Efficiency Analysis Based on DEA from Multiple Perspectives

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Preface

Data envelopment analysis (DEA) is a powerful nonparametric methodology in analyzing the efficiency of a group of decision making units (DMU), which have similar constructions. The research object “DMU” can be considered as stores, hospitals, plants, departments *etc*. Conventional parametric analysis methods are meeting difficulties in confirming appropriate parameters in analysis, which are usually mixed with human factors affecting the validity of analysis results, whereas the appearance of DEA dramatically improved the weakness of parametric analysis methods. The analysis results obtained by DEA is impersonal preponderant, and thus more meaningful to the real world.

DEA is currently playing a vital role in financial world, such as analyzing stocks, futures, and banking efficiency. The analysis process mainly includes two procedures, namely, efficiency evaluation and efficiency improvement. The efficiency evaluation about banking industry has been mentioned in many DEA studies where most of them concern the efficiency ranking, clustering analysis and application of existent DEA models. As bank is a very complicated object with many financial factors, it is difficult to analyze the efficiency of banks from only one input/output classification method. Thus we attempt to analyze banks from different perspectives which have different understanding about the attributes of bank. Another aspect of our research concerns the procedure of efficiency improvement which is rarely surveyed in related studies. As different perspectives have different understanding about the same bank, it is rather difficult to seek an optimal approach to improve the efficiency of a bank. The adjustment of an attribute may satisfy one perspective, but at the same time incurs discontentment of another perspective. How to give attention to multiple perspectives, and seek an appropriate improving scheme is the kernel mission of the current research.

In this thesis, we employ Nash bargaining game (NBG) theory to evaluate a group of banks from the perspectives of management, customer, stakeholder and employee. The evaluation DEA model gives an identical weight assignment scheme which might be meaningful to guide the reformation of the banks sectors in future, moreover, the evaluation results concern different perspectives playing distinctive roles in affecting the efficiency of the bank. As to the improvement for banks, we try to figure out an outlet for each inefficient bank enclosed by multiple efficiency frontiers, so that the bank can obtain maximum efficiency considering multiple perspectives and their different market statuses.

20 Chinese banks and 65 Japanese banks are used as concrete case studies in our research. Although we only concern five attributes and several perspectives of these banks in our research, other attempts with different number of attributes and perspectives can also be carried out by readers. The nonlinear model proposed are transformed into linear one which provides approximate solutions. Actually readers can utilize other algorithms of tools besides R in solving nonlinear problems.

Acknowledgements

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I appreciate the finance support from Japanese Government (Monbukagakusho: MEXT) Scholarship Program, it creates a favorable environment for my studying and living in Japan in the past three and a half years. I also acknowledge the financial support from Morita Lab. of Osaka University, which allows me to attend several international conferences and learn a lot from researchers from different countries.

Love is the most cherished thing in my philosophy of life. I appreciate the love from all my friends, even if we may not keep in touch frequently. I appreciate the love from all the people who helped or paid attention to me, although we may even not remember the names of each other clearly. I appreciate the love from my family, as they are always my secure backing supporting me tenderly. I also appreciate the love from my forthcoming baby, because it is his coming that kindles the light of hope for my future lovely.

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Contents

Chapter 1

Introduction to the thesis

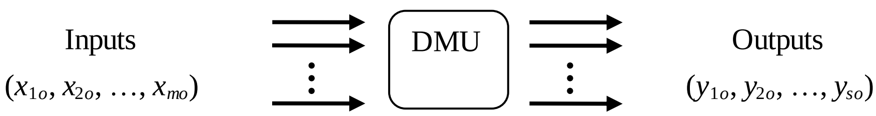
The thesis focuses on efficiency evaluation and improvement of systems from multiple perspectives by developing new approaches based on the data envelopment analysis (DEA) and Nash bargaining game theories. The research concerns the following three aspects: (1) improve the existing efficiency evaluation model in DEA research by utilizing the desirable and undesirable attribute classification method; (2) address the arising issue of efficiency evaluation from multiple perspectives; (3) improve inefficient systems to the state of Pareto Optimality for multiple perspectives. Many numerical case studies are also given to demonstrate the advantages of our research.

Chapter 1 begins with introducing the background about DEA, the concept of perspective and NBG theory.

1.1 Background on DEA

Data Envelopment Analysis (DEA) is initially developed by Charnes, Cooper and Rhodes [1] based on the work of Farrell [2] and others. It has become a comprehensive research field which intersects management, finance, mathematics, computer science, *et al* [3-13]. As a powerful nonparametric tool to evaluate and compare the relative efficiencies of a collection of entities, namely "Decision Making Units" (DMUs) with similar properties, numerous researchers are focusing on improvement of various DEA models or actual applications of concreted methodological models.

In DEA literature, DMU is defined as a black box structure consisting of two parts, namely inputs and outputs. The main mission of DMU is producing outputs with inputs, whereas we do not concern about the interior function of the DMU. For each DMU being evaluated, we denote it as DMU_o and suppose that there are m inputs and s outputs which can be denoted by $(x_{1o}, x_{2o}, \ldots, x_{mo})$ and $(y_{1o}, y_{2o}, \ldots, y_{so})$ respectively.



A general assumption in DEA is that we do not know each attribute plays a what kind of role in affecting the efficiency of DMU$_o$. We assume that inputs and outputs have two sets of weights like (v_1, v_2, …, v_m) and (u_1, u_2, …, u_s) in deciding the efficiency of DMU$_o$. Thus its virtual inputs and outputs can be written in a weighted format as follows.

$$\text{Virtual input} = v_1\, x_{1o} + v_2\, x_{2o} + \ldots + v_m\, x_{mo}$$
$$\text{Virtual output} = u_1\, y_{1o} + u_2\, y_{2o} + \ldots + u_s\, y_{so}$$

As the weights are unknown and we want to maximize the efficiency of DMU$_o$, the process of obtaining the efficiency score of DMU$_o$ is determined by maximizing the ratio

$$\frac{\text{virtual output}}{\text{virtual input}}$$

The optimal weights may vary with different DMUs and the main aim in DEA research is obtaining the optimal weights that maximizing the efficiency for given DMU.

Before introduction of substantial DEA models, we would like to introduce two simple examples to show some other important concepts in DEA and the necessity of DEA. We list eight corporations (labeled from A to H at the head of each column) in the following Table 1.1 where we assume that each corporation has one input and one output, viz. “number of workers” (measured in 100 persons) and “production value” (measured in million dollars per season).

Table 1.1 An example of eight corporations with one input and one output

Corporation	A	B	C	D	E	F	G	H
Number of workers	4	3	3	2	8	6	5	5
Production value	3	2	3	1	5	3	4	2
Productivity	0.75	0.667	1	0.5	0.625	0.5	0.8	0.4

The input “number of workers” and output “production value” for each corporation are listed in each column. The efficiency of a corporation is often expressed by productivity which is calculated as production value per worker, as shown in the last row in Table 1.1. We plot the eight corporations in Figure 1.1 where the slope of line passing through each point and the origin indicates the productivity of the corporation. We can find out that corporation C has the highest productivity comparing with other corporations therefore the line passing through C and the origin is defined as “efficient frontier” in DEA research. Any other points having lower or the same productivity are located in the area under or on the line passing C and the origin, which is called *production possibility set* (PPS). All data points are enveloped in PPS. That is also why such data analysis methodology is named data envelopment analysis.

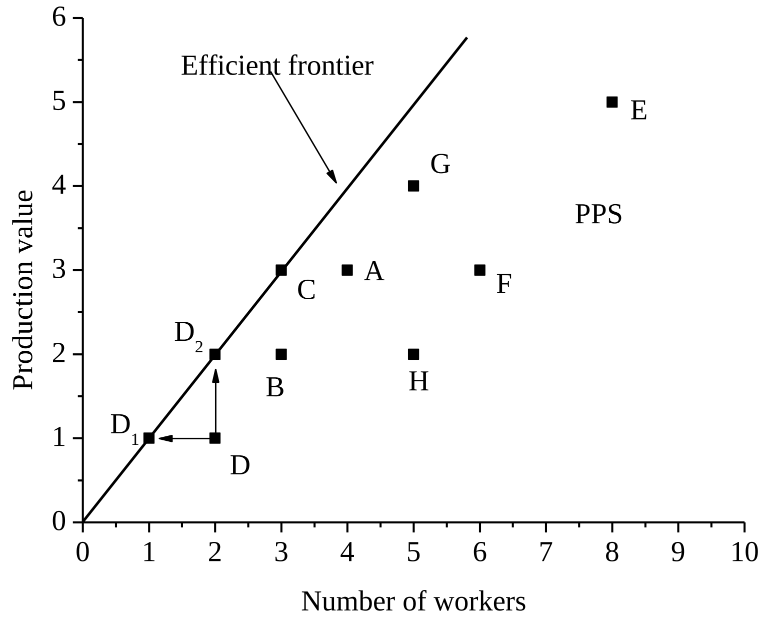


Figure 1.1 Improvement of corporation D

For an inefficient corporation like D the improving scheme is provided. As shown in Figure 1.1, there are many ways to improve corporation D to be efficient. One is achieved by reducing the input (number of workers) from 2 to 1 keeping the output (production value) invariable, namely moving D to D_1. Another way is keeping the input invariable and raising the output from 1 to 2. Actually any points on the segment D_1D_2 can be considered as improving targets for D. But if we improve D to the points on D_1D_2 other than D_1 and D_2, we have change the input and output simultaneously.

The above example is a beginning case in efficiency analysis about a group of data points, through which some basic concepts in DEA are introduced. We will continue to show a simple example with two inputs and one output. As shown in Table 2, nine corporations are listed from A to I in the first row. Each corporation has two inputs “number of workers” and “number of branches” (unit: 100 persons), and one output “production value” (unit: million dollars) which are listed in the following rows. In order to plot these points in a two dimensional plane, the production value of each corporation is unitized to 1 under the *constant returns to scale* (CRS) assumption, which means a constant ration between input and output. Therefore input values are normalized to values for getting 1 unit of production value. The nine corporations are plotted in Figure 1.2 where the horizontal axis is defined as number of workers per production value, and the vertical axis is defined as the number of braches per production value.

Table 1.2 An example of nine corporations with two inputs and one output

Corporation	A	B	C	D	E	F	G	H	I
Number of workers	4	4	2	6	7	7	3	8	5
Number of branches	3	2	4	2	3	4	4	1	3
Production value	1	1	1	1	1	1	1	1	1

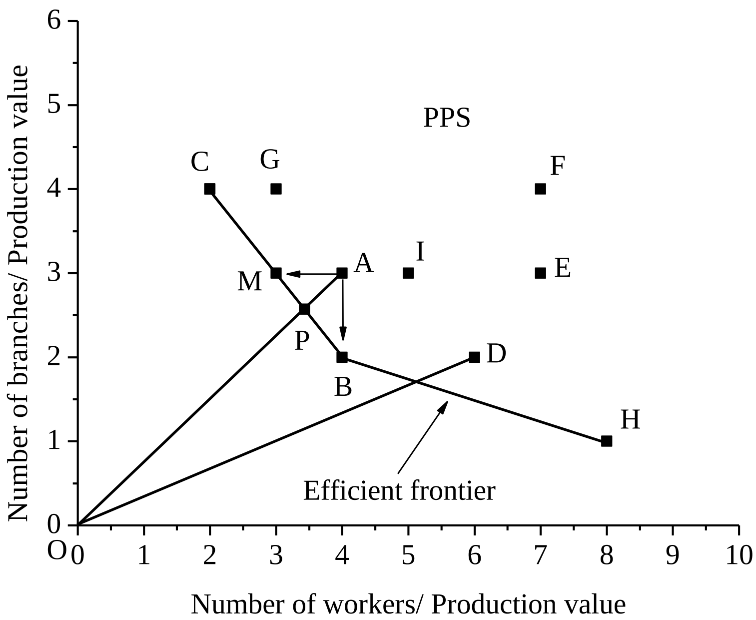


Figure 1.2 Improvement of corporation A

Usually a corporation with fewer workers and branches while producing one unit value is considered to be more efficient. Given this, the efficient frontier in this example consists of the segments connecting C and B, and the segment connecting B and H. All data points are enveloped in the area enclosed by the efficient frontier. We assume that the points located on the efficient frontier gain the highest efficiency score equaling to 1, thus other inefficient points enclosed by the efficient frontier can be measured referring to the efficient frontier. Take A as an example, if we connect the origin O and A, the line intersects line CB at point P. The coordinate of P is (3.429, 2.571) which can be obtained by the equations of line CB and OA. Let length of segment OA denote the inefficiency of point A, thus the efficiency score of A can be denoted as follows.

$$\frac{\text{OP}}{\text{OA}} = \frac{d(\text{O, P})}{d(\text{O,A})} = \frac{\sqrt{3.429^2 + 2.571^2}}{\sqrt{4^2 + 3^2}} 0.857$$

which means corporation A is assessed by the linear combination of efficient corporations C and B, which is called the *reference set* in DEA literature. Different corporations may have different reference set. For example, the reference set of corporation D consists of B and H, as the line connecting the origin and D intersects the efficient frontier at the segment BH.

As to the improvement for an inefficient corporation like A, we can either move A to point M through decreasing the number of workers or move A to point B through decreasing the number of branches. Any other points between MB are considered to be possible improving schemes for corporation A, whereas the shortest way to improve A is moving A to point P, which is the intersection point of the line CB and the line connecting the origin and A. The improving process can also be interpreted from another viewpoint as follows

$$0.857 \times A(4,3) = P(3.429, 2.571)$$

which means corporation A has to decrease both of its inputs by 0.857 to bring coincidence with the coordinate of P, the point located on the efficient frontier used to evaluate A.

There are also many other cases with multiple inputs and multiple outputs we may meet in efficiency analysis, which are impossible to plot on a two dimensional plane to analyze geometrically. In such cases, we need to expand the concepts we mentioned in the above two examples to develop a more general methodology, namely DEA. DEA can provide evaluation for a collection of DMUs with similar inputs and outputs. Based on the evaluation result DEA portrays the efficiency frontier of these entities, and then present the improving approach towards benchmark for DEA inefficient DMUs.

Back to the beginning of Section 1.1, following the definitions of DMU and virtual input and output, we start the introduction of the first DEA model, CCR model, which is the abbreviation of three authors' last names Charnes, Cooper and Rhodes, and considered to be the basis of DEA theory. Suppose that there are n DMUs each with m inputs and s outputs: DMU_1, DMU_2, …, DMU_n. Basically the selection of data set for these DMUs should comply with the following rules:

a. Numerical data are available for each input and output, with the data assumed to be positive for all DMUs.

b. The items (inputs, outputs and the choice of DMUs) should reflect an analyst's or a manager's interest in the components that will enter into the relative efficiency evaluations of the DMUs.

c. In principle, smaller input amounts are preferable and larger output amounts are preferable so the efficiency score should reflect these principles.

d. The measurement units of the different inputs and outputs need not be congruent. Some may involve number of persons, or areas of floor space, money expended, etc.

Suppose that the inputs and outputs for the j_{th} DMU is denoted by vectors (x_{1j}, x_{2j}, …, x_{mj}) and (y_{1j}, y_{2j}, …, y_{sj}), respectively. The inputs and outputs for all DMUs in the current system can be expressed by the following two matrixes $\boldsymbol{X}$ and $\boldsymbol{Y}$.

$$\boldsymbol{X} = \overbrace{\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}}^{\text{Inputs of } n \text{ DMUs}} \qquad \boldsymbol{Y} = \overbrace{\begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{s1} & y_{s2} & \cdots & y_{sn} \end{pmatrix}}^{\text{Outputs of } n \text{ DMUs}}$$

For the DMU$_o$ being evaluated, utilizing the definitions of virtual input and output, its efficiency score is expressed by the following fractional programming problem, of which the objective function captures the optimal set of weights for the inputs and outputs in the process of maximizing the efficiency of DMU$_o$.

$$\begin{aligned} \max_{\boldsymbol{v},\boldsymbol{u}} \ \theta = & \frac{u_1 y_{1o} + u_2 y_{2o} + \ldots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \ldots + v_m x_{mo}} \\ s.t. \quad & \frac{u_1 y_{1j} + u_2 y_{2j} + \ldots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \ldots + v_m x_{mj}} \leq 1 \ (j = 1, 2, \ldots, n) \\ & v_1, v_2, \ldots, v_m \geq 0 \\ & u_1, u_2, \ldots, u_s \geq 0 \end{aligned} \tag{1.1}$$

where vectors $\boldsymbol{v} = (v_1, v_2, \ldots, v_m)$ and $\boldsymbol{u} = (u_1, u_2, \ldots, u_s)$ are defined as sets of weights for the inputs and outputs of DMU$_o$. "o" ranges over 1, 2, …, n, which ensures that efficiency evaluation executes for each DMU in the system. The first constraint ensures the ratio between weighted inputs and outputs should not exceed one, which assumes that efficiency score for each DMU is always under (or equal to) one. The second and the third constraint assume that all weights should be nonnegative, which is in accord with actual situation.

The efficiency scores of all DMUs can be obtained through the above CCR model. The optimal weights while a DMU is attaining its efficiency score are also obtained. But the objective function and the first constraint are nonlinear, which is difficult in actual calculation. But it can be transformed into to a solvable linear one as follows.

$$
\begin{aligned}
&\max_{\boldsymbol{v},\boldsymbol{u}} \; u_1 y_{1o} + u_2 y_{2o} + \ldots + u_s y_{so} \\
&s.t. \quad v_1 x_{1o} + v_2 x_{2o} + \ldots + v_m x_{mo} = 1 \\
&\qquad u_1 y_{1j} + u_2 y_{2j} + \ldots + u_s y_{sj} \le v_1 x_{1j} + v_2 x_{2j} + \ldots + v_m x_{mj} \\
&\qquad (j = 1, 2, \ldots, n) \\
&\qquad v_1, v_2, \ldots, v_m \ge 0 \\
&\qquad u_1, u_2, \ldots, u_s \ge 0
\end{aligned}
\tag{1.2}
$$

where the objective function is changed as the numerator of the former one, and a new constraint which ensures the former denominator equalling to unit is added. It has been proved that Eq. (1.2) equals to Eq. (1.1), which we will not give more description here. Also the first constraint in Eq. (1.1) is transformed from a fractional form to an inequation. In the above two examples, the units we used include "100 persons", "1 branch" and "million dollars". An important feature of the model is units invariance. Thus in the above examples we can also use "1000 persons", "10 branches" and "1000 dollars" or any other units, but the calculation results are the same.

We talked about the production possibility set (PPS) in the mentioned examples. Suppose that a system including n DMUs and each can be denoted by $(\boldsymbol{x}_j, \boldsymbol{y}_j)$ (j = 1, 2, …, n), where the vectors $\boldsymbol{x}_j$ and $\boldsymbol{y}_j$ are the inputs and outputs for DMU j, and all inputs and outputs should be nonnegative. The properties of PPS are summarized as the following four points.

a. All observed DMUs belong to PPS.

b. If a DMU ($\boldsymbol{x}, \boldsymbol{y}$) is included in PPS, the DMU ($k\boldsymbol{x}$, $k\boldsymbol{y}$) is also in PPS for any positive scalar k. This property is also named as *constant returns to scale* assumption.

c. For a DMU ($\boldsymbol{x}$, $\boldsymbol{y}$) enclosed in PPS, any DMUs with inputs no less than **x** in any components and with outputs no greater than y in any components are also enclosed in PPS.

d. Any semi-positive linear combination of DMUs in PPS also belongs to PPS. Here semi-positive means all values are nonnegative but at least one should be positive.

Based the properties of PPS above, the concept of PPS is defined as follows.

$$PPS = \{(\boldsymbol{x}, \boldsymbol{y}) \mid \boldsymbol{x} \geq \boldsymbol{X\lambda}, \boldsymbol{y} \leq \boldsymbol{Y\lambda}, \boldsymbol{\lambda} \geq 0\}$$

where $\boldsymbol{\lambda}$ is a semi-positive vector $\boldsymbol{R}^n$.

Basically there are two forms of a DEA model in DEA research, namely multiplier form and envelopment form. Using vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ for input and output multipliers respectively, the multiplier form of CCR model is

$$\begin{aligned}
&\textit{Multiplier form} \\
&\max_{\boldsymbol{v},\boldsymbol{u}} \ \boldsymbol{u}\boldsymbol{y}_{\boldsymbol{o}} \\
&s.t. \quad \boldsymbol{v}\boldsymbol{x}_{\boldsymbol{o}} = 1 \\
&\qquad -\boldsymbol{vX} + \boldsymbol{uY} \leq 0 \\
&\qquad \boldsymbol{v}, \boldsymbol{u} \geq 0
\end{aligned} \tag{1.3}$$

The model (1.3) is the same as (1.2) except that (1.3) is expressed by vectors and matrixes. The dual problem of (1.3) is obtained as follows

$$\begin{aligned}
&\textit{Envelopment form} \\
&\min_{\theta,\lambda} \ \theta \\
&s.t. \quad \theta\boldsymbol{x}_{\boldsymbol{o}} - \boldsymbol{X\lambda} \geq 0 \\
&\qquad -\boldsymbol{y}_{\boldsymbol{o}} + \boldsymbol{Y\lambda} \geq 0 \\
&\qquad \boldsymbol{\lambda} \geq 0
\end{aligned} \tag{1.4}$$

which is called envelopment form of CCR model. Variables $\boldsymbol{v}$ and $\boldsymbol{u}$ are replaced by variable θ and a nonnegative vector $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)^{\mathrm{T}}$ in model (1.4). We talked about the concept of reference set in the second example. Actually the values of the components of vector $\boldsymbol{\lambda}$ correspond with the reference set of DMU$_o$ under evaluation, which can be defined as follows.

$$R_o = \{ j \mid \lambda_j^* > 0, j = 1, 2, \ldots, n \}$$

There are two kinds of CCR-efficiency for model (1.3) and (1.4) respectively. For the multiplier form (1.3), DMU$_o$ is defined as CCR-efficiency, if $\boldsymbol{u}^*\boldsymbol{y}_o = 1$ and at least one optimal set of weights ($\boldsymbol{v}^*$, $\boldsymbol{u}^*$) exists with $\boldsymbol{v}^* > 0$ and $\boldsymbol{u}^* > 0$, otherwise DMU$_o$ is called CCR-inefficiency. The definition of CCR-efficiency implies that there exist two cases of CCR-inefficiency. The first case is $\boldsymbol{u}^*\boldsymbol{y}_o < 1$, and the second one is $\boldsymbol{u}^*\boldsymbol{y}_o = 1$, but at least one component of $\boldsymbol{v}^*$ or $\boldsymbol{u}^*$ is zero.

For the envelopment form (1.4), ($\boldsymbol{X\lambda}$, $\boldsymbol{Y\lambda}$) outperforms ($\theta\boldsymbol{x}_o$, $\boldsymbol{y}_o$) when $\theta^* < 1$. It means the inputs of DMU$_o$ $\boldsymbol{x}_o$ are simultaneously reduced to $\theta\boldsymbol{x}_o$ with the outputs unvaried. Suppose that the *input excesses* and the *output shortfalls* compared with ($\boldsymbol{X\lambda}$, $\boldsymbol{Y\lambda}$) are $\boldsymbol{s}^- \in \boldsymbol{R}^m$ and $\boldsymbol{s}^+ \in \boldsymbol{R}^s$ respectively. Vectors $\boldsymbol{s}^-$ and $\boldsymbol{s}^+$ are also named as *slack* vectors which can be defined as follows:

$$\boldsymbol{s}^- = \theta \boldsymbol{x_o} - \boldsymbol{X\lambda}, \quad \boldsymbol{s}^+ = \boldsymbol{Y\lambda} - \boldsymbol{y_o}$$

The judgment of CCR-efficiency by model (1.4) is implemented by a two phase process, which can be expressed as:

Phase Ⅰ

$$\begin{aligned} \max_{\theta,\boldsymbol{\lambda}} \quad & \theta \\ s.t. \quad & \theta \boldsymbol{x_o} - \boldsymbol{X\lambda} \ge 0 \\ & -\boldsymbol{y_o} + \boldsymbol{Y\lambda} \ge 0 \\ & \boldsymbol{\lambda} \ge 0 \end{aligned}$$

The envelopment form of CCR model is solved to obtain an optimal objective value, which is denoted by θ^* and will be used in the succeeding phase.

Phase Ⅱ

$$\begin{aligned}
\max_{\boldsymbol{\lambda},\boldsymbol{s}^-,\boldsymbol{s}^+} \quad & \boldsymbol{es}^- + \boldsymbol{es}^+ \\
s.t. \quad & \boldsymbol{s}^- = \theta^* \boldsymbol{x_o} - \boldsymbol{X\lambda} \text{ ....\textit{input excesses}} \\
& \boldsymbol{s}^+ = \boldsymbol{Y\lambda} - \boldsymbol{y_o} \text{ ....\textit{output shortfalls}} \\
& \boldsymbol{\lambda}, \boldsymbol{s}^-, \boldsymbol{s}^+ \geq 0
\end{aligned} \tag{1.5}$$

Given the preceding introduction, DMU$_o$ is CCR-efficiency if θ^* equals to one and is zero slack ($\boldsymbol{s}^{-*}$ = **0**, $\boldsymbol{s}^{+*}$ = **0**). It has been proved that the CCR-efficiency of multiplier form and envelopment form are the same, which we will not give more introductions.

As to the inefficiency of DMU$_o$, it can be summarized into two types: *technical inefficiency* (is also referred to as *radial inefficiency, weak inefficiency or Farrell inefficiency*) and *mix inefficiency*. The technical inefficiency can be reduced by decreasing inputs radially, whereas the mix inefficiency has to be reduced by changing the input proportions.

