Trade Pattern and Economic Growth in a Small Open Economy

Akihiko Kaneko

Department of Social Engineering
Tokyo Institute of Technology

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Contents

Chapter 1

Introduction

The theory of international trade has been analyzed mainly within the static theory framework. In the theory, economic growth means an outward shift of a country's production possibility frontier caused by the exogenous change of capital stock or technology. Thus, the traditional trade theory cannot explain how trade affects the economic growth of a country. Several authors attempted to incorporate the trade theory into a neoclassical growth theory (Oniki and Uzawa (1965) or Stiglitz(1970)). In those papers, determination of trade pattern and factor price equalization are analyzed. However, those papers do not investigate the relationship between trade and the growth rate, because the neoclassical growth theory predicts that the level of income per capita and that the long-run growth rate becomes zero since the capital-labor ratio converges to some long-run equilibrium value, as do the real wages, the rate of return to capital.

Recent empirical research has provoked controversy whether per capita incomes in different countries are converging or not. Romer (1989), Dowrick (1992) and Quah (1993) present the evidence that the growth rates do not converge among countries. No-convergence of long-run growth rates poses one of the central questions in macroeconomics. Why is it that the poor countries as a group are not catching up with the rich countries in the same way that, for example, the low income states in the United States have been catching up with the high income states? The endogenous growth theory has been developed as an answer to this question. The basic model of the endogenous growth theory presents an economy in which diminishing returns to a broad concept of capital does not apply. Development of the endogenous growth model enables to examine the effects of several policies on the long-run growth rate.

In this paper, we construct a three-sector model – two commodity producing sectors and one human capital producing sector – in which human capital accumulation is the engine of growth. We adopt a human capital accumulation as the engine of growth, since it's importance is emphasized in both theoretical and empirical research. Our model makes it possible to clear the relationship between the growth rates and trade.

Moreover, we reexamine a prediction of a dynamic trade model that even a slight technological difference leads to complete specialization. Chapter 5 investigates a two-sector model in which there is an externality of each firm's investment and we show that complete specialization does not occur in this model. The degree of labor quality increases as an externality of firm's investment. Namely, there is a learning-by-doing effect as a by-product of each firm's investment. The learning-by-doing effect becomes the engine of growth. There also exist scale effects: the more labor input is employed, the higher the marginal product of capital is. The scale effects break the dichotomy between production side and demand side, and prevent a small country from becoming specialized.

The plan of paper is organized as follows. In the next chapter, we survey various types of the endogenous growth model and dynamic trade models. In chapter 3, we show how trade affects the long-run growth rate, depending in which kind of commodity a small country has a comparative advantage. In chapter 4, we examine the effects of various policies (specifically tariffs and a production tax) on the long-run growth rate. Finally, in chapter 5, we present a learning-by-doing model with scale effects and demonstrate that the scale effects prevent a small country from becoming specialized.

Chapter 2

A Survey on the Endogenous Growth Theory and the Dynamic Trade Theory

Since late 1980s, a large number of studies have been made on an endogenous growth theory and a dynamic trade theory. In the endogenous growth theory, determinants of long-run growth are investigated theoretically and empirically. Existing studies endogenize technical innovations in various ways. According to the way of endogenization we can classify the endogenous growth models into several basic models: an AK model, a learning-by-doing model, a public-goods model, a human capital accumulation model and an R&D model. Many useful surveys of a endogenous growth model have been done.[1] We here present only two models which concern with the chapters 3, 4 and 5 –a learning-by-doing model and a human capital accumulation model. Then, endogenous growth models in an open economy are surveyed in subsection 2.1.3.

In the dynamic trade theory, optimal saving and investment behaviors are incorporated into a static Heckscher-Ohlin (H-O) model and determinants of a trade pattern are reexamined. The model is investigated under several cases: a small country model or a two-country model, if there exists an international financial market or not. We shall survey each case in section 2.2.

2.1 The Endogenous Growth Theory

Endogenous growth models emphasize production structure, whereas household's behavior is the same in all basic models.

[1] For example, Sala-i-Martin (1990 a and b) Lucas (1993), Shibata (1993), Hammond and Rodríguez-Clare (1993), Dinopoulos (1994), Thompson (1994), Barro and Sala-i-Martin (1995) and Aghion and Howitt (1998).

We assume that there is a representative agent who lives infinitely in an economy. His instantaneous utility function is:

$$\begin{array}{ll} \dfrac{C(t)^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \ln C(t) & \text{if } \sigma = 1, \end{array}$$

where C is consumption. Then, he maximizes his life time utility, subject to the budget constraint.

$$\begin{aligned} U &= \int_0^\infty \frac{C(t)^{1-\sigma}-1}{1-\sigma} \exp(-\rho t)\, dt \\ \text{s.t.} \qquad \dot{a} &= Ra - wL - C, \end{aligned}$$

where ρ is the subjective discount rate, a is assets, R is the interest rate, w is the wage rate, L is labor force. A dot always means time derivatives. The optimal condition for this problem under the perfect foresight is given by the Euler condition as follows:

$$\sigma \frac{\dot{C}}{C} = R - \rho. \tag{2.1}$$

2.1.1 The Learning-By-Doing Model

There is a considerable evidence that technology in many firms and industries exhibits a *learning curve* in which productivity is related to past production levels. Arrow (1962) argues that the acquisition of knowledge is related to experience giving an example of the airframe industry. Romer (1986) follows this idea by postulating that by increasing physical capital a firm obtains know-how of efficient production of a certain commodity. This positive effect of experience is called a learning-by-doing. The Arrow-Romer type production function is given by

$$F(K, L) = AK^{\alpha}L^{1-\alpha}\bar{K}^{\eta}, \tag{2.2}$$

where $\bar{K}$ is the aggregate capital stock. The last term $\bar{K}^{\eta}$ represents an idea that the private marginal product increases as the aggregate capital stock grows. Suppose that $1 - \alpha = \eta$. Then, equation (2.2) is

$$F(K, L) = AK^{\alpha}\left(L\bar{K}\right)^{1-\alpha}.$$

If a capital market is competitive, the profit maximization equates the marginal productivity of physical capital to the real interest (assume no population growth).

$$\frac{\partial F}{\partial K} = \alpha AK^{\alpha-1}\bar{K}^{1-\alpha}L^{1-\alpha}.$$

In the equilibrium $K = K$,

$$\frac{\partial F}{\partial K} = \alpha A L^{1-\alpha}. \tag{2.3}$$

Thus, the real interest remains constant. If we substitute (2.3) into (2.1), the growth rate is given by

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}[\alpha A L^{1-\alpha} - \rho]. \tag{2.4}$$

If the labor stock is sufficient, the economy enjoys the positive long-run growth rate.

In this model, social marginal product is not equal to private one, so competitive equilibrium is not social optimum. In fact, a social production function is

$$F^s = AK^\alpha K^{1-\alpha} L^{1-\alpha} = AKL^{1-\alpha}.$$

A social marginal product of capital is $AL^{1-\alpha}$, which is larger than private one. A competitive firm would achieve the lower growth rate than the social optimal one because it fails to internalize the spillover in production.

Note that the model implies a scale effect. An expansion of aggregate labor force, L, raises the real interest rate and the growth rate. This effect results from increasing returns to K and L at the social level. We extend the model to two commodity two-factor model to investigate the validity of a predication of a dynamic trade model.

2.1.2 Two-Sector Models of Endogenous Growth

Suppose that a production function takes the form of a Cobb-Douglas production function, $AK^\alpha L^{1-\alpha}$. The marginal product of capital is $A\alpha K^{\alpha-1} L^{1-\alpha}$ and it declines as capital accumulates. Thus, from the Euler equation (2.1), the growth rate converges to zero in the long run. The decline of the marginal product is due to the fixed labor force. In the two-sector growth model, households can increase the effective labor force.

In a two-sector endogenous growth model, the labor force of households is considered as human capital and there is a sector with a capability to produce new skills or knowledge (new human capital). Households can enhance the effective labor force by acquiring new human capital. Lucas (1988) is the first to postulate an economy in which inputs of final commodity sector are both physical and human capital, and where a human capital producing sector exists. However, in his model, only human capital is engaged in production of new human capital. A model which employs also physical in addition to human capital for producing new human capital was first developed by King, Plosser and Rebelo (1988). Rebelo (1991) and Mino (1996) also adopt this version of the two-sector endogenous growth model.

Production functions of all firms exhibit a constant returns to scale. Specifically,

$$Y_1 = f(k_1)H_1, \tag{2.5}$$

$$\dot{H} = g(k_2)H_2. \tag{2.6}$$

where H is total human capital, k_i is the ratio of physical capital to human capital devoted to sector i, H_i is the human capital devoted to sector i. Sector 1 produces a commodity which is used for capital and consumption. Sector 2 produces new human capital. Sector 2 can be considered as a private school or a vocational school. It produces new skills and new knowledge.

Commodity 1 is numeraire. Let us denote the price of new human capital q, the nominal rent on physical capital R, and the nominal rent on new human capital V. The profit maximization by competitive firms equates the marginal productivity of each factor input with its rent. Thus, while all sectors are in operation, we obtain following equations:

$$f' = qg' = R, \tag{2.7}$$

$$f - k_1 f' = q[g - k_2 g'] = V. \tag{2.8}$$

We assume that $f(k_1)$ and $g(k_2)$ satisfy the standard neoclassical assumptions and Inada conditions.

In this economy, there are two kinds of capital. Thus, a no-arbitrage condition must hold:

$$R = \frac{V}{q} + \frac{\dot{q}}{q}. \tag{2.9}$$

The left hand side of the condition is the return from holding physical capital. The right hand side is the return from holding human capital. Since commodity 1 is numeraire, the return from holding human capital consists of its rent and capital gains.

The steady state growth equilibrium is realized when Y_1, K (the total capital stock), H and C, grow at the same constant rate. Thus, the relative price remains constant ($\dot{q} = 0$). Making use of (2.7) and (2.8), the steady state expression of the no-arbitrage condition is

$$f_1' = q[g - k_2 g']. \tag{2.10}$$

Introducing the equation above into the Euler equation (2.1), in the steady state growth equilibrium we obtain

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{H}}{H} = \frac{1}{\sigma}[f_1' - \rho] - g\gamma_2, \tag{2.11}$$

where $c \equiv C/H$ and $\gamma_2 \equiv H_2/H$.[2] From (2.10) and (2.11), the long-run growth rate of this economy expressed as $\frac{q}{\sigma}[g - k_2 g] - \rho$. Thus, the productivity of sector 2 ($g - k_2 g$) is high enough so that the economy enjoys the positive growth rate.

[2] We omit derivation of the dynamic system here, because we shall add one more sector to the model and develop full derivation of the autonomous dynamic system in chapter 3.

The two-sector endogenous growth model has transitional dynamics. Several studies have been devoted to studies on it. Mulligan and Sala-i-Martin (1993) develop an efficient algorithm to characterize the transitional path. Caballé and Santos (1993) use a physical capital production function which employs both physical and human capital, and a human capital production function which employs only human capital. They characterize the set of steady states as a ray from the origin and show the global convergence of every off-balanced path to some point on this ray. Mino (1996) examines the two-sector endogenous growth model, in which both sectors employ human capital as well as physical capital. He demonstrates that the dynamic behavior of the economy and effects of a tax policy depend on the magnitude of factor intensity in each sector.

2.1.3 Endogenous Growth Models in an Open Economy

There are some studies which examine the relationship between the trade and the growth rate in the endogenous growth theory.

Young (1991) points out the possibility that opening trade would slow down the growth rate of the developing country. In his model, the production of a commodity has the bounded spillover effect that enhances the productivity of both the own and the other industry. The bounded spillover effect implies that the spillover effect of the production of some industry would be exhausted when the productivity in the industry reaches a certain point. The developed country has a higher initial level of knowledge. Under free trade, the developing country specializes in commodities that had already exhausted the spillover effect.

Using a model in which finite-lived individuals invest in human capital, Stokey (1991) demonstrates that the free trade can lower the rate of investment in human capital. Investments have a positive external effect on the accumulation of human capital of later cohorts. When a small country has a highly initial knowledge relative to the rest of the world, the opportunity cost of investment in human capital is large and the rate of the accumulation of human capital slows down. On the other hand, if it has a much lower initial knowledge, the small country decides to import the high quality goods and lowers the accumulation of human capital.

Matsuyama (1992) addresses the role of agricultural productivity in a two-sector model of endogenous growth in which the engine of growth is a learning-by-doing in the manufacturing sector. He assumes that preferences are non-homothetic and the income elasticity of demand for the agricultural good is less than unity. He concludes that for the closed economy, the model predicts a positive link between agricultural productivity and economic growth and that for a small open economy, the country specializes in a good in which it has a comparative advantage and the model predicts a negative link between agricultural productivity and economic growth. The results depend on the assumption that the engine of growth is the learning-by-doing in the manufacturing sector.

Majumdar and Mitra (1995) consider a two-sector model in which a capital commodity sector exhibits an initial phase of increasing returns in production and a production function of a consumption commodity sector exhibits constant returns. Both sectors employ only one factor: capital commodity. They find that an autarky economy may face a poverty trap due to the increasing returns. It can escape from the trap by engaging in free trade. Opening trade caused unbounded economic growth.

Dinopoulos and Syropoulos (1997) examine the effect of tariffs in a multi-country Schumpeterian growth model. The endogenous growth arises because of endogenous innovations. Each country produces one tradeable commodity and one non-tradeable commodity. The technical innovation occurs in both productions. A reduction in tariff shifts the demand of households from non-tradeable commodity to tradeable commodity. Thus, it reduces R&D investment in the non-tradeable commodity sector while it raises R&D investment in the tradeable commodity sector. The effect of tariff changes on the long-run growth rate is ambiguous.

In the above mentioned models, authors adopt one factor model of production. Recently, several articles present a model which incorporate a Heckscher-Ohlin structure into the endogenous growth model. Both Mino (1994) and Pecorino (1994) introduce a human capital producing sector into a $2 \times 2 \times 2$ model and develop a similar two-country model to demonstrate the relationship among taxation, specialization and growth. Mino (1994) show how tax policies of each country affect both the long-run growth rate and specialization pattern depending on the international tax system: a resident rule and a source rule. The conditions for specialization patterns depend on the comparative advantage and the relative magnitude of stocks of human capital. In Pecorino's (1994) model, only a comparative advantage plays a role in determining specialization, due to the assumption of identical capital intensity across sectors in both countries .

We present several models with the expansion of variety of intermidiate commodities as an engine of growth. Wälde (1994) examines a two-factors, two-final-commodities Heckscher-Ohlin model of endogenous growth, in which expanding variety of intermediate commodities is the engine of growth He concentrates on the relationship between factor price equalization and specialization. Under his assumption (each country has a different subjective discount rate and immobile physical capital), holds a situation of incomplete specialization and different factor rewards.

Grossman and Helpman (1991 ch.6) investigate an economy where the engine of growth is highly human capital intensive R&D technology. In their model, R&D implies the expansion of variety of intermediate commodities. There are two tradeable final commodities. Since each sector employs different inputs (skilled labor and unskilled labor), both sectors operate even after opening trade. If a country has a comparative advantage in a human capital intensive final commodity, after opening up to trade it reallocates human capital toward that sector from R&D sector. It decreases the long-run growth rate. Rivera-Batiz and Romer (1991) examine an R&D endogenous growth model. They assume that the expansion of variety of capital commodities and R&D activity for new types of capital commodities exhibits increasing returns to scale. Because the flow of capital commodities avoids redundant effort of R&D and encourages the exploitation of increasing returns to scale in the R&D activity, international trade can increase the long-run growth rate.

