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The method of the microscopic calculation

of the cross section in the distorted wave Born approximation

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Contents

Abstract:

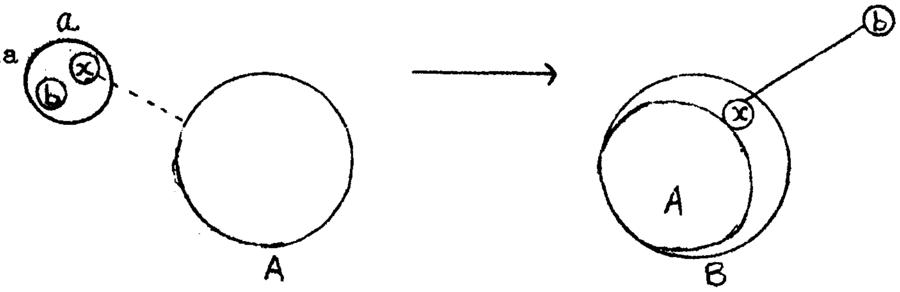
We develop a new method for the calculation of the cross section in the distorted wave Born approximation (DWBA). The form factor in the transition matrix element is expanded with harmonic oscillator wave functions. This method is convinient not only for the microscopic calculation using shell model wave functions, but also for the study of the recoil effect of the target nucleus. This can be conveniently applied to the investigation about the term which is assumed to be zero in the usual DWBA calculation and about the heavy particle stripping term.

The numerical calculations are performed for the heavy particle stripping term on the reaction $^{11}B(d,n)^{12}C$. We found that the damping of the matrix elements with increase of the nodes of harmonic oscillator wave functions is very rapid. We also found that the recoil effect plays an important role about the angular distribution of the calculated cross section. The result obtained using the reasonable interaction is much smaller than that of the usual stipping process. We can conclude that even if we calculate the heavy particle stripping term microscopically, the heavy particle stripping contribution on the reaction of $^{11}B(d,n)^{12}C$ is very small contrary to the result of Owen and Madansky.

1. Introduction

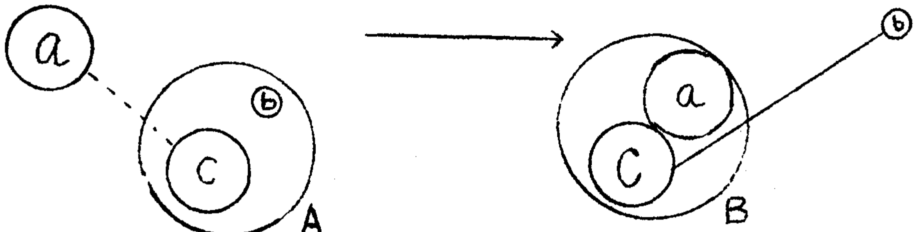
The theory of the duteron stripping reaction given by Butler[1] has had remarkable success in explaining forward peaks of angular distributions. For the stripping raction A(a,b)B, the main feature of the theory is shown in Fig. 1a.

Fig. 1a



This reaction is induced by the interaction between the particles b and x written with the solid line in the final channel, or by the interaction between the particle x and A written with the dotted line in the initial channel. However, some experiments extending the measurement of the angular distribution to the backward angle show a large component in the backward direction. To explain these results, Owen, Madansky[2] and others[3] have taken into account of the heavy particle stripping mechanism in addition to the usual stripping porcess. The featureof the heavy particle stripping mechanism is shown in Fig. 1b.

Fig. 1b



This reaction is induced by the interaction between the particles b and C written with the solid line in the final channel, or by the interaction between the particles a and C written with the dotted line in the initial channel. Owen and

Madansky calculated the cooss sections of the reaction $^{11}B(d,n)^{12}C$ in the plane wave Born approximation and obtained fairly good fits to the experimental results shown in Fig. 2. But to obtain good results, they had to use the parameter Λ_2/Λ_1 describing how much the heavy particle stripping process contributes to the cross sections.

On the other nand, the analysis of deuteron stripping reactions using the distorted wave Born approximation (DWBA) have obtained more remarkable success than the analysis using the plane wave Born approximation and have explained experimental results not only qualitatively but also quantitatively. In spite of such brilliant success of stripping reaction using DWBA, there have been only a few calculations[4] of the heavy particle stripping mechanism using DWBA. The reasons are as follows:

a) In D.W.B.A. calculations, distortion effects reduce the heavy particle strippin cross section[5].

b) In the actual calculations of the heavy particle stripping term, if we use the pane wave Born approximation, the six-dimensional integration of the transition matrix can be factorized into the product of two three-dimensional integration. But if we use D.W.B.A., this factorization cann't be done. Furthermore, the calculation of the heavy particle stripping term is more laborious than that of the usual stripping owing to the complicated form factor of the heavy particle stripping term.

Almost all the calculations done up to now are treated by the assumption that ^{10}B in the calculation of the heavy particle stripping term on the reaction $^{11}B(d,n)^{12}C$ is a single cluster and that interaction inducing reactions only depend on the relative distance between the center-of-mass of ^{10}B and the neutron. We call the calculation made by the above assumption as the macroscopic calculation, while we call the calculation without the above assumption, that is, considering ^{10}B as not a single cluster but assembly of nucleons as the microscopic calculation.

At the stage in the development of the study of the heavy particle stripping term, it is appropriate to present calculations of it using DWBA as consistently as possible. We develop a new method for the calculation of the cross section for this purpose. In §2, we give a treatmetn of the heavy particle stripping term introducing the discussion of Rodberg, and also give our numerical calculation on the reaction $^{11}B(d,n)^{12}C$ which Owen and Madansky have treated. We give a description of the cross section using form factors expanded with harmonic oscillator wave functions in §3. This method is applied to the calculation of the heavy particle stripping term on the reaction $^{11}B(d,n)^{12}C$. Results and discussions are given in §4. Concluding remarks are given in §5.

2. Treatment of the heavy particle stripping process

2.1 Derivation of T^{DWBA} matrix

The reaction

$$A(a,b)B \tag{2.1}$$

is considered.

We write the Hamiltonian of the total system H as

$$H=H_a+H_A+K_{aA}+V_{aA}, \tag{2.2a}$$

$$H=H_b+H_B+K_{bB}+V_{bB}, \tag{2.2b}$$

where Ha is the intrinsic Hamiltonian of the particle a, K_{aA} the kinetic energy operator between particles a and A, and V_{aA} the interaction between particles a and A. We define the intrinsic state of the particle a by Φ a, the intrinsic state of the nucleus A by Ψ A and the plane wave of the relative motion between particles a and A by ϕa. If we write the total energy by E, we obtain

$$(H_a+H_A+K_{aA}-E)\,\phi_a\Phi_a\Psi_A = 0\,, \tag{2.3a}$$

$$(H_b+H_B+K_{bB}-E)\,\phi_b\Phi_b\Psi_B = 0\,. \tag{2.3b}$$

The transition matrix T_{ba} can be written as

$$T_{ba} = \langle \phi_b \Phi_b \Psi_B | V_{bB} | \Psi_a^{(+)} \rangle$$

$$= \langle \phi_b \Phi_b \Psi_B | V_{bB} \Omega_a^{(+)} | \phi_a \Phi_a \Psi_A \rangle \tag{2.4}$$

where

$$\Omega_a^{(+)} = 1 + \frac{1}{E - H + i\epsilon} V_{aA}, \tag{2.5a}$$

$$\Psi_a^{(+)} \rangle = \Omega_a^{(+)} | \phi_a \Phi_a \Psi_A \rangle. \tag{2.5b}$$

We definee the operator $\omega_a^{(+)}$ and the distorted wave $\chi_a^{(+)}$ as

$$\omega_a^{(+)} = 1 + \frac{1}{E - H_a - H_A - K_{aA} - U_{aA} + i\epsilon} U_{aA}, \tag{2.6a}$$

$$| \chi_a^{(+)} \Phi_a \Psi_A \rangle = \omega_a^{(+)} | \phi_a \Phi_a \Psi_A \rangle, \tag{2.6b}$$

$$\omega_b^{(-)} = 1 + \frac{1}{E - H_b - H_B - K_{bB} - U_{bB}^* - i\epsilon} U_{bB}^*, \tag{2.7a}$$

$$| \chi_b^{(-)} \Phi_b \Psi_B \rangle = \omega_b^{(-)} | \phi_b \Phi_b \Psi_B \rangle. \tag{2.7b}$$

From Eq. (2.7a), we obtain the equality as

$$V_{bB} \Omega_a^{(+)} = \left(\omega_b^{(-)*} (V_{bB} - U_{bB}) + V_{bB} \left(1 - \frac{1}{E - H_b - H_B - K_{bB} - U_{bB} + i\epsilon} (V_{bB} - U_{bB})\right)\right) \Omega_a^{(+)} \tag{2.8}$$

If we use the equality

$$\frac{1}{E - H_b - H_B - K_{bB} - U_{bB} + i\epsilon} - \frac{1}{E - H + i\epsilon} = - \frac{1}{E - H_b - H_B - K_{bB} - U_{bB} + i\epsilon} (V_{bB} - U_{bB}) \frac{1}{E - H + i\epsilon}, \tag{2.9}$$

the second term of Eq. (2.8) becomes

$$V_{bB} \left(1 - \frac{1}{E - H_b - H_B - K_{bB} - U_{bB} + i\epsilon} (V_{bB} - U_{bB})\right) \Omega_a^{(+)}$$

$$= V_{bB} \frac{1}{E - H_b - H_B - K_{bB} - U_{bB} + i\epsilon} (E - H + i\epsilon) \left(1 + \frac{1}{E - H + i\epsilon} V_{aA}\right)$$

$$= V_{bB} \frac{1}{E - H_b - H_B - K_{bB} - U_{bB} + i\epsilon} (E - H_a - H_A - K_{aA} + i\epsilon)$$

$$= (\omega_b^{(-)*} - 1) \, i\epsilon. \tag{2.10}$$

Under the condition

$$\lim_{\epsilon \to 0} \; i\epsilon(\omega_b^{(-)*} - 1) = 0, \tag{2.11}$$

we obtain T_{ba} as

$$T_{ba} = \langle \chi_b^{(-)} \Phi_b \Psi_B | (V_{bB} - U_{bB}) | \Psi_a^{(+)} \rangle. \tag{2.12}$$

If we use the relation obtained from Eqs. (2.5a) and (2.6a)

$$\Omega_a^{(+)} = \left(1 + \frac{1}{E - H + i\epsilon} \; (V_{aA} - U_{aA})\right) \omega_a^{(+)}, \tag{2.13}$$

we obtain the amplitude of DWBA as

$$T_{ba}^{DWBA} = \langle \chi_b^{(-)} \Phi_b \Psi_B | V_{bB} - U_{bB} | \chi_a^{(+)} \Phi_a \Psi_A \rangle. \tag{2.14}$$

2.2 Exchange effect

We follow the antisymmetric treatment of the transition matrix made by Tobocman[6]. The state $\Psi_a^{(+)}$ is the solution of the total Hamiltonian with the boundary condition that in the channel a there are the incoming wave and the out-going scattered wave and in other open channels b there are outgoing scattered waves. We can write the asymptotic behabior of $\Psi_a^{(+)}$ using transition matices T_{ba} in Eq. (2.4) as

$$\Psi_a^{(+)} \sim \phi_a \Phi_a \Psi_A - \sum_b T_{ba} \frac{\mu_b}{2\pi\hbar^2} \Phi_b \Psi_B \frac{e^{ik_b r_{bB}}}{r_{bB}}, \tag{2.15}$$

where μ_b is the reduced mass of the channel b. This is not antisymmetric wave function. From Eq. (2.15), we obtain the antisymmetric solution of the total Hamiltonian H as

$$\tilde{\Psi}_a^{(+)} = N^{-1/2} \sum_n \epsilon_n P_n \Psi_a^{(+)}, \tag{2.16}$$

where N is the number of identical nucleons, P_n is the permutation operator and ϵ_n is its parity. The sum over n extends all possible permutations of N particles. The asymptotic behabior of Eq. (2.16) is

$$\Psi_a^{(+)} \sim \phi_a N^{-1/2} \sum_n \epsilon_n P_n (\Phi_a \Psi_A)$$

$$- \sum_{nb} N^{-1/2} \epsilon_n T_{ba} \frac{\mu_b}{2\pi\hbar^2} P_n (\Phi_b \Psi_B) \frac{e^{ik_b r_{bB}}}{r_{bB}} \qquad (2.17)$$

Since the sum over b includes all possible permutations, we can replace b by P_{nb}^{-1} in the summand for each value of n. Thus

$$\Psi_a^{(+)} \sim \phi_a N^{-1/2} \sum_n \epsilon_n P_n (\Phi_a \Psi_A) - \sum_{nb} N^{-1/2} \epsilon_n T_{P_n^{-1} ba} \frac{\mu_b}{2\pi\hbar^2} \Phi_b \Psi_B \frac{e^{ik_b r_{bB}}}{r_{bB}}$$

$$= \phi_a N^{-1/2} \sum_n \epsilon_n P_n (\Phi_a \Psi_A) - \sum_b T_{b\tilde{a}} \frac{\mu_b}{2\pi\hbar^2} \Phi_b \Psi_B \frac{e^{ik_b r_{bB}}}{r_{bB}}, \qquad (2.18)$$

where

$$T_{b\tilde{a}} = \sum_n N^{-1/2} \epsilon_n T_{bP_n a} . \qquad (2.19)$$

The differential cross section $\frac{d\sigma_{\tilde{b}\tilde{a}}}{d\Omega}$ can be written as

$$\frac{d\sigma_{\tilde{b}\tilde{a}}}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \sum_n | T_{P_n b \tilde{a}} |^2 = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b N}{k_a} | T_{b\tilde{a}} |^2$$

$$= \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \left| \sum \epsilon_n T_{b P_n a} \right|^2 \qquad (2.20)$$

In Eq. (2.20), we sum the cross section over P_{nb} since we cannot distiguish between $\Phi_b \Psi_B$ and $P_n(\Phi_b \Psi_B)$.

2.3 Application to (d,n) (or (d,p)) reaction

Tobocman gave explicit expression for the (d,p) reaction antisymmetrizing protons and neutrons seperately. Here we give explicit txpression for the (d,n) reaction treating protons and neutrons as the same for the following calculations. For simplicity, we write the operator $\sum_n \epsilon_n P_n$ as

$$P = \sum_n \epsilon_n P_n . \tag{2.21}$$

Then we may write $\sum_n \epsilon_n T^{DWBA}_{bP_n a}$ explicitly as

$$\sum_n \epsilon_n T^{DWBA}_{bP_N A} = \langle \chi_b^{(-)}(b_1)\Phi_b(b_1)\Psi_B(b_3, b_4, \ldots, b_N) | (\sum_i V_{b_1 b_i} - U_{b_1 B})P$$

$$| \Psi_A(b_3, b_4, \ldots, b_N)\Phi_a(b_1, b_2)\chi_a^{(+)}(b_1, b_2) \rangle , \tag{2.22}$$

where b_1, b_2 . . . b_N in the wave function $\Phi_b\Psi_B$ and etc. are N equivalent nucleons. $V_{b_1 b_i}$ represents the two-body interaction between particles b_1 and b_i, and $U_{b_1 B}$ represents the distorting potential which the particle b_1 feels. We operate the P to the initial channel states, and use the relation $P = \frac{P^2}{N!}$

$$P \, |\Psi_A(b_3, b_4, \ldots b_N)\Phi a(b_1, b_2)\chi_a^{(+)}(b_1, b_2)\rangle$$

$$= \frac{P}{\sqrt{2!}\sqrt{(N-2)!}} | \tilde{\Psi}_A(b_3, B_4, \ldots b_N)\tilde{\Phi}_a(b_1, b_2)\chi_a^{(+)}(b_1, b_2)\rangle , \tag{2.23}$$

where $\tilde{\Psi}_A(b_3, \ldots, b_N)$ is the normalized antisymmetrized wave function of $\Psi_A(b_3, \ldots, b_N)$. Next we operate the P to the interaction and the final state.

$$\sum \epsilon_n T^{DWBA}_{bP_n a} = \sqrt{\frac{(N-1)!}{2!(N-2)!}} \langle \chi_b^{(-)}(b_1)\Phi_b(b_1)\tilde{\Psi}_B(b_2, \ldots, b_N)$$

$$|(\sum_{i=2}^{N} V_{b_1 b_i} - U_{b_1 B})(-)^{1+i} P_{b_1 b_i} | \tilde{\Psi}_A(b_3, \ldots, b_N)\Phi_a(b_1, b_2)\chi_a^{(+)}(b_1, b_2)\rangle$$

$$= \sqrt{\frac{N-2}{2}} \Big\{ 2\langle\chi_b^{(-)}(b_1)\Phi_b(b_1)\tilde{\Psi}_B(b_2, \ldots, b_N) | (\sum_{i=2}^{N} V_{b_1 b_i} - U_{b_1 B})$$

$$|\tilde{\Psi}_A(b_3, \ldots, b_N)\Phi_a(b_1, b_2)\chi_a^{(+)}(b_1, b_2)\rangle$$

$$+ (N-2)\langle\chi_b^{(-)}(b_3)\Phi_b(b_3)\Psi_B(b_1, b_2, b_4, \ldots b_N) | (\sum_{i\neq 3}^{N} V_{b_3 b_i} - U_{b_3 B})$$

$$|\tilde{\Psi}_A(b_3, \ldots b_N)\tilde{\Phi}_a(b_1, b_2)\chi_a^{(+)}(b_1, b_2)\rangle \Big\} \tag{2.24}$$

If we drop the suffix b_i which distiguishes between N nucleons, Eq. (2.24) becomes

$$\sum \epsilon_n T_{bP_n a} = \sqrt{\frac{N-1}{2}} \Big\{ 2\langle\chi_b^{(-)} \Phi_b \tilde{\Psi}_B | V_{bx} + V_{bA} - U_{bB} | \tilde{\Psi}_A \tilde{\Phi}_a \chi_a^{(+)}\rangle$$
$$+ (N-2)\langle\chi_b^{(-)} \Phi_b \tilde{\Psi}_B | V_{ba} + V_{bC} - U_{bB} | \tilde{\Psi}_A \tilde{\Phi}_a \chi_a^{(+)}\rangle, \tag{2.25}$$

where x is the captured particle in the particle a, and C is the remainder obtained by removing the particle b from the nucleus A. Thus the transition amplitude is seen to consist of three direct terms and three excahnge terms. The first term of the direct amplitude is called the stripping term, the first term of the exchange amplitude is called the knock on term and the second term of the exchange amplitude is called the heavy particle stripping term.

2.4 Treatment of the heavy particle stripping term in DWBA

As we have mentioned in §1, one of the reason why there have been very few calculations in DWBA is the cancelation of the heavy particle stripping term and the distorting optical potential. We introduce the treatment of Rodberg et al.[5] Their conclusion is that the heavy particle stripping contribution to the cross section is of order $\frac{1}{A}$ compared with the usual stripping contribution. We explain it in detail. They derived the amplitude of DWBA as

$$T_{ba}^{DWBA} = \langle\chi_b^{(-)} \Phi_b \Psi_B | V_{bx} + V_{bA} - U_{bB} | \Psi_A \Phi_a \chi_a^{(+)}\rangle \tag{2.14}$$

in the reaction

$$a+A \rightarrow (b+x)+A \rightarrow b+(x+A) \rightarrow b+B \tag{2.26}$$

They call the transition amplitude $\langle\chi_b^{(-)} \Phi_b \Psi_B | V_{bA} - U_{bB} | \Psi_A \Phi_a \chi_a^{(+)}\rangle$ in T_{ba}^{DWBA} of Eq. (2.14) the heavy particle stripping term. If the interaction V_{bA} is assumed as a one-body Woods-Saxon potential with the strength V_o equal to that of the optical potential U_{bB}

$$V_{bA} = \frac{V_o}{1 + e^{(r_{bA} - r_o A^{1/3})/a}} \tag{2.27}$$

and the distorting optical potential U_{bB} is assumed as

$$U_{bB} = \frac{V_o}{1 + e^{(r_{bB} - r_o (A+1)^{1/3})/a}}$$

then their heavy particle stripping term is

$$(V_{bA} - U_{bB}) \sim \frac{V}{a} \frac{e^{(r_{bB} - r_0(A+1)^{1/3})/a}}{1+e^{(r_{bB} - r_0(A+1)^{1/3})/a}} \frac{1}{A}\left(\frac{2\mathbf{r}_{bB}\cdot\mathbf{r}_{aA}}{r_{bB}} - r_{bB} - \frac{1}{3}r_0(A+1)^{1/3}\right)$$
$$\longrightarrow \frac{1}{A}\left(\frac{2\mathbf{r}_{bB}\cdot\mathbf{r}_{aA}}{r_{bB}} - r_{bB} - \frac{1}{3}r_0(A+1)^{1/3}\right) V_0 \delta\left(r_{bB} - \frac{1}{3}r_0(A+1)^{1/3}\right). \qquad (2.29)$$

The residual interaction in Eq. (2.29) increases with the increasing target radius. These introduce into the cross section a factor propotional to the volume and thus to A. Combined with the factor of $\frac{1}{A^2}$ coming from the residual interaction, this causes the resulting heavy particle stripping cross section to be of order $\frac{1}{A}$ compared with the deuteron stripping cross section. We remark that the heavy particle stripping term of their definition is the contribution of the second and the third direct terms of Eq. (2.25), while the usual heavy particle stripping term is the contribution of the second and third exchange terms of Eq.(2.25). Of course the same discussion can be applied to the usual heavy particle stripping calculation under the assumption that the interaction V_{bC} is a one-body Woods-Saxon potential with the same strength of the optical potential U_{bB}.