Up to this point, we have introduced the multiplier and envelopment forms of CCR model, also the judgment of CCR efficiency. Besides the basic knowledge, another concern is about the orientation of the CCR model. The models introduced until now are *input-oriented* CCR model, which means minimizing the scale of inputs while keeping the minimum scale of outputs. The *output-oriented* CCR model, having opposite views about input/output adjustment, can be defined as follows.

$$\begin{aligned}
\min_{\boldsymbol{p},\boldsymbol{q}} \quad & \boldsymbol{px_o} \\
s.t. \quad & \boldsymbol{qy_o} = 1 \\
& -\boldsymbol{pX} + \boldsymbol{qY} \leq 0 \\
& \boldsymbol{p}, \boldsymbol{q} \geq 0
\end{aligned} \tag{1.6}$$

Eq. (1.6) is the multiplier form of output-oriented CCR model, whose dual problem can be expressed as

$$\begin{aligned}
\max_{\theta,\lambda} \quad & \eta \\
s.t. \quad & \boldsymbol{x_o} - \boldsymbol{X}\mu \geq 0 \\
& \eta \boldsymbol{y_o} - \boldsymbol{Y}\mu \leq 0 \\
& \mu \geq 0
\end{aligned} \tag{1.7}$$

If we define $\boldsymbol{\lambda} = \boldsymbol{\mu} / \eta$, and $\theta = 1 / \eta$. Eq. (1.7) can be transformed into the following equation.

$$\begin{aligned} \min_{\theta,\lambda} \ & \theta \\ s.t. \ & \theta \boldsymbol{x_o} - \boldsymbol{X\lambda} \geq 0 \\ & -\boldsymbol{y_o} + \boldsymbol{Y\lambda} \geq 0 \\ & \boldsymbol{\lambda} \geq 0 \end{aligned}$$

which is the input-oriented CCR model. Thus an optimal solution of output-oriented model relates with an optimal solution of the input-oriented model, which can be expressed as $\eta^* = 1 / \theta^*$, and $\boldsymbol{\mu}^* = \boldsymbol{\lambda}^* / \theta^*$.

BCC model is utilized in Chapter 2, which can be expressed by the following equation.

$$\begin{aligned} \min_{\theta,\lambda} \ & \theta \\ s.t. \ & \theta \boldsymbol{x_o} - \boldsymbol{X\lambda} \geq 0 \\ & -\boldsymbol{y_o} + \boldsymbol{Y\lambda} \geq 0 \\ & \boldsymbol{e\lambda} = 1 \\ & \boldsymbol{\lambda} \geq 0 \end{aligned}$$

Actually BCC only adds one more constraint compared with CCR model. The constraint $\boldsymbol{e\lambda} = 1$ makes differences on two aspects compared with CCR. Firstly, it ensures that the PPS of BCC is convex. Secondly, the efficient frontier of BCC is VRS (Variable Returns to Scale) which is different with the CRS (Constant Returns to Scale) of CCR model.

There are also other different DEA models, such as ADD, SBM, FDH *et al.*, which we will not give detailed introduction due to limited space. In our research, the CCR model is utilized to incorporate with NBG and the concept of multiple perspectives.

1.2 Single perspectives vs. multiple perspectives

Besides comparison and frontier analysis, another significant bestowal from DEA should be the classifying methodology about the attributes of DMU, which has been presented by various DEA models. In traditional DEA research, efficiency analysis is based on a *single perspective*, namely, a unique input/output classification scheme about the attributes of DMU. Generally the classification scheme used to determine whether an attribute should be considered an input or an output is determined by the perspective before efficiency analysis. If the value of an attribute is considered the more the better from the perspective, it is determined as an output. On the contrary, it is considered to be an input. This input/output classification scheme may be determined by a group of specialists who are very familiar with the background of the case study, or may be determined through a mathematical method which is mainly used in methodological research. In some cases, it is difficult to evaluate a group of DMUs from a unique viewpoint.

In the case of *multiple perspectives* an attribute may play different roles from different perspectives. Take a retail store as an example, suppose that there are three attributes: "number of employees", "area" and "the average price of goods" for each store. As shown in the following Table 1.3, from the perspective of management, a store with fewer employees, smaller area and higher price may be preferred as such a store produces high profit with low investment. In contrast, from the perspective of customer they may consider a store with more employees, larger area and lower price very efficient as such a sore provides much better services and shopping environment.

Table 1.3 Input/output classifications from two perspectives

Attribute	Management	Customer
Employee	Input	Output
Area	Input	Output
Price	Output	Input

Thus different perspectives may have different input/output classifications based on their preference about the attributes of DMU. Given this, the efficiency score of DMU$_o$ estimated by the CCR model varies with different perspectives, as they would use different input/output classifications.

The process of efficiency evaluation under multiple perspectives is quite different with the case under a single perspective in many aspects, which we would like to explain through a concrete example. As shown in Table 1.4, we assume that there are nine retail stores labeled A through I at the head of each column. Each store has three attributes, namely, the number of employees (unit: 10 persons), area (unit: 1,000 m^2), and price (average retail price, unit: 100 dollars), which are as recorded in each column. We utilize the two perspectives indicated in Table 1.3, namely management and customer. The customer and management classify attributes in a perfectly contradictory manner. Note that the data in Table 1.4 is pretreated in order to plot the nine stores in a two dimensional plane. The value of attributes "Employee" and "Area" of each store are divided by the corresponding value of price of this store. Thus the price is unitized to "1", so the values of employee and area are normalized to values for one unit of price.

Table 1.4 Nine stores and corresponding attributes

Store	A	B	C	D	E	F	G	H	I
Employee	4	7	8	4	2	5	6	5.5	6
Area	3	3	1	2	4	2	4	2.5	2.5
Price	1	1	1	1	1	1	1	1	1

Table 1.5 Efficiency scores of nine stores from two perspectives

Store	A	B	C	D	E	F	G	H	I
Management	0.857	0.632	1	1	1	0.923	0.6	0.774	0.75
Customer	0.75	1	1	0.6	1	0.706	1	0.8	0.853

We take "employee/price" and "area/price" as two axes and plot the stores in Figure 1.3. The efficiency scores of nine DMUs for perspectives of management and customer are shown in Table 1.5, which are calculated respectively by input-oriented CCR model.

In Figure 1.3, the black line which is constructed by DMU *E*, *D* and *C* is the efficient frontier from the perspective of management, and the efficient frontier of customer consists of DMU *E*, *G*, *B* and *C*. This can also be verified by the results shown in Table 1.5, where the DMUs located on two efficient frontiers achieve the highest efficiency score "1" compared to other DMUs. DMU *C* and *E* are evaluated as efficient DMUs by two perspectives simultaneously, which are also reflected in Figure 1.3 as the crossing points of two perspectives.

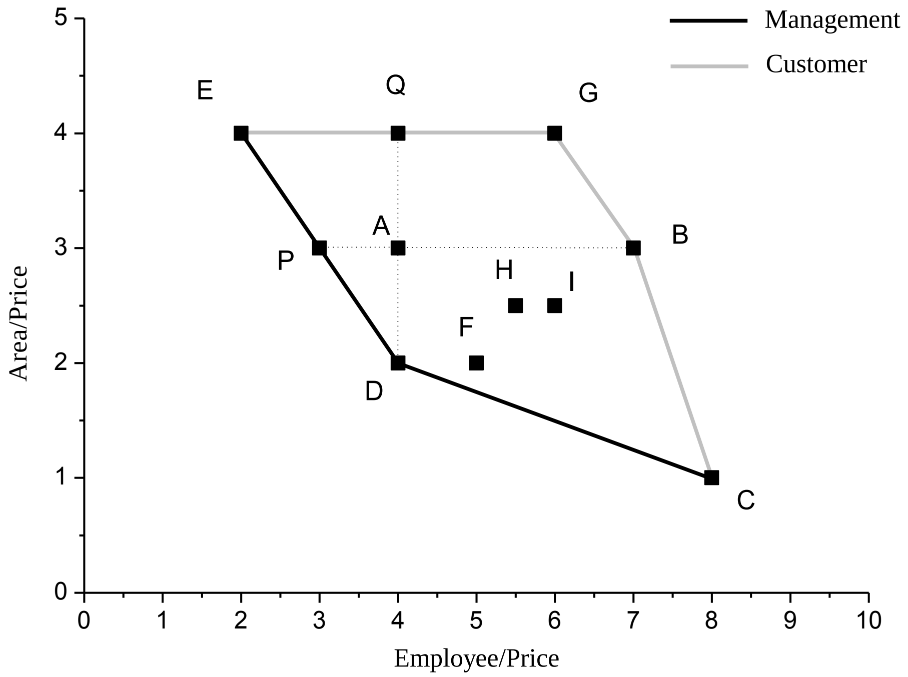


Figure 1.3 Efficiency analysis under two perspectives

From the above example we recognize the properties of efficiency evaluation under multiple perspectives in the following aspects.

a. ***Number of efficient frontiers:*** There is only one efficient frontier under a single perspective, but there exist multiple efficient frontiers under multiple perspectives.

b. ***Benchmark of efficiency evaluation:*** In the case of single perspective, DMU$_o$ obtains a unique set of optimal weight assignment which can maximize its efficiency score, but there are multiple sets of optimal weight assignments from multiple perspectives. As we can select different facets from different frontiers, it becomes rather difficult to evaluate a DMU objectively.

c. ***The area of PPS:*** The area of PPS under a single perspective is a open set starting from the efficient frontier without end points. But the PPS in the case of multiple perspectives data points are surrounded by several efficient frontiers, which narrows the scope of PPS under multiple perspectives.

d. ***The concepts of efficiency and inefficiency:*** The concepts of “efficiency” and “inefficiency” are not appropriate for case of multiple perspectives. In the case of single perspective, the efficiency of a DMU is judged by whether it is located on the efficient frontier. But in the case of multiple perspectives, a DMU may be located on one or several efficient frontiers (the crossing point of several frontiers), which is difficult to say efficient or inefficient. Whereas the DMU which is the point of intersection by all efficient frontiers, is absolutely efficient.

e. ***Efficiency improvement of DMU$_o$:*** Suppose that DMU$_o$ is not efficient for all perspectives. All DMUs need improvement in the case of multiple perspectives except the DMUs located at the crossing points of all efficient frontiers (Such DMUs are efficient for all perspectives.). The efficiency improvement for DMU$_o$ is much more complicated compare with the case under a single perspective. In the case of a single perspective, the method to effectively improve DMU$_o$ (assume that DMU$_o$ is not efficient) is to move DMU$_o$ to a point located on the efficient frontier, which is the linear combination of the points in its reference set. Basically the movement of DMU$_o$ can be summarized as either decreasing inputs while keeping the *status quo* for outputs (for input-oriented CCR model) or increasing outputs and keeping inputs (for output-oriented CCR model)[14]. But for the case of multiple perspectives, as an attribute considered to be input from one perspective may be considered to be output from another, it is difficult to determine whether to increase or decrease its value in order to improve DMU$_o$.

f. ***Reference set:*** The reference set of DMU$_o$ under multiple perspectives is difficult to define. The reference set of DMU$_o$ under single perspective is always obtained by solving Eq. (1.4), which is the linear combination of DMUs on an efficient facet. Whereas in the case of multiple perspectives, the linear combination of efficient DMUs on one efficient facet may not still locate on the facet, thus it is impossible to find out a reference set to improve DMU$_o$.

In the current research, we propose a concept of "perspective" to depict a general opinion of most stakeholders. For the domain of industrial production, the abstract concept perspective can be visualized to a type of potential market, namely a representative group of consumers. Every perspective may be corresponding with a given type of market tendency, which can guide the trend of production of an enterprise in next season. From the viewpoint of mathematical constitution, a perspective can be considered as a combination of different states of attributes. As each attribute for a DMU has 2 states, for DMUs with n attributes, 2^m perspectives exist. As the processes of efficiency evaluation and efficiency improvement are quite different with the case in traditional DEA research, the current research focuses on incorporating NBG theory to analyze the efficiency of DMUs under multiple perspectives.

1.3 Nash bargaining game

Nash bargaining game belongs to the realm of *Social Welfare Function*, which is about how to assign welfare among different individuals. The beginning research about social welfare function is a kind of earlier *neoclassical welfare theory* which only cares about maximizing the total utility of the society, neglecting balanced assignment of the welfare. Nash improved the neoclassical welfare theory by considering the two-person bargaining problem with fixed disagreement payoffs. It was considered by Nash [15] in a paper that provided the foundation of modern bargaining theory. The Nash two-person solution to this problem can easily be generalized to the n-person case.

The general two-person bargaining game may be stated as follows: Two players try to divide some good or some amount of money, and the NBG theory focuses on seeking an equilibrium solution between two players who want to divide the surplus value of cooperation. We assume two players, A and B, who want to divide the surplus value produced through cooperation. If each of these players operates his own business without cooperation, A will obtain payoff a, and B will obtain payoff b. (We also call a and b breakpoints, that means if the bargaining game does not yield an agreement.) If the players cooperate, they will obtain total value V, which is greater than $a + b$. The surplus value is generated because of their cooperation. This added value is why the players want to cooperate. The surplus value $s = V - a - b$. Here, $x = a + w_A s$, $y = b + w_B s$. Let $u_A = w_A s$ and $u_B = w_B s$ be the utility function for player A and B respectively, and w_A and w_B denote the market weights of A and B, respectively. The function of NBG takes the following form:

$$max\ u_A^{w_A} u_B^{w_B}$$

The payoff vector, $\boldsymbol{\alpha} = (u_A, u_B)$ is an element of a two dimensional *bargaining set* $\boldsymbol{P}$ which is defined as follows.

$$\boldsymbol{P} = \{(u_A, u_B) : u_A + u_B \le V - a - b, u_A \ge 0, u_B \ge 0\}$$

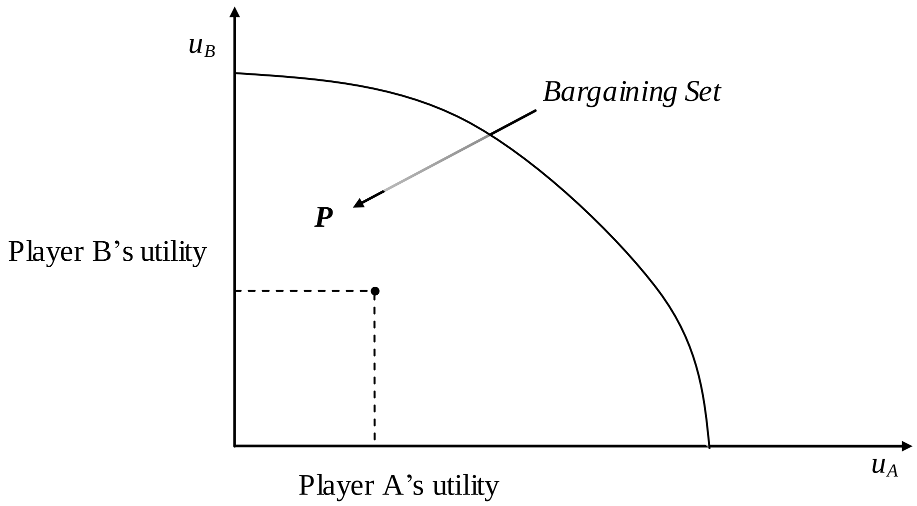


Figure 1.4 Payoff space of two-player bargaining game

The bargaining set $\boldsymbol{P}$ is assumed to be compact and convex as shown in Figure 1.4.

Suppose that vector $\boldsymbol{d}$ = (a, b) represents the point of disagreement. A bargaining solution is defined as,

$$F : (\boldsymbol{P}, \boldsymbol{d}) \rightarrow \boldsymbol{\alpha}$$

Suppose that $\boldsymbol{\alpha}^{*} = (u_A{}^{*}, u_B{}^{*})$ is the optimal solution of NBG. Nash [15] proposed that a reasonable solution should satisfy the following axioms:

a. ***Individual rationality (IR):*** No person will agree to accept a payoff lower than the one guaranteed to him under disagreement, namely $x^{*} \geq a$ and $y^{*} \geq b$ ($\boldsymbol{\alpha}^{*} \in \boldsymbol{P}$).

b. ***Pareto optimality:*** The agreement will represent a situation that could not be improved on to both persons' advantage, which means all points on the boundary of the bargaining set are *Pareto Optimal solutions*. In a bargaining situation, players would like to settle at a Pareto optimal outcome, because if they settle at an outcome which is not Pareto optimal, then there exists another outcome where at least one player is better off without hurting the interest of the other players. Pareto optimal solutions are not unique in most of the cases.

c. ***Invariant to affine transformations:*** An affine transformation $\tau : \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^2$ is defined by a matrix $\boldsymbol{A}$ and a vector b of the following form:

$$\boldsymbol{A} = \begin{bmatrix} \beta_1 & \\ & \beta_1 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}$$

Now the transformation can be defined as $\tau\,(\boldsymbol{\alpha}) = \boldsymbol{A\alpha} + \boldsymbol{b}$. A bargaining solution is invariant to an affine transformation:

$$\forall \boldsymbol{A}, \boldsymbol{b} \; if \;\; F(\boldsymbol{P}, \boldsymbol{d}) = \boldsymbol{\alpha}$$

$$then \; F(\tau(\boldsymbol{P}), \tau(\boldsymbol{d})) = \tau(\boldsymbol{\alpha})$$

d. ***Independent from Irrelevant Alternatives:*** If $\boldsymbol{\alpha}$ is the Nash bargaining solution for a bargaining set $\boldsymbol{P}$, then for any subset $\boldsymbol{Q}$ of $\boldsymbol{P}$ containing $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}$ continues to be the Nash Bargaining Solution. This axiom of Nash is slightly controversial unlike the previous axioms, since more alternatives present better bargaining power. However, this can be intuitively justified, by the following argument:

Let us consider that the set $\boldsymbol{Q}$ has a Nash bargaining solution $\boldsymbol{\alpha}'$ and $\boldsymbol{\alpha}$ be another Nash bargaining solution of $\boldsymbol{P}$ as shown in Figure 1.5. Now $\boldsymbol{\alpha}' \in \boldsymbol{Q}$, $\boldsymbol{\alpha} \in \boldsymbol{Q}$ and $\boldsymbol{\alpha}' \in \boldsymbol{P}$, $\boldsymbol{\alpha} \in \boldsymbol{P}$. In both the bargaining sets $\boldsymbol{P}$ and $\boldsymbol{Q}$, both the options $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$ are available to the players. They should be expected to settle to the same outcomes. The presence of irrelevant alternatives in $\boldsymbol{P}$ should not influence the bargaining solution. The axiom can be expressed as follows.

$$if \;\; F(\boldsymbol{P}, \boldsymbol{d}) = \boldsymbol{\alpha}$$
$$and \; \boldsymbol{Q} \subset \boldsymbol{P}, \boldsymbol{\alpha} \in \boldsymbol{Q}, \boldsymbol{d} \in \boldsymbol{Q}$$

$$\Rightarrow F(\boldsymbol{Q}, \boldsymbol{d}) = \boldsymbol{\alpha}$$

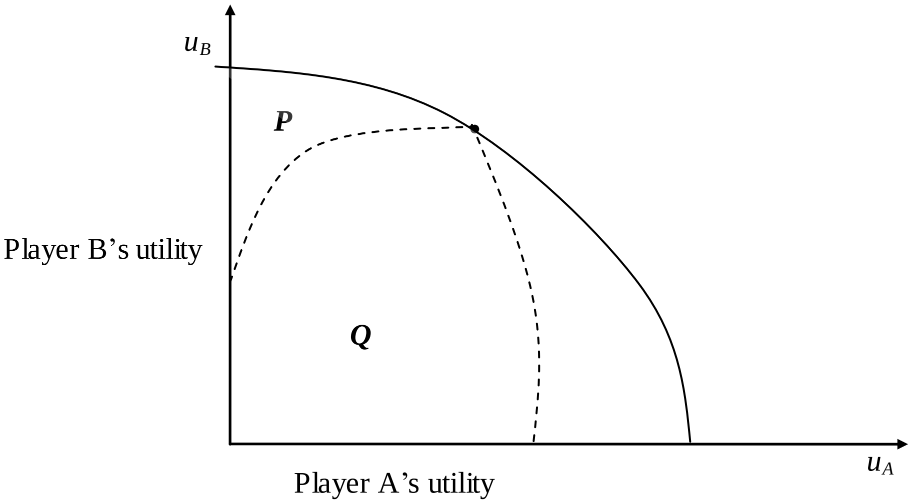


Figure 1.5 Independence from irrelevant alternatives

e. ***Symmetry:*** The principle of symmetry says that symmetric utility functions should ensure symmetric payoffs. Payoff should not discriminate between the identities of the players. It should only depend on their payoff functions. Simply put, symmetry implies the bargaining solution for region $\boldsymbol{P} = u_A + u_B \leq 1$, $u_A \geq 0$, $u_B \geq 0$, $\boldsymbol{d} = (0, 0)$, should be (1/2, 1/2) as shown in Figure 1.6. If both players have the same utility functions, then symmetry demands that both get equal payoffs.

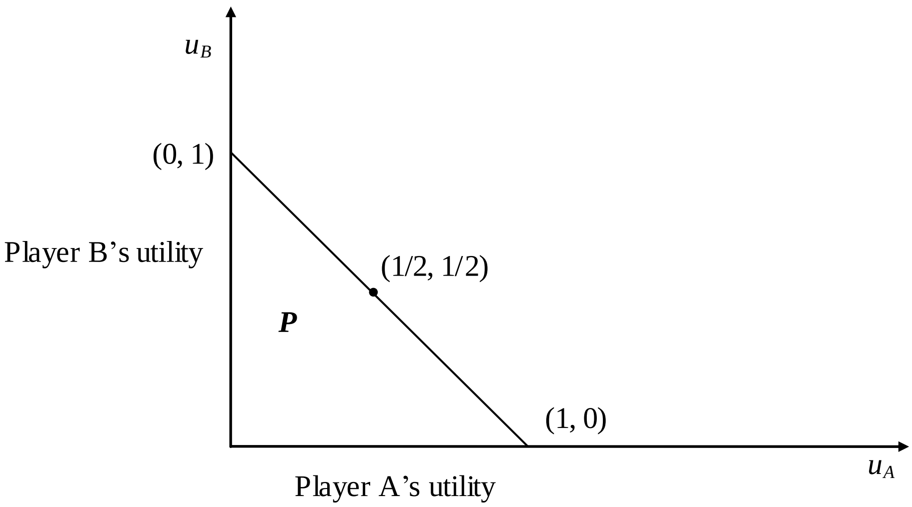


Figure 1.6 Symmetry

Nash characterized the Nash bargaining solution and proved that there is a unique solution satisfying the above axioms given by Nash. In our research, each perspective is considered to be a player, and Nash bargaining game theory is used to reconcile the conflict among multiple perspectives in the process of efficiency analysis, which we will explain the details in the succeeding chapters.

1.4 Synopsis of the thesis

The main text of this thesis consists of three parts: preliminary studies, efficiency evaluation under multiple perspectives and efficiency improvement under multiple perspectives, respectively.

Chapter 1 of this thesis illustrates the two basic methodologies we utilized in our research, namely DEA and NBG. To introduce DEA, Section 1.1 starts from two simple examples with one input and one output, two inputs and one output respectively, to explain how DEA models are created. Some concepts and definitions being used in DEA literature, such as *DMU*, *PPS*, *CRS* and *reference set* are explained to lay the foundations for the following introduction about concrete DEA models. Given this, we continue to introduce the most important DEA model, CCR, and the inefficiency constitution of it, namely technical inefficiency and mix inefficiency. Then a two-phase CCR model used to judge the efficiency of DMU is introduced. Following the basic description about CCR model, two kinds of its transformation, input-oriented and output-oriented are also introduced. As to the NBG methodology, we mainly focus on illuminating the axioms proposed by Nash.

Chapter 2 focuses on preliminary studies about CCR model, concerning the concept "multiple perspectives". This chapter is a tentative research about "multiple perspectives" in three aspects: (a) classification method about attributes of DMU, (b) iterative calculation model to obtain more precise evaluation results of DMUs, and (c) initiatory consideration about how to improve inefficient DMUs under multiple perspectives. Many methods and applications are used as preliminary attempt to solve problems in efficiency analysis under multiple perspectives, and that is why we call it "preliminaries".

In Chapter 3, we study about evaluating DMUs in the case of multiple perspectives. As each perspective tends to assign a different set of weights to the attributes of DMU that is most beneficial from its own viewpoint, there exist multiple benchmarks even in evaluating the same DMU. To reconcile the conflicts among multiple perspectives, and give an objective assessment for each DMU, this chapter proposes a DEA evaluation model based on an identical weight assignment scheme. We also rank the efficiencies of 20 Chinese banks based an identical weight assignment scheme.

Given the foregoing three chapters, we dedicate to study how to improve inefficient DMUs under multiple perspectives in Chapter 4. The NBG theory is also utilized in selecting a most appropriate direction to improve DMUs. Firstly, we propose the improving DEA model under multiple perspectives, then based on this we give the concrete calculation method in transforming nonlinear model into linear one. The chapter also follows with an application of 65 Japanese banks.

Finally, the whole thesis ends with Chapter 5 which is conclusions and subsequent research about the current thesis.

Chapter 2

Preliminaries

In Chapter 1, we introduce the theoretical basis of the research, DEA and NBG. The concepts and definitions in DEA and NBG are presented as the introduction part of the following chapters. For the DEA theory, we center around introducing the CCR model, based on which Chapters 3 and 4 are launched. For the NBG theory, we concentrate on the formation of its equation and useful axioms, which is incorporated in CCR model in succeeding chapters.

In this chapter, we reexamine the concepts "desirable" and "undesirable" that have been appeared in many DEA related research papers. Based on such classification about the attributes of DMU, there are many new problems needing to be resolved, like how to obtain a rather exact solution by solving the nonlinear model in order to evaluate DMUs more accurately. A numerical case study is given as concrete application of the method we propose.

