Calculation of Dose Distributions in Radiation Therapy by a Digital Computer
II. Computation of Dose Distrsbution in Radium Therapy

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ディジタル型電子計算機による線量分布の計算
第2報 ラジウム治療における線量分布の計算

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最近，電子計算機を用いて小線源治療の線量分布を計算する方法が発表されているが，それらを検討して癌研としての方式を開発したので報告する．

線量計算には，ラジウム容器である白金および組織によるラジウムーガンマ線の吸収をそれぞれ Mayneord および Meredith 他の実験結果を基にして多項式で表わし，Sievert の積分を Simpson の法則を用いて数値積分する方法をとつた．

ラジウムの位置は外部照射の治療計画用に開発したシミュレーターによる直角2方向撮影フィルムを用いて計算する．

計算結果の表示は，計算点とその点の線量率を表でプリントアウトすると同時に，等線量図を記号を用いてプリントアウトする方法をとつた．等線量曲線はそれらを基にして容易に手で作図できる．又，出力の精度をチエックするために，計算機により計算したラジウム管の長さと真の長さを比較して，その値をプリントアウトした．

20mgのラジウム管，婦人科領域の治療用アプリケーター，およびその臨床例についての線量分布図を示した．

計算時間はＧＥ 635を用いて1本のラジウムにつき 841点の線量を約7秒である．

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Introduction

In radium therapy, it is important to determine the positions of the radiation sources which are inserted into a patient and to estimate the dose resulting from the treatment. Calculation of the dose distribution throughout an implant is so laborious that it is rarely done by manual methods except for model cases.

It is now possible to calculate complete isodose distributions for individual patients by the use of a computer. In recent years, computer methods for interstitial and intracavitary therapy have been reported.[1)3)-5)8)-10)13)15)-19)24)]

The present paper reports a programme which is used in the Cancer Institute Hospital for the calculation of dose distributions and for the localization of radium implants with the aid of a digital computer. In this programme, the dose rates are calculated by numerical integration of rows of point source placed along the axis of hollow platinum cylinders and the positions of radium tubes are obtained from two radiographs at right angles taken by a simulator developed for the treatment planning in teletherapy.

Method

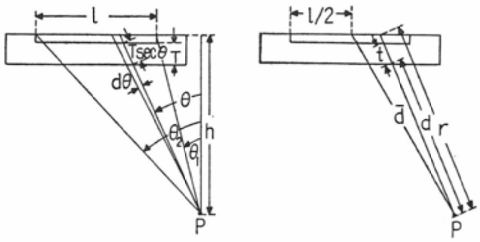
1. The dose distribution around a linear source

Referring to Fig. 1, tbe exposure rate, I, at a point, P, in air at a distance, h, from the axis of a screened linear source of activity, M, active length, l, and wall thickness, T, is given by the following equation (the so-called "Sievert integral"):

$$I = \frac{M\Gamma}{lh}\int_{\theta_1}^{\theta_2} \exp\,(-\mu T \sec\,\theta)\; d\theta \tag{1}$$

where θ_1 and θ_2 are defined by the diagram. The constant Γ is the specific gamma ray constant and μ is the effective absorption coefficient of the wall material.

Fig. 1. Geometry of dose calculation for linear radium sources



Since 12-gamma rays are emitted from radium and its daughters, the exposure rate for a radium tube is expressed as

$$I = \frac{M}{lh}\int_{\theta_1}^{\theta_2} \sum_{i=1}^{12} \Gamma_i\; \exp\,(-\mu_i\, T\, \sec\,\theta)\; d\theta = \frac{M}{lh}\int_{\theta_1}^{\theta_2} \Phi d\theta \tag{2}$$

where

$$\Phi = \sum_{i=1}^{12} \Gamma_i\; \exp\,(-\mu_i\; T\; \sec\;\theta) \tag{3}$$

Γ_i and μ_i are the specific gamma ray constant and effective absorption coefficient for each of 12-gamma rays, respectively.

