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Fundamental Studies on the Electron Penetration in Radiotherapy

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放射線治療における電子線透過の基礎的研究

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4から32MeV電子線の水中における透過現象を確率論的および定性的に解析することにより，放射線治療時の物理的基礎データ：path lengthと平均深さの関係，そして各深さにおけるエネルギー・スペクトル，平均角度分布，charge rate，current，深部量百分率，ラド変換係数Cγを得た。ビーム・ジオメトリは単一エネルギー，無限平面のビームが単一方向に水に入射する理想的な状態を考えた。我々の結果は，モーメント法によるKessarisの結果と比較すると，より満足すべき結果であった。

Summary

The penetration of electron beams of energy 4 to 32 MeV in water has been theoretically investigated with a stochastics and qualitative analysis and with a view toward applications in electron-irradiation therapy. The data concerning the average penetration depth as a function of the pathlength, as well as the energy spectrum, average angular distribution, charge rate, current, flux, percentage depth dose and rad conversion factor Cγ as a function of depth were obtained. The beam geometry was considered in such a way as infinite monodirectional beam was embedded in a water of infinite extent. This result was compared with a moment method result of N.D. Kessaris and other results, and it had more sufficient generality.

Introduction

In radiotherapy of high energy electrons, it is essential to have an accurate knowledge of the dose delivered at specified points in the patient. There are many reports for calculational approaches in the electron penetration by various authors. However the data in most of these reports have not been used in radiotherapy because the electron energy is low (below 5 MeV) and the medium is not water. N.D. Kessaris has mathematically investigated the penetration of energy 10 to 20 MeV in water by solving the Lewis equation by the moment method under the continuous slowing-down approximation. M.J. Berger and S.M. Seltzer also calculated energy and charge deposition and flux in water with mainly 20 MeV electrons by Monte Carlo methods. However, the agreement on the results of both authors is not perfect. Fig. 1 shows a comparison between a moment method result of Kessaris and a Monte Carlo result of Berger pertaining to the charge rate distribution for a 20 MeV beam, and also shows our experimental data.
The measurement in our experiment has been carried out for the current distribution by a process using the direct collection of electrons absorbed in a water phantom. The charge rate distribution has been obtained from the current data. The beam in our experiment was nearly monochromatic. However, it was not perfect because of the presence of unavoidable scattering materials, such as a window of a beryllium film in 0.3 mm thick at the outlet of the accelerated electrons, and a monitor consisting of five sheets of aluminim leaves in 0.02 mm thick. Our experimental results agreed well with a Monte Carlo result of Berger rather than a moment method result of Kessaris.

This theoretical treatment on the penetration of electrons will be done with a stochastics and qualitative analyses in multiple scattering, secondary electron emission and bremsstrahlung emission. This paper presents information of the penetration of 4 to 32 MeV electrons into a water phantom, according to the pioneering work of Kessaris in radiotherapy. The energy range which we chose in this theoretical treatment can be produced by the betatron in Kyoto Univ. Hospital. The incident beam is monoenergetic, monodirectional and broad, and it is embedded in water of infinite extent.

**Theory and calculation results**

1. The number of collisions

The number of electrons in a medium is changed by multiple scattering. This phenomena are applied to three physics conceptions: a) it is non-continuous process; b) each collision occurs independently; c) at least $n_e$...
collisions are necessary to stop an initial electron: The probability that an electron has \( n \) collisions in \( t \) path length, \( P(n,t) \), is expressed by a Poisson distribution.

\[
P(n,t) = e^{-\nu} \frac{(\nu t)^n}{n!} \quad (n \neq 0) \\
= e^{-\nu} \quad (n = 0)
\]

where \( \nu \) is the average number of collisions in a unit path length (cm\(^2\)/g), \( t \) is the path length of an electron (g/cm\(^2\)), and \( \nu t \) is the average number of collisions which are made by one electron in \( t \) path length. Since an electron makes a large number of collisions in traversing a fraction of its path length and \( \nu t \) value is large, the Poisson formula (1) (2) is evaluated numerically by Gaussian distribution.

\[
P(n,t) = \frac{1}{\sqrt{2\pi \nu t}} e^{-\frac{\nu t - n^2 t}{2\nu t}} \quad (n \neq 0) \quad (3)
\]

\[
= \frac{1}{\sqrt{2\pi \nu t}} e^{-\frac{\nu t}{2}} \quad (n = 0) \quad (4)
\]

2. The energy loss

Since \( \nu t \) value is large, it is possible to describe the energy loss of electrons with reasonable accuracy by continuous slowing down approximation. The energy loss per one collision in water, \( \Delta E \), is referred to the energy loss per unit path length, \( -(dE/dt) \), as the total stopping power, \( (s/p)\)\(_{E,w} \). The electron energy, \( E_k \), with which a particle is scattered \( k \) times is given by

\[
E_k = E_{k-1} - \Delta E \\
= E_{k-1} - \frac{(s/p)\) \(_{E,w}}{\nu} \quad (5)
\]

where \( (s/p)\) \(_{E,w} \) is referred to the \( (k-1) \)th collision.