2.2 The Dynamic Trade Theory

In a static trade model, the most dominant international trade theory is a Heckscher-Ohlin (H-O) model. In the H-O model, a trade pattern is determined by a ratio of two factor endowments: physical capital and labor. The model predicts that the country that is relatively capital abundant exports the relatively capital-intensive commodity and that the other country exports the relatively labor-intensive commodity. Recently a neoclassical dynamic trade model has been developed incorporating optimal saving and investment behavior into the static H-O model. There are significant differences between the static trade theory and the dynamic one. First, in the dynamic trade theory, capital stock is a state variable which changes according to investment behavior. Second, while in the static theory, both trade and production pattern are analyzed with no international financial market, and consequently current account is always balanced. However, in the dynamic theory we can investigate asset transactions as a result of optimal behavior of households. Moreover, as we incorporate intertemporal maximization, there is one condition which does not appear in the static model but does in the dynamic model: the Euler equation which dictates the law of consumption.

First, we point out the difference between a static model and a dynamic one in an autarky economy. Then, we survey main results of the dynamic trade model in subsections 2.2.2 and 2.2.3.[3]

2.2.1 The Autarky Economy

Main difference between a static trade model and dynamic one can be seen in an autarky economy, which we are about to point out.

The production structure is the same as a static H-O model. Two sectors operate in an economy. Sector 1 produces a capital commodity and sector 2 produces a consumption commodity. Both sectors employ physical capital and labor as input. The production functions exhibit a constant returns to scale.

$$Y_1 = f(k_1)L_1. \tag{2.12}$$

$$Y_2 = f(k_2)L_2. \tag{2.13}$$

Letting R express the interest rate, w the wage rate and p the relative price of commodiy

[3] Ono (1993) presents a comprehensive survey of recent development of the dynamic trade theory.

2, as a result of each firm's competitive behavior we have

$$f_1'(k_1) = pf_2'(k_2) = R, \tag{2.14}$$
$$f_1(k_1) - k_1 f_1'(k_1) = p[f_2(k_2) - k_2 f_2'(k_2)] = w. \tag{2.15}$$

From (2.14) and (2.15), we know k_i, R and w are functions of p:

$$k_i = k_i(p), \; R = R(p), \; W = W(p), \; k_i' \equiv \frac{dk_i}{dp} \gtrless 0 \;\; if \;\; k_1 \gtrless k_2, \; (i = 1, 2) \tag{2.16}$$

A household has a log-liner instantaneous utility function and maximizes it's life time utility. Formally,

$$U = \int_0^\infty [\alpha \ln C_1 + (1-\alpha) \ln C_2] \exp(-\rho t)\, dt,$$

subject to the flow budget constraint:

$$\dot{K} = RK + wL - C_1 - pC_2,$$

where C_i $(i = 1, 2)$ represents the consumption level of commodity i, K the total physical capital stock, and L the labor force. Set up the Hamiltonian of this model, we have usual first order conditions:

$$p = \frac{1-\alpha}{\alpha} \cdot \frac{C_1}{C_2}, \tag{2.17}$$
$$\frac{\dot{C}_1}{C_1} = R - \rho. \tag{2.18}$$

(2.17) is the equality between the intratemporal marginal rate of substitution and the relative price, which also appears in the static H-O model. The Euler equation (2.18) is a specific condition in the dynamic trade model.

The autonomous dynamic system consists of the Euler equation and the flow budget constraint. The saddle point stability of this economy is satisfied under usual assumptions of the production functions. Thus, the economy reaches the steady state equilibrium in the long run. The growth rate of the consumption level and the capital (asset) accumulation will be zero. From (2.16) and (2.18), the unique interest rate and the unique relative price are determined by ρ. Namely, we demonstrate that there is unique relative price, $\bar{p}$, at which both sectors can operate in the steady state equilibrium. Put another way, dichotomy between production side and demand side holds in a dynamic trade model In chapters 3 and 5, we shall discuss it in detail.

2.2.2 The Small Country Model

A small country is defined as a country for which relative prices of commodities in world markets are given. First, we study a case in which there is no international financial market, and then we investigate a case in which households can have access to international financial markets.

When there is no international financial market, even after opening trade households accumulate the same assets which they accumulated in the autarky. Thus, from the saddle point property and (2.18) the interest rate is determined domestically in the long run. The growth rate of the consumption level and the asset accumulation are zero in the long run. Then, there is unique relative price, $\bar{p}$, at which both sectors can operate in the steady state equilibrium. Unless the relative price of commodities happens to be equal to the autarky price, $\bar{p}$, the small country specializes in a commodity in which it has a comparative advantage in production. Formal proof is as follows:

Proof. For a small country, the relative price of commodity is set at the world price, p^w. Sector 2 operates and sector 1 cannot survive in the steady state with p^w if and only if:

$$\mathrm{MPL}_1 \equiv f_1(k_1) - k_1 f_1'(k_1) < p^w \left[f_2(k_2) - k_2 f_2'(k_2) \right] \equiv \mathrm{MPL}_2 \tag{2.19}$$

$$\text{when } R = f_1'(k_1) = p^w f_2'(k_2) = \rho. \tag{2.20}$$

Sector 2 cannot oprate if the value of marginal product of labor (MPL) for sector 2 is larger than that for sector 1, taking into consideration that the capital rent equals ρ.

Given the world price, p^w, making use of (2.20) and (2.19), we obtain that

$$\frac{dMPL_j}{dR} = -k_j < 0. \quad (j = 1, 2) \tag{2.21}$$

Since $f_1'' < 0$, from (4.4) we know that

$$\frac{df_1'(k_1(p))}{dp} \gtrless 0 \text{ if } k_1 \lessgtr k_2. \tag{2.22}$$

Both sectors operate in the steady state equilibrium before opening to trade, that is, the condition below holds:

$$f_1'(k_1) = \bar{p} f_2'(k_2) = \rho. \tag{2.23}$$

Considering (2.23) and (2.21), if $p^w > (<) \bar{p}$ the equation below holds:

$$MPL_1 < (>) MPL_2. \tag{2.24}$$

This immediately implies (2.19). ■

In chapter 3, we shall prove that the same result holds in an endogenous growth model with a human capital accumulation.

Next, we investigate a case with an international financial market. The trade pattern is determined according to the same principle as the one in the previous case: A small country specializes in a commodity in which it has a comparative advantage in the long run. The asset accumulation, however, is affected by international financial market. Since households can accumulate foreign assets, as a result of arbitrage the domestic interest rate, R, is fixed to the world interest rate, R^*. Additionally, from (2.18) we know that if $R^* \neq \rho$, there is not a steady state equilibrium. If $R^* < \rho$, the economy enjoys a high level of consumption early on and the level of consumption asymptotically approaches zero. If $R^* > \rho$, the consumption level of the economy grows unboundedly. This case is analyzed in Baxter (1992).

Note that in the static H-O model there is a range of the relative world price in which a small country does not specializes completely. In the dynamic trade model, incomplete specialization never occurs, unless the relative world price coincides with the autarky price.

2.2.3 The Two-Country Model

We first treat a case in which there is no international financial market. As in the small country case, the interest rate of each country is determined by the subjective discount rate in the autarky equilibrium. Thus, the two countries with the identical preference, identical technology and identical initial endowment have the same level of per capita income in the long run. Even if they open to trade, no trade occurs. If the initial endowment differs, the country that is relatively capital abundant in the initial period remains relatively capital abundant and exports the relatively capital-intensive commodity, while the other country remains relatively labor abundant and exports the relatively labor-intensive commodity in the long run. Namely, the prediction of the static Heckscher-Ohlin holds in this case. Because in a trading world the factor prices are equalized, households in both countries have the same incentive to accumulate their respective capital stocks. Therefore, the initial ratio of endowments and trade pattern remains unchanged in the long run. This case is studied in Chen (1992).

Under the same conditions (free trade and no international financial market), Oniki and Uzawa (1965) examine the specialization pattern by assuming constant but different saving rate of each country. It is the earliest study on the dynamic trade theory. Stiglitz (1970) extends Oniki and Uzawa (1965) by incorporating a household's optimizing behavior. These two models assume that two countries have the same technology but the different subjective discount rate. In this setting, a country with the lower subjective discount rate keeps accumulating physical capital. Thus, in the long run, the capital-labor ratio becomes so large that it breaks the imperfect specialization trade equilibrium.

Next, we present models which assume free trade in commodities and financial assets. The models are Baxter (1992) and Ono and Shibata (1993). As a source of specialization, they emphasize asymmetry in the technological level of two countries. If we assume that both countries with different technologies specialize incompletely, contradiction occures as follows. The interest rates in two countries are equal to each other as a result of arbitrage. On the other hand, as a static Heckscher-Ohlin model predicts, if imperfect specialization of both countries is assumed, the rent for physical capital in the two countries with different technology must differ. Thus, supposing the imperfect specialization of both countries yields a contradiction. Baxter (1992) concludes with a Ricardian implication that at least one of the two countries must specialize completely when there is only a minor difference in the technological level. Ono and Shibata (1993) obtain conditions for which various steady-state specialization patterns hold in the two-country model. Intuitively, the relative labor abundant country produces both commodities and the relative labor scarce country produces a commodity in which it has a comparative advantage. When the two counties are roughly similar in the endowment of labor, each of them specializes in a commodity in which it has a comparative advantage.

As we have seen, it's Ricardian flavor of prediction (a small difference in technology causes complete specialization) is a special feature of a dynamic trade model.

2.3 Conclusion

We briefly presented a relevant literature on the endogenous growth theory and the dynamic trade theory. Considerable number of studies have been made on both theories.

The endogenous growth theory endogenizes knowledge accumulation or technological innovation and realizes the long-run positive growth rate. The dynamic trade model incorporates intertemporal optimal behavior of households and firms and examines trade pattern and asset accumulation.

Following the development of the endogenous growth theory enables us to examine the effects of various policies on the long run growth. As we have seen in 2.1.3, many papers investigate the effect of free trade on the economic growth and some of them point out the possibility of negative effect of free trade on the economic growth. Most of them adopt one factor model of production, whereas such model do not grasp the importance of a production of a capital commodity. In order to overcome this deficiency, we adopt a two-factor and two-commodity model and demonstrate that free trade never lowers the long-run growth rate of a small country. Moreover, the magnitude of the effect of free trade on the growth rate depends on what kind of commodity the country produces.

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Chapter 3

Terms of Trade, Economic Growth and Trade Pattern: A Small Country Case

Abstract

By incorporating endogenous human capital accumulation into a dynamic trade model, we examine the relationship between the growth rate and the specialization pattern in a growing economy. It is found that as long as its autarky price differs from the world price, a small country eventually specializes in an industry and that the growth rate depends on which commodity it specializes in. Specifically, if a country specializes in a capital commodity, the growth rate is unaffected by changes in the terms of trade. In contrast, if it specializes in a consumption commodity, its growth rate is significantly influenced by the terms of trade.

3.1 Introduction

In the literature on endogenous growth, several factors are emphasized as engines of growth, such as non-essential resources, learning-by-doing, human capital and technical progress. But the relationship between trade patterns and the growth rate is almost always ignored. Using cross-country data, Barro and Sala-i-Martin (1995) empirically investigate determinants of economic growth. Without providing theoretical reasoning, they find that the growth rate in real per capita GDP is positively correlated with the improvement in the world price. In a small-country context, conventional static trade theory demonstrates that an improvement in the terms of trade raises the 'absolute level' of national income. However, this framework cannot be used to investigate the effects of the terms of trade on the growth rate. In contrast, using a dynamic trade model of a small country, this paper investigates the relationship between an improvement in the terms of trade and the 'growth rate' of national income. We find that it is the trade pattern that determines the effect of the terms of trade on the growth rate of national income.

In this paper we use an endogenous growth model with two inputs, physical and human capital, and two commodities, a pure consumption commodity and a commodity used for both consumption and investment. Human capital accumulation is an engine of growth. In the literature on endogenous growth generated by human capital accumulation, such as Lucas (1988), King, Plosser and Rebelo (1988), King and Rebelo (1990), Mulligan and Sala-i-Martin (1993), Mino (1996), there is only one commodity that is used for both consumption and investment. Therefore, these papers cannot investigate the effects of changes in the terms of trade. There are also two-commodity dynamic trade models with optimizing agents (e.g., Chen (1992), Baxter (1992), Ono and Shibata (1994)), but they do not incorporate endogenous growth.

There does exsit a literature where the effect of trade on the growth rate is analyzed. Grossman and Helpman (1991 ch.6) investigate an economy with two final commodities where the engine of growth is R&D technology. There are two final production sectors as in our model. Each sector employs different inputs so that both sectors operate even after opening trade. In contrast, in our model, opening trade causes perfect specialization because two final production sectors employ the same inputs. Rivera-Batiz and Romer (1991) examine an R&D endogenous growth model. Assuming a specific R&D technology (which they call a lab equipment model), they find that trade raises the demand for intermediate goods and that this effect increases the growth rate. Since there is only one final product in their model, they cannot investigate the relationship between the trade pattern and the growth rate.

Lee (1995) introduces the 'AK' technology into a two-commodity dynamic trade model. He points out that opening trade enhances the long-run growth rate when a country imports cheaper capital commodity and increases the efficiency of capital accumulation. In his model, a small country always imports both commodities for investment and consumption because it is assumed that both commodities are composites of domestic and foreign commodities. In our model, we do not adopt such an assumption and a small country must specialize perfectly. We can, therefore, find a relationship between trade patterns and economic growth.

Pecorino (1994) and Bond, Trask and Wang (1997) analyze a two-country version of an endogenous growth model with human capital accumulation. Pecorino (1994) examines a relationship between a tax system and the growth rate.[1] He explores whether taxation on earnings of physical capital influences the long-run growth rate. Bond, Trask and Wang (1997) show that on a balanced growth path, a physical capital abundant country exports a physical capital intensive commodity but that initial factor endowment does not determine a long-run pattern of trade. They also examine the relationship between the autarky price and autarky physical to human capital ratio.

[1] Mino (1994) also analyzes a relationship between a tax system and the long-run growth rate in a two-country model of endogenous growth.

The plan of this paper is as follows. In section 3.2, we examine which specialization pattern obtains depending on the terms of trade. In section 3.3, we investigate the effect of an improvement in the terms of trade on the growth rate under each specialization pattern. In the last section, we discuss the contributions of this paper.

3.2 Specialization Patterns

We first examine the condition for each specialization pattern to hold. The first subsection describes firms' behavior in a closed economy as a bench mark, and the second subsection discusses the effect of opening trade on the specialization pattern.

3.2.1 The Optimal Conditions of Firms

We consider a small country with three sectors. Sector 1 produces commodity 1 (capital commodity) which is used for both investment and consumption. We treat it as numeraire. Sector 2 produces commodity 2 (consumption commodity) which is used only for consumption. Sector 3 produces commodity 3 (human capital Q). We assume that the production functions satisfy constant returns to scale:

$$\begin{aligned} Y_1 &= f_1(k_1) H_1, \\ Y_2 &= f_2(k_2) H_2, \\ Q &= g(k_3) H_3, \end{aligned}$$

where Y_i is the output of commodity i, k_i the ratio of physical capital to human capital employed in the ith sector, and H_i the stock of human capital devoted to the ith sector.[2] $f_i(k_i)$ and $g(k_3)$ satisfy the standard neoclassical assumptions and Inada conditions.

$$\begin{gathered} f_i'(k_i),\ g'(k_3) > 0,\ f_i''(k_i),\ g''(k_3) < 0, \\ \lim_{k_i \to o} f_i'(k_i) = \infty,\quad \lim_{k_i \to \infty} f_i'(k_i) = 0, \\ \lim_{k_3 \to o} g'(k_3) = \infty, \text{ and } \lim_{k_3 \to \infty} g'(k_3) = 0, \end{gathered}$$

where $f_i' = \dfrac{df_i}{dk_i}$, $f_i'' = \dfrac{df_i'}{dk_i}$, $g' = \dfrac{dg}{dk_3}$, $g'' = \dfrac{dg'}{dk_3}$ $(i = 1, 2)$.