Considering the absorption and scattering in soft tissue, the exposure rate I′ at a point P in soft tissue in the vicinity of a linear radium source is given by

$$I' = \frac{M}{lh}\int_{\theta_1}^{\theta_2} \Phi \cdot \mathrm{WAR}\; d\theta \tag{4}$$

where WAR is the ratio of the exposure in water and in air, which is called the "water-air exposure ratio" in this paper. This may be given by

$$\mathrm{WAR} = \sum_{i=1}^{12} \exp\;(-\mu'_i d) \tag{5}$$

where μ'_i and d are the effective absorption coefficient of tissue for each of 12-gamma rays and thickness of tissue, respectively.

To carry out this integration, the mathematical expressions for Φ and WAR may be found which approximate the experimental results, and equation (4) can be integrated numerically by Simpson's rule.

1) Absorption of radium gamma rays in platinum

Some experimental values[7][11] for the exposure rate at 1 cm from 1 mg source of radium filtered by various thicknesses of platinum are shown in Fig. 2, where the value for radium filtered by 0.5 mm of platinum is normalized to 8.25 Rcm^2/mgh[6]. These results are in good agreement with each other. Also shown in this graph are the exposure rates[21] calculated from the spectrum of radium using total absorption coefficient (μ_0) and true absorption coefficient ($\mu_0-\sigma_s$)of platinum, where the specific gamma ray constant of radium corrected back to zero filtration is adopted as 9.08 Rcm^2/mgh, which is calculated from the result by Young et al.[24]

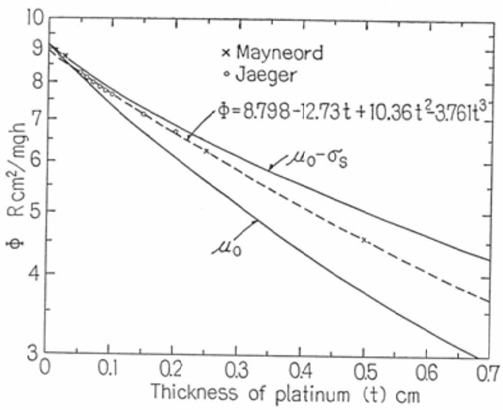
Young and others used the values calculated for true absorption to extrapolate to greater absorber thickness in their calculation, as the transmission factors measured by Whyte and by Keyser are close to the curve calculated for true absorption over the range of their measurements (up to 2 mm of platinum). However, the data measured by Mayneord et al.[11] for large thicknesses of absorber lie between the two calculated curves, μ_0 and $\mu_0-_{\sigma s}$, as shown in Fig. 2. The data of Mayneord et al. have been used in the present paper. Their results are expressed with reasonable accuracy by the following equation:

$$\Phi=8.798 - 12.73\,t + 10.36\,t^2 - 3.761\,t^3 \ (Rcm^2/mgh) \quad (6)$$

where t is the thickness of a platinum wall (in cm).

The results calculated by this equation are shown as a dotted line in Fig. 2.

Fig. 2. Transmission of radium gamma rays through platinum
Curve (μ_0): calculated for total absorption
Curve ($\mu_0-\sigma_s$): calculated for true absoprtion



2) Water-air exposure ratio

Several authors[12][14][20][22] have published results on experimental measurements of the water-air exposure ratio for radium gamma rays. These results are in fair agreement with each other except for the data by Wootton et al.[22] Although Batho et al.,[2] Adams et al.,[1] and Laughlin et al.[9] used the data of Wootton et al. in their calculation, the data of Ponnunni et al.,[14] and of Meredith et al.[12] have been used in the present paper. Their results are approximated by the equation

$$WAR(d) = 1.000 - 0.01161\,d - 0.0004435\,d^2 \quad (7)$$

where d is the thickness of tissue (in cm).

The results calculated by this equation (as a solid line) and values taken from several papers are shown in Fig. 3. Substituting (6) and (7) into (4), the dose distributions in soft tissue around a linear radium source of any type and activity can be calculated using repeated Simpson's rule.

In practical calculation, however, since μ'_i is small, a rough approximation can be used such as

$$I' = \frac{M}{lh} \text{WAR}(\bar{d}) \int_{\theta_1}^{\theta_2} \Phi d\theta \quad (8)$$

where $\bar{d}$ is the thickness of tissue from midpoint of source to the point P. Maximum difference in results between equations (4) and (8) for a radium of active length 1.5 cm is less than 1%.