By the use of approximate slowing down model, the relationship between the electron path length and its residual energy can be obtained\(^9\). However, this relationship is not defined unique since electrons experience discrete and sometimes large energy losses as they traverse a material. This deviation of the energy loss of the electrons from the average is known as straggling\(^9\). For straggling, we shall denote the distribution of the energy loss of electrons by a simple exponential curve with extensive simplification rather than by Gaussian curve\(^1\). This distribution is given by

\[
SF(\Delta E) = \frac{N_{\Delta E}}{N_{\Delta E_0}} \quad (7)
\]

\[
= e^{-\frac{\Delta E}{\Delta E_0}} \quad (8)
\]

where \( \Delta E_0 \) is the most probable energy loss, \( \Delta E_{av} \) is the average energy loss, and \( SF(\Delta E) \) is a straggling factor on \( \Delta E \) energy loss. This factor is expressed by the ratio of the number of electrons, \( N_{\Delta E} \) with \( \Delta E \) energy loss to the number of electrons, \( N_{\Delta E_0} \) with \( \Delta E_0 \) energy loss. In the text book by Segre\(^9\), \( \Delta E_p \) and \( \Delta E_{av} \) have been given by

\[
\Delta E_p = \frac{2\pi Ne^4 zx}{m_0c^2} \left( \log \frac{x}{a_0} - 0.37 \right) \quad (9)
\]

\[
\Delta E_{av} = \frac{2\pi Ne^4 zx}{m_0c^2} \left( \log \frac{E}{2m_0c^2T} + \frac{1}{8} \right) \quad (10)
\]
However, since these formulas (9) (10) are not exact, $\Delta E_p$ and $\Delta E_{av}$ in water are obtained by using the ratio of (11) and (10), and $(s/p)_{E, w}$.

\[
\Delta E_p = \frac{\log \frac{x}{a_e} - 0.37}{\log \frac{E_0}{2m_e c^2 I_{ho}} + \frac{1}{8}} \cdot (s/p)_{E, w} \tag{11}
\]

\[
\Delta E_{av} = \frac{(s/p)_{E, w}}{v} \tag{12}
\]

From formulas (5), (8), (11), (12), the electron energy with the $k$th collision is given by

\[
E_{k}(SF) = E_{(k-1)} - \Delta E(SF)
\]

\[
= E_{(k-1)} - (s/p)_{E, w} \cdot \left( \frac{\log \frac{1}{E_{k-1}} - 0.37}{\log \frac{E_{k-1}}{2m_e c^2 I_{ho}} + \frac{1}{8}} \right)
- \log SF \cdot \left( 1 - \frac{\log \frac{1}{E_{k-1}} - 0.37}{\log \frac{E_{k-1}}{2m_e c^2 I_{ho}} + \frac{1}{8}} \right) \tag{14}
\]

where $a_e=0.52917 \times 10^{-8}$ cm (Bohr radius)

$m_e c^2=0.510976$ MeV (rest energy of an electron)

$I_{ho}=65.1$ eV (mean excitation energy for water)

In this calculation, SF has been changed from 1.00 to 0.05 at intervals 0.05.

3. The scattering angle

As electrons traverse a scattering medium, their energy are lost by multiple interactions, but also are deflected laterally from their original paths as they make multiple discrete changes in direction. These deflections are to be the results of a single scattering, multiple scattering and plural scattering. The analytical treatment of this process is difficult without excessive simplification. Therefore, we think of a particle being scattered a certain number of times, $k$, after which it has the average direction, $\vec{\theta}_k$. For the calculation of $\vec{\theta}_k$, it is assumed that the average collision number, $m(\vec{\theta}_{st})$, of a single scattering and the plural scattering at the $k$th collision are given by

\[
m(\vec{\theta}_{st}) = \frac{\Delta(E_{k-1}) - E_k}{E_k} \tag{15}
\]

where $\vec{\theta}_{st}$ is the average angle for a single scattering and the plural scattering at the $k$th collision, and it is larger than an angle $\vec{\theta}_n$, and $a$ is a constant value. This probability for a single scattering and a plural scattering is independent from the probability for the multiple scattering. The values of angle $\vec{\theta}_n$ for the multiple scattering have been recommended by ICRU.

The path length in a medium is not equal to the depth because of the multiple scattering of the electron. The depth, $x(n)$, in which an electron has $n$ collisions is approximated as follows

\[
x(n) \approx \sum_{k=0}^{n-1} (t_a(c\cos \vec{\theta}_n)!) \tag{16}
\]
where \( t_k \) is the path length with \( k \) th collision. By using the average path length at each collision, \( t(n)/n \), the formula (16) is given by

\[
x(n) = \frac{t(n)}{n} \sum_{k=1}^{n-1} \left( \cos \delta_k \right)!
\]

(17)

The relation between the depth, \( x(n) \), and the path length, \( t(n) \), is given by the formula (17). From (5), (4) and (17), we can get the probability that an electron has \( n \) collisions in \( x(n) \) depth.

\[
P(n, x) = \frac{1}{\sqrt{2\pi x(n)/b(n)}} e^{-\left(\frac{x(n) - b(n)}{2(b(n)/dx)}\right)^2} \quad (n \neq 0)
\]