Using the above properties, we describe firms' behavior in a closed economy. As a result of each firm's competitive behavior, the value of the marginal product of each factor input is equalized to its rental price. Thus, letting p the price of commodity 2,

[2] Like the standard two sector model, the factor intensity in each sector is very important to secure the saddle-point property of equilibrium. But we assume that the condition for the saddle-point equilibrium is satisfied and do not mention the factor intensity condition.

q the price of human capital, R the rent on physical capital, and V the rent on human capital, we have

$$R = f_1'(k_1) = pf_2'(k_2) = qg'(k_3), \tag{3.1}$$

$$V = f_1(k_1) - k_1 f_1'(k_1) = p[f_2(k_2) - k_2 f_2'(k_2)] = q[g(k_3) - k_3 g'(k_3)]. \tag{3.2}$$

From (3.1) and (3.2), we obtain each variable as a function of only the relative rental ratio, $\omega\,(= V/R)$:

$$k_i = k_i(\omega) \quad (i = 1, 2, 3), \tag{3.3}$$

$$p = \frac{f_1'(k_1(\omega))}{f_2'(k_2(\omega))} = p(\omega), \tag{3.4}$$

$$q = \frac{f_1'(k_1(\omega))}{g'(k_3(\omega))} = q(\omega), \tag{3.5}$$

$$R = R(\omega),\ V = V(\omega), \tag{3.6}$$

$$k_i' \equiv \frac{dk_i}{d\omega} > 0\,(i = 1, 2, 3). \tag{3.7}$$

Note that (3.5) and (3.6) can be obtained from the profit maximization conditions of only sectors 1 and 3.

In this economy there are two kinds of capital, K (physical capital) and Q (human capital). Thus, a no-arbitrage condition must always hold[3]:

$$R = \frac{V}{q} + \frac{\dot{q}}{q}. \tag{3.8}$$

A dot means a time derivative. Equation (3.8) holds when the household accumulates two kinds of capital: physical and human capital. Thus, even after opening trade, if it cannot accumulate another type of asset (e.g., foreign asset), a no-arbitrage condition is still given by (3.8).

3.2.2 Determination of the Specialization Pattern

In this subsection, we demonstrate that perfect specialization occurs when free trade starts.

We assume that human capital is non-tradeable. The terms of trade are defined as the ratio of the export price to the import price. Since only capital and consumption commodities are traded, the world price p^w represents the terms of trade when the small country exports commodity 2. On the other hand , $1/p^w$ does when it exports commodity 1. Many countries actually face a credit constraint for various reasons. We focus on this

[3] This condition is derived from the household's optimization formally in subsection 3.1. Otherwise, the household accumulates only one kind of capital.

case and assume that financial assets on installed physical capital are not internationally traded[4]. Under these assumptions, we have the following lemmata.[5]

Lemma 1 *Suppose that a small country is in a steady state growth equilibrium. If sectors 1 and 3 operate, then the relative rental ratio ω is also that which emerges under autarky. Furthermore, only sectors 1 and 3 operate whenever the autarky price of commodity 2 is higher than its world price.*

Proof.

First, we derive the steady state level of the relative rental ratio, the rent on physical and human capital, and the price of commodity 2 under autarky. The steady state growth equilibrium is realized when Y_1, Q, K, H, C_1, and C_2 grow at the same constant rate. Thus, q stays constant in the steady state growth equilibrium[6] ($\dot{q} = 0$). From (3.5), (3.6) and (3.8), we obtain

$$R(\omega) = V(\omega)/q(\omega). \tag{3.9}$$

From (3.4), (3.6) and (3.9), the steady state level of ω (which we shall call $\bar{\omega}$), $R(\bar{\omega})$, $V(\bar{\omega})$ and $p(\bar{\omega})$ under autarky are determined.

Now, the economy opens trade. Since there is no international financial market and the household cannot have foreign assets, the time path and the equilibrium level of ω are determined by no-arbitrage condition (3.8) under free trade as well as under autarky.

If sectors 1 and 3 operate in the steady state growth equilibrium even after opening trade, the conditions below hold:

$$f_1'(k_1) = qg'(k_3) = R. \tag{3.10}$$

$$f_1(k_1) - k_1 f_1'(k_1) = q[g(k_3) - k_3 g'(k_3)] = V. \tag{3.11}$$

Equations (3.10) and (3.11) are the conditions of the firms' optimization, from which we derive k_i ($i = 1, 3$), q, R, and V as functions of ω. They are the same as (3.3), (3.5) and (3.6), since they are derived from firms' optimization of only sectors 1 and 3. In the steady state growth equilibrium, from (3.8), (3.10) and (3.11) we have

$$R(\omega) = V(\omega)/q(\omega). \tag{3.12}$$

4 It implies that a country cannot borrow capital from a foreign country at all. Even if a fixed amount of borrowing from abroad is allowed, the following arguments hold (see the appendix). If there is no restriction on foreign borrowing from abroad, ω and the long run growth rate would be fixed at an exogenous world level. The case in which there is an international financial market which the household in a small country can freely access is treated in Mino (1994).

5 We do not consider a case in which the world price coincides with the autarky price. If it occurs, the three sectors are in operation as in the closed economy and the small country does not trade with the world market.

6 The steady-growth equilibrium of this types of economy is locally saddle point stable (see Mino (1996)).

Since (3.12) is the same as (3.9), the steady state level of ω, R, V and q are the same as those under autarky ($\bar{\omega}$, $R(\bar{\omega})$, $V(\bar{\omega})$ and $q(\bar{\omega})$).

Finally, we show that if $p^w < p(\bar{\omega})$, then sector 2 cannot survive and hence only sectors 1 and 3 operate. If $p^w < p(\bar{\omega})$, from (3.1) and (3.3) the equation below holds:

$$f_1'(k_1(\bar{\omega})) = p(\bar{\omega}) f_2'(k_2(\bar{\omega})) > p^w f_2'(k_2(\bar{\omega})). \tag{3.13}$$

(3.13) implies that sector 2 cannot operate, whereas sectors 1 and 3 can. ■

Lemma 2 *Suppose that a small country is in a steady state growth equilibrium. If sectors 2 and 3 operate after trade opens, then the relative rental ratio ω is positively correlated with the world price. Moreover, if the world price is higher than the autarky price, a small country always specializes in sectors 2 and 3.*

Proof.

Suppose that sectors 2 and 3 operate in the steady state growth equilibrium. The profit maximizing conditions for sectors 2 and 3 hold:

$$p^w f_2'(k_2) = qg'(k_3) = R, \tag{3.14}$$

$$p^w [f_2(k_2) - k_2 f_2'(k_2)] = q[g(k_3) - k_3 g'(k_3)] = V. \tag{3.15}$$

The above two equations show that variables k_i $(i = 2, 3)$, R, V, and q are functions of ω. From (3.8) we have $R = V/q$ in the steady state growth equilibrium. Using (3.14), (3.15), and this property we obtain

$$p^w f_2'(k_2(\omega)) = g(k_3(\omega)) - k_3(\omega) g'(k_3(\omega)). \tag{3.16}$$

Given the world price p^w, equation (3.16) determines the steady state level of ω (which we shall call $\tilde{\omega}$). Formally,

$$\tilde{\omega} = \tilde{\omega}(p^w). \tag{3.17}$$

Totally differentiating (3.16), we obtain

$$\frac{d\tilde{\omega}}{dp^w} = -\frac{f_2'}{p^w f_2'' k_2' + k_3 g'' k_3'} > 0, \tag{3.18}$$

which implies that in the steady state growth equilibrium; ω is positively correlated with p^w Taking (3.18) into account, from (3.16) we find that the steady state level of the marginal productivity of capital for sector 2, $p^w f_2'(k_2(\tilde{\omega}(p^w)))$, is positively correlated with p^w. Formally,

$$\begin{aligned}\frac{dp^w f_2'(k_2(\tilde{\omega}(p^w)))}{dp^w} &= \frac{d[g(k_3(\omega)) - k_3(\omega) g'(k_3(\omega))]}{d\omega} \frac{d\omega}{dp^w} \\ &= -k_3 g'' k_3' \cdot \frac{d\omega}{dp^w} > 0. \end{aligned} \tag{3.19}$$

From (3.16) and (3.17), we have

$$\tilde{\omega}\left(p\left(\bar{\omega}\right)\right)=\bar{\omega}. \tag{3.20}$$

That is, when the world price happens to be equal to the autarky price, the relative rental ratio is also equal to the autarky level. Since $k_1^{'}>0$ and $f_1^{''}<0$, from (3.19) and (3.20) we find that if $p^w > p\left(\bar{\omega}\right)$

$$p^w f_2'\left(k_2\left(\tilde{\omega}\left(p^w\right)\right)\right)>p\left(\bar{\omega}\right)f_2'\left(k_2\left(\bar{\omega}\right)\right)=f_1'\left(k_1\left(\bar{\omega}\right)\right)>f_1'\left(k_1\left(\tilde{\omega}\right)\right), \tag{3.21}$$

which implies that sector 1 cannot operate under free trade. ■

These lemmata show that the relative rental ratio ω remains at the autarky level when the small country specializes in sectors 1 and 3, whereas it is positively correlated with the world price when the small country specializes in sectors 2 and 3. The economic intuition behind these results is as follows.

When the consumption commodity sector (sector 2) operates, the value of the product of sector 2 rises as p^w increases. Since the input price (or the price of a capital commodity) remains unchanged, the rent on physical capital increases. In the steady state, the (real) rent on physical capital must equal the real rent on human capital as a result of arbitrage ($R = V/q$). The real rent on human capital increases following an increase of the (real) rent on physical capital. From (3.7) we know $d\left(V/q\right)/d\omega = d[g\left(k_3\left(\omega\right)\right)-k_3\left(\omega\right)g'\left(k_3\left(\omega\right)\right)]/d\omega > 0$, which implies the relative rental ratio (ω) must increase, as stated in lemma 2. However, under specialization in the capital commodity (sector 1), the input price always equals the value of the product of sector 1, because sector 1 produces the capital commodity. Thus, even when p^w changes, the rent on physical capital does not change and the relative rental ratio remains unchanged, as shown in lemma 1.

Note that in a small country version of a static Heckscher-Ohlin (H-O) model (two factor inputs, two tradeable commodities, constant returns to scale), free trade does not cause perfect specialization as long as the difference between the autarky price and the world price is not too large. In contrast, lemmata 1 and 2 show that even though the production structure of our model is the same as the static H-O model, in our model free trade must lead to complete specialization.[7] This Ricardian flavor of our model is due to a no-arbitrage condition as below.

As shown in the proof of lemma 1, using only the steady state expression of the no-arbitrage condition and the optimal conditions of firms, we derive the unique steady state level of the autarky price of commodity 2. Only at this price, the small country can produce commodities 1 and 2 in the steady state growth equilibrium. This explains the Ricardian flavor of the model. Thus, any world price below (above) the autarky

[7] This property is pointed out in Baxter (1992) and Ono and Shibata (1994) in a neoclassical growth model.

price, $p(\bar{\omega})$, must lead to complete specialization in sector 1 (sector 2). In the static Heckscher-Ohlin model, there is no condition for a steady state equilibrium similar to the no-arbitrage condition. Thus, the relative rental ratio and the relative price of commodity can change according to the world price and the both two sectors can operate.

Moreover, the no-arbitrage condition establishes the dichotomy between production side (which determines all prices) and demand side (which determines the corresponding quantities). We derive the prices (p, q, R and V) from the no-arbitrage condition and the optimal conditions of firms. If we introduce household's behavior and market clearing conditions, the corresponding quantities (output of each sector, stock of physical and human capital, physical and human capital employed in each sector, consumption level of each commodity) are determined. This is another aspect of the Ricardian flavor in our model.

3.3 The Long-Run Growth Rate

In this section, first we demonstrate the household's optimizing behavior. Then, we investigate the relationship between the specialization pattern and the long-run growth rate for the two cases: perfect specialization in the capital commodity, and perfect specialization in the consumption commodity.

3.3.1 Household's Behavior

We assume that the representative household has a log-linear utility function and a constant subjective discount rate ρ. The household owns the physical and human capital. It receives the rents from two kinds of capital which is used for the consumption of two commodities at the world price p^w, and the accumulation of both physical and human capital. To accumulate human capital means to embody new skills or knowledge and to increase the degree of labor quality. The household maximizes the following lifetime utility

$$U = \int_0^{\infty} [\alpha \ln C_1 + (1-\alpha) \ln C_2] \exp(-\rho t)\, dt,$$

subject to the flow budget constraint:

$$\dot{K} + q\dot{H} = RK + VH - C_1 - p^w C_2, \tag{3.22}$$

where C_i $(i = 1, 2)$ represents the consumption level of commodity i, K the total physical capital, and H the total human capital. Defining A as total assets $(K + qH)$, the flow budget constraint can be rewritten as

$$\dot{A} = RK + VH - C_1 - p^w C_2 + \dot{q}H.$$

From the Hamiltonian function of this problem:

$$J = \alpha \ln C_1 + (1-\alpha)\ln C_2 + \lambda_1 (RK + VH - C_1 - p^w C_2 + \dot{q}H) + \lambda_2 (A - K - qH),$$

we derive the first order conditions for interior solutions:

$$\frac{\partial J}{\partial C_1} = 0: \ \ \frac{\alpha}{C_1} - \lambda_1 = 0, \tag{3.23}$$

$$\frac{\partial J}{\partial C_2} = 0: \ \ \frac{1-\alpha}{C_2} - \lambda_1 p^w = 0, \tag{3.24}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial J}{\partial A}: \ \ \dot{\lambda}_1 = \rho\lambda_1 - \lambda_2, \tag{3.25}$$

$$\frac{\partial J}{\partial K} = 0: \ \ R\lambda_1 - \lambda_2 = 0, \tag{3.26}$$

$$\frac{\partial J}{\partial H} = 0: \ \ \lambda_1 (V + \dot{q}) - \lambda_2 q = 0. \tag{3.27}$$

Combining (3.23) with (3.24), we have

$$p^w = \frac{1-\alpha}{\alpha} \cdot \frac{C_1}{C_2}. \tag{3.28}$$

Introducing (3.26) and (3.23) into (3.25), we obtain

$$\frac{\dot{C}_1}{C_1} = R - \rho. \tag{3.29}$$

(3.28) represents the equality between the intratemporal marginal rate of substitution and the relative price, and (3.29) is the Euler equation.

Using (3.26), (3.27) is rewritten as

$$\lambda_1 (V + \dot{q} - Rq) = 0.$$

Since from (3.23) we know that λ_1 is always positive, to satisfy the above equation a no-arbitrage condition must always hold:

$$R = \frac{V}{q} + \frac{\dot{q}}{q}.$$

Otherwise, the household accumulates only one kind of capital: When $(V + \dot{q})/q > R$, $\partial J/\partial H > 0$ and the household wants to accumulate only human capital. When $(V + \dot{q})/q < R$, $\partial J/\partial H < 0$ and it wants to accumulate only physical capital. In each case, the asset market does not clear.

The transversality condition is

$$\lim_{t\to\infty} \lambda_1 A \exp(-\rho t) = 0. \tag{3.30}$$

No matter which commodity the small country specializes in, the household's behavior is described as equations (3.28), (3.29) and (3.30).

3.3.2 The Case of Perfect Specialization in a Capital Commodity

We derive here the relationship between the steady state growth rate and the terms of trade under perfect specialization in the capital commodity. We shall find it useful to express the system in terms of variables that will be constant in the steady state. The steady state growth equilibrium is realized when H, K, C_1 grows at the constant rate. A specification that facilitates dynamic analysis involves the ratios, ω, $c_1 \equiv C_1/H$ and $k \equiv K/H$. These three variables remain constant in the steady state growth equilibrium.