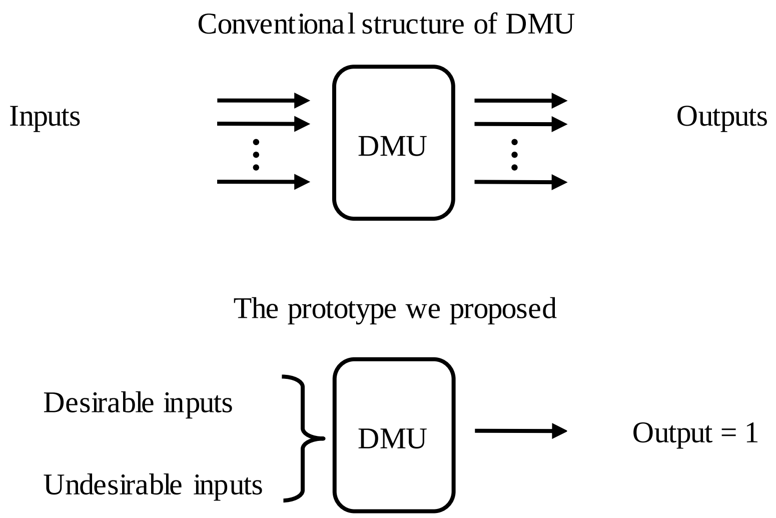
We also present a DEA model based on CCR, which is suitable for desirable and undesirable attributes of DMU simultaneously. An efficiency improvement DEA model is showed in the later part of this chapter as a preliminary attempt in improving DMUs under multiple perspectives.

2.1 Desirable and undesirable attributes

The research about classification to the attributes of DMU has met a widespread interest in DEA research [16-24]. In the paper of Bougnol *et al* [25], they assigned two trends to all attributes of a DMU, viz. "desirable" or "undesirable", which are referred to as *isotonic* and *anti-isotonic* in the terminology in Dyson *et al* [26].

In traditional DEA research, inputs of DMU are usually investment to the system, for which a smaller scale is considered to be better. The outputs of DMU are outcome of the system, for which a larger scale is considered to be better. But we sometimes meet with special situations in which some outputs of the DMU may be considered the less the better. For instance efficiency evaluation about a group of coal-fired power stations, the output power capacity is surely preferred, whereas the pollutants discharged during the process of power generation are considered not preferred. Such outputs are classified as undesirable outputs to DMU. Similarly, we sometimes face the situation with desirable inputs, such as the "equity capital" of a bank in evaluating a group of banks, which is considered to be investment to a bank but is preferred.

Actually, not for all systems we can distinguish inputs and outputs. Especially in the case of multiple perspectives, an attribute may be considered as input by one perspective, whereas considered as output by another perspective. For instance, the banking system with various financial attributes, the distinction between input and output seems to be blurred and difficult to define. As the traditional input/output oriented DEA model may not make much sense if applied to a different perspective [25], we develop a new DEA model by transforming the BCC model. Namely make the outputs of all DMUs equals to unit, and incorporate the concept "desirable" and "undesirable" into inputs. The preferred attributes are classified as desirable inputs, and the attributes playing negative roles in affecting the efficiency of the system is defined as undesirable inputs, which can be shown as the following figure.



Where the above figure is the structure of DMU in conventional DEA research, and the below one is the prototype of the DMU we proposed in the current research.

As each attribute of DMU has 2 states (desirable or undesirable), for DMUs with m attributes, 2^m combinations exist. In our research, we name the combination "perspective". As the processes of efficiency estimation and improvement are all based on special viewpoint of a corporation, organization, or maybe executive individual. In order to make the processes more objective, namely uninfluenced by given viewpoints, we propose the concept of "perspective" in this thesis to embody different perceptions from various viewpoints.

Setting the values of outputs of all DMUs as "1" and incorporating the concept "desirable" and "undesirable" into it, thus the differences generated by outputs can be neglected, so that a DMU can be considered as a system with a constant output. As an input-oriented CCR model with a single constant output coincides with an input-oriented BCC model without outputs [27], the CCR model we selected finally is transformed into an input-oriented BCC model without outputs as below.

$$\min \theta$$
$$s.t. \begin{cases} \theta \boldsymbol{x_o} \geq \boldsymbol{\lambda X} \\ \boldsymbol{\lambda e^T} = 1 \\ \boldsymbol{\lambda} \geq 0 \end{cases}$$

The model proposed above can be used to assess a group of DMUs with the same attributes of inputs, but only for undesirable inputs, namely the consumption of a DMU which is the always mentioned traditional concept in numerous DEA papers published. In order to process the desirable attributes with a similar treatment to undesirable attributes, the actual values will be changed to their opposites. We assume ω to be the desirable attributes for DMUs, then:

$$-\theta \boldsymbol{x_{o\omega}} \geq -\sum_{i=1}^{n} \lambda_i \boldsymbol{x_{i\omega}}, (\omega: \textit{positive attributes for } D)$$

Where vector $\boldsymbol{x_o}$ indicates the inputs of DMU$_o$ that is under assessment, n is the number of DMUs. The original concept of desirable attribute means the more the better. After substituted by their opposites, the actual meaning of desirable attributes can be assessed by their opposites, which means the less the better. However, due to some mathematical restraints, it is impossible to solve this model with negative values. So we have to change the inequation to the following format:

$$\theta \boldsymbol{x_{o\omega}} \leq \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_{i\omega}, (\omega: \textit{positive attributes for } D)$$

Up to this point, the desirable inputs become reverse inputs, where the only difference with the former one is the direction of the sign of inequality. The problem is changed into seeking solution for reverse inputs.

2.2 An iterative DEA model

In this section, we mainly aim at improving the existing DEA models and solving the efficiency assessment problem by utilizing an iterative method. By incorporating the concepts "desirable" and "undesirable", we transform the traditional BCC model. And we also get more precise results of efficiency evaluation by virtue of iterative computation.

Adopting the classifying method of desirable and undesirable introduced by Bougnol *et al* [25], the inputs of DMU include two parts, namely the desirable inputs (which are also named as reverse inputs) and undesirable inputs. In order to solve the reverse inputs problem, we utilize the method proposed by Lewis and Sexton [28], assuming η to be the reciprocal of θ, the BCC model incorporating desirable inputs is expressed as follows.

$$
\begin{aligned}
&\min\ \theta \\
&s.t.\quad \theta \boldsymbol{x}_{o\varphi} \geq \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_{i\varphi}, (\varphi \in U) \\
&\qquad \eta \boldsymbol{x}_{o\omega} \leq \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_{i\omega}, (\omega :\in D) \\
&\qquad \theta\eta = 1 \\
&\qquad \sum_{i=1}^{n} \lambda_i = 1 \\
&\qquad \theta, \eta \geq 0 \ and \ \lambda_i \geq 0, (i = 1, \ldots, n)
\end{aligned}
\tag{2.1}
$$

where θ is the efficiency score of DMU$_o$. Vectors $\boldsymbol{x}_{o\varphi}$ and $\boldsymbol{x}_{o\omega}$ represent the undesirable and desirable attributes of DMU$_o$ respectively, taking φ and ω as the indices of classifying undesirable and desirable attributes. Sexton *et al.* argue that the model incorporates both η and θ and adds a constraint ensures that the two variables have the proper relationship, that is, the inverse efficiency θ equals the multiplicative inverse of the efficiency score η. Thus the objective function of Eq. (2.1) can also be replaced by "Max η" to obtain a equivalent formulation. One advantage of the model (2.1) is that it balances input reductions and output enhancements simultaneously. But one disadvantage is that the model contains a nonlinear constraint, which renders the optimization process more difficult.

Model (2.1) seems to be a trim and solvable model at the first glimpse, however because of the existence of the nonlinear constraint "$\theta\eta$ = 1", which makes the model change into a nonlinear problem it is difficult to seek an optimal solution. Thus the problem is now trapped with how to solve this transformed nonlinear programming. Fortunately, Lewis and Sexton [28] proposed a linear approximate solution to this problem through first-order approximation of Taylor Series Expansion, as follows:

$$\begin{aligned}
&\theta\eta = 1 \\
&\eta = \frac{1}{\theta} = f(\theta) \cong f(\xi) + f'(\xi)(\theta - 1) \\
&f'(\theta) = -\frac{1}{\theta^2}
\end{aligned}$$

Using Taylor Series Expansion, at the point $\xi = 1$, such that:

$$\begin{aligned}
&f(1) = 1, f'(1) = -1 \\
&\therefore \eta \cong 1 - 1(\theta - 1) = 2 - \theta \\
&\theta + \eta \cong 2
\end{aligned}$$

The model now is transformed to be a solvable linear programming, that is:

$$\begin{aligned}
&\min \ \theta \\
&s.t. \quad \theta \boldsymbol{x_{o\varphi}} \geq \sum_{i=1}^{n} \lambda_i \boldsymbol{x_{i\varphi}}, (\varphi : attributes \ for U \ ) \\
&\qquad \eta \boldsymbol{x_{o\omega}} \leq \sum_{i=1}^{n} \lambda_i \boldsymbol{x_{i\omega}}, (\omega : attributes \ for \ D) \\
&\qquad \sum_{i=1}^{n} \lambda_i = 1 \\
&\qquad \theta + \eta = 2 \\
&\qquad \lambda_i \geq 0, (i = 1, ..., n)
\end{aligned} \tag{2.2}$$

This transformed model can not only be used in the process of efficiency assessment of a group of DMUs with positive undesirable and desirable attributes, but also feasible for DMUs with negative attributes, that although in this thesis we will not deal with, due to the restricts of actual meaning of attributes in the bank systems we are considering. However it should be useful in some domains. We observe that the approximating constraint $\theta + \eta = 2$ is equivalent to $\theta - 1 = 1 - \eta$. Thus, in the model (2.2), the radial increase in outputs equals to the radial decrease in inputs. In other words, variables θ and η have equal distances from 1.

The Eq. (2.2) is an approximate solution obtained from Taylor Series Expansion expanded at the point $\xi = 1$. As shown in Figure 2.1, as the relationship between θ and η is reciprocal the exact point should be on the hyperbola. If we use the equation $\theta + \eta = 2$ to approximate the nonlinear constraint $\theta\eta = 1$, the result of θ and η is accurate around the point A (1, 1). Whereas if the point is far from A (1, 1), especially at the infinite position close to coordinates, it will recur inaccurate results of θ and η.

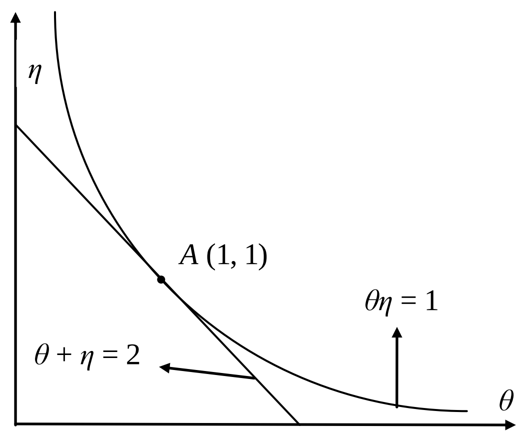


Figure 2.1 Approximation by hyperbola

In order to assess the efficiency of DMUs more exactly, in this section we propose an iterative method incorporating with BCC model, viz. *iterative DEA model* to seek precise solutions for θ and η. The process of approximation is shown in Figure 2.2. The flow will be easier to understand if we explain it by the relationship of lines and hyperbola as shown in Figure 3. We assume that the result obtained from Step 1 is point A, actually in Step 2 we can select any tangent line between L1 and L2 as the restrict condition. But not each tangent line between L1 and L2 is the most efficient, namely the least times of iteration. So we need to find the most optimal tangent line in the cluster of tangent lines. We assume line L to be the most optimal after the process of calculating with L1.

We can prove that point C can fulfill the constraint of the last step of iteration. So the value of θ we get in the next step of iteration must be less than point A. As the minimum θ is under the horizontal line L2, and the value of hyperbola H2 is less than H1, the value of the hyperbola through point C should be nearer to unit than H2. Based on the explanation above, we should select the tangent line through point B. If we select the tangent line passing through the tangent point above point B, the value of θ is augmented. Otherwise if we select the tangent line passing through the tangent point below point B, the times of iteration will probably increased.

Step1. Solve the following

$$\begin{aligned}
&\min\ \theta \\
&s.t.\quad \theta \boldsymbol{x}_{o\varphi} \geq \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_{i\varphi}, (\varphi : \textit{positive attributes for}\ U) \\
&\qquad \eta \boldsymbol{x}_{o\omega} \leq \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_{i\omega}, (\omega : \textit{positive attributes for}\ D) \\
&\qquad \sum_{i=1}^{n} \lambda_i = 1 \\
&\qquad \theta + \eta = 2 \\
&\qquad \theta, \eta \geq 0\ \textit{and}\ \lambda_i \geq 0, (i = 1, ..., n)
\end{aligned}$$

$\theta n \cong 1$

Yes → STOP

No → Loop

Step2. $\theta' = \theta$, $\eta' = \eta$, and solve the following LP

$$\begin{aligned}
&\min\ \theta \\
&s.t.\quad \theta \boldsymbol{x}_{o\varphi} \geq \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_{i\varphi}, (\varphi : \textit{positive attributes for}\ U) \\
&\qquad \eta \boldsymbol{x}_{o\omega} \leq \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_{i\omega}, (\omega : \textit{positive attributes for}\ D) \\
&\qquad \sum_{i=1}^{n} \lambda_i = 1 \\
&\qquad \theta + \theta'^2 \eta - 2\theta' = 0 \\
&\qquad \theta, \eta \geq 0\ \textit{and}\ \lambda_i \geq 0, (i = 1, ..., n)
\end{aligned}$$

Figure 2.3 Flow of iteration process

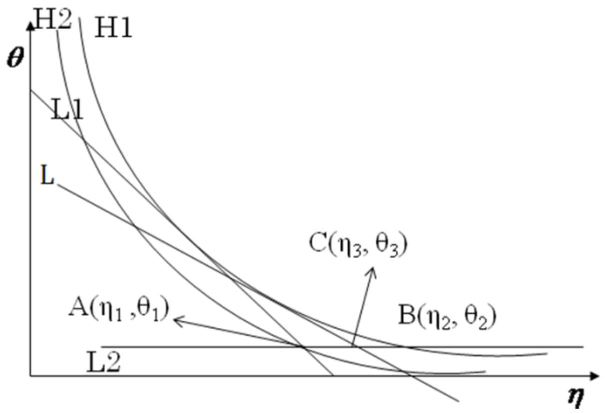


Figure 2.2 Selection of tangent line

In order to determine the weight allocation strategy for attributes of DMU, we resort to the multiplier form of the iterative DEA model. Although we assume that the outputs of every DMU equal to unit in the proposed envelopment model, to deal with the dual problem, we need to allow the output to grow or decrease. Thus we need to add the following constraint for output:

$$\sum_{i=1}^{n} \lambda_i - \eta \geq 0, (\textit{ for the output})$$

Then we give the LP_o problem as follows (Envelopment form):

$$\begin{aligned}
&\langle LP_o \rangle \\
&\min \ \theta \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.3) \\
&s.t. \quad -\sum_{i=1}^{n} \lambda_i \mathbf{x}_{i\varphi} + \theta \mathbf{x}_{o\varphi} \geq 0, (\varphi : \textit{attributes for} U \ ) \\
&\qquad\ \ \sum_{i=1}^{n} \lambda_i \mathbf{x}_{i\omega} - \eta \mathbf{x}_{o\omega} \geq 0, (\omega : \textit{attributes for } D) \\
&\qquad\ \ \sum_{i=1}^{n} \lambda_i - \eta \geq 0, (\textit{ for the output}) \\
&\qquad\ \ \theta + \theta^{*2}\eta - 2\theta^{*} = 0 \\
&\qquad\ \ \theta, \eta \geq 0 \textit{ and } \lambda_i \geq 0, (i = 1, \ldots, n)
\end{aligned}$$

$$
\begin{aligned}
&\langle DLP_o \rangle \\
&\max \ 2\theta^{*}q \\
&s.t. \quad -\sum_{i=1}^{m} x_{i\varphi}v_{i\varphi} + \sum_{i=1}^{s} x_{i\omega}v_{i\omega} + u \le 0 \\
&\qquad \sum_{i=1}^{m} x_{o\varphi}v_{o\varphi} + q \le 1 \qquad\qquad\qquad\qquad\qquad (2.4) \\
&\qquad -\sum_{i=1}^{s} x_{o\omega}v_{o\omega} - u + \theta^{*2}q \le 0 \\
&\qquad v_{\varphi}, v_{\varphi}, u \ge o \ \textit{and } q \textit{ unrestricted in sign}
\end{aligned}
$$

Where θ^{*} denotes the optimal solution of LP_o obtained from the above iterative DEA model. As the relationship between θ and η is reciprocal and $\theta \le 1$, thus $\eta \ge 1$. Also as LP_o problem contains the restrict condition $\sum_{i=1}^{n} \lambda_i - \eta \ge 0, (\textit{for the output})$ thus it is not necessary to add $\sum_{i=1}^{n} \lambda_i = 1$ in LP_o. And the corresponding dual problem is expressed as (2.4) (Multiplier form), where v_{φ} and v_{ω} are weights for undesirable and desirable inputs respectively. And u is the weight for output. Note the classification of undesirable and desirable for inputs mentioned in the above LP problems is under a given perspective. Thus the classification changes for different perspectives. Also note even for the same perspective, the value of θ^{*} varies with different DMU, as for different DMU the approximation condition $\theta\eta = 1$ is different.

2.3 Evaluating DMUs under multiple perspectives

The efficiency scores attained by envelopment form of the iterative DEA model can be used to compare efficiencies of given DMUs under the circumstance of multiple perspectives. We can take a simple example to introduce it under three perspectives. Postulating the efficiency scores of DMU_o for three perspectives are EF_{1o}, EF_{2o} and EF_{3o}, respectively, and the market weights (market status or market share) for the three perspectives are w_1, w_2, …, w_8 (Note that the efficiency score varies with different DMU, however weights not.), then we can assess the overall efficiency for all perspectives by the weighted sum or the weighted mean value of the efficiencies, such as:

$$\sum_{i=1}^{3} w_i EF_{io} \quad \text{OR} \quad \sum_{i=1}^{3} w_i EF_{io} \Big/ 3$$

Anyway we can adopt lots of other approaches to assess the efficiency of DMU$_o$ under multiple perspectives besides the methods listed above. But the results we got from the envelopment form of iterative DEA model only focus on efficiency scores, that is why even though the overall efficiency score of DMU$_o$ is the highest one, the resource (attribute) allocation scheme is different for each perspective, viz. in the multiplier form of the model the weights before inputs are not consistent for each perspective. Usually, customers pay attention to the efficiency condition of an enterprise but not the inner constitution of it, as they want to select the most efficient enterprise. Whereas the entrepreneur or manager focuses on the inner constitution of the enterprise, as they want to improve the efficiency of the enterprise through research its inner constitution. Thus it is important to study the resource allocation scheme, namely the dual problem proposed as model (2.4) in Section 2.4. In the status quo of multiple perspectives, we need to seek an identical input allocation scheme for all perspectives by virtue of the multiplier form. Moreover we should maximize the sum of efficiency scores for all perspectives in order to satisfy more customers.

From the viewpoint of multiplier form in model (2.4), it means we focus on seeking the optimal weights v_φ, v_ω and u which can maximize the sum of efficiency scores for all perspectives. Moreover the optimal weights v_φ, v_ω and u are identical for all existing perspectives, as we try to provide an improving scheme for an enterprise under the case of multiple perspectives. Suppose that there are L perspectives which are worthy of our research in the system. For DMU$_o$, we suppose it is efficient for a perspective $p \in (1, L)$, thus the efficiency score under perspective p is $\theta_p^* = 1$. As the optimal solutions of LP_o and DLP_o are identical, we get $\theta_p^* = 1 = 2\theta_p^* q_p$ by model (2.3) and (2.4), where q_p denotes the value of q under perspective p. Then $q_p = 1/2$, and according to model (2.4) we get $v_\varphi x_{o\varphi} = 1/2$ and $v_\omega x_{o\omega} + u_p = 1/2$. But in the case of multiple perspectives, as the value of v_φ and v_ω we are seeking should not only satisfy the efficient perspective p, but also satisfy all other perspectives. Namely from the viewpoint of enterprise the weights of v_φ and v_ω should maximize as more perspectives as possible. Thus the actual value of q_p we get finally may not be 1/2. Then we utilize the following DEA model to get the gross value of efficiency scores for all perspectives.

$$
\begin{aligned}
& max \;\; \sum_{k=1}^{L} 2\theta^{*}{}_{k} q_k \\
& s.t. \quad -v_{\varphi} X_{\varphi} + v_{\omega} X_{\omega} + u_k \le 0 \\
& \qquad\;\; v_{\varphi} x_{o\varphi} + q_k \le 1 \\
& \qquad\;\; -v_{\omega} x_{o\omega} - u_k + \theta^{*}{}_{k}{}^{2} q_k \le 0 \\
& \qquad\;\; \varphi \in U_k, \omega \in D_k \\
& \qquad\;\; v_{\varphi}, v_{\omega}, u_k \ge o \text{ and } q_k \text{ unrestricted in sign}
\end{aligned}
\tag{2.5}
$$

In the above DEA model (2.5), our target is seeking global value of v_{φ} and v_{ω} that can maximize the sum of efficiency scores for all perspectives. θ_k^* denotes the efficiency score of DMU$_o$ obtained from LP_o under the viewpoint of perspective $k \in (1, L)$, φ and ω denote U and D inputs respectively. And U_k, D_k denote the corresponding classifying scheme to inputs under perspective k. Note if the number of DMU is n, in the process of assessing efficiency of DMU$_o$ there are $(n$+1+1$)p$ constraints in all. The above DEA model based on multiple perspectives incorporates the merits of both envelopment form and multiplier form, which can give us a unique allocation scheme of attributes and maximize the sum of efficiency scores for all perspectives simultaneously.

As the classification for desirable and undesirable inputs differs for each perspective, U_k and D_k also vary corresponding to different perspectives, whereas the weight v before each component of x never changes for all perspectives. And this is the key differences of efficiency estimation between single perspective and multiple perspectives. Under a single perspective which is the same with various traditional DEA models proposed in DEA literature, there is no appreciable distinction between envelopment form and multiplier form, as both of the two forms supply us an approach about efficiency estimation or comparison. When we emphasize the ranking of efficiency scores of DMUs, we utilize the envelopment form, and when we emphasize the allocation of attributes, we utilize the latter one. Actually, even though in the process of utilizing envelopment form to rank efficiency scores, we can get its dual problem (multiplier form), which gives the allocation strategy easily. However under the case of multiple perspectives, we can not only utilize the envelopment form to assess efficiencies of DMUs, as for different perspectives, the weights are also different.

Besides the global solution (identical weight assignment scheme) for multiple perspectives, another result we can get from model (2.5) is ranking of DMUs' efficiency score under multiple perspectives, from which we can find out which DMU holds the maximum value of efficiency score for multiple perspectives, namely the sum of each perspective's efficiency score under identical weight assignment scheme ($max\sum_{k=1}^{L}2\theta_k^* q_k$). The ranking result reflects that to what extent a DMU can satisfy the 4 perspectives. Always the DMUs ranking top places are meaningful for other DMUs, as it can be selected as exemplar in this industry and thus provide significant guiding information for others. Note the maximum efficiency score under multiple perspectives of model (2.5) is 4, if a DMU obtains the efficiency score equaling to unit for each single perspective. In such case $q_k = 1/2$, (k $\in$ (1, L)) which is the most optimal solution.

2.4 A concrete application

In this section, we apply the model we proposed to Chinese banking systems to evaluate a bank under multiple perspectives. There exist many papers surveying the efficiency analysis about banking industry in the literature of DEA, but few researchers pay attention to the case of multiple perspectives. In this section we define undesirable and desirable attributes according to different perceptions of multiple perspectives. Based on the classification method of attributes and selection of perspectives, we perform the efficiency evaluation for each bank. Given this, Section 2.5 shows a preliminary attempt to build a model for improving inefficient DMUs under multiple perspectives.

2.4.1 Definition of U & D and selection of perspectives

We mainly selected twenty banks in China, and assign them with five representative attributes which can characterize a general banking system. Then we analyze and compare the efficiencies of these banks under four perspectives. Before assessing the efficiency of every bank under a specific perspective, we have to decide the specific perspectives. Actually, for every stakeholder, there might be a detailed perspective.

We form the first perspective by recourse to the classification methodology of inputs and outputs proposed by Avkiran, N.K. and H. Morita [29] (Inputs and outputs are separately defined as C_1 and C_2 in the terminology of this paper.). Variables for each bank are categorized into Ds and Us, which are provided for years 2001-2006. For C_1 attributes we correspond them with Us, and for C_2 attributes, correspond them with Ds accordingly, as shown in the column of perspective 1 of Table 2.1. Perspective 1, 2, 3 and 4 denote different classification methods from the typical viewpoints of shareholders, customers, managements and employees respectively. The four perspectives are different classification opinions about five typical attribute fields of a bank: soundness, credit quality, profitability, efficiency and valuation. Both perspectives consider soundness and credit quality as desirable attributes (gray cells as shown in Table 2.1), whereas they have different opinions for other three attributes.

Table 2.1 Five performance attributes and corresponding desirability with four perspectives

Category	Parameter	Description	Perspective			
			1	2	3	4
Soundness	CAR (%)	Capital adequacy ratio	D	D	D	D
Credit Quality	Equity / Impaired loans (%)	Equity per Impaired loans	D	D	U	D
Profitability	ROAE (%)	Return on average equity	U	U	D	D
Efficiency	Income / Cost (%)	Cost per Income	D	U	D	U
Valuation	DPS	Dividends per share	U	U	U	U

Different perspectives (classification opinions) of attributes are generated according to different groups of stakeholders. For example, Dividends per share (DPS), which is part of the measure of shareholder value created by a bank, is considered undesirable by most of stakeholders. The customers of stakeholders often interpret such apportioning of wealth as financed from the fees and charges levied by the bank on services and products. The executive managements of stakeholders are also likely to treat higher DPS as undesirable, thus, becoming an input into DEA, because dividends reduce an inexpensive source of internal funds that can otherwise be reinvested in the business for growth. Similarly, the bank employee group of stakeholders regards higher dividends as taking away funds that could otherwise be invested to improve their working conditions. Actually other kinds of perspectives might also exist besides these perspectives listed. Each given perspective is assumed to provide a different classification method abide by which we can distinguish U or D for every attribute of a bank system. But in our research we mainly focus on the four typical perspectives in banking system.