(18)

\[
P(n, x) = \frac{1}{\sqrt{2\pi x(n)/b(n)}} e^{-\left(\frac{x(n) - b(n)}{2(b(n)/dx)}\right)^2} \quad (n = 0)
\]

(19)

where

\[
b(n) = \frac{1}{n} \sum_{k=1}^{n-1} \left( \cos \delta_k \right)!
\]

(20)

\[
b(n) = 1 \quad (n = 0)
\]

(21)

The average penetration depth, \( \langle x \rangle_{av} \), in which an electron with a path length, \( t(n) \), has reached is given by

\[
\langle x \rangle_{av} = \frac{\sum_{k=0}^{n-1} \{ P(n, t; t(n)/b(n)) \}}{\sum_{k=0}^{n-1} P(n, t)}
\]

(22)

In Fig. 2, our calculation result of the average penetration depth, \( \langle x \rangle_{av} \), as a function of the path length, \( t(n) \), is shown for \( E_0 = 20 \text{ MeV} \). \( E_0 \) is the electron energy at the surface of a water medium and is called as the initial electron energy.

![Graph](image)

**Fig. 2.** The average penetration depth, \( \langle x \rangle_{av} \), as a function of the path length, \( t(n) \), for initial electron energy \( E_0 = 20 \text{ MeV} \) in water. Solid curve is from the present work, and the dotted curve from a moment method calculation by Kessaris9.
4. The primary electron distribution

We shall denote the number of the incident electrons by $N^e$. In this paper it is normalized with one incident electron ($N^e = 1$). From (18), (20), the charge rate $Q_{pm}(x)$ and current $I_{pm}(x)$, for that the primary electrons are stopped at $x$ depth to a plane surface in water, are given by

$$Q_{pm}(x) = N^e P(n_o, x)$$
$$I_{pm}(x) = -\int_x \frac{Q_{pm}(x)}{x} dx$$

(23) (24)

The values of $n_o$ and $m$ are unknown. In this calculation, we used the fitting method by using the experimental data and Berger's result for the charge rate distribution of the initial energy 20 MeV. (see Table 1 and Fig. 10) This method is the only fruitful one. The $n_o$ value is calculated from the $u$ value, and $v$ is changed with $SF$. We got $u$ value at $SF = 1.0$ by using the fitting method.

$$v = 17.0$$
(at $SF = 1.0$)  

(25)

In this calculation, $v$ value was changed with $SF$ value from 17.0 to 49.0 for the initial electron energy $E_o = 20$ MeV. The value of $n_o$ depends on $v$ and $(s/P)_{E_W}$. We got $n_o$ value with $E_o$ approximately.

$$n_o \approx 9.0 \cdot 10^{1.27 - 0.0994 \ln E_o}$$

(26)

The $m$ value depends on $a$ and $\theta_{\phi, h}$ values. In this calculation, since $\theta_{\phi, h}$ was used as a constant value ($\theta_{\phi, h} = 90^\circ$), we got $a = 4.0$ by using the fitting method.

The charge rate distribution and the current distribution for $E_o = 20$ MeV are plotted in Fig. 3 and Fig. 4, respectively.

Fig. 3. The rate of charge accumulation as a function of the depth in water for initial electron energy $E_o = 20$ MeV. The total rate which is the sum of the primary rate and the secondary rate is seen to become negative for small depths. The point A and B are respected to the peak position and the polarity altering point on the total distribution.

Fig. 4. The current distribution as a function of the depth in water for initial electron energy $E_o = 20$ MeV. The total current is the sum of the primary current and the secondary current. The point C is respected to the polarity altering point on the total distribution.
The fluence, $\Phi$, of particles is defined as quotient of $dN$ by $da$:

$$\Phi = \frac{dN}{da}$$  \hspace{1cm} (27)

where $dN$ is the number of particles which enter a sphere cross-sectional area $da$ of a volume element. For a parallel broad beam, this definition reduces to the quotient of the number of particles crossing a plane surface perpendicular. The primary electron spectrum, that is distribution of the primary electron number in energy, at depth $x$ to this plane surface, $N_{pri,E}(x)$, is given by

$$N_{pri,E}(x) \approx N_0' P_k(x)$$  \hspace{1cm} (28)

The energy spectra of the primary electrons for $E_e = 20$ MeV in water are plotted in Fig. 5. The distributions of the most probable energy $^{10}$, $(E_{p,\nu})$, and the mean energy $^{10}$, $(\bar{E})_e$, of the spectrum for $E_e = 20$ MeV in water are plotted in Fig. 6. The flux of the primary electrons at depth $x$, $\Phi_{pri}(x)$, is given by

$$\Phi_{pri}(x) = \int_0^E N_{pri,E}(x) \, dE $$

$$\approx N_0 \sum \frac{P_k}{x}$$  \hspace{1cm} (29)

The flux distribution of the primary electrons for $E_e = 20$ MeV in water is plotted in Fig. 7.