From lemma 1, if $p(\bar{\omega}) > p^w$ the small country specializes in the capital commodity and the steady state growth level of ω equals $\bar{\omega}$. Equations (3.10) and (3.11) in the proof of lemma 1 show that k_1, k_3, and q depend on only ω. Since we have $\bar{\omega}$ as the steady state growth level of ω,

$$k_1 = k_1(\bar{\omega}),\ k_3 = k_3(\bar{\omega}),\ q = q(\bar{\omega}),\ k_i' > 0,\ (i = 1, 3). \tag{3.31}$$

The market clearing conditions for physical and human capital are respectively

$$K/H \equiv \bar{k} = \bar{\gamma}_1 k_1(\bar{\omega}) + \bar{\gamma}_3 k_3(\bar{\omega}), 1(= H/H) = \bar{\gamma}_1 + \bar{\gamma}_3, \tag{3.32}$$

where

$$\gamma_i = H_i/H (i = 1, 3),$$

and a bar ($^-$) means the steady state growth value. In view of (3.32) and (3.32), γ_1 and γ_3 are expressed as functions of $\bar{k}$ and $\bar{\omega}$.

$$\bar{\gamma}_1 = \bar{\gamma}_1(\bar{k}, \bar{\omega}),\ \bar{\gamma}_3 = \bar{\gamma}_3(\bar{k}, \bar{\omega}). \tag{3.33}$$

The market clearing conditions for commodities 1 and 3 are given by

$$f_1(k_1(\bar{\omega})) H_1 = C_1 + I_P + NEX_1, \tag{3.34}$$
$$Q = g(k_3(\bar{\omega})) H_3 = I_H, \tag{3.35}$$

where I_P is physical capital investment, NEX_1 is the net export of commodity 1, and I_H is human capital investment. As the small country specializes in commodity 1, it exports only commodity 1. Moreover, no foreign asset is allowed to be accumulated, and hence NEX_1 is equal to the consumption of commodity 2. Because we do not consider any adjustment costs for investment, I_P and I_H imply an increase in each capital:

$$I_P = \dot{K},$$
$$I_H = \dot{H}. \tag{3.36}$$

As the production function satisfies constant returns to scale, its product is equal to the payment for its factor inputs. Thus, in sector 1 we have

$$Y_1 = RK_1 + VH_1, \tag{3.37}$$

where K_i is the stock of physical capital devoted to sector i ($i = 1, 3$). Similarly, the value of output in sector 3, qQ ($= q\dot{H}$ from (3.35) and (3.36)), equals the payment for its factor inputs:

$$q\dot{H} = RK_3 + VH_3. \tag{3.38}$$

Substituting equations (3.28), (3.37), and (3.38) into flow budget constraint (3.22) gives the dynamics of aggregate physical capital:

$$\dot{K} = Y_1 - C_1/\alpha.$$

Since in the steady state, physical capital and human capital grow at the same constant rate, the ratio of physical capital to human capital is constant, that is, $\dot{k} = 0$. We rewrite the above equation in per human capital terms using (3.33), (3.35), and (3.36) as

$$\dot{k} = \left[\bar{\gamma}_1\left(\bar{k}, \bar{\omega}\right) \cdot f_1\left(k_1\left(\bar{\omega}\right)\right) - \frac{\bar{c}_1}{\alpha}\right] - \bar{\gamma}_3\left(\bar{k}, \bar{\omega}\right) \cdot g\left(k_3\left(\bar{\omega}\right)\right)\bar{k} = 0, \tag{3.39}$$

where $c_1 = C_1/H$.

Substituting (3.10) and (3.31) into (3.29), we obtain the dynamic equation of aggregate consumption:

$$\dot{C}_1/C_1 = f_1'\left(k_1\left(\omega\right)\right) - \rho.$$

Since $\dot{c}_1/c_1 = \dot{C}_1/C_1 - \dot{H}/H$, $\dot{c}_1/c_1 = 0$ in the steady state. $\dot{H}/H$ is obtained from (3.33), (3.35), and (4.39). In consequence, the differential equation of c_1 in the steady state is expressed as

$$\dot{c}_1 = \bar{c}_1\left[f_1'\left(k_1\left(\bar{\omega}\right)\right) - \rho - \bar{\gamma}_3\left(\bar{k}, \bar{\omega}\right) \cdot g\left(k_3\left(\bar{\omega}\right)\right)\right] = 0. \tag{3.40}$$

Combining (3.31) with the steady state expression for the no-arbitrage condition (3.12), we obtain

$$f_1'\left(k_1\left(\bar{\omega}\right)\right) - \left[g\left(k_3\left(\bar{\omega}\right)\right) - k_3\left(\bar{\omega}\right)g'\left(k_3\left(\bar{\omega}\right)\right)\right] = 0. \tag{3.41}$$

(3.39), (3.40) and (3.41) constitute the steady state growth equilibrium conditions.

From (3.33), (3.35) and (4.39), the steady state growth rate of this economy is expressed as

$$\dot{H}/H = \bar{\gamma}_3\left(\bar{k}, \bar{\omega}\right) \cdot g\left(k_3\left(\bar{\omega}\right)\right).$$

Therefore, from equation (3.40) the growth rate of the steady state growth equilibrium is

$$f_1'(k_1(\bar{\omega})) - \rho, \tag{3.42}$$

which depends on only $\bar{\omega}$. Lemma 1 shows that the steady state growth level of ω is not influenced by the terms of trade $(1/p^w)$ and is always $\bar{\omega}$. Formally, we state:

Proposition 1 *As long as a country specializes in a capital commodity, which occurs when $p(\bar{\omega}) > p^w$, its growth rate is not influenced by the terms of trade.*

3.3.3 The Case of Perfect Specialization in a Consumption Commodity

From lemma 2, if $p(\bar{\omega}) < p^w$, a small country specializes in a consumption commodity and the steady growth level of ω $(\tilde{\omega})$ is a function of p^w. We can obtain similar optimal conditions and the market clearing conditions to the previous analysis in a straightforward way. From the proof of lemma 2, we obtain

$$k_2 = k_2(\tilde{\omega}), \ k_3 = k_3(\tilde{\omega}), \ q = q(\tilde{\omega}), \ k_i' > 0 \ (i = 2, 3),$$

where $k_2(\tilde{\omega})$, $k_3(\tilde{\omega})$ and $q(\tilde{\omega})$ are respectively obtained from (3.14) and (3.15). The optimal conditions of the household are the same as (3.28), (3.29) and (3.30) because its lifetime utility and the flow budget constraint are the same.

The market clearing conditions for physical and human capital can be written analogously to (3.32) and (3.32):

$$\tilde{k} = \tilde{\gamma}_2 k_2(\tilde{\omega}) + \tilde{\gamma}_3 k_3(\tilde{\omega}),$$
$$\tilde{\gamma}_2 + \tilde{\gamma}_3 = 1,$$

where a tilde ($\tilde{}$) means the steady state growth value and $\gamma_i = H_i/H$ $(i = 2, 3)$. From the above equations we can express $\tilde{\gamma}_i$ as functions of $\tilde{k}$ and $\tilde{\omega}$: $\tilde{\gamma}_2\left(\tilde{k}, \tilde{\omega}\right)$ and $\tilde{\gamma}_3\left(\tilde{k}, \tilde{\omega}\right)$.

The market clearing conditions for commodities 2 and 3 are

$$f_2(k_2(\tilde{\omega}))H_2 = C_2 + NEX_2,$$
$$g(k_3(\tilde{\omega}))H_3 = I_H,$$

where NEX_2 is the net export of commodity 2. Since the small country specializes in commodity 2, it exports only commodity 2. As no foreign asset is accumulated, NEX_2 is equal to the import of commodity 1 which is used for capital accumulation and consumption.

3.3.4 The Definition of Steady State Growth

Following the same procedure as the previous subsection, we obtain the steady state conditions of this economy[8]:

$$p^w \tilde{\gamma}_2 \left(\tilde{k}, \tilde{\omega}\right) \cdot f_2 \left(k_2 \left(\tilde{\omega}\right)\right) - \frac{\tilde{c}_1}{\alpha} - \tilde{\gamma}_3 \left(\tilde{k}, \tilde{\omega}\right) \cdot g \left(k_3 \left(\tilde{\omega}\right)\right) \tilde{k} = 0. \tag{3.43}$$

$$\tilde{c}_1 [p^w f_2' \left(k_2 \left(\tilde{\omega}\right)\right) - \rho - \tilde{\gamma}_3 \left(\tilde{k}, \tilde{\omega}\right) \cdot g \left(k_3 \left(\tilde{\omega}\right)\right)] = 0. \tag{3.44}$$

$$p^w f_2' \left(k_2 \left(\tilde{\omega}\right)\right) - [g \left(k_3 \left(\tilde{\omega}\right)\right) - k_3 \left(\tilde{\omega}\right) g' \left(k_3 \left(\tilde{\omega}\right)\right)] = 0. \tag{3.45}$$

The long-run growth rate,

$$\dot{H}/H = \tilde{\gamma}_3 \left(\tilde{k}, \tilde{\omega}\right) \cdot g \left(k_3 \left(\tilde{\omega}\right)\right),$$

can be expressed as

$$p^w f_2' \left(k_2 \left(\tilde{\omega}\right)\right) - \rho, \tag{3.46}$$

from (3.44). From (3.19), an increase (decrease) in p^w raises (lowers) the long-run growth rate. Since the small country exports commodity 2 and imports commodity 1, p^w stands for the terms of trade itself. Consequently, as the terms of trade improve, the growth rate rises.

Proposition 2 *As long as a country specializes in a consumption commodity, which occurs when $p(\tilde{\omega}) < p^w$, the growth rate is positively correlated with the terms of trade in the long run.*

The relationship between the terms of trade and the growth rates is depicted in Figure 1. The reason why the terms of trade do not affect the growth rate in one case while they do in the other case as follows. As indicated in equations (3.42) and (3.46), the growth rate in the steady state growth equilibrium is determined by the rent on physical capital R, which is equal to the value of the marginal product of physical capital in the operating sectors (see equations (3.10) and (3.14)). The higher R is, the more capital accumulation is promoted and the higher the growth rate is. In the case of perfect specialization in the consumption commodity, the improvement of the terms of trade raises the value of the product of sector 2 with the value of physical capital unchanged. This effect increases the value of the marginal product of capital in sector 2, $p^w f_2' (k_2)$ $(= R)$, as we show in the end of section 2. This effect increases the long-run growth rate.

In the case of perfect specialization in the capital commodity (sector 1), the input price (the price of physical capital) always equals the output price (the price of the product of

[8] Note that replacing $\bar{\gamma}_1 \left(\bar{k}, \bar{\omega}\right) f_1 \left(k_1 \left(\bar{\omega}\right)\right)$ and $f_1' \left(k_1 \left(\bar{\omega}\right)\right)$ of (3.39), (3.40) and (3.41) by $p^w \tilde{\gamma}_2 \left(\tilde{k}, \tilde{\omega}\right) f_2 \left(k_2 \left(\tilde{\omega}\right)\right)$ and $p^w f_2' \left(k_2 \left(\tilde{\omega}\right)\right)$ yields the following equations (3.43), (3.44) and (3.45).

sector 1). Therefore, the terms of trade do not affect the marginal product of physical capital in sector 1 (measured in terms of capital commodity), $f_1'(k_1)$ $(= R)$. Thus, capital accumulation is not affected.

The key that differentiates lemmata 1 and 2 or propositions 1 and 2 is whether a country produces a capital commodity or not. If a small country produces a capital commodity, the output price (the price of a capital commodity) is always kept equal to the input price even under a change in the terms of trade. If not, a change in the terms of trade influences only the output price (the price of a consumption commodity) with the input price unchanged, causing the marginal product of capital to increase and the growth rate to rise.[9]

3.4 Conclusion

This paper has developed a three-sector growth model of international trade with intertemporal optimizing behavior and endogenous physical and human capital accumulation. Like the one-sector neoclassical model, the two-sector neoclassical model has a positive growth rate along the transition path to the steady state (Baxter (1992)) but the growth rate becomes zero in the steady state. In our model, the economy has a positive growth rate not only along the transition path but also in the steady state growth equilibrium. Due to this property, we find that the effect of the terms of trade on the long-run growth rate drastically differs according to the trade pattern.

In addition, our model provides new insight into 'endogenous growth theory'. So far, the endogenous growth theory which considers human capital accumulation has emphasized enrollment of education quality or productivity of human capital accumulation as an engine of growth. However, from the discussion of this paper the terms of trade prescribe the growth rate of a country which specializes in a consumption commodity: the higher the terms of trade are, the higher the growth rate is, and vice versa. On the other hand, if it specializes in a capital commodity, the terms of trade do not affect the growth rate. Thus, an industrial policy which affects comparative advantage can cause a dramatic difference in growth: it derives either a steady state growth rate or a fluctuation of a growth rate.

[9] We should note that these asymmetric effects do not result from the choice of numeraire. When a consumption commodity is treated as numeraire and the relative price of commodity 1 is p, the no-arbitrage condition in a closed economy and the flow budget constraint in an open economy can be written as

$$\frac{\dot{p}}{p} + \frac{R}{p} = \frac{V}{q} + \frac{\dot{q}}{q},$$
$$p^w \dot{K} + q\dot{H} = RK + VH - p^w C_1 + C_2,$$

We can derive the same lemmata and propositions using the above equations.

Appendix

In this appendix, we treat the case where creditors refuse to lend to a small country beyond a certain ratio of physical capital and demonstrate that the same lemmata and propositions hold as before when the amount of capital that the small country borrows reaches the limit level.

Naturally only physical capital can be used as collateral for international borrowing. Thus, we assume that the household of the small country faces a credit constraint:

$$B \leq \beta K,$$

where B is a debt and β is a constant positive value.

As is well known, if ρ is large sufficiently, this economy chooses to accumulate an ever-growing stock of debt and eventually the constraint becomes binding on the world credit market (see Hamada (1967), Detragiache (1992) and Barro, Mankiw and Sala-i-Martin (1995)). Thus, we focus on this case.[10] When the amount of debt reaches the limit,

$$B = \beta K.$$

The household cannot accumulate foreign debt any more and pays interest on the accumulated debt. His modified flow budget constraint is

$$\dot{K} + q\dot{H} - \dot{B} = RK + VH - iB - C_1 - p^w C_2, \tag{A.1}$$

where i is the world interest.

Defining net total asset $K + qH - B$ as A^c, equation (A.1) is rewritten as

$$\dot{A}^c = RK + VH - C_1 - p^w C_2 - iB + \dot{q}H.$$

From the Hamiltonian of this problem:

$$\begin{aligned} L = {} & \alpha \ln C_1 + (1-\alpha)\ln C_2 + \lambda_1 \left(RK + VH - C_1 - p^w C_2 - iB + \dot{q}H\right) \\ & + \lambda_2 \left(A^c - K - qH + B\right) + \lambda_3 \left(\beta K - B\right), \end{aligned}$$

we can obtain the Euler equation and a no-arbitrage condition:

$$\frac{\dot{C}_1}{C_1} = R + \frac{\beta}{1-\beta}(R - i) - \rho. \tag{A.2}$$

$$R + \frac{\beta}{1-\beta}(R - i) = \frac{\dot{q}}{q} + \frac{V}{q}. \tag{A.3}$$

[10] If ρ is sufficiently small, a country would eventually accumulate enough assets to violate the small country assumption.

When the credit constraint binds, a no-arbitrage condition is modified. The left hand side of (A.3) indicates the rate of return on physical capital. It has a premium ($[\beta/(1-\beta)] \cdot (R-i)$), because only physical capital can be used as collateral. Instead of no-arbitrage condition (3.8), using (A.3) we can proceed the same proofs and have the same lemmata in subsection 3.2.2: If the autarky price of commodity 2 is higher than the world price, a small country specializes in capital commodity and the relative rental ratio ω stays at its autarky level. If the world price is higher than the autarky price, then a small country always specializes in sectors 2 and 3, and the relative rental ratio ω is positively correlated to the world price.[11]

In the following argument, we refer the case in which the autarky price of commodity 2 is higher than the world price as case 1 and the other case as case 2.