But the approximation of replacing the interaction between complex particles by one-body interaction has several question.

a. state dependence:

The potential seen in the bound state cannot be the same as that seen in the scattering state.

b. energy dependence of the optical potential:

Even in the same scattering states, the potentials seen by the particle with different energies are different. To avoid the energy dependence of optical potential we must use the nonlocal optical potential and introduce the imaginary part of the optical potential. It is not correct to assume that the interaction between a nucleon and a complex nucleus is a one-body potential having the same strength of the optical potential. The reason is as follows: If we assume that the interaction $V_{bB}=V_{bx}+V_{bA}$ in Eq. (2.14) is one-body Woods-Saxon potential having the same strength of the optical potential U_{bB}, we obtain

$$T_{ba}^{DWBA} = 0,$$

while $T_{ba}^{DWBA} = \langle \chi_b^{(-)} \Phi_b \Psi_B | V_{bx} | \Psi_A \Phi_a \chi_a^{(+)} \rangle$ can explain many experimental results. In such case, it is important to examine how much the second direct term or the second exchange term contribute to the total cross section without assuming that the interaction between a nucleon and a complex nucleus is a one-body optical potential. For this prupose our method of the calculation is very useful. So we calculate the cross section of the second exchange term i.e. the usual heavy particle stripping term on the reaction $^{11}B(d,n)^{12}C$.

2.5 Macroscopic calculation[7)]

If we dare to assume the interacion to be a one-body type, it is reasonable to determine the interaction type according to its role inducing the reaction.

a. The interaction V_{bx} combines particles b and x, and makes the bound state a. We assume that the interaction is a Gaussian form and that its strength is adjusted to give the binding energy of the particle a.

b. The interaction V_{bC} combined particles b and C and makes the bound state A. It is replaced by a one-body Woods-Saxon potentials, and its strength is chosen to give a right seperation energy of the particle b from the nucleus A.

c. The interaction V_{bA} is the interaction between particles which are not bound, and then it is reaxonable to replace the interaction V_{bA} by the one-body Woods-Saxon potential having the strength equal to that of the optical potential U_{bB}.

d. The interaction V_{ba} is the interaction between particles which are not bound. But as the number of nucleons in the particle a is much smaller than that of the nucleus B, we cannot treat this interaction like the interaction

V_{bA}. We assume that the interaction V_{ba} is Gaussian type and that its magnitude is equal to that of the interaction V_{bx}. From the above consideration, the calcuaation of the second and the third direct terms of Eq. (2.25) is the same as that of Rodberg et al, neglecting the contribution of the imaginary part of the optical potential. In the calculation of the exchange amplitude of Eq. (2.25), the interaction V_{bC} and U_{bB} are the same Woods-Saxon potentials, but the strength of them may be different.

Next we introduce the outline of our macroscopic calculations. The differential cross section is derived from the transition matrix in Eq. (2.25) and can be written as

$$\frac{d\sigma_{ba}}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \frac{2J_B+1}{(2J_A+1)(2s_a+1)} \sum_m \Big| A^{(D)}_{lsj} \left(\beta^{(bx)}_{lm}(\theta) + \beta^{(bA)}_{lm}(\theta) - \beta^{(bB)}_{lm}(\theta) \right)$$

$$+ \sum_{l_b l_a} A^{(E)\, l_b l_a}_{lsj} \left(\beta^{(ba)\, l_b l_a}_{lm}(\theta) + \beta^{(bC)\, l_b l_a}_{lm}(\theta) - \beta^{(bB)\, l_b l_a}_{lm}(\theta) \right) \Big|^2, \qquad (2.30)$$

where J_A and J_B are the spins of the target and the residual nuclei respectively and s_a is the spin of the incident nucleon a. The quantities μ_a and μ_b are the spectroscopic amplitudes of the direct and exchange processes respectively depending upon the internal nuclear structure, where l_b and l_a specify the orbital angular momentum of the bound nucleon in the initial and final states resp pectively. The spectroccopic amplitudes may be defined as

$$A^{(D)}_{sj} = i^{l} \sqrt{\frac{2s_a+1}{2s+1}} \quad a(s)\, J_{BA}(lsj), \qquad (2.31a)$$

$$A^{(E)}_{lsj} = i^{l} (-)^{l_b} J_{BC}(l_a s_a j_a)\, J_{AC}(l_b s_b j_b)\, (-)^{J_C - J_A + j - j_a}$$
$$\times [(2j+1)(2l+1)(2s+1)(2j_a+1)(2j_b+1)]^{1/2}$$
$$\times W(J_A J_B j_b j_a ; J\, J_C) \times \begin{pmatrix} j & j_a & j_b \\ l & l_a & l_b \\ s & s_a & s_b \end{pmatrix} \qquad (2.31b)$$

$J_{BA}(lsj)$ and $a(s)$ in Eqs. (2.31) are the reduced widths and may be defined in the form

$$\Psi_{J_B M_B}(\xi, \mathbf{r}, \sigma) = \sum_{\substack{\ell s j \\ m \mu \\ J_A M_A}} (J_A j M_A M_B - M_A | J_B M_B)(\ell s m \mu | j M_B - M_A)$$

$$\times J_{BA}(\ell s j) \, \psi_\ell(r) \, Y_{\ell m}(\hat{r}) \, \chi_{s\mu}(\sigma) \, \Psi_{J_A M_A}(\xi), \tag{2.32a}$$

$$\chi_{s_a m_a}(r_{bx}, \sigma_b, \sigma_x) = \sum_{\substack{s \mu \\ s_b m_b}} (s_b s m_b \mu | s_a m_a) \, a(s) \, \Phi(r_{bx}) \, \chi_{s_b m_b}(\sigma_b) \, \chi_{s\mu}(\sigma_x), \tag{2.32b}$$

where the wave function $\psi_\ell(r)$ is the radial wave function of the particles x, $\chi_{s_a m_a}(r_{bx}, \sigma_b, \sigma_x)$ is the spin wave function of the particle x, Φ is the spatial part of the intrinsic wave function of the particle a, σ is the spin coordinate and ξ is the intrinsic coordinate of the nucleus A. In Eq. (2.30), $\beta_{lm}^{(\alpha)}$ is written as

$$(2\ell+1)^{1/2} i^\ell \beta_{\ell m}^{(\alpha)} = J^{(\alpha)} \int d\mathbf{r}_{aA} \int d\mathbf{r}_{bB} \, \chi_b^{(-)}(\mathbf{k}_b, \mathbf{r}_{bB}) \, f_{\ell m}^{(\alpha)}(\mathbf{r}_{bB}, \mathbf{r}_{aA}) \, \chi_a^{(+)}(\mathbf{k}_a, \mathbf{r}_{aA}), \tag{2.33}$$

where $J^{(D)}$ and $J^{(E)}$ are the Jacobians in the transformation of the coordinates of $\mathbf{r}_{xA}$ and $\mathbf{r}_{bx}$ to $\mathbf{r}_{aA}$ and $\mathbf{r}_{bB}$ and of $\mathbf{r}_{aC}$ to $\mathbf{r}_{aA}$ and $\mathbf{r}_{bB}$ respectively. They are expressed as

$$J^{(D)} = \left[\frac{aB}{x(A+a)}\right]^3, \tag{2.34a}$$

$$J^{(E)} = \left[\frac{AB}{C(B+b)}\right]^3, \tag{2.34b}$$

where a, b, x, A, B, and C are the masses of corresponding particles a, b, x, A, B and C. In Eq. (2.33), $f_{lm}^{(\alpha)}(\mathbf{r}_{bB}, \mathbf{r}_{aA})$ may be written as

$$f_{lm}^{(bx)}(\mathbf{r}_{bB}, \mathbf{r}_{aA}) = \psi_\ell(r_{xA}) \, Y_{lm}^*(\hat{\mathbf{r}}_{xA}) \, V(r_{bx}) \, \psi_{oo}(r_{bx}), \tag{2.35a}$$

$$f_{lm}^{(bA)}(\mathbf{r}_{bB}, \mathbf{r}_{aA}) = \psi_\ell(r_{xA}) \, Y_{lm}^*(\hat{\mathbf{r}}_{xA}) \, V(r_{bA}) \, \psi_{oo}(r_{bx}), \tag{2.35b}$$

$$f_{lm}^{(bB)}(\mathbf{r}_{bB}, \mathbf{r}_{aA}) = \psi_\ell(r_{xA}) \, Y_{lm}^*(\hat{\mathbf{r}}_{xA}) \, U(r_{bB}) \, \psi_{oo}(r_{bx}), \tag{2.35c}$$

$$f_{lm}^{(ba)l_b l_a}(\mathbf{r}_{bB},\mathbf{r}_{aA}) = \sum_{\mu_a\mu_b}(l_a l_b \mu_a - \mu_b | l m)\ (-)^{\mu_b}\varphi_{l_a}(r_{aC})\ Y^*_{l_a\mu_a}(\hat{\mathbf{r}}_{aC})$$
$$V(r_{ba})\varphi_{lb}(r_{bC})\ Y_{l_b\mu_b}(\hat{\mathbf{r}}_{bC}), \tag{2.35d}$$

$$f_{lm}^{(bC)l_b l_a}(\mathbf{r}_{bB},\mathbf{r}_{aA}) = \sum_{\mu_a\mu_b}(l_a l_b \mu_a - \mu_b | l m)\ (-)^{\mu_b}\varphi_{la}(r_{aC})Y^*_{la\mu a}(\hat{\mathbf{r}}_{aC})$$
$$V(r_{bc})\varphi_{lb}(r_{bC})\ Y_{l_b\mu_b}(\hat{\mathbf{r}}_{bC}), \tag{2.35e}$$

$$f_{lm}^{(bB)l\ l_a}(\mathbf{r}_{bB},\mathbf{r}_{aA}) = \sum_{\mu_b\mu_b}(l_a l_b \mu_a - \mu_b | l m)\ (-)^{\mu_b}\varphi_{la}(r_{aC})\ Y^*_{l_a\mu_a}(\hat{\mathbf{r}}_{aC})$$
$$U(r_{bB})\varphi_{l_b}(r_{aC})\ Y_{l_b\mu_b}(\hat{\mathbf{r}}_{bC}), \tag{2.35f}$$

From Eq. (2.30), we can write for the cross section of each term for simplicity as

$$\frac{d\sigma^{(\alpha)}}{d\Omega} = C\ V^{(\alpha)2}\,|\,n^{(\alpha)}\,A^{(\alpha)}_{lsj}\,|^2\sum_m|\,\beta^{(\alpha)}_{lm}\,|^2, \tag{2.36}$$

if we neglect the interference term. The notation c is the factor independent on α, and $n^{(\alpha)}$ is the factor due to the equivalent nucleons.

From Eq. (2.25), $n^{(D)}$ is equal to $2\sqrt{\frac{N-1}{2}}$ and $n^{(E)}$ is equal to $(N-2)\sqrt{\frac{N-2}{2}}$.

We calculate the cross sections of Eq. (2.36) on the reaction $^{11}B(d,n)^{12}C$. The bound states $\varphi_l(r)$ of Eqs. (2.35) are solved in the Woods-Saxon well by giving right seperation energies, and the bound state $\varphi_{oo}(r)$ of Eqs. (2.35) is solved in the Gaussian well by giving the binding energy of the particle a. (in this case a is the deuteron). The spectroscopic factors are calculated from the assumed configuration[8]:

$$^{11}B_{gnd}:\ -0.672\left[\Psi_{P[43]}\,{}^{22}\Gamma_{[\tilde{43}]}\right]_{3/2} + 0.741\left(\Psi_{D[43]}\,{}^{22}\Gamma_{[\tilde{43}]}\right]_{3/2}, \tag{2.37a}$$

$$^{12}C_{gnd}:\ \left[\Psi_{S[44]}\,{}^{11}\Gamma\right]_0, \tag{2.37b}$$

where $\varphi_{L[f]}$ is the spatial part of the wave function with the symmetry [f] and the angular momentum L, and ${}^{2T+1\,2S+1}\Gamma$ is the spin-isospin part of the wave function with the symmetry $[\tilde{f}]$ which is dual to the symmetry [f], the spin angular momentum S and the isospin angular momentum T. The suffix under the ritht side of the bracket is the total angular momentum. We show the result in Table 2 and Fig. 3. From Table 2, we can see that because of the very small spectroscopic amplitudes of exchange terms, the contribution of the heavy particle strippin terms is very small.

This result seems to be contrary to the result of Owen and Madansky[1]. But as we mentioned in §1, they used the parameter Λ_1/Λ_2 which correspond to $\frac{\eta^{(E)} A^{(E)\,l_b l_a}_{lsj}}{\eta^{(D)} A^{(D)}_{lsj}}$ in our calculation. According to their calculation Λ_1/Λ_2 is set to be of order unit, but in our calculation $\frac{\eta^{(E)} A^{(E)\,l_b l_a}_{lsj}}{\eta^{(D)} A^{(D)}_{lsj}}$ is of order $\frac{1}{10}$ for the capture of the deuteron into D state and is of order $\frac{1}{100}$ for the capture of the deuteron into S state. This difference arises from the following reasons:

a. The wave functions in their calculation are only the products of orbital parts and spin parts, and have not the total angular momentum. Then the recoupling factors such as Racah coefficients and X-coefficients in Eq. (2.31b) do not appear in the exchange term.

b. Their symmetric orbital wave function for two nucleons in the P-shell is described as

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}}\left(\psi_p(\mathbf{r}_1) + \psi_p(\mathbf{r}_2)\right),$$

but usually it must be described

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \psi_p(\mathbf{r}_1)\,\psi_p(\mathbf{r}_2).$$

Using the latter type wave function, the spectroscopic amplitude of the exchange term reduces more rapidly than that of the direct term.

We see from table 2 that the values of $\beta^{(\alpha)}_{lm}$ vary with the change of optical parameters but that the ratio $\frac{\beta^{(bC)\,l_b l_a}_{lm}}{\beta^{(bx)}_{lm}}$ does not seem to be more larger than the value of order 10 in the $^{11}B(d,n)^{12}C$. From Fig. 3, the angular pattern of the heavy particle stripping term has not backward peak in DWBA because of the distortion effect. So the main component of the reaction may be the usual stripping term and it may have the possibility of explaining the large backward peak due to the distortion effect. The angular patterns of the stripping and heavy particle stripping terms calculated by using harmonic oscillator bound state wave functions are shown in Fig. 4. We carry out this calculateion to compare with the microscopic calculation of §4. From Fig. 4 we see that the results obtained by using harmonic oscillator bound states are similar to those obtained by using Woods-Saxon bound state about the angular pattern. However the heavy particle stripping cross section obtained by using harmonic oscillator bound states are much more reduced. This tends to make the ratio $\frac{\beta_{\ell m}^{(bC)l_b l_a}}{\beta_{\ell m}^{(bx)}}$ smaller.

Table 1. Optical parameter

	incident channel (a)	incident channel (b)	exit channel
v	50.0	71.32	43.0
w	36.0	10.58	12.0
r_o	1.5	1.5	1.25
a_R	0.65	0.7	0.65
a_I	0.65	0.7	——
b	——	——	0.98

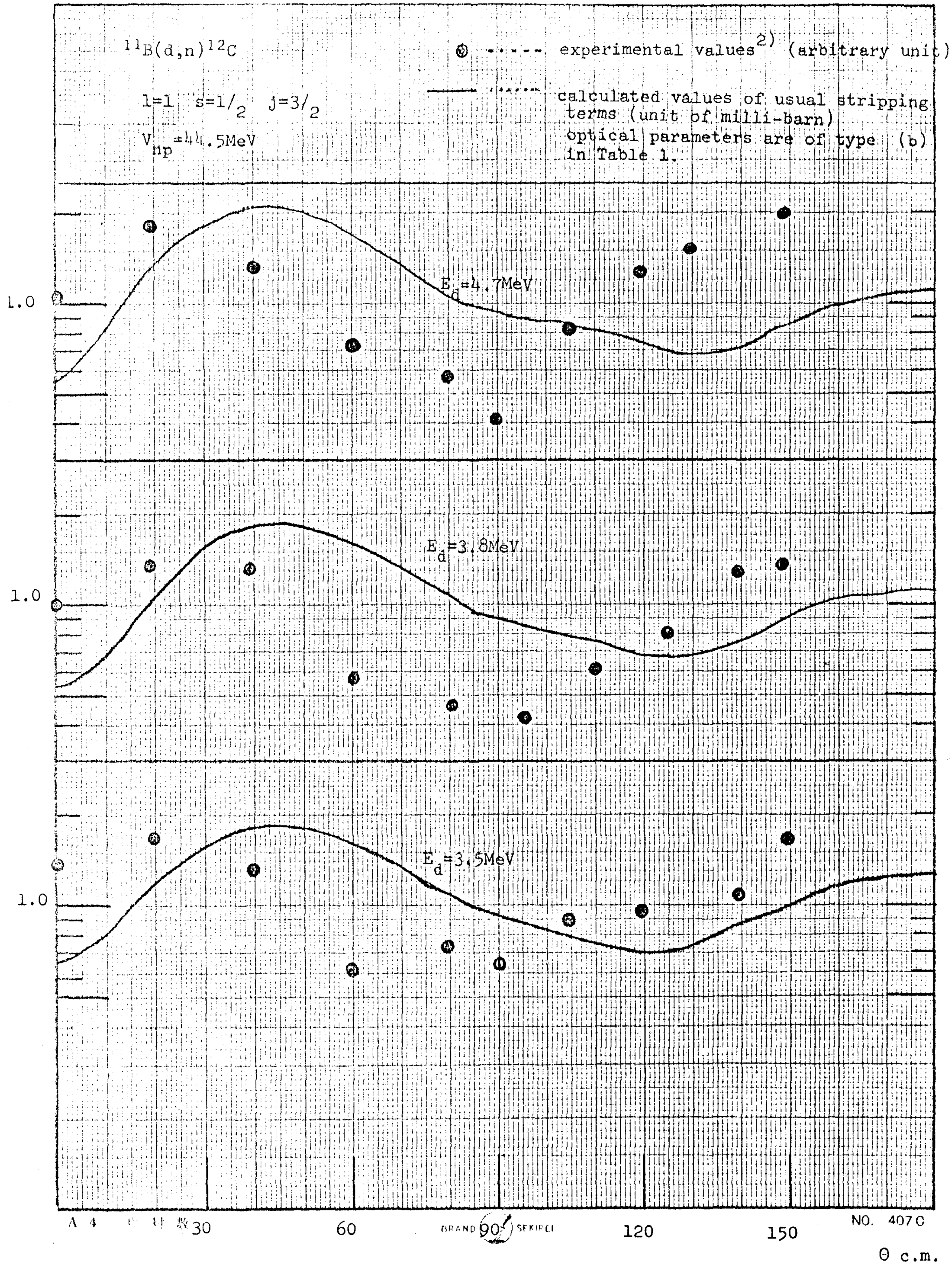
The notations v, w, a_R, a_I and b are defined by Eq. (3.13) and r_o is defined by $R=r_o A^{1/3}$, where A is the mass number of the target nucleus. The deuteron optical parameters of the type (a) are given by Hodgson[14] and those of the type (b) are given by Nelson et al[15].

