T$-matrix. It should be emphasized again that in the present approach the $T$-matrix is obtained at the pion-nucleon threshold, $\nu = m_\pi$, $t = 0$, $k^2 = k'^2 = m_\pi^2$, while in the PCAC-plus-current-algebra approach it is at the Weinberg point, i.e., $\nu = t = k^2 = k'^2 = 0$.

The calculated scattering lengths are tabulated in table 3.1, where the observed scattering lengths are also shown for comparison. The scattering lengths calculated with the dimension-three operator, the entries in the first column, are surprisingly close to the experimental values, which is exactly the same result as in refs. [43, 44]. The contribution of the dimension-four operator in the isospin-even component of the $T$-matrix, the entry in the second column, is much larger than the experimental value. This is similar to the fact that the $T$-matrix at the Weinberg point in the PCAC-plus-current-algebra approach is much larger than the experimental value at the pion-nucleon threshold. The latter difference is usually attributed to sources such as resonance and/or smooth background contributions, which cannot be determined by the symmetry argument alone. Therefore, a natural question is what cancels the sigma term contribution to the $T$-matrix at the pion-nucleon threshold in the

[4] The derivation of the Tomozawa-Weinberg relation presented here is not exactly parallel to that of the Gell-Mann-Oakes-Renner relation in ref. [13]. In appendix B we present more parallel derivation.

Table 3.1: Calculated and observed pion-nucleon and kaon-nucleon scattering lengths in the unit of fm. Experimental values are taken from ref. [49] for the pion-nucleon channel and ref. [50] for the kaon-nucleon channel. In the parentheses are shown the scattering lengths calculated without $\Lambda(1405)$ contribution. We explicitly showed the errors in the calculated scattering lengths due to the $\bar{\mathrm{K}}\mathrm{N}\Lambda(1405)$ coupling constant. Certainly, there are other errors in the calculated and observed scattering lengths. However, we cannot specify explicit numbers for those errors except for the observed pion-nucleon scattering lengths, which are small anyway.

| | dim. 3 | $\leq$ dim. 4 | $\leq$ dim. 6 | Expt. |
|---|---|---|---|---|
| $a_{\pi\mathrm{N}}^{(+)}$ | 0 | $-0.07$ | $-0.07$ | $-0.01$ |
| $a_{\pi\mathrm{N}}^{(-)}$ | 0.11 | 0.11 | 0.12 | 0.13 |
| $a_{\mathrm{Kp}}^{(+)}$ | $-0.67 \pm 0.54$ | $-0.97 \pm 0.54$ | $-1.02 \pm 0.54$ | $-0.50$ |
| | (0) | ($-0.29$) | ($-0.34$) | |
| $a_{\mathrm{Kp}}^{(-)}$ | $-0.28 \pm 0.54$ | $-0.28 \pm 0.54$ | $-0.22 \pm 0.54$ | $-0.17$ |
| | (0.39) | (0.39) | (0.45) | |
| $a_{\mathrm{K}n}^{(+)}$ | 0 | $-0.26$ | $-0.31$ | 0.10 |
| $a_{\mathrm{K}n}^{(-)}$ | 0.20 | 0.20 | 0.23 | 0.27 |

present approach. There are two possibilities: one is higher order terms in the OPE and the other is the continuum contribution above the pion-nucleon threshold in the spectral function. We will come back to this point later.

### 3.3.2 Kaon-nucleon scattering lengths

Let us turn to the kaon-nucleon channel. The contribution of $\Lambda(1405)$ is taken into account by the following replacement [46]:

$$\begin{aligned} T_{\mathrm{K^-p}} &\to T_{\mathrm{K^-p}} - g_{\Lambda^*}^2 \frac{m_{\mathrm{K}}^2}{m_{\mathrm{K}}+M_{\mathrm{N}}-M_{\Lambda^*}} \left(\frac{1}{M_{\Lambda^*}-M_{\mathrm{N}}}\right)^2, \\ T'_{\mathrm{K^-p}} &\to T'_{\mathrm{K^-p}} + g_{\Lambda^*}^2 \frac{m_{\mathrm{K}}}{(M_{\Lambda^*}-M_{\mathrm{N}})^2} \left(\frac{1}{M_{\Lambda^*}-M_{\mathrm{N}}-m_{\mathrm{K}}}\right)^2 \left\{2(M_{\Lambda^*}-M_{\mathrm{N}})-m_{\mathrm{K}}\right\}, \end{aligned} \tag{3.33}$$

where $g_{\Lambda^*}$ is the $\bar{\mathrm{K}}\mathrm{N}\Lambda(1405)$ coupling constant, $M_{\Lambda^*}$ is the mass of $\Lambda(1405)$. In eq. (3.33) we neglected higher order terms in the binding energy, $m_{\mathrm{K}}+M_{\mathrm{N}}-M_{\Lambda^*}$. Having done this modification, we obtain the $T$-matrices and their derivatives as

$$\begin{aligned} T_{\mathrm{Kp}}^{(+)} &= \frac{\sigma_{\mathrm{Kp}}}{f_{\mathrm{K}}^2} + \frac{g_{\Lambda^*}^2}{2} \frac{m_{\mathrm{K}}^2}{m_{\mathrm{K}}+M_{\mathrm{N}}-M_{\Lambda^*}} \left(\frac{1}{M_{\Lambda^*}-M_{\mathrm{N}}}\right)^2, \\ T_{\mathrm{Kn}}^{(+)} &= \frac{\sigma_{\mathrm{Kn}}}{f_{\mathrm{K}}^2}, \\ T_{\mathrm{Kp}}^{(-)} &= -\frac{m_{\mathrm{K}}}{f_{\mathrm{K}}^2} + \frac{g_{\Lambda^*}^2}{2} \frac{m_{\mathrm{K}}^2}{m_{\mathrm{K}}+M_{\mathrm{N}}-M_{\Lambda^*}} \left(\frac{1}{M_{\Lambda^*}-M_{\mathrm{N}}}\right)^2, \\ T_{\mathrm{Kn}}^{(-)} &= -\frac{m_{\mathrm{K}}}{2f_{\mathrm{K}}^2}, \\ T_{\mathrm{Kp}}^{\prime(+)} &= -\frac{g_{\Lambda^*}^2}{2} \frac{m_{\mathrm{K}}}{(M_{\Lambda^*}-M_{\mathrm{N}})^2} \left(\frac{1}{M_{\Lambda^*}-M_{\mathrm{N}}-m_{\mathrm{K}}}\right)^2 \left\{2(M_{\Lambda^*}-M_{\mathrm{N}})-m_{\mathrm{K}}\right\}, \\ T_{\mathrm{Kn}}^{\prime(+)} &= 0, \\ T_{\mathrm{Kp}}^{\prime(-)} &= -\frac{1}{f_{\mathrm{K}}^2} - \frac{g_{\Lambda^*}^2}{2} \frac{m_{\mathrm{K}}}{(M_{\Lambda^*}-M_{\mathrm{N}})^2} \left(\frac{1}{M_{\Lambda^*}-M_{\mathrm{N}}-m_{\mathrm{K}}}\right)^2 \left\{2(M_{\Lambda^*}-M_{\mathrm{N}})-m_{\mathrm{K}}\right\}, \\ T_{\mathrm{Kn}}^{\prime(-)} &= -\frac{1}{2f_{\mathrm{K}}^2}, \end{aligned} \tag{3.34}$$

where $T_{\mathrm{KN}}^{(\pm)} = \frac{1}{2}(T_{\mathrm{K^-N}} \pm T_{\mathrm{K^+N}})$.

The results for the kaon-nucleon channel are similar to those for the pion-nucleon channel except for the contribution of $\Lambda(1405)$: the dimension-three operator gives the Tomozawa-Weinberg term to $T^{(-)}$ and the dimension-four operator gives the sigma term to $T^{(+)}$.

The calculated and observed scattering lengths are tabulated in table 3.1. If we believe the experimental values for the kaon-nucleon scattering lengths, the original Tomozawa-Weinberg results, the entries in the first column for $a_{\rm Kn}^{(\pm)}$ and those in the parentheses in the first column for $a_{\rm Kn}^{(\pm)}$, are not so successful as in the pion-nucleon channel. In the present approach, the leading terms are those with the contribution of $\Lambda(1405)$ included. Though the error of the $\Lambda(1405)$ contribution due to the experimental uncertainty of the $\bar{\rm K}{\rm N}\Lambda(1405)$ coupling constant is quite large, the inclusion of the $\Lambda(1405)$ contribution seems to improve the agreement with the observed values. However, we should also keep in mind that the experimental situation concerning the $\rm K^-p$ scattering length is still controversial. Namely, the scattering lengths determined by the atomic experiment, ref. [45], differ in sign from those shown in table 3.1, which are determined by the scattering experiment. Sound experimental determination of the scattering length as well as the $\bar{\rm K}{\rm N}\Lambda(1405)$ coupling constant is needed.

## 3.4 Contribution of higher order terms of OPE

Let us now consider the effect of the higher order terms of the OPE. The OPE of the correlation function is given up to dimension six as follows:

$$
\begin{aligned}
\Pi_{\rm 0odd}(\omega^2) &= 0, \\
\Pi_{\rm 0even}(\omega^2) &= \frac{3}{8\pi^2}(m_1+m_2)^2\ln(-\omega^2) - (m_1+m_2)\,\langle\bar{q}_1 q_1 + \bar{q}_2 q_2\rangle_0 \frac{1}{\omega^2} \\
&\quad + \frac{1}{8}(m_1+m_2)^2\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle_0\frac{1}{\omega^4}, \\
\Pi_{\rm Nodd}(\omega^2) &= \langle q_1^\dagger q_1 - q_2^\dagger q_2\rangle_{\rm N}\frac{1}{\omega^2} + (m_1+m_2)^2\langle q_1^\dagger q_1 - q_2^\dagger q_2\rangle_{\rm N}\frac{1}{\omega^4}, \\
\Pi_{\rm Neven}(\omega^2) &= -(m_1+m_2)\langle\bar{q}_1 q_1 + \bar{q}_2 q_2\rangle_{\rm N}\frac{1}{\omega^2} \\
&\quad -(m_1+m_2)^2\Big\{2\,(i\langle\bar{q}_1\mathcal{S}(\gamma_0 D_0)q_1\rangle_{\rm N} + i\langle\bar{q}_2\mathcal{S}(\gamma_0 D_0)q_2\rangle_{\rm N})
\end{aligned}
$$

$$-\frac{1}{8}\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle_{\rm N}-\frac{1}{2}\langle\frac{\alpha_s}{\pi}\mathcal{S}(G_{0\nu}G_0^{\nu})\rangle_{\rm N}\Big\}\frac{1}{\omega^4}, \tag{3.35}$$

where $\mathcal{S}[A_\mu B_\nu]$ means the symmetric and trace-less tensor made of $A_\mu$ and $B_\nu$. By substituting eq. (3.35) into eq. (3.17) we obtain

$$\begin{aligned}2m_\varphi^2 f_\varphi^2\frac{1}{M^2}\exp\left(-\frac{m_\varphi^2}{M^2}\right) &= -(m_1+m_2)\langle\bar{q}_1q_1+\bar{q}_2q_2\rangle_0\frac{1}{M^2}\\ &\quad+(m_1+m_2)^2\left\{\frac{3}{8\pi^2}-\frac{1}{8}\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle_0\frac{1}{M^4}\right\}.\end{aligned} \tag{3.36}$$