Case (1)

As sectors 1 and 3 operate, we obtain

$$
\begin{gathered}
f_1'(k_1) = qg'(k_3) = R, \\
f_1(k_1) - k_1 f_1'(k_1) = q\left[g(k_3) - k_3 g'(k_3)\right] = V.
\end{gathered}
$$

Since we have (A.1) instead of (3.22), using (3.4) we obtain the formula that determines $\dot{k}$:

$$
\dot{k} = \frac{1}{(1-\beta)}\left[\hat{\gamma}_1\left(\hat{k},\hat{\omega}\right) f_1\left(k_1\left(\hat{\omega}\right)\right) - i\beta\hat{k} - \frac{\hat{c}_1}{\alpha}\right] - \hat{\gamma}_3\left(\hat{k},\hat{\omega}\right) g\left(k_3\left(\hat{\omega}\right)\right)\hat{k} = 0, \tag{A.6}
$$

where a hat ($\hat{}$) means the steady state growth value. Making use of the Euler equation (A.2) and (A.6), we obtain

$$
\frac{\dot{c}_1}{\hat{c}_1} = \frac{1}{1-\beta} f_1'\left(k_1\left(\hat{\omega}\right)\right) - \frac{\beta}{1-\beta} i - \rho - \hat{\gamma}_3\left(\hat{k},\hat{\omega}\right) \cdot g\left(k_3\left(\hat{\omega}\right)\right) = 0. \tag{A.7}
$$

From (A.3) the steady state growth expression of no-arbitrage condition is

$$
f_1'\left(k_1\left(\hat{\omega}\right)\right) + \frac{\beta}{1-\beta}\left[f_1'\left(k_1\left(\hat{\omega}\right)\right) - i\right] - \left[g\left(k_3\left(\hat{\omega}\right)\right) - k_3\left(\hat{\omega}\right) g'\left(k_3\left(\hat{\omega}\right)\right)\right] = 0. \tag{A.8}
$$

Equations (A.6), (A.7) and (A.8) consist of the steady state growth conditions.

The growth rate of this economy is expressed as

$$
\frac{1}{1-\beta}\left[f_1'\left(k_1\left(\hat{\omega}\right)\right) - \beta i\right] - \rho,
$$

[11] In the appendix, we use the term 'autarky' to refer to the situation that only capital market is open.

which depends only $\hat{\omega}$. Thus the growth rate in this case is not influenced by the terms of trade, which proposition 1 states.

Case (2)

We can obtain the steady state growth conditions for this case by replacing $\hat{\gamma}_1\left(\hat{k}, \hat{\omega}\right) f_1\left(k_1\left(\hat{\omega}\right)\right)$ and $f_1'\left(k_1\left(\hat{\omega}\right)\right)$ by $p^w \gamma_2^*\left(k^*, \omega^*\right) f_2\left(k_2\left(\omega^*\right)\right)$ and $p^w f_2'\left(k_2\left(\omega^*\right)\right)$ respectively.

$$p^w \gamma_2^*\left(k^*, \omega^*\right) f_2\left(k_2\left(\omega^*\right)\right) - i\beta k^* - \frac{c_1^*}{\alpha} - (1-\beta)\,\hat{\gamma}_3\left(k^*, \omega^*\right) g\left(k_3\left(\omega^*\right)\right) k^* = 0.$$

$$\frac{1}{1-\beta} p^w f_2' k_2\left(\omega^*\right) - \beta i - \rho - \hat{\gamma}_3\left(k^*, \omega^*\right) \cdot g\left(k_3\left(\omega^*\right)\right) = 0.$$

$$p^w f_2'\left(k_2\left(\omega^*\right)\right) + \frac{\beta}{1-\beta}\left[p^w f_2'\left(k_2\left(\omega^*\right)\right) - i\right] - \left[g\left(k_3\left(\omega^*\right)\right) - k_3\left(\omega^*\right) g'\left(k_3\left(\omega^*\right)\right)\right] = 0.$$

An asterisk $(*)$ means the steady state growth value. Therefore, the long-run growth rate is $\frac{1}{1-\beta}\left[p^w f_2' k_2\left(\omega^*\right)\right] - \beta i - \rho$. Since $p^w f_2'\left(k_2\left(\omega^*\right)\right)$ is an increasing function of p^w, we derive proposition 2.

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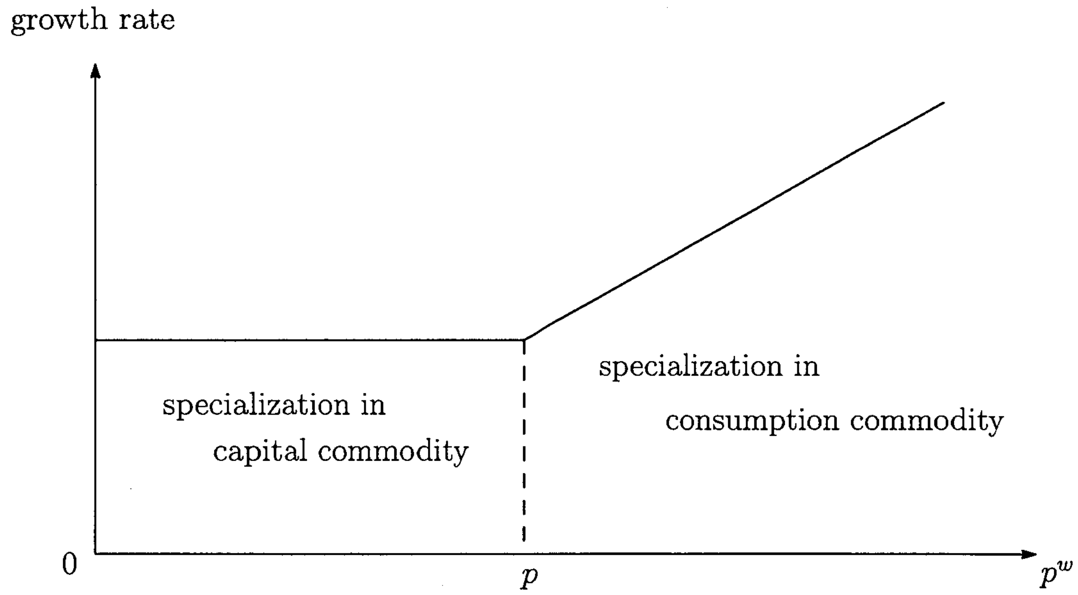

Figure 1

Chapter 4

Tariffs, Production Taxes and the Growth Rate in a Small Open Economy

Abstract

Incorporating human capital accumulation into a dynamic trade model, we analyze the relationship between tax policies (specifically, tariffs and production taxes) and the growth rate. After opening trade, a small country must specialize in one of traded commodities. If a small country specializes in a capital commodity, tariffs do not affect the long-run growth rate. In contrast, if it specializes in a consumption commodity, tariffs reduce the long-run growth rate. Whichever commodity it specializes in, production taxes decrease the long-run growth rate.

4.1 Introduction

In chapter 3, we examined the effect of the terms of trade on the long-run growth rate. In this chapter, we study the effects of several policies on the long-run growth rate.

The effect of trade policies (e.g., tariffs and production taxes) on the growth rate has been pointed out by several studies, but only a few theoretical analyses have been made.[1] In a small country model, the static trade theory shows that a rise in a tariff must reduce real income. However, it cannot investigate the 'growth' effect but only the 'level' effect on income. In this paper, we introduce a human capital accumulation sector into a dynamic trade model of a small economy and investigate the relationship between the trade policies and growth.

[1] Using cross-section data of countries, Barro and Sala-i-Martin (1995) find a negative relationship between tariffs and growth.

A number of literature has discussed the effects of tariffs on both growth and welfare. Grossman and Helpman (1991 ch.6) examine the relationship between tariffs and long-run growth in an R&D growth model. Since the final production sectors employ different factor inputs in their model, incomplete specialization occurs after opening trade. In contrast, in our model the same factor inputs are employed and opening trade, therefore, causes complete specialization. Osang and Pereira (1996) use a very similar model to ours – two consumable commodities, human capital accumulation and a balanced trade account in a small country – and demonstrate that a tariff on imports of the consumption commodity has no impact on steady-state growth and that a tariff on the capital commodity reduces growth. Dinopoulos and Syropoulos (1997) examine the effect of tariffs on both growth and welfare in a multi-country Schumpeterian growth model. They find that a reduction in the tariff rate increases the long-run growth under certain conditions. In Osang and Pereia's model, the small country always imports both the commodities for pure consumption and investment because they assume that both capital and consumption commodities are composites of domestic and foreign commodities. In our model, we do not adopt such an assumption and the small country must specialize perfectly. As the production structure is Ricardian in Dinopoulos and Syropoulos' model, they do not focus on the determination of the specialization. In our model, we show that the determinants of specialization pattern and the effect of tariff policies differs according to the specialization pattern.

In this paper, we use the model presented in the previous chapter – a human capital accumulation model with two factors and two commodities. Using this model, we show that given the world price and the tariff rate, a small country must specialize either in a capital commodity or in a consumption commodity. Moreover, we obtain the following results: 1) When the small country specializes in a capital commodity, tariffs do not affect the long-run growth rate. 2) When the small country specializes in a consumption commodity, tariffs decrease the long-run growth rate. 3) Whichever commodity the small country specializes in, production taxes decrease the long-run growth rate.

The plan of this paper is as follows. In section 4.2, we examine which specialization pattern obtains depending on the world price and the tariff. In subsection 4.3, we investigate the effect of tariff policies on the growth rates under each specialization pattern. In subsection 4.4, we show that production taxes always reduce the long-run growth rate. In the last section, we discuss the contributions of this paper.

4.2 Specialization Patterns

In this section, we discuss the determination of specialization pattern in a small country. The procedure to show the determinants of specialization is similar to the one in chapter 3.

We first describe the behavior of firms and a no-arbitrage condition in a closed economy. Then, we investigate which specialization pattern obtains after opening trade.

4.2.1 Firms Behavior and a No-arbitrage Condition

To investigate the determination of specialization pattern in the next subsection, we show the optimal behavior of firms and a no-arbitrage condition in a closed economy.

Except for the existence of government, the structure of economy is the same as in chapter 3. Three sectors operate in an economy. Sector 1 produces a commodity which is used for consumption and investment. We call it a capital commodity. Sector 2 produces a commodity which is used for only consumption. We call it a consumption commodity. Sector 3 produces a human capital commodity. The production functions of all sectors exhibit constant returns to scale:

$$\begin{aligned} Y_1 &= f_1\left(k_1\right) H_1, \\ Y_2 &= f_2\left(k_2\right) H_2, \\ Q &= g\left(k_3\right) H_3, \end{aligned}$$

where Y_i is the output of commodity i, Q the output of the human capital commodity, k_i the ratio of physical capital to human capital employed in the ith sector, and H_i the stock of human capital devoted to the ith sector. $f_i\left(k_i\right)$ and $g\left(k_3\right)$ satisfy the standard neoclassical assumptions and Inada conditions.

As a result of each firm's competitive behavior, the value of the marginal product of each factor input is equalized to its rent. Thus, letting p the price of commodity 2, q the price of the human capital commodity, R the rent on physical capital, and V the rent on human capital, we have

$$f_1'\left(k_1\right) = p f_2'\left(k_2\right) = q g'\left(k_3\right) = R, \tag{4.1}$$

$$\begin{aligned} f_1\left(k_1\right) - k_1 f_1'\left(k_1\right) &= p\left[f_2\left(k_2\right) - k_2 f_2'(k_2)\right] \\ &= q\left[g\left(k_3\right) - k_3 g'\left(k_3\right)\right] = V, \end{aligned} \tag{4.2}$$

where commodity 1 is taken as numeraire. From (4.1) and (4.2), we obtain each variable as a function of only the relative rental ratio, $\omega\,(\equiv V/R)$:

$$k_i = k_i\left(\omega\right), \tag{4.3}$$

$$k_i' \equiv \frac{dk_i}{d\omega} > 0 \quad (i = 1, 2, 3). \tag{4.4}$$

From (4.1) and (4.2), we also have

$$p = \frac{f_1'\left(k_1\left(\omega\right)\right)}{f_2'(k_2\left(\omega\right))} = p\left(\omega\right), \tag{4.5}$$

$$q = \frac{f_1'\left(k_1\left(\omega\right)\right)}{g'(k_3\left(\omega\right))} = q\left(\omega\right), \tag{4.6}$$

$$R = R\left(\omega\right), \quad V = V\left(\omega\right). \tag{4.7}$$

In this economy there are two kinds of capital, K (physical capital) and H (human capital). Thus, a no-arbitrage condition must always hold[2]:

$$R = \frac{V}{q} + \frac{\dot{q}}{q}. \tag{4.8}$$

A dot ($\dot{}$) means a time derivative.

4.2.2 Determination of Specialization Patterns

In this subsection, we demonstrate that perfect specialization occurs in a small country after opening up to trade.

We assume that human capital commodity is non-tradeable and that there is no international financial market as in chapter 3. The government imposes a tariff on a commodity which the small country exports.[3] We shall call the relative price in an open economy the domestic price. Under these assumptions, the following two lemmata hold.

Lemma 3 *Suppose that a small country is in a steady state growth equilibrium after opening trade. If sectors 1 and 3 operate, then the value of the marginal product of physical capital is also that which emerges under autarky. Furthermore, only sectors 1 and 3 operate if and only if the price of commodity 2 under autarky is higher than the domestic price of commodity 2 after opening trade.*

Proof.

First, we obtain the relative rental ratio, the rent for physical and human capital and the price of commodity 2 of the closed economy. Since q stays constant ($\dot{q} = 0$) in the steady state growth equilibrium,[4] from (4.6), (4.7) and (4.8) we have

$$R(\omega) = V(\omega)/q(\omega). \tag{4.9}$$

From the above equation, we obtain the steady state growth level of the relative rental ratio under autarky (which we shall call $\bar{\omega}$). In addition, (4.5) and (4.7) give $R(\bar{\omega})$, $V(\bar{\omega})$, $p(\bar{\omega})$. $p(\bar{\omega})$ is the price of commodity 2 under autarky.

Now, the country opens up to trade. Since there is not the international financial market, the household does not accumulate foreign assets and the same no-arbitrage condition as (4.8) holds. We let τ_1 represent the ad valorem rate of the export tariff. When sectors 1 and 3 operate after opening trade, the domestic price of commodity 2

---

2 As pointed out in chapter 3, this condition is derived from the household's optimization formally

3 It is not necessary to analyzes the effect of import taxes separately. Lerner symmetry theorem implies the equivalence of taxation on imports and on exports.

4 We assume that the conditions for saddle stable are satisfied (see Mino (1996) for detail).

is set at $(1+\tau_1)p^w$, where p^w is the world price of commodity 2. We shall define the following expression:

$$p^{\tau_1} \equiv (1+\tau_1)p^w. \tag{4.10}$$

The conditions of the profit maximization are written as

$$f_1'(k_1) = qg'(k_3) = R, \tag{4.11}$$
$$f_1(k_1) - k_1 f_1'(k_1) = q\left[g(k_3) - k_3 g'(k_3)\right] = V. \tag{4.12}$$

From (4.11) and (4.12), we know that $k_i \ \ (i=1,3)$, q, R and V are functions of ω.

$$k_i = k_i(\omega), q = q(\omega), R = R(\omega), V = V(\omega), k_i' > 0, \ \ (i=1,3). \tag{4.13}$$

Since the above functions are obtained from the profit maximization of sectors 1 and 3, they are the same functions as (4.3), (4.6) and (4.7). In the steady state growth equilibrium, from no-arbitrage condition (4.8) and (4.13) we obtain

$$R(\omega) = V(\omega)/q(\omega). \tag{4.14}$$

Since (4.14) is the same function as (4.9), when the small country specializes in sectors 1 and 3, ω in the steady state growth equilibrium is equal to $\bar{\omega}$. Thus, from (4.13) the value of the marginal product of physical capital, $f_1'(k_1)$, is equal to $f_1'(k_1(\bar{\omega}))$ which is the one under autarky. It is not influenced by a change in the domestic price of commodity 2, p^{τ_1}, or a change in the tariff rate.