The generation of a perspective can be gained through mathematical or statistical (questionnaires) approaches, such as collecting original research data through questionnaires here and there or resort to mathematical analysis. It also means the data from experienced bank managements, or some other veterans in the bank system. In one word, we can get a variety of objective perspectives that we are interested in.

2.4.2 Efficiency analysis based on four typical perspectives

In this section, the model we proposed is validated through analyzing and comparing the efficiencies of twenty banks in China from the viewpoints of four perspectives. As shown in Table 2.2, we list twenty Chinese banks and the corresponding attributes we focus on. Bank 1, …, 4 are the four big banks in China which hold the largest scale. Bank 4, …, 11 are the joint stock commercial banks, and bank 12, …, 16 are the city commercial banks. The last four are local incorporated foreign banks. The five typical attributes of the banking system we select are as follows: CAR (capital adequacy ratio) which belongs to the category of soundness, equity / impaired loans (%) which indicates the credit quality of a bank, ROAE (return on average equity) which stands for the profitability, income / cost (%) denotes the efficiency and DPS (dividends per share) computed as the ratio of dividend paid to number of outstanding shares as sourced from BankScope respectively.

Table 2.2 Twenty Chinese banks with corresponding attributes

Bank Code	Name of Bank	CAR	Equity/Impaired loans%	ROAE	Income/ Cost	DPS
1	Bank of China	13	500.00	14	222.22	45.45
2	China Construction Bank	13	500.00	18	250.00	44.00
3	Industrial and Commercial Bank of China	13	476.19	16	277.78	55.42
4	Bank of Communications	14	588.24	18	294.12	35.71
5	Bank of Nanjing	31	1666.67	14	294.12	48.39
6	China CITIC Bank	15	1000.00	14	285.71	23.26
7	China Merchants Bank	11	666.67	25	277.78	26.92
8	China Minsheng Banking	11	714.29	18	217.39	13.54
9	Huaxia Bank	8	188.68	17	250.00	22.00
10	Industrial Bank	12	833.33	31	270.27	18.29
11	Shanghai Pudong Development Bank	9	357.14	21	263.16	16.51
12	Bank of Beijing	20	833.33	18	416.67	19.05
13	Bank of Ningbo	21	5000.00	17	270.27	46.51
14	BOC Hong Kong	13	5000.00	17	277.78	61.00
15	Chong Hing Bank	14	3333.33	8	158.73	56.00
16	Fubon Bank (Hong Kong)	14	3333.33	11	156.25	48.00

17	Bank of East Asia	13	2500	15	196.08	62.00
18	Dah Sing Banking Group	16	3333.33	9	232.56	46.00
19	Hang Seng Bank	11	5000	39	384.62	65.00
20	Wing Hang Bank	17	3333.33	21	256.41	50.00

The analysis result is demonstrated in Table 2.3, from which we can classify the banks into four classes:

a. Bank 8 is considered efficient by all perspectives. Such bank may be rewarded and becomes exemplar of the banking system. Also it will attract more customers.

b. As shown in the last row of Table 2.3, NULL means these 13 banks are considered inefficient by all perspectives. Hereby there may be serious defects in these bank systems, which need to be improved.

c. Moreover we can find there are some inefficient banks (bank 5, 12 and 19 as shown in red numbers in the last row of Table 2.3) even though they achieve the efficiency score $\theta = 1$. The presence of nonzero slacks in several attributes mean they are not Pareto-Koopmans efficient. We can improve such banks through decreasing the surplus quantity of attributes.

d. The remaining banks which are partially efficient for some perspectives can be categorized into class four. Also we can classify the perspectives into two classes in terms of banks: (i) Perspective 2 evaluating 5 banks as efficiency which is the most number of efficient banks compared with other perspectives. Banks could collaborate with such stakeholders for public relations and promotional purposes. (ii) Perspective 1, 3 and 4 whose number of efficient banks are fewer. Bank management could spend more effort in satisfying such stakeholders.

To sum up, there are several routes listed above, through which we can survey the relationship between banks and perspectives. Bank management may be interested in how to tally with a universal perspective through improving certain inefficient attributes. Also, customers may be interested in selecting an appropriate bank by categorizing self to a given perspective exactly beforehand.

Table 2.3 Efficiency Report in Terms of Perspectives

Perspective	Efficient for Banks								
1	8	6	15	18					
2	8	6	15	16	18				
3	8	9	11						
4	8	16							
NULL	1	2	3	4	**5**	7	10	**12**	13
	14	17	**19**	20					

2.4.3 Weight assignment under multiple perspectives

Suppose that the four perspectives we focus on have the equal importance or the same status from the view of market, we utilize the efficiency estimating model proposed in model (2.5) to get the identical weight assignment scheme under multiple perspectives. As shown in Table 2.4, we list the results of efficiency estimation under single perspective and multiple perspectives. The results for perspective 1, …, 4 are obtained from the iterative DEA model proposed in Section 2.2. The gray cell indicates this DMU can attain the efficiency score equaling to unit for corresponding perspective, and compared with bank 8 the only difference is these banks include nonzero slacks in some attributes. Actually we can transform the DMUs with nonzero slacks into efficient ones easily through decreasing the surplus quantity of the corresponding attributes. Thus bank 5, 12 and 19 can be improved to be efficient through trimming its nonzero slacks in some attributes.

Utilizing the efficiency score under single perspectives (θ_k^*, k = (1, …, 4) in DLP_o) and model (2.5), we list the efficiency analysis result based on multiple perspectives as shown in the last column of Table 2.4. Table 2.5 indicates the ranking of efficiency score under single perspective 1, …, 4 and multiple perspectives respectively. We can classify the banks into 5 classes through comparing their efficiency score under single perspective with their scores under multiple perspectives.

a. Ignoring the nonzero slacks, we find out in the case of multiple perspectives, the efficiency scores of bank 8 and 12 are locating on the top level (maximum value 4) in descending order, moreover both of these banks get the efficiency score equaling to 1 in the case of single perspective. Thus such banks may satisfy all perspectives to the highest extent or degree.

b. Although for each single perspective, the efficiency scores of bank 5 and 19 are the same with bank 8, equaling to unit, under the case of multiple perspectives their efficiency scores are ranking after bank 8, which shows their excellent performance for single perspectives and ordinary performance in the case of multiple perspectives.

c. Bank 11 and 10 rank the fourth and the fifth places in the case of multiple perspectives, whereas they rank lower places in most of the single perspectives. Such banks are beneficial in the case of multiple perspectives.

d. Bank 4, 17, 2, 1 and 3 are ranking lower places in both of the single perspective and multiple perspectives. Such banks are adaptive for neither the single perspective nor the case of multiple perspectives. And they should be the emphasis to be improved.

e. Most of the left banks which make no typical sense rank medium places. The majority are always holding such places like in most other industries.

Table 2.6 demonstrates the weight assignment scheme under multiple perspectives, from which we can find out bank 8 and 12 (bold numbers) which are the exemplars of banking system provide significant weight assignment schemes. As bank 12 has nonzero slacks in some attributes, it can improve its efficiency referring to bank 8. In Table 2.7, we show the comparison of efficiency score in single perspective with the case in multiple perspectives. Bank 8 and 12 are efficient in both cases and their efficiency scores equal to unit. Although Bank 5 and 19 get efficiency score equaling to unit in single perspective, when we seek the global solution in multiple perspectives their efficiency are lost. And bank 12 in spite of nonzero slacks does not lose its efficiency in either case, that’s why we mentioned bank 12 can also be considered as an exemplar of banking system.

The obvious result from the application of the DEA model based on multiple perspectives we developed is the efficiency scores of the majority of banks under single perspective changed when we consider the problem of efficiency estimation again under multiple perspectives. We try to illustrate how and to what extent the efficiency scores of DMUs will change in the process of seeking an identical weight assignment scheme for all perspectives. Through efficiency comparison, exhibition of weight assignment and ranking method, we also get the exemplars for banking system which may provide reference information for others.

Table 2.4 Efficiency Analysis Result of twenty Chinese Banks

Bank Code	Name of Bank	P1	P2	P3	P4	Multiple Perspectives
1	Bank of China	0.8166	0.877	0.9651	0.8784	2.0701
2	China Construction Bank	0.7608	0.8123	0.985	0.8842	2.101
3	Industrial and Commercial Bank of China	0.8502	0.7582	0.9841	0.8127	1.7684
4	Bank of Communications	0.8304	0.8242	0.9666	0.8403	2.5039
5	Bank of Nanjing	1	1	1	1	3.0317
6	China CITIC Bank	1	1	0.798	0.9274	3.2262
7	China Merchants Bank	0.7153	0.7954	0.9755	0.9042	2.8174
8	China Minsheng Banking	1	1	1	1	**4**
9	Huaxia Bank	0.9109	0.931	1	0.8747	3.0295
10	Industrial Bank	0.9171	0.9107	1	1	3.4523
11	Shanghai Pudong Development Bank	0.93	0.8481	1	0.9245	3.5052
12	Bank of Beijing	1	1	1	1	**4**
13	Bank of Ningbo	1	1	0.8622	1	3.3092
14	BOC Hong Kong	1	1	0.7062	1	2.8377
15	Chong Hing Bank	1	1	0.5659	0.9936	2.4839
16	Fubon Bank (Hong Kong)	0.9452	1	0.6096	1	2.7382
17	Bank of East Asia	0.7546	0.8502	0.637	0.9461	2.3494
18	Dah Sing Banking Group	1	1	0.66	0.931	2.845
19	Hang Seng Bank	1	1	1	1	2.8475
20	Wing Hang Bank	0.8344	0.8956	0.8372	0.9977	2.8647
Number of DMUs whose Efficiency Score = 1		9	10	7	8	

Table 2.5 Efficiency Ranking from Different Perspectives

Bank Code	Weights				
	CAR	Equity/Impaired loans%	ROAE	Income/Cost	DPS
1	0.0067	0.0001	0	0.0001	0.0152
2	0.006	0.0001	0.0002	0.0001	0.0149
3	0.0033	0.0001	0.0022	0.0001	0.0124
4	0.0098	0.0001	0	0	0.0175
5	0.0122	0	0	0	0.0128
6	0.0131	0	0.0051	0	0.0213
7	0.0002	0.0001	0	0.0004	0.0181
8	**0.0226**	**0**	**0**	**0**	**0.0369**
9	0	0	0.0013	0	0.0247
10	0.0166	0.0001	0	0	0.0283
11	0	0	0.0001	0	0.0315
12	**0.025**	**0**	**0**	**0**	**0.0262**
13	0.0043	0.0001	0.003	0.0001	0.009
14	0	0.0001	0.0023	0.0003	0.0071
15	0	0.0001	0.0038	0.0003	0.0087
16	0	0.0001	0.004	0.0003	0.0095
17	0	0.0001	0.0022	0.0002	0.0083
18	0.0021	0.0001	0.0039	0.0003	0.0095
19	0	0.0001	0.0036	0.0002	0.0069
20	0.0042	0.0001	0.0013	0.0001	0.0096

Table 2.6 Identical Weight Assignment Scheme under Multiple Perspectives

Perspective	Efficiency Ranking
P1	**8** = **12** = 5 = 19 = 6 = 13 = 14 = 15 = 18 > 16 > 11 > 10 > 9 > 3 > 20 > 4 > 1 > 2 > 17 > 7
P2	**8** = **12** = 5 = 19 = 6 = 13 = 14 = 15 = 16 = 18 > 9 > 10 > 20 > 1 > 17 > 11 > 4 > 2 > 7 > 3
P3	**8** = **12** = 5 = 19 = 9 = 10 = 11 > 2 > 3 > 7 > 4 > 1 > 13 > 20 > 6 > 14 > 18 > 17 > 16 > 15
P4	**8** = **12** = 5 = 19 = 13 = 10 = 14 = 16 > 20 > 15 > 17 > 18 > 6 > 11 > 7 > 2 > 1 > 9 > 4 > 3
Multiple Perspectives	**8** = **12** > 13 > 11 > 10 > 6 > 5 > 9 > 20 > 19 > 18 > 14 > 7 > 16 > 4 > 15 > 17 > 2 > 1 > 3

Table 2.7 Comparison of Efficiency Score under Single Perspective with Multiple Perspectives

Bank Code	P1	P2	P3	P4	Multiple Perspectives				
					P1	P2	P3	P4	Sum
1	0.8166	0.877	0.9651	0.8784	0.5063	0.5115	0.5416	0.5107	2.0701
2	0.7608	0.8123	0.985	0.8842	0.5207	0.5237	0.5828	0.4737	2.101
3	0.8502	0.7582	0.9841	0.8127	0.4738	0.3627	0.4872	0.4447	1.7684
4	0.8304	0.8242	0.9666	0.8403	0.6222	0.6176	0.6492	0.6149	2.5039
5	1	1	1	1	0.7579	0.7579	0.7579	0.7579	3.0317
6	1	1	0.798	0.9274	0.8652	0.8652	0.804	0.6918	3.2262
7	0.7153	0.7954	0.9755	0.9042	0.7351	0.6365	0.7222	0.7236	2.8174
8	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**4**
9	0.9109	0.931	1	0.8747	0.7903	0.7891	0.6687	0.7814	3.0295
10	0.9171	0.9107	1	1	0.8853	0.8792	0.8759	0.8119	3.4523
11	0.93	0.8481	1	0.9245	0.8862	0.8081	0.9242	0.8867	3.5052
12	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**4**
13	1	1	0.8622	1	0.9351	1	0.4766	0.8976	3.3092
14	1	1	0.7062	1	0.7464	0.8943	0.3799	0.8171	2.8377
15	1	1	0.5659	0.9936	0.6401	0.8677	0.3148	0.6613	2.4839
16	0.9452	1	0.6096	1	0.7586	0.9058	0.3681	0.7057	2.7382
17	0.7546	0.8502	0.637	0.9461	0.6814	0.7071	0.4213	0.5396	2.3494
18	1	1	0.66	0.931	0.731	0.9171	0.4286	0.7683	2.845
19	1	1	1	1	0.8184	0.642	0.4607	0.9264	2.8475
20	0.8344	0.8956	0.8372	0.9977	0.8197	0.8311	0.5029	0.711	2.8647

2.5 Improving DMUs for perspectives

Based on the efficiency evaluation results of model (2.5), we attempt to construct a model to improve inefficient DMUs under multiple perspectives in this section. The premises of considering such a DEA model under multiple perspectives mainly based on the following two points: First we have to insure the efficiency of the perspectives that DMU_o considers efficient, because credit is very important for a company especially for a bank. Maybe the perspectives DMU_o owns only possess very little market, but these perspectives may possess large market in future, thereby we can not discard them. Moreover if so, DMU_o will destroy its reputation and no other perspectives will believe it henceforth. Second, we do not need to improve DMU_o to be efficient for all perspectives, because it is really a difficult thing to cater for all tastes.

From the viewpoint of market, each perspective stands for a group of consumers or a kind of market trend, so they have different market share, some perspectives possessing primary share have important influences on market, and others may stand for a little stream of market. In order to explain this problem simply, we use the concept of Market Weight to denote its share of a perspective in the market. We assume the value of market weight for each perspective as vector $\boldsymbol{w} = (w_1, w_2, w_3, w_4)$. The vector $\boldsymbol{w}$ is mainly decided by some organizations of market research, and not correlative to any perspective or DMU.

The efficiency score of DMU_o for all perspectives is denoted as vector $\boldsymbol{\theta}_\mathbf{o} = (\theta_1, \theta_2, \theta_3, \theta_4)$, and each item corresponds to an efficiency score for a perspective. The vector $\boldsymbol{\theta}_\mathbf{o}$ means the efficiency scores of DMU_o from different viewpoints of perspectives. We denote N as the set of perspectives whose efficiency score is 1, i.e., $N = \{\text{k} \mid \theta_k = 1, k = 1, 2, 3, 4\}$.

We improve DMU_o to be efficient for a given perspective through increasing or decreasing the attributes of DMU_o. We denote the change of attributes for DMU_o by the vector $\boldsymbol{s} = (s_1, s_2, ..., s_m)$, where m is the number of the attributes. For a single perspective k, we estimate the efficiency of DMU_o through the following model for a given attribute change vector s, where U_k and D_k denote the classification of attributes by perspective k. The sign of inequality $\leftrightarrow_k$ also varies with U or D attributes by different classification of perspective k.

$$
\begin{aligned}
&\alpha_k(s) = \min_{\lambda_k, \theta_k} \theta_k \\
&s.t. \ -\boldsymbol{X}_j \lambda_k + \xi_{jk}(x_{oj} + s_j) \leftrightarrow_k 0 \\
&\qquad \xi_{jk} = \begin{cases} \theta_k & j \in U_k \\ \dfrac{1}{\theta_k} & j \in D_k \end{cases} \\
&\qquad \leftrightarrow_k = \begin{cases} \geq & j \in U_k \\ \leq & j \in D_k \end{cases} \\
&\qquad \theta_k > 0, k : \text{index of perspective} \\
&\qquad \sum_{i=1}^{n} \lambda_{ik} = 1, \lambda_{ik} \geq 0, (i = 1, \ldots, n.)
\end{aligned}
\tag{2.6}
$$

Here we show the process of improving DMU$_o$.

Step1. Keeping Efficiency of Efficient Perspectives

Denote the efficient perspectives belonging to DMU$_o$ as set N. For each perspective k included in N, we assume that while $\boldsymbol{s} \in S^*$ DMU$_o$ can keeps its efficiency for efficient perspectives, namely $S^* = \{ \boldsymbol{s} \mid \alpha_k(s) = 1 \text{ for all } k \in N \}$, where S^* denotes the common range of attribute change for all efficient perspectives. And DMU$_o$ can keep its efficiency for all efficient perspectives in this range. Please note the classification of U and D varies with different perspectives, whereas the attributes change for each U and D does not.

Step2. Selecting Target Perspective

For the perspectives which DMU$_o$ is not efficient for, we compare their weights and select the maximum one towards which to improve DMU$_o$. We denote it as t.

Step3. Improving DMU$_o$ for Perspective t

During the process of improving DMU$_o$ towards perspective t, the change of attributes of DMU$_o$ should also be constrained in S^*. We try to seek appropriate $\boldsymbol{s} \in S^*$ which can maximize the value of efficiency score for perspective t. The process can be embodied by the following model:

$$
\begin{aligned}
&\max_{s} \alpha_t(\boldsymbol{s}) \\
&s.t. \ \ \boldsymbol{s} \in S^*
\end{aligned}
$$

Step4. Selecting New Target

After improving DMU$_o$ towards perspective t, if θ_t equals to unit, then insert t into set N, and select a new target from the left perspectives whose efficiencies are not unit, then the succeeding process is similar. If θ_t does not equal to unit, we only keep its maximum value and continue to select new target. The basic methodology is improving DMU$_o$ towards an inefficient perspective at a time, and finally improving it for other inefficient perspectives step by step.

The process above can ensure the efficiency of the perspectives which DMU$_o$ is efficient for, moreover we can get the maximum efficiency score of perspective t while changing the attributes of DMU$_o$ in set S^*. By virtue of this model, we also get appropriate value of $\boldsymbol{s} \in S^*$ which supply us an improving schema for perspective t, which owns the largest market share. And we apply the model for DMU$_o$ iteratively until the efficiency score of each perspective is unit or maximum.

2.6 Conclusions

The chapter presents an iterative DEA model incorporating the classifying method "desirable" and "undesirable", by which we can get precise efficiency score in the process of efficiency estimation. By the dual problem of iterative DEA model we developed the DEA model based on multiple perspectives, from which we can get an identical weight assignment scheme for all perspectives. As it is difficult to choose lots of perspectives from numerous stakeholders, we mainly aim at four typical types of perspectives.

Through studying the problem of efficiency estimation from the viewpoints of multiple perspectives, a new classifying methodology is developed. The method is meaningful to guide the market, and improve the inefficient DMUs. And also provide significance for practice reference. The last part of this chapter mainly focuses on an application of twenty Chinese banks. The result showed how to get an identical weight allocation for each bank and which bank rank top places in single and multiple perspectives. From the view of banking system, the result may be meaningful in illustrating its market status, namely the bank is efficient for what kind of customers and what kind of customers are still not satisfied. Thus the bank can set its goal for next season. Moreover, the result provides detailed weight assignment scheme of exemplary banks for the inefficient banks. From the view of customers, as each bank has its main group of customers, the result may be meaningful in surveying a given bank is catering for which group of people, and which bank is the most appropriate for themselves. Thus they can select the most appropriate bank from the mass banks as their business partner.

As a preliminary attempt in the research of efficiency improvement under multiple perspectives, Section 2.5 presents a nonlinear model to satisfy multiple perspectives step by step. We do not give detailed description about the solving process as a more efficient method will be introduced in Chapter 3.

The outcome is also significant for market analysis, investment and merchandise planning for large-scale companies, especially multinational companies with tremendous varieties of products or multiple branches. By reference to the weight assignment scheme of other DMUs, the manager can increase production of the variety of products tallying with market requirements, and at the same time decrease production of the variety of products contradicting with primary market requirements. It is also a vital problem for a manager to improve its existing products to contend for more perspectives. In one word, we endeavor to provide guidelines for decision makers and market researchers, and then optimize the production system accordingly.

Chapter 3

Efficiency evaluation under multiple perspectives

Chapter 2 summarizes the preliminary studies we have done in the research of efficiency analysis based on DEA from multiple perspectives. The chapter begins from the introduction about an iterative DEA model, in which we incorporate the undesirable and desirable concepts considering from multiple perspectives. Also we utilize the dual problem of the iterative DEA model to obtain the model (2.5) which provides a method of evaluating DMUs. That is we use the sum of efficiency scores of multiple perspectives to assess whether a DMU is efficient for multiple perspectives. Moreover the method also gives a classifying method about DMUs from the efficiency scores of multiple perspectives. But the result does not consider the market statuses of different perspectives, which means important perspectives may get rather low efficiency scores and minor perspectives may obtain high scores conversely. Moreover the improving method for inefficient DMUs proposed in Section 2.6 is only a methodological model which gives no concrete solvable approaches.

To solve these problems, we incorporate NBG in this chapter to obtain an equilibrium solution for multiple perspectives considering their different market weights. Based on this, we introduce an applicable method about improving inefficient DMUs in Chapter 4.

3.1 Introduction

Many studies of efficiency analysis for banking industry employ the concept of DEA [30-37]. In traditional DEA models, we consider only one perspective, which provides only one input/output classification, where an output refers to an attribute for which a higher value is considered to be an improvement and an input refers to an attribute for which a lower value is considered to be an improvement. However, the problem of multiple perspectives, which was addressed by Bougnol *et al.* [25], is often encounter. Dyson *et al.* [26, 28, 38, 39] discussed the problem of classifying DEA attributes by introducing desirable input versus undesirable output, which means more input is better or, analogously less output is preferred. They also discuss the modeling and computational complexity in the research.

A previous study of multiple perspectives can also be found in Sarrico *et al* [40], where each type of students is referred to as one perspective. Different student types define universities' attributes as either inputs or outputs depending on their age, ability, aptitude, future job prospects, etc. Different types of students may have different evaluation even for the same university. In the research of Bougnol *et al.* [25], they conduct an investigation into the Memphis I-40 public project, in which perspective is referred to as constituency. Constituencies stand for individuals living in the area relevant to projects. 64 constituencies listed have different designations of attributes (9 in all) to input/output depending on their positions. 26 projects are evaluated by 64 constituencies in order to compare the efficiencies of these projects. In the research of M. Garcia-Cestona *et al.* [41] they propose a DEA model to evaluate Spanish savings banks with multiple goals depending on their different ownership structures. Their study indicates each type of ownership structures has different goal priorities and efficiency levels. Another relevant use of the concept of multiple perspectives appears in the research of N.K. Avkiran [42], which surveys efficiency evaluation of Chinese banks by multiple stakeholders such as customers, management, employees and regulators. Different stakeholders have different input/output classifications that lead to different efficiency evaluation even for the same bank.

In all related studies illustrated above, there is no study that specifically deals with the efficiency evaluation based on an identical weight assignment scheme. Weight assignment scheme based on a given perspective can not evaluate DMUs objectively. In order to evaluate DMUs fairly in the case of multiple perspectives, the current study follows the concept of multiple perspectives and incorporates the methodology of NBG, which can balance different perspectives and present an appropriate weight assignment scheme.

The remainder of the chapter is organized as follows. In Section 3.2, we introduce the efficiency evaluation DEA model under multiple perspectives which includes two parts, the reason of incorporating NBG and the concrete two phase evaluation model. In Section 3.3, we demonstrate an application of efficiency analysis involving approximately 20 Chinese banks, each having five attributes under two typical perspectives. Finally, Section 3.4 presents the conclusions of the chapter.

3.2 Efficiency evaluation DEA model

For a complicated banking system with various financial attributes, there are usually different classifications of inputs and outputs from the perspectives of different stakeholders. In order to obtain the highest efficiency score, different perspectives tend to select different weight assignment schemes, even in evaluating the same bank. In order to balance multiple perspectives (Pareto Optimality) based on their market statuses and evaluate DMU more objectively, we propose a new DEA model incorporating Nash bargaining game (NBG) theory, which focuses on seeking an identical weight assignment scheme to cater to multiple perspectives.