2. Localization of radium sources in a patient

The positions of the radium sources in a patient are obtained from two radiographs at right angles taken by the Universal Planning X-ray Apparatus manufactured by the Tokyo Shibaura Electric Co., Ltd.

This apparatus has been developed as a simulator for various kinds of teletherapy units, and its details have been reported by Yamashita et al.[23] This simulator has an acryl plate with orthogonal opaque scale at 1 cm intervals. The acryl plate is situated perpendicular to the axis of X-ray beam and its position can be varied to suit that of diaphragm of teletherapy units. The distance from focus to center of rotation is also variable.

Fig. 3. Water-air exposure ratio versus distance from radium source

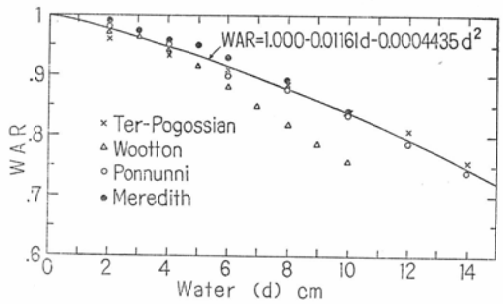
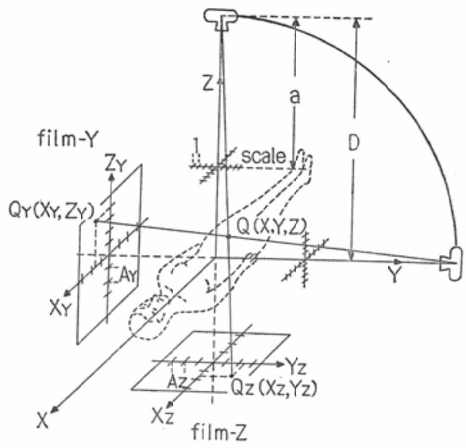


Fig. 4. Geometrical construction of two perpendicular radiographs of a point object Q and normal position of the patient in Cartesian co-ordinates



Referring to Fig. 4, the co-ordinates of a point Q (X, Y, Z) are obtained from the co-ordinates of a point Q_y (X_y, Z_y) and a point Q_z (X_z, Y_z) on the film-Y and -Z, respectively, using the formulae:

$$X = \frac{DX_y (F_z - Y_z)}{F_y F_z - Z_y Y_z}, \quad Y = \frac{DY_z (F_y - Z_y)}{F_y F_z - Z_y Y_z}, \quad Z = \frac{DZ_y (F_z - Y_z)}{F_y F_z - Z_y Y_z} \quad (9)$$

where D is the distance from focus to center of rotation, and F_y and F_z are the distances from focus to film-Y and -Z, respectively. When the distance from focus to the acryl plate with scale is a, F_y and F_z are given by

$$F_y = a\ A_y,\ F_z = a\ A_z \tag{10}$$

where A_y and A_z are the magnifications of the scale on the film-Y and -Z, respectively.

The practical calculation of the dose distribution is done in a co-ordinate system whose origin is transformed into an arbitrary point, and the locations of the planes of calculation with respect to the sources can be selected arbitrarily.

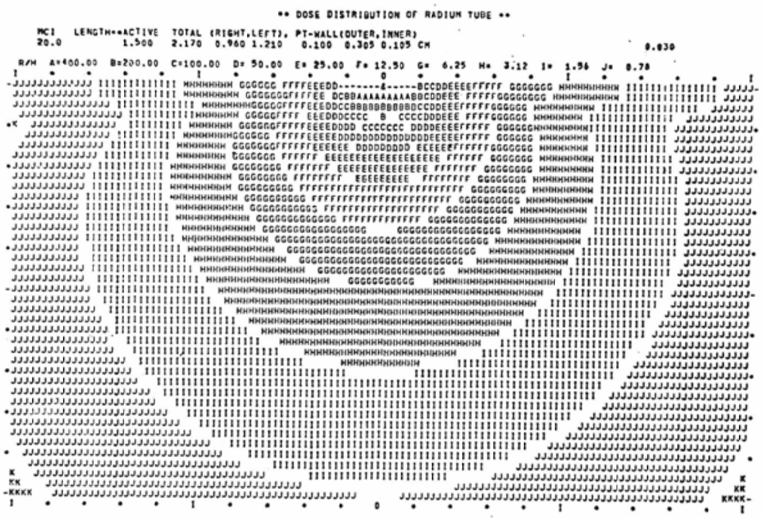
The results of the calculations are printed using the tabular and scale print-out methods.