Finally, we show that sectors 1 and 3 operate in the steady state growth equilibrium when the autarky price of commodity 2 is larger than the domestic price of commodity 2 after opening trade. The domestic price of commodity 2 is set at p^{τ_1}. If $p^{\tau_1} < p(\bar{\omega})$, the following equations hold from (4.1) and (4.3).

$$f_1'(k_1(\bar{\omega})) = p(\bar{\omega}) f_2'(k_2(\bar{\omega})) > p^{\tau_1} f_2'(k_2(\bar{\omega})). \tag{4.15}$$

(4.15) implies that sector 2 cannot operate and hence only sectors 1 and 3 operate. ■

Lemma 4 *Suppose that a small country is in a steady state growth equilibrium after opening trade. If sectors 2 and 3 operate, then the value of the marginal product of physical capital is positively correlated to the domestic price of commodity 2. Furthermore, only sectors 2 and 3 operate if and only if the domestic price of commodity 2 is higher than the price of commodity 2 under autarky.*

Proof.

Suppose that sectors 2 and 3 operate. We let τ_2 represent the ad valorem rate of the export tariff. The domestic price of commodity 2 becomes $[1/(1+\tau_2)]p^w$ because of the export tariff. We shall define the following expression:

$$p^{\tau_2} \equiv \frac{1}{1+\tau_2} p^w. \tag{4.16}$$

The profit maximizing conditions of sectors 2 and 3 are

$$p^{\tau_2} f_2' (k_2) = qg'(k_3) = R. \tag{4.17}$$

$$p^{\tau_2} [f_2 (k_2) - k_2 f_2'(k_2)] = q [g (k_3) - k_3 g' (k_3)] = V. \tag{4.18}$$

From the above conditions, we know that $k_i \, (i = 2, 3)$, q, R and V are functions of only ω.

The household cannot accumulate foreign assets. Thus, the no-arbitrage condition in the steady state growth equilibrium is $R = V/q$. From (4.17) and (4.18), we have

$$p^{\tau_2} f_2' (k_2 (\omega)) = g (k_3 (\omega)) - k_3 (\omega) g' (k_3 (\omega)) . \tag{4.19}$$

Given the domestic price of commodity 2, p^{τ_2}, (4.19) determines the steady state growth level of the relative rental ratio, ω, (which we shall call $\tilde{\omega}$):

$$\tilde{\omega} = \tilde{\omega}(p^{\tau_2}) . \tag{4.20}$$

Totally differentiating (4.19), we have

$$\frac{d\omega}{dp^{\tau_2}} = -\frac{f_2'}{p^{\tau_2} f_2'' k_2' + k_3 g'' k_3'} > 0, \tag{4.21}$$

which means that ω is an increasing function of p^{τ_2}. Furthermore, concerning (4.21), (4.19) gives

$$\begin{aligned} \frac{dp^{\tau_2} f_2' (k_2 (\tilde{\omega} (p^{w})))}{dp^{\tau_2}} &= \frac{d[g (k_3 (\omega)) - k_3 (\omega) g' (k_3 (\omega))]}{d\omega} \frac{d\omega}{dp^{\tau_2}} \\ &= -k_3 g'' k_3' \cdot \frac{d\omega}{dp^{\tau_2}} > 0. \end{aligned} \tag{4.22}$$

The value of the marginal product of physical capital in sector 2, $p^{\tau_2} f_2' (k_2 (\tilde{\omega} (p^{\tau_2})))$, is also an increasing function of p^{τ_2} in the steady state growth equilibrium.

Finally, we show that if $p^{\tau_2} > p (\bar{\omega})$, the small country specializes in sectors 2 and 3. From (4.19) and (4.20),

$$\tilde{\omega} (p (\bar{\omega})) = \bar{\omega}. \tag{4.23}$$

That is, when the domestic price of commodity 2 happens to be equal to the autarky price of commodity 2, the relative rental ratio is equal to the autarky one. From (4.22), (4.23), $k_1' > 0$ and $f_1'' < 0$, if $p^{\tau_2} > p (\bar{\omega})$, then

$$p^{\tau_2} f_2' (k_2 (\tilde{\omega} (p^{\tau_2}))) > p (\bar{\omega}) f_2' (k_2 (\bar{\omega})) = f_1' (k_1 (\bar{\omega})) > f_1' (k_1 (\tilde{\omega})) \tag{4.24}$$

holds and therefore sector 1 cannot operate. ■

These lemmata show that when the government imposes a tariff, the relative price of commodity faced by the firm is crucial in the determination of specialization pattern. This is another aspect of the dichotomy pointed out in chapter 3. A tariff also changes the relative price faced by a household. However, since the dichotomy between the production side and the demand side holds, we do not need to consider the change of demand caused by the tariff.

Moreover, we find that if we allow an export subsidy (a negative tariff), the government can alter the specialization pattern dramatically. But, in this paper we do not argue the government's intervention in the specialization pattern. We only discuss the changes in the long-run growth caused by tax policies.

4.3 Tariffs and the Long-run Growth Rates

In this subsection, we discuss the relationship between tariffs and the growth rate under each specialization pattern. We also introduce a tariff on a commodity which the small country exports.

4.3.1 Specialization in a Capital Commodity

We describe here the conditions of the steady state growth equilibrium and examine the relationship between tariff policies and the long-run growth rate under specialization in a capital commodity. As the small country exports the capital commodity, the domestic price of commodity 2 is p^{τ_1}.

Firms Behavior

The profit maximization conditions for the operating sectors are

$$f_1'(k_1) = qg'(k_3) = R, \tag{4.25}$$

$$f_1(k_1) - k_1 f_1'(k_1) = q\left[g(k_3) - k_3 g'(k_3)\right] = V. \tag{4.26}$$

The above equations show that $k_i\,(i = 1, 3)$, R, V and q are functions of ω. From lemma 1, the steady state growth level of ω is not influenced by the tariff rate and remains $\bar{\omega}$. Thus, the steady state growth level of $k_i\,(i = 1, 3)$, R, V and q are expressed as below:

$$k_i = k_i(\bar{\omega}),\ R = R(\bar{\omega}),\ V = V(\bar{\omega}),\ q = q(\bar{\omega}),\ k_i' > 0, \quad (i = 1, 3). \tag{4.27}$$

Households Behavior

The household has a log-liner instantaneous utility function and maximizes its lifetime utility. It receives the rent from both physical and human capital and spends on consumption and accumulation of two kinds of capital. Formally,

$$\max U = \int_0^\infty [\alpha \ln C_1 + (1-\alpha)\ln C_2] \exp(-\rho t)\, dt,$$

subject to the flow budget constraint:

$$\dot{K} + q\dot{H} = RK + VH - C_1 - p^{\tau_1} C_2 + T, \tag{4.28}$$

where C_i $(i = 1, 2)$ represents the consumption level of commodity i, K the total physical capital, and H the total human capital and T the lump-sum transfer from the government. Defining A as total assets $(K + qH)$, the flow budget constraint can be rewritten as

$$\dot{A} = RK + VH - C_1 - p^{\tau_1} C_2 + \dot{q}H + T.$$

From the Hamiltonian function of this problem:

$$\begin{aligned} J = {} & \alpha \ln C_1 + (1-\alpha)\ln C_2 \\ & + \lambda_1 (RK + VH - C_1 - p^{\tau_1} C_2 + \dot{q}H + T) + \lambda_2 (A - K - qH), \end{aligned}$$

we derive the first order conditions for interior solutions as in chapter 3.

$$p^{\tau_1} = \frac{1-\alpha}{\alpha} \cdot \frac{C_1}{C_2}. \tag{4.29}$$

$$\frac{\dot{C}_1}{C_1} = R - \rho. \tag{4.30}$$

$$R = \frac{V}{q} + \frac{\dot{q}}{q}. \tag{4.31}$$

(4.29) represents the equality between the intratemporal marginal rate of substitution and the relative price, (4.30) the Euler equation, and (4.31) the no-arbitrage condition.

The transversality condition is

$$\lim_{t \to \infty} \lambda_1 A \exp(-\rho t) = 0. \tag{4.32}$$

Market Clearing Conditions

From lemma 1, we know that the level of the relative rental ratio is $\bar{\omega}$ in the steady state growth equilibrium. When the small country is in the steady state growth equilibrium, the market clearing conditions for physical and human capital are

$$\bar{k} = \bar{\gamma}_1 k_1(\bar{\omega}) + \bar{\gamma}_3 k_3(\bar{\omega}), \tag{4.33}$$

$$1(= H/H) = \bar{\gamma}_1 + \bar{\gamma}_3, \tag{4.34}$$

where a bar ($^-$) means the steady state growth value, $k \equiv K/H$, and $\gamma_i \equiv H_i/H \ \ (i = 1, 3)$. From (4.33) and (4.34), $\bar{\gamma}_1$ and $\bar{\gamma}_3$ are functions of $\bar{k}$ and $\bar{\omega}$:

$$\bar{\gamma}_1 = \bar{\gamma}_1\left(\bar{k}, \bar{\omega}\right), \bar{\gamma}_3 = \bar{\gamma}_3\left(\bar{k}, \bar{\omega}\right). \tag{4.35}$$

The market clearing conditions for commodities 1 and 3 are

$$f_1\left(k_1\left(\bar{\omega}\right)\right) H_1 = C_1 + I_P + NEX_1, \tag{4.36}$$
$$Q = g\left(k_3\left(\bar{\omega}\right)\right) H_3 = I_H, \tag{4.37}$$

where I_P is physical capital investment, NEX_1 the net export of commodity 1, and I_H human capital investment. Since there is no adjustment cost for investment, I_P and I_H respectively imply the increments of physical and human capitals:

$$I_P = \dot{K}, \tag{4.38}$$
$$I_H = \dot{H}. \tag{4.39}$$

Government

The government remits the tax revenue in a lump-sum fashion to the household and its budget always balances:

$$\tau_1[f_1\left(k_1\left(\omega\right)\right) H_1 - C_1 - I_P] = T. \tag{4.40}$$

As no foreign asset is accumulated, the net export of commodity 1 equals the consumption of commodity 2:

$$f_1\left(k_1\left(\omega\right)\right) H_1 - C_1 - I_P = p^w C_2. \tag{4.41}$$

Thus, (4.40) is rewritten as

$$\tau_1 \cdot p^w C_2 = T. \tag{4.42}$$

the Steady State Growth Equilibrium

Next, we describe the steady state growth equilibrium. As the production functions of sectors 1 and 3 exhibit constant returns to scale, the payment to the factor inputs is always equal to the value of the product:

$$Y_1 = RK_1 + VH_1. \tag{4.43}$$

From (4.37) and (4.39)

$$q\dot{H} = RK_3 + VH_3. \tag{4.44}$$

Introducing (4.33), (4.34), (4.42), (4.43) and (4.44) into the flow budget constraint (4.28), we obtain

$$\dot{K} = Y_1 - C_1 - p^w C_2. \tag{4.45}$$

We rewrite the above equation in per human capital terms using (4.35), (4.37), and (4.39) as

$$\dot{k} = \bar{\gamma}_1\left(\bar{k}, \bar{\omega}\right) \cdot f_1\left(k_1\left(\bar{\omega}\right)\right) - \frac{\alpha\tau_1 + 1}{\alpha(\tau_1 + 1)}\bar{c}_1 - \bar{\gamma}_3\left(\bar{k}, \bar{\omega}\right) \cdot g\left(k_3\left(\bar{\omega}\right)\right)\bar{k} = 0. \tag{4.46}$$

Using (4.11) and (4.27), the Euler equation (4.30) gives

$$\dot{C}_1/C_1 = f_1'\left(k_1\left(\bar{\omega}\right)\right) - \rho. \tag{4.47}$$

Since $\dot{c}_1/c_1 = \dot{C}_1/C_1 - \dot{H}/H$, in the steady state growth equilibrium using (4.35), (4.37) and (4.39) the above equation is rewritten as

$$\dot{c}_1 = \bar{c}_1\left[f_1'\left(k_1\left(\bar{\omega}\right)\right) - \rho - \bar{\gamma}_3\left(\bar{k}, \bar{\omega}\right) \cdot g\left(k_3\left(\bar{\omega}\right)\right)\right] = 0. \tag{4.48}$$

From (4.14) and (4.27), the steady state growth expression of the no-arbitrage condition is

$$f_1'\left(k_1\left(\bar{\omega}\right)\right) - \left[g\left(k_3\left(\bar{\omega}\right)\right) - k_3\left(\bar{\omega}\right)g'\left(k_3\left(\bar{\omega}\right)\right)\right] = 0. \tag{4.49}$$

(4.46), (4.48) and the above equation are the conditions of the steady state growth equilibrium.

From (4.35) and (4.37), the long-run growth rate of this economy is

$$\dot{H}/H = \bar{\gamma}_3\left(\bar{k}, \bar{\omega}\right) \cdot g\left(k_3\left(\bar{\omega}\right)\right).$$

Using (4.48), we can express the above equation as $f_1'\left(k_1(\bar{\omega})\right) - \rho$. The growth rate is the difference between the value of the marginal product of physical capital and the subjective discount rate. Lemma 1 shows that a change in the domestic price of commodity 2 or the tariff rate does not affect the steady state growth level of the value of the marginal product of physical capital under specialization in a capital commodity. Thus, the tariff policy does not influence the long-run growth rate.

Proposition 3 *If a small country specializes in a capital commodity, a tariff policy does not affect the long-run growth rate.*

4.3.2 Specialization in a Consumption Commodity

We consider here the case in which the small country specializes in a consumption commodity. As the small country exports the consumption commodity, the domestic price of commodity 2 is $[1/(1 + \tau_2)] \cdot p^w (\equiv p^{\tau_2})$, where τ_2 represents the ad valorem rate of the export tariff.

Firms Behavior

Due to the export tariff, the profit maximizing conditions of each firms become

$$p^{\tau_2} f_2'(k_2) = qg'(k_3) = R, \tag{4.50}$$

$$p^{\tau_2}[f_2(k_2) - k_2 f_2'(k_2)] = q[g(k_3) - k_3 g'(k_3)] = V. \tag{4.51}$$

As shown in lemma 2, k_i $(i = 2, 3)$, R, V, and q depend on only ω and the steady state growth value of ω is defined as $\tilde{\omega}$. The steady state growth value of each variable is expressed as

$$k_i = k_i(\tilde{\omega}), \; R = R(\tilde{\omega}), \; V = V(\tilde{\omega}), \; q = q(\tilde{\omega}), \; k_i' > 0, \quad (i = 2, 3). \tag{4.52}$$

Household's Behavior

Since the domestic consumer price of commodity 2 is p^{τ_2}. The flow budget constraint of the household is

$$\dot{K} + q\dot{H} = RK + VH - C_1 - p^{\tau_2} C_2 + T. \tag{4.53}$$

The first order conditions of utility maximization are

$$p^{\tau_2} = \frac{1-\alpha}{\alpha} \cdot \frac{C_1}{C_2}, \tag{4.54}$$

$$\frac{\dot{C}_1}{C_1} = R - \rho, \tag{4.55}$$

$$R = \frac{V}{q} + \frac{\dot{q}}{q}. \tag{4.56}$$

$$\lim_{t \to \infty} \lambda A \exp(-\rho t) = 0. \tag{4.57}$$

Market Clearing Conditions

The market clearing conditions for physical and human capital are analogous to (4.33) and (4.34):

$$\tilde{k} = \tilde{\gamma}_2 k_2(\tilde{\omega}) + \tilde{\gamma}_3 k_3(\tilde{\omega}), \tag{4.58}$$

$$\tilde{\gamma}_2 + \tilde{\gamma}_3 = 1, \tag{4.59}$$

where $\gamma_2 \equiv H_2/H$. From the above equations $\tilde{\gamma}_i$ $(i = 2, 3)$ are functions of $\tilde{k}$ and $\tilde{\omega}$:

$$\tilde{\gamma}_2 = \tilde{\gamma}_2\left(\tilde{k}, \tilde{\omega}\right), \tilde{\gamma}_3 = \tilde{\gamma}_3\left(\tilde{k}, \tilde{\omega}\right). \tag{4.60}$$

The market clearing conditions for commodities 2 and 3 are

$$f_2(k_2(\tilde{\omega})) H_2 = C_2 + NEX_2, \tag{4.61}$$

$$g(k_3(\tilde{\omega})) H_3 = I_H. \tag{4.62}$$

where NEX_2 represents the net export of commodity 2.