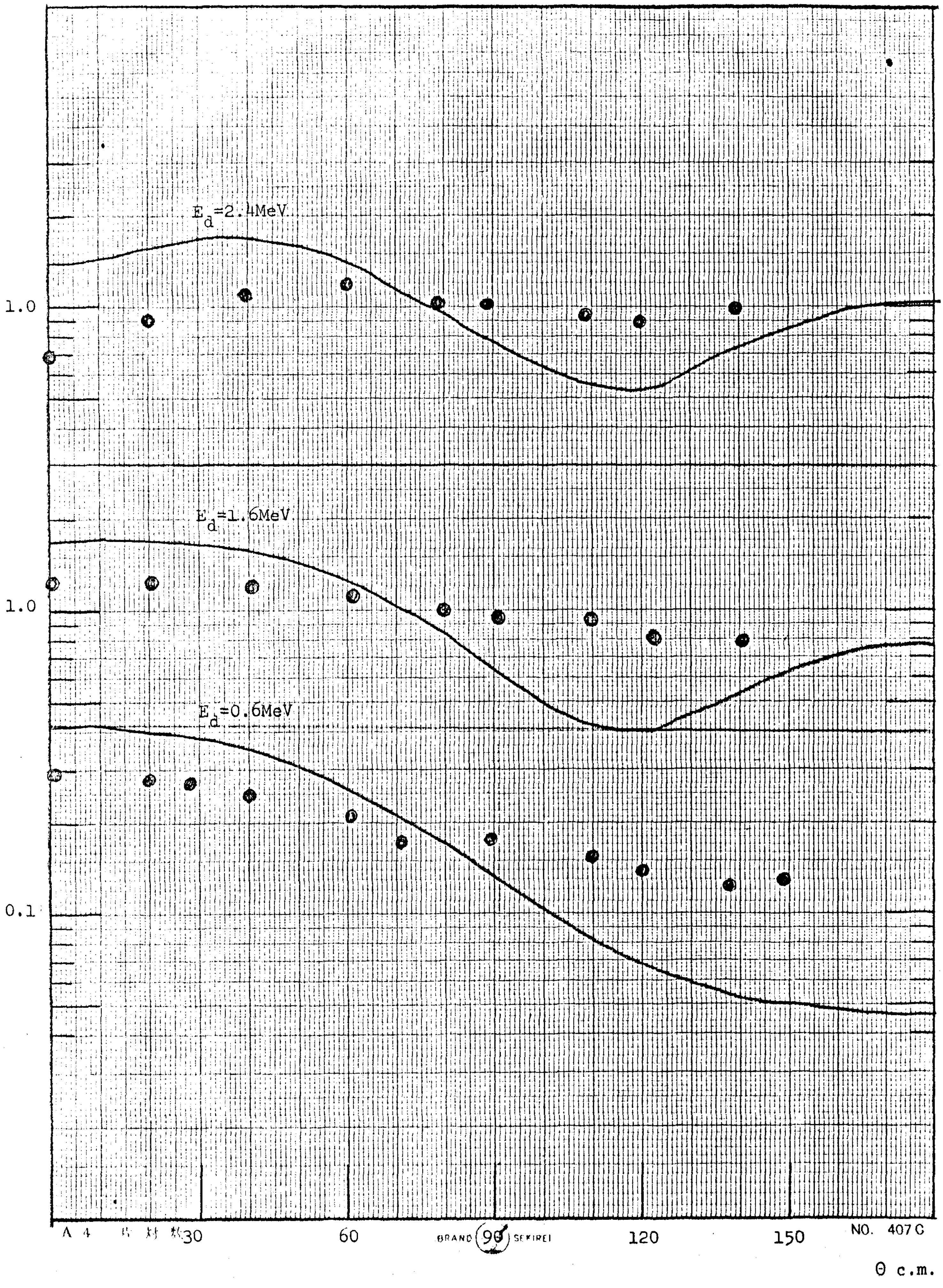
Table 2. Cross sections of each process

inducing interaction		strength	$n\ Aesj^{2}$	$\lvert\beta_{lm}\rvert^2$ (a)	$\lvert\beta_{lm}\rvert^2$ (b)	magnitude of cross section (a)	magnitude of cross section (b)
Direct	V_{bx}	44.5	3.936	21.8	110	2.78	13.9
	V_{bA}	43	3.936	11.1	230	1.22	27.4
	U_{bB}	43	3.936	14.5	263	1.72	41.3
	iU_{bB}	12	3.936	16.0	188	0.148	1.74
Exchange capture of deuteron into S- state	V_{ba}	44.5	2.557×10^{-3}	2.31	15.5	1.92×10^{-4}	13.1×10^{-4}
	V_{bc}	52.4	2.557×10^{-3}	16.6	15.6	19.1×10^{-4}	17.9×10^{-4}
	U_{bB}	43	2.557×10^{-3}	15.7	13.3	12.1×10^{-4}	12.1×10^{-4}
	iU_{bB}	12	2.557×10^{-3}	7.57	9.84	0.457×10^{-4}	0.594×10^{-4}
Exchange capture of deuteron into D-state	V_{ba}	44.5	1.679×10^{-1}	0.811	6.05	4.41×10^{-3}	43.9×10^{-3}
	V_{bC}	52.4	1.679×10^{-1}	4.93	6.06	37.1×10^{-3}	45.7×10^{-3}
	U_{bB}	43	1.679×10^{-1}	4.90	5.48	24.9×10^{-3}	27.8×10^{-3}
	iU_{bB}	12	1.779×10^{-1}	1.53	2.91	0.605×10^{-3}	1.15×10^{-3}

The results (a) and (b) correspond to the optical parameters (a) and (b) respectively.

Fig. 2 Experimental values and calculated values of the cross sections





E_d=2.4MeV
E_d=1.6MeV
E_d=0.6MeV
1.0
1.0
0.1
30
60
90
120
150
θ c.m.
A 4
NO. 407 C
BRAND
SEKIREI

Fig. 3a $^{11}B(d,n)^{12}C$

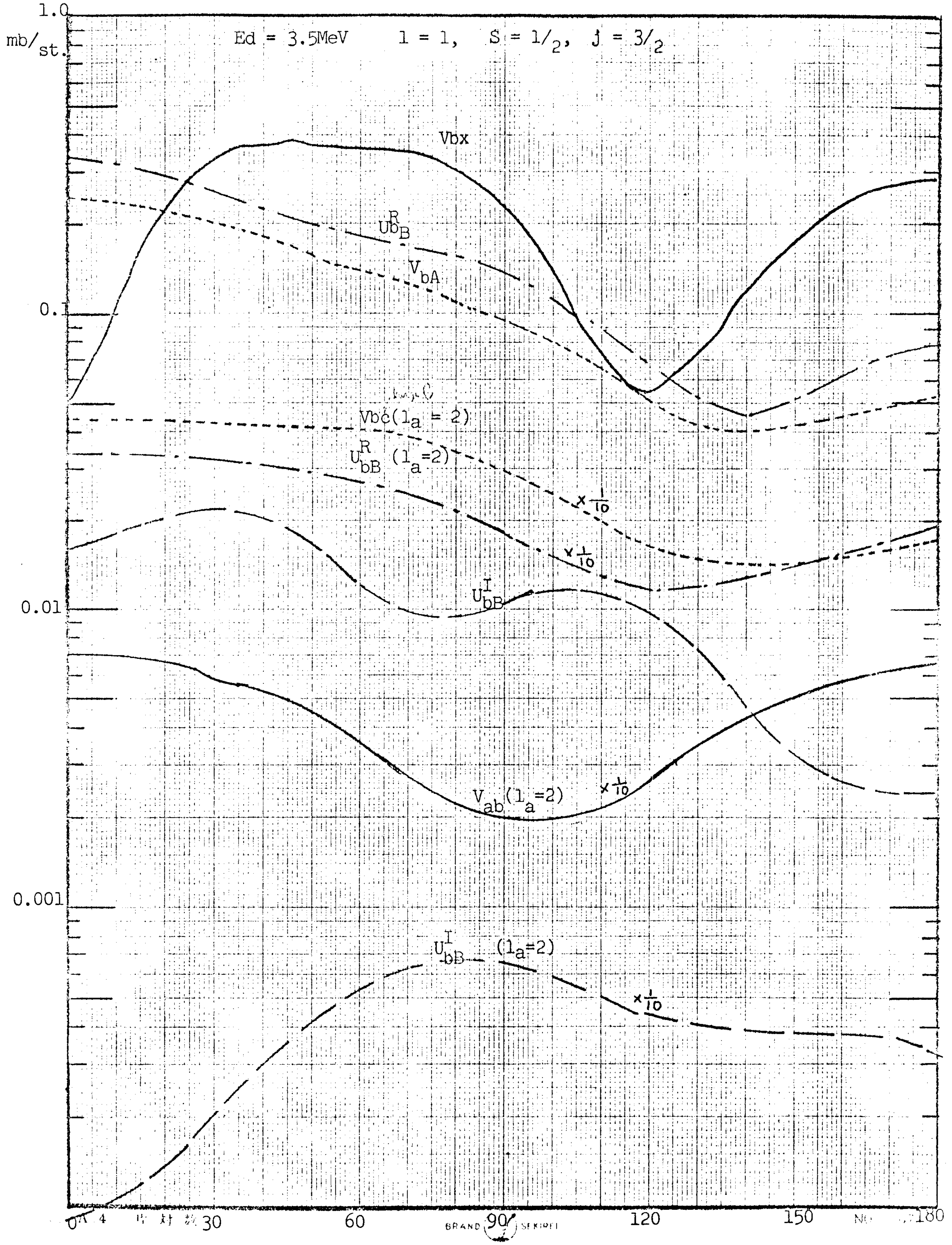


Fig. 3b $^{11}B(d,n)^{12}C$

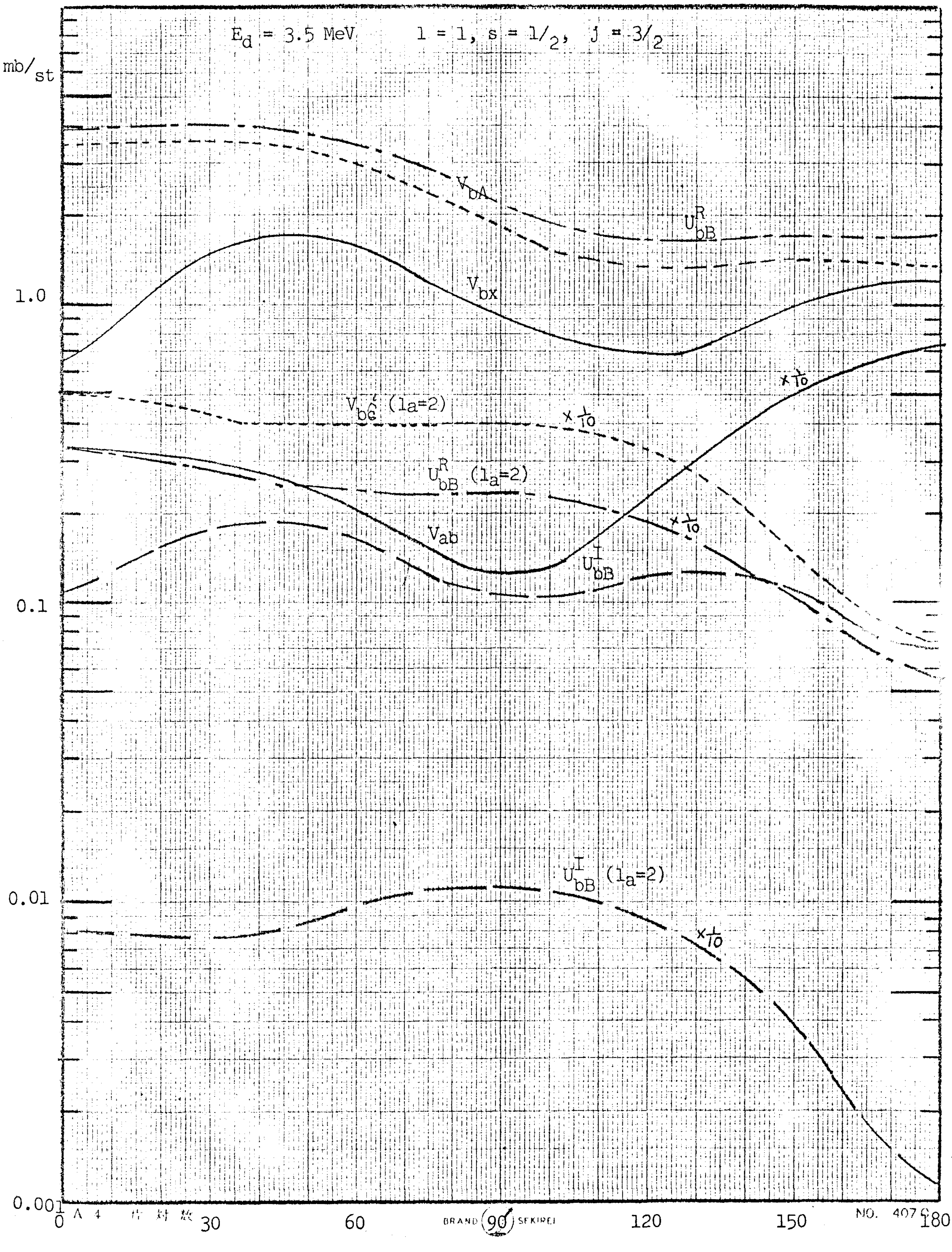
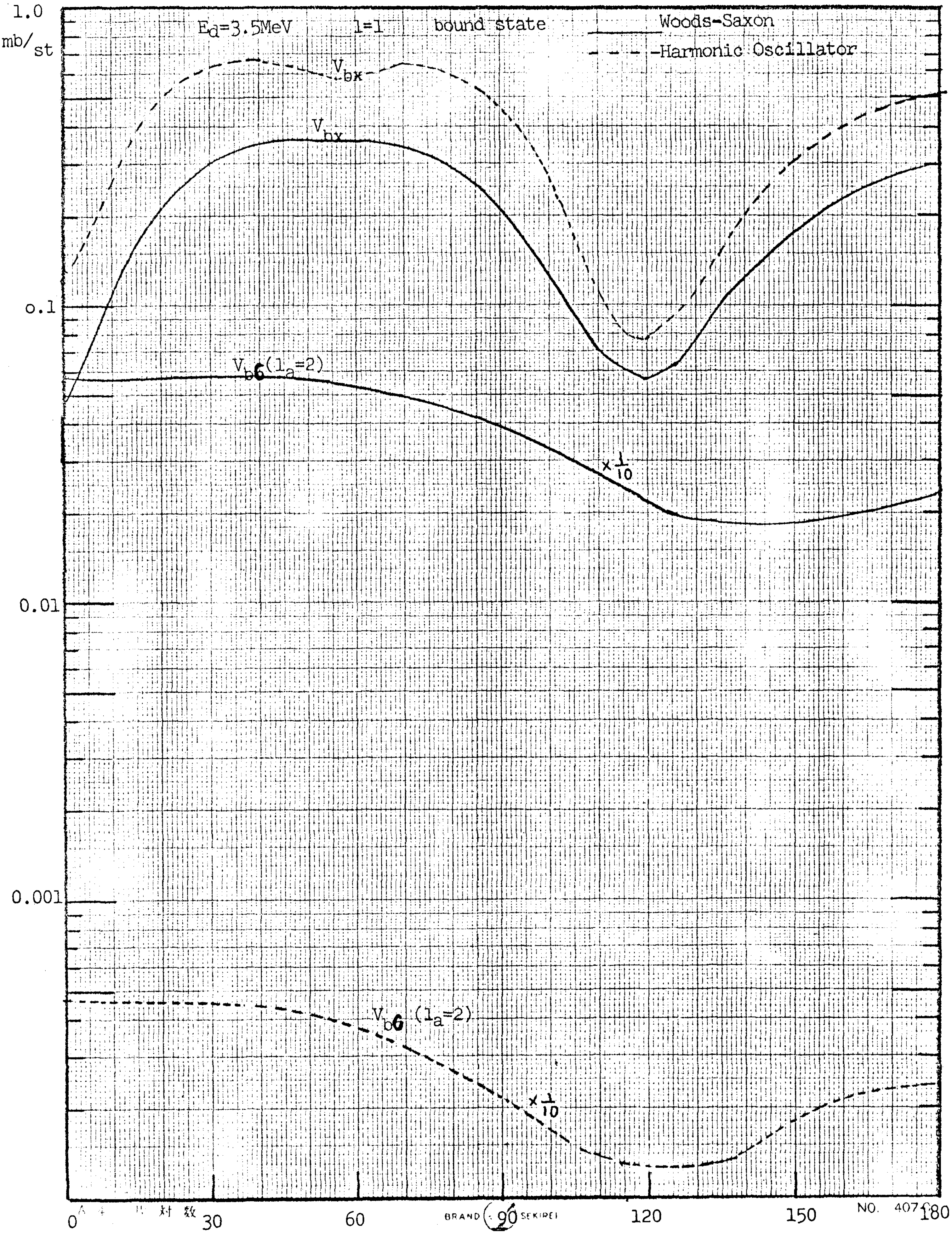


Fig.4 $^{11}B(d,n)^{12}C$



3. Method of the calculation

3.1 Method

The reaction A(a,b)B is considered. The corresponding transition amplitude can be written as

$$T_{ba} = \int d\mathbf{r}_{aA} \int d\mathbf{r}_{bB}\, \chi_b^{(-)*}(\mathbf{k}_b, \mathbf{r}_{bB}) \langle J_B M_B s_b m_b | V | J_A M_A s_a m_a \rangle \chi_a^{(+)}(\mathbf{k}_a, \mathbf{r}_{aA}), \quad (3.1)$$

where J_A, J_B, s_a and s_b are the spins of the particles, and M_A, M_B, m_a and m_b are their corresponding Z-components, $\chi_b^{(-)}$ and $\chi_a^{(+)}$ are the distorted incoming and outgoing waves respectively and H is the Jacobian defined in Eqs. (2.24a) and (2.34b). If we use the completeness of the harmonic oscillator wave functions $\psi_{NLM}(\nu, \mathbf{r})$ the transition amplitude (3.1) becomes[9]

$$\begin{aligned} T_{ba} = J \sum_{\substack{N_a L_a M_{La} \\ N_b L_b M_{Lb}}} & \int d\mathbf{r}_{bB}\, \chi_b^{(-)*}(\mathbf{k}_b, \mathbf{r}_{bB})\, \psi_{N_b L_b M_{Lb}}\left(\frac{bB}{B+b}\nu, \mathbf{r}_{bB}\right) \\ \times & \int d\mathbf{r}'_{bB} \int d\mathbf{r}'_{aA}\, \psi^*_{N_b L_b M_{Lb}}\left(\frac{bB}{B+b}\nu, \mathbf{r}'_{bB}\right) \langle J_B M_B s_b m_b | V | J_A M_A s_a m_a \rangle \psi_{N_a L_a M_{La}}\left(\frac{aA}{A+a}\nu, \mathbf{r}'_{aA}\right) \\ \times & \int d\mathbf{r}_{aA}\, \psi^*_{N_a L_a M_{La}}\left(\frac{aA}{A+a}\nu, \mathbf{r}_{aA}\right) \chi_a^{(+)}(\mathbf{k}_a, \mathbf{r}_{aA}) \end{aligned} \quad (3.2)$$

The harmonic oscillator wave functions $\psi_{NLM}(\nu, \mathbf{r})$ can be written as

$$\psi_{NLM}(\nu, \mathbf{r}) = \psi_{NL}(\nu, r)\, Y_{LM}(\hat{\mathbf{r}}),$$

$$\begin{aligned} \psi_{NL}(\nu, r) = & \left[\frac{2^{L-N+2}\,(2L+2N+1)!!\,\nu^{L+3/2}}{N!\,\{(2L+1)!!\}^2\,\pi^{1/2}} \right]^{1/2} r^L e^{-\frac{1}{2}\nu r^2} \\ & \times \sum \binom{N}{k} \frac{(2L+1)!!}{(2L+2k+1)!!} (-2\nu r^2)^k . \end{aligned} \quad (3.3)$$

We expand the form factor using multipole terms which correspond to the transfer to the target with definite angular momenta $\mathbb{j}$, $\mathbb{l}$ and $\mathbb{s}$.

$$\mathbb{j} = \mathbb{J}_B - \mathbb{J}_A , \qquad \mathbb{s} = \mathbb{s}_a - \mathbb{s}_b , \qquad \mathbb{j} = \boldsymbol{\ell} + \mathbb{s} . \tag{3.4}$$

The transition amplitude becomes

$$T_{ba} = J \sum_{\substack{N_a L_a M_{La} \\ N_b L_b M_{Lb}}} \int d\mathbf{r}_{bB} \int d\mathbf{r}_{aA}\, \chi_b^{(-)*}(\mathbf{k}_b, \mathbf{r}_{bB}) \sum_{\ell s j} (J_A\, j\, M_A\, m_j | J_B M_B)\, (\ell\, s\, m\, m_s | j\, m_j)$$

$$\times\, (S_a S_b\, m_a - m_b | S\, m_s)\, (-)^{S_b - m_b}\, i^{-\ell}\, G_{\ell s j m}^{(N_b L_b M_{Lb} N_a L_a M_{La})}(\mathbf{r}_{bB}, \mathbf{r}_{aA})\, \chi_a^{(+)}(\mathbf{k}_a, \mathbf{r}_{aA}) \tag{3.5}$$

Using the orthogonality of Clebsh-Gordan coefficients, we obtain the inverted form as

$$G_{\ell s j m}^{(N_b L_b M_{Lb} N_a L_a M_{La})}(\mathbf{r}_{bB}, \mathbf{r}_{aA})$$

$$= J \left(\frac{2\ell+1}{2J_B+1}\right) i^{\ell} \sum_{\substack{M_A M_B m_j I \\ m_a m_b m_s \mu \\ j_a j_b m_{ja} m_{jb}}} (J_A\, j\, M_A\, m_j | J_B M_B)\, (\ell\, s\, m\, m_s | j\, m_j)\, (S_a S_b\, m_a - m_b | S\, m_s)$$

$$\times\, (-)^{S_b + m_b} (j_a J_A\, m_{ja} M_A | I \mu)(j_b J_B\, m_{jb} M_B | I \mu)(L_a S_a M_{La} m_a | j_a m_{ja})(L_b S_b M_{Lb} m_b | j_b m_{jb})$$

$$\times\, \varphi_{N_b L_b M_{Lb}}\!\left(\frac{bB}{B+b}\nu, \mathbf{r}_{bB}\right) \varphi^{*}_{N_a L_a M_{La}}\!\left(\frac{aA}{A+a}\nu, \mathbf{r}_{aA}\right) \left\langle ((L_b S_b) j_b\, J_B) I \mu \right| V \left| ((L_a S_a) j_a\, J_A) I \mu \right\rangle , \tag{3.6}$$

where

$$\left\langle ((L_b S_b) j_b\, J_B) I \mu \right| V \left| ((L_a S_a) j_a\, J_A) I \mu \right\rangle$$

$$= \sum_{\substack{M_{La} M_{Lb} m_a m_b \\ m_{ja} m_{jb} M_A M_B}} (L_a S_a M_{La} m_a | j_a m_{ja})(L_b S_b M_{Lb} m_b | j_b m_{jb})(j_a J_A m_{ja} M_A | I \mu)(j_b J_B m_{jb} M_B | I \mu)$$

$$\times \int d\mathbf{r}'_{bB} \int d\mathbf{r}'_{aA}\, \varphi^{*}_{N_b L_b M_{Lb}}\!\left(\frac{bB}{B+b}\nu, \mathbf{r}'_{bB}\right) \langle J_B M_B S_b m_b | V | J_A M_A S_a m_a \rangle\, \varphi_{N_a L_a M_{La}}\!\left(\frac{aA}{A+a}\nu, \mathbf{r}'_{aA}\right).$$

[illegible] couple to j_a and [illegible] to I.