Similarly, by substituting eq. (3.35) into eqs. (3.21) and (3.22) and dividing them by eq. (3.36) we obtain

$$T_{\varphi{\rm N}}=\frac{m_\varphi^2}{M^2}\frac{m_\varphi\langle q_1^\dagger q_1-q_2^\dagger q_2\rangle_{\rm N}-(m_1+m_2)\langle\bar{q}_1q_1+\bar{q}_2q_2\ \rangle_{\rm N}-\langle\tilde{\mathcal{O}}_1\rangle_{\rm N}\left(\frac{1}{m_\varphi^2}+\frac{1}{M^2}\right)}{-\frac{3}{8\pi^2}(m_1+m_2)^2+(m_1+m_2)\langle\bar{q}_1q_1+\bar{q}_2q_2\ \rangle_0\frac{1}{M^2}+(m_1+m_2)^2\frac{1}{8}\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle_0\frac{1}{M^4}}, \tag{3.37}$$

$$T'_{\varphi{\rm N}}=\frac{m_\varphi}{M^2}\frac{m_\varphi\langle q_1^\dagger q_1-q_2^\dagger q_2\rangle_{\rm N}\left(1+\frac{2m_\varphi^2}{M^2}\right)-\frac{2m_\varphi^2}{M^2}(m_1+m_2)\langle\bar{q}_1q_1+\bar{q}_2q_2\rangle_{\rm N}-\langle\tilde{\mathcal{O}}_2\rangle_{\rm N}\left(\frac{1}{m_\varphi^2}+\frac{1}{M^2}\right)}{-\frac{3}{8\pi^2}(m_1+m_2)^2+(m_1+m_2)\langle\bar{q}_1q_1+\bar{q}_2q_2\rangle_0\frac{1}{M^2}+(m_1+m_2)^2\frac{1}{8}\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle_0\frac{1}{M^4}},$$

where

$$\begin{aligned}\tilde{\mathcal{O}}_1 = \ & (m_1+m_2)^2\Big[m_\varphi\left(q_1^\dagger q_1-q_2^\dagger q_2\right)\\ & -\left\{2\left(i\bar{q}_1\mathcal{S}(\gamma_0D_0)q_1+i\bar{q}_2\mathcal{S}(\gamma_0D_0)q_2\right)-\tfrac{1}{8}\tfrac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}-\tfrac{1}{2}\tfrac{\alpha_s}{\pi}\mathcal{S}(G_{0\nu}G_0^{\nu})\right\}\Big],\end{aligned} \tag{3.38}$$

$$\begin{aligned}\tilde{\mathcal{O}}_2 = \ & (m_1+m_2)^2\Big[3m_\varphi\left(q_1^\dagger q_1-q_2^\dagger q_2\right)\\ & -2\left\{2\left(i\bar{q}_1\mathcal{S}(\gamma_0D_0)q_1+i\bar{q}_2\mathcal{S}(\gamma_0D_0)q_2\right)-\tfrac{1}{8}\tfrac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}-\tfrac{1}{2}\tfrac{\alpha_s}{\pi}\mathcal{S}(G_{0\nu}G_0^{\nu})\right\}\Big].\end{aligned} \tag{3.39}$$

The scattering lengths with higher order terms included are shown in the third column of table 3.1. The matrix elements of the operators with respect to the nucleon are taken from ref. [51], and are

$$\begin{aligned}i\langle\mathcal{S}[\bar{u}\gamma_\mu D_\nu u]\rangle_{\rm p} &= i\langle\mathcal{S}[\bar{d}\gamma_\mu D_\nu d]\rangle_{\rm n}=222\ {\rm MeV},\\ i\langle\mathcal{S}[\bar{d}\gamma_\mu D_\nu d]\rangle_{\rm p} &= i\langle\mathcal{S}[\bar{u}\gamma_\mu D_\nu u]\rangle_{\rm n}=95\ {\rm MeV},\end{aligned}$$

$$
\begin{aligned}
i\langle \mathcal{S}[\bar{s}\gamma_\mu D_\nu s]\rangle_{\rm p} &= i\langle \mathcal{S}[\bar{s}\gamma_\mu D_\nu s]\rangle_{\rm n} = 18 \text{ MeV}, \\
\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu}\rangle_{\rm N} &= -738 \text{ MeV}, \qquad \langle \frac{\alpha_s}{\pi} \mathcal{S}[G_{\mu 0} G^{\mu 0}]\rangle_{\rm N} = -50 \text{ MeV}. \quad (3.40)
\end{aligned}
$$

Some of the values are obtained from the measurement of the structure function. It should be noted that these matrix elements both with the nucleon and the vacuum are for the renormalization scale, 1 GeV. We checked the Borel mass stability of the results and found that the results are very stable in the wide range of the Borel mass. For instance, the pion-nucleon scattering lengths change only 5 % when $M^2$ changes from 0.5 to 2.5 GeV$^2$. In table 3.1 the results with $M^2 = 1$ GeV$^2$ are shown.

In the pion-nucleon channel the effect of the dimension-five and six operators is very small. Also in the kaon-nucleon channel the effect is small but not so extreme as in the pion-nucleon channel. This is because the kaon mass is not so small as the pion mass.

## 3.5 Continuum contribution

We now come back to the question concerning the discrepancy in the isospin-even component of the $T$-matrix in the pion-nucleon channel. Since we have observed that the effect of the dimension-five and six terms of the OPE is small, it is unlikely that the difference is explained by higher order terms of the OPE. Therefore, we speculate that the continuum contribution near above the pion-nucleon threshold should be responsible for the discrepancy.

In order to confirm the above speculation, we estimate the continuum contribution, the last term of eq. (3.15), employing the non-linear sigma model [52], which is known to describe the low-energy pion-nucleon scattering well. The relevant in-

teraction lagrangian density of the non-linear sigma model is given by

$$\mathcal{L}_{\rm int} = \frac{1}{4f_\pi^2}\bar{\psi} i\gamma^\mu \tau\psi \cdot (\phi \times \partial_\mu \phi) + \frac{g}{2M_{\rm N}}\bar{\psi}\gamma_5\gamma^\mu\tau\psi\cdot\partial_\mu\phi, \tag{3.41}$$

where $\psi$ is the nucleon filed, $\phi$ is the pion filed and the coupling constants $g$ is taken to be 13.5. In order to obtain Im $T$ we use the optical theorem,

$$\text{Im } T_{ii} = -\frac{1}{2}\sum_n (2\pi)^4\delta^4(p_n + k_n + p + k)|\mathcal{T}_{ni}|^2, \tag{3.42}$$

and calculate the off-shell T-matrix, $\mathcal{T}_{ni}$, at the tree-level for the interaction lagrangian density, eq. (3.41), as shown in fig. 3.1.

The calculated results are $-0.060$ fm and $-0.016$ fm for the isospin-even and isospin-odd channels, respectively where the Borel mass is taken to be 1 GeV. If we add these contributions to the experimental scattering lengths, the results agree well with the calculated values.

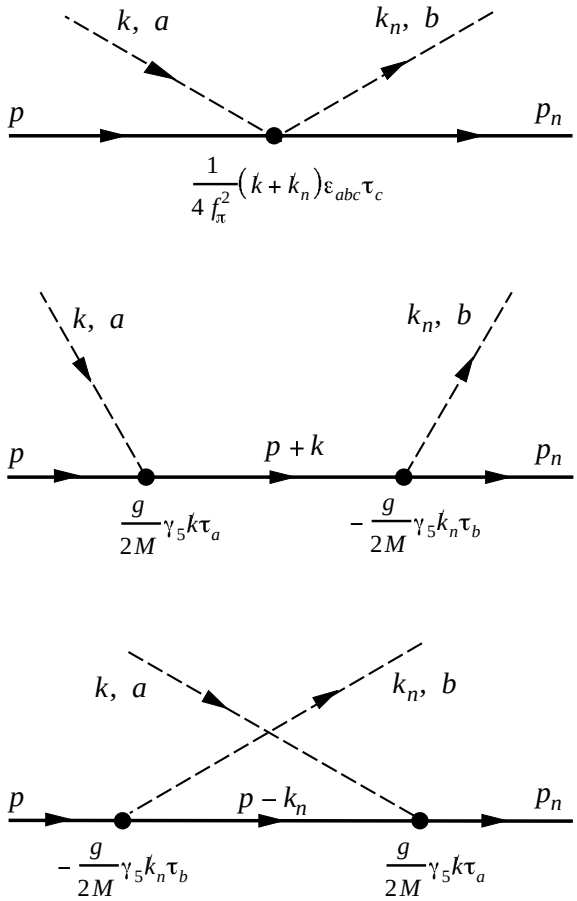


Figure 3.1: Tree-level Feynman diagrams for the pion-nucleon $T$-matrix in the non-linear sigma model.

# 4 Quark momentum fraction for pseudo-scalar mesons

## 4.1 Introductory remarks

The quark distribution in hadron has been studied in the QCD sum rule method. Belyaev and Ioffe have studied the Bjorken variable $\xi$ dependence of the spin-averaged parton distribution $f_1(\xi, \mu^2)$ of the proton based on QCD sum rules, and have shown that their method is applicable in the intermediate region of the Bjorken variable $\xi$ [53]. Later the result was confirmed by Singh and Pasupathy [54]. They have also shown that the QCD sum rule calculation of the longitudinally polarized parton distribution $g_1(\xi, \mu^2)$ of the proton agrees with experiments in the intermediate region of the Bjorken variable [54]. More recently Ioffe and Khodjamirian made the QCD sum rule calculation of the chiral-odd transversely polarized parton distribution $h_1(\xi, \mu^2)$ of the proton [55], to be measure in RHIC.

The moments of the parton distributions have been also studied in QCD sum rules by considering bi-local power corrections (BPC) to OPE [56]. Alternatively BPC can be taken into account by introducing constant external fields [57]. Using this technique, Belyaev and Blok calculated $M_2^{\rm s}$ values for the pion and the nucleon in the limit of massless quark [58]. There has been variety of calculations using this technique, and these calculations indicate that QCD sum rules are a useful tool in calculating the moments of structure functions.