Applications

1. The dose distribution around a radium source

Fig. 5 shows an example of the dose distribution in air for a 20 mg radium tube, whose active length is 1.5 cm and filtration is 0.1 cm of platinum, calculated by the eleven-source Simpson integration. The larger the point sources are considered, the higher will be the accuracy of the resultıng dose rate, but the computer time required will also increase. Hope et al.[5] have indicated that beyond 0.5 cm from the tube eleven sources were sufficient for every point and that only a small region on the end perpendicular required more than five to calculate with error less than 1%. In clinical cases, therefore, the five source Simpson integration is used.

Fig. 5. Computer print-out of dose distribution for a 20-mg radium tube using an eleven-point source integration



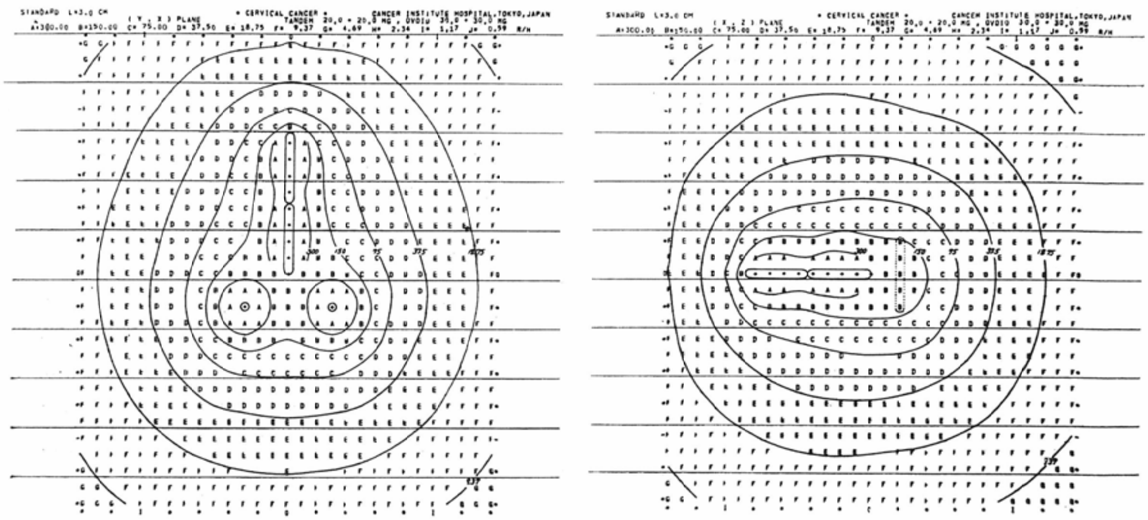
The symbols used in Fig. 5 indicate some ranges of dose rate as shown at the top of the figure. A = 400, B = 200, and C = 100, for instance, are used for the ranges greater than 400, 400—200, and 200—100 R/h, respectively. The computation of dose at points on a 1.67 × 2 mm grid is done.

2. The dose distribution for standard arrangement in cervix radium treatment

Fig. 6 shows an example of the standard arrangement in the cervix radium therapy (tandem 20 + 20 mg, ovoids 30 + 30 mg). The computation of dose is done at points on a 5 mm grid.

By drawing lines through equal symbols isodose curves can easily be obtained. Solid lines in the figure are drawn by hand. If a more accurate value is desired, the tabular print-out may be used.

Fig. 6. Computer print-out of dose distribution for standard arrangement in cervix radium therapy. Lines are drawn by hand.