The average angular distribution of the primary electron, $N_{pri,\delta}(x)$, is given by

$$N_{pri,\delta}(x) \approx N_1 \cdot P_k(x)$$  \hspace{1cm} (30)

Fig. 5. The energy spectrum of primary electrons at various indicated depths of penetration in water. The initial electron energy is 20 MeV.

Fig. 6. Decrease of the most probable energy, $(E_{p,\nu})$, and mean energy, $(\bar{E})_e$, with the depth in water. The straight line is from the relation $^{10}$; $(\bar{E})_e = E_e (1 - x/R_p)$, where $R_p$ is the practical range. The initial electron energy is 20 MeV.
These distributions for $E_e=20$ MeV in water are plotted in Fig. 8.

The absorbed dose, $D$, is defined as the quotient of $dE$ by $dm^{13}$

$$D = \frac{dE}{dm}$$  \hspace{1cm} (32)

where $dE$ is the mean energy imparted by ionizing radiation to the matter in a volume element and $dm$ is the mass of the matter in that volume element. The depth dose at depth $x$ in water for the primary electrons $D_{pr}(x)$ is closely approximated by $^{10}$

$$D_{pr}(x) = \int_0^x N_{pr, e} \cdot (s/p)_{e, 1, A=}\text{d}E$$  \hspace{1cm} (33)

where $(s/p)_{e, 1, A=}$ is the restricted collision mass stopping power in water for electrons of energy $E$. The formula (33) is written by

$$D_{pr}(x) \approx \sum_{q=1}^{\infty} N_0 \cdot P(k, x) \cdot \frac{(s/p)_{e, 1, A=}}{\nu}$$  \hspace{1cm} (34)

In this calculation, the cut-off energy $A$ was used as $A=10$ keV and the electron energy was calculated from the formula (14). The depth dose distribution of the primary electrons for $E_e=20$ MeV is plotted in Fig. 9.

5. The secondary electrons

The assumption that fast (primary) electrons continuously dissipate their energy is invalid because large energy transfers to atom give rise to the projection of secondary electrons in electron-electron knock-on collisions. However, due to relatively short electron ranges, the part of the spectrum below 10 keV of secondary

Fig. 7. The flux distributions as a function of the depth in water with initial electron energy $E_e=10, 20, 30$ MeV. The total flux is the sum of the primary flux and the secondary flux. Distribution is resulting from an incident current of one electron.

Fig. 8. The average angular distribution of the primary electron at various indicated depths of penetration in water. The initial electron energy is 20 MeV.
electrons may be easily attenuated at the given depth. For this reason, the cut off energy $A$ in this paper is used as $A = 10$ keV. The total energy of secondary electrons at the $k$th collision of a primary electron, $E_{e,k}$, is given by

$$E_{e,k} = (s/p)_{h} \cdot w \cdot (s/p)_{h} \cdot w$$

where $(s/p)_{h} \cdot w$ is the collision mass stopping power in water.

For the secondary electron, we use the assumptions that, a) the number of secondary electrons is emitted on one collision of the primary electrons, and the number, $N_{s,k,m}(x)$, and the average energy, $E_{s,k,m}(x)$, of the secondary electrons at the depth $x$ are given by

$$N_{s,k,m}(x) = N_{p,k}(x) \cdot m$$

$$E_{s,k,m}(x) = \frac{E_{e,k}(x)}{m}$$

b) the $\delta$ rays emitted by the secondary electrons do not contribute much to the total distribution. c) the path length of the secondary electrons is expressed by the c.s.d.a. range, $r_{a}(E_{s},k,m)$; d) the scattering angle of the secondary electron, $\theta_{s,k,m}$, is given by the next approximated formula.

$$\cos \theta_{s,k,m} = \cos \theta_{s} \cdot \sqrt{\frac{E_{e,k}(x)}{E_{a}}}$$

From these assumptions, we have calculated the secondary (electron) distributions. The charge rate, $Q_{s}(x)$, and current, $I_{s}(x)$ of the secondary electrons in water are given by

$$Q_{s}(x) = \sum_{k=0}^{N_{k,m}} \left\{ N_{s,k,m}(x-r_{a}(E_{s},k,m) \cos \theta_{s,k,m}) - N_{s,k,m}(x) \right\}$$

$$I_{s}(x) = - \int_{0}^{x} Q_{s}(x) \, dx$$
where \( N_{k,h,m}(x) \) is the number of the secondary electrons that is emitted at the depth \( x \) and it is minus charge rate. \( N_{k,h,m}(x) \) is the number of the secondary electrons are emitted at the depth \( x \), and it is plus charge rate. In this calculation, we got \( m \) value by using the fitting method.

\[
m = 0.15
\]

(41)

The secondary charge rate distribution and the secondary current distribution for \( E_p=50 \) MeV are plotted in Fig. 3 and Fig. 4, respectively.

The flux of the secondary electron at depth \( x \) in water, \( \Phi_\phi(x) \), is given by

\[
\Phi_\phi(x) = \sum_{k=1}^{10} \left( \frac{r_0(E_{\phi,h,m})}{v} \right) N_{k,h,m}(x)
\]

(42)

and this distribution for \( E_p=20 \) MeV is plotted in Fig. 7.