In the present study, the DMU is complicated banking system in which the same attributes may be interpreted differently based on multiple perspectives of stakeholders. One attribute that is considered to be an input from one perspective may be considered to be an output from another perspective. For example, the attribute "profitability" of a bank is usually considered to be an input from the perspective of the customer, because most customers regard the higher profitability of banks, which is achieved at their expense in the form of higher fees and charges [42], as an input. However, from the perspective of management, profitability may be defined as an output, because management considers higher profitability to mean higher salary and bonuses. Thus, different perspectives have different input/output classifications. In the present research, each input/output classification is referred to as a "perspective" from the viewpoint of a given group of stakeholders. In traditional DEA research, we can recommend the most appropriate weight assignment (the set of weights that maximizes the efficiency score) to DMU_o. However, in the case of multiple perspectives, it is difficult to fix the optimal weight assignment, because different perspectives tend to select different weight assignments in order to ensure that the bank obtains the highest efficiency score from their own perspective. If we select a weight assignment randomly from one perspective, other perspectives may receive a low efficiency, which may result in dissatisfaction with the bank. Thus, there is a need for a DEA model that can provide a rational identical weight assignment scheme for multiple perspectives. The different market weights of multiple perspectives (percentages of the entire market that perspectives possess) should also be taken into consideration in obtaining the identical weight assignment. Specifically, the weight assignment should have two main characteristics in the case of multiple perspectives. First, the weight assignment is an equilibrium solution that satisfies all perspectives to the highest extent (Pareto optimality) according to their market weights. In the case of multiple perspectives, we intend not to sacrifice any perspective while ranking the efficiencies of DMUs, and the goal in the case of multiple perspectives is to provide an objective ranking result. Second, the weight assignment should be an identical weight assignment for all perspectives, because we intend to clarify the influences of different attributes.

The Nash bargaining game (NBG) [15] is such a conventional method in dealing with equilibrium solutions to problems involving multiple players. This chapter describes a new DEA model that incorporates NBG theory, in which we define each perspective as a player. The proposed DEA efficiency model is a cooperative model. Under an identical weight assignment, multiple perspectives negotiate for a higher efficiency score. We assume that the breakdown point for each DMU (player) is 0, which means that if these perspectives do not cooperate in fixing an identical weight assignment, each of them will receive an efficiency score of 0. Then, we use the NBG method to fix a rational identical weight assignment, which can maximize the efficiency of multiple perspectives according to their market weights.

3.2.1 Why incorporating NBG

Weight assignment research in the case of multiple perspectives is based on the DEA model of a single perspective. We extend this model for the case of multiple perspectives and combine the NBG method to seek a rational weight assignment for multiple perspectives. Before the introduction of the DEA model based on the NBG, we present a simple example to show the relationship between weight assignment and efficiency in the case of multiple perspectives.

In Table 3.1, we assume that there are nine branch stores labeled A through I at the head of each column. Each store has three attributes, namely, the number of employees (unit: 10 persons), area (unit: 1,000 m^2), and price (average retail price, unit: 100 dollars), which are as recorded in each column. In Table 3.2, we assume that there are two perspectives: management and customer. From the viewpoint of management, a branch store that consumes fewer resources and has higher prices is considered to be more efficient. Thus, the number of employees and the area are regarded as inputs, and the price is regarded as an output. However, from the viewpoint of the customer, a store having more resources and lower prices is considered to be more efficient. Thus, the customer and management classify attributes in a perfectly contradictory manner. Note that the price is unitized to “1”, and so the values of employee and area are normalized to values for one unit of price. We take “employee/price” and “area/price” as two axes and plot the stores in Figure 3.1.

Table 3.1 Nine stores and corresponding attributes

Store	A	B	C	D	E	F	G	H	I
Employee	4	7	8	4	2	5	6	5.5	6
Area	3	3	1	2	4	2	4	2.5	2.5
Price	1	1	1	1	1	1	1	1	1

Table 3.2 Input/output classification from two perspectives

Attribute	Management	Customer
Employee	Input	Output
Area	Input	Output
Sale	Output	Input

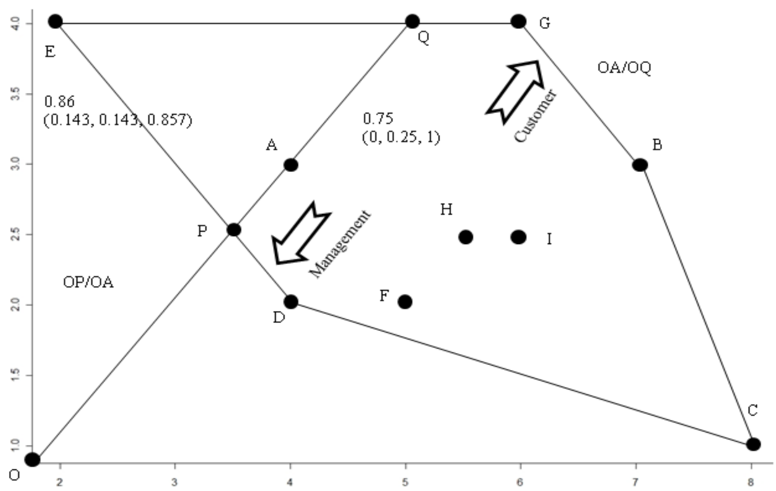


Figure 3.1 Efficiency analysis from two perspectives

$$\begin{aligned} \min_{\boldsymbol{v},\,\boldsymbol{u}} \quad & \boldsymbol{u}\boldsymbol{y}_{\boldsymbol{o}} \\ s.t. \quad & \boldsymbol{v}\boldsymbol{x}_{\boldsymbol{o}} = 1 \\ & -\boldsymbol{v}\boldsymbol{X} + \boldsymbol{u}\boldsymbol{Y} \le 0 \\ & \boldsymbol{v} \ge 0, \ \ \boldsymbol{u} \ge 0 \end{aligned} \qquad (3.1)$$

We estimate the efficiency score of each DMU by the CCR model (3.1), where $\boldsymbol{X}$ and $\boldsymbol{Y}$ denote the input and output matrixes respectively, and $\boldsymbol{v}$, $\boldsymbol{u}$ denote the weights of inputs and outputs for DMU_o. As for different perspectives $\boldsymbol{X}$ and $\boldsymbol{Y}$ in (3.1) are also different. Efficiency scores of DMU_o for multiple perspectives can be obtained by corresponding transformations of (3.1). In Figure 1, we plot two efficiency frontiers from the viewpoints of management and customers, respectively. From the perspective of management, stores C, D, and E are located on the efficiency frontier, and the efficiency score of A is 0.86, which is calculated as OP/OA. The reference set of A is {D, E}. The weight assignment we used while evaluating A from perspective of management is (0.143, 0.143, 0.857), which is calculated by model (1). If we consider the perspective of the customer, stores E, G, B, and C are located on the efficiency frontier, and the efficiency score of A is 0.75, which is calculated as OA/OQ. The reference set of A is {E, G}. The weight assignment we used while evaluating A from perspective of the customer is (0, 0.25, 1), which is also calculated by model (3.1).

In the case of multiple perspectives, we may find that we are using different weight assignments even when evaluating the same DMU. Usually, we need to determine an identical weight assignment for a DMU in order to clarify how each attribute affects the efficiency of the DMU. In such a case, it is difficult for A to make a choice as to whether select the weight assignment from the perspective of management or from the perspective of the customer. If we select the weight assignment from the perspective of management, namely (0.143, 0.143, 0.857), then management will have the highest efficiency score. However, the perspective of the customer will have an efficiency score of 0.7 (smaller than 0.75, as evaluated by (0, 0.25, 1)), namely OA/OR, as shown in Figure 2. Note here that the efficiency frontier of the perspective of the customer is changed to BG, the slope of which is the same as (0.143, 0.143, 0.857) (slope of ED). On the other hand, if we select the weight assignment from the perspective of the customer, namely (0, 0.25, 1), then the customer has the highest efficiency score of 0.86, whereas management has an efficiency score of only 0.3 (much smaller than 0.86, as evaluated by (0.143, 0.143, 0.857)), namely OS/OA, as shown in Figure 3. Note here that the efficiency frontier of the perspective of management is changed to SC, the slope of which is the same as (0, 0.25, 1) (slope of EG).

In Figure 3.1, the efficiency score of A is estimated by different weight assignment, namely the slopes of ED and EG respectively. And the reference DMUs for management and customer are P and Q respectively. In Figure 3.2 both of the two perspectives evaluate A by ED from the view of management perspective. ED is parallel with RB, which is the efficient frontier of customer under the weight assignment scheme of ED.

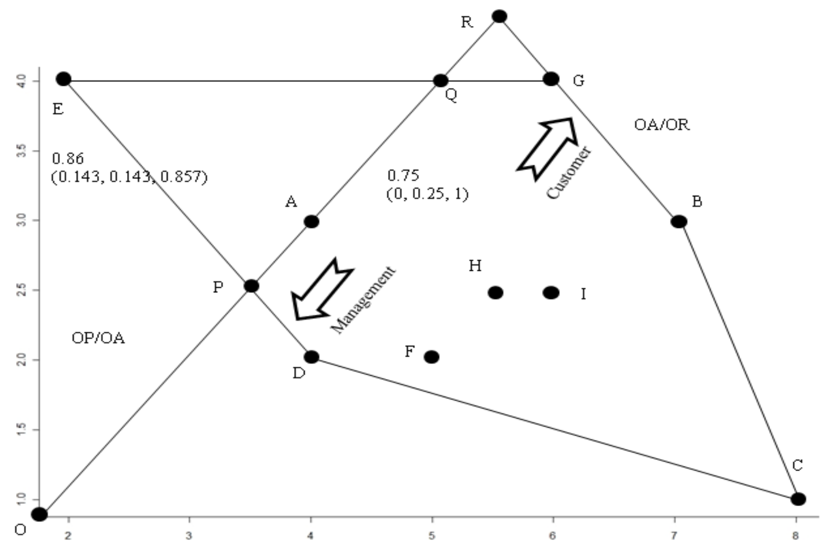


Figure 3.2 Efficiency estimation from the perspective of management

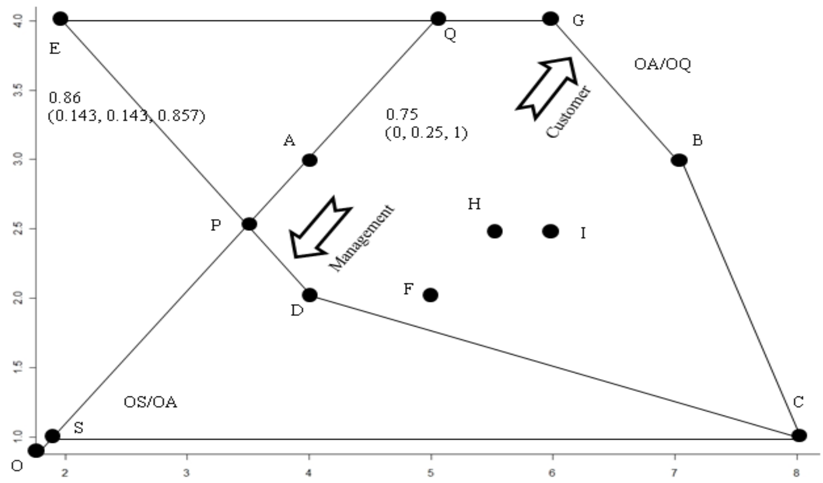


Figure 3.3 Efficiency estimation from the perspective of customer

The reference DMU for management is still P, but the reference DMU for customer becomes R. In Figure 3.3 both of the two perspectives evaluate A by EG from the view of customer perspective. EG is parallel with SC, which is the efficient frontier of management under the weight assignment scheme of EG. The reference DMU for customer is still Q, but the reference DMU for management becomes S.

Therefore, it is necessary to select a rational line between ED and EG for which the slope of the line denotes the identical weight assignment for the two perspectives. As two perspectives negotiate for a higher efficiency score, the line should be selected so as to balance the efficiency scores of the two perspectives. In the present study, each perspective represents one group of stakeholders, and it is necessary to consider how to seek an identical weight assignment to satisfy all perspectives in ranking the efficiencies of DMUs. Here, “satisfy” means that an identical weight assignment balances the efficiency scores among different perspectives while maximizing the total efficiency score for multiple perspectives. Different perspectives may have inconsistent opinions concerning the selected identical weight assignment, because a perspective occupying a higher market weight tends to assume that the weight assignment should result in a higher efficiency score.

In order to solve this problem, we incorporate the NBG methodology into DEA to obtain an identical weight assignment (equilibrium solution) in the case of multiple perspectives. Each perspective is considered to be one player in the NBG, and the game mode is the DEA model with identical weight assignment. In Section 3.2, we will introduce how to incorporate the NBG into DEA in order to balance the efficiency scores of multiple perspectives. Since the efficiency scores of multiple perspectives are based on identical weight assignment, in the following section we focus on how to obtain an identical weight assignment.

3.2.2 A two phase DEA model with identical weight assignment

The Nash bargaining game theory focuses on seeking an equilibrium solution between two players who want to divide the surplus value of cooperation. The function takes the following form:

$$max\ (x-a)^c(y-b)^d \qquad (3.2)$$

We assume two players, A and B, who want to divide the surplus value produced through cooperation. If each of these players operates his own business without cooperation, A will obtain payoff a, and B will obtain payoff b. Parameters a and b are also named breakpoints, which means if the bargaining game does not yield an agreement. If the players cooperate, they will obtain total value V, which is greater than $a + b$. The surplus value is generated because of their cooperation. This added value is why the players want to cooperate. The surplus value $s = V - a - b$. Here, $x = a + cs$, $y = b + ds$, where x and y denote the values of A and B, respectively. c and d denote the market weights of A and B, respectively. The above model expresses the optimal assignment of surplus value between two players. Nash proposed that a reasonable solution should satisfy the following axioms: 1) invariant to affine transformations or invariant to equivalent utility representations, 2) Pareto optimality, 3) independence of irrelevant alternatives, and 4) symmetry. Nash also presented a unique solution called the Nash solution, which satisfies the above axioms and can be obtained by solving the following equation, which is an extension of model (3.2) to multiple perspectives:

$$\max \prod_{k=1}^{p} (E_k - 0)^{w_k} = \max \prod_{k=1}^{p} E_k^{\ w_k} \qquad (3.3)$$

where w_k denotes the market share of perspective k, and E_k denotes the efficiency score of perspective k in estimating a DMU. (We assume that there are p perspectives in the system that are worthy of examination.) We assume the breakpoint for each perspective to be 0, which means that if A and B cooperate to seek an identical weight assignment, both players can benefit according to their market share; otherwise one of the players will receive an efficiency score of 0. Next, we will describe how to obtain efficiency score E_k with identical weight assignment based on multiple perspectives.

One characteristic of the efficiency frontier in DEA research is that the DMU that receives the highest efficiency score must be located on the efficiency frontier in the process of estimating DMU$_o$, regardless of the weight assignment used to estimate DMU$_o$. As shown in Figure 3.1, from the perspective of management, the DMUs located on the efficiency frontier are C, D, and E. In the process of estimating A, regardless of the slope of the line, the DMU with the highest efficiency score must be a DMU in the set {C, D, E}. For a given weight assignment, the other DMUs on the efficiency frontier are less efficient than the DMU that receives the highest efficiency score. Therefore, under the identical weight assignment for multiple perspectives, the DMU gaining the highest efficiency score for each perspective must be located on the efficiency frontier of the corresponding perspective. We denote the DMUs on the efficiency frontier of perspective

k as set $\boldsymbol{S_k}$ and denote the DMU having the highest efficiency score under identical weight assignment for perspective k as DMU_{tk}. The other DMUs in $\boldsymbol{S_k}$ are less efficient than DMU_{tk}. Under perspective k, the efficiency score of DMU_o is denoted as $\boldsymbol{E_{ok}}$ / $\boldsymbol{E_{tk}}$. Here, $\boldsymbol{E_{ok}}$ and $\boldsymbol{E_{tk}}$ are the efficiency scores of DMU_o and DMU_t under perspective k, respectively. In addition, the efficiency score of DMU_o is unitized by the efficiency score of DMU_t. Therefore, Eq. (3.3) can be rewritten as follows:

$$\begin{aligned} &\Omega = max \prod_{k=1}^{p} E_{ok} / E_{tk}{}^{w_k} \\ &s.t.\ E_{tk} \geq E_{jk} \\ &\quad (\forall DMU_{jk} \in S_k, DMU_{jk} \neq DMU_{tk}) \end{aligned} \tag{3.4}$$

As different perspectives have different classification of inputs and outputs for the attributes of DMU, we use the method of input/output selection proposed by N.C.P. Edirisinghe and X. Zhang [43]. Consider a system with m DMUs, each with n attributes, which may be classified as either inputs or outputs by different perspectives. Let the binary vector $\boldsymbol{x_k} \in \boldsymbol{R}^{2n}$ be used to identify the classification of perspective k, where for i = 1,…, n. Parameter i is an input if $x_i = 1$ and is an output if $x_{n+i} = 1$. The condition $x_i + x_{n+i} = 1$ is always valid so as to ensure that an attribute is either an input or an output. Thus, the set of classifications corresponds to the set of vectors $\boldsymbol{x_k}$, which can be denoted by Φ and is given as follows:

$$\Phi := \{x_k : x_i + x_{n+i} = 1, x_i, x_{n+i} \in \{0,1\}, i = 1,...,n\} \tag{3.5}$$

Incorporating the method of input/output selection, the multiplier form of Eq. (3.4) can be written as follows:

$$
\begin{aligned}
& \textit{phase } 1 \\
& \Omega = max \ \prod_{k=1}^{p} E_{ok} / E_{tk}{}^{w_k} \\
& s.t. \quad E_o(x_k) = \frac{\sum_{i=1}^{n} u_i x_{(n+i)k} \xi_{io}}{\sum_{i=1}^{n} u_i x_{ik} \xi_{io}} \\
& \qquad E_t(x_k) = \frac{\sum_{i=1}^{n} u_i x_{(n+i)k} \xi_{it}}{\sum_{i=1}^{n} u_i x_{ik} \xi_{it}} \qquad\qquad (3.6) \\
& \qquad \frac{\sum_{i=1}^{n} u_i x_{(n+i)k} \xi_{it}}{\sum_{i=1}^{n} u_i x_{ik} \xi_{it}} \geq \frac{\sum_{i=1}^{n} u_i x_{(n+i)k} \xi_{ij}}{\sum_{i=1}^{n} u_i x_{ik} \xi_{ij}} \\
& \qquad (\forall DMU_{jk} \in S_k, DMU_{jk} \neq DMU_{tk}) \\
& \textit{phase } 2 \\
& \max_{q=1,\ldots,h} \ \Omega
\end{aligned}
$$

where the objective function is denoted as NBG (incorporating the CCR model) in order to maximize the efficiency score for multiple perspectives. Here, u_i denotes the identical weight assignment for multiple perspectives, and ξ_i denotes the data of attribute i for the given DMU$_o$. The first and second restriction conditions indicate the multiplier forms of the efficiencies of DMU$_o$ and DMU$_t$, respectively. The 3rd restriction condition indicates that other DMUs on the frontier should be less efficient than DMU$_t$. In phase 1, we assume that DMU$_{tk}$ is the most efficient DMU in $\boldsymbol{S_k}$. Since DMU$_{tk}$ changes for different weight assignments, we assume that h is the number of combinations of elements in $\boldsymbol{S_k}$, where $h = \prod_{k=1}^{p} \mathbf{s}_k$. Namely, we need to solve h optimization sub-problems as in phase 1, and let q denote the number of execution of phase 1. Here, x_{ik} and $x_{(n+i)k}$ denote the classification of input/output for attribute i under a given vector $\boldsymbol{x_k}$ (classification vector from perspective k).

Since the classification for input/output differs for each perspective k, x_{ik} and $x_{(n+i)k}$ also vary corresponding to different perspectives, whereas the weight u_i before each component of ξ_i remains unchanged for all perspectives. This is one key difference in efficiency estimation between a single perspective and multiple perspectives. Under a single perspective, which is the same for various traditional DEA models proposed in

DEA literature, there is no appreciable distinction between envelopment form and multiplier form, because both of these forms provide an approach to efficiency estimation or comparison. When we are interested primarily in the ranking of efficiency scores of DMUs, we use the envelopment form, and when we are interested primarily in the allocation of attributes, we use the multiplier form. Even in the process of using the envelopment form to rank efficiency scores, we can get its dual problem (multiplier form), which allows the allocation strategy to be easily obtained. However, in the case of multiple perspectives, not only do we use the envelopment form to get $\boldsymbol{S_k}$ for each perspective, but also use the multiplier form to obtain identical weight assignment for multiple perspectives.

In addition to the identical weight assignment scheme for multiple perspectives, Eq. (3.6) also indicates the ranking of the efficiency scores of DMUs under multiple perspectives, based on which we can determine which DMU has the maximum gross efficiency score for multiple perspectives. The ranking result indicates the extent to which a DMU can satisfy multiple perspectives. The top-ranked DMU is always meaningful for other DMUs, because the top-ranked DMU can be used as an example in industry, thereby providing significant guiding information in weight assignment for other inefficient DMUs.

3.3 Measuring the efficiency of Chinese banks

In this section, we also use the 20 Chinese banks as a concrete case study, which has been used in Chapter 2 (Please refer to the data of these banks listed in Chapter 2). The model we proposed is validated by analyzing and comparing the efficiencies of 20 banks from the viewpoints of two perspectives. The 20 Chinese banks consist of different types and scales. The five typical attributes of the banking system we selected are as follows: 1) capital adequacy ratio (CAR), which belongs to the category of soundness, 2) equity/impaired loans (%), which indicates the credit quality of a bank, 3) return on average equity (ROAE), which is an indication of profitability, 4) income/cost (%) which is an indication of efficiency, and 5) dividends per share (DPS) computed as the ratio of dividend paid to the number of outstanding shares as sourced from BankScope.

Being different with the case study in Chapter 2, in this section we only study the simple case of two perspectives. Perspective 1 and 2 denote the classification methods from the typical viewpoints of customers and employees, respectively as shown in Table 3.3. These two perspectives assign different classifications regarding the five typical attribute fields of a bank, namely, soundness, credit quality, profitability, efficiency, and valuation. Both perspectives consider soundness and credit quality as O attributes and tend to consider efficiency and valuation as I attributes. However, these perspectives have different opinions regarding the classification of profitability (gray cells shown in Table 3.3).

Table 3.3 Five attributes and corresponding input/output classifications with two perspectives

Category	Parameter	Description	Stakeholder Perspective 1	Stakeholder Perspective 2
Soundness	CAR (%)	Capital adequacy ratio	O	O
Credit Quality	Equity / Impaired loans (%)	Equity per Impaired loans	O	O
Profitability	ROAE (%)	Return on average equity	I	O
Efficiency	Income / Cost (%)	Cost per Income	I	I
Valuation	DPS	Dividends per share	I	I

Different perspectives (classification opinions) of attributes are generated according to different groups of stakeholders. For example, dividends per share (DPS), which is, in part, a measure of shareholder value created by a bank, is classified as I by both perspectives. Customers often interpret bank profits as being obtained through fees and charges levied by the bank for services and products. Bank employees are also likely to consider higher DPS to be undesirable, because employees regard higher dividends as being taken from funds that could otherwise be used to improve their working conditions. Customers consider higher profitability to be increased profits for the bank, which are taken from the profits of customers. Thus, customers regard higher profitability to be an I attribute. In contrast, employees consider higher profitability as enabling an increase in salary. Perspectives other than the two perspectives considered herein might also exist. Each perspective is assumed to provide a different classification method, by which each attribute of a bank system can be classified as either an I attribute or an O attribute. However, in the present study, we focus primarily on the two typical perspectives in banking system.

In order to simplify the process of computation, we assume that two perspectives have equal market weight 1, i.e., $w_1 = w_2 = 1$. The results of the efficiency analysis based on two perspectives are shown in Table 3.4. More specifically, the number of DMUs on the frontier of perspective 1 is six, and the number of DMUs on the frontier of perspective 2 is seven:

$$DMU_{t1} \in \{5,12,13,15,16,18\}$$

$$DMU_{t2} \in \{5,8,10,12,13,16,19\}$$

The number of possible combinations of DMU_{t1} and DMU_{t2} is 42. Thus, h = 42, which means that, in phase 2, we need to solve 42 sub-problems. Therefore, we compare the results of these sub-problems as shown in phase 2 of Eq. (3.6), and select the maximum one as the optimal solution for the DMU.

Table 3.4 Efficiency Analysis by Two Perspectives

Bank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
P1					1							1	1		1	1		1		
P2					1			1		1		1	1			1			1	

Table 3.5 illustrates the weight assignment scheme under multiple perspectives, in which we recommend different weight assignment schemes for different banks in order to satisfy multiple perspectives. Table 3.6 shows the efficiency scores for each perspective under NBG, from which we can determine that Banks 5 and 16 are the most efficient banks in the case of multiple perspectives. The NBG value (value of the objective function of the NBG) is the product of the efficiency scores for P1 and P2 and indicates the overall efficiency score of a DMU for the case of multiple perspectives. We can rank the NBG value in order to determine how banks satisfy Pareto optimality. Banks that have a higher NBG value generate a more equilibrium solution for multiple perspectives. In Table 7, we also find that the efficiency score of Bank 13 (Bank of Ningbo) for P2 is 0.999999, which is approximately equal to 1. Since we use the approximation calculation method in calculating the efficiency score, we can consider the efficiency score to be 1, which means that Bank 13 is also an efficient bank in the case of multiple perspectives. In addition, the weight assignment of Bank 13 can also be considered as an exemplar by other inefficient DMUs.

Table 3.6 lists the gross efficiency scores of the 20 banks under multiple perspectives. We can classify the banks into three classes by comparing their efficiency scores.

(a) In the case of multiple perspectives, the efficiency scores of Banks 5, 13, and 16 are located on the top level (maximum value: 1). Thus, such banks may satisfy all perspectives to the highest extent or degree.

(b) Banks 10 and 12 are efficient with only one perspective. Such banks have excellent performance for a single perspective and average performance in the case of multiple perspectives.

(c) Most of the remaining banks which are adaptive for neither a single perspective nor multiple perspectives, rank medium places. The improvement of these banks should be a priority.