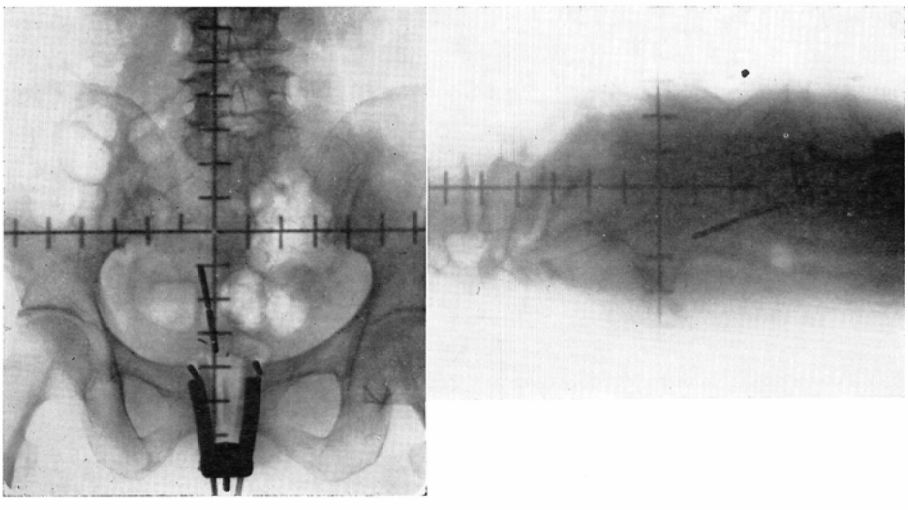
3. Examples of clinical use

To avoid a mixed position technique, the patient always keeps her left hand towards the film in the lateral view, and in the antero-posterior view she keeps her back towards the film.

In Fig. 4 the normal position of the patient and Cartesian axes are shown. The zero point of the coordinate system is identical with the crossing point of the central axes of the two perpendicular views. In actual calculation the zero point is transformed into a desired point, but the directions of Cartesian axes remain the same except for particular cases.

Figs. 7 (a) and (b) are the antero-posterior and lateral radiographs, respectively, taken by the simula-

Fig. 7. The antero-posterior and lateral radiographs of an intracavitary implant of cervical applicators

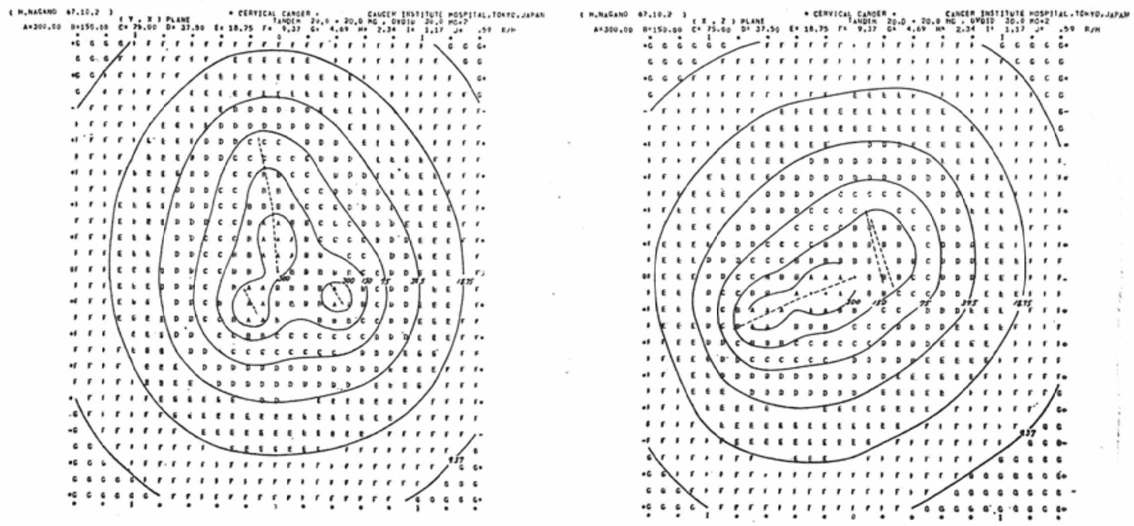


tor. The images of the scales are shown. By these scales the co-ordinates of the radium tubes can be easily measured.

Figs. 8 (a) and (b) are the resultant dose distributions in the (X-Y) and(X-Z) planes, respectively. The lower point of tandem is taken as the origin.