Government

The government always remits its tax revenue to the households in a lump-sum manner:

$$(p^w - p^{\tau_2}) \cdot NEX_2 = (1 - \frac{1}{1+\tau_2})p^w (f_2(k_2(\omega))H_2 - C_2) = T. \tag{4.63}$$

the Steady State Growth equilibrium

Proceeding the same procedure as subsection 3.1, we obtain the conditions of the steady state growth equilibrium:

$$p^w \tilde{\gamma}_2 \left(\tilde{k}, \tilde{\omega}\right) \cdot f_2 \left(k_2 \left(\tilde{\omega}\right)\right) - \frac{1+\tau_2(1-\alpha)}{\alpha} \tilde{c}_1 - \tilde{\gamma}_3 \left(\tilde{k}, \tilde{\omega}\right) \cdot g \left(k_3 \left(\tilde{\omega}\right)\right) \tilde{k} = 0. \tag{4.64}$$

$$p^{\tau_2} f_2' \left(k_2 \left(\tilde{\omega}\right)\right) - \rho - \tilde{\gamma}_3 \left(\tilde{k}, \tilde{\omega}\right) \cdot g \left(k_3 \left(\tilde{\omega}\right)\right) = 0. \tag{4.65}$$

$$p^{\tau_2} f_2' \left(k_2 \left(\tilde{\omega}\right)\right) - \left\{g \left(k_3 \left(\tilde{\omega}\right)\right) - k_3 \left(\tilde{\omega}\right) g' \left(k_3 \left(\tilde{\omega}\right)\right)\right\} = 0. \tag{4.66}$$

From (4.65), the steady state growth rate of the economy is

$$\dot{H}/H = \tilde{\gamma}_3 \left(\tilde{k}, \tilde{\omega}\right) \cdot g \left(k_3 \left(\tilde{\omega}\right)\right) = p^{\tau_2} f_2' \left(k_2 \left(\tilde{\omega}\right)\right) - \rho, \tag{4.67}$$

which shows that the long-run growth rate depends on the tariff. How does a change in the tariff influences the long-run growth rate? A rise in the tariff rate lowers the domestic relative price p^{τ_2}. From lemma 2, we know that a decline in the domestic price of commodity 2 decreases the value of the marginal product of physical capital in sector 2, $p^{\tau_2} f_2' \left(k_2 \left(\tilde{\omega}\right)\right)$ and hence from (4.67) a rise in the tariff reduces the long-run growth rate. Formally, we state

Proposition 4 *When a small country specializes in a consumption commodity, a rise in a tariff lowers the long-run growth rate.*

The key that differentiates propositions 1 and 2 is the following. As is well known, the effect of a tariff can be decomposed into the effect of a production tax on an export industry and the effect of a consumption tax on an import commodity. When a small country specializes in a capital commodity, a tariff on the export commodity (capital commodity) means both a production tax on the capital commodity sector (sector 1) and a consumption tax on the consumption commodity, that is, a subsidy on the purchase of the capital commodity. Thus, a decline in the value of the product of sector 1 (a decline in the output price) caused by the production tax and a decline in the price of capital commodity (a decline in the input price) caused by the subsidy offset each other, and the value of the marginal product of physical capital remains unchanged. As in (4.47), the steady state growth rate is determined by the value of the marginal product of physical capital. Thus, the long-run growth rate is not affected by the tariff policy.

On the other hand, under specialization in a consumption commodity an export tariff corresponds to a production tax on the consumption commodity sector (sector 2) coupled with a consumption tax on the capital commodity. Thus, a rise in the tariff decreases the value of the product of sector 2 (the output price) and increases the price of the capital commodity (the input price), which makes the value of the marginal product of physical capital lower. As in (4.47), the lower the value of the marginal product of physical capital, the less capital accumulation is promoted. Thus, the long-run growth rate decreases.

To put it another way, when a small country produces a commodity used as a production factor, a tariff does not change the long-run growth rate. When it produces a pure consumption commodity, a tariff decreases the long-run growth rate.

4.4 A Production Tax and the Long-run Growth Rate

For comparison, we analyze the influence of production taxes on growth rates under specialization in a capital commodity.[5] The analysis of the production tax makes the economic implication of propositions 1 and 2 clear. A tax is levied on the production of the tradeable commodity.[6] Let t_1 denote the ad valorem rate of the production tax, so that sector 1 receives $(1-t_1)$ for every unit of output that it sells. The profit maximization conditions of sectors 1 and 3 are

$$(1-t_1)f_1'(k_1) = qg'(k_3) = R, \tag{4.68}$$

$$(1-t_1)[f_1(k_1) - k_1 f_1'(k_1)] = q[g(k_3) - k_3 g'(k_3)] = V, \tag{4.69}$$

from which we know that $k_i\ (i = 1, 3)$ and q are functions of only ω.

$$k_1 = k_1(\omega^*), k_3 = k_3(\omega^*), q = q(\omega^*), k_i' > 0 \quad (i = 1, 3). \tag{4.70}$$

An asterisk $(*)$ means the steady state growth value.

The production tax does not influence the domestic prices of commodities which the households face.

$$\dot{K} + q\dot{H} = RK + VH - C_1 - p^w C_2 + T. \tag{4.71}$$

The government's receipt from the production tax on sector 1 is transferred to the household in lump-sum fashion. The government budget is balanced at every moment.

$$t_1 f_1(k_1) = T. \tag{4.72}$$

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5 For the sake of brevity, we do not consider the case in which the small country specializes in a consumption commodity. By a procedure similar to the following one, we find that the production tax would also reduce the long-run growth rate under specialization in a consumption commodity.

6 We set this assumption to compare the effect of the production taxes with that of tariffs easily. The production tax levied on the production of sector 3 also reduces the long-run growth rate.

The market clearing conditions for commodities and two kinds of capital are the same as those in subsection 4.3.1. By an analysis which is analogous to the one of subsection 4.3.1, we can obtain the conditions for the steady state growth equilibrium:

$$\left[\gamma_1^*\left(k^*, \omega^*\right) \cdot f_1\left(k_1\left(\omega^*\right)\right)-\frac{c_1^*}{\alpha}\right]-\gamma_3^*\left(k^*, \omega^*\right) \cdot g\left(k_3\left(\omega^*\right)\right) k^*=0. \tag{4.73}$$

$$(1-t_1) f_1'\left(k_1\left(\omega^*\right)\right)-\rho-\gamma_3^*\left(k^*, \omega^*\right) \cdot g\left(k_3\left(\omega^*\right)\right)=0. \tag{4.74}$$

$$(1-t_1) f_1'\left(k_1\left(\omega^*\right)\right)-\left[g\left(k_3\left(\omega^*\right)\right)-k_3\left(\omega^*\right) g'\left(k_3\left(\omega^*\right)\right)\right]=0. \tag{4.75}$$

From (4.74), the long-run growth rate of this economy is

$$\dot{H}/H=\gamma_3\left(k^*, \omega^*\right) \cdot g\left(k_3\left(\omega^*\right)\right)=(1-t_1) f_1'\left(k_1\left(\omega^*\right)\right)-\rho.$$

Since the value of the marginal product of physical capital, $(1-t_1)f_1'\left(k_1\left(\omega^*\right)\right)$, is a decreasing function of t_1, the production tax lowers the long-run growth rate.[7]

Proposition 5 *When a small country specializes in a capital commodity, a production tax lowers the long-run growth rate.*

As stated in proposition 1, a tariff does not affect the long-run growth rate under specialization in a capital commodity. In contrast, a rise in the production tax reduces the long-run growth rate. Recall that the effect of a tariff consists of the effect of a production tax and that of a consumption tax. It makes a difference between propositions 1 and 3. Even under specialization in the capital commodity, the production tax reduces the value of the product of sector 1 with the price of the capital commodity unchanged. Thus, the value of the marginal product of physical capital decreases and the long-run growth rate lowers.[8]

4.5 Conclusion

In this paper, we analyze the effect of trade policies on growth rates in a dynamic trade model with human capital accumulation. If a small country specializes in a capital commodity, tariffs have no effect on the growth rate. If it specializes in a consumption

[7] From (4.75)

$$\frac{d\omega}{dt_1}=\frac{f_1'}{(1-t_1)f_1''k_1'+k_3g''k_3'}<0.$$

Concerning the above equation, differentiating (4.75) gives

$$\frac{d(1-t_1)f_1'(k_1)}{dt_1}=-k_3g''k_3'\cdot\frac{d\omega}{dt_1}<0.$$

[8] Grossman and Helpman (1991, Ch.10) show the difference between the effect of a production tax and that of a tariff on the growth rate in a quality ladder model.

commodity, a rise in the tariff rate decreases the growth rate. A production tax has a negative effect on the growth rate, whichever commodity it specializes in.

It is well known that free trade is Pareto superior to autarky in the small country model. Since every policy which we analyze here distorts the relative price of commodities, the social welfare must deteriorate. However, the propositions we derive here imply that the government has a preference over the specialization pattern and the practicability of the government policy differs under each specialization pattern.

If the small country specializes in a consumption commodity, tariff policies change the long-run growth rate. Since the government cannot distinguish the effect of policy on the growth rate from that of a change in the efficiency of a human capital sector, the government which aims at the stability of the growth rate must prefer specialization in a capital commodity to specialization in a consumption commodity.

Moreover, if the government wants to change specialization pattern from a consumption commodity into a capital commodity, that is, to emerge a capital industry and rises tariff rates, then its long-run growth rate lowers during the policy. If the people judge the government policy by the growth rate, it would be difficult for the government to execute the tariff policy.

On the other hand, if a county specializes in a capital commodity, the growth rate is not affected by the tariff policy. It is more likely that the government finances a given level of government spending from the tariff revenue than under specialization in a consumption commodity. The above discussion suggests that the practicability of the tariff policy should differ according to the specialization pattern.

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Chapter 5

Scale Effects and the Dichotomy in a Dynamic Trade Model

Abstract

This paper presents a two-sector learning-by-doing model with 'scale effects'. Scale effects mean that the marginal product of capital increases with the increase in the level of labor input. While a standard dynamic trade model predicts a Ricardian property (complete specialization in a small country), opening trade does not cause complete specialization in our model. This property results from scale effects. Because of scale effects, the marginal product of capital and the autarky price are determined by both production and demand side, which prevents a small country from becoming specialized.

5.1 Introduction

This paper examines a two-sector model in which there is an externality of each firm's investment and shows that complete specialization does not occur in our model. Firm's investment increases the degree of labor quality as externalities. Namely, there is a learning-by-doing effect in each firm's investment. There also exists scale effect: the more labor is employed, the higher the marginal product of capital is. We demonstrate that the scale effect breaks the dichotomy between production side and demand side, and prevents a small country from becoming specialized.

There is a significant difference between the result of a static Heckscher-Ohlin (H-O) model and the one of a dynamic H-O model. Since capital accumulation is allowed in a dynamic trade model, the main source of international trade is technological difference. Although the structure of production is the same as a static H-O model (two commodities, two inputs and constant returns to scale), if there is a slight difference of technology between trading countries, complete specialization occurs in at least one country. In a small country context, the technological difference causes the gap between the relative price before opening trade (the autarky price) and the world price. Thus, as long as the autarky price differs from the world price, the small country must specialize completely. Baxter(1992), and Ono and Shibata (1993) demonstrate this property in a neoclassical growth model. In an endogenous growth model with human capital accumulation, the same property also holds as we see in chapters 3 and 4. Complete specialization in a two-tradeable commodity and two-input model seems to be widely accepted.

The scale effects can be recognized in some endogenous growth models (for example the learning-by-doing model, the public goods model and the expanding product variety model). In these models, the generated knowledge can be used in nonrival manner. Thus, the model exhibits increasing returns to scale in all factor inputs. The model presented here, however, also implies constant returns to scale in factor inputs chosen by an individual firm.

Some authors develop a dynamic trade model in which no specialization occurs. Lee (1995) and Osang and Pereira (1996) examine the relationship between trade and the long-run growth rate of a two-commodity (capital and consumption commodities) small open economy. In these papers, small open economy always produces two commodites. A household's consumption and firm's investment assume the form of composite of foreign and domestic commodities in their model. Namely, they assume the structure of economy after opening trade is different from the one before opening trade. We do not adopt such an assumption. Grossman and Helpman (1991 ch.6) also presents a small open economy in which complete specialization never occurs. But, they assume each sector employs different inputs. Thus, the production structure is different from a two-commodity and two-input model. The model which we present here is a two-commodity and two-input model.

The paper proceeds as follows: Section 2 presents a closed economy and explains why the scale effects break the Ricardian property. In section 3, we investigate the opening economy and show that opening trade does not cause complete specialization. Section 4 discusses the contribution.

5.2 The Closed Economy

5.2.1 Firms

In this paper, two firms operate in a closed economy. Sector 1 produces a commodity which is used for investment and consumption (we call it capital commodity). Sector 2 produces a pure consumption commodity. Production functions of firm i in sector 1 and firm j in sector 2 are follows:

$$Y_1^i = A\left(K_1^i\right)^{\alpha_1}\left(L_1^i\right)^{1-\alpha_1}\bar{K}_1{}^{1-\alpha_1}, \tag{5.1}$$

$$Y_2^j = B\left(K_2^j\right)^{\alpha_2}\left(L_2^j\right)^{1-\alpha_2}\bar{K}_2{}^{1-\alpha_2}, \tag{5.2}$$

where $0 < \alpha_1,\ \alpha_2 < 1$. $\bar{K}_1$ and $\bar{K}_2$ represent aggregate capital stock of sector 1 and 2 respectively. These production functions demonstrate that each firm's investment enhances knowledge of labor as a by-product and that the knowledge of each firm, once invested, spills over across the whole sector. In other words, the knowledge is accumulated as an externality. The degree of the spillover differs from each sector.[1]

We assume that the contribution of each firm's investment is negligible. Hence, each firm takes aggregate capital of each sector as given. As a result of competitive behavior, the value of the marginal product of each factor input is equalized to the factor price. Thus, letting p the price of commodity 2, R the rent on physical capital, and w the wage rate, we have

$$R = A\alpha_1 \left(K_1^i\right)^{\alpha_1-1} \left(L_1^i\right)^{1-\alpha_1} \bar{K}_1^{\,1-\alpha_1} = pB\alpha_2 \left(K_2^j\right)^{\alpha_2-1} \left(L_2^j\right)^{1-\alpha_2} \bar{K}_2^{\,1-\alpha_2}, \quad (5.3)$$
$$w = A(1-\alpha_1) \left(K_1^i\right)^{\alpha_1} \left(L_1^i\right)^{-\alpha_1} \bar{K}_1^{\,\alpha_1} = pB(1-\alpha_2) \left(K_2^j\right)^{\alpha_2} \left(L_2^j\right)^{-\alpha_2} \bar{K}_2^{\,1-\alpha_2}. \quad (5.4)$$

From the above equations, we have

$$p = \frac{A\alpha_1 \left(K_1^i\right)^{\alpha_1-1} \left(L_1^i\right)^{1-\alpha_1} \bar{K}_1^{\,1-\alpha_1}}{B\alpha_2 \left(K_2^j\right)^{\alpha_2-1} \left(L_2^j\right)^{1-\alpha_2} \bar{K}_2^{\,1-\alpha_2}}. \quad (5.5)$$

All individual firms are identical and the number of firms in each sector is normalized to one. The market equilibrium requires that the external effects satisfy $\bar{K}_l = K_l^m (l = 1,\ 2,\ m = i,\ j)$ The factor prices and the relative price in the equilibrium are

$$R = A\alpha_1 \left(L_1^i\right)^{1-\alpha_1} = pB\alpha_2 \left(L_2^j\right)^{1-\alpha_2}, \quad (5.6)$$
$$w = A(1-\alpha_1)K_1 \left(L_1^i\right)^{-\alpha_1} = pB(1-\alpha_2)K_2 \left(L_2^j\right)^{-\alpha_2}, \quad (5.7)$$
$$p = \frac{A\alpha_1 \left(L_1^i\right)^{1-\alpha_1}}{B\alpha_2 \left(L_2^j\right)^{1-\alpha_2}}. \quad (5.8)$$

Having rational expectations, a household correctly anticipates that these interest rate, wage rate and price hold in the equilibrium. Note that the more labor is employed, the higher the marginal product of capital is (see 5.6). This is what we call a 'scale effect'. In other words, the marginal product of capital changes through the level of labor input. Since every firm is identical, we omit the symbols which indicate firm's identity without ambiguity.