In this section, we apply the technique to investigate quark momentum fractions of the pion and the kaon including the effect of finite quark mass [16].

## 4.2 Quark momentum fraction in QCD sum rule

The quark momentum fraction $M_2^{\rm s}$ is defined by the hadronic matrix element of the symmetrized energy-momentum tensor for the quarks,

$$M_2^{\rm s}(\mu^2)p_\mu p_\nu = \frac{1}{2}\langle {\rm hadron}(p)|\Theta^{\rm q}_{\mu\nu}|{\rm hadron}(p)\rangle. \tag{4.1}$$

Here $|{\rm hadron}(p)\rangle$ is the momentum eigenstate of a hadron, $\mu$ is the renormalization point. $\Theta^{\rm q}_{\mu\nu}$ denotes the symmetrized energy-momentum tensor for the quarks,

$$\Theta^{\rm q}_{\mu\nu}(x) \equiv \frac{i}{4}\left(\bar\psi(x)\overleftrightarrow{D}_\mu\gamma_\nu\psi(x) + \bar\psi(x)\overleftrightarrow{D}_\nu\gamma_\mu\psi(x)\right), \tag{4.2}$$

where $\psi(x)$ and $D_\mu$ are the quark field and the covariant derivative, respectively. $M_2^{\rm s}$ can also be represented in terms of spin-averaged quark distribution functions $f_1^{\rm q}(\xi,\mu^2)$:

$$M_2^{\rm s}(\mu^2) = \int_0^1 {\rm d}\xi\ \xi\ \sum_{\rm q}[f_1^{\rm q}(\xi,\mu^2) + f_1^{\bar{\rm q}}(\xi,\mu^2)]. \tag{4.3}$$

To calculate $M_2^{\rm s}$ in the QCD sum rule, we first consider the following three-point function:

$$\Pi_{\mu\nu\rho\lambda}(q) = i\int d^4x d^4y\, {\rm e}^{iqx}\langle 0|T[\Theta^{\rm q}_{\mu\nu}(y), A_\mu(x), A^\dagger_\nu(0)]|0\rangle, \tag{4.4}$$

where $A^(_\mu x)$ is the axial vector currents eq. (3.2).

According to the QCD sum rule method, we calculate $\Pi_{\mu\nu\rho\lambda}(q)$ in two different ways. In the phenomenological side, we express $\Pi_{\mu\nu\rho\lambda}(q)$ in terms of $M_2^{\rm s}$, using the spectral representation. On the other hand, in the theoretical side, we apply OPE and expand $\Pi^1_{\mu\nu\rho\lambda}(q)$ into terms with various condensates. In the region of $Q^2$ where these two calculations are fiducial, we can equate both sides of the QCD sum rule and can obtain the QCD sum rule for $M_2^{\rm s}$. In actual calculations we perform the Borel transformation on both sides of QCD sum rule, to get more stable result.

## 4.3 Phenomenological parameterization

In low $Q^2$ region, the spectral density of the correlation function $\Pi_{\mu\nu\rho\lambda}(q)$ are dominated by the pion pole contribution. With the help of reduction formula, we find the pion pole consists of the two types of contributions: the double-pole term and the single-pole term, and $M_2^{\rm s}$ appears as the coefficient of the double pole. In the narrow resonance approximation, we have,

$$\begin{aligned}\frac{1}{\pi}{\rm Im}\Pi_{\mu\nu\rho\lambda}(q) &= \langle 0|A_\mu(0)|\varphi^-\rangle\langle\varphi^-|\Theta^{\rm q}_{\rho\lambda}|\varphi^-\rangle\langle\varphi^-|A^\dagger_\nu(0)|0\rangle\delta'(q^2-m_\varphi^2) \\ &\quad + B_{\mu\nu\rho\lambda}\delta(q^2-m_\varphi^2) + \theta(q^2-s_0)C_{\mu\nu\rho\lambda} \qquad (4.5)\end{aligned}$$

where the contribution of the higher resonances are approximated to the continuum starting from $s_0$; $C_{\mu\nu\rho\lambda}$ should be determined to match the asymptotic behavior of the theoretical side. $B_{\mu\nu\rho\lambda}$ is a constant tensor, which represent matrix elements of $\Theta^{\rm q}_{\rho\lambda}$ which are not diagonal in $\varphi$. By substituting eq. (4.1) and eq. (3.14), eq. (4.5) gives

$$\begin{aligned}\frac{1}{\pi}{\rm Im}\Pi_{\mu\nu\rho\lambda}(q) &= 4f_\varphi^2 M_2^{\rm s} q_\mu q_\nu q_\rho q_\lambda \delta'(q^2-m_\varphi^2) \\ &\quad + B_{\mu\nu\rho\lambda}\delta(q^2-m_\varphi^2) + \theta(q^2-s_0)C_{\mu\nu\rho\lambda}. \qquad (4.6)\end{aligned}$$

In eq. (4.6), $M_2^{\rm s}$ is appearing as the coefficient of $q_\mu q_\nu q_\rho q_\lambda$. So we extract the coefficient of $q_\mu q_\nu q_\rho q_\lambda$, $\Pi'(q^2)$, from $\Pi_{\mu\nu\rho\lambda}(q)$. With the help of the dispersion relation,

$$\Pi'(q^2) = \frac{1}{\pi}\int \frac{{\rm Im}\Pi'(s)\ {\rm d}s}{s+Q^2}, \qquad (4.7)$$

we get

$$\Pi'(q^2) = -4f_\varphi^2 \frac{M_2^{\rm s}}{(Q^2+m_\varphi^2)^2} q_\mu q_\nu q_\rho q_\lambda + \frac{B'}{(Q^2+m_\varphi^2)} \qquad (4.8)$$

where we have move the continuum contribution to the theoretical side. By applying the Borel transformation $\hat{L}_M$, we obtain

$$\hat{L}_M\left[\Pi'(-Q^2)\right] = -4f_\varphi^2 M_2^{\rm s}\frac{e^{-m_\varphi^2/M^2}}{M^4} + \frac{e^{-m_\varphi^2/M^2}}{M^2}B' \qquad (4.9)$$

as the phenomenological side of the QCD sum rule.

## 4.4 Theoretical calculation

In the OPE side of the QCD sum rule, we apply OPE to the T-product in the integrand of eq. (4.4). Since one of the external momenta of the three-point correlation function $\Pi(q)$ are zero, we have to take the bi-local power correction (BPC) into account as well as the ordinary local power correction [56]:

$$\Pi'^{\mathrm{OPE}}(q^2) = \sum_n C_n(q^2)\langle O_n\rangle + \sum_m C_m^{\mathrm{BL}}(q^2)\Delta_m(0), \tag{4.10}$$

where the second term of the r.h.s represents BPC.

To be consistent with the phenomenological side, we consider only the coefficient of $q_\mu q_\nu q_\rho q_\lambda$, and then perform the Borel transformation. After some calculations, we obtain,

$$\begin{aligned}\hat{L}_M\left[\Pi'(-Q^2)\right] &= -\frac{1}{2\pi^2}\frac{1}{M^2} - \frac{1}{18}\langle\alpha_{\mathrm{s}}FF\rangle\frac{1}{M^6} - 2\frac{m_{\mathrm{u}}\langle\bar{u}u\rangle + m_{\mathrm{s}}\langle\bar{s}s\rangle}{M^6} \\ &\quad + \frac{1}{3}\frac{m_{\mathrm{u}}\langle\bar{u}\sigma_{\mu\nu}F^{\mu\nu}u\rangle + m_{\mathrm{s}}\langle\bar{s}\sigma_{\mu\nu}F^{\mu\nu}s\rangle}{M^8} \\ &\quad - \frac{16\pi}{27}\alpha_{\mathrm{s}}\frac{\langle\bar{u}u\rangle^2 + \langle\bar{s}s\rangle^2}{M^8} + \frac{64\pi}{27}\alpha_{\mathrm{s}}\frac{\langle\bar{u}u\rangle\langle\bar{s}s\rangle}{M^8}.\end{aligned} \tag{4.11}$$

Here, we calculate up to dimension-6 condensate terms and the first order in quark masses, whereas in ref. [58] they neglect quark masses. Wilson coefficients are obtained by calculating diagrams depicted in fig. 4.1. Note that BPC contributes to dimension-6 condensate terms, but they are small compared to the other dimension-6 condensate terms [58]. The smallness of this term is intuitively understandable, since low momentum quarks have less contributions to the momentum fraction.

Equating eq. (4.9) and eq. (4.11), we get the QCD sum rule for $M_2^{\mathrm{s}}$. But beforehand we have to perform the QCD evolution to get $M_2^{\mathrm{s}}$ for arbitrary renormalization

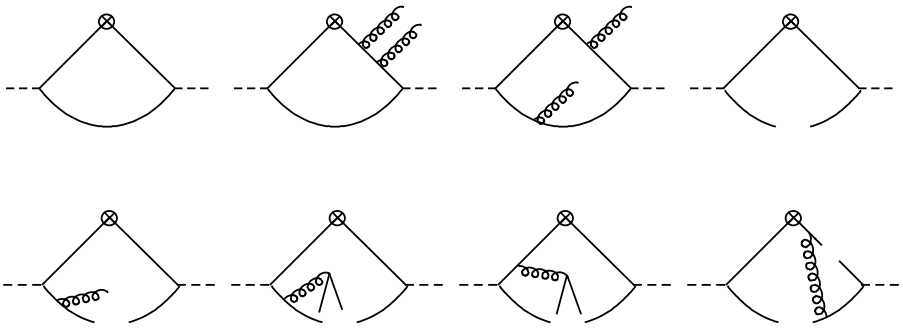

Figure 4.1: Typical diagrams that contribute to $\Pi'(Q^2)$. The curry lines represent the background gluon fields. The cross mark stand for the operator $\Theta^{\rm q}_{\rho\lambda}$

point. In the present case, the QCD evolution equations are given by

$$M_2^{\rm s}(\mu^2) + M_2^{\rm G}(\mu^2) = 1 \tag{4.12}$$

$$\frac{16}{25}M_2^{\rm s}(M^2) - \frac{9}{25}M_2^{\rm G}(M^2) = (\frac{16}{25}M_2^{\rm s}(\mu^2) - \frac{9}{25}M_2^{\rm G}(\mu^2))L^{-\frac{50}{81}}, \tag{4.13}$$

where $M_2^{\rm G}$ is the momentum fraction carried by the gluons:

$$\begin{aligned} M_2^{\rm G}(\mu^2)p_\mu p_\nu &= \frac{1}{2}\langle\varphi^-(p)|\Theta^{\rm G}_{\mu\nu}|\varphi^-(p)\rangle \\ \Theta^{\rm G}_{\mu\nu} &= F_{\mu\alpha}F^{\alpha}_{\ \nu} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}, \end{aligned} \tag{4.14}$$

and

$$L = \ln(\frac{M^2}{\Lambda^2})/\ln(\frac{\mu^2}{\Lambda^2}). \tag{4.15}$$

After the QCD evolution, we get the QCD sum rule for $M_2^{\rm s}(\mu^2)$

$$\begin{aligned} &\frac{9}{25}(1-L^{50/81}) + L^{50/81}\frac{{\rm e}^{m_\varphi^2/M^2}}{f_\varphi^2}\left[\frac{1}{8\pi^2}M^2(1-{\rm e}^{-s_0/M^2})\right. \\ &+\frac{1}{72}\frac{\langle\frac{\alpha_{\rm s}}{\pi}FF\rangle}{M^2} + \frac{1}{2}\frac{m_{\rm u}\langle\bar{u}u\rangle + m_{\rm s}\langle\bar{s}s\rangle}{M^2} \\ &-\frac{1}{12}\frac{m_{\rm u}\langle\bar{u}\sigma_{\mu\nu}F^{\mu\nu}u\rangle + m_{\rm s}\langle\bar{s}\sigma_{\mu\nu}F^{\mu\nu}s\rangle}{M^4} \\ &\left.+\frac{4\pi}{27}\alpha_{\rm s}\frac{\langle\bar{u}u\rangle^2+\langle\bar{s}s\rangle^2}{M^4} - \frac{16\pi}{27}\alpha_{\rm s}\frac{\langle\bar{u}u\rangle\langle\bar{s}s\rangle}{M^4}\right] \\ =\ & M_2^{\rm s}(\mu^2) + CM^2, \end{aligned} \tag{4.16}$$

where $C$ is a constant, which originates from the second term of the r.h.s. of eq. (4.9). In eq. (4.16), we have moved the contribution of the continuum to the l.h.s. Note that the four-quark condensate terms are slightly different from those previously obtained by Belyaev and Blok in the pion case [58].

For a given $\mu^2$, the r.h.s. of eq. (4.16) is a linear function of the Borel mass squared $M^2$ with $M_2^{\rm s}(\mu^2)$ as a constant term, whereas the l.h.s. of eq. (4.16) is a very complicated function of $M^2$. We approximate the latter by a linear function in the region of $M^2$ where the QCD sum rule is applicable. This region of $M^2$ is obtained according to the following condition; (i) the order of the contribution of the highest order term of OPE is less than 10 % of the l.h.s., and (ii) contribution of the continuum is less than 50 % of the other. In this way, we calculated $M_2^{\rm s}(\mu^2)$ for various values of the continuum threshold $s_0$, and find stable $s_0$, which has the least influence on the result.

## 4.5 Results and discussion

We plot the r.h.s. of eq. (4.16) in Fig. 4.2 for both the pion and the kaon cases. Here the values of the continuum thresholds, which are determined in the way explained above, are $s_0 = 0.8{\rm GeV}^2$ for the pion and $s_0 = 1.2{\rm GeV}^2$ for the kaon. We thus obtain the results,

$$M_2^{\rm s}(\mu^2 = 49{\rm GeV}^2) = 0.39 \pm 0.04 \qquad \text{for the pion,} \tag{4.17}$$

$$M_2^{\rm s}(\mu^2 = 49{\rm GeV}^2) = 0.41 \pm 0.04 \qquad \text{for the kaon,} \tag{4.18}$$

Here we put $\mu^2 = 49{\rm GeV}$ in order to compare our result with the available lattice calculation.

We get the similar $M_2^{\rm s}$ value for the pion eq. (4.17) and for the kaon eq. (4.18). We first compare our result with the analysis of experimental data and other theoretical

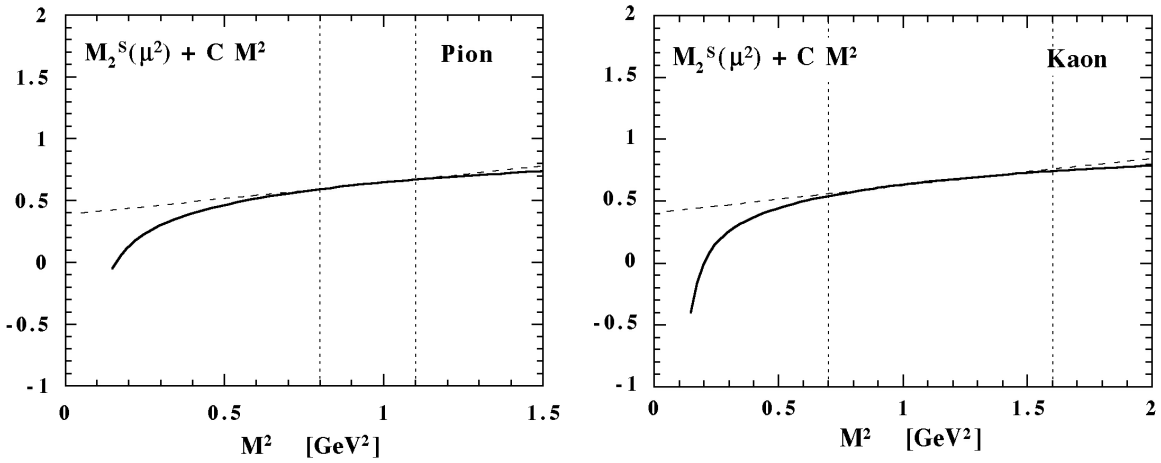


Figure 4.2: The solid lines indicate the OPE side of the QCD sum rule at $\mu^2 = 49\text{GeV}^2$. We approximate the solid lines by linear lines in the region of $M^2$ in between the dotted lines.

calculations, and then discuss in particular about the flavor dependence of our result.

Our result for the pion case can be compared with the following values.

(i) NLO analysis of the Drell-Yan data [59]

$$2\int_0^1 \mathrm{d}\xi\ \xi f_{\text{valence}}(\xi, \mu^2 = 49\text{GeV}^2) = 0.40 \pm 0.02. \tag{4.19}$$

(ii) Lattice calculation [60]

$$M_2^{\text{s}}(\mu^2 = 49\text{GeV}^2) = 0.46 \pm 0.07. \tag{4.20}$$

(iii) Calculation in the Nambu and Jona-Lasinio (NJL) model [61]

$$2\int_0^1 \mathrm{d}\xi\ \xi f_{\text{valence}}(\xi, \mu^2 = 49\text{GeV}^2) = 0.41. \tag{4.21}$$

Our result for $M_2^{\text{s}}$ can be compared with the momentum fraction carried by the valence quarks, $2\int_0^1 \mathrm{d}\xi\ \xi f_{\text{valence}}$, since in OPE calculations, we calculate only the

lowest order contribution in the strong coupling constant $\alpha_{\rm s}$ for each condensate term and do not explicitly include the contributions of the sea quarks (the quark loops), which are higher order terms in $\alpha_{\rm s}$. Similar comment can be applied to the result of the lattice calculation, eq. (4.20). Since in ref. [60], calculations have been performed in the quenched approximation [60], the sea quarks are not included. Taking these comments into account, they are all consistent with our result, eq. (4.17).

For the kaon case, there is no available experimental analysis nor the lattice calculation for the momentum fraction. There only exist calculation based on the NJL model [61]. The NJL calculation gives the value for the kaon similar to the value for the pion, and support our result. The momentum fraction carried by the valence quarks at $\mu^2 = 49{\rm GeV}^2$ is 0.42 [62]. This value is also consistent with our result eq. (4.18).

Let us now consider in detail the flavor (quark contents) dependence of our result. Though the two values, (4.17) and (4.18), are close to each other, it does not imply that their dynamical origins are similar, as we can see below. In the sum rules for these two mesons, differences stem from the input values of the decay constants, the meson masses and the quark mass dependent terms ($m_{\rm q}\langle\bar{q}q\rangle$ and $m_{\rm s}\langle\bar{s}s\rangle$).

In the pion case, even if we put the pion mass and/or quark mass to be zero, the result is almost unchanged. This can be understood from the fact that the values of the pion mass and the quark mass are much smaller than the Borel mass $M$. Therefore the meson/quark masses play a minor role in the calculations for the pion.

On the other hand, non-zero values of them are essential in the kaon case. For example, if we simply neglect the terms with the quark mass in the resulting sum rule eq. (4.16), we have 0.51 as $M_2^{\rm s}$ for the kaon. This estimate is useful to ensure

that we can not omit the strange quark mass dependent terms in OPE calculations. In the same way, if we simply neglect the kaon mass, we have 0.26 as $M_2^{\rm s}$ for the kaon. This estimate indicates that to neglect the kaon mass in the phenomenological side of the QCD sum rule gives serious change in the result. The point we want to stress is one has to consistently take into account the quark mass, the meson mass and the decay constant in QCD sum rule calculation, in order to get physically meaningful result. In this way, the difference in each of the input values gives an important change to the sum rules. However, our results, (4.17) and (4.18), indicate that if we consistently choose these input values, these effects cancel among themselves to give the $M_2^{\rm s}$ for the kaon, which turns out to be similar to that for the pion.

In the next section, we apply similar technique, as was used in this section, to anther topic: strangeness contents of the proton.

# 5 Strangeness content of the proton and direct instantons

The strangeness content of the nucleon is a good measure for the braking of the OZI rule in the nucleon. The success of the quark models without strange quark indicate small OZI violation in the nucleon. In QCD sum rule for the nucleon, we do not consider the explicit strange degree of freedom. Some experiments, however, suggest large braking of the OZI rule: The measurement of the polarized parton distribution $g_1(\xi, \mu^2$ shows the strange quark carries about 10% of the proton spin with opposite direction [63]; There is DIS experiment which indicate about 4% of the proton momentum is carried by the strange quark [64]. Thus theoretical and experimental situation on the strangeness content of the nucleon remains controversial.

In this section, we calculate the strange scalar density in the proton in QCD sum rule.

## 5.1 Borel sum rule for strange scalar density in the proton

First we consider the following three-point correlation function $\Pi(q)$,

$$\Pi(q) = -\int \mathrm{d}^4x\, \mathrm{d}^4y\, e^{iqx} \langle 0|T[\bar{s}s(y), \eta(x), \bar{\eta}(0)]|0\rangle, \tag{5.1}$$

where $\eta$ is the proton interpolating field.