The depth dose of the secondary electrons at the depth \( x \) in water, \( D_\phi(x) \), is given by

\[
D_\phi(x) = \int E_\phi N_{k,h,m}(x) \cdot (s/p)_{E_\phi,k,h} \cdot dE
\]

(43)

Since \( E_{\phi,h,m}(x) \) calculated from the formula (37) distribute below about 0.26 MeV for initial energy 1 to 32 MeV and the \( \delta \) rays do not contribute to the distribution, the formula (43) is closely approximated by

\[
D_\phi(x) = \sum_{k=1}^{10} \left( \frac{s/p}{v} P(k,h) \cdot E_{\phi,k,h} \right)
\]

(44)

The depth dose distribution of the secondary electrons calculated with the formula (44) is plotted for \( E_p=20 \) MeV in Fig. 9. The contribution by the secondary electrons on the total depth dose changes from about 24% on the surface to 30% on the peak depth for the initial electron energy 20 MeV.

6. The bremsstrahlung photons

The total energy of bremsstrahlung photons emitted at the \( k \) th collision of a primary electron, \( E_{\phi,k,h} \), is given by

\[
E_{\phi,k,h} = \frac{(s/p)_{E_\phi,k,h}}{v}
\]

(45)

where \( (s/p)_{E_\phi,k,h} \) is the mass stopping power for energy loss due to bremsstrahlung production in water. The spectral distributions of bremsstrahlung photons with energy are continuous. However in this paper, we assume that the photon energy can be expressed by the monoenergty, \( E_\phi \), of the primary electrons at \( k \) th collision that is given by the formula (14) with extensive simplification. The depth dose of bremsstrahlung photons that are produced at the \( k \) th collision of a primary electron, \( D_{\phi,k,h}(x) \) is approximated by

\[
D_{\phi,k,h}(x) \simeq \int_0^x (\mu_{\phi,\mu} E_{\phi,k,h} N_{\phi,\mu,\mu}(x) \cdot e^{-\mu_{\phi,\mu} x} x^{-1} dx
\]

(46)

where \( (\mu_{\phi,\mu})_{E_{\phi,k,h}} \) and \( (\mu/\rho)_{E_{\phi,k,h}} \) are the mass absorption coefficient and the mass attenuation coefficient of water for a photon with \( E_{\phi} \) energy respectively, and \( x \) is the distance from the point where a photon is produced to the point of depth \( x \). The total depth dose of bremsstrahlung photons at depth \( x \), \( D_{\phi}(x) \), is given by
\[ D_0(x) = \sum_{k=1}^{n_k} \frac{E_{\text{in},k}}{N_k \cdot P(k,x)} \cdot \sum_{\mu} \int_{\mu}^{\infty} \frac{\mu_{\text{in},k}}{E_{\text{in},k} \cdot N_k \cdot P(k,x)} \cdot e^{-\mu_{\text{in},k} \cdot x} \, dx \]  

(48)

The contribution of \( D_0(x) \) that was calculated, is plotted for \( E_0 = 20 \text{ MeV} \) in Fig. 9.

7. The total distribution

The total distribution of the charge rate \( Q(x) \), current \( I(x) \) and flux \( \Phi(x) \) are given by

\[ Q(x) = Q_{\text{in}}(x) + Q_s(x) \]  

(49)

\[ I(x) = I_{\text{in}}(x) + I_s(x) \]  

(50)

\[ \Phi(x) = \Phi_{\text{in}}(x) + \Phi_s(x) \]  

(51)

These distributions for \( E_0 = 10, 20, 30 \text{ MeV} \) are plotted in Fig. 3, 4, 7, respectively. The peak position \( A \) in Fig. 3, and positions \( B, C \) in Fig. 3, 4 where the polarity alter in the charge rate as well as current distributions, and width at half maximum of the charge rate distribution in water are shown in Table 1 and Fig. 10.

The total distribution of the depth dose, \( D(x) \), is given by

\[ D(x) = D_{\text{in}}(x) + D_s(x) - D_0(x) \]  

(52)

And, the percentage depth dose, \( D_{\text{percentage}}(x) \) is

\[ D_{\text{percentage}}(x) = 100 \cdot \frac{D(x)}{D_p} \]  

(53)

where \( D_p \) is a peak depth dose. The percentage depth dose distribution for \( E_0 = 10, 20, 30 \text{ MeV} \) in water are plotted in Fig. 9, and the depth at 50, 80, 100 per cent depth dose is plotted as a function of the initial electron energy \( E_0 \) in Fig. 11.