Letting $k_l = K_l/L_l\,(l = 1, 2)$, from (5.6) and (5.7) we obtain,

$$k_1 = \frac{\alpha_1}{1-\alpha_1}\omega \equiv k_1(\omega),\ k_2 = \frac{\alpha_2}{1-\alpha_2}\omega \equiv k_2(\omega). \quad (5.9)$$

[1] Benhabib and Farmer (1996) introduces sector-specific externalities into a version of real business cycle model. They find that indeterminacy arises even if the externality of each sector is mild.

5.2.2 A Household

We assume a representative household has a log-liner instantaneous utility function and maximizes his life time utility. Aggregate labor is normalized to one.

$$\max U = \int_0^\infty [\delta \ln C_1 + (1-\delta) \ln C_2] \exp(-\rho t)\, dt, \tag{5.10}$$

$$s.t.\ \dot{A} = RA + w - C_1 - pC_2, \tag{5.11}$$

A represents total assets. Set up the Hamiltonian function that

$$H = \delta \ln C_1 + (1-\delta) \ln C_2 + \lambda(RA + w - C_1 - pC_2) \tag{5.12}$$

The first order conditions give

$$p = \frac{1-\delta}{\delta}\frac{C_1}{C_2}, \tag{5.13}$$

$$\frac{\dot{C}_1}{C_1} = R - \rho. \tag{5.14}$$

The transversality condition is

$$\lim_{t\to\infty} \lambda A \exp(-\rho t) = 0. \tag{5.15}$$

5.2.3 Market Clearing Conditions

We assume that markets for the capital stock, the labor, the commodities and the asset are always clearing.

$$K_1 + K_2 = K. \tag{5.16}$$

$$L_1 + L_2 = 1. \tag{5.17}$$

$$BK_2^{\alpha_2} L_2^{1-\alpha_2} \bar{K}_2^{\,1-\alpha_2} = C_2. \tag{5.18}$$

$$A = K. \tag{5.19}$$

From (5.9), (5.16) and (5.17)

$$L_1 = \frac{K - k_2(\omega)}{k_1(\omega) - k_2(\omega)} = \frac{1 - \frac{\alpha_2}{1-\alpha_2}\frac{\omega}{K}}{\frac{\alpha_1-\alpha_2}{(1-\alpha_1)(1-\alpha_2)}\frac{\omega}{K}}, \tag{5.20}$$

$$L_2 = \frac{K - k_1(\omega)}{k_2(\omega) - k_1(\omega)} = \frac{1 - \frac{\alpha_1}{1-\alpha_1}\frac{\omega}{K}}{\frac{\alpha_2-\alpha_1}{(1-\alpha_1)(1-\alpha_2)}\frac{\omega}{K}}. \tag{5.21}$$

We find that functions L_1 and L_2 are homogeneous of the first degree on K and ω:

$$L_1 = L_1\left(\frac{\omega}{K}\right),\ L_2 = L_2\left(\frac{\omega}{K}\right) \tag{5.22}$$

From (5.8), we know that the equilibrium price is also homogeneous of the first degree on K and ω:

$$p = p\left(\frac{\omega}{K}\right). \tag{5.23}$$

Introducing (5.13), (5.21) and (5.23) into (5.18), we obtain

$$Bk_2(\omega)\left[L_2\left(\frac{\omega}{K}\right)\right]^{2-\alpha} = \frac{1-\delta}{\delta}\frac{C_1}{p\left(\frac{\omega}{K}\right)}. \tag{5.24}$$

From the above equation, function ω depends on C_1 and K:

$$\omega = \omega(C_1, K). \tag{5.25}$$

From (5.6) and (5.14), making use of (5.22) and (5.25), we obtain

$$\frac{\dot{C}_1}{C_1} = A\alpha_1 L_1\left(\frac{\omega(C_1, K)}{K}\right)^{1-\alpha_1} - \rho. \tag{5.26}$$

Substituting (5.13), (5.19), (5.22) and (5.25) into (5.11) gives

$$\frac{\dot{K}}{K} = \frac{Ak_1(\omega(C_1, K))\left[L_1\left(\frac{\omega(C_1,K)}{K}\right)\right]^{2-\alpha_1}}{K} - \frac{C_1}{K}. \tag{5.27}$$

Consequently, we obtain a complete dynamic system constituted by (5.26) and (5.27) that describes behaviors of C_1 and K.

5.2.4 The Role of Scale Effects in the General Equilibrium

Equations (5.6) and (5.8) indicate that the interest rate and the relative price of commodity are determined by labor input of each sector. To determine the level of labor input, we need the household's demand function.

Let us integrate the flow budget constraint (5.11) from 0 to ∞ using (5.13) and the transversality condition (5.15). This gives

$$(1-\delta)\rho[K(0) + h(0)] = pC_2(0), \tag{5.28}$$

where $h(0) \equiv \int_0^\infty w(t)\exp\left(\int_0^t -R(s)ds\right)dt$. $h(0)$ is called human wealth. $X(0)$ denotes the value of variable X at time 0. (5.28) represents the demand for commodity 2. This economy always stays in the steady state equilibrium (see Appendix), and L_1 and L_2 is always constant. Making use of this property, and introducing (5.6), (5.7) and (5.9) into (5.28), we obtain

$$h(0) = \omega(0). \tag{5.29}$$

Substituting (5.9), (5.28) and (5.29) into the market clearing condition for commodity 2, (5.18), we can see that

$$B\frac{\alpha_2}{1-\alpha_2}\omega(0)\left[L_2(0)\right]^{2-\alpha_2} = (1-\delta)\frac{\rho\left[K(0)+\omega(0)\right]}{p(0)}. \tag{5.30}$$

Note that functions L_2 and p are homogeneous of the first degree on K and ω. Thus, we obtain $\omega(0)/K(0)$ from (5.30). Introducing $\omega(0)/K(0)$ into (5.22), labor input of each sector is determined. Then, from (5.6) and (5.8) we obtain the interest rate and the relative price of commodity.

The existence of externalities precludes the diminishing returns and realizes the constant interest rate. What determines the interest rate is the labor input of each sector. As in (5.30), labor input itself depends on how much the household demands each commodity. It means that the interest rate and the relative price are not determined by only production side. As in chapter 3, the dichotomy between production side and demand side holds in a dynamic trade model. The interest rate and the relative price of commodity are determined by only production side.

Note that this property does not result from only externalities. It is scale effects that cause the property. We have the following production functions as the example of a two-sector learning-by-doing model without scale effects.

$$Y_1 = AK_1^{\alpha_1}L_1^{1-\alpha_1}\left(\frac{\bar{K}_1}{L_1}\right)^{1-\alpha_1}, \tag{5.31}$$

$$Y_2 = BK_2^{\alpha_2}L_2^{1-\alpha_2}\left(\frac{\bar{K}_2}{L_2}\right)^{1-\alpha_2}. \tag{5.32}$$

These production functions captures an idea that the learning and spillovers arise from interacting freely with the average person who possesses the average level of skills and knowledge, rather than from the accumulated knowledge. Proceeding the same procedure as in subsection 5.2.1, the equilibrium interest rate is $A\alpha_1$ and the equilibrium relative price of commodity 2 is $A\alpha_1/B\alpha_2$. The interest rate and the relative price are determined by only the production functions.

In a dynamic model without scale effects (see the above example and the model in chapter 3), the relative price and the interest rate are determined by only production side. The dichotomy between production side and demand side holds. The model exhibits a Ricardian property even if there are two factor inputs. Thus, free trade leads complete specialization as in a static Ricardian trade model. The existence of scale effects requires the development of the general equilibrium to determine the interest rate and the relative price. The model lacks the Ricardian property.

5.3 The Small Open Economy

In this section, we show this economy does not specialize completely under free trade. When the two sectors operate, each sector's value of the marginal product of capital and labor must be equal and the market clearing conditions for two factors of production, K and L, are satisfied. We demonstrate this condition holds with scale effects under free trade.

Proposition 6 *When there exist externalities of firms' investment with scale effects, free trade does not lead to complete specialization. Without scale effects, an economy specializes completely under free trade.*

Proof.

When the economy opens to trade, the relative price of commodity 2 is given exogenously as p^w. Given p^w, the competitive firm equates the rent of physical capital to its marginal product. From (5.1) and (5.2), the marginal product of capital in sector 1 is $A\alpha_1 L_1^{1-\alpha_1}$ and that in sector 2 is $B\alpha_2 L_2^{1-\alpha_2}$. Thus, in the equilibrium of imperfect specialization, the equation below holds:

$$(R =)\, A\alpha_1 L_1^{1-\alpha_1} = p^w B\alpha_2 L_2^{1-\alpha_2}. \tag{5.33}$$

Since the total labor force is 1, the left hand side (LHS) of (5.33) is monotonously increasing in L_1 and the right hand side (RHS) is monotonously decreasing in L_1. When L_1 is 0, the LHS is larger than the RHS. When L_1 is 1, the LHS is smaller than the RHS. Thus, (5.33) determines the unique labor inputs of each sector. In the equilibrium, the value of the marginal product of labor input for each sector is equal to each other:

$$A(1-\alpha_1)K_1 L_1^{-\alpha_1} = p^w B(1-\alpha_2)K_2 L_2^{-\alpha_2}. \tag{5.34}$$

Since at any point in time the total capital stock is given, (5.34) determines the unique capital input of each sector as (5.33) determines the labor input. From (5.33) and (5.34), the labor input and the capital input of each sector are determined and the equilibrium of imperfect specialization holds.

Suppose that there is no scale effect and that two sectors with production functions (5.31) and (5.32) operate in autarky economy. The value of marginal product of capital is equal to its rent:

$$R = A\alpha_1 = pB\alpha_2. \tag{5.35}$$

The equilibrium autarky price p is $A\alpha_1/B\alpha_2$. When the economy opens trade, if $p^w < (>)p$, $A\alpha_1 > (<)p^w B\alpha_2$. Unless the world price happens to be equal to the autarky price, the value of the marginal product of capital in each sector differs from each other. If the world price is lower (higher) than the autarky price, sector 2 (1) cannot operate. Thus, the complete specialization must occur under free trade. ■

Because of the scale effect, the value of marginal product of each input in each sector can be equated thorough changing the labor inputs of each sector. Thus, both sectors can operate. Without the scale effect, the marginal product of capital is determined by the parameters of the production function. Thus, once the relative price is given exogenously, the value of the marginal product differs in each sector and one of the two sectors can not survive.

Next, let us examine the long-run growth in the open economy. Proceeding the same procedure as in the closed economy, we obtain the complete dynamic system of the open economy.

$$\frac{\dot{C}_1}{C_1} = A\alpha_1 L_1^{1-\alpha_1} - \rho. \tag{5.36}$$

$$\frac{\dot{K}}{K} = \frac{Ak_1(\omega(C_1,\, K))[L_1(K,\, \omega(C_1,\, K)]^{2-\alpha_1}}{K} + \frac{p^w Bk_2(\omega(C_1,\, K))[L_2(K,\, \omega(C_1,\, K)]^{2-\alpha_2}}{K} - \frac{C_1}{\delta K}. \tag{5.37}$$

Once the labor input of each economy is determined, from (5.36) the long-run growth rate is determined.[2] Making use of (5.33), we obtain

$$\frac{dL_1}{dp^w} = \frac{B\alpha_2 L_2^{1-\alpha_2}}{A\alpha_1(1-\alpha_1)L_1^{-\alpha_1} + p^w B\alpha_2(1-\alpha_2)L_2^{-\alpha_2}} > 0. \tag{5.38}$$

Thus, the lower the relative price of the capital commodity is, the higher the long-run growth rate is. When the relative price of capital commodity is high enough, the economy enjoys the positive long-run growth rate.

5.4 Conclusion

In this paper, we demonstrate that when production functions exhibit 'scale effects', free trade does not cause complete specialization in a small economy. Once the scale effects are introduced into a dynamic trade model, the prediction of a dynamic trade model does not hold. We find that the externalities themselves does not rule out the complete specialization, but that scale effects do. Our model in chapter 3 and 4 shows complete specialization. The human capital accumulation model does not exhibit the

[2] We can demonstrate that the existence and stability of the steady state growth equilibrium as in the closed economy.

scale effects, because the human capital belongs to only an individual who accumulates. While knowledge is nonrival in the learning-by-doing-model, human capital is a rival good. Thus, in the endogenous growth model with human capital accumulation there is no scale effect. The result of complete specialization in chapter 3 and 4 depends on this property.

There exists a literature on the empirical validity of scale effects. The model with the scale effects seems to predict that countries with more labor tend to grow faster in per capita terms. While Backus, Kahoe and Kahoe (1992) and Jones (1995) examine the scale effects empirically, the subject has yet to be analyzed exhaustively since it is difficult to measure appropriate labor force.

Appendix

We examine here stability and existence of a steady state growth equilibrium. Suppose that there exists a steady state growth equilibrium and denote the growth rate is ϵ. Then, by definition we have

$$\epsilon = \frac{\dot{C}_1}{C_1} = \frac{\dot{K}}{K}.$$

Dividing (5.16) by total labor ($\equiv 1$) differentiating with respect to time logarithmically, and making use of (5.9), we obtain

$$\frac{\dot{K}}{K} = L_1\frac{\dot{k}_1}{k_1} + L_2\frac{\dot{k}_2}{k_2} = \frac{\dot{\omega}}{\omega}.$$

As shown in (5.21), functions L_1 and L_2 are homogeneous of the first degree on K and ω. Thus, K and ω always grow at the same rate and L_1 and L_2 are constant. Introducing (5.13) into (5.18) and differentiating logarithmically, we obtain

$$\frac{\dot{\omega}}{\omega} + (1-\alpha)\frac{\dot{L}_2}{L_2} = \frac{\dot{C}_1}{C_1}. \tag{A.1}$$

Considering (A.1), if the steady state growth equilibrium exists, the economy always stays there. Since the economy always stays in the steady state growth equilibrium, the transversality condition (4.32) is automatically satisfied.

Define a new variable as

$$x \equiv C_1/K.$$

Then, from (5.26) and (5.27) we have

$$\frac{\dot{x}}{x} = \frac{\dot{C}_1}{C_1} - \frac{\dot{K}}{K} = AL_1^{1-\alpha_1}\left(\alpha_1 - \frac{L_1k_1}{K}\right) - \rho + x.$$

or

$$\dot{x} = \left[AL_1^{1-\alpha_1}\left(\alpha_1 - \frac{L_1k_1}{K}\right) - \rho + x\right]x.$$

Denote the steady state value of x as x^*. To realize a positive x^*, the following condition must be satisfied:

$$\rho > AL_1^{1-\alpha_1}\left(\alpha_1 - \frac{L_1k_1}{K}\right) = AL_1^{1-\alpha_1}\left(\alpha_1 - L_1\frac{\alpha_1}{1-\alpha_1}\frac{\omega}{K}\right). \tag{A.2}$$

As shown in subsection 5.2.4, (5.30) determines the equilibrium levels of ω/K and labor input, L_1. We should assume that ω/K fulfills (A.2).

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