The proton interpolating field without derivatives can be generally expressed as

$$\eta(x) = \eta_1(x) + t\,\eta_2(x), \tag{5.2}$$

where

$$\eta_1(x) = \varepsilon^{abc}[u^{aT}(x)Cd^b(x)]\gamma_5 u^c(x), \tag{5.3}$$

$$\eta_2(x) = \varepsilon^{abc}[u^{aT}(x)C\gamma_5 d^b(x)]u^c(x), \tag{5.4}$$

and $t$ is a parameter. $t$ may be determined to maximize the overlap of $\eta$ with the proton state,

$$\langle 0|\,\eta(0)\,|\mathrm{p}(\mathbf{p},s)\rangle = \lambda_{\mathrm{p}} u(\mathbf{p},s), \tag{5.5}$$

where $u$ is the proton spinor normalized as $\sum_s u(\mathbf{p},s)\bar{u}(\mathbf{p},s) = \not{p} + M_{\mathrm{p}}$. The so-called Ioffe's current [65] is corresponding to the case $t=-1$,

$$\begin{aligned} \eta_{\mathrm{Ioffe}}(x) &= 2\left(\eta_1(x) - \eta_2(x)\right) \\ &= \varepsilon^{abc}[u^{aT}(x)C\gamma_\mu d^b(x)]\gamma_5\gamma^\mu u^c(x). \end{aligned} \tag{5.6}$$

Because of its Lorentz structure, $\Pi(q)$ can be decomposed into the Lorentz scalar part $\Pi_m(q^2)$ and the Lorentz vector part $\Pi_q(q^2)$,

$$\Pi(q) = \Pi_m(q^2) + \not{q}\,\Pi_q(q^2). \tag{5.7}$$

Since $\Pi_m(q^2)$ and $\Pi_q(q^2)$ are analytic functions of $Q^2$, we have the following dispersion relations,

$$\Pi_{m,q}(-Q^2) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi_{m,q}(s)\,ds}{s+Q^2} + \text{subtraction terms}, \tag{5.8}$$

where $Q^2 = -q^2$. $\mathrm{Im}\Pi_{m,q}(s)$ contains the contributions of the proton pole as well as the poles of the higher resonances. In the low $Q^2$ region, due to the suppression factor $\dfrac{1}{s+Q^2}$, the proton pole gives the dominant contribution to the integral. In the section 5.2, we will explicitly show the phenomenological parameterization of $\mathrm{Im}\Pi(q)$. On the other hand, we theoretically calculate the correlation function $\Pi(q)$ in high $Q^2$ region in section 5.3. By applying the Borel transformation to these two expressions for $\Pi(q)$, i.e. phenomenological parameterization and theoretical calculation, we obtain the Borel sum rule:

$$\hat{L}_M\left[\Pi_{m,q}(-Q^2)\right] = \frac{1}{\pi M^2}\int ds\,\mathrm{Im}\Pi_{m,q}(s)e^{-s/M^2}. \tag{5.9}$$

## 5.2 Phenomenological parameterization

In the low $Q^2$ region, the proton pole is the dominant contribution to the dispersion integral, eq. (5.8). As in the last section, section 4, the proton pole gives the two types of contributions: the double-pole term and the single-pole term, and $\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle$ appears as the coefficient of the double pole. In the narrow resonance approximation, we have,

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi(q) &= \lambda_{\mathrm{p}}^2\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle(\not{q}+M_{\mathrm{p}})\delta'(q^2-M_{\mathrm{p}}^2)+(B_1+\not{q}\,B_2)\delta(q^2-M_{\mathrm{p}}^2)\\ &\quad+\theta(q^2-s_0)(C_1+\not{q}\,C_2),\end{aligned} \tag{5.10}$$

where as usual the contribution of the higher resonances are approximated to the continuum starting from $s_0$; $C_1$ and $C_2$ should be determined to match the asymptotic behavior of the theoretical side. $B_1$ and $B_2$ are constants. In the following we move the continuum contribution to the theoretical side. Substituting eq. (5.10), the dispersion relation eq. (5.8) gives

$$\Pi(q)=\lambda_{\mathrm{p}}^2\frac{\not{q}+M_{\mathrm{p}}}{(Q^2+M_{\mathrm{p}}^2)^2}\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle+\frac{C_1+\not{q}\,C_2}{Q^2+M_{\mathrm{p}}^2}. \tag{5.11}$$

Performing the Borel transformation to $\Pi_{m,q}(-Q^2)$, we have

$$\hat{L}_M\left[\Pi_q(-Q^2)\right] = \lambda_{\mathrm{p}}^2\frac{e^{-M_{\mathrm{p}}^2/M^2}}{M^4}\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle+C_2\frac{e^{-M_{\mathrm{p}}^2/M^2}}{M^2}, \tag{5.12}$$

$$\hat{L}_M\left[\Pi_m(-Q^2)\right] = \lambda_{\mathrm{p}}^2M_{\mathrm{p}}\frac{e^{-M_{\mathrm{p}}^2/M^2}}{M^4}\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle+C_1\frac{e^{-M_{\mathrm{p}}^2/M^2}}{M^2}. \tag{5.13}$$

Using eq. (5.12) and their derivatives with respect to $M^2$, we obtain the following relations,

$$\begin{aligned}\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle &= -\frac{M^2}{\lambda_{\mathrm{p}}^2e^{-M_{\mathrm{p}}^2/M^2}}\left\{(M^2-M_{\mathrm{p}}^2)\hat{L}_M\left[\Pi_q(-Q^2)\right]\right.\\ &\quad\left.+M^4\frac{d}{dM^2}\,\hat{L}_M\left[\Pi_q(-Q^2)\right]\right\},\end{aligned} \tag{5.14}$$

$$\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle = -\frac{M^2}{\lambda_{\mathrm{p}}^2 M_{\mathrm{p}} e^{-M_{\mathrm{p}}^2/M^2}} \left\{ (M^2 - M_{\mathrm{p}}^2)\hat{L}_M \left[\Pi_m(-Q^2)\right] + M^4 \frac{d}{dM^2}\, \hat{L}_M \left[\Pi_m(-Q^2)\right] \right\}. \quad (5.15)$$

## 5.3 Theoretical calculation

In theoretical calculation of $\Pi(q)$, we take into account the direct instanton contribution [22] as well as the conventional OPE contribution. The direct instantons have been shown to be essential to reproduce the light pseudoscalar mesonic nonet in QCD sum rules [40]. Also, it has been shown that the direct instantons provide important effects to improve the stability of the QCD sum rules for the nucleon [66].

OPE of three-point correlation function $\Pi(q)$ consists of the perturbative terms and its local and bi-local power correction: We neglect he perturbative and the local power correction terms, since they are strongly suppressed ($O(\alpha_s^2)$). The Wilson coefficients of BPC, $C_n^{\mathrm{BL}}(q)$, are obtained from the OPE calculation of the proton two-point correlation function,

$$N(q) = i \int \mathrm{d}^4x\; e^{iqx} \langle 0|T[\eta(x), \bar{\eta}(0)]|0\rangle = \sum_m C_m^{\mathrm{BL}}(q) O_m, \quad (5.16)$$

and $\Delta_m$ is the bi-local condensate defined as

$$\Delta_m(0) = \lim_{q\to 0} i \int \mathrm{d}^4x\, e^{iqx} \langle 0|T\left[\bar{s}s(x), O_m(0)\right]|0\rangle. \quad (5.17)$$

### 5.3.1 Direct instanton contribution

In order to calculate the direct instanton contribution to the three-point correlation function, we first consider the quark propagator in the instanton background. The quark propagator in Euclidean space can be expanded in terms of the eigen states of the Hermitian operator $i\not{D}$,

$$S(x,y) = -i\langle 0|T[q(x)\bar{q}(y)]|0\rangle \underset{x_0\to -ix_4}{\longrightarrow} \sum_n \frac{\psi_n(x)\psi_n^\dagger(y)}{\lambda_n + im_q}, \quad (5.18)$$

where $\lambda_n$ is the eigen values of $i\not{D}$,

$$i\not{D}\psi_n = \lambda_n \psi_n. \tag{5.19}$$

For the light quarks, the 't Hooft zero mode (ZM) [67], $\psi_0$, gives the dominant contribution to the sum over $n$ in eq. (5.18), where $\psi_0$ satisfies $i\not{D}\psi_0 = 0$. If we take the non-vanishing quark condensates into account, ZM contribution to the quark propagator is given by [68]

$$S_{\rm ZM}^{I(\bar{I})}(x,y) = \frac{\psi_0^{I(\bar{I})}(x)\psi_0^{I(\bar{I})\,\dagger}(y)}{im_q^*}, \tag{5.20}$$

where,

$$m_q^* = m_q - \frac{2}{3}\pi^2\rho^2\langle\bar{q}q\rangle. \tag{5.21}$$

The indices $I(\bar{I})$ denotes the contribution under the instanton (anti-instanton) background. Other contributions to the quark propagator are of order $O(\rho m^*)$ compared to the leading term.

By substituting the explicit expressions for the ZM wave function $\psi_0$, the quark propagator, eq. (5.20), is given by,

$$\begin{aligned} S_{\rm ZM}^{I}(x,y) &= \frac{1}{2\pi^2}\frac{1}{m^*}\frac{\rho^2}{(x'^2+\rho^2)^{3/2}(y'^2+\rho^2)^{3/2}} \\ &\qquad \frac{\not{x}'}{|x'|}\frac{1+\gamma_5}{2}\left(1+\frac{i}{4}\bar{\eta}_{a\,\mu\nu}\sigma_{\mu\nu}\tau^a\right)\frac{\not{y}'}{|y|'}, \end{aligned} \tag{5.22}$$

$$\begin{aligned} S_{\rm ZM}^{\bar{I}}(x,y) &= \frac{1}{2\pi^2}\frac{1}{m^*}\frac{\rho^2}{(x'^2+\rho^2)^{3/2}(y'^2+\rho^2)^{3/2}} \\ &\qquad \frac{\not{x}'}{|x'|}\frac{1-\gamma_5}{2}\left(1+\frac{i}{4}\eta_{a\,\mu\nu}\sigma_{\mu\nu}\tau^a\right)\frac{\not{y}'}{|y|'}. \end{aligned} \tag{5.23}$$

Here $x' = x - z$, $y' = y - z$. $\eta$ and $\bar{\eta}$ are the 't Hooft symbols. $z$ is the center of the (anti-)instanton and $\rho$ is the size of the (anti-)instanton.