An indirect method of the initial electron energy determination is very often preferred using well established empirical relationship between the initial electron energy, \( E_\mu \), and the practical range, \( R_p \). In Fig. 11, \( R_p \) values that are obtained from the data of the percentage depth dose are shown, and that is expressed by the relations.

\[ \rho \cdot R_p = 0.398 E_0 - 1.79 \quad (10 \leq E_0 \leq 32) \]  

(54)

\[ \rho \cdot R_p = 0.49 E_0 - 0.65 \quad (4 \leq E_0 \leq 10) \]  

(55)

| Table 1. Calculated and experimental parameters of the charge rate distribution for initial electron energy \( E_0 = 20 \text{ MeV} \) in water |
|-------------------------------------------------|-----------------|-----------------|
| Peak Position \( (g/cm^2) \) | Full Width at Half-max \( (g/cm^2) \) |
| This result  ; calculation | 8.60 | 3.30 |
| Our result\(^{1}\) ; experiment | 8.30 | 3.60 |
| Berger\(^{1}\) ; calculation | 8.54 | 3.12 |
| Kessarist\(^{1}\) ; calculation | 7.99 | 1.74 |
| Alexander\(^{1}\) ; experiment | 8.08 | 3.95 |
| Laughlin\(^{1}\) ; experiment | 7.71 | 3.30 |
Fig. 10. The peak position A on the charge rate distribution and the positions B, C where the polarity alters or the charge rate and current distributions (see Fig. 3, 4), and width at half maximum of the charge rate distribution with initial electron energy. Solid curve is from the present work and points, O, □, △, are from our experiment\(^9\) for A, B, C positions, respectively. The dotted curve is from the present work and point, x, is from our experiment for width at half maximum of the charge rate distribution.

Fig. 11. The depth at 50, 80, 100 per cent depth dose and the practical range, \(R_p\), with initial electron energy from the present work. The straight line is the recommended \(R_p\) value from the formula (74) by ICRU\(^{10}\). The dotted line is the average depth at 80 per cent depth dose from the practical therapy machines\(^{15}\).

8. The conversion factor, \(C_E\)

In practical electron dosimetry, an ionization chamber calibrated as an exposure meter for high energy photons (\(^{60}\)Co or 2 MV X-rays) is available. The absorbed dose at the point of measurement in water phantom using an ionization chamber is given by\(^{10}\)

\[
D(x) = M(x) \cdot N_e \cdot C_E
\]

where \(M(x)\) is the instrument reading; \(N_e\) is the exposure calibration factor and \(C_E\) is the overall conversion factor to absorbed dose in water. In ICRU Report\(^{10}\), \(C_E\) is given by

\[
C_E = A_e \cdot s_{w,a} \cdot p_{w,a} \cdot \bar{\omega}
\]

where \(A_e\) is the attenuation factor; \(s_{w,a}\) is the stopping power ratio; \(p_{w,a}\) is the perturbation ratio; \(\bar{\omega}\) is the average energy expended in the gas per ion pair formed and \(e\) is the charge of the electron. Since \(C_E\) recommended by ICRU\(^{110}\) is the overall conversion factor, this value depends on the shape and the wall material of the ionization chamber, and is changed with \(s_{w,a}\) and \(p_{w,a}\) values. In this paper, supposing that the wall material is water and the chamber cavity is a small air-filled cavity (the Bragg-Gray cavity), \(C_E\) values were calculated by using values of \(s_{\text{water, air}}\) and by taking \(p_{\text{water, air}} = 1.30\). The stopping power ratio \(s_{\text{water, air}}\), which is the essential constituent of the Bragg-Gray relation, is given by
\[ S_{\text{water}}(x) = \frac{D_{\text{water}}(x)}{D_{\text{air}}(x)} \]  

(58)

where \( D_{\text{water}}(x) \) is the depth dose \( D(x) \) calculated by the formula (52), and \( D_{\text{air}}(x) \) is the absorbed dose in the small air-filled cavity at depth \( x \). \( D_{\text{air}}(x) \) is given by

\[
D_{\text{air}}(x) = \frac{\sum_{k=0}^{N_0} \left( N_0 \cdot P(k, x) \cdot \frac{1}{v} \cdot (s/p)_{k, \text{air}} \right)}{\sum_{k=0}^{N_0} \left( N_0 \cdot P(k, x) \cdot \frac{1}{v} \cdot (s/p)_{k, \text{air}} \right)} 
+ \frac{\sum_{k=0}^{N_0} \left( N_0 \cdot P(k, x) \cdot \frac{1}{v} \cdot [(s/p)_{k, \text{col,air}}] \right)}{\sum_{k=0}^{N_0} \left( N_0 \cdot P(k, x) \cdot \frac{1}{v} \cdot [(s/p)_{k, \text{col,air}}] \right)} 
+ \frac{\sum_{k=0}^{N_0} \left( \int_0^x N_0 \cdot P(k, x) \cdot (\mu_{\text{et}}/\rho)_{k, \text{air}} \cdot \frac{1}{v} \cdot (s/p)_{k, \text{Rad,air}} \cdot e^{-[(s/p)_{k, \text{Rad,air}}] x} dx \right)}{\sum_{k=0}^{N_0} \left( \int_0^x N_0 \cdot P(k, x) \cdot (\mu_{\text{et}}/\rho)_{k, \text{air}} \cdot \frac{1}{v} \cdot (s/p)_{k, \text{Rad,air}} \cdot e^{-[(s/p)_{k, \text{Rad,air}}] x} dx \right)}
\]  

(59)

where \((s/p)_{k, \text{col,air}}\) is the collision mass stopping power in air; \((s/p)_{k, \text{col,air}}\) is the restricted collision stopping power in air; \((\mu_{\text{et}}/\rho)_{k, \text{air}}\) is the mass absorption coefficient of air for a photon energy with \( E_0 \). Calculated values of \( C_E \) by taking \( A_0=0.985 \), \( \rho_{\text{water,air}}=1.00 \), \( \mu/\rho = 0.569 \) and using formulas (57) (58) are shown in Fig. 12 for \( E_0=10, 20, 30 \) MeV and in Table 2 for \( E_0=4-32 \) MeV.