The information of the reaction type is included in this matrix element. If we sum over Z-components in Eq. (3.6) and write $G_{\ell s j m}^{(N_b L_b M_{Lb} N_a L_a M_{La})}(\mathbf{r}_{bB}, \mathbf{r}_{aA})$ as the product of two factors

$$G_{lsjm}^{(N_b L_b M_{LB} N_a L_a M_{La})}(\mathbb{r}_{bB},\mathbb{r}_{aA}) = A_{lsj}^{(N_b L_b N_a L_a)} \cdot f_{lm}^{(N_b L_b M_{lb} N_a L_a M_{la})}(\mathbb{r}_{bB},\mathbb{r}_{aA}), \qquad (3.7)$$

we obtain

$$A_{lsj}^{(N_b L_b N_a L_a)} = \sum_{j_a j_b} i^{\ell} (-)^{J_A + j_a - I + L_a + \ell} \frac{(2I+1)\sqrt{(2\ell+1)(2S+1)(2j+1)(2j_a+1)(2j_b+1)}}{\sqrt{2J_B+1}}$$
$$\times\, w(j_a J_A j_b J_B; I_j) \begin{Bmatrix} L_a & S_a & j_a \\ L_b & S_b & j_b \\ l & S & j \end{Bmatrix}$$
$$\times \langle ((L_b S_b) j_b J_B) I\mu | V | ((L_a S_a) j_a J_A) I\mu \rangle, \qquad (3.8)$$

defining the form factor

$$f_{\ell m}^{(N_b L_b M_{Lb} N_a L_a M_{La})}(\mathbb{r}_{bB},\mathbb{r}_{aA}) = (L_b -M_{Lb}\, L_a M_{La} | \ell m)\, \psi^{*}_{N_b L_b -M_{Lb}}\left(\frac{bB}{B+b} \nu, \mathbb{r}_{bB}\right) \psi^{*}_{N_a L_a M_{La}}\left(\frac{aA}{A+a} \nu, \mathbb{r}_{aA}\right). \qquad (3.9)$$

We neglect the spin-orbit coupling in the distorted wave and define the patial amplitude by

$$(2\ell+1)^{1/2} i^{\ell} \beta_{\ell m}^{(N_b L_b N_a L_a)}(\theta) = \sum_{M_{La} M_{Lb}} \int d\mathbb{r}_{bB} \int d\mathbb{r}_{aA}\, \chi_b^{(-)*}(\mathbb{k}_b,\mathbb{r}_{bB})\, f_{\ell m}^{(N_b L_b M_{Lb} N_a L_a M_{La})}(\mathbb{r}_{bB},\mathbb{r}_{aA})\, \chi_a^{(+)}(\mathbb{k}_a,\mathbb{r}_{aA}). \qquad (3.10)$$

To calculate Eq. (3.10), the distorted wave $\chi^{(\pm)}(\mathbb{k},\mathbb{r})$ is expanded in partial waves as

$$\chi_a^{(+)}(\mathbb{k}_a,\mathbb{r}_{aA}) = \frac{4\pi}{k_a r_{aA}} \sum_{L_a M_{La}} i^{L_a} \chi_{L_a}(\mathbb{k}_a, r_{aA})\, Y_{L_a M_{La}}(\hat{\mathbb{r}}_{aA})\, Y^{*}_{L_a M_{La}}(\hat{\mathbb{k}}_a), \qquad (3,11a)$$

$$\chi_b^{(-)}(\mathbb{k}_b,\mathbb{r}_{bB}) = \frac{4\pi}{k_b r_{bB}} \sum_{L_b M_{Lb}} i^{-L_b} \chi_{L_b}(\mathbb{k}_b, r_{bB})\, Y_{L_b M_{Lb}}(\hat{\mathbb{r}}_{bB})\, Y^{*}_{L_b M_{Lb}}(\hat{\mathbb{k}}_b), \qquad (3.11b)$$

where $\hat{\mathbb{r}}$ and $\hat{\mathbb{k}}$ denote the polar angles of the vectors $\mathbb{r}$ and $\mathbb{k}$. The function $Y_{LM}(\hat{\mathbb{r}})$ is the spherical harmonics. The partial-distorted wave $\chi(kr)$ is the solution of a radial Schrodinger equation with a optical potential U(r)

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{2\eta k}{r} - \frac{2\mu}{\hbar^2} U(r) - \frac{L(L+1)}{r^2}\right] \chi_L(kr) = 0, \qquad (3.12)$$

where η is the Coulomb parameter $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$. For U(r) we take a Gaussian absorption form or a Woods-Saxon absorption form,

$$U(r) = \frac{V}{1+\exp((r-R)/a_R)} + iW\exp[-(\frac{r-R}{b})^2] \tag{3.13a}$$

or

$$U(r) = \frac{V}{1+\exp((r-R)/a_R)} + i\frac{W}{1+\exp((r-R)/a_I)} \tag{3.13b}$$

Inserting Eqs. (3.9) and (3.11) into Eq. (3.10) and performing the angular part of the intrgration, we obtain

$$\beta_{lm}^{(N_bL_bN_aL_a)}(\theta) = \frac{4\pi}{k_a k_b}\, i^{L_a+L_b-l}\,(2l_b+1)^{1/2}\,(L_b m\, l -m \,|\, L_a 0)\left[\frac{(L_b-m)!}{(L_b+m)!}\right]^{1/2} P_{L_b}(\theta)\, I_{lL_bL_a}^{(N_bN_a)}, \tag{3.14}$$

where

$$I_{lL_bL_a}^{(N_bN_a)} = \int_0^\infty r_{aA}\,dr_{aA}\,\psi_{N_aL_a}\!\left(\frac{aA}{A+a}\nu, r_{aA}\right)\chi_{L_a}(k_a r_{aA}) \int_0^\infty r_{bB}\,dr_{bB}\,\psi_{N_bL_b}\!\left(\frac{bB}{B+b}\nu, r_{bB}\right)\chi_{L_b}(k_b, r_{bB}). \tag{3.15}$$

The function $\psi_{NL}(\nu r)$ is the radial part of the harmonic oscillator wave function defined by Eq. (3.3). If we use the relation

$$|(j_1j_2)J_{12}\,(j_3j_4)J_{34};J\rangle = \sqrt{(2J_{12}+1)(2J_{34}+1)(2J_{13}+1)(2J_{24}+1)}\begin{Bmatrix} j_1 & j_2 & J_{12} \\ j_3 & j_4 & J_{34} \\ J_{13} & J_{24} & J \end{Bmatrix} |(j_1j_3)J_{13}(j_2j_4)J_{24};J\rangle,$$

we obtain for the spectroscopic amplitude $A_{lsj}^{(N_bL_bN_aL_a)}$ in L-S representation the following equation

$$\begin{aligned} A_{lsj}^{(N_bL_bN_aL_a)} = \sum{}' &(J_B|L_BS_B)(J_A|L_AS_A)\, i^{l}(-)^{J_A+j_a-I+L_A+1}(2I+1)(2\Lambda_L+1) \\ &\times (2\Lambda_s+1)\,(2j_a+1)(2j_b+1)\sqrt{(2J_A+1)(2l+1)(2s+1)(2j+1)} \\ &\times w\,(j_aJ_Aj_bJ_B;Ij) \\ &\times \begin{Bmatrix} L_a & S_a & j_a \\ L_b & s_b & j_b \\ l & s & j \end{Bmatrix} \begin{Bmatrix} L_a & s_a & j_a \\ L_A & S_A & J_A \\ \Lambda_L & \Lambda_S & I \end{Bmatrix} \begin{Bmatrix} L_b & S_b & j_b \\ L_B & S_B & J_B \\ \Lambda_L & \Lambda_S & I \end{Bmatrix} \\ &\times \langle (L_bL_B)\Lambda_L(S_bS_B)\Lambda_s I M | V | (L_aL_A)\Lambda_L(s_as_A)\Lambda_s I M\rangle. \end{aligned} \tag{3.16}$$

The notation $(J_B|L_BS_B)$ is defined by the equation

$$\Psi_{J_B M_B} = \sum_{\substack{M_{LB} \\ M_{SB}}} (J_B | L_B S_B)(L_B S_B M_{LB} M_{SB} | J_B M_B) \Psi_{L_B M_{LB}} \chi_{S_B M_{SB}}$$

The matrix element $\langle (L_b L_B)\Lambda_L (S_b S_B)\Lambda_s I\mu | V | (L_a L_A)\Lambda_L (s_a s_A)\Lambda_s I\mu \rangle$ can be written as

$$\langle (L_b L_B)\Lambda_L (S_b S_B)\Lambda_S I\mu | V | (L_a L_A)\Lambda_L (S_a S_A)\Lambda_S I\mu \rangle$$

$$= \langle (L_b L_B)\Lambda_L (S_b S_B)\Lambda_S | V | (L_a L_A)\Lambda_L (S_a S_A)\Lambda_S \rangle$$

$$= (t_b T_B m_{tb} M_{TB} | \Lambda_T M_{\Lambda T})(t_a T_A m_{ta} M_{TA} | \Lambda_T M_{\Lambda T}) \langle (L_b L_B)\Lambda_L (S_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (L_a L_A)\Lambda_L (S_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle, \tag{3.17}$$

where t_a, t_b, T_A and T_B are the isospins of the particle a, b, A, and B respectively, and m_{ta}, m_{tb}, M_{T_A} and M_{T_B} their Z-components respectively. The differential cross section is then given by

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \frac{2J_B+1}{2J_A+1} \sum_{lsjm} \frac{1}{2S_a+1} \left| \sum_{\substack{N_b L_b \\ N_a L_a}} A_{lsj}^{(N_b L_b N_a L_a)} \beta_{lm}^{(N_b L_b N_a L_a)}(\theta) \right|^2, \tag{3.18}$$

where μ_a is the reduced mass between particles a and A.

3.2 Recoil effect

It is difficult to calculate the matrix elementin Eq. (3.17) numerically We define $F^{Shell}(N_b L_b m_{L_b} : N_A L_A m_{L_a})$ by

$$F^{shell}(N_b L_b m_{L_b} : N_a L_a m_{L_a})$$

$$= \sum (L_b L_B m_{Lb} M_{LB} | \Lambda_L M_{\Lambda L})(S_b S_B m_{Sb} M_{SB} | \Lambda_S M_{\Lambda S})(L_a L_A m_{La} M_{LA} | \Lambda_L M_{\Lambda L})(S_a S_A m_{Sa} M_{SA} | \Lambda_S M_{\Lambda S})$$

$$\times \langle (L_b L_B)\Lambda_L (S_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (L_a L_A)\Lambda_L (S_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle^{shell}, \tag{3.19}$$

and define $F(n_b l_b m_{l_b} : n_a l_a m_{l_a})$ by

$$F(n_b l_b m_{l_b} : n_a l_a m_{l_a})$$

$$= \sum (l_b L_B m_{l_b} M_{L_B} | \Lambda_L M_{\Lambda_L})(s_b S_B m_{s_b} M_{S_B} | \Lambda_S M_{\Lambda_S})(l_a L_A m_{l_a} M_{L_A} | \Lambda_L M_{\Lambda_L})(s_a S_A m_{s_a} M_{S_A} | \Lambda_S M_{\Lambda_S})$$

$$\times \langle (l_b L_B)\Lambda_L (s_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (l_a L_A)\Lambda_L (s_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle, \qquad (3.20)$$

The wave functions in the matrix element $\langle (l_b L_B)\Lambda_L (s_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (l_a L_A)\Lambda_L (s_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle$ are intrinsic wave functions Ψ_A and Ψ_B. If we assume that:

$$\Psi_A^{shell} = \varphi_{000}(A\nu, \mathbf{r}_A)\Psi_A, \qquad (3.21a)$$

$$\Psi_B^{shell} = \varphi_{000}(B\nu, \mathbf{r}_B)\Psi_B, \qquad (3.21b)$$

and perform the integration over the coordinate of the center-of-mass of the total particles, we obtain the realtion

$$F^{shell}(N_b L_b m_{L_b} : N_a L_a m_{L_a}) = \sum_{\substack{n_a l_a m_{l_a} \\ n_b l_b m_{l_b} \\ \tilde{N} \tilde{L} \tilde{M}}} (\tilde{L} l_a \tilde{M} m_{l_a} | L_a m_{L_a})(\tilde{L} l_b \tilde{M} m_{l_b} | L_b m_{L_b})$$

$$\times (N_a L_a 0 0; L_a | a : A | \tilde{N} \tilde{L} \; n_a l_a ; L_a)(N_b L_b 0 0; L_b | b : B | \tilde{N} \tilde{L} n_b l_b ; L_b) F(n_b l_b m_{l_b} : n_a l_a m_{l_a}). \qquad (3.22)$$

The notation $(n_1 l_1 n_2 l_2 ; L | m_1 : m_2 | \tilde{N} \tilde{L} \; n l ; L)$ is the Talmi coefficient. The validity of the approximation written by Eqs. (3.21) was discussed by Elliot et al[10]. We insert Eqs. (3.22) and (3.20) into the inverted form of Eq. (3.19) We finally obtain the equation

$$\langle (L_b L_B)\Lambda_L (S_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (L_a L_A)\Lambda_L (S_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle^{shell}$$

$$= \sqrt{(2L_a+1)(2L_b+1)} \sum_{\substack{n_a l_a n_b l_b \\ \tilde{N} \tilde{L} \Lambda_L'}} (2\Lambda_L'+1) W(\tilde{L} l_b \Lambda_L L_B ; L_b \Lambda_L') W(\tilde{L} l_a \Lambda_L L_A ; L_a \Lambda_L')$$

$$\times (N_a L_a 0 0; L_a | a : A | \tilde{N} \tilde{L} n_a l_a ; L_a)(N_b L_b 0 0; L_b | b : B | \tilde{N} \tilde{L} n_b l_b ; L_b)$$

$$\times \langle (l_b L_B)\Lambda_L' (s_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (l_a L_A)\Lambda_L (s_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle. \qquad (3.23)$$

Using shell model wave functions for the target and residual nucleus states, we can calculate matrices $\langle (L_b L_B)\Lambda_L (s_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (L_a L_A)\Lambda_L (s_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle^{\text{shell}}$ and obtain matrices $\langle (L_b L_B)\Lambda_L (s_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (L_a L_A)\Lambda_L (s_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle$ in Eq. (3.16). If we take only one term $\tilde{N}=\tilde{L}=0$ in the summation of Eq. (3.23), we obtain the equation

$$\langle (L_b L_B)\Lambda_L (s_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (L_a L_A)\Lambda_L (s_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle$$

$$= \left(\frac{a+A}{A}\right)^{\frac{2N_a+L_a}{2}} \left(\frac{b+B}{B}\right)^{\frac{2N_b+L_b}{2}} \langle (L_b L_B)\Lambda_L (s_b S_B)\Lambda_S (t_b T_B)\Lambda_T | V | (L_a L_A)\Lambda_L (s_a S_A)\Lambda_S (t_a T_A)\Lambda_T \rangle^{\text{shell}} \tag{3.23a}$$

The removal of the center-of-mass motion of the microscopic calculation is similar to that of the center-of-mass motion in the calculation of the reduced width for nucleon clusters in the shell model[11]. We show the numerical results for the heavy particle stripping term of the reaction $^{11}B(d,n)^{12}C$ in §4.

3.3 Comparison our method with the usual calculation

We show that our method is identical to the usual calculation in the strippin reaction. We consider the reaction

$$a+A \rightarrow (b+x) + A \rightarrow b + (b+A) \rightarrow b+B$$

The matrix element $\langle ((L_b s_b) j_b J_B) I\mu | V_{bx} | ((L_a s_a) j_a J_A) I\mu \rangle$ in Eq. (3.8) can be written as

$$\langle ((L_b s_b) j_b J_B) I\mu | V_{bx} | ((L_a s_a) j_a J_A) I\mu \rangle$$

$$= (-)^{j_a+J_A-I} J_{BA}(l_x s_x j_x) \sqrt{(2J_B+1)(2j_b+1)(2j_x+1)(2j_a+1)(2L_a+1)(2s_a+1)}$$

$$\times W(j_a J_A j_b J_B; I j_x) \begin{pmatrix} L_b & s_b & j_b \\ l_x & s_x & j_x \\ L_a & s_a & j_a \end{pmatrix} \langle [\psi_{N_b L_b}(\tfrac{bB}{B+b}\nu, \mathbf{r}_{bB}) \psi_{l_x}(\mathbf{r}_{xA})]_{L_a} | V(r_{bx}) | \psi_{N_a L_a}(\tfrac{aA}{a+A}\nu, \mathbf{r}_{ax}) \Phi_{\mathbf{q}} \tag{3.24}$$

where $\varphi_{N_bL_b}(-\frac{bB}{B+b}\nu, \mathbf{r}_{bB})$ is the harmonic oscillator wave function including the spherical harmonics of rank L_b and Φ_a is the intrinsic spatial wave function of the particle a. Substituting Eq. (3.24) into Eq. (3.8) yeilds the result as

$$A_{\ell sj}^{(N_bL_bN_aL_a)}$$

$$= i^{\ell}(-)^{L_a+\ell}\frac{\sqrt{(2L_a+1)(2s_a+1)}}{\sqrt{(2\ell+1)(2s+1)}}J_{BA}(\ell sj)a(s)\left\langle\left[\varphi_{N_bL_b}(-\frac{bB}{B+b}\nu,\mathbf{r}_{bB})\varphi_{\ell}(\mathbf{r}_{xA})\right]_{L_a}\middle|V_{bx}\middle|\varphi_{N_aL_a}(\frac{aA}{A+a}\nu,\mathbf{r}_{aA})\Phi_a\right\rangle.$$

(3.25)

From the Eqs . (3.9) (3.10) and (3.25), the quantity $\sum_{\substack{N_aL_a\\N_bL_b}} A_{\ell sj}^{(N_bL_bN_aL_a)}\beta_{\ell m}^{(N_bL_bN_aL_a)}(\theta)$ in Eq. (3.18) becomes

$$\sum_{\substack{N_aL_a\\N_bL_b}} A_{\ell sj}^{(N_bL_bN_aL_a)}\beta_{\ell m}^{(N_bL_bN_aL_a)}(\theta)$$

$$= \sum_{\substack{N_aL_am_{La}\\N_bL_bm_{Lb}}} i^{\ell}(-)^{L_a+\ell}\sqrt{\frac{(2L_a+1)(2s_a+1)}{(2\ell+1)(2s+1)}}\,J_{BA}(\ell sj)\,a(s)$$

$$\times\int d\mathbf{r}'_{aA}\int d\mathbf{r}'_{bB}\left[\varphi_{N_bL_b}(-\frac{bB}{B+b}\nu,\mathbf{r}'_{bB})\varphi_{\ell}(\mathbf{r}'_{xA})\right]^{*}_{L_am_{La}}V(\mathbf{r}'_{bx})\,\varphi_{N_aL_a}(\frac{aA}{A+a}\nu,\mathbf{r}'_{aA})\,\Phi_a(\mathbf{r}'_{bx})$$

$$\times i^{-\ell}\frac{1}{\sqrt{2\ell+1}}\int d\mathbf{r}_{aA}\int d\mathbf{r}_{bB}\,\chi_b^{(-)*}(\mathbf{k}_b,\mathbf{r}_{bB})(L_bL_a-m_{Lb}m_{La}|\ell m)\varphi^{*}_{N_bL_b-m_{Lb}}(\frac{bB}{B+b}\nu,\mathbf{r}_{bB})\,\varphi^{*}_{N_aL_am_{La}}(\frac{aA}{A+a}\nu,\mathbf{r}_{aA})\,\chi_a^{(+)}(\mathbf{k}_a,\mathbf{r}_{aA})$$

$$=\sqrt{\frac{2s_a+1}{2s+1}}\,J_{BA}(\ell sj)\,a(s)\,\frac{1}{\sqrt{2\ell+1}}$$

$$\times\int d\mathbf{r}_{aA}\int d\mathbf{r}_{bB}\,\chi_b^{(-)*}(\mathbf{k}_b,\mathbf{r}_{bB})\,\varphi^{*}_{\ell m}(\mathbf{r}_{xA})\,V(\mathbf{r}_{bx})\,\Phi_a(\mathbf{r}_{bx})\,\chi_a^{(+)}(\mathbf{k}_a,\mathbf{r}_{aA}),$$