Using ZM contribution to the quark propagator, eq.(5.22) and eq.(5.23), we calculate the three-point correlation function $\Pi(q)$. From the Pauli principle, maximally three quark propagators can be zero mode as in fig. 5.1 It gives the dominant

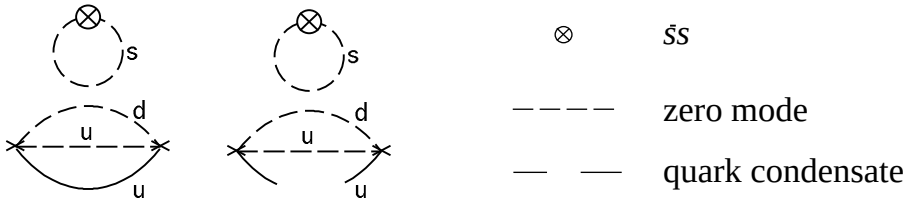


Figure 5.1: The dominant direct instanton contribution to the three point function $\Pi(q)$.

contribution of the direct instanton to $\Pi(q)$:

$$\Pi^{I+\bar{I}}(q) = -2\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{d}^4z\,d\rho\,n(\rho)\,e^{iqx}\frac{1}{m_{\mathrm{q}}^{*\,2}m_{\mathrm{s}}^*}\left(\frac{\rho}{\pi}\right)^6 \frac{1}{(x'^2+\rho^2)^3}\frac{1}{(y'^2+\rho^2)^3}\frac{1}{(z'^2+\rho^2)^3}\left[\frac{c_1}{\pi^2}\frac{\not{x}}{x^4}+\frac{2c_2}{3}i\langle\bar{q}q\rangle\right], \quad (5.24)$$

where $c_1 = 6(t^2-1)$ and $c_2 = \frac{1}{8}\Big[13(t^2+1)+10t\Big]$. $n(\rho)$ is the instanton density. Note that from the simple consideration on the chirality of the strange quark, the diagram like fig. 5.1 do not contribute to the QCD sum rule for, e.g., strangeness momentum fraction [21].

For the instanton size density, we adopt the one corresponding to the instanton liquid model proposed by Shuryak [69],

$$n(\rho) = n_c\delta(\rho-\rho_c), \quad (5.25)$$

with $\rho_c \simeq 1/3\,\mathrm{fm}$ and $n_c \simeq 1/2\,\mathrm{fm}^{-4}$. The instanton liquid picture was later confirmed by Dyakonov and Petrov by using the variational method [70].

By substituting eq. 5.25, $\Pi_m^{\mathrm{Inst}}(-Q^2)$ and $\Pi_q^{\mathrm{Inst}}(-Q^2)$ are given by

$$\Pi_q^{I+\bar{I}}(-Q^2) = -\frac{n_c c_1}{32\pi^2}\frac{\rho_c^2}{m_{\mathrm{q}}^{*\,2}m_{\mathrm{s}}^*}\left[\frac{1}{Q^2}\int_0^{Q^2} dP^2\,P^4\,(K_1(P\rho_c))^2 - \frac{1}{2Q^4}\int_0^{Q^2} dP^2\,P^6 + \frac{1}{2}\int_{Q^2}^{\infty} dP^2\,P^2\,(K_1(P\rho_c))^2\right], \quad (5.26)$$

$$\Pi_m^{I+\bar{I}}(-Q^2) = \frac{n_c c_2}{6}\frac{\rho_c^2}{m_{\mathrm{q}}^{*\,2}m_{\mathrm{s}}^*}\langle\bar{q}q\rangle Q^2\,(K_0(Q\rho_c))^2\,, \quad (5.27)$$

(5.28)

where $K_n(z)$ is the modified Bessel function of the second kind known as the McDonald Function.

Applying the Borel transformation, we obtain the direct instanton contribution to the Borel sum rule:

$$\begin{aligned}
\hat{L}_M\left[\Pi_q^{I+\bar{I}}(-Q^2)\right] &= -\frac{n_c c_1}{32\pi^2}\frac{1}{m_{\rm q}^{*2}m_{\rm s}^*}\frac{1}{\rho_c^2}\left[\frac{16}{5}\frac{1}{z^2}-\frac{384}{35}\frac{1}{z^4}\right. \\
&\quad +e^{-z^2/2}\left(K_0(z^2/2)(\frac{48}{35}+\frac{2}{7}z^2+\frac{1}{70}z^4)\right. \\
&\quad \left.\left.+K_1(z^2/2)(\frac{192}{35}\frac{1}{z^2}+\frac{8}{7}+\frac{8}{35}z^2-\frac{1}{70}z^4)\right)\right], \quad (5.29)\\
\hat{L}_M\left[\Pi_m^{I+\bar{I}}(-Q^2)\right] &= \frac{n_c c_2}{24}\frac{1}{m_{\rm q}^{*2}m_{\rm s}^*}\langle\bar{q}q\rangle z^4 e^{-z^2/2}\left(K_0(z^2/2)+K_1(z^2/2)\right), \quad (5.30)
\end{aligned}$$

where $z=\rho_c M$. Eq. (5.30) drops exponentially as $M$ becomes larger. On the hand, in eq. (5.30), the first and the second term of r.h.s do not drop exponentially; they are proportional to $\frac{1}{M^2}$ and $\frac{1}{M^4}$, respectively. It is interesting that the two terms appearing in r.h.s. of the phenomenological side, eq. (5.30), also behave like $\frac{1}{M^2}$ and $\frac{1}{M^4}$ in large $M^2$ region.

### 5.3.2 Bilocal power corrections

Using the QCD calculation of the proton two-point correlation function by Forkel and Banerjee [66], we obtain BPC:

$$\begin{aligned}
\hat{L}_M\left[\Pi_q^{\rm BPC}(-Q^2)\right] &= (5+2t+5t^2)\frac{1}{2^{10}\pi^2}\langle\langle\frac{\alpha_s}{\pi}FF\rangle\rangle, \quad (5.31)\\
\hat{L}_M\left[\Pi_m^{\rm BPC}(-Q^2)\right] &= (1-t^2)\frac{5}{2^6\pi^2}M^2\langle\langle\bar{q}q\rangle\rangle \\
&\quad -\frac{n_c c_2}{24}\frac{1}{m_{\rm q}^{*2}}\langle\langle\bar{q}q\rangle\rangle z^4 e^{-z^2/2}\left(K_0(z^2/2)+K_1(z^2/2)\right), \quad (5.32)
\end{aligned}$$

where $\langle\langle\cdots\rangle\rangle$ are the bilocal condensates:

$$\langle\langle\bar{q}q\rangle\rangle = \lim_{q\leftarrow 0} i\int {\rm d}^4x\, e^{iqx}\langle 0|T\left[\bar{q}q(x),\bar{s}s(0)\right]|0\rangle, \quad (5.33)$$

$$\langle\langle\frac{\alpha_s}{\pi}FF\rangle\rangle \;=\; \lim_{q\leftarrow 0} i\int \mathrm{d}^4x\, e^{iqx}\langle 0|T\left[\frac{\alpha_s}{\pi}FF(x),\bar{s}s(0)\right]|0\rangle. \tag{5.34}$$

$\langle\langle\frac{\alpha_s}{\pi}FF\rangle\rangle$ can be obtained from the QCD low energy theorem [22]:

$$\begin{aligned}&\lim_{q\leftarrow 0} i\int \mathrm{d}^4x\, e^{iqx}\langle 0|T\left[\mathcal{O}(x),\frac{\beta(\alpha_s)}{4\alpha_s}FF(0)\right]|0\rangle\\ &=(-d)\langle\mathcal{O}\rangle[1+\text{terms linear in the quark masses}],\end{aligned} \tag{5.35}$$

where $\mathcal{O}(x)$ is a local operator with canonical dimension $d$. Neglecting higher order terms in the quark mass, we have

$$\langle\langle\frac{\alpha_s}{\pi}FF\rangle\rangle = 8/3 < \bar{s}s > . \tag{5.36}$$

We estimate $\langle\langle\bar{q}q\rangle\rangle$ from QCD sum rule for

$$i\int \mathrm{d}^4x\, e^{iqx}\langle 0|T\left[\bar{q}q(x),\bar{s}s(0)\right]|0\rangle, \tag{5.37}$$

and obtain

$$\langle\langle\bar{q}q\rangle\rangle = (0.29\mathrm{GeV})^2. \tag{5.38}$$

## 5.4 Results and Discussion

By substituting the theoretical calculations of the correlation function, eqs. (5.29) and (5.31), eq. (5.14) gives the $\not{q}$-sum rule:

$$\begin{aligned}&\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle =\\ &\frac{e^{M_\mathrm{N}^2/M^2}}{2M_\mathrm{N}\lambda_\mathrm{N}^2}\left[\frac{n_c c_1}{32\pi^2}\frac{1}{m_\mathrm{q}^{*2}m_\mathrm{s}^*}\frac{1}{\rho_c^6}\left\{\frac{384}{35}-\frac{16}{5}M_\mathrm{N}^2\rho_c^2+\frac{384}{35}\frac{M_\mathrm{N}^2\rho_c^2}{z^2}+\right.\right.\\ &\quad -e^{-z^2/2}\left(K_0(z^2/2)(\frac{3}{14}z^8+(\frac{24}{35}+\frac{1}{70}M_\mathrm{N}^2\rho_c^2)z^6\right.\\ &\qquad\qquad +(\frac{58}{35}+\frac{2}{7}M_\mathrm{N}^2\rho_c^2)z^2+\frac{48}{35}M_\mathrm{N}^2\rho_c^2z^2)\\ &\qquad +K_1(z^2/2)(\frac{2}{7}z^8+(\frac{36}{35}-\frac{1}{70}M_\mathrm{N}^2\rho_c^2)z^6+(\frac{96}{35}+\frac{8}{35}M_\mathrm{N}^2\rho_c^2)z^4\\ &\qquad\qquad \left.\left.+(\frac{192}{35}+\frac{8}{7}M_\mathrm{N}^2\rho_c^2)z^2+\frac{192}{35}M_\mathrm{N}^2\rho_c^2)\right)\right\}\\ &\quad \left.-\frac{c_3}{128\pi^2}\langle\langle\frac{\alpha_s}{\pi}FF\rangle\rangle M^2(M^2-M_\mathrm{N}^2)\right]\end{aligned} \tag{5.39}$$

Similarly, by substituting eqs. (5.30) and (5.32), eq. (5.15) gives the $m$-sum rule:

$$\begin{aligned}
&\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle = \\
&\frac{e^{M_\mathrm{N}^2/M^2}}{2M_\mathrm{N}^2\lambda_\mathrm{N}^2}\left[\frac{n_c c_2}{24}\frac{1}{m_\mathrm{q}^{*\,2}m_\mathrm{s}^*}\frac{1}{\rho_c^4}(\langle\bar{q}q\rangle - m_\mathrm{s}^*\langle\langle\bar{q}q\rangle\rangle)\times\right. \\
&\qquad e^{-z^2/2}\left(K_0(z^2/2)(z^{10}-3z^8+M_\mathrm{N}^2\rho_c^2 z^6)\right. \\
&\qquad\qquad \left.+K_1(z^2/2)(z^{10}-2z^8+M_\mathrm{N}^2\rho_c^2 z^6)\right) \\
&\qquad -\langle\langle\bar{q}q\rangle\rangle\frac{c_4}{16\pi^2}\left(M_\mathrm{N}^2M^4(1-(1+\frac{s_0}{M^2})e^{-s_0/M^2})\right. \\
&\qquad\qquad \left.\left.-2M^6(1-(1+\frac{s_0}{M^2}+\frac{s_0^{2\prime}}{2M^4})e^{-s_0/M^2}\right)\right] \qquad (5.40)
\end{aligned}$$