9. Approximation of \( \bar{\theta}_n \), \( (s/p) \), \( (\mu/\rho) \) and \( (\mu_{\text{et}}/\rho) \)

In this study, approximate estimations of the angle for the multiple scattering, the mass stopping power, c.s.d.a. range for the secondary electron, the mass absorption coefficient and the mass attenuation coefficient were made with the published pertinent data:

\[
\bar{\theta}_n = \sqrt{3.39 \cdot \rho \cdot l \cdot E^{-0.394} \ln (\rho^{-1/0.98}) + 50.0 \cdot e^{-0.78 E + 833.0 \cdot e^{-2.3E}}}
\]

(60)

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Fig. 12. Comparison of \( C_E \) values between our calculated result and a moment method result of Kessaris that is recommended by ICRU.
<table>
<thead>
<tr>
<th>Depth in Water (d/cm)</th>
<th>Initial Electron Energy E_e/MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.920</td>
</tr>
<tr>
<td>1.0</td>
<td>0.983</td>
</tr>
<tr>
<td>1.5</td>
<td>0.954</td>
</tr>
<tr>
<td>2.0</td>
<td>0.955</td>
</tr>
<tr>
<td>2.5</td>
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</tr>
<tr>
<td>3.0</td>
<td>0.953</td>
</tr>
<tr>
<td>3.5</td>
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</tr>
<tr>
<td>4.0</td>
<td>0.943</td>
</tr>
<tr>
<td>4.5</td>
<td>0.938</td>
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<tr>
<td>5.0</td>
<td>0.933</td>
</tr>
<tr>
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<tr>
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<tr>
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<tr>
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<tr>
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<td>0.912</td>
</tr>
<tr>
<td>8.5</td>
<td>0.909</td>
</tr>
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<td>0.907</td>
</tr>
<tr>
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<tr>
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<td>0.900</td>
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<tr>
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</tr>
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<tr>
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</tr>
<tr>
<td>17.5</td>
<td>0.892</td>
</tr>
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</table>
where $\theta$ is expressed with radian unit; $\rho$ is the density of water ($\rho_{\text{H}_2\text{O}}$); $l$ is the path length given by $l/\nu$ in this paper; $E$ is the electron energy given by the formula (14)

\[
(s/\rho)_{E,\text{water}} = 0.04671|E-1.4|^{0.375} + 1.866 + 0.3e^{-0.5E} + 9.34e^{-27.0E} \quad (61)
\]

\[
(s/\rho)_{E,\text{col},E/2,\text{water}} = 0.018|E-1.5|^{0.3}e^{-0.216(l)} + 1.852 + 1.919e^{-4.615E} + 3.7e^{-12.6E} \quad (62)
\]

\[
(s/\rho)_{E,\text{col},E/2,\text{air}} = 0.035|E-1.3|^{0.3}e^{-0.3115|E-1.3|^{0.3}} + 1.650 + 2.850e^{-6.6E} \quad (63)
\]

\[
(s/\rho)_{E,\text{KoV},\text{water}} = 0.013E^{-1.138} + 0.004 \quad (64)
\]

\[
\text{with Berger's data}^9
\]

\[
(s/\rho)_{E,\text{col},E/2,\text{water}} - (s/p)_{E,\text{col},E/2,\text{KoV},\text{water}} = 0.0799 \ln E + 0.396 \quad (67)
\]

\[
\text{with Berger's data}^9 \text{ and ICRU Report 16}^{112}
\]

\[
r_0 = 0.446E^{0.1412}e^{0.0085|E-0.02389}e^{-0.0435|E-0.4|} \quad (38)
\]

\[
= 0.577E^{0.1415} \quad (E \leq 0.3 \text{ MeV}) \quad (39)
\]

\[
\text{with Berger's data}^9
\]

By using these formulas (66)–(69), an accuracy is below 1% for energy 0.1 to 35 MeV.

\[
(\mu/\rho)_{E,\text{water}} = 0.0152 + 0.00001E + 0.0174e^{-0.118E} + 0.0347e^{-0.433E} - 0.0529e^{-1.37E} \quad (70)
\]

\[
(\mu/\rho)_{E,\text{water}} = 0.0150 + 0.0225e^{-0.347E} \quad (71)
\]

\[
(\mu/\rho)_{E,\text{air}} = 0.0145 + 0.0199e^{-0.402E} \quad (72)
\]

By using (70)–(72), an accuracy is below 1% for energy 1 to 35 MeV.

**Discussion**

The practical circumstance of the electron beams in the betatron and the linear accelerator are usually more complex than assumed in this calculation. However, we believe that our calculational approach may be of help to dosimetry for electron-beam radiation therapy.