(3.26)

where we use the completeness of the harmonic oscillator wave function and the relation

$$\langle [\psi_{N_bL_b}(\tfrac{bB}{B+b}\nu, \mathbf{r}_{bB}')\, \psi_l(\mathbf{r}_{xA}')]_{L_a m_{La}} | V_{bx} | \psi_{N_aL_am_{La}}(\tfrac{aA}{A+a}\nu, \mathbf{r}_{aA}')\, \Phi_a(\mathbf{r}_{bx}')\rangle (L_b l\, m_{Lb} m | L_a m_{La})$$

$$= \sum_{L_a' m_{La}'} \langle [\psi_{N_bL_b}(\tfrac{bB}{B+b}\nu, \mathbf{r}_{bB}')\, \psi_l(\mathbf{r}_{xA}')]_{L_a m_{La}} | V_{bx} | \psi_{N_aL_a'm_{La}'}(\tfrac{aA}{A+a}\nu, \mathbf{r}_{aA}')\, \Phi_a(\mathbf{r}_{bx})\rangle (L_b l\, m_{Lb} m | L_a' m_{La}').$$

3.4 Calculation of the matrix elements in the form factor

We use the notations $\langle (bB)\Lambda | V | (aA)\Lambda \rangle$ and $\langle [\psi_{N_bL_b}(b\nu, \mathbf{r}_b)\, \phi_b(\nu_b)\, {}^{2t_b+1\,2s_b+1}\Gamma\ \Psi_{LB}\, {}^{2T_B+1\,2S_B+1}\Gamma]_{\Lambda_L\Lambda_S\Lambda_T} | V | [\psi_{N_aL_a}(a\nu, \mathbf{r}_a)\, \phi_a(\nu_a)\, {}^{2t_a+1\,2s_a+1}\Gamma\ \Psi_{LA}\, {}^{2T_A+1\,2S_A+1}\Gamma]_{\Lambda_L\Lambda_S\Lambda_T}\rangle$ as the simplified form and the explicit form of the matrix element $\langle (L_bL_B)\Lambda_L (s_bS_B)\Lambda_S (t_bT_B)\Lambda_T | V | (L_aL_A)\Lambda_L (s_aS_A)\Lambda_S (t_aT_A)\Lambda_T\rangle$ respectively. The notation Ψ_{LB} is the orbital part of the wave function of the nucleus B and ${}^{2T_B+1\,2S_B+1}\Gamma$ is the spin-isospin part of the wave function of the nucleus B. $\phi_b(\nu_b)$ is the orbital part of the internal wave function of the particle b. In our calculation $\phi_b(\nu_b)$ is assumed as follows:

$$\phi_b(\nu_b) = \frac{\psi_{000}(\nu_b, \mathbf{r}_1)\, \psi_{000}(\nu_b, \mathbf{r}_2) \cdots\cdots \psi_{000}(\nu_b, \mathbf{r}_b)}{(\text{wave function of center-of-mass motion of particle b})}, \tag{3.27}$$

where $\psi_{000}(\nu_b, \mathbf{r})$ is the harmonic oscillator wave function defined by Eq. (3.3). For example, if the particle b is composed of three nucleons

$$\phi_b(\nu_b) = \psi_{000}(\tfrac{2}{3}\nu_b, \mathbf{r}_{1-(2,3)})\, \psi_{000}(\tfrac{1}{2}\nu_b, \mathbf{r}_{2-3}), \tag{3.28a}$$

where $\mathbf{r}_{1-(2,3)}$ is the distance between the particle 1 and the center-of-mass of the particles 2 and 3, and $\mathbf{r}_{2-3}$ is the distance between particles 2 and 3. If the particle b is composed of four nucleons

$$\phi_b(\nu_b) = \psi_{000}(\tfrac{1}{2}\nu_b, \mathbf{r}_{1-2})\,\psi_{000}(\tfrac{1}{2}\nu_b, \mathbf{r}_{3-4})\,\psi_{000}(\nu_b, \mathbf{r}_{(1,2)-(3,4)})$$

$$= \psi_{000}(\tfrac{1}{2}\nu_b, \mathbf{r}_{1-2})\,\psi_{000}(\tfrac{2}{3}\nu_b\,\mathbf{r}_{3-(1,2)})\,\psi_{000}(\tfrac{3}{4}\nu_b, \mathbf{r}_{4-(1,2,3)}) \, . \qquad (3.28b)$$

The matrix element $\langle(bB)\Lambda|V|(aA)\Lambda\rangle$ written here is the matrix element $\langle(bB)\Lambda|V|(aA)\Lambda\rangle^{shell}$ in Eq. (3.27), but we remove the suffix shell for simplicity. We give the expression for the matrix element $\langle(bB)\Lambda|V|(aA)\Lambda\rangle$ according to the six terms in Eq. (2.25)

a. $V = V_{bx}$; stripping term

We transform the orbital part of the wave function of the particle a as

$$\psi_{N_aL_am_{La}}(a\nu, \mathbf{r}_a)\,\phi_a(\nu_a)$$

$$= \sum a_{bx}(n)\,(n_b' \ell_b\, n_x\, \ell_x ; L_a | \mu_b : \mu_x | N_a L_a\, n\, 0 ; L_a)\left[\psi_{N_bL_b}(b\nu, \mathbf{r}_b)\,\phi_b(\nu_a)\,\psi_{n_x l_x}(x\nu, \mathbf{r}_x)\,\phi_x(\nu_a)\right]_{L_a m_{La}},$$

$$(3.29)$$

where

$$a_{bx}(n) = \int \psi^*_{n00}(\alpha\,\nu, \mathbf{r}_{bx})\,\psi_{000}(\alpha\,\nu_a, \mathbf{r}_{bx})\,d\mathbf{r}_{bx} \, , \qquad (3.30)$$

and α is determined by Eqs. (3.28). Next we transform the wave function of the nucleus B as

$$\Psi_{L_B}{}^{2T_B+1\,2S_B+1}\Gamma = \sum_{Ax'} \langle B\{|Ax'\rangle \left[\Psi_{L_A}{}^{2T_A+1\,2S_A+1}\Gamma\ \Psi_{l_{x'}}{}^{2t_{x'}+1\,2s_{x'}+1}\Gamma\right]_{L_B S_B T_B} .$$

Here $\langle B\{|Ax'\rangle$ is the coefficient of fractional parantage and it is explicitly written as

$$\langle B\{|Ax'\rangle = \langle \alpha_B L_B S_B T_B \{| \alpha_A L_A S_A T_A\ \alpha_{x'} l_{x'} s_{x'} t_{x'} \rangle. \qquad (3.31)$$

The notation α_B denotes the quantum number except for the angular momentum quantum numbers. By using these equations, we obtain the matrix element

$$\langle (bB)\Lambda | V_{bx} | (aA)\Lambda \rangle$$

$$= \sum_{\substack{n b' \\ x x'}} a_{bx}(n)(n_{b'} l_{b'} n_x l_x ; L_a | \mu_b : \mu_x | N_a L_a n 0 ; L_a) \langle B \{| A x' \rangle \langle S_a t_a \{| S_{b'} t_{b'} S_x t_x \rangle \langle b(Ax')B;\Lambda | (b'x)aA;\Lambda \rangle$$

$$\times \left\langle \left(\left[\psi_{N_b L_b}(b\nu, R_b)\, \phi_b(\nu_b)^{2t_b+1\,2S_b+1} \Gamma \Phi_{l_{x'}}^{2t_{x'}+1\,2S_{x'}+1} \Gamma \right] \right)_{L_a S_a t_a} \middle| V_{bx} \middle| \left[\psi_{n_{b'} l_{b'}}(b\nu, R_b)\, \phi_{b'}(\nu_a)^{2t_{b'}+1\,2S_{b'}+1} \Gamma\, \psi_{n_x l_x}(x\nu, R_x)\, \phi_x(\nu_a)^{2t_x+1\,2S_x+1} \Gamma \right]_{L_a S_a t_a} \right\rangle.$$

The recoupling factor in Eq. (3.32) is given by the following equation:

$$\langle b(Ax')B ; \Lambda | (b'x)aA ; \Lambda \rangle$$

$$= \langle L_b(L_A l_{x'})L_B ; \Lambda_L | (l_{b'} l_x) L_a L_A ; \Lambda_L \rangle \langle S_b(S_A S_{x'})S_B ; \Lambda_S | (S_{b'} S_x) S_a S_A ; \Lambda_S \rangle \langle t_b(T_A t_{x'})T_B ; \Lambda_T | (t_{b'} t_x) t_a T_A ; \Lambda_T \rangle. \tag{3.33}$$

The letters a, b, . . . in Eqs. (3.31), (3.32) and (3.33) denote all the quantum numbers of the particles a, b, ... The matrix element $\langle (bx')a | V_{bx} | (b'x)a \rangle$ in Eq. (3.32) is the two-body matrix element in the reaction (d,p) and (d,n). If the particles x and b are composed of more than two nucleons, we take out one nucleon from the particles x and b using the coefficients of fractional parantage and the Talmi coefficients. The matrix element $\langle (bx')a | V_{bx} | (b'x)a \rangle$ becomes as follows

$$\langle (bx')a | V_{bx} | (b'x)a \rangle$$

$$= \sum_{\substack{b_1 b_2 b_2' \\ x_1 x_2 x_2' \\ n' n'' n''' \\ a_1}} b_{b_1 b_2}(n')\, (n_{b_1} l_{b_1} n_{b_2} l_{b_2} ; L_b | \mu_{b_1} : 1 | N_b L_b n' 0 ; L_b) \langle S_b t_b \{| S_{b_1} t_{b_1} \tfrac{1}{2}\ \tfrac{1}{2} \rangle$$

$$\times \langle (b_1 b_2) b x' ; a | b_1 (b_2 x') a_1 ; a \rangle \langle b_2 (x_1 x_2') x' ; a_1 | (b_2 x_2') a_2\, x_1 ; a_1 \rangle \langle x' \{| x_1 x_2' \rangle$$

$$\times\, a_{b_1 b_2}(n'')\, n(b)\, (n_{b_1} l_{b_1} n_{b_2'} l_{b_2'}, l_{b'} | \mu_{b_1} : 1 | n_{b'} l_{b'} n'' 0 ; l_{b'}) \langle s_{b'} t_{b'} \{| s_{b_1} t_{b_1} \tfrac{1}{2} \tfrac{1}{2} \rangle \langle (b_1 b_2') b' x ; a | b_1 (b_2' x) a_1 ; a \rangle$$

$$\times\, a_{x_1 x_2}(n'')\, n(x)\, (n_{x_1} l_{x_1} n_{x_2} l_{x_2} ; l_x | \mu_{x_1} : 1 | n_x l_x n'' 0 ; l_x) \langle s_x t_x \{| s_{x_1} t_{x_1} \tfrac{1}{2} \tfrac{1}{2} \rangle$$

$$\times \langle b_2' (x_1 x_2) x ; a | (b_2' x_2) a_2 x_1 ; a_1 \rangle \left(W + (-)^{s_{a_2}+1} B + (-)^{t_{a_2}} H + (-)^{s_{a_2}+t_{a_2}+1} M \right)$$

$$\times \left\langle \left[\psi_{n_{b_2} l_{b_2}}(\nu, \mathbf{r}_{b_2}) \psi_{n_{x_2'} l_{x_2'}}(\nu_A, \mathbf{r}_{x_2}) \right]_{L_{a_2}} \middle| V_{b_2 x_2} \middle| \left[\psi_{n_{b_2'} l_{b_2'}}(\nu, \mathbf{r}_{b_2}) \psi_{n_{x_2} l_{x_2}}(\nu, \mathbf{r}_{x_2}) \right]_{L_{a_2}} \right\rangle$$

$$\times \int \phi_{b_1}(\nu_b)^* \phi_{b_1}(\nu_a)\, d\xi_{b_1} \int \Phi_{l_{x_1}}^* \phi_{x_1}(\nu_a) \psi_{n_{x_1} l_{x_1}}(x_1 \nu, \mathbf{r}_{x_1})\, d\mathbf{r}_{x_1}\, d\xi_{x_1} \,, \tag{3.34}$$

where

$$b_{b_1 b_2}(n) = \int \psi_{n00}^*(\beta\nu, \mathbf{r}_{b_1 b_2}) \psi_{000}(\beta\nu_b, \mathbf{r}_{b_1 b_2})\, d\mathbf{r}_{b_1 b_2} \,, \tag{3.35a}$$

$$a_{b_1 b_2}(n) = \int \psi_{n00}^*(\beta\nu, \mathbf{r}_{b_1 b_2}) \psi_{000}(\beta\nu_a, \mathbf{r}_{b_1 b_2})\, d\mathbf{r}_{b_1 b_2} \,, \tag{3.35b}$$

$$a_{x_1 x_2}(n) = \int \psi_{n00}^*(\gamma\nu, \mathbf{r}_{x_1 x_2}) \psi_{000}(\gamma\nu_a, \mathbf{r}_{x_1 x_2})\, d\mathbf{r}_{x_1 x_2} \,. \tag{3.35c}$$

The parameters β and γ is determined from Eqs. (3.28) and ν_A is the parameter adjusted to the separation energy of the particle x from the nucleus B. The notations n(B) and n(x) is the nucleon numbers contained in the particle b and x respectively. We take the two body interaction as

$$V = f(r_{12}) \left(W + B P^{\sigma} - H P^{\tau} - M P^{\sigma} P^{\tau} \right) , \quad W + B + H + M = 1 \,.$$

b. $V = V_{bA}$

We transform the states of the particles B, a and A into the following forms

$$\Psi_{L_B}{}^{2T_B+1\,2S_B+1}\Gamma = \sum_{A'x'} \langle B \{| A' x' \rangle \left[\Psi_{L_A'}{}^{2T_{A'}+1\,2S_{A'}+1}\Gamma\; \Psi_{l_x'}{}^{2t_{x'}+1\,2s_{x'}+1}\Gamma \right]_{L_B S_B T_B} , \tag{3.36a}$$

$$\psi_{N_a L_a}(a\nu, \mathbf{r})\,\phi_a(\nu_a)^{2t_a+1\,2S_a+1}\Gamma$$

$$= \sum_{n b' x} (n_x l_x n_{b'} l_{b'}; L_a | \mu_x : \mu_b | N_a L_a n 0; L_a)\, a_{xb}(n) \langle S_a t_a \{| S_x t_x S_{b'} t_{b'} \rangle$$

$$\times \left[\psi_{n_x l_x}(x\nu, \mathbf{r}_x)\,\phi_x(\nu_a)^{2t_x+1\,2S_x+1}\Gamma\; \psi_{n_{b'} l_{b'}}(b\nu, \mathbf{r}_b)\,\phi_b(\nu_a)^{2t_{b'}+1\,2S_{b'}+1}\Gamma \right]_{L_a S_a t_a}$$

(3.36b)

$$\Psi_{L_A}{}^{2T_A+1\,2S_A+1}\Gamma = \sum_{C,y} \langle A \{| C y \rangle \left[\Psi_{L_C}{}^{2T_C+1\,2S_C+1}\Gamma\;\; \Psi_{l_y}{}^{22}\Gamma \right]_{L_A S_A T_A}. \quad (3.36c)$$

By using these equations, we obtain

$$\langle (bB)\Lambda \,|\, V_{bA} \,|\, (aA) \rangle$$

$$= \sum_{\substack{n A' x B' \\ C y y' b' d}} \langle B \{| A x \rangle \langle b(A'x)B; \Lambda | x(bA')B; \Lambda \rangle \langle A' \{| C y \rangle \langle b(Cy)A'; B' | (by)d\, C; B' \rangle$$

$$\times\; a_{xb}(n)\, (n_x l_x n_{b'} l_{b'}; L_a | \mu_x ; \mu_{b'} | N_a L_a\, n 0; L_a) \;\; \langle S_a t_a \{| S_x t_x S_{b'} t_{b'} \rangle$$

$$\times \langle (xb')a\, A; \Lambda | x(b'A)B'; \Lambda \rangle \langle b'(Cy')A; B' | (b'y')d\, C; B' \rangle \langle A \{| C y' \rangle$$

$$\times \langle \left[\psi_{N_b L_b}(b\nu, \mathbf{r}_b)\,\phi_b(\nu_b)^{2t_b+1\,2S_b+1}\Gamma\; \Psi_{l_x}{}^{22} \right]_{L_d S_d t_d} \,|\, V_{by} \,|$$

$$\left[\psi_{n_{b'} l_{b'}}(b\nu, \mathbf{r}_b)\,\phi_b(\nu_a)^{2t_{b'}+1\,2S_{b'}+1}\Gamma\; \Psi_{l_{y'}}{}^{22}\Gamma \right]_{L_d S_d t_d} \rangle.$$

(3.37)

The matrix element $\langle (by)d \,|\, V_{by} \,|\, (b'y')d \rangle$ in Eq. (3.37) is a two-body matrix element if the particle b is a single nucleon, but if the particle b is composed of more than two nucleons, we take out the single nucleon b_2 from the particle b. The matrix element $\langle (by)\,d \,|\, V_{by} \,|\, (b'y')d \rangle$ then becomes

$$\langle (by)d \,|\, V_{by} \,|\, (b'y')d \rangle$$

$$= \sum_{\substack{n' n^{\circ} \\ b_1 b_2 b_2' e}} b_{b_1 b_2}(n')\, n(b)\;\; (n_{b_1} l_{b_1} n_{b_2} l_{b_2}; L_b | \mu_{b_1} : \| N_b L_b\, n' 0; L_b) \langle S_b t_b \{| S_{b_1} t_{b_1} \tfrac{1}{2} \tfrac{1}{2} \rangle$$

$$\times \langle (b_1 b_2) b\, y ; d \,|\, b_1 (b_2 y) e ; d \rangle\, a_{b_1 b_2}(n'')\, (n_{b_1} l_{b_1} n_{b_2'} l_{b_2'} ; l_{b'} \,|\, \mu_{b_1} : 1 \,|\, n_{b'} l_{b'} n'' 0 ; l_{b'})$$

$$\times \langle s_{b'} t_{b'} \{| s_{b_1} t_{b_1} \tfrac{1}{2} \tfrac{1}{2} \rangle \langle (b_1 b_2') b\, y ; d \,|\, b_1 (b_2' y) e ; d \rangle$$

$$\times \left(W + (-)^{s_e+1} B + (-)^{t_e} H + (-)^{t_e+s_e+1} M \right)$$

$$\times \langle [\psi_{n_{b_2} l_{b_2}}(\nu, \mathbf{r}_{b_2})\, \psi_{n_y l_y}(\nu_A \mathbf{r}_y)]_{l_a} \,|\, V_{b_2 y} \,|\, [\psi_{n_{b_2'} l_{b_2'}}(\nu, \mathbf{r}_{b_2})\, \psi_{n_{y'} l_{y'}}(\nu_A, \mathbf{r}_y)]_{l_a} \rangle . \tag{3.38}$$

c. $V = U(r_{bB})$.