For numerical calculations, we chose $t = -1.1$ as Forkel and Banerjee did in their QCD sum rule calculation for the proton mass [66]. The mass and decay constant of the proton are also taken from their analysis. In figs 5.2 and 5.3, we show the Borel curve for $\not{q}$-sum rule and $m$-sum rule, respectively. In both sum rules the leading contributions come from direct instanton. The instanton contributions to the borel cures (the dashed lines) are more sable for the $\not{q}$-sum rule, since there are constant terms in the r.h.s of eq. (5.39). They originate from first and the second term of r.h.s in eq. (5.30)

After taking BPC into account, both curves are stable within the Borel window $0.8\,\mathrm{GeV} \leq M \leq 1.2\,\mathrm{GeV}$. The values at $M = 1\,\mathrm{GeV}$ are

$$\frac{\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle}{2M_\mathrm{p}} = 0.3 \qquad (\not{q}\text{ sum rule}), \qquad (5.41)$$

$$\frac{\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle}{2M_\mathrm{p}} = -0.2 \qquad (m\text{ sum rule}). \qquad (5.42)$$

These values are corresponding to the y-parameter eq. (3.31),

$$y = -0.04 \qquad (\not{q}\text{ sum rule}), \qquad (5.43)$$

$$y = 0.04 \qquad (m\text{ sum rule}), \qquad (5.44)$$

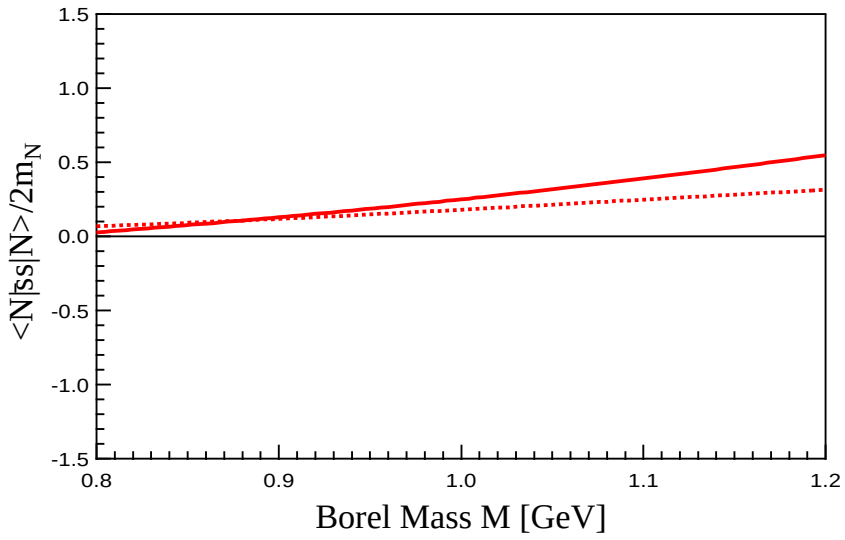


Figure 5.2: Borel curve for the $\not{q}$ sum rule. The dashed line is the result without BPC.

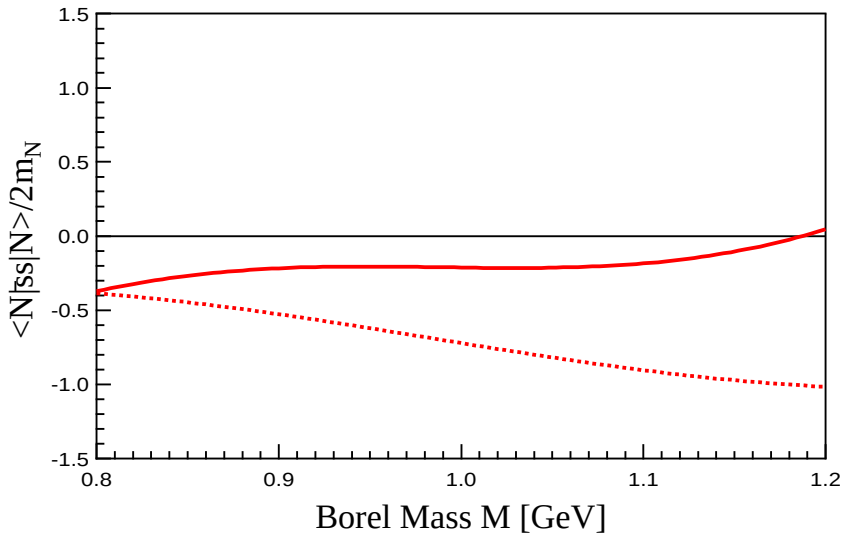


Figure 5.3: Borel curve for the sum rule for $\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle$. The dashed line is the result without BPC. For the continuum threshold for BPC, we take $s_0 = 2.9\mathrm{GeV}^2$.

where we used the phenomenological analysis of the $\pi$-N sigma term in ref. [18] and the quark masses.

The results obtained from $\not{q}$ sum rule and $m$ sum rule are differ in sign. However the magnitudes are quite small compared with other theoretical and phenomenological results: (i) Semi-phenomenological analysis using the $\pi$-N scattering data and chiral perturbation theory gives $y \approx 0.2$ [18]; (ii) Recent lattice calculation gives $\dfrac{\langle p \mid \bar{s}s \mid p \rangle}{2M_{\rm p}} = 2.84(44)$ and $y = 0.66(15)$ [19]; (iii) The calculation based on the NJL model with the 't Hooft interaction [67] gives $y = 0.118$ [71]. The result of small strange scalar density corresponding to small OZI violation in the proton.

# 6 Summary

We have studied the quark contents in hadrons by the QCD sum rule method. QCD sum rule is a model independent method to investigate the non-perturbative aspects of QCD. By using OPE and the spectral representation, we obtain QCD sum rules: analytic relations between physical quantities and the matrix element of local operators. Taking the matrix elements as inputs, we can calculate various properties of hadrons.

We have applied first the QCD sum rule to the pion-nucleon and kaon-nucleon scattering lengths. We have shown that the leading and the next-to-leading order terms of OPE give rise to the Tomozawa-Weinberg and sigma terms, respectively. The higher order terms of OPE have been estimated, where the moments of the parton distributions are used as inputs. We have discussed phenomenological contributions, which should be added to the experimental scattering lengths to be compared with the theoretical calculation by the OPE: We have estimated the $\Lambda(1405)$ contribution in the kaon-nucleon channel and the continuum contribution above the threshold in the pion-nucleon channel. It turned out that the results of the QCD sum rule for the pion-nucleon scattering lengths are consistent with those of the low energy theorem and therefore with experiments. On the other hand, those for the kaon-nucleon scattering lengths differ from the results of the naive PCAC-plus-current-algebra approach by the contribution of $\Lambda(1405)$.

Next we have studied the quark momentum fraction and its dependence on the flavor structure of hadrons. The quark momentum fraction is express as the second moment $M_2^s$ of the flavor-singlet spin-averaged quark distribution function $f_1^s(\xi, \mu^2)$. We have calculated the quark momentum fractions of the pion and the kaon in the framework of the QCD sum rule. Structure functions of these mesons play an

important role in, e.g., the QCD sum rule at finite temperature, since they are main excitation modes at low temperature. Our calculations have given similar values of the quark momentum fractions of these two mesons, in spite of mass differences of the pion and the kaon or that of the u-, d-quarks and the s-quark. The quark momentum fraction of the pion is consistent with the experimental analysis, the lattice calculation and the value based on the NJL model. The value for the kaon is consistent with the calculation based on the NJL model. The pion mass and the quark mass turned out to be unimportant in the calculation of the quark momentum fraction of the pion. In contrast, the kaon mass and the quark mass have a large effect on the quark momentum fraction of the kaon, and non-zero values of them are essential. In spite of these differences, these effects seem to cancel among themselves to give almost the same quark momentum fractions for the pion and for the kaon.

Finally we have investigated the strangeness content of the proton, which measures the OZI violation in the proton. We have calculated the strange scalar density in the proton $\langle \mathrm{p}|\bar{s}s|\mathrm{p}\rangle$. We have considered the three-point correlation function composed of the interpolating fields of the proton and the strange scalar density, and have obtained two independent sum rules: the $\not{q}$ sum rule and the $m$ sum rule. In theoretical calculations of the correlation function, we have taken into account the direct instanton and bi-local power correction term of OPE. We neglect the perturbative and the local power corrections term of OPE, since they are strongly suppressed ($O(\alpha_s^2)$). We have obtained a stable Borel curves from both sum rules within the Borel window $0.8\mathrm{GeV} \leq M \leq 1.2\mathrm{GeV}$. We have obtained the results $y = -0.04$ from $\not{q}$ sum rule and $y = 0.04$ from $m$ sum rule. Though they differ in sign, the QCD sum rule calculation predicts rather small value for the strange scalar density in the proton compared with other semi-phenomenological and theoretical

calculations.

Using QCD sum rules, we can calculate various parton distributions and their moments which are unambiguously comparable with experiments and with lattice calculations. Thus QCD sum rules provide a useful framework for the studies of the quark distributions inside hadrons.

# APPENDICES

## A Borel transformation

In this appendix, we explain the Borel transformation as the method to obtain a better convergent series. We first consider a series,

$$f(z) = \sum_{n=0}^{\infty} a_n z^n, \tag{A.1}$$

and another series,

$$\phi(z) = \sum_{n=0}^{\infty} \frac{1}{n!} a_n z^n. \tag{A.2}$$

$\phi(z)$ is called the Borel function corresponding to $f(z)$. The series $\phi(z)$ converges better than $f(z)$. So even if $f(z)$ is a divergent series, there is a possibility that $\phi(z)$ becomes a convergent series for some region of $z$. And if the series $\phi(z)$ converges, $f(z)$ is called Borel summable. $f(z)$ and $\phi(z)$ are related through the following equation:

$$f(z) = \frac{1}{z} \int_0^{\infty} e^{-t/z} \phi(t)\, dt. \tag{A.3}$$

We can check eq. (A.3) order by order by substituting eq. (A.1) and eq. (A.2) [5].