In this calculation, $\nu$ has been used as the average number of collisions in unit path length, and this value has been obtained by using the fitting method. However, it can be considered that it is not the practical number of collisions but is only a value that decides the shape of the electron distribution. From this consideration, $n_0$ value calculated from the formula (25) is not the practical collision number to stop an initial electron. The same discussion can be applied to $\sigma$ and $\mu$ values. The width at half-maximum and the peak position $A$ on the charge rate distribution depend mainly upon the values of $\nu$ and $\sigma$, respectively. The number of secondary electrons and the positions $B$ and $C$, where the polarity alter on the change and current distribution, depend
mainly upon the \( m \) value.

The comparisons of the charge rate and current distributions in water for the initial energy of 20 MeV, that include the data on Berger's report\(^6\) and our experimental data\(^3\) as well as calculated values from the present work, are given in Table 1 and Fig. 10. The agreement of the present work with experimental data is not perfect, because it is very difficult to satisfy the monochromatic condition in the experiment. The comparison clearly indicates that values of Kessaris are different from other data. The reason is that in continuous slowing down approximation on the moment method by Kessaris, it was assumed that the electrons lose energy continuously along their path so that their energy is a deterministic rather than a stochastic function of the path length traversed, the energy loss at each point of their track being assumed equal to the mean loss. In this paper, we have taken into account energy-loss fluctuations (straggling). This difference is shown in Fig. 2.

The most probable energy, \( (E_p)_w \) of electrons in carbon, measured by Harder\(^9\), decreases linearly with absorber thickness. However, \( (E_p)_w \) in water, calculated in this work, does not decrease linearly. (In Fig. 6) On the contrary, the mean energy of spectrum, \( \langle E \rangle \) in water decreases linearly and is approximately expressed by the relation recomended by ICRU\(^10\):

\[
\langle E \rangle = E_0 \left( 1 - \frac{x}{\rho \cdot R_p} \right) \tag{73}
\]

where \( \rho \cdot R_p \) is the practical range given by the formula (54) (55).

Secondary electrons are produced not only as the result of a knock-on collision but also as the result of an interaction of a bremsstrahlung photon with water. In this paper we neglected the later because the contribution of bremsstrahlung photons in the depth dose is not large.

The comparison between the depth at 80 per cent depth dose obtained from our calculation and the average depth\(^15\) at this depth; dose obtained by the practical therapy machines is shown in Fig. 11. In this comparison, it is clear that such differences can be attributed to contamination of the beam by scattered electrons originating from the therapy machine and its accessories, especially the collimator device. Since the depth dose curves in electron radiotherapy show variations by scattered electrons, the own data of the depth dose curves for practical radiotherapy must be obtained by a dosimeter.

The relations between \( E_0 \) and \( R_p \) in water was expressed by the formulas (54) (55). However, ICRU\(^10\) have recommended the next formula.

\[
\rho \cdot R_p = 0.521 \cdot E_0 - 0.576 \tag{74}
\]

This formula was derived from the experimental data of Markus, B. On the other hand, Hoshino, K., et al.\(^16\) have experimentally determined the relation between energy and \( R_p \) for 10-30 MeV electrons, and reported that the relation between \( (E_p)_w \) and \( R_p \) may be approximated by a different formula, and the relations between \( E_0 \) or \( E_w \) and \( R_p \) will not be given by a single formula. These difference are resulted from the beam condition, especially, from that our calculated beam condition is monoenergetic. Thus, in the practical dosimetry, this relation between energy and \( R_p \) must be used with care.

In this paper, \( C_v \) values have been calculated by using the Bragg-Gray relation. However, in practical cases the Bragg-Gray condition is not met completely and the contributions made to \( D_{av}(x) \) by secondary electrons produced either in air (probe material), in the wall material of the chamber and in water (surrounding media) must be considered. It has been the aim of the cavity theory by Burlin\(^15\) to calculate these boundary effects and thereby to drive the stopping power ratio, especially for wall-less air cavity embedded in various media. However, this theory is not perfect\(^10\). For practical purposes, ICRU\(^16\) recommended two ways of suppressing
such boundary effects; a) the atomic composition of the sensitive probe material of the wall should closely resemble the composition of the material of the surrounding medium, or b) the probe wall may take over the function of suppressing boundary effects. Therefore, it can be more useful to calculate $C_E$ values by supposing the Bragg-Gray Cavity. Fig. 12 shows the comparison of $C_E$ values between our calculated result and a moment method result of Kessaris recommended by ICRU\textsuperscript{10}. The agreement is good except values at large depths. Otherwise, Antoku, S., et al.\textsuperscript{19} has experimentally determined $C_E$ values by using a Frick dosimeter and reported that these have been also agreed with Kessaris data. However, a; large depths, these were dependent on the effective center position of the chamber. There are no data at the depths deeper than $r_e$ in Kessaris report. This difference at large depths is resulted from that Kessaris has calculated by the simplified function-fitting method with a Wick-type argument\textsuperscript{20} at large depths.

**Acknowledgement**

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