Table 3.5 Weight assignment under multiple perspectives

Bank Code	Name of Bank	Weights				
		CAR	Equity/Impaired loans%	ROAE	Income/Cost	DPS
1	Bank of China	0.036725	0	0.018801	10.12393	5.850087
2	China Construction Bank	1.057981	0	1.058655	2.532548	0.060985
3	Industrial and Commercial Bank of China	1.148278	0	1.171602	3.034554	0
4	Bank of Communications	0	0	0	2.349733	1.383453
5	**Bank of Nanjing**	**0.925724**	**0.002084**	**0**	**1.478171**	**2.540701**
6	China CITIC Bank	3.12209	0.007769	0	0.353801	7.236468
7	China Merchants Bank	0.001477	0	0.001841	2.286988	1.458645
8	China Minsheng Banking	11427.66	19.56615	19535.49	22576.28	275.0048
9	Huaxia Bank	0	0	0	2.587117	1.616158
10	Industrial Bank	0.002279	0	1.160711	1.539877	1.975364
11	Shanghai Pudong Development Bank	0.866827	0.002157	0	0.18464	3.776455
12	Bank of Beijing	1.96989	0.015098	1.139465	0	7.25973
13	Bank of Ningbo	0.428779	0.004976	1.238618	1.212146	1.774923
14	BOC Hong Kong	0	0.032285	4.52792	0.936543	1.407642
15	Chong Hing Bank	0.995826	0.009222	0	1.859073	2.617927
16	**Fubon Bank (Hong**	**0.999696**	**0.927725**	**1.02261**	**0.85279**	**1.19966**

	Kong)					
17	Bank of East Asia	0.272347	0.000342	0.352509	2.858083	0.664074
18	Dah Sing Banking Group	3.514901	0.010038	0.02563	0.000274	0.001674
19	Hang Seng Bank	0	0.00254	0.853141	1.580413	2.241797
20	Wing Hang Bank	0.000216	0	0.000934	2.379044	1.709672

Table 3.6 Efficiency score under multiple perspectives

Bank Code	Bank Name	Multiple Perspectives		NBG Value
		P1	P2	
1	Bank of China	0.847315	0.811927	0.687958
2	China Construction Bank	0.488366	0.777289	0.379602
3	Industrial and Commercial Bank of China	0.442346	0.653963	0.289278
4	Bank of Communications	0.462311	0.950421	0.439390
5	**Bank of Nanjing**	**1**	**1**	**1**
6	China CITIC Bank	0.838939	0.838939	0.703819
7	China Merchants Bank	0.807566	0.963428	0.778032
8	China Minsheng Banking	0.442522	0.804799	0.356141
9	Huaxia Bank	0.649981	0.31737	0.206284
10	Industrial Bank	0.554653	1	0.554653
11	Shanghai Pudong Development Bank	0.601175	0.601175	0.361411
12	Bank of Beijing	1	0.972801	0.972801
13	Bank of Ningbo	1	0.999999	0.999999
14	BOC Hong Kong	0.935152	0.920726	0.861019
15	Chong Hing Bank	0.942146	0.942146	0.887639
16	**Fubon Bank (Hong Kong)**	**1**	**1**	**1**
17	Bank of East Asia	0.705339	0.872112	0.615135
18	Dah Sing Banking Group	0.999993	0.780101	0.780096
19	Hang Seng Bank	0.693791	0.999796	0.693649
20	Wing Hang Bank	0.795489	0.982328	0.781431

The result of the application of the proposed DEA model based on identical weight assignment is that the efficiency scores of the majority of banks under a single perspective change when we consider the problem of efficiency estimation from the viewpoint of identical weight assignment under multiple perspectives. We attempted to illustrate how and to what extent the efficiency scores of DMUs change in the process of seeking an identical weight assignment scheme for all perspectives. By comparing the efficiency and investigating the weight assignment and ranking method, we classify DMUs into three classes according how a DMU can satisfy multiple perspectives. The efficient DMUs for multiple perspectives may be considered to be exemplars for other inefficient DMUs in efficiency estimation.

3.4 Concluding remarks

In this chapter, we proposed a DEA model with identical weight assignment for multiple perspectives. We first incorporated a method of input/output classification that is more appropriate for multiple perspectives and then used the NBG theory to balance the efficiency scores among multiple perspectives. By incorporating the NBG with the DEA model, we developed a new DEA model, from which we were able to obtain an optimal identical weight assignment scheme for all perspectives.

In addition, the efficiency ranking is based on the identical weight assignment scheme for all DMUs. Since it is difficult to choose several perspectives from among numerous stakeholders and the calculation will become more complicated, we focused on two typical perspectives and considered how to satisfy these perspectives to the highest extent by an identical weight assignment while ranking efficiencies. The model is meaningful for guiding the market and improving the inefficient DMUs. The model also provides a practice reference about ranking a group of DMUs in the case of multiple perspectives. Finally, we considered an application involving 20 Chinese banks. The results reveal how to obtain an identical weight assignment for each bank in the case of two perspectives, and which bank is ranked highest. From the viewpoint of the banking system, the results may be meaningful in illustrating the market status of a back, namely, the types of customers who consider the bank to be efficient, the types of customers who are not satisfied by the bank, and how to improve the weight assignment in order to satisfy multiple groups of stakeholders. Moreover, the results provide a detailed weight assignment scheme of exemplary banks for inefficient banks.

In Eq. (3.6), $h=\prod_{k=1}^{p}\mathbf{s}_k$ denotes the number of optimization sub-problems we need to solve in phase 2. As the number of efficient DMUs calculated by model (3.1) for each perspective is related with lots of factors, such as the number of attributes, perspectives and DMUs. The computational complexity is increasing along with the increase of these factors. When the number of perspectives and DMUs increase infinitely, the computational process becomes a NP problem. But always the Bargaining game is carried out by two or three perspectives which will not lead to the NP problem. On the other hand, the weight assignment calculated by Eq. (3.6) sometimes is not unique, which is a common situation in DEA literature. DMU can select an appropriate weight assignment scheme by itself in such case.

The results are also useful for market analysis, investment, and merchandise planning for large-scale companies, especially multinational companies with tremendous varieties of products or multiple branches. An identical weight assignment can provide an optimal resource allocation in order to cater to multiple customers or branches. By referencing the identical weight assignment in the case of multiple perspectives, the manager can clarify which attributes are more important and which are less important. Improving existing products in order to exploit new market is also crucial. In summary, we have provides guidelines for decision makers and market researchers and have optimized the production system accordingly.

Chapter 4

Efficiency improvement under multiple perspectives

Based on the introduction about multiple perspectives, input/output classification methods, basic DEA models, and the results of efficiency evaluation in the forenamed chapters, we mainly focus on addressing the problem of efficiency improvement under multiple perspectives in Chapter 4. This chapter can be considered as the extension of Chapter 3, as we also use NBG to improve DMUs under multiple perspectives. We firstly rank DMUs from the viewpoint of each single perspective then the DMUs with low efficiency are selected as the targets to be improved.

In Chapter 3, we attempt to evaluate a DMU with identical weight assignment under multiple perspectives, but in this chapter we relax the constraint of identical weight assignment. Actually whether to use identical weight assignment or not is determined by the viewpoint from which we consider the problem of efficiency evaluation. This is similar with the two forms of DEA models, namely multiplier form and envelopment form, which we have introduced in Chapter 1. The multiplier form gives optimal weights which are useful when we want to analyze each input/output plays a what kind of role in affecting the efficiency of a DMU, whereas envelopment form only concerns about the efficiency score neglecting the weight analysis.

If we evaluate a DMU from the viewpoint of managerial staffs, they may be interested in considering about adopting what kind of managerial scheme to balance different perspectives (Who might be customers.). Thus the identical weight assignment which can make clear that each attribute plays what kind of role in affecting the whole efficiency of the DMU may be necessary for managerial staff. That is what we focus on in the research of evaluating Chinese banks in last chapter.

But if we evaluate a DMU from different perspectives, who are not interested in the managerial details (Here we can consider the multiple perspectives are different customers who only want to select the most efficient DMU as their investment target, but not concern about the interior factors of the DMU.), the weight assignment is not that important. Thus we can use different weight assignments from different perspectives to evaluate the same DMU, as each perspective only pays attention to the efficiency score of the DMU while evaluating from itself.

As we have studied the efficiency evaluation problem from the managerial viewpoint in Chapter 3, we mainly consider the efficiency improvement from multiple perspectives in this chapter (Of course, we can also study the process of efficiency improvement using identical weight assignment, that means a consideration from managerial level. But we will not give explanation here.).

Actually we mentioned a model to improve DMUs towards a perspective in Section 2.5, but it is only a methodological model which is difficult to realize and perform. Thus we propose a feasible model to improve inefficient DMUs under multiple perspectives in this chapter. The chapter begins with the introduction about literature review related with bargaining game theory, follows with the definition of input/output classification method, and concerns about concrete calculation details at last.

4.1 Introduction

Back to the differences of single perspective and multiple perspectives we mentioned in 1.2, the efficiency evaluation method under single perspective is not suitable for the case of multiple perspectives. Especially for the banking systems which are the objects of our research in this chapter, as the same attributes may be interpreted differently based on the multiple perspectives of various stakeholders. That's why we propose the new evaluation model for the case of multiple perspectives in last chapter. Following the adjustment of evaluation model, the improving scheme for an inefficient DMU is also necessary to be reconsidered.

There are some studies incorporating game theory into DEA models[44-48]. Du *et al.* [47, 49] and Liang *et al.* [44, 45, 50-52] proposed a two-stage network DEA model in which they view each stage as a player and the two-stage DEA model is a cooperative game model. The bargaining game dealt with the conflict between two stages which is caused by the intermediate measures. On the other hand, a model based on identical weight assignment is proposed in last chapter to show a more objective evaluation based on an identical weight assignment scheme. The method changes the selection strategy of inputs and outputs for different perspectives and evaluates DMUs fairly which can balance the views of multiple perspectives. A meaningful result of efficiency evaluation for 20 Chinese banks is also given to compare the overall efficiency scores of DMUs from multiple perspectives.

Most of the previous studies related to multiple perspectives focused on efficiency evaluation and comparison for DMUs, whereas no study deals with efficiency improvement for DMUs, which are not efficient for all perspectives. There are many differences between a single perspective and multiple perspectives in improving an inefficient DMU. The concept of "efficiency" or "inefficiency" is not appropriate for the case of multiple perspectives. All DMUs need improvement in the case of multiple perspectives except the DMUs located at the crossing points of all efficient frontiers (Such DMUs are efficient for all perspectives.). On the other hand, in the case of single perspective, the linear combination of points in the reference set is still an efficient point, which can be set as the target to improve the inefficient DMU. Whereas in the case of multiple perspectives, the linear combination of efficient DMUs on one efficient facet may not still locate on the facet, thus it is impossible to find out a reference set to improve inefficient DMUs.

Nash bargaining game (NBG) [15, 53] is a popular method in dealing with equilibrium solutions to problems involving multiple players. In the current study, each perspective of a stakeholder is defined as a player. We use NBG to determine (1) the appropriate value for each attribute, namely, whether to increase or decrease an attribute of DMU_o and to what extent to improve an attribute; (2) selecting which attribute would improve DMU_o by comparing the NBG results of its various attributes. The proposed game mode of NBG is cooperative. Multiple perspectives negotiate for a higher efficiency score in fixing the appropriate value of an attribute of DMU_o.

The current research is based on the efficiency evaluation by CCR model from multiple perspectives, for which we will give detailed description in Section 4.2. For the DMUs which are already efficient for all perspectives, there is no need to improve them. For the DMUs partially efficient for some perspectives, we attempt to improve their efficiency by looking at all attributes and identifying the most effective one as the way to improve DMUs.

4.2 Efficiency improvement DEA model

4.2.1 Classification of attributes

As the precondition of efficiency improvement, efficiency evaluation is processed by CCR model. Consider a set of n DMUs to be analyzed, of which input and output vectors are represented by an ($m \times n$) matrix $\boldsymbol{X}$ and an ($s \times n$) output matrix $\boldsymbol{Y}$. The number of inputs and outputs are denoted by m and s respectively. Thus the efficiency score of DMU$_o$ being evaluated is shown as follows:

$$\begin{aligned} \max_{\boldsymbol{v},\boldsymbol{u}} \quad & \boldsymbol{u}\boldsymbol{y}_o \\ s.t. \quad & \boldsymbol{v}\boldsymbol{x}_o = 1 \\ & -\boldsymbol{v}\boldsymbol{X} + \boldsymbol{u}\boldsymbol{Y} \le 0 \\ & \boldsymbol{v} \ge 0,\ \boldsymbol{u} \ge 0 \end{aligned} \tag{4.1}$$

where $\boldsymbol{x}_o = (x_{1o}, x_{2o}, \ldots, x_{mo})^{\mathrm{T}}$ and $\boldsymbol{y}_o = (y_{1o}, y_{2o}, \ldots, y_{so})^{\mathrm{T}}$ are input and output vectors of DMU$_o$. Row vectors $\boldsymbol{v}$ and $\boldsymbol{u}$ denote the weights of inputs and outputs. The objective function in Eq. (4.1) captures the maximum weighted output of DMU$_o$ under the constraint $\boldsymbol{v}\boldsymbol{x}_o = 1$. Eq. (4.1) is a traditional efficiency evaluation model for single perspective, where each attribute is specified as either an input or an output. For multiple perspectives, as different perspectives have different perceptions about input/output classification, we incorporate the method of input/output selection into Eq. (4.1). In order to make the model simple and easier to be understood, we redefine the input/output classification method by diagonal matrixes which has been mentioned in Chapter 3.

Consider a system with n DMUs each with r attributes, the whole data set can be denoted by a ($r \times n$) matrix $\boldsymbol{A}$. Matrix $\boldsymbol{A}$ equals to ($\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n$) where the attributes of DMU$_o$ under evaluation is denoted by column vector $\boldsymbol{a}_o$. Assume that the number of perspectives is q which has different input/output classifications for r attributes. Let the following two ($r \times r$) diagonal matrixes $\boldsymbol{P}_j^{OUT}$ and $\boldsymbol{P}_j^{IN}$ be used as an example to identify the classification from a given perspective j.

$$\boldsymbol{P}_j^{OUT} = diag(1,0,1,0,\ldots,0)$$

$$\boldsymbol{P}_j^{IN} = \boldsymbol{I}_r - \boldsymbol{P}_j^{OUT} = diag(0,1,0,1,\ldots,1)$$

where we use the diagonal element p_{ij}^{OUT} to specify whether an attribute is input or output. A given attribute i is considered to be output by perspective j if $p_{ij}^{OUT} = 1$ ($i = 1,\ldots, r$), and is an output if $p_{ij}^{OUT} = 0$. We call $\boldsymbol{P}_j^{OUT}$ the output matrix. In the above example, perspective j considers the first and the third attributes of DMU as inputs, and other attributes as outputs. As an attribute is classified as either output or input, we can also obtain the input matrix $\boldsymbol{P}_j^{IN}$ by subtracting $\boldsymbol{P}_j^{OUT}$ from the identity matrix $\boldsymbol{I}_r$. In the input matrix $\boldsymbol{P}_j^{IN}$, the diagonal elements with value "1" indicate inputs, and elements with value "0" indicate outputs.

The condition $\boldsymbol{P}_j^{OUT} + \boldsymbol{P}_j^{IN} = \boldsymbol{I}_r$ is always valid so as to ensure that an attribute is either an input or an output. Thus, the set of all possible input/output classifications corresponds to the set of diagonal matrixes $\boldsymbol{P}_j$, which can be denoted by $\boldsymbol{\Phi}$, and is given as follows:

$$\boldsymbol{\Phi} := \left\{ \boldsymbol{P}_j : \boldsymbol{P}_j^{OUT} + \boldsymbol{P}_j^{IN} = \boldsymbol{I}_r, p_{ij}^{OUT}, p_{ij}^{IN} \in \{0,1\}, i = 1,\dots,r \right\} \qquad (4.2)$$

Incorporating the method of input/output classification ($\boldsymbol{P}_j^{OUT}$ and $\boldsymbol{P}_j^{IN}$) for all attributes from a given perspective j. Eq. (4.1) can be rewritten in the form of multiple perspectives as follows:

$$\begin{aligned} & \max_{\boldsymbol{u}_j} \ E_{oj} = \frac{\boldsymbol{u}_j \boldsymbol{P}_j^{OUT} \boldsymbol{a}_o}{\boldsymbol{u}_j \boldsymbol{P}_j^{IN} \boldsymbol{a}_o} \\ & s.t. \ \frac{\boldsymbol{u}_j \boldsymbol{P}_j^{OUT} \boldsymbol{A}}{\boldsymbol{u}_j \boldsymbol{P}_j^{IN} \boldsymbol{A}} \le 1 \ \ (j = 1,\dots,q) \\ & \qquad \boldsymbol{u}_j \ge 0 \end{aligned} \qquad (4.3)$$

in which row vector $\boldsymbol{u}_j$ denotes the weight assignment for each attribute under perspective j. Unlike traditional CCR model, the objective function E_{oj} in Eq. (4.3) is expanded from a single perspective into multiple perspectives, which means the maximum efficiency score of DMU$_o$ for a given perspective j. To obtain the efficiency scores of DMU$_o$ for multiple perspectives, Eq. (4.3) should be executed q times.

Before the introduction of efficiency improvement for multiple perspectives, we present a simple example to show the situation of DMUs under multiple perspectives, and illustrate what kind of DMUs need efficiency improvement. In this example, the total number of DMUs is nine, each with three attributes. We survey the situation of DMUs under two perspectives.

As shown in Table 4.1, we assume that there are nine branch stores labeled A through I at the head of each column. Each store has three attributes, namely, the number of employees (unit: 10 persons), area (unit: 1,000 m^2), and price (average retail price, unit: 100 dollars), which are as recorded in each column. In Table 4.2, we assume that there are two perspectives: management and customer. From the viewpoint of management, a branch store that consumes fewer resources and has higher prices is considered to be more efficient. Thus, the number of employees and the area are regarded as inputs, and the price is regarded as an output. However, from the viewpoint of the customer, a store having more resources and lower prices is considered to be more efficient. Thus, the customer and management classify attributes in a perfectly contradictory manner. Note that the price is unitized to "1", and so the values of employee and area are normalized to values for one unit of price.

Table 4.1 Nine stores and corresponding attributes

Store	A	B	C	D	E	F	G	H	I
Employee	4	7	8	4	2	5	6	5.5	6
Area	3	3	1	2	4	2	4	2.5	2.5
Price	1	1	1	1	1	1	1	1	1

Table 4.2 Input/output classification from two perspectives

Attribute	Management	Customer
Employee	Input	Output
Area	Input	Output
Price	Output	Input

We take "employee/price" and "area/price" as two axes and plot the stores in Figure 4.1. The efficiency scores of nine DMUs for perspectives of management and customer are calculated by Eq. (4.3) as shown in Table 4.3. In Figure 4.1, the black line which is constructed by DMU *E*, *D* and *C* is the efficient frontier from the perspective of management, and the efficient frontier of customer consists of DMU *E*, *G*, *B* and *C*. This can also be verified by the results shown in Table 4.3, where the DMUs located on two efficient frontiers achieve the highest efficiency score "1" compared to other DMUs. DMU *C* and *E* are evaluated as efficient DMUs by two perspectives simultaneously, which are also reflected in Figure 4.1 as the crossing points of two perspectives. Efficient DMUs for all perspectives are out of our consideration. The DMUs surrounded by efficient frontiers, such as *A*, *F*, *H* and *I* in the above example, which are partially efficient for multiple perspectives are the targets of our consideration in the following process of efficiency improvement.

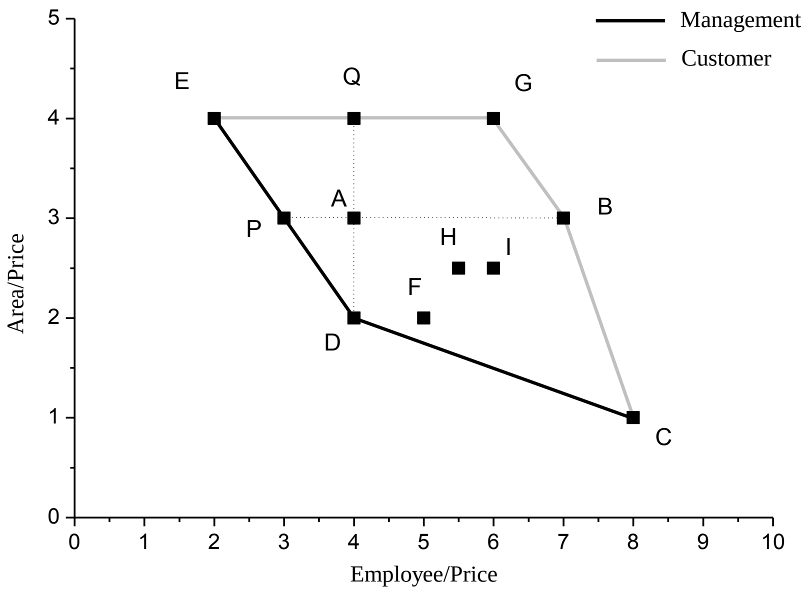


Figure 4.1 Efficiency analysis under two perspectives

Table 4.3 Efficiency scores of nine stores from two perspectives

Store	A	B	C	D	E	F	G	H	I
Management	0.857	0.632	1	1	1	0.923	0.6	0.774	0.75
Customer	0.75	1	1	0.6	1	0.706	1	0.8	0.853

4.2.2 Improving process

Here, we utilize the NBG theory [15] again in the improving process in order to seek an equilibrium solution for multiple perspectives. We review the NBG theory at first. The theory starts with two players who want to divide the surplus value of cooperation. We assume two players, *A* and *B*, who want to divide the surplus value produced through cooperation. If each of these players operates his own business without cooperation, *A* will obtain payoff *a*, and *B* will obtain payoff *b*. *a* and *b* are breakpoints which means if the bargaining game does not yield an agreement. If the players cooperate, they will obtain total value *V*, which is greater than $a + b$. The surplus value is generated because of their cooperation. This added value is why the players want to cooperate. The surplus value $s = V - a - b$. Here, $x = a + w_A s$, $y = b + w_B s$, where x and y denote the utility functions of player A and B, respectively, and w_A and w_B denote the market weights of A and B, respectively. The function of NBG takes the following form:

$$max\left(x-a\right)^{w_A}\left(y-b\right)^{w_B} \qquad (4.4)$$

The above model expresses the optimal assignment of surplus value between two players. Nash [15] proposed that a reasonable solution should satisfy the following axioms: (a) invariant to affine transformations or invariant to equivalent utility representations, (b) Pareto optimality, (c) independence of irrelevant alternatives, and (d) symmetry.

Incorporating the NBG theory into efficiency improvement problem under multiple perspectives, we suppose that each perspective is a player and multiple perspectives bargain for an appropriate scheme in improving DMU$_o$. Assume that multiple perspectives reach a final agreement about the improving scheme for DMU$_o$. Namely the change of attributes for DMU$_o$ is along the direction vector $\boldsymbol{s}$ and the according weight assignment for the attributes of DMU$_o$ by a given perspective j is denoted by vector $\boldsymbol{u}_j$. (Note we assume that different perspectives have different weight assignments for the attributes of DMU$_o$.) Thus the efficiency score of perspective j can be denoted as $E_j(\boldsymbol{s}, \boldsymbol{u}_j)$. An optimal solution called the Nash solution satisfies the above four axioms and can be obtained by solving the following equation:

$$max\prod_{j=1}^{q}(E_j(\boldsymbol{s},\boldsymbol{u}_j)-E_{min,j})^{w_j} \qquad (4.5)$$

where the weight variable w_j denotes the market share of perspective j. And $E_{min,j}$ is the lowest efficiency score of perspective j while improving DMU$_o$ along direction $\boldsymbol{s}$. The breakpoint for each perspective is set to $E_{min,j}$, which means that if they cooperate to seek an optimal solution $\boldsymbol{s}$ in adjusting the attributes of DMU$_o$, both perspectives can maximize their efficiency scores according to their market shares; otherwise each perspective receives a lowest efficiency score $E_{min,j}$ in the direction.

Suppose that the attributes of DMU$_o$ after improvement is $\boldsymbol{a}_o^*$, the direction $\boldsymbol{s}$ from $\boldsymbol{a}_o$ to $\boldsymbol{a}_o^*$ is denoted by vector $(\theta_1, \theta_2, \ldots, \theta_r)$ where each element is the angle between $\boldsymbol{s}$ and each attribute of DMU$_o$. The Euclidean distance from $\boldsymbol{a}_o$ to $\boldsymbol{a}_o^*$ can be expressed as

$$d = \|\boldsymbol{a}_o^* - \boldsymbol{a}_o\|$$

Thus $\boldsymbol{s}$ can be denoted as follows

$$\boldsymbol{s} = (\boldsymbol{a}_o^* - \boldsymbol{a}_o) / \|\boldsymbol{a}_o^* - \boldsymbol{a}_o\| = (\cos\theta_1, \cos\theta_2, \ldots, \cos\theta_r)$$

The improved value for DMU$_o$ along direction $\boldsymbol{s}$ can be denoted by

$$d\boldsymbol{s} = (\boldsymbol{a}_o^* - \boldsymbol{a}_o) = (d\cos\theta_1, d\cos\theta_2, \ldots, d\cos\theta_r)$$

The product of direction vector $\boldsymbol{s}$ and changed distance d, viz. "$d\boldsymbol{s}$", is an equilibrium solution corresponding to the rule of Pareto Optimality (PO) which can maximize the product of efficiency scores of multiple perspectives. Each element of $d\boldsymbol{s}$ is the component of $(\boldsymbol{a}_o^* - \boldsymbol{a}_o)$ on each attribute of DMU. $E_j(\boldsymbol{s}, \boldsymbol{u}_j)$ can be rewritten as the following envelopment form using input/output classification method (4.2) under multiple perspectives:

$$E_j(\boldsymbol{s}, \boldsymbol{u}_j) = \frac{\boldsymbol{u}_j \boldsymbol{P}_j^{OUT}(\boldsymbol{a}_o + d\boldsymbol{s})}{\boldsymbol{u}_j \boldsymbol{P}_j^{IN}(\boldsymbol{a}_o + d\boldsymbol{s})} \qquad (4.6)$$

Incorporating the above expression of $E_j(\boldsymbol{s}, \boldsymbol{u}_j)$ into Eq. (4.5) allows us to show the whole process of efficiency improvement about DMU$_o$ using the following Eq. (4.7).

$$\begin{aligned}
&\max_{\boldsymbol{s}, d, \boldsymbol{u}_j} \Omega = \prod_{j=1}^{q} \left(\frac{\boldsymbol{u}_j \boldsymbol{P}_j^{OUT}(\boldsymbol{a}_o + d\boldsymbol{s})}{\boldsymbol{u}_j \boldsymbol{P}_j^{IN}(\boldsymbol{a}_o + d\boldsymbol{s})} - E_{min,j} \right) \\
&s.t. \ \frac{\boldsymbol{u}_j \boldsymbol{P}_j^{OUT}(\boldsymbol{a}_o + d\boldsymbol{s})}{\boldsymbol{u}_j \boldsymbol{P}_j^{IN}(\boldsymbol{a}_o + d\boldsymbol{s})} \le 1 \ (j = 1, \ldots, q) \ldots\ldots\ldots\ldots\ldots(a) \\
&\quad \frac{\boldsymbol{u}_j \boldsymbol{P}_j^{OUT}(\boldsymbol{a}_o + d\boldsymbol{s})}{\boldsymbol{u}_j \boldsymbol{P}_j^{IN}(\boldsymbol{a}_o + d\boldsymbol{s})} \ge E_{min,j} \ (j = 1, \ldots, q) \ldots\ldots(b) \\
&\quad \frac{\boldsymbol{u}_j \boldsymbol{P}_j^{OUT} \boldsymbol{A}}{\boldsymbol{u}_j \boldsymbol{P}_j^{IN} \boldsymbol{A}} \le 1 \ (\boldsymbol{a}_o \notin \boldsymbol{A}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(c) \\
&\quad \boldsymbol{u}_j \ge 0 \ (j = 1, \ldots, q)
\end{aligned} \qquad (4.7)$$

Different perspectives have different weight assignments, which are categorized by the subscript j. The objective function uses NBG (incorporating the CCR model) theory to get an equilibrium solution $\boldsymbol{s}$ and d in improving DMU$_o$, which specifies along which direction and to what extent to improve DMU$_o$. The equilibrium solution maximizes the efficiency score of DMU$_o$ for multiple perspectives to avoid invoking discontentment of some perspectives.

Constraint (a) indicates that the ratio of "weighted output" vs. "weighted input" of DMU$_o$ should not exceed 1 for all perspectives. Each perspective hopes to obtain a higher efficiency score better than the breakpoint in the bargaining process, which is expressed by constraint (b). Constraint (c) means that the ratio of "weighted output" vs. "weighted input" of other DMUs (except DMU$_o$) should not exceed 1 for all perspectives. As there are n DMUs and q perspectives in the current system, the total number of constraints in improving DMU$_o$ is $n \times q$. The market share of each perspective is set to 1, which means all perspectives have the same status. For the improving scheme of DMU$_o$, the optimal direction $\boldsymbol{s}^*$ and change distance d are obtained, which can balance n DMUs to be the status of PO.

4.2.3 Calculation method

For a system with many attributes, usually multiple perspectives may reach an agreement about some attributes, such as the number of visitors of a store. Management and customer both consider a store having a large number of visitors as more efficient. That means multiple perspectives all regard the number of visitors as an output for a store. Of course there are also attributes considered as input commonly by multiple perspectives. Such attributes (inputs or outputs) are categorized as *common attributes* for multiple perspectives. To improve the efficiency of a DMU through changing the common attribute is fairly easy as there are basically only two ways: either decreasing the common input attributes or increasing the common output attributes.

Accordingly, in many situations we meet *conflictive attributes* which are perceived differently from multiple perspectives that can not reach an agreement. Take the price attribute of a store as an example. Management prefers higher price in order to make a profit, but from the perspective of customer lower price is preferred because of economy. To improve a DMU in conflictive attributes is rather difficult, as either increasing or decreasing the attribute some of perspectives will get a lower efficiency score. Increasing the price attribute will help management get a higher efficiency but at the same time decrease the efficiency of customer perspective. In our research, the three attributes employee, area and price list in the above example, for example, are the target attributes we consider in the case of multiple perspectives.

Another concern we have is to what extent we should improve a target attribute. Take DMU A in Figure 1 as an example, if we fix the value of "price", basically there are two ways to improve A, (1) adjusting the number of employees or (2) adjusting the area of the store. From the perspective of management, decreasing the number of employees is preferred, whereas from the perspective of customer the opinion is the opposite. As different perspectives have different opinion about the adjustment of attribute in improving the efficiency of a DMU, to be in conformity with one perspective may invoke discontentment of other perspectives. In order to ensure the equilibrium for multiple perspectives, we use the NBG theory to balance viewpoints of multiple perspectives and determine the optimal change for each attribute.

We mainly consider the following two aspects in efficiency improvement: (a) target attributes to be improved, and (b) improving the selected attribute by NBG. Eq. (4.7) shows a general method of improving DMU_o along a random direction, and the objective function of Eq. (4.7) is a product of q terms, which is nonlinear. But we can transform the nonlinear programming into a linear one in the process of improving DMU_o along a conflictive attribute.

The first step of calculating Eq. (4.7) is confirming the distance d and direction vector $\boldsymbol{s}$. The direction $\boldsymbol{s}$ is fixed while we improve DMU_o along a conflictive attribute. Therefore the objective function of Eq. (4.7) becomes a function of variable d. Suppose that the change interval of d is (d_{min}, d_{max}). Here we take the example shown in Figure 1 to interpret how to fix lower and upper bounds in improving efficiency of DMU_o along an attribute under multiple perspectives. The lower and upper bounds in adjusting "employee" attribute of A(4, 3) are fixed as P(3, 3) and B(7, 3). Even if we continually decrease "employee" after exceeding point P, the efficiency of management perspective still remains "1", and the efficiency of customer perspective keeps decreasing. On the other hand, the efficiency of customer perspective remains "1" even if we continually increase "employee" after exceeding point B, and the efficiency of management perspective keeps decreasing. Thus adjusting "employee" attribute outside the segment PB does not yield meaningful results. In this example, A is DMU_o under evaluation compared with Eq. (4.7) and attribute "employee" denotes the direction towards which DMU_o is being improved. As DMU_o is improved along the conflictive attribute "employee", the direction vector $\boldsymbol{s}$ is ($\cos 0^{\circ}$, $\cos 90^{\circ}$, $\cos 90^{\circ}$), thus $\boldsymbol{s}$ = ($\cos 0^{\circ}$, $\cos 90^{\circ}$, $\cos 90^{\circ}$) = (1, 0, 0) and $\boldsymbol{a}_o$ = (4, 3, 1), also $3 \le 4 + d \le 7$, $-1 \le d \le 3$ is obtained as the lower and upper bounds of d in adjusting "employee". The case is similar for adjusting "area" attribute, the range of which is the segment DQ. The calculated lower and upper bounds of d in adjusting "area" attribute are (-1, 1).

The above is an example in the case of 2 perspectives. In the case of multiple perspectives, the universal method of determining lower and upper bounds of "d" is generalized as follows: Suppose that the variable α denotes the attribute of DMU_o being improved, which can be denoted as $a_{\alpha o}$ (α_{th} element in vector $\boldsymbol{a}_o$). The number of perspectives considering $a_{\alpha o}$ as input is "n_a", and the number of perspectives considering $a_{\alpha o}$ as output is "n_b". Suppose that "L" is the first point which can ensure "n_a" perspectives simultaneously obtain the efficiency "1" while decreasing $a_{\alpha o}$ of DMU_o. Similarly we suppose that "U" is the first point which can ensure "n_b" perspectives simultaneously obtain the efficiency "1" while increasing $a_{\alpha o}$ of DMU_o. As $a_{\alpha L} \leq a_{\alpha o} + d \leq a_{\alpha U}$, we give the following definition.

Definition: While improving DMU_o along a conflictive attribute $a_{\alpha o}$, the lower bound of d is defined as $a_{\alpha L}$ -$a_{\alpha o}$, and the upper bound of d is defined as $a_{\alpha U}$ - $a_{\alpha o}$.

In a given direction $\boldsymbol{s}$, the calculation process of searching optimal result for d in ($a_{\alpha L}$ - $a_{\alpha o}$, $a_{\alpha U}$ - $a_{\alpha o}$) is as follows: The lower bound "$a_{\alpha L}$ -$a_{\alpha o}$" is set as the initial value, then we increase d by a very small positive number ε (such as 0.001 or even more smaller one, depends on the steps of calculation.) for each step t, namely, $d_t = -(a_{\alpha L} - a_{\alpha o}) + \varepsilon t$, t = 1, 2, … until the upper bound "$a_{\alpha U}$ - $a_{\alpha o}$" is reached. For a given step t, d_t is a fixed value, and $\boldsymbol{u}_j$ varies for different perspectives, thus $E_j(d_t, \boldsymbol{u}_j)$ for different perspectives in the objective function of Eq. (4.7) are mutually independent terms. So the objective function of Eq. (4.7) for each step t, equals to the function below:

$$\max_{d_t, \boldsymbol{u}_j} \prod_{j=1}^{q} \left(E_j(d_t, \boldsymbol{u}_j) - E_{min,j}\right) = \prod_{j=1}^{q} \max_{d_t, \boldsymbol{u}_j} \left(E_j(d_t, \boldsymbol{u}_j) - E_{min,j}\right) \qquad (4.8)$$

Eq. (4.8) is a linear function of which each term $\max_{d_t, \boldsymbol{u}_j} \left(E_j(d_t, \boldsymbol{u}_j) - E_{min,j}\right)$ can be calculated by Eq. (4.3) respectively. The maximum value of Ω in Eq. (4.7) is determined through comparing the results of all steps. Therefore the optimal solution of d in adjusting a given attribute is obtained.

4.4 Concluding remarks

This chapter illustrates a method about how to select an appropriate scheme to improve the efficiency of a DMU from multiple perspectives. As different perspectives have different preferences in increasing or decreasing an attribute of DMU_o, we use NBG value to describe the efficiency score of DMU_o for multiple perspectives. Thus the NBG value is an equilibrium solution which can avoid incurring discontentment of some perspectives in improving DMU_o. NBG values for all attributes of DMU_o are calculated by Eq. (4.7), and the most appropriate improvement scheme is selected through comparing NBG values of all improving directions.

As the main methodological section of the improving model, Eq. (4.7) can be modified in some parameters to get more significant results.

a. w_j denoting market weight of perspective j is set as "1" in the current study, which means each perspective has the same market status. For other studies which might have perspectives with different market weights, the model is still adaptive by replacing the value of w_j.

b. Breakpoint of perspective j is set as $E_{min, j}$, which means each perspective has a bottom efficiency score in the improving direction. In future study, we may have special request about some perspectives, for example, a bank may request its efficiency score for stakeholder to be above 0.9 in the process of NBG. In such case, the breakpoints of according perspectives should be added into Eq. (4.7).

The research follows the input/output classifications and the concept of multiple perspectives that have been frequently referred to in the recent DEA literature. More important is we give an improvement scheme based on the results of efficiency evaluation from multiple perspectives, which may be a new method for other researchers who are interested in performing DEA from multiple perspectives.

Chapter 5

A case study on Japanese banking industry

We talked about the process of efficiency improvement under multiple perspectives in last chapter, and presented an improving DEA model incorporating NBG theory to balance multiple perspectives in the improving process. The model not only obtains a NBG value which makes multiple perspectives reach to an equilibrium state, but also gives suggestions about improving a DMU along which direction and how to adjust its attributes.

Following the methodological research in Chapter 4, we would like to use the data of Japanese banking industry to demonstrate a concrete case study in this chapter. The case study shows how to improve the attributes of an inefficient bank concerning multiple perspectives, which have different input/output classifications for its attributes.

5.1 Data and attribute classification

In this section, we evaluate 65 Japanese banks from the viewpoints of four perspectives. Based on the evaluation result, we select the banks which are not efficient for all perspectives as the targets of efficiency improvement. The processes of efficiency improvements along all attributes are compared to select the most appropriate attribute as the final scheme to improve the inefficient banks. We can also set a special direction besides the attribute directions if the vector of improving direction $\boldsymbol{s}$ is given by decision maker, which we will not give detailed explanations.

As shown in Table 5.1, five typical attributes of the banking system we selected are as follows:

(1) capital adequacy ratio (CAR), which belongs to the category of soundness

(2) Net impaired assets per Shareholders' equity (NIA/SE), which indicates the credit quality of a bank

(3) return on average equity (ROAE), which is an indication of profitability

(4) cost per income (C/I), which is an indication of efficiency

(5) dividends per share (DPS) computed as the ratio of dividend paid to the number of outstanding shares.

We form four perspectives by referring to the classification method of inputs and outputs proposed by Avkiran and Morita [29]. Variables for each bank are categorized as Inputs and Outputs, for years 2001 through 2006 as shown in Table 5.1. There are four perspectives such as, stakeholders, customers, managements and employees which denote different classification methods from four typical viewpoints, respectively. Four perspectives assign different classifications regarding the five typical attribute fields of a bank, namely, soundness, credit quality, profitability, efficiency, and valuation. As attribute CAR is considered to be output by all perspectives, efficiency improvement by it can be obtained simply by increasing its value. Therefore CAR is a common attribute which is out of our consideration. The other four attributes for which four perspectives have different input/output classifications are considered as conflictive attributes, such attributes are the targets of our consideration.

Table 5.1 Five performance attributes and corresponding input/output classification with Four Perspectives

Category	Parameter	Description	Perspectives			
			S[a]	C	M	E
Soundness	CAR (%)	Capital adequacy ratio	Output	Output	Output	Output
Credit Quality	NIA/SE (%)	Net impaired assets per Shareholders' equity	Output	Input	Output	Input
Profitability	ROAE (%)	Return on average equity	Output	Input	Output	Output
Efficiency	C/I (%)	Cost per Income	Input	Output	Input	Output
Valuation	DPS	Dividends per share	Output	Input	Input	Input

[a] S, C, M and E denote the group of stakeholder, customer, management and employee, respectively.

Different perspectives have different input/output classifications about the four conflictive attributes in order to obtain more benefits from the viewpoint of their own groups. For example, dividends per share (DPS) in Table 5.1, which is, in part, a measure of shareholder value created by a bank, is classified as input by three perspectives. Customers often interpret it as being obtained through fees and charges levied by the bank for services and products. Managements treat DPS as input because dividends reduce a source of internal funds that can otherwise be reinvested in the business for growth. Bank employees are also likely to consider DPS to be input, as employees regard higher dividends as being taken from funds that could otherwise be used to improve their working conditions[42]. Only stakeholders have a conflictive viewpoint comparing with other three perspectives. Each perspective is assumed to provide a different classification method, by which each attribute of a bank system can be classified as either input or output.

In the present study, we focus primarily on the four conflictive attributes and four typical perspectives in banking systems. We attempt to improve an inefficient bank on four conflictive attributes, and select the most optimal attribute as the final improving scheme by comparing the NBG values obtained from Eq. (4.7).

In the following process of concrete computation, we assume that four perspectives have equal market weight, namely stakeholders, customers, managements and employees have the same status in the bargaining process. Table 5.2 indicates 65 Japanese banks and 5 corresponding attributes. Taking privacy into account, we use bank codes instead of real bank names.

5.2 Efficiency improvement for an inefficient bank

In Table 5.3, the efficiency scores of 65 Japanese banks evaluated by CCR model from multiple perspectives are listed. The NBG value for these banks before improvement is calculated as the product of CCR values of four perspectives. Here the breakpoints for each perspective is set to be "0" in order to discover which banks obtain non-equilibrium efficiency for multiple perspectives. And these banks are considered to be the targets in the improving process.

We rank banks by ascending order of NBG value. Bank 20 and 23 which are efficient for all perspectives do not need improvement. Other banks which are either partially efficient for some perspectives or inefficient for all perspectives are the targets of our consideration. Due to the limited space, here we only give the improvement schemes for banks ranking middle places (Bank 58, 57, 1, 49 and 32 which are ranking from 31 to 35.) and last places (Bank 61, 39, 62, 54 and 11 which are ranking from 61 to 65.).

Table 5.2 65 Japanese banks and corresponding attributes

Bank Code	Attributes				
	CAR	NIA/SE	ROAE	C/I	DPS
1	13.14	0.395908	2.87	64.29	3.96
2	10.84	0.245343	4.44	62.98	44.78
3	12.41	0.471256	3.52	66.21	2.94
4	13.2	0.785792	5.55	47.17	2.82
5	11.8	0.30037	5.55	61.98	3.48
6	11.9	0.390477	5.71	46.01	66.58
7	13.74	0.23448	3.64	71.52	28.78
8	11.71	0.2477	4.65	57.49	4.99
9	9.7	0.233714	4.06	66.19	3.49
10	11.32	0.39096	6.68	58.49	24.66
11	9.49	0.651953	7.22	58.69	12.52
12	11.2	0.269918	9.26	42.21	6.41
13	11.63	0.343818	9.61	43.69	5.49
14	9.47	0.508708	8.12	50.41	2.8
15	9.83	0.624146	1.82	68.49	2.48
16	13.68	0.294851	5.03	53.28	7.96
17	10.13	0.516477	6.74	71.36	2.49
18	10.78	0.477081	4.41	57.4	2.49
19	12	0.35292	4.18	50.77	2.99
20	9.15	1.034239	15.47	65.91	1.53
21	10.86	0.623065	3.97	63.5	2.49
22	9.21	0.577191	2.81	74.62	2.48
23	10.21	1.081472	6.7	45.16	1.56
24	12.14	0.468906	4.96	49.06	4.37
25	13.58	0.379838	4.84	45.05	5.39
26	10.68	0.540855	7.46	54.88	4.99
27	12.26	0.230483	3.79	63.24	3.48
28	10.39	0.346746	6.98	52.28	2.99
29	13.45	0.505728	1.28	60.6	2.88
30	10.77	0.598766	7.06	65	2.99
31	10.89	0.293222	3.95	65.65	3.49
32	10.58	0.369043	6.23	67.91	2.95

33	12.55	0.261668	4.47	49.86	3.49
34	11.98	0.28665	5.37	47.03	3.23
35	10.55	0.401909	5.47	59.75	3.48
36	10.06	0.62715	5.28	62.09	2.84
37	13.72	0.252266	4.09	59.09	3.49
38	11.05	0.401813	8.8	55.5	3.98
39	10.17	0.617024	4.96	60.66	37.36
40	12.67	1.041665	4.6	65.34	4.47
41	10.07	0.256771	6.25	54.59	3.53
42	8.55	0.666096	3.48	65.83	2.48
43	9.46	0.655718	7.57	58.19	4.03
44	10.94	0.344507	5.01	70.15	3.5
45	8.33	0.504109	3.87	65.93	2.49
46	10.64	0.431264	7.47	51.6	29.93
47	9.44	0.72972	2.46	67.55	2.43
48	11.15	0.529269	2.74	67.97	2.95
49	10.71	0.471519	4.73	42.05	3.64
50	10.15	0.356296	5.08	64.51	2.96
51	14.24	0.34547	4.76	60.34	3.41
52	12.2	0.215601	3.48	69.07	2.99
53	9.85	0.59728	4.63	64.66	2.98
54	10.67	0.621865	4.56	73.16	34.84
55	14.55	0.257155	5.26	48.82	7.87
56	10.49	0.517004	12.32	47.58	5.96
57	9.84	0.504465	3.74	66.29	2.39
58	11.26	0.419057	5.54	51.14	3.48
59	10.59	0.636048	5	74.98	3.24
60	9.86	0.91957	3.83	68.97	2.76
61	10.17	0.648808	3.12	54.59	3.98
62	10.25	0.783271	9.44	62.13	24.97
63	10.64	0.55704	2.64	59.79	2.47
64	13.07	0.251755	4.22	65.43	2.92
65	12.03	0.438404	4.39	58.63	3.36

Table 5.3 NBG ranking under multiple perspectives

NBG ranking	NBG value	Bank code	Efficiency scores of perspectives			
			Stakeholder	Customer	Management	Employee
1	1	20	1	1	1	1
2	1	23	1	1	1	1
3	0.978	55	0.994	0.995	0.99	1
4	0.837	4	1	0.917	1	0.912
5	0.729	12	1	0.731	0.997	1
…	…	…	…	…	…	…
31	0.333	58	0.770	0.705	0.803	0.764
32	0.328	57	0.521	0.998	0.647	0.976
33	0.327	1	0.682	0.896	0.729	0.734
34	0.326	49	0.891	0.640	0.893	0.641
35	0.326	32	0.557	0.906	0.646	1
…	…	…	…	…	…	…
61	0.150	61	0.685	0.587	0.680	0.547
62	0.077	39	0.695	0.489	0.616	0.367
63	0.076	62	0.837	0.305	0.738	0.404
64	0.074	54	0.583	0.617	0.521	0.394
65	0.069	11	0.687	0.366	0.668	0.411

Take B61 as an example, we use the model proposed in Chapter 4 to improve its efficiency. At first, as attribute CAR is considered to be output by all perspectives, we do not need to improve this attribute. In other words, even if we want to improve the efficiency of B61 by adjusting its CAR attribute, we just need to increase its CAR value until multiple perspectives can reach the efficiency score 1. Here we mainly take the other four conflictive attributes as the directions to improve B61. Firstly we try to improve B61 along the direction of NIA/SE. The improving process can be divided into the following three steps according to the model we proposed in Chapter 4.

Step 1: Calculate the lower bound and upper bound of adjusted attribute.

We decrease the attribute NIA/SE of B61 step by step (a very small value). As perspectives customer and employee regard NIA/SE as input, when we decrease this attribute to a value which is small enough, perspectives customer and employee obtain the efficiency score 1. The value of NIA/SE at this point is calculated as 0.7572, which is defined as the lower bound of adjusting NIA/SE of B61 as shown in Figure 5.1.

Similarly, we can calculate the upper bound of NIA/SE for B61, which is the value that makes perspectives stakeholder and management efficient while we increase NIA/SE of B61 step by step. The upper bound is 1.3085 as shown in Figure 5.1.

Step 2: Calculate the breakpoints for each perspective.

Now the four perspectives bargain about improving NIA/SE of B61 to what extant in order to satisfy all of them. While NIA/SE varies in the range [0.7572, 1.3085], each perspective obtains a lowest efficiency score for B61. It is rational that each perspective does not want to reach to the worst efficiency score, and each of them expects an improvement. Thus the lowest efficiency score for each perspective is considered to be the breakpoint, and we can calculate its value as shown in Table 5.4

Step 3: Compare the NBG values of all steps and select the maximum one.

The range [0.7572, 1.3085] is divided in to 500 steps in the process of calculation. For each step, we calculate the payoffs of all perspectives, and the NBG value which is the product of values of four payoffs. Compare the NBG values we can find out at the step 257 where the value of NIA/SE is 0.1788, the NBG value obtains highest value 2.8158e-06 as shown in Figure 5.1. Thus 0.1788 is determined as the optimal solution in improving NIA/SE of B61.

Table 5.4 Breakpoints and payoffs of multiple perspectives in adjusting NIA/SE of B61

	Stakeholder	Customer	Management	Employee
Breakpoints	0.6180	0.5683	0.6547	0.4914
Eff. Score	0.7220	0.5779	0.7167	0.5369
Payoff	0.104	0.0096	0.062	0.0455
Max NBG	2.8158e-06			

Similarly, we can calculate the optimal solutions for B61 while improving along the other three conflictive attributes ROAE, C/I and DPS, the results of which are expressed in Figures 5.2, 5.3 and 5.4.

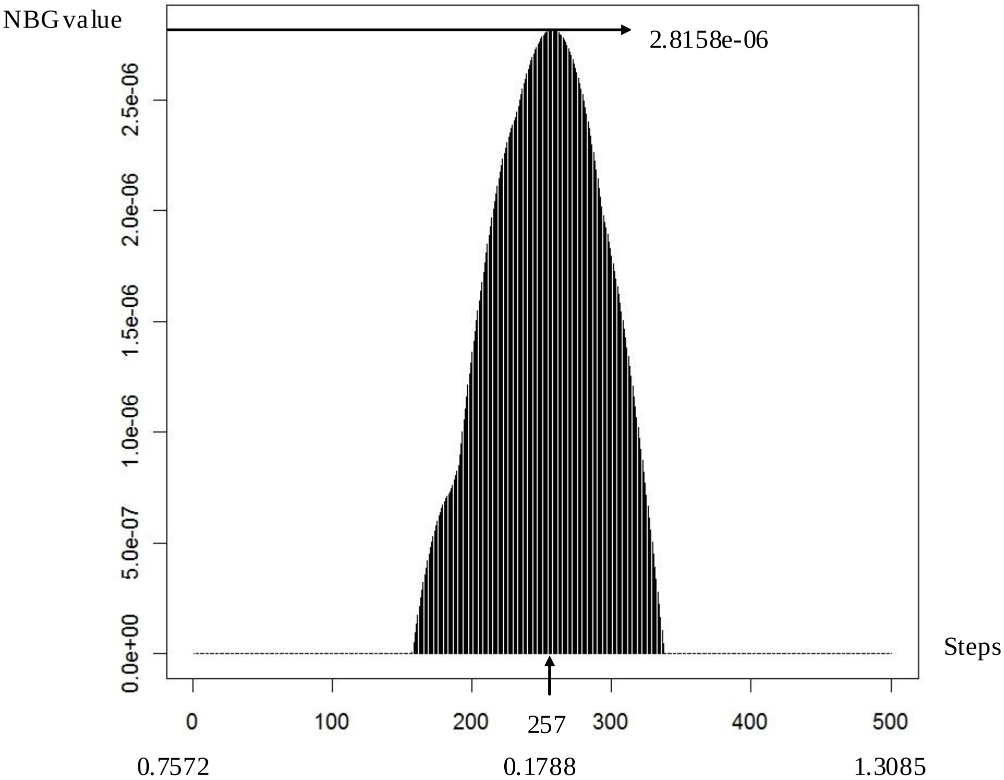


Figure 5.1 Improve NIA/SE of B61

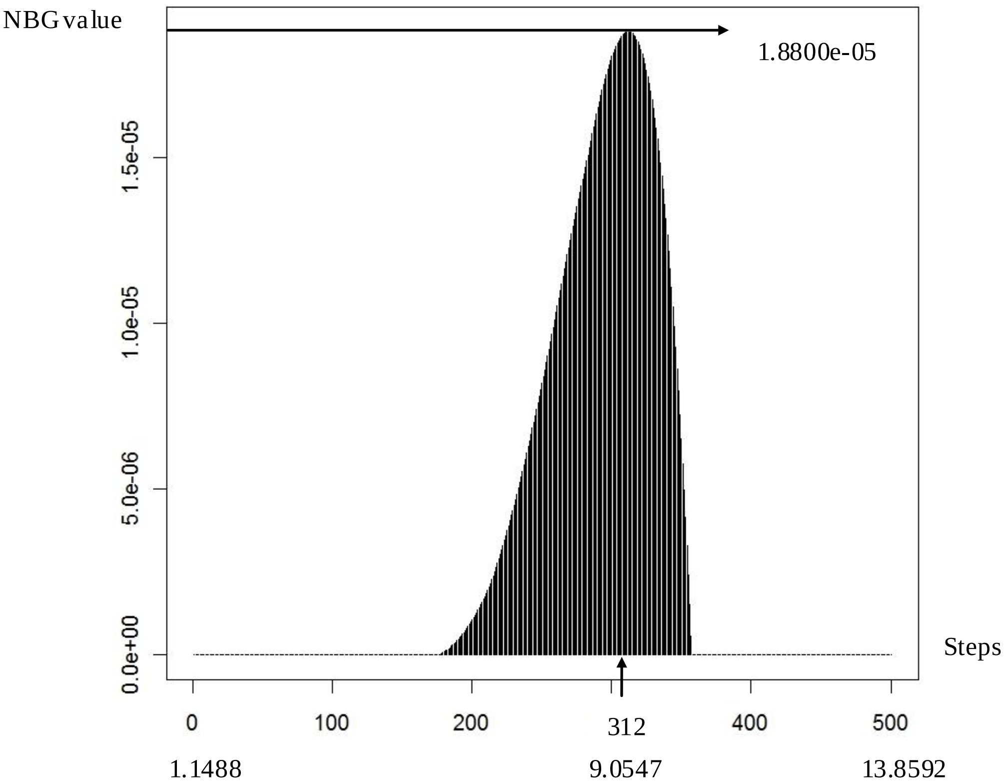


Figure 5.2 Improve ROAE of B61

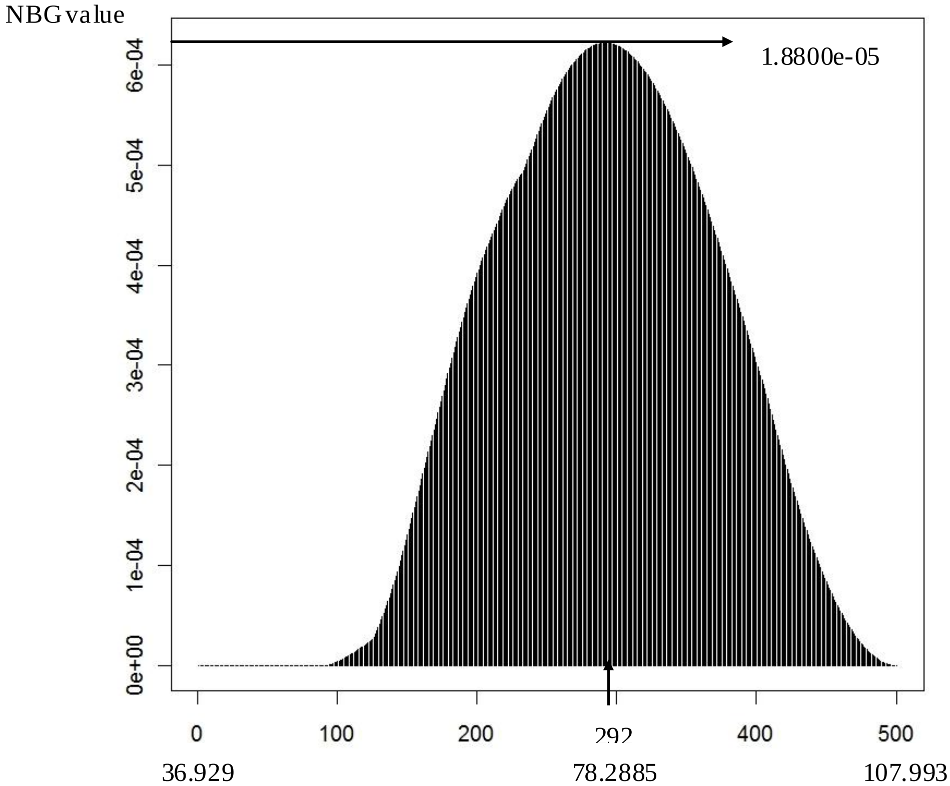


Figure 5.3 Improve C/I of B61

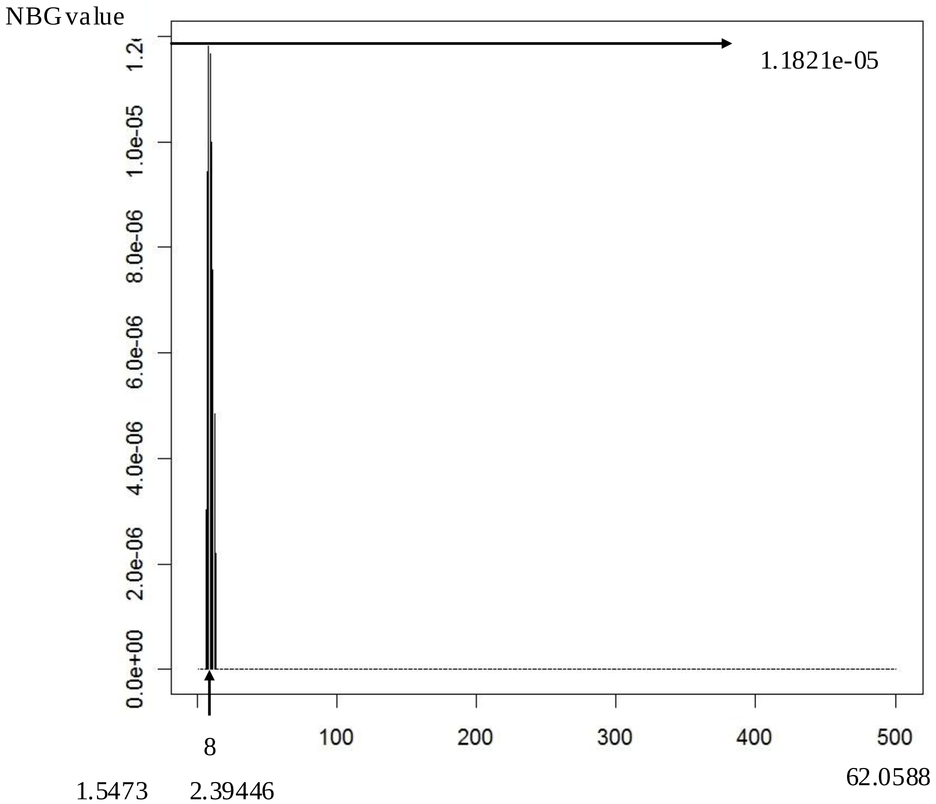


Figure 5.4 Improve DPS of B61

In the way that we treat B61, we obtain Table 5.5 illustrating the efficiency improvement for banks ranking middle places, such as B58, B57, B1, B49 and B32. Improvement schemes are categorized by four conflictive attributes: NIA/SE, ROAE, C/I and DPS, as shown in the first column. For each attribute of a bank ranking middle places, we utilize the three steps mentioned in Section 5.2. We set its breakpoints for multiple perspectives as the lowest efficiency score while improving along an attribute. Then we compare the breakpoints and efficiency scores of all perspectives after improvement and list the pay off of each perspective. The NBG values for all steps are compared to select the maximum one, and the corresponding value of the attribute is defined as the optimal value in adjusting the attribute.

Take B58 as an example, NBG values after improvement along four attributes are listed in the "NBG" row, such as 7.129e-06, 1.202e-08, 6.725e-05 and 6.173e-08, which are calculated by Eq. (4.7). The breakpoints of multiple perspectives are set differently while improving along four attributes, as multiple perspectives have different lowest efficiency scores in different improving directions. Selecting different attribute to improve the efficiency of B58, we can get four efficiency scores for the four perspectives. The decision maker may decide the final improving scheme by considering the efficiency scores of multiple perspectives and their breakpoints.

Take the NIA/SE as the example direction to improve B58, the maximum NBG value of all steps is 7.129e-06. When the NIA/SE attribute of B58 varies from 0.197 to 1.225, which is expressed by arrow, B58 obtains the highest NBG value at 0.905. That means when the NIA/SE attribute of B58 equals to 0.905, the efficiency scores of multiple perspectives can be balanced to the maximum extent. The varying range of NIA/SE attribute of B58 is calculated as [0.197, 1.225] which has another meaning. It means when the NIA/SE value of B58 is lower than 0.197, the perspectives considering NIA/SE to be input (namely customer and employee) as shown in Table 5.1 all obtain efficiency score "1". Similarly when NIA/SE of B58 is above 1.225, the perspective considering NIA/SE to be output (namely management and stakeholder) obtain efficiency score "1". Thus the change range of NIA/SE for B58 outside the range [0.197, 1.225] is meaningless. The improving schemes for other 4 banks are listed in the following four columns in Table 5.5.

Table 5.6 lists the processes of efficiency improvement for banks occupying last 5 places, whose NBG values are considered to be worst. The calculating process is the same with Table 5.6.

Table 5.5 Efficiency improvement for banks ranking middle places[a]

Attribute	P	Efficiency variation of each perspective while improving banks along conflictive attributes									
		B58		B57		B1		B49		B32	
NIA/SE (%)	S[c]	0.758 0.875	0.117	0.514 0.514	0[d]	0.678 0.744	0.066	0.865 0.945	0.08	0.551 0.669	0.118
	C	0.622 0.645	0.023	0.955 1	0.441	0.714 0.717	0.003	0.599 0.614	0.015	0.777 0.785	0.008
	M	0.803 0.871	0.068	0.647 0.647	0	0.729 0.741	0.012	0.893 0.940	0.047	0.646 0.666	0.02
	E	0.601 0.641	0.04	0.683 1	0.317	0.607 0.680	0.073	0.592 0.610	0.018	0.626 0.716	0.09
	NBG[b]	7.129e-06		0[e]		1.823e-07		9.322e-07		1.796e-06	
	R	0.197→1.225		0.448→1.589		0.230→1.541		0.187→0.845		0.212→1.628	
	OV	0.905		NA		0.820		0.705		1.036	
ROAE (%)	S	0.747 0.768	0.021	0.520 0.570	0.05	0.682 0.697	0.015	0.879 0.910	0.031	0.526 0.992	0.466
	C	0.705 0.705	0	0.976 0.976	0	0.734 0.734	0	0.636 0.638	0.002	0.906 0.906	0
	M	0.779 0.796	0.017	0.647 0.670	0.023	0.729 0.737	0.008	0.880 0.914	0.034	0.636 1	0.364
	E	0.705 0.752	0.047	0.976 0.978	0.002	0.734 0.808	0.074	0.636 0.693	0.057	0.906 1	0.094
	NBG[b]	1.202e-08		6.908e-10		4.669e-09		6.836e-08		1.594e-17	
	R	1.111→ 13.246		3.599→17.167		2.166→16.656		1.018→9.492		2.633→17.596	
	OV	5.383		6.883		5.817		5.611		17.447	
C/I (%)	S	0.449 0.528	0.079	0.501 0.582	0.081	0.453 0.583	0.13	0.401 0.500	0.099	0.504 0.589	0.085
	C	0.705	0.146	0.846	0.098	0.896	0.104	0.640	0.164	0.778	0.088

		0.851		0.944		1		0.804		0.866	
	M	0.542 0.599	0.057	0.631 0.696	0.065	0.567 0.660	0.093	0.487 0.559	0.072	0.597 0.678	0.081
	E	0.764 0.865	0.101	0.840 0.901	0.061	0.734 0.792	0.058	0.641 0.807	0.166	0.919 0.980	0.061
	NBG	6.725e-05		3.123e-05		7.388e-05		1.920e-04		3.726e-05	
	R	39.366→87.724		34.332→68.813		43.686→96.669		37.425→93.351		37.634→75.059	
	OV	74.667		59.296		75.157		74.895		64.205	
DPS	S	0.770 0.771	0.001	0.520 0.521	0.001	0.682 0.683	0.001	0.890 0.891	0.001	0.557 0.557	0
	C	0.508 0.575	0.067	0.685 0.864	0.179	0.868 0.892	0.024	0.504 0.661	0.157	0.574 0.641	0.067
	M	0.770 0.783	0.013	0.520 0.610	0.09	0.682 0.698	0.016	0.890 0.895	0.005	0.557 0.604	0.047
	E	0.534 0.675	0.141	0.426 0.855	0.429	0.566 0.586	0.02	0.431 0.662	0.231	0.722 0.816	0.094
	NBG	6.173e-08		2.534e-06		1.279e-08		5.349e-08		1.050e-07	
	R	1.705→74.013		1.503→95.934		2.002→93.037		2.003→49.112		1.604→98.284	
	OV	4.453		2.825		6.189		3.510		4.504	

[a] We improve each bank ranking middle places along four conflictive attributes (namely rows NIA/SE, ROAE, C/I and DPS), and two columns of data are listed for each attribute. The first column indicates breakpoints before improvement and efficiency scores after improvement of the bank for multiple perspectives along each conflictive attribute. And the second column calculates the corresponding changed values.

[b] NBG values along four attributes are calculated based on different selection of breakpoints.

[c] P denotes perspective, S, C, M and E denote the group of stakeholder, customer, management and employee, respectively.

[d] We retain 3 digits after the decimal point, which causes the same value with breakpoints. But actually the efficiency score is a litter higher than breakpoint.

[e] The NBG value might be “0” when the DMU being improved is very near or on the boundary of weak efficiency facet. As even very little excess across the weak efficiency facet will cause the efficiency score invariable, thus the pay off obtains “0” which also makes the NBG value “0”.

Table 5.6 Efficiency improvement for banks ranking middle places[a]

Attribute	P	Efficiency variation of each perspective while improving banks along conflictive attributes									
		B61		B39		B62		B54		B11	
NIA/SE (%)	S[c]	0.618 0.722	0.104	0.622 0.887	0.265	0.710 0.880	0.17	0.528 0.785	0.257	0.609 0.724	0.115
	C	0.568 0.578	0.01	0.314 0.412	0.098	0.239 0.293	0.054	0.345 0.485	0.14	0.264 0.344	0.08
	M	0.655 0.717	0.062	0.577 0.688	0.111	0.655 0.773	0.118	0.493 0.616	0.123	0.598 0.695	0.097
	E	0.491 0.537	0.046	0.156 0.252	0.096	0.231 0.356	0.125	0.140 0.242	0.102	0.257 0.358	0.101
	NBG	2.816e-06		2.746e-04		1.343 e-04		4.464 e-04		9.017e-05	
	R	0.179→1.309		0.188→1.454		0.193→1.467		0.227→1.754		0.182→1.407	
	OV	0.757		0.899		0.888		1.015		0.78	
ROAE (%)	S	0.685 0.796	0.111	0.695 0.816	0.121	0.699 0.893	0.194	0.583 0.675	0.092	0.631 0.752	0.121
	C	0.547 0.552	0.005	0.307 0.343	0.036	0.248 0.277	0.029	0.367 0.407	0.04	0.281 0.313	0.032
	M	0.680 0.830	0.15	0.603 0.697	0.094	0.641 0.794	0.153	0.520 0.597	0.077	0.605 0.717	0.112
	E	0.547 0.752	0.205	0.307 0.480	0.173	0.248 0.438	0.19	0.367 0.524	0.157	0.281 0.476	0.195

	NBG	1.880e-05		7.053e-05		1.673 e-04		4.429e-05		8.485e-05	
	R	1.149→13.859		1.274→21.178		1.312→26.874		1.539→21.341		1.23→19.956	
	OV	9.055		8.558		10.821		8.707		9.057	
C/I (%)	S	0.346 0.478	0.132	0.213 0.394	0.181	0.207 0.460	0.253	0.214 0.394	0.18	0.193 0.402	0.209
	C	0.549 0.827	0.278	0.410 0.864	0.454	0.271 0.555	0.284	0.447 0.913	0.466	0.319 0.626	0.307
	M	0.408 0.498	0.09	0.189 0.349	0.16	0.183 0.406	0.223	0.191 0.352	0.161	0.196 0.391	0.195
	E	0.539 0.727	0.188	0.321 0.542	0.221	0.359 0.543	0.184	0.320 0.543	0.223	0.369 0.544	0.175
	NBG	6.216e-04		28.878 e-04		29.434 e-04		30.231 e-04		21.967 e-04	
	R	36.929→ 107.994		37.112→ 197.743		45.731→ 250.94		37.838→199.31		38.927→ 209.229	
	OV	78.289		107.147		113.037		108.24		100.235	
DPS	S	0.680 0.681	0.001	0.616 0.617	0.001	0.738 0.740	0.002	0.521 0.521	0	0.668 0.669	0.001
	C	0.549 0.909	0.36	0.489 0.826	0.337	0.305 0.530	0.225	0.617 0.906	0.289	0.366 0.620	0.254
	M	0.680 0.758	0.078	0.616 0.671	0.055	0.738 0.755	0.017	0.521 0.618	0.097	0.668 0.699	0.031
	E	0.279 0.847	0.568	0.367 0.788	0.421	0.404 0.673	0.269	0.394 0.886	0.492	0.408 0.677	0.269
	NBG	1.182e-05		4.745e-06		2.245e-06		7.277e-06		2.174e-06	
	R	1.547→62.059		1.543→78.441		1.592→56.289		1.624→105.786		1.458→70.874	
	OV	2.394		2.773		4.436		2.874		3.541	

[a] The structure of this table is the same with Table 5.5 except that this table focuses on the analysis results about banks ranking the last five places.

5.3 Factors impacting efficiency improvement schemes

There are many factors which can impact the final selection of efficiency improvement for an inefficient bank. Firstly, the selection of breakpoints for multiple perspectives is correlated with the final selection of improvement scheme. In Section 5.2, we take B61 as an example to improve its efficiency score along four different conflictive attributes. But please note that we select different breakpoints for different attribute, which makes it difficult to compare the improving effect for these attributes. To make it possible to compare different attributes, we need to use an identical breakpoint for different attributes. We assume the breakpoint of a perspective is defined as its lowest efficiency score while we improve along all different attributes, which is shown as Table 5.7. Thus we have identical breakpoints of multiple perspectives while improving B61 along four attributes from which we can compare the improving effects. We compare the NBG value of the four attributes, and find out that if we improve B61 along DPS attribute B61 obtains the highest NBG value 0.06465, which makes multiple perspectives obtain highest payoffs. Thus improving the value of DPS to 1.547 is considered to be the final improving scheme.

Secondly, DEA provides an improving method for inefficient DMUs based on the evaluation result. But it does not show how to change a specific attribute for a inefficient DMU, especially for some financial factors that we can not directly adjust. In Table 5.5 and Table 5.6 we may select the most appropriate attribute to improve efficiency for each inefficient bank. CAR, NIA/SE and C/I are commonly encountered attributes in financial field that we can directly adjust. Whereas DPS can be calculated by the following formula:

$$DPS = (D - SD) / S$$

where D denotes the sum of dividends over a period (usually one year), SD denotes special dividend which is declared as a one-off payment by banking system, and S means shares outstanding for the period. Improvement schemes for inefficient banks in Table 5.5 and Table 5.6 indicate decreasing DPS which can be achieved by increasing SD or decreasing D. Usually increasing D and SD indicates prosperous future of a bank, however excessive amount of SD will weaken the current assets and ability of paying debts. Moreover a dividend decrease is not always a signal that the bank has a pessimistic view of its near financial future. The bank may free up cash to keep the business afloat. Besides D, SD and S, DPS is also affected by many other factors such as, tax, debt or management policy[54, 55].

Table 5.7 Compare improving schemes along four attributes for B61

Attributes		Stakeholder	Customer	Management	Employee
	Breakpoints	0.346	0.547	0.408	0.279
	Eff. Score	0.618	1.000	0.655	1.000
	Payoff	0.272	0.453	0.247	0.721
NIA/SE	Range	0.179 → 1.309			
	Optimal Value	0.179			
	Max NBG	0.02193			
	Eff. Score	0.685	1.000	0.680	0.547
	Payoff	0.339	0.453	0.272	0.268
ROAE	Range	1.149 → 13.859			
	Optimal Value	1.149			
	Max NBG	0.01123			
	Eff. Score	0.571	0.692	0.567	0.617
	Payoff	0.225	0.145	0.159	0.338
C/I	Range	36.929 → 107.994			
	Optimal Value	65.497			
	Max NBG	0.00176			
	Eff. Score	0.68	1.00	1.00	1.00
	Payoff	0.334	0.453	0.592	0.721
DPS	Range	1.547 → 62.059			
	Optimal Value	1.547			
	Max NBG	0.06465			

ROAE is calculated by the ration of “Net Income after Tax” to “Average Shareholders’ Equity”, which measures a bank's profitability by revealing how much profit a bank generates with the money shareholders have invested over a fiscal year. The denominator is usually computed as the sum of the equity value at the beginning and the end of the year divided by two. Improvement schemes recommend some inefficient banks in Tables 5.5 and 5.6 to increase their ROAEs. A better ROAE can be obtained through consistent dividend payments, share buyback as well as mergers and acquisitions. As higher dividend payout and share buybacks will reduce reserves, which lead to an ROAE improvement. Another ways to enhance ROAEs include raising their operating profit margins and recurring non-interest income activities that can produce better profit opportunities. Also the relationship between ROAE and leverage originated from DuPont Analysis has attracted plenty of interest. Positive or negative leverage can also affect ROAE depending on the actual conditions of the bank [56, 57].

5.4 Concluding remarks

In this chapter, we take 65 Japanese banks as a concrete case study to explain how to use the models that we proposed in Chapter 4 to improve inefficient banks under multiple perspectives.

In our case study, we suppose that each perspective has the same market status, which means their weights in the equation of NBG are set as “1”. In actual applications, the decision maker can assign concrete values to the market weights. We take B61 as an example to show how to calculate varying range and breakpoints for multiple perspectives, and how to get the optimal value for an attribute under improvement. The process of calculation employs an approximate method to transform the nonlinear computation into linear one.

We also give a further consideration about selection of breakpoint in the end of this chapter, which might provide a method to compare different improving schemes, and allow the decision maker to select the most appropriate one.

Chapter 6

Conclusions and future directions

This thesis studies how to incorporate NBG theory into existent DEA models in evaluating and improving DMUs under multiple perspectives. As the basic knowledge and initial research, Chapter 1 introduces the concepts and principles of DEA and NBG. Given this, Chapter 2 sums up the preliminary research we did, attempting to seek an appropriate method to analyze DMUs under multiple perspectives. Chapters 4 and 5 introduce the specific methods of efficiency evaluation and improvement in order to satisfy multiple perspectives. Finally, in the last chapter we talk about the contributions of this research, and future research directions.

6.1 Contributions

The current research follows the studies about desirable/undesirable classifications to attributes of DMU, classification methods, and many other existing DEA models. The contribution of this research is proposing a new method incorporating game theory to solve the problems of efficiency evaluation and improvement for DMUs under multiple perspectives. We summarize the contributions as the following points:

a. Preliminary research in classification method of desirable/undesirable about attributes of DMUs, and proposal of initial analysis models. This is included in Chapter 2.

b. Proposed a new DEA model integrating NBG to evaluate DMUs under the circumstance of multiple perspectives. Most of the detailed introduction is appeared in Chapter 3.

c. Solved the problem of efficiency improvement based on “b” concerning how to satisfy multiple perspectives by NBG.

6.2 Directions for future research

Although we represent a systematic method about efficiency analysis based on DEA under multiple perspectives, there still exist some uncompleted tasks and valuable subsequent research topics, which can be summed up in the following aspects:

a. Chapter 4 deals with the problem of improving inefficient DMUs in the attribute directions, which is not suitable for the case improving a DMU in other directions. Thus we will endeavor to develop a more integral improving DEA model to deal with improving DMUs in random directions.

b. The thesis employ game theory as it is difficult to fix the weights of different perspectives. However the proposed DEA model integrating NBG still seems not very appropriate for two reasons: Firstly, some perspective, like employee, may not really have enough power or status to join in the bargaining game except management and customer. Secondly, the proposed model met some difficulties in seeking optimal solutions as it is a nonlinear model. Based on the two reasons, it is necessary to consider other methods to fix the weights of perspectives.

c. Some studies focusing on the clustering analysis in DEA based on input/output classification, different reference sets, and layers of efficiency frontiers [58-62]. Actually the problem of clustering analysis still exists in the case of multiple perspectives. We have done some methodological research about this problem, which might be a meaningful research topic in future.

We hope the research we have been dedicating to is helpful for the development of DEA research, and can be applied realistically by the readers.

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