Applying this technique to a OPE series $g(Q^2)$,

$$g(Q^2) = \sum_{n=1}^{\infty} \frac{a_n}{Q^{2n}}, \tag{A.4}$$

we obtain the a series,

$$G(M^2) = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \frac{a_n}{M^{2n}}. \tag{A.5}$$

[5]For asymptotic (divergent) series like the perturbation series, the validity of the eq. (A.3) is not obvious. But for finite series, we can safely use eq. (A.3) (and eqs. (A.6) and (A.7) below).

The transformation from $g(Q^2)$ to $G(M^2)$ is called the Borel transformation, which we denote by $\hat{L}_M$. $\hat{L}_M$ is given by

$$\begin{aligned} G(M^2) &= \hat{L}_M \left[ g(Q^2) \right] \\ &= \frac{1}{2\pi i} \frac{1}{M^2} \int_{a-i\infty}^{a+i\infty} e^{Q^2/M^2} g(Q^2)\, dQ^2. \end{aligned} \tag{A.6}$$

where the integration contour runs to the right of all singularities of $g(Q^2)$ in the $Q^2$-plane. The inverse Borel transformation is given by

$$\begin{aligned} g(Q^2) &= \hat{L}_Q^{-1} \left[ G(M^2) \right] \\ &= \int_0^{\infty} e^{-Q^2/M^2} \frac{G(M^2)}{M^2}\, d\left( \frac{1}{M^2} \right) \end{aligned} \tag{A.7}$$

The Borel transformation can also be expressed as a differential operator,

$$\hat{L}_M \equiv \lim_{\substack{Q^2,n\to\infty \\ \frac{Q^2}{n}=M^2}} \frac{1}{(n-1)!} (Q^2)^n (-\frac{d}{dQ^2})^n, \tag{A.8}$$

which was introduced by Shifman, Vainshtein and Zakharov in the original paper of QCD sum rules [13].

It is important to note that the Borel transformation eq. (A.6) is connected with the inverse Laplace transformation:

$$\hat{L}_M \left[ g(Q^2) \right] = \frac{1}{M^2} \hat{\mathcal{L}}^{-1}_{1/M^2} \left[ g(s) \right], \tag{A.9}$$

where $\hat{\mathcal{L}}^{-1}_{1/M^2}$ is the inverse Laplace transformation given by,

$$\hat{\mathcal{L}}^{-1}_t \left[ g(s) \right] \equiv \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{st} g(s)\, ds. \tag{A.10}$$

Here we show some relations which are familiar in the calculations of QCD sum rules.

$$\hat{L}_M \left[ \frac{1}{(Q^2)^k} \right] = \frac{1}{(k-1)!} \frac{1}{(M^2)^k}, \tag{A.11}$$

$$\hat{L}_M\left[(Q^2)^k\right] = 0, \tag{A.12}$$

$$\hat{L}_M\left[(Q^2)^k \ln Q^2\right] = (-)^{k+1} k! M^{2k}, \tag{A.13}$$

$$\hat{L}_M\left[\frac{1}{(Q^2+s)^k}\right] = \frac{1}{(k-1)!}\frac{1}{(M^2)^k} e^{-\frac{s}{M^2}}, \tag{A.14}$$

$$\hat{L}_M\left[\frac{1}{(Q^2+s)(Q^2+s')}\right] = \frac{1}{M^2}\frac{1}{s'-s}(e^{-\frac{s}{M^2}} - e^{-\frac{s'}{M^2}}). \tag{A.15}$$

The following equations are useful in the calculations involving the direct instanton.

$$\hat{L}_M\left[(K_1(\rho Q))^2\right] = \frac{1}{2} e^{-\rho^2 M^2/2} K_1\left(\frac{\rho^2 M^2}{2}\right), \tag{A.16}$$

$$\begin{aligned}\hat{L}_M\left[Q^2 (K_1(\rho Q))^2\right] &= \frac{\rho^2 M^4}{4} e^{-\rho^2 M^2/2} \\ &\quad \times \left(K_0\left(\frac{\rho^2 M^2}{2}\right) + K_1\left(\frac{\rho^2 M^2}{2}\right)\right),\end{aligned} \tag{A.17}$$

$$\begin{aligned}\hat{L}_M\left[Q^4 (K_1(\rho Q))^2\right] &= \frac{\rho^2 M^6}{4} e^{-\rho^2 M^2/2} \\ &\quad \times \left(K_0\left(\frac{\rho^2 M^2}{2}\right) \times (-3+\rho^2 M^2)\right. \\ &\quad \left. + K_1\left(\frac{\rho^2 M^2}{2}\right) \times (-2+\rho^2 M^2)\right),\end{aligned} \tag{A.18}$$

where $K_n(z)$ is the modified Bessel function of the second kind known as the McDonald Function. They can be derived by using the integral representation of $(K_1(\rho\sqrt{s}))^2$:

$$\left(K_1(\rho\sqrt{s})\right)^2 = \frac{1}{2}\int_0^\infty K_1\left(\frac{\rho^2}{2t}\right) \exp\left(-st - \frac{\rho^2}{2t}\right)\frac{dt}{t}. \tag{A.19}$$

# B Order of corrections to LET in QCD sum rules

In ref. [13] Shifman, Vainshtein and Zakharov showed that the form of the OPE in the vacuum requires a massless pion in the limit $m_q = 0$, if $\langle 0|\bar{q}q|0\rangle \neq 0$ and that the leading order term in the OPE is identified with a contribution of the pion state.

Let us briefly follow the discussion of ref. [13].

$$\begin{aligned}\Pi_0(\omega) &= \sum_n \left\{ \frac{|\langle n|A_0(0)|0\rangle|^2}{\omega - E_n} - \frac{|\langle n|A_0^\dagger(0)|0\rangle|^2}{\omega + E_n} \right\} \\ &= \sum_P \frac{m_P^2 f_P^2}{\omega^2 - m_P^2} \\ &= -\frac{(m_1+m_2)\langle 0|\bar{q}_1 q_1 + \bar{q}_2 q_2|0\rangle}{\omega^2} + \mathcal{O}(m_q^2), \end{aligned} \tag{B.1}$$

holds only if there exists a pseudoscalar state satisfying the conditions

$$m_P^2 = O(m_q), \qquad f_P = O(m_q^0), \tag{B.2}$$

while all the states with a non-vanishing mass decouple in the chiral limit,

$$f_P = O(m_q) \qquad \text{if} \qquad m_P = O(m_q^0). \tag{B.3}$$

The Gell-Mann-Oakes-Renner relation is just the $\mathcal{O}(m_q)$ term in eq. (B.1).

Similarly,

$$\begin{aligned}\Pi_N(\omega) &= \sum_n \left\{ \frac{|\langle n|A_0(0)|N\rangle|^2}{\omega - (E_n - M_N)} - \frac{|\langle n|A_0^\dagger(0)|N\rangle|^2}{\omega + (E_n - M_N)} \right\} \\ &= \frac{\langle N|q_1^\dagger q_1 - q_2^\dagger q_2|N\rangle}{\omega} - \frac{(m_1+m_2)\langle N|\bar{q}_1 q_1 + \bar{q}_2 q_2|N\rangle}{\omega^2} + \mathcal{O}(m_q^2), \end{aligned} \tag{B.4}$$

holds only if their exists a state satisfying conditions,

$$\langle n|A_0^{(\dagger)}(0)|N\rangle = \mathcal{O}(m_q^0) \qquad \text{if} \qquad E_n - M_N \to 0 \qquad (m_q \to 0), \tag{B.5}$$

while all other states decouple in the chiral limit,

$$\langle n|A_0^{(\dagger)}(0)|N\rangle = \mathcal{O}(m_q) \qquad \text{if} \qquad E_n - M_N = \mathcal{O}(m_q^0). \tag{B.6}$$

The state which survives is that of pion-nucleon at the threshold and the matrix element of the pion-nucleon intermediate state with proper normalization has the following structure,

$$\begin{aligned}&\langle\pi(\mathbf{k})N(-\mathbf{k})|A_0^\dagger(0)|N(\mathbf{0})\rangle\\&= i\sqrt{2}m_\pi f_\pi(2\pi)^3\delta^3(\mathbf{k}) - i\frac{1}{\sqrt{2}}f_\pi T_+\delta(\omega_\mathbf{k}-m_\pi)+\theta(\omega_\mathbf{k}-m_\pi)F_+(\omega_\mathbf{k}),\end{aligned} \tag{B.7}$$

where $\omega_\mathbf{k}=\sqrt{m_\pi^2+\mathbf{k}^2}+\sqrt{M_N^2+\mathbf{k}^2}-M_N$, $F_+(\omega)=O(m_q)$ and $T_+/m_\pi=O(m_q^0)$, as can be explicitly seen later.

We split the correlation function into even and odd parts as in eq. (3.5) and take the combination, $\tilde{\Pi}_N(\omega^2)=\Pi_N(\omega^2)+(\omega^2-m_\pi^2)\frac{d}{d\omega^2}\Pi_N(\omega^2)$, for both parts. Then, we obtain the following relations

$$-\frac{2m_\pi f_\pi^2 T^{(-)}}{(\omega^2-m_\pi^2)^2} = m_\pi^2\frac{\langle q_1^\dagger q_1 - q_2^\dagger q_2\rangle_N}{\omega^4}+\mathcal{O}(m_q^2), \tag{B.8}$$

$$-\frac{2m_\pi^2 f_\pi^2 T^{(+)}}{(\omega^2-m_\pi^2)^2} = -m_\pi^2\frac{(m_1+m_2)\langle\bar{q}_1 q_1+\bar{q}_2 q_2\rangle_N}{\omega^4}+\mathcal{O}(m_q^2). \tag{B.9}$$

Eq. (B.8) shows that in $\tilde{\Pi}_{N\,\mathrm{odd}}(\omega^2)$ the leading order term in $\frac{1}{\omega^2}$ is identified with the contribution of the pion-nucleon state at the threshold,

$$-2m_\pi f_\pi^2 T^{(-)} = m_\pi^2\langle q_1^\dagger q_1 - q_2^\dagger q_2\rangle_N+\mathcal{O}(m_q^2), \tag{B.10}$$

which is nothing but the Tomozawa-Weinberg relation, while eq. (B.9) shows that in $\tilde{\Pi}_{N\,\mathrm{even}}(\omega^2)$ the leading order term in $\frac{1}{\omega^2}$ contains not only the contribution of the pion-nucleon state at the threshold but also that of the states with higher energies

$$2f_\pi^2 T^{(+)} = (m_1+m_2)\langle\bar{q}_1 q_1+\bar{q}_2 q_2\rangle_N+\mathcal{O}(m_q). \tag{B.11}$$

Namely, the order of the correction to the sigma term in the isospin-even pion-nucleon scattering length is the same as that o

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