In the numerical calculation, we set V = 1 and multiply the final channel distorted wave function $\chi_{L_b}(k_b \mathbf{r}_{bB})$ by the optical potential $U(r_{bB})$. Then we obtain

$$\langle (bB)\Lambda \,|\, (aA)\Lambda \rangle$$

$$= \sum a_{bx}(n)\, (n_b l_b n_x l_x ; L_a \,|\, \mu_b : \mu_x \,|\, N_a L_a n 0 ; L_a) \langle s_a t_a \{| s_b t_b s_x t_x \rangle$$

$$\times \langle B \{| A x \rangle \langle b (A x) B ; \Lambda \,|\, (b x) a A ; \Lambda \rangle \int \phi_b^*(\nu_b)\, \phi_b(\nu_a)\, d\xi_b \int \Phi_{L_x}^*\, \psi_{n_x l_x}(x\nu, \mathbf{r}_x)\, \phi_x(\nu_a)\, d\mathbf{r}_x\, d\xi_x \tag{3.39}$$

d. $V = V_{ba}$; knock on term

The matrix element $\langle (bB)\Lambda | V_{ab} | (aA)\Lambda \rangle$ can be written in the following form:

$$\langle (bB)\Lambda \,|\, V_{ba} \,|\, (aA)\Lambda \rangle$$

$$= \sum \langle B \{| C \alpha' \rangle \langle b (C \alpha') B ; \Lambda \,|\, (b \alpha') d\, C ; \Lambda \rangle \langle A \{| C b' \rangle \langle a (C b') A ; \Lambda \,|\, (b' a) d\, C ; \Lambda \rangle$$

$$\times \langle [\psi_{N_b L_b}(b \nu_b, \mathbf{r}_b)\, \phi_b(\nu_b)\, {}^{2t_b+1\, 2s_b+1}\Gamma\;\; \Phi_{L_{a'}}\, {}^{2t_{a'}+1\, 2s_{a'}+1}\Gamma]_{l_d s_d t_d}$$

$$\times | V_{ba} | [\Phi_{L_{b'}}\, {}^{2t_{b'}+1\, 2s_{b'}+1}\Gamma\;\; \psi_{N_a L_a}(a\nu, \mathbf{r}_a)\, \phi_a(\nu_a)\, {}^{2t_a+1\, 2s_a+1}\Gamma]_{l_d s_d t_d} \rangle . \tag{3.40}$$

If the particle a is a single nucleon, we set $\phi_a(\nu_a)$ equal to 1. It is also the same about the particle b. If it is not a single nucleon, the matrix element in Eq. (3.40) can be written as the linear combination of two-body elements

$$
\begin{aligned}
&\langle (ba')d \mid V_{ba} \mid (b'a)d \rangle \\
&= \sum_{\substack{nn'b_1b_2b_2' \\ eCa_1a_2a_2'}} \langle (b_1b_2')ba;d \mid b_1(b_2'a)e;d \rangle \\
&\times a_{a_1a_2}(n)\,(n_{a_1}l_{a_1}n_{a_2}l_{a_2};L_a \mid \mu_{a_1}:1 \mid N_aL_a\,n0;L_a)\,\langle s_at_a\{\mid s_{a_1}t_{a_1}\tfrac{1}{2}\tfrac{1}{2}\rangle \\
&\times \langle b_2'(a_1a_2)a;e \mid (b_2'a_2)C\,a_1;e \rangle \\
&\times b_{b_1b_2}(n')\,(n_{b_1}l_{b_1}n_{b_2}l_{b_2};L_b \mid \mu_{b_1}:1 \mid N_bL_b\,n'0;L_b)\,\langle s_bt_b\{\mid s_{b_1}t_{b_1}\tfrac{1}{2}\tfrac{1}{2}\rangle \\
&\times \langle (b_1b_2)ba;d \mid b_1(b_2a)e;d \rangle \langle \alpha'\{\mid a_1a_2'\rangle \langle b_2(a_1a_2')a;e \mid (b_2a_2')C\,a_1;e \rangle \\
&\times n(b)\,n(a)\,\left(W + (-)^{s_c+1}B + (-)^{t_c}H + (-)^{s_c+t_c+1}M\right) \\
&\times \langle [\psi_{n_{b_2}l_{b_2}}(\nu,\mathbf{r}_{b_2})\,\psi_{n_{a_2'}l_{a_2'}}(\nu_A,\mathbf{r}_{a_2})]_{l_c} \mid V_{b_2a_2} \mid [\psi_{n_{b_2'}l_{b_2'}}(\nu_A,\mathbf{r}_{b_2})\,\psi_{n_{a_2}l_{a_2}}(\nu,\mathbf{r}_{a_2})]_{l_c}\rangle \\
&\times \int \psi^*_{n_{b_1}l_{b_1}}(b_1\nu,\mathbf{r}_{b_1})\,\phi^*_{b_1}(\nu_b)\,\Phi_{l_{b_1}}\,d\mathbf{r}_{b_1}d\xi_{b_1} \int \Phi^*_{l_{a_1}}\,\psi_{n_{a_1}l_{a_1}}(a_1\nu,\mathbf{r}_{a_1})\,\phi_{a_1}(\nu_a)\,d\mathbf{r}_{a_1}d\xi_{a_1}.
\end{aligned}
\tag{3.41}
$$

e. $V = V_{bC}$ heavy particle stripping term

The matrix element can be written as

$$\langle (bB)\Lambda | V_{bC} | (aA)\Lambda \rangle$$

$$= \sum \langle B\{|C'a\rangle \langle b(C'a)B;\Lambda | a(C'b)A;\Lambda\rangle \langle A\{|C''y\rangle \int \Psi^{*}_{La} \psi_{N_aL_a}(a\nu_a, \mathbf{r}_a)\phi_a(\nu_a)\,d\mathbf{r}_a d\xi_a$$

$$\times\ n(c)\ \langle C'\{|Dy\rangle \langle (Dy)C'b;\Lambda | D(yb)d;\Lambda\rangle$$

$$\times \left\langle \left(\left[\psi_{n_y\ell_y}(\nu_A, \mathbf{r}_y)\ {}^{22}\Gamma\ \psi_{N_bL_b}(b\nu, \mathbf{r}_b)\,\phi_b(\nu_b)\ {}^{2t_b+1\,2s_b+1}\Gamma\right]\right)_{\ell_d s_d t_d} \Big\| V_{yb} \Big\| \left(\psi_{n_{y'}\ell_{y'}}(\nu_A \mathbf{r}_y)\ {}^{22}\Gamma\ \Psi_{\ell_{b'}}\ {}^{2t_{b'}+1\,2s_{b'}+1}\Gamma\right)_{\ell_d s_d t_d} \right\rangle .$$

(3.42)

If the particle b is composed of more than two nucleons the matrix element $\langle (yb)d | V_{yb} | (y'b')d \rangle$ is expressed using two-body matrix elements.

$$\langle (yb)d | V_{yb} | (y'b')d \rangle$$

$$= \sum b_{b_1b_2}(n)\,(n_{b_1}\ell_{b_1} n_{b_2}\ell_{b_2}; L_b | \mu_{b_1} : 1 | N_bL_b\, n\, 0; L_a)\,\langle s_b t_b\{| s_{b_1} t_{b_1} \tfrac{1}{2}\tfrac{1}{2}\rangle$$

$$\times \langle y(b_1b_2)b;d | (yb_2)c\, b_1;d\rangle \langle b'\{| b_1 b_2'\rangle \langle y'(b_1b_2')b;d | (yb_2')c\, b_1;d\rangle$$

$$\times\ n(b)\ \left(W + (-)^{s_c+1}B + (-)^{t_c}H + (-)^{s_c+t_c+1}M \right) \int \psi^{*}_{n_{b_1}\ell_{b_1}}(b_1\nu, \mathbf{r}_{b_1})\,\phi^{*}_{b_1}(\nu_b)\,\Psi_{\ell_{b_1}} d\mathbf{r}_b d\xi_{b_1}$$

$$\times \left\langle \left[\psi_{n_y\ell_y}(\nu_A, \mathbf{r}_y)\,\psi_{n_{b_2}\ell_{b_2}}(\nu, \mathbf{r}_{b_2}) \right]_{\ell_c} \Big| V_{yb_2} \Big| \left[\psi_{n_{y'}\ell_{y'}}(\nu_A, \mathbf{r}_y)\,\psi_{n_{b_2'}\ell_{b_2'}}(\nu_A \mathbf{r}_{b_2}) \right]_{\ell_c} \right\rangle .$$

(3.43)

f. $V = U(r_{bB})$

As in the case c, we multiply the final channel distorted wave function $\chi_{L_b}(r_{bB})$ by the optical potential $U(r_{bB})$. The overlap integral $\langle (bB)\Lambda | (aA)\Lambda \rangle$ in the exchange configuration becomes

$$\langle (bB)\Lambda | (aA)\Lambda \rangle$$

$$= \sum_{C,d} \langle B\{|Ca\rangle \langle b(Ca)B;\Lambda | (ba)d\, C;\Lambda\rangle \langle A\{|Cb\rangle \langle a(Cb)A;\Lambda | (ba)dC;\Lambda\rangle$$

$$\times \int \psi^{*}_{N_bL_b}(b\nu, \mathbf{r}_b)\, \phi^{*}_b(\nu_b)\, \Psi_{L_b}\, d\mathbf{r}_b\, d\xi_b \int \Psi^{*}_{L_a}\, \psi_{N_aL_a}(a\nu, \mathbf{r}_a)\, \phi_a(\nu_a)\, d\mathbf{r}_a\, d\xi_a \, . \tag{3.34}$$

4. Numerical Calculation of the heavy particle stripping term on the reaction $^{11}B(d,n)^{12}C$ and discussion

4.1 Numerical calculation of the matrix element in the form factor

As we have mentioned in §1, the heavy particle stripping mechanism is shown in Fig. 2. In the case of the reaction $^{11}B(d,n)^{12}C$, we take a to be a deuteron, b to be a neutron, A to be ^{11}B, B to be ^{12}C and C to be ^{10}B. Then wave functions describing ^{11}B and ^{12}C for this calculation are given in Eqs. (2.27). Since the core C is composed of four S-shell nucleons and six P-shell nucleons, the matrix element in Eq. (3.23) can be written as

$$\langle (L_bL_B)\Lambda_L (s_bS_B)\Lambda_S (t_bT_B)\Lambda_T | V_{bC} | (L_aL_A)\Lambda_L (s_aS_A)\Lambda_S (t_aT_A)\Lambda_T \rangle^{shell}$$

$$= 4 \langle [\psi_{N_bL_b}(\nu, \mathbf{r}_b)\, {}^{22}\Gamma_{[\tilde{1}]}\, \Psi_{S[44]}\, {}^{11}\Gamma_{[\tilde{44}]}]_{\Lambda_L\Lambda_S\Lambda_T} | V_{sb} |$$

$$[\psi_{N_aL_a}(2\nu, \mathbf{r}_a)\, \psi_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, {}^{13}\Gamma_{[\tilde{2}]}\, \Psi_{L_A[43]}\, {}^{22}\Gamma_{[\tilde{43}]}]_{\Lambda_L\Lambda_S\Lambda_T} \rangle^{shell}$$

$$+\ 6\left\langle\left[\varphi_{N_bL_b}(\nu,\mathbf{r}_b)\,{}^{22}\Gamma_{[1]}\,\Psi_{S[44]}\,{}^{11}\Gamma_{[\tilde{44}]}\right]_{\Lambda_L\Lambda_S\Lambda_T}\middle|V_{pb}\right|$$

$$\left.\left[\varphi_{N_aL_a}(2\nu,\mathbf{r}_a)\,\varphi_{00}(\tfrac{1}{2}\nu_d,\mathbf{r})^{13}\Gamma_{[2]}\,\Psi_{L_A[43]}\,{}^{22}\Gamma_{[\tilde{43}]}\right]_{\Lambda_L\Lambda_S\Lambda_T}\right\rangle^{\text{shell}}, \tag{4.1}$$

where V_{sb} is the interaction between particle b and a nucleon in the S-shell, V_{pb} the interaction between the particle b and a nucleon in the P-shell, and $\mathbf{r}$ the coordinate of the internal motion of the deuteron. The parameter ν_d represents the extension of the intrinsic state of the deuteron and should be adjusted to the binding energy of the deuteron. The notation $\left[\Psi_{L_1[f_1]}\,{}^{2T_1+1\,2S_1+1}\Gamma_{[\tilde{f}_1]}\Psi_{L_2[f_2]}\,{}^{2T_2+1\,2S_2+1}\Gamma_{[\tilde{f}_2]}\right]_{\Lambda_L\Lambda_S\Lambda_T}$ denotes the state constructed by coupling the two states to have the total spatial angular momentum Λ_L, the total spin Λ_s and the total iso-spin Λ_T. To simplify the notation, we write the first term of Eq. (4.1) as $F_s^{\text{shell}}(N_bL_bL_BN_aL_aL_A)$ and the second term as $F_p^{\text{shell}}(N_bL_bL_BN_aL_aL_A)$. Using the table of Jahn and Wierigern(12), we obtain for the decomposition the following expressions:

$^{12}C \rightarrow {}^{10}B + d$:

$$\Psi_{S[44]}\,{}^{11}\Gamma_{[\tilde{44}]} = \frac{\sqrt{2}}{\sqrt{3}\sqrt{7}}\left[\Psi_{S[2]}\,{}^{13}\Gamma_{[\tilde{2}]}\,\Psi_{S[42]}\,{}^{13}\Gamma_{[\tilde{42}]}\right]_{000}$$

$$+\frac{1}{2\sqrt{2}\sqrt{7}}\left[\Psi_{D[2]}\,{}^{13}\Gamma_{[\tilde{2}]}\,\Psi_{D_I[42]}\,{}^{13}\Gamma_{[\tilde{42}]}\right]_{000}+\frac{\sqrt{5}}{2\sqrt{2}\sqrt{3}}\left[\Psi_{D[2]}\,{}^{13}\Gamma_{[\tilde{2}]}\,\Psi_{D_I[42]}\,{}^{13}\Gamma_{[\tilde{42}]}\right]_{000}, \tag{4.2a}$$

$^{11}B \rightarrow {}^{10}B + n$

$$\Psi_{P[43]}\,{}^{22}\Gamma_{[\tilde{43}]}$$

$$= -\frac{\sqrt{2}}{\sqrt{3}\sqrt{7}}\left[\Psi_{P[1]}\,{}^{22}\Gamma_{[1]}\,\Psi_{S[42]}\,{}^{13}\Gamma_{[\tilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}}+\frac{\sqrt{2}}{\sqrt{3}\sqrt{7}}\left[\Psi_{P[1]}\,{}^{22}\Gamma_{[1]}\,\Psi_{S[42]}\,{}^{31}\Gamma_{[\tilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}}$$

$$-\frac{1}{2\sqrt{2}\sqrt{7}}\left[\Psi_{P[1]}\,{}^{22}\Gamma_{[1]}\,\Psi_{D_I[42]}\,{}^{13}\Gamma_{[\tilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}}+\frac{1}{2\sqrt{2}\sqrt{7}}\left[\Psi_{P[1]}\,{}^{22}\Gamma_{[1]}\,\Psi_{D_I[42]}\,{}^{31}\Gamma_{[\tilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}}$$

$$- \frac{\sqrt{5}}{2\sqrt{2}\sqrt{3}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{D_I[42]}^{13}\Gamma_{[\tilde{42}]} \right]_{1\frac{1}{2}\frac{1}{2}} + \frac{\sqrt{5}}{2\sqrt{2}\sqrt{3}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{D_{II}[42]}^{31}\Gamma \right]_{1\frac{1}{2}\frac{1}{2}}$$

$$+ \frac{1}{2\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{P[33]}^{11}\Gamma_{[\tilde{33}]} \right]_{1\frac{1}{2}\frac{1}{2}} + \frac{3}{2\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{P[33]}^{33}\Gamma_{[\tilde{33}]} \right]_{1\frac{1}{2}\frac{1}{2}} \tag{4.2b}$$

$$\Psi_{D[43]}^{22}\Gamma_{[\tilde{43}]}$$

$$= \frac{5}{2\sqrt{2}\sqrt{3}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{D_I[42]}^{13}\Gamma_{[\tilde{42}]} \right]_{2\frac{1}{2}\frac{1}{2}} - \frac{\sqrt{5}}{2\sqrt{2}\sqrt{3}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{D_I[42]}^{31}\Gamma_{[\tilde{42}]} \right]_{2\frac{1}{2}\frac{1}{2}}$$

$$+ \frac{1}{2\sqrt{2}\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{D_{II}[42]}^{13}\Gamma_{[\tilde{42}]} \right]_{2\frac{1}{2}\frac{1}{2}} - \frac{1}{2\sqrt{2}\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{D_{II}[42]}^{31}\Gamma_{[\tilde{42}]} \right]_{2\frac{1}{2}\frac{1}{2}}$$

$$+ \frac{\sqrt{3}}{2\cdot 5} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{P[33]}^{11}\Gamma_{[\tilde{33}]} \right]_{2\frac{1}{2}\frac{1}{2}} + \frac{3\sqrt{3}}{2\cdot 5} \left[\Psi_{P[1]}^{22}\Gamma \Psi_{P[33]}^{33}\Gamma_{[\tilde{33}]} \right]_{2\frac{1}{2}\frac{1}{2}}$$

$$- \frac{1}{2\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{F[33]}^{11}\Gamma_{[\tilde{33}]} \right]_{2\frac{1}{2}\frac{1}{2}} - \frac{3}{5\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{F[33]}^{33}\Gamma \right]_{2\frac{1}{2}\frac{1}{2}}$$

$$+ \frac{\sqrt{2}}{\sqrt{3}\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma \Psi_{F[42]}^{13}\Gamma_{[\tilde{42}]} \right]_{2\frac{1}{2}\frac{1}{2}} - \frac{\sqrt{2}}{\sqrt{3}\sqrt{7}} \left[\Psi_{P[1]}^{22}\Gamma_{[\tilde{1}]} \Psi_{F[42]}^{31}\Gamma_{[\tilde{42}]} \right]_{2\frac{1}{2}\frac{1}{2}} . \tag{4.2c}$$

We recouple the final channel wave function $|b(a'C)B;\Lambda\rangle$ to $|a'(bC)A;\Lambda\rangle$ and separate the spatial part of the wave function of the particle a' into the center-of-mass motion and the internal motion using the Talmi coefficient. The letters a', b, A and C in the wave function $|b(a'c)B;\Lambda\rangle$ represent the total quantum numbers of each particle. The non-vanishing matrix elements in Eq. (4.1) can be written for $L_A = 1$ in the following form,

$$F_p^{\mathrm{shell}}(N_b 1 0 N_a 0 1) = \frac{1}{3\sqrt{7}} \left\langle \left[\psi_{N_b l}^{22}\Gamma \Psi_{S[42]}^{13}\Gamma_{[\tilde{42}]} \right]_{1\frac{1}{2}\frac{1}{2}} \middle| V_{bp} \middle| \Psi_{P[43]}^{22}\Gamma_{[\tilde{43}]} \right\rangle$$

$$\times\left\{\int \psi^{*}_{N_a0}(2\nu, \mathbf{r}_a)\, \psi_{10}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \int \psi^{*}_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r}\right.$$

$$\left. - \int \psi^{*}_{N_a0}(2\nu, \mathbf{r}_a)\, \psi_{00}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \int \psi^{*}_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{10}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r}\right\}, \tag{4.3a}$$

$F_p^{shell}(N_b 1 0 N_a 2 1)$

$$= \left\{ \frac{1}{4\sqrt{3}\sqrt{5}\sqrt{7}} \left\langle \left[\psi_{N_b 1}\, {}^{22}\Gamma\, \Psi_{D\mathrm{I}[42]}\, {}^{13}\Gamma_{[\widetilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}} \middle| V_{bp} \middle| \Psi_{P[43]}\, {}^{22}\Gamma_{[\widetilde{43}]} \right\rangle \right.$$

$$\left. + \frac{1}{4\cdot 3} \left\langle \left[\psi_{N_b 1}\, {}^{22}\Gamma\, \Psi_{D\mathrm{II}[42]}\, {}^{13}\Gamma_{[\widetilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}} \middle| V_{bp} \middle| \Psi_{P[43]}\, {}^{22}\Gamma_{[\widetilde{43}]} \right\rangle \right\}$$

$$\times \int \psi^{*}_{N_a2}(2\nu, \mathbf{r}_a)\, \psi_{02}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \cdot \int \psi^{*}_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r}, \tag{4.3b}$$

$F_p^{shell}(N_b 3 0 N_a 2 1)$

$$= \left\{ \frac{1}{4\sqrt{5}\cdot 7} \left\langle \left[\psi_{N_b 3}\, {}^{22}\Gamma_{[1]}\, \Psi_{D\mathrm{I}[42]}\, {}^{13}\Gamma_{[\widetilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}} \middle| V_{pb} \middle| \Psi_{P[43]}\, {}^{22}\Gamma_{[\widetilde{43}]} \right\rangle \right.$$

$$\left. + \frac{1}{4\sqrt{3}\sqrt{7}} \left\langle \left[\psi_{N_b 3}\, {}^{22}\Gamma_{[1]}\, \Psi_{D\mathrm{II}[42]}\, {}^{13}\Gamma_{[\widetilde{42}]}\right]_{1\frac{1}{2}\frac{1}{2}} \middle| V_{pb} \middle| \Psi_{P[43]}\, {}^{22}\Gamma_{[\widetilde{43}]} \right\rangle \right\}$$

$$\times \int \psi^{*}_{N_a2}(2\nu, \mathbf{r}_a)\, \psi_{02}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \cdot \int \psi_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r} \tag{4.3c}$$

and for $L_A = 2$

$F_p^{shell}(N_b 1 0 N_a 2 2)$

$$= \left\{ \frac{1}{4\cdot 3\sqrt{7}} \left\langle \left[\psi_{N_b 1}\, {}^{22}\Gamma_{[1]}\, \Psi_{D\mathrm{I}[42]}\, {}^{13}\Gamma_{[\widetilde{42}]}\right]_{2\frac{1}{2}\frac{1}{2}} \middle| V_{pb} \middle| \Psi_{D[43]}\, {}^{22}\Gamma_{[\widetilde{43}]} \right\rangle \right.$$

$$+\frac{\sqrt{5}}{4\cdot 3\sqrt{3}}\left\langle\left[\psi_{N_b1}{}^{22}\Gamma_{[1]}\,\Psi_{D\text{II}[42]}{}^{13}\Gamma_{[\tilde{42}]}\right]_{2\,\frac{1}{2}\frac{1}{2}}\middle|V_{pb}\middle|\Psi_{D[43]}{}^{22}\Gamma_{[\tilde{43}]}\right\rangle\Bigg\}$$

$$\times\int\psi^*_{N_a2}(2\nu,\mathbf{r}_a)\,\psi_{02}(2\nu_A,\mathbf{r}_a)\,d\mathbf{r}_a\cdot\int\psi^*_{00}(\tfrac{1}{2}\nu_a,\mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A,\mathbf{r})\,d\mathbf{r}, \tag{4.3d}$$

$F_p^{shell}(N_b30N_a22)$

$$=\Bigg\{\frac{1}{4\sqrt{3}\cdot 7}\left\langle\left[\psi_{N_b3}{}^{22}\Gamma_{[1]}\,\Psi_{D\text{I}[42]}{}^{13}\Gamma_{[\tilde{42}]}\right]_{2\,\frac{1}{2}\frac{1}{2}}\middle|V_{pb}\middle|\Psi_{D[43]}{}^{22}\Gamma_{[\tilde{43}]}\right\rangle$$

$$+\frac{\sqrt{5}}{4\sqrt{3}\sqrt{7}}\left\langle\left[\psi_{N_b3}{}^{22}\Gamma_{[1]}\,\Psi_{D\text{II}[42]}{}^{13}\Gamma_{[\tilde{42}]}\right]_{2\,\frac{1}{2}\frac{1}{2}}\middle|V_{pb}\middle|\Psi_{D[43]}{}^{22}\Gamma_{[\tilde{43}]}\right\rangle\Bigg\}$$

$$\times\int\psi^*_{N_a2}(2\nu,\mathbf{r}_a)\,\psi_{02}(2\nu_A\,\mathbf{r}_a)\,d\mathbf{r}_a\int\psi^*_{00}(\tfrac{1}{2}\nu_a,\mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A,\mathbf{r})\,d\mathbf{r}. \tag{4.3e}$$

The matrix element $F_s^{shell}(N_bL_bL_BN_aL_aL_A)$ can be obtained by substituting V_{pb} for the interaction V_{sb} in the expression $F_p^{shell}(N_bL_bL_BN_aL_aL_A)$. We use the notation $E_p(N_bL_bL_BN_aL_aL_A)$ for the remainder obtained by removing the overlap integrals from the matrix element $F_p^{shell}(N_bL_bL_BN_aL_aL_A)$ in Eqs. (4.3). If we insert Eqs. (4.2b) and (4.2c) intc Eqs. (4.3), the matrix element $E_p(N_bL_bN_aL_aL_A)$ can be written as a linear combination of the matrix elements of the following type.

$$\langle [\psi_{N_b L_b}{}^{22}\Gamma_{[1]}\, \Phi_{L_C[f_C]}{}^{2T_C+1\,2S_C+1}\Gamma_{[\tilde{f}_C]}]_{L_A S_A T_A} | V_{bp} | [\psi_{01}{}^{22}\Gamma\, \Phi_{L'_C[f'_C]}{}^{2T'_C+1\,2S'_C+1}\Gamma_{[\tilde{f}'_C]}]_{L_A S_A T_A} \rangle$$

We may use the simple notation $\langle (bC)A | V_{bp} | (b'C')A \rangle$ for the above matrix element.

We seperate the matrix element $E_p(N_b L_b L_B N_a L_a L_A)$ into two parts. One of them contains only the matrix elements of the type as $\langle (bC)A | V_{bp} | (b'C)A \rangle$ and is denoted by $E_p^{(D)}(N_B L_B L_B N_a L_a L_A)$ and the other contains the matrix elements of the type as $\langle (bC)A | V_{bp} | (b'C') \rangle$ $(C' \neq C)$ and is denoted by $E_p^{(ND)}(N_b L_b L_B N_a L_a L_A)$. If we take the interaction depending only on the relative distance r_{bC}, as we made in the macroscopic calculation of § 2, the term $E_p^{(ND)}(N_B L_b L_B N_a L_a L_A)$ vanishes. But as we shall see later (see Table 6), according to this microscopic calculation using the antisymmetrized wave functions for the target and residual nuclei and the two-body interaction of Rosenfeld type, the term $E_p^{(D)}(N_b L_b L_B N_a L_a L_A)$ vanishes. If we express the matrix elements $\langle (bC)A | V_{bp} | (b'C)A \rangle$ using two-body matrix elements, we obtain the final results such as Eq. (3.42) expressed by the linear combination of two-body matrix elements for the term $E_p(N_b L_b L_B N_a L_a L_A)$.

In the step of this calculation, we show the intermediate states appearing in this calculation in Fig. 5. The matrix elements $E_p(N_b L_b L_B N_a L_a L_A)$ are expressed in terms of the two-body matrix elements in Table 3.

Fig. 5 Intermediate States

$B \rightarrow C+a$ $\qquad$ $A \rightarrow C'+b$

$C \rightarrow D+$ one nucleon $\qquad$ $C' \rightarrow D +$ one nucleon

C	D	C'
$[42]^{13}S$	$[41]^{22}P$	$[42]^{13}S$
	$[32]^{22}P$	$[42]^{31}S$
$[42]^{13}D_I$	$[32]^{24}P$	$[42]^{13}D_I$
		$[42]^{31}D_I$
	$[41]^{22}D$	$[42]^{13}D_{II}$
		$[42]^{31}D_{II}$
$[42]^{13}D_{II}$	$[32]^{22}D$	$[33]^{11}P$
	$[32]^{24}D$	$[33]^{33}P$
	$[41]^{22}F$	$[33]^{11}P$
		$[33]^{33}F$
	$[42]^{22}F$	$[42]^{13}F$
	$[32]^{24}F$	$[42]^{31}F$

Table 3 Representation of Ep(NbLbL$_B$NaLaL$_A$) by two-body matrix elements

	$E_p^{(D)}$(Nb10Na01)	$E_p^{(ND)}$(Nb10Na01)	$E_p^{(D)}$(Nb10Na21)	$E_p^{(ND)}$(Nb10Na21)	$E_p^{(D)}$(Nb10Na22)	$E_p^{(ND)}$(Nb10Na22)
$\langle [4_{Nb1} 4_{01}]_0 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_0 \eta_1 \rangle$	-20	-22	-2932	-620	2420	-1700
$\langle [4_{Nb1} 4_{01}]_1 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_1 \eta_1 \rangle$	-60	33	-4494	22442	6270	-3570
$\langle [4_{Nb1} 4_{01}]_2 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_2 \eta_1 \rangle$	-100	-11	-6254	-1822	9310	5270
$\langle [4_{Nb1} 4_{01}]_0 \eta_2 \lvert V \rvert [4_{01} 4_{01}]_0 \eta_2 \rangle$	-16	40	-1856	-64	2080	800
$\langle [4_{Nb1} 4_{01}]_1 \eta_2 \lvert V \rvert [4_{01} 4_{01}]_1 \eta_2 \rangle$	-48	-60	-6792	-1416	5880	4920
$\langle [4_{Nb1} 4_{01}]_2 \eta_2 \lvert V \rvert [4_{01} 4_{01}]_2 \eta_2 \rangle$	-80	20	-2296	1480	6440	-5720
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_0 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_0 \eta_4 \rangle$		18		-1440		2160
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_1 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_1 \eta_4 \rangle$		27		2052		-2700
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_2 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_2 \eta_4 \rangle$		-45		-612		540
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_0 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_0 \eta_5 \rangle$		-18		1440		-2160
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_1 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_1 \eta_5 \rangle$		-27		-2052		2700
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_2 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_2 \eta_5 \rangle$		45		612		-540
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_0 \eta_2 \lvert V \rvert [4_{01} 4_{01}]_0 \eta_6 \rangle$		72		-5760		8640
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_1 \eta_2 \lvert V \rvert [4_{01} 4_{01}]_1 \eta_6 \rangle$		108		8208		-10800
$\frac{1}{\sqrt{3}} \langle [4_{Nb1} 4_{01}]_2 \eta_2 \lvert V \rvert [4_{01} 4_{01}]_2 \eta_6 \rangle$		-180		-2448		2160
$\langle [4_{Nb1} 4_{01}]_0 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_0 \eta_3 \rangle$		54		2592		720
$\langle [4_{Nb1} 4_{01}]_1 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_1 \eta_3 \rangle$		-27		-2052		2700
$\langle [4_{Nb1} 4_{01}]_2 \eta_1 \lvert V \rvert [4_{01} 4_{01}]_2 \eta_3 \rangle$		81		7668		-14220
common factor	$\frac{1}{[illegible]2\sqrt{2}\cdot 243\sqrt{3}\cdot 7}$	$\frac{1}{[illegible]2\sqrt{2}\cdot 243\sqrt{3}\cdot 7}$	$\frac{1}{64\sqrt{2}\cdot 243\sqrt{3}\sqrt{5}\cdot 7}$	$\frac{1}{64\sqrt{2}\cdot 243\sqrt{3}\sqrt{5}\cdot 7}$	$\frac{1}{64\sqrt{2}\cdot 243\sqrt{3}\cdot 5\sqrt{5}\sqrt{7}}$	$\frac{1}{64\sqrt{2}\cdot 243\sqrt{3}\, 5\sqrt{5}\sqrt{7}}$
			E_p(Nb30Na21)		E_p(Nb30Na22)	
$\langle [4_{Nb3} 4_{01}]_2 \eta_1 \lvert \quad \rvert [4_{01} 4_{01}]_2 \eta_1 \rangle$			-262		1170	
$\langle [4_{Nb3} 4_{01}]_2 \eta_2 \lvert \quad \rvert [4_{01} 4_{01}]_2 \eta_2 \rangle$			312		2280	
$\frac{1}{\sqrt{3}} \langle [4_{Nb3} 4_{01}]_2 \eta_1 \lvert \quad \rvert [4_{01} 4_{01}]_2 \eta_4 \rangle$			234		1710	
$\frac{1}{\sqrt{3}} \langle [4_{Nb3} 4_{01}]_2 \eta_1 \lvert \quad \rvert [4_{01} 4_{01}]_2 \eta_5 \rangle$			-234		-1710	
$\frac{1}{\sqrt{3}} \langle [4_{Nb3} 4_{01}]_2 \eta_2 \lvert \quad \rvert [4_{01} 4_{01}]_2 \eta_6 \rangle$			936		6840	
$\langle [4_{Nb3} 4_{01}]_2 \eta_1 \lvert \quad \rvert [4_{01} 4_{01}]_2 \eta_3 \rangle$			418		-30	
common factor			$\frac{1}{16\sqrt{2}\cdot 81\sqrt{5}\cdot 7\sqrt{7}}$		$\frac{1}{16\sqrt{2}\, 27\sqrt{3}\cdot 5\sqrt{5}\cdot 49}$	

In Table 3, the spin-isospin wave functions are defined by

$$\eta_1 = \frac{1}{4}\left({}^{13}\Gamma - \sqrt{3}\,{}^{33}\Gamma + \sqrt{3}\,{}^{11}\Gamma - 3\,{}^{31}\Gamma\right), \tag{4.4a}$$

$$\eta_2 = \frac{1}{2}\left({}^{13}\Gamma - \sqrt{3}\,{}^{33}\Gamma\right), \tag{4.4b}$$

$$\eta_3 = \frac{1}{4}\left({}^{31}\Gamma - \sqrt{3}\,{}^{33}\Gamma + \sqrt{3}\,{}^{11}\Gamma - 3\,{}^{13}\Gamma\right), \tag{4.4c}$$

$$\eta_4 = \frac{1}{4}\left({}^{11}\Gamma - \sqrt{3}\,{}^{13}\Gamma + \sqrt{3}\,{}^{31}\Gamma - 3\,{}^{33}\Gamma\right), \tag{4.4d}$$

$$\eta_5 = \frac{1}{4}\left({}^{33}\Gamma + \sqrt{3}\,{}^{31}\Gamma + \sqrt{3}\,{}^{13}\Gamma + 3\,{}^{11}\Gamma\right), \tag{4.4e}$$

$$\eta_6 = \frac{1}{2}\left({}^{33}\Gamma + \sqrt{3}\,{}^{13}\Gamma\right). \tag{4.4f}$$

For example, $E_p^{(D)}(N_b 1 0 N_a 0 1)$ is defined by

$$\begin{aligned}
E_p^{(D)}(N_b 1 0 N_a 0 1) = \frac{1}{2\sqrt{2}\;243\sqrt{3}\cdot 7}\Big(&-20\langle(\psi_{N_b1}\psi_{01})_0\eta_1|V|(\psi_{01}\psi_{01})_0\eta_1\rangle \\
&-60\langle(\psi_{N_b1}\psi_{01})_1\eta_1|V|(\psi_{01}\psi_{01})_1\eta_1\rangle - 60\langle(\psi_{N_b1}\psi_{01})_2\eta_1|V|(\psi_{01}\psi_{01})_2\eta_1\rangle \\
&-16\langle(\psi_{N_b1}\psi_{01})_0\eta_2|V|(\psi_{01}\psi_{01})_0\eta_2\rangle - 48\langle(\psi_{N_b1}\psi_{01})_1\eta_2|V|(\psi_{01}\psi_{01})_1\eta_2\rangle \\
&-80\langle(\psi_{N_b1}\psi_{01})_2\eta_2|V|(\psi_{01}\psi_{01})_2\eta_2\rangle\Big).
\end{aligned}$$

We check on this step of the calculation in the following way.

a. We have used the relation that the matrix elements $<(bC)A|V|(b'C')A>$ are expressed by the linear combinations of the two-body interaction. If we make the interaction V_{pb} equal to unity and the parameter ν equal to ν_A, we obtain

$$\langle[\psi_{N_bL_b}\,{}^{22}\Gamma\,\Phi_{L_c[f_c]}\,{}^{2T_c+1\,2S_c+1}\Gamma_{[\tilde{f}_c]}]_{L_AS_AT_A}|[\psi_{01}\,{}^{22}\Gamma\,\Phi_{L_c'[f_c']}\,{}^{2T_c'+1\,2S_c'+1}\Gamma_{[\tilde{f}_c']}]_{L_AS_AT_A}\rangle$$

$$= \delta_{N_b0}\,\delta_{L_b1}\,\delta_{L_cL_c'}\,\delta_{S_cS_c'}\,\delta_{T_cT_c'}\,\delta_{[f_c][f_c']}\,.$$

For example, to obtain the results in Table 3, the following relation was used.

$$\langle [\psi_{01}{}^{22}\Gamma\, \Phi_{S[42]}\, {}^{13}\Gamma_{[42]}]_{1\frac{1}{2}\frac{1}{2}} | V_{pb} | [\psi_{01}{}^{22}\Gamma\, \Phi_{S[42]}\, {}^{13}\Gamma_{[42]}]_{1\frac{1}{2}\frac{1}{2}} \rangle$$

$$= \frac{1}{81}\Big\{ 5 \langle (\psi_{01}\psi_{01})_0 \eta_1 | V_{pb} | (\psi_{01}\psi_{01})_0 \eta_1 \rangle + 15 \langle (\psi_{01}\psi_{01})_1 \eta_1 | V_{pb} | (\psi_{01}\psi_{01})_1 \eta_1 \rangle$$

$$+ 25 \langle (\psi_{01}\psi_{01})_2 \eta_1 | V_{pb} | (\psi_{01}\psi_{01})_2 \eta_1 \rangle + 4 \langle (\psi_{01}\psi_{01})_0 \eta_2 | V_{pb} | (\psi_{01}\psi_{01})_0 \eta_2 \rangle$$

$$+ 12 \langle (\psi_{01}\psi_{01})_1 \eta_2 | V_{pb} | (\psi_{01}\psi_{01})_1 \eta_2 \rangle + 20 \langle (\psi_{01}\psi_{01})_2 \eta_2 | V_{pb} | (\psi_{01}\psi_{01})_2 \eta_2 \rangle \Big\}.$$

To examine whether above relation is correct or not, we set $V_{pb}=1$. Then the above equation is examined to be correct, as its right side is unity.

b. We express the matrix element $E_p(N_b L_b L_B N_a L_a L_A)$ by the two-body matrix elements of the type

$$\langle (\psi_{N_b 1}\psi_{01})_L{}^{2T+1\,2S+1}\Gamma\, | V_{pb} | (\psi_{01}\psi_{01})_L{}^{2T+1\,2S+1}\Gamma \rangle$$

Owing to the symmetry of the wave function $[\psi_{01}\psi_{01}]_L{}^{2T+1\,2S+1}\Gamma$, the two-body matrix elements such as

$$\langle (\psi_{N_b 1}\psi_{01})_0{}^{11}\Gamma | V_{pb} | (\psi_{01}\psi_{01})_0{}^{11}\Gamma \rangle \text{ and } \langle (\psi_{N_b 1}\psi_{01})_1{}^{13}\Gamma | V_{pb} | (\psi_{01}\psi_{01})_1{}^{13}\Gamma \rangle \text{ etc.}$$

are not contained in the matrix element $E_p(N_b L_b L_B N_a L_a L_A)$.

Finally, if we take the two-body interaction

$$V(r) = \big(V_W + V_\sigma (\sigma_1\cdot\sigma_2) + V_\tau (\tau_1\cdot\tau_2) + V_{\sigma\tau} (\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2) \big) f(r), \quad (4.5a)$$

$$f(r) = V_0\, e^{-\frac{r^2}{\xi^2}}, \quad (4.5b)$$

the matrix element $F_p^{shell}(N_b L_b L_B N_a L_a L_A)$ becomes as followes:

$$F_p^{(D)\,\mathrm{shell}}(\mathrm{Nb10\,Na01})$$

$$= -\frac{1}{2\sqrt{2}\;243\sqrt{3}\cdot 7}\left\{(-36V_W+24V_\sigma)S_1^1+(-108+72V_\sigma)P_1^1+(-180V_W+120V_\sigma)D_1^1\right\}$$

$$\times\left\{\int \psi^*_{Na0}(2\nu,\mathbf{r}_a)\,\psi_{10}(2\nu_A\cdot\mathbf{r}_a)\,d\mathbf{r}_a\int\psi^*_{00}(\tfrac{1}{2}\nu_a,\mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A,\mathbf{r})\,d\mathbf{r}\right.$$

$$\left.-\int \psi^*_{Na0}(2\nu,\mathbf{r}_a)\,\psi_{00}(2\nu_A,\mathbf{r}_a)\,d\mathbf{r}_a\int\psi^*_{00}(\tfrac{1}{2}\nu_a,\mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A,\mathbf{r})\,d\mathbf{r}\right\},$$

(4.6a)

$$F_p^{(ND)\,\mathrm{shell}}(\mathrm{Nb10\,Na01})$$

$$= -\frac{1}{2\sqrt{2}\;243\sqrt{3}\cdot 7}\left\{(18V_W+66V_\sigma-54V_T+54V_{T\sigma})S_1^1+(-27V_W-153V_\sigma-81V_T-243V_{\sigma T})P_1^1\right.$$

$$\left.+(9V_W+87V_\sigma+135V_T+513V_{\sigma T})D_1^1\right\}\cdot$$

$$\times\left\{\int \psi^*_{Na0}(2\nu,\mathbf{r}_a)\,\psi_{10}(2\nu_A,\mathbf{r}_a)\,d\mathbf{r}_a\int\psi^*_{00}(\tfrac{1}{2}\nu_a,\mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A,\mathbf{r})\,d\mathbf{r}\right.$$

$$\left.-\int \psi^*_{Na0}(2\nu,\mathbf{r}_a)\,\psi_{00}(2\nu_A,\mathbf{r}_a)\,d\mathbf{r}_a\int\psi^*_{00}(\tfrac{1}{2}\nu_a,\mathbf{r})\,\psi_{10}(\tfrac{1}{2}\nu_A\,\mathbf{r})\,d\mathbf{r}\right\},$$

(4.6b)

$$F_p^{(D)\,\mathrm{shell}}(\mathrm{Nb10\,Na21})$$

$$= \frac{1}{64\sqrt{2}\;243\sqrt{3}\cdot\sqrt{5}\sqrt{7}}\left\{(-4{,}788V_W+4{,}008V_\sigma)S_1^1+(-11{,}286V_W+2{,}196V_\sigma)P_1^1\right.$$

$$\left.+(-8{,}550V_W+10{,}212V_\sigma)D_1^1\right\}\int\psi^*_{Na2}(2\nu,\mathbf{r}_a)\,\psi_{02}(2\nu_A,\mathbf{r}_a)\,d\mathbf{r}_a\cdot\int\psi^*_{00}(\tfrac{1}{2}\nu\,\mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A,\mathbf{r})\,d\mathbf{r},$$

(4.6c)

$$F_p^{(ND)\,\mathrm{shell}}{}_{(N_b 10\; N_a 21)}$$

$$= \frac{1}{64\sqrt{2}\;243\sqrt{3}\;\sqrt{5}\cdot 7}\Big\{(-684\,V_W + 2{,}616\,V_\sigma + 4{,}320\,V_T + 16{,}416\,V_{\sigma T})\,S_1'$$

$$+ (1{,}026\,V_W - 8{,}352\,V_\sigma - 6{,}156\,V_T - 18{,}486\,V_{\sigma T})\,P_1' + (-342\,V_W + 5{,}736\,V_\sigma + 1{,}836\,V_T + 26{,}676\,V_{\sigma T})\,D_1'\Big\}$$

$$\times \int \psi^*_{Na2}(2\nu, \mathbf{r}_a)\,\psi_{02}(2\nu_A, \mathbf{r}_a)\,d\mathbf{r}_a \cdot \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\,d\mathbf{r}, \tag{4.6d}$$

$$F_p^{\mathrm{shell}}{}_{(N_b 30\; N_a 21)}$$

$$= \frac{1}{16\sqrt{2}\;81\sqrt{5}\;7\sqrt{7}}\,(50\,V_W + 602\,V_\sigma - 705\,V_T - 146\,V_{\sigma T})\,D_3'$$

$$\times \int \psi^*_{Na2}(2\nu, \mathbf{r}_a)\,\psi_{02}(2\nu_A, \mathbf{r}_a)\,d\mathbf{r}_a \cdot \int \psi^*_{80}(\tfrac{1}{2}\nu_d, \mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\,d\mathbf{r}, \tag{4.6e}$$

$$F_p^{(D)\,\mathrm{shell}}{}_{(N_b 10\; N_a 22)}$$

$$= \frac{1}{64\sqrt{2}\;243\sqrt{3}\;5\sqrt{5}\sqrt{7}}\Big\{(4{,}500\,V_W - 2{,}760\,V_\sigma)\,S_1'$$

$$+ (12{,}150\,V_W - 6{,}660\,V_\sigma)\,P_1' + (15{,}750\,V_W - 12{,}180\,V_\sigma)\,D_1'\Big\}$$

$$\times \int \psi^*_{Na2}(2\nu, \mathbf{r}_a)\,\psi_{02}(2\nu_A, \mathbf{r}_a)\,d\mathbf{r}_a \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\,\psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\,d\mathbf{r} \tag{4.6f}$$

$$F_p^{(ND)\,\mathrm{shell}}{}_{(N_b 10\; N_a 22)}$$

$$= \frac{1}{64\sqrt{2}\;243\sqrt{3}\;5\sqrt{5}\sqrt{7}}\Big\{(-900\,V_W + 2{,}040\,V_\sigma - 6{,}480\,V_T - 10{,}800\,V_{\sigma T})\,S_1'$$

$$+ (1{,}350\, V_W + 14{,}760\, V_\sigma + 8{,}100\, V_T + 24{,}300\, V_{\sigma T})\, P_1^1$$

$$+ (-450\, V_W - 16{,}800\, V_\sigma - 1{,}620\, V_T - 45{,}900\, V_{\sigma T})\, D_1^1 \Big\}$$

$$\times \int \psi^*_{N_a 2}(2\nu, \mathbf{r}_a)\, \psi_{02}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r},$$

(4.6g)

$$F_p^{\text{shell}}(N_b 3 0 N_a 2 2) = \frac{1}{16\sqrt{2}\; 27\sqrt{3}\; 5\sqrt{5}\cdot 49} (3{,}450\, V_W - 1{,}770\, V_\sigma - 5{,}130\, V_T - 10{,}350\, V_{\sigma T})\, D_3^1$$

$$\times \int \psi^*_{N_a 2}(2\nu, \mathbf{r}_a)\, \psi_{02}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A\, \mathbf{r})\, d\mathbf{r},$$

(4.6h)

$$F_s^{\text{shell}}(N_b 1 0 N_a 0 1) = (-)\frac{\sqrt{2}\, V_W}{3\sqrt{3}\cdot 7}\, P_1^0 \Big\{ \int \psi^*_{N_a 0}(2\nu, \mathbf{r}_a)\, \psi_{10}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})$$

$$\psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r}) \int \psi^*_{N_a 0}(2\nu, \mathbf{r}_a)\, \psi_{00}(2\nu_A, \mathbf{r})\, d\mathbf{r}_a \cdot \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r} \Big\}$$

(4.6i)

$$F_s^{\text{shell}}(N_b 1 0 N_a 2 1) = \frac{(-)\, 19\, V_W}{4\sqrt{2}\; 3\sqrt{3}\sqrt{5}\cdot 7}\, P_1^0 \int \psi^*_{N_a 2}(2\nu, \mathbf{r}_a)\, \psi_{02}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r}$$

(4.6j)

$$F^{\text{shell}}(N_b 1 0 N_a 2 2) = \frac{\sqrt{5}\, V_W}{4\sqrt{2}\; 3\sqrt{3}\; \sqrt{7}}\, P_1^0 \int \psi^*_{N_a 2}(2\nu, \mathbf{r}_a)\, \psi_{02}(2\nu_A, \mathbf{r}_a)\, d\mathbf{r}_a \int \psi^*_{00}(\tfrac{1}{2}\nu_d, \mathbf{r})\, \psi_{00}(\tfrac{1}{2}\nu_A, \mathbf{r})\, d\mathbf{r},$$

(4.6k)

where we use the notation for the two-body matrix element as follows

$$L_m^n = \langle [\psi_{N_b m}(\nu, \mathbf{r}_1)\, \psi_{0n}(\nu_A, \mathbf{r}_2)]_L \,|\, f(r_{12}) \,|\, [\psi_{01}(\nu_A, \mathbf{r}_1)\, \psi_{0n}(\nu_A, \mathbf{r}_2)]_L \rangle.$$

These equations are the explicit expression of Eq. (3.42) for the reaction $^{11}B(d,n)^{12}C$. If we put $V_{sb}=V_{pb}=1$, $\nu_d=\nu_A=\nu$, we obtain the non-vanishing overlap integrals. From Eqs. (4.6a), (4.6b) and (4.6i)

$$F_p^{shell}(010101)=F_s^{shell}(010101)=(-)\frac{\sqrt{2}}{3\sqrt{3}\cdot 7}=(-)3888 \times 10^{-5},$$

from Eqs.(4.6c), (4.6d)and (4.6j)

$$F_p^{shell}(101021)=F_s^{shell}(101021)=(-)\frac{19}{4\sqrt{2}\,3\sqrt{3}\,\sqrt{5}\,7}=(-)\cdot 4129 \times 10^{-5},$$

from Eq. (4.6f) (4.6g) and (4.6k)

$$F_p^{shell}(101022)=F_s^{shell}(101022)=\frac{5}{4\sqrt{2}\,3\sqrt{3}\,\sqrt{7}}=2875 \times 10^{-5}.$$

We give the parameters used for the calculation in Table 4. Table 5 shows the values of V_w, V_σ, V_τ and $V_{\sigma\tau}$ for the four types of the interaction.

4.2 Discussion about the matrix elements in the form factor and the cross section

In table 6, we give the numerical results of the matrix elements $6F_p^{shell}(N_bL_bL_BN_aL_aL_A) + 4F_s^{shell}(N_bL_bL_BN_aL_aL_A)$, for four types of the nuclear force such as Wigner, Serber, Rosenfeld and Gillet types. The matrix elements $F_p^{(D)shell}(N_bL_bL_BN_aL_aL_A)$ for the Rosenfeld interaction vanish, although the matrix elements $F_p^{shell}(N_bL_bL_BN_aL_aL_A)$ have the same trend for four types of the interaction. To see the damping of the matrix elements $F^{shell}(N_bL_bL_BN_aL_aL_A)$ with the increase of the node N_b, we show the feature of $F^{shell}(N_bL_bL_BN_aL_aL_A)$ for the Rosenfeld force by the solid line of Fig. 6. We also calculated the matrix elements for $\lambda=0.75$ and 0.80. The results obtained for these values of λ are similar to those for λ equal to 0.7.

From Table 6, the matrix elements $4F_s(N_bL_bL_BN_aL_aL_A)$ are of the same order as the matrix elements $6F_p(N_bL_bL_BN_aL_aL_A)$ in the case of the Wigner and Serber forces. But in our calculations, this S shell effect is considered to be small implicitly. If the effect of nucleons in the S shell is large, we must take into account the neutron going out from S shell in addition to from P-shell. In other wards we must take as the antisymmetric wave functions for the description of the target ^{11}B(antisymmetric wave function of four nucleons in the S shell and seven nucleons in the P shell) in stead of (antisymmetric wave function of four nucleons in the S shell) x (antisymmetric wave function of seven nucleons in the P shell).

The contribution of the S shell in the case of the Rosenfeld type happens to vanish due to the fact that it has only V_w term as can be seen from Eqs. (4.6i) (4.6j) and (4.6k). We compare the result for the Rosenfeld interaction with that of the macroscopic calculation.

We show the results of the microscopic and the macroscopic calculation in Fig. 8. The result of the microscopic calculation with the reasonable strength of the two-body Rosenfeld interaction is larger than that of the macroscopic calculation by the factor 10. But it is much smaller than that of the usual stripping calculation. The orders of the experimental results are few milli-barns and the results of the usual stripping calculations are of the same order as the experimental results. We can conclude that even if we calculate the heavy particle stripping term microscopically, the heavy particle stripping contribution on the reaction of $^{11}B(d,n)^{12}C$ is very small.

We also find the following results. The cross section of the capture of the deuteron into S state is as large as that into D state. The contribution of the D state in the wave function of the ^{11}B is very small as in the case of the macroscopic calculation. This result is due to the recoupling factors of Eq. (3.16) in spite of the fairly large matrix elements shown in Table 6.

4.3 Recoil effect

To obtain $F(N_bL_bL_BN_aL_aL_A)$, we solve Eq. (3.23) by using the electric computer HITAC-5020. The results are shown by the dotted line in Fig. 6 for the Rosenfeld force. In Fig. 6, we show only the matrix elements $F(N_bL_bL_BN_aL_aL_A)$. But in addition to the above matrix elements, many matrix elements with the same order of values appear, for example, about 100 matrix elements in the case L_A=1. The main contributions of these matrix elements for the Rosenfeld force are shown in Fig. 7. These matrix elements play an important role to affect on the pattern of the angular distribution of the cross section. In Fig. 9, we show the cross sections calculated with and without the recoil effect. In the macroscopic calculation without the recoil effect, the form factor is as follows

$$f_{\ell m}^{(bC)}(\mathbf{r}_{bB}, \mathbf{r}_{aA}) = \sum_{\mu_a \mu_b} (l_a l_b \mu_a - \mu_b | \ell m)(-)^{\mu_b} \mathcal{Y}_{l_a}(r_{aA}) Y^*_{l_a \mu_a}(\hat{\mathbf{r}}_{aA}) V(r_{bB}) \mathcal{Y}_{l_b}(r_{bB}) Y_{l_b \mu_b}(\hat{\mathbf{r}}_{bB})$$

in stead of Eq. (2.35e).

We calculated the matrix elements for Eq. (3.23a) and obtained nearly the same values as the exact solutions concerning the matrix elements $F(N_b L_b L_B N_a L_a L_A)$ having the same quantum numbers as the non-vanishing matrix elements $F^{shell}(N_b L_b L_B N_a L_a L_A)$. But we can obtain no matrix elements having other quantum numbers by solving Eq. (3.23a).

From this discussion, we may say the following conclusions about the recoil effect.

a. The recoil effect makes the value of the cross section large by the factor about $\left(\frac{a+A}{A}\right) \times \left(\frac{b+B}{B}\right) = 1.5$ (in this case) because the main contribution to the cross section comes from the matrix element F(010021). This can be seen from Fig. 9.

b. The exact recoil effect change the angular pattern because of the large contribution of the matrix elements such as $F(N_b 0011)$ in Fig. 7.

5. Concluding remarks

The main work of this paper is to develop the method of the DWBA microscopic calculation. We remark as follows:

Application:

a. This method can be conveniently applied to the investigation about the term which is assumed to be zero in the usual DWBA calculation and about the heavy particle stripping term.

b. This method can also be applied to the microscopic calculation on the reaction (t,p) and (d,t)etc.

Problem of the convergence:

c. We find that , in the numerical calculation on $^{11}B(d,n)^{12}C$, the matrix elements in the form factor decrease very rapidly with the increase of the nodes of the harmonic oscillator wave functions used for the expansion of the form factor.

Recoil effect:

d. This method is very convenient to include the recoil effect in the microscopic calculation. This effect makes the magnitude of the cross section large, and varies the angular pattern.

Cross section of the heavy particle strippin process on $^{11}B(d,n)^{12}C$:

e. The value of the cross section of the heavy particle stripping process is much smaller than that of the usual stripping process. From Fig. 2, the usual stripping calculations with more reasonable optical parameters may explain the backward peak of the experimental results due to the distortion effect.

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Table 4 Parameter of the harmonic oscillator wave function

Symbol	Meaning	Value	Validity of the value of the parameter
	the force range of the Gaussian interaction	2.0fm	
νA	the parameter of the bound harmonic oscillator wave function of P-shell nucleon in ^{12}C and ^{11}B	$0.24fm^{-2}$	From the Kuruth's calculation(13) about the energy of P-shell nuclei, the range of the parameter λ extends from 0.69 to 0.77. In our calculation, we take λ = 0.7 so $\nu_A = \frac{2\lambda^2}{\xi^2} = 0.24$
νd	the parameter of the bound state harmonic oscillator wave function describing the internal bound state of the deuteron	$0.24fm^{-2}$	C. L. Lin and S. Yoshida have used the parameter $\nu_d=0.213fm^{-2}$ from the electron scattering. But to simplify our calculation, we have taken $\nu_d=0.24fm^{-2}$
ν	the parameter of the harmonic oscillator wave function used to expand the form factor	$0.24fm^{-2}$	To simplify the calculation we have taken $\nu = \nu_A$

Table 5 Type of the force used for our calculation

	v_W	v_σ	v_τ	$v_{\sigma\tau}$
Wigner	1.0			
Serber	3/8	-1/8	-1/8	-1/8
Rosenfeld			-1/10	-7/30
Gillet		-1/5	-3/10	-1/10

Table 6 Value of the expansion coefficient of the form factor

	N_b	Wigner	Serber	Rosenfeld	Gillet
$6F_p^{(D)}(N_b10101)$	0	-2270	-1040	0	-302.7
	1	-858.2	-393.4	0	-114.4
	2	-243.0	-104.0	0	-30.27
	3	-70.44	-141.7	0	-4.006
	4	+13.09	+4.708	0	+1.742
	5	+14.55	+6.664	0	+1.939
$6F_p^{(ND)}(N_b10101)$	0	164.9	-462.2	-798.8	-888.6
	1	18.22	-170.4	-229.9	-219.6
	2	22.68	-3.808	-15.93	-14.33
	3	-21.98	19.26	28.89	51.88
	4	-13.63	16.40	25.31	38.51
	5	-7.124	9.536	15.02	21.66
$6F_p(N_b10101)$	0	-2105	-1502	-798.8	-1191
	1	-840.0	-563.8	-229.9	-334.0
	2	-1.616	-107.9	-15.93	-15.93
	3	-92.43	5.484	28.89	47.87
	4	-0.537	21.10	25.31	40.24
	5	7.430	16.19	15.02	23.60

	N_b	Wigner	Serber	Rosenfeld	Gillet
$6F_p^{(D)}(N_b10021)$	0	-2412	-1739	0	-377.5
	1	-91.7	-413.6	0	-127.7
	2	-241.0	-105.7	0	-24.46
	3	-31.93	-9.971	0	3.2
	4	13.88	9.263	0	6.473
	5	15.44	8.589	0	4.474
$6F_p^{(ND)}(N_b10021)$	0	-87.63	-718.5	-1087	-711.1
	1	-9.68	-159.8	-270.6	-143.2
	2	12.04	30.37	15.79	40.29
	3	11.67	51.68	62.40	59.28
	4	6.346	36.23	46.54	40.12
	5	3.783	19.96	26.24	21.79
$6F_p(N_b10021)$	0	-2499	-1857	-1087	-1088
	1	-921.3	-573.4	-270.6	-270.9
	2	-228.9	-75.33	15.71	15.83
	3	-20.26	41.71	62.40	62.48
	4	20.22	45.49	46.54	46.59
	5	19.22	28.55	26.24	26.26
$6F_p(N_b30021)$	0	-16.16	-16.12	-33.81	-34.17
	1	-11.51	-11.48	-24.07	-24.33
	2	-6.401	-6.385	-13.38	-13.53
	3	-3.163	-3.156	-6.616	-6.688
	4	-1.453	-1.450	-3.640	-3.[illegible]73
	5	-0.6365	-0.6349	-1.331	-1.345

	N_b	Wigner	Serber	Rosenfeld	Gillet
$6F_p^{(D)}(N_b10022)$	0	1679	772.4	0	228.6
	1	634.6	291.2	0	85.13
	2	167.9	76.52	0	21.70
	3	22.23	9.772	0	2.313
	4	-9.669	-4.684	0	1.691
	5	-10.74	-5.063	0	-1.644
$6F_p^{(ND)}(N_b10022)$	0	-61.02	399.9	679.5	445.1
	1	-6.732	100.3	170.8	94.21
	2	8.001	-5.15	0.086	-21.34
	3	8.130	-22.55	-32.39	-34.72
	4	7.508	-16.91	-9.341	-23.88
	5	2.634	-9.553	-8.509	-13.04
$6F_p(N_b10022)$	0	1617	1172	679.5	673.7
	1	627.8	391.5	170.8	179.3
	2	175.9	71.37	0.086	0.36
	3	30.36	-12.77	-32.39	-32.40
	4	-2.161	-21.59	-9.341	-22.18
	5	-8.106	-14.61	-8.509	-14.68
$6F_p(N_b30022)$	0	-145.9	-145.9	-123.89	-123.89
	1	-103.9	-103.9	-88.20	-88.20
	2	-57.79	-57.79	-49.05	-49.05
	3	-28.56	-28.56	-24.24	-24.24
	4	-13.12	-13.12	-11.14	-11.14
	5	-5.747	-5.747	-4.878	-4.878

	N_b	Wigner	Serber		N_b	Wigner	Serber
$4F_s(N_b10101)$	0	-1948	-730.7	$4F_s(N_b10101)$ + $6F_p(N_b10101)$	0	4218	-1770
	1	-1033	-387.7		1	-1891	-781.8
	2	-458.9	-121.1		2	-701.9	-225.1
	3	-188.4	-70.68		3	-258.8	-212.3
	4	-74.21	-27.83		4	-87.30	-23.12
	5	-28.38	-10.64		5	-42.93	-3.976
$4F_s(N_b10021)$	0	-1749	-655.9	$4F_s(N_b10021)$ + $6F_p(N_b10021)$	0	-4248	-1588
	1	-928.0	-348.0		1	-1849	-586.6
	2	-411.9	-154.4		2	-640.8	-155.1
	3	-169.1	-63.44		3	-189.3	-18.68
	4	-66.61	-24.98		4	-46.39	-10/36
	5	-25.47	-9.553		5	-6.26	-10.75
$4F_s(N_b10022)$	0	1751	540.2	$4F_s(N_b10022)$ + $6F_p(N_b10022)$	0	3368	1712
	1	764.5	286.6		1	1392	678.1
	2	339.3	172.2		2	515.2	198.5
	3	139.3	52.26		3	169.6	39.49
	4	54.78	20.57		4	52.70	-1.02
	5	20.98	7.869		5	12.87	-6.741

Fig. 6 Value of $F(NbLbL_B NaLaL_A) = 6F_p(NbLbL_B NaLaL_A) + 4Fs(NbLbL_B NaLaL_A)$

Rosenfeld Force

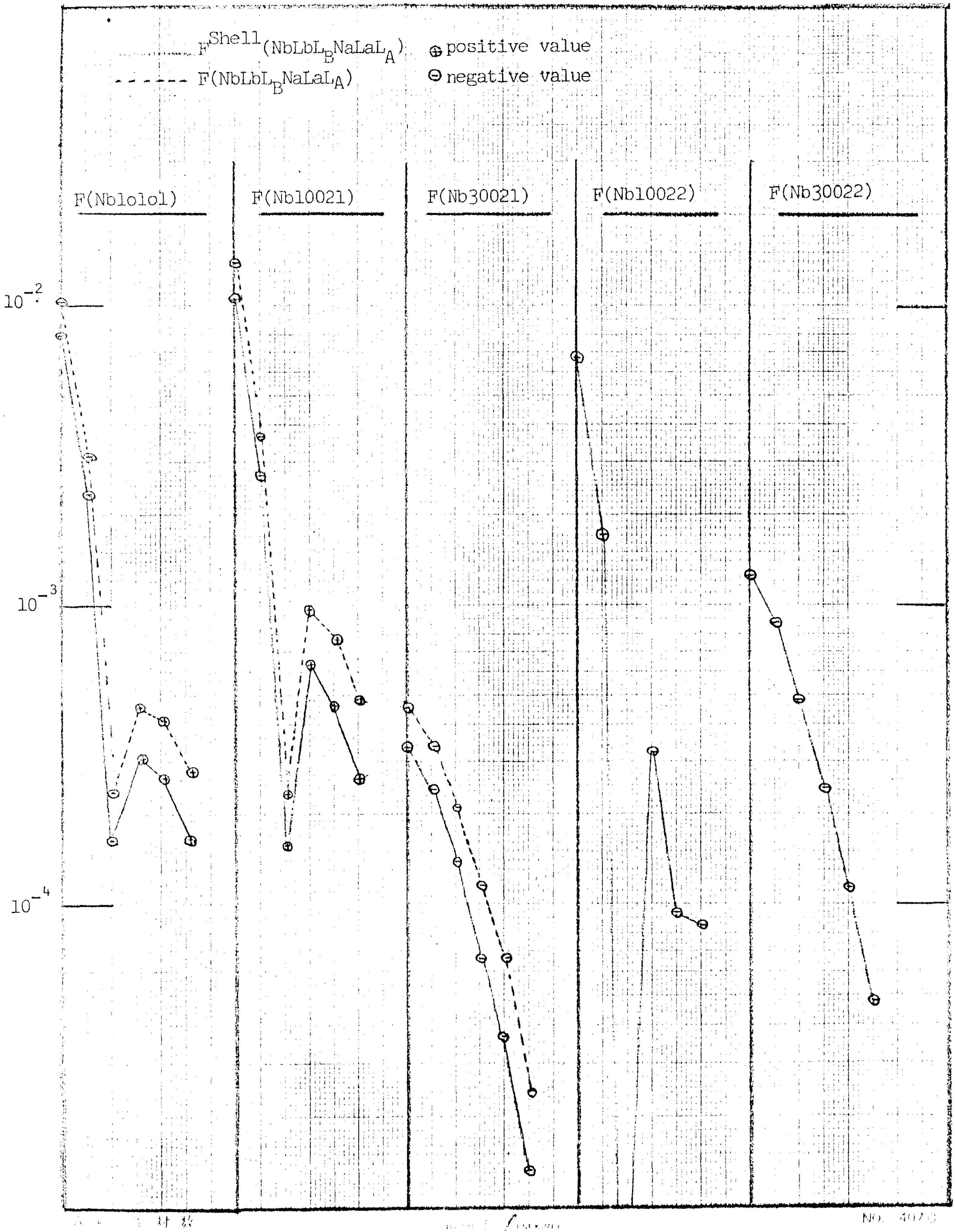


Fig. 7 Value of $F(N_bL_bL_BN_aL_aL_A)=6F_p(N_bL_bL_BN_aL_aL_A)+4F_s(N_bL_bL_BN_aL_aL_A)$ appearing by solving Eq. (3.23). Rosenfeld force