Fig. 8. Computer print-out of dose distribution for a cervix radium treatment
(a) Dose distribution in X-Y plane, (b) Dose distribution in X-Z plane

(a) (b)



For a given implant, accuracy of the output is limited by the accuracy of the input data. As a check on accuracy, the co-ordinates of the tube end-points are used to calculate the total length of the radium tube. These values, together with the actual length, are printed out. If the permissible error (10%) is exceeded, the programme is rejected. In general, the error of the computed total length has been less than 3%.

This programme requires about seven seconds of computer time in the GE 635 for a plane of 841 points per tube.

Summary

A digital computer method is described for calculating the dose distribution in the vicinity of radium insertion using measured values of the absorption of radium gamma rays in platinum and soft tissue. The arrangement of the sources is measured on antero-posterior and lateral radiographs and is transferred to the computer. The output may take the form of a tabular listing of co-ordinate points and corresponding dose rates, and of a plot containing isodose information. Examples of the computer print-put of a 20-mg radium tube, standard arrangement in cervix radium therapy, and its clinical use were shown.

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References

1) Adams, G.D., and Meurk, M.L.: Phys. Med. Biol. 9(1964), 533–540.
2) Batho, H.F., and Young, M.E.J.: Brit. J. Radiol. 37 (1964), 689–692.
3) Batten, G.W., and Shalek, R.J.: Technical reports series No. 57., Computer calculation of dose distributions in radiotherapy, IAEA, Vienna, 1966, 100–106.
4) Busch, M.: Technical reports series No. 57., IAEA, Vienna, 1966, 83–99.
5) Hope, C.S., Laurie, J., Orr, J.S., and Walters, J.H.: Phys. Med. Biol. 9 (1964), 345–357.
6) International Commission on Radiological Units and Measurements, 1959, N.B.S. Handbook 78.
7) Jaeger, R.G.: Dosimetrie und Strahlenshutz, p. 80. Georg Thieme Verlag, Stuttgart, 1959.
8) Lane, Jr., F.W., Shaw, R.K., and Stibitz, G.R.: Amer. J. Roentgenol. 100 (1967), 870–877.
9) Laughlin, J.S., Siler, W.M., Holodny, E.I., and Ritter, F.W.: Amer. J. Roentgenol. 89 (1963), 470–490.
10) Masaki, N.: Nippon Acta Radiol. 23 (1963), 197–204.
11) Mayneord, W.V., and Roberts, J.E.: Brit. J. Radiol. 10 (1937), 365. (Evans, R.D.: Nucleonics 1 (1947), 32–43).
12) Meredith, W.J., Greene, D., and Kawashima, K.: Brit. J. Radiol. 39 (1966), 280–286.
13) Nelson, R.F., and Meurk, M.L.: Radiology 70 (1958), 90.
14) Ponnunni, K.I., Kenney, G.N., and Cameron, J.R.: Amer. J. Roentgenol. 90 (1966), 66–69.
15) Powers, W.E., Bogardus, C.R., White, W., and Gallagher, T.: Radiology 85 (1965), 135–142.
16) Rose, J.A., Bloedorn, F.G., and Robinson, J.E.: Amer. J. Roentgenol. 97 (1966), 1032–1040.
17) Schlessinger, J.E., and Porter, E.H.: Brit. J. Radiol. 39 (1966), 147–150.
18) Shalek, R.J., and Stovall, M.: Radiology 76 (1961), 119–120.
19) Stovall, M., and Shalek, R.J.: Radiology 78 (1962), 950–954.
20) Ter-Pogossian, M., Ittner, III, W.B., and Aly, S.M.: Nucleonics 10 (1952), 50–52.
21) Tsukamoto, K., and Onai, Y.: Nippon Acta Radiol. 14 (1954), 190–196.
22) Wootton, P., Shalek, R.J., and Fletcher, G.H.: Amer. J. Roentgenol. 71 (1954), 683–690.
23) Yamashita, H., Onai, Y., Igari, S., Yamada, M., Makino, S., and Nagai, K.: Nippon Acta Radiol. 24 (1964), 858 (abstract).
24) Young, M.E.J., and Batho, H.F.: Brit. J. Radiol. 37 (1964), 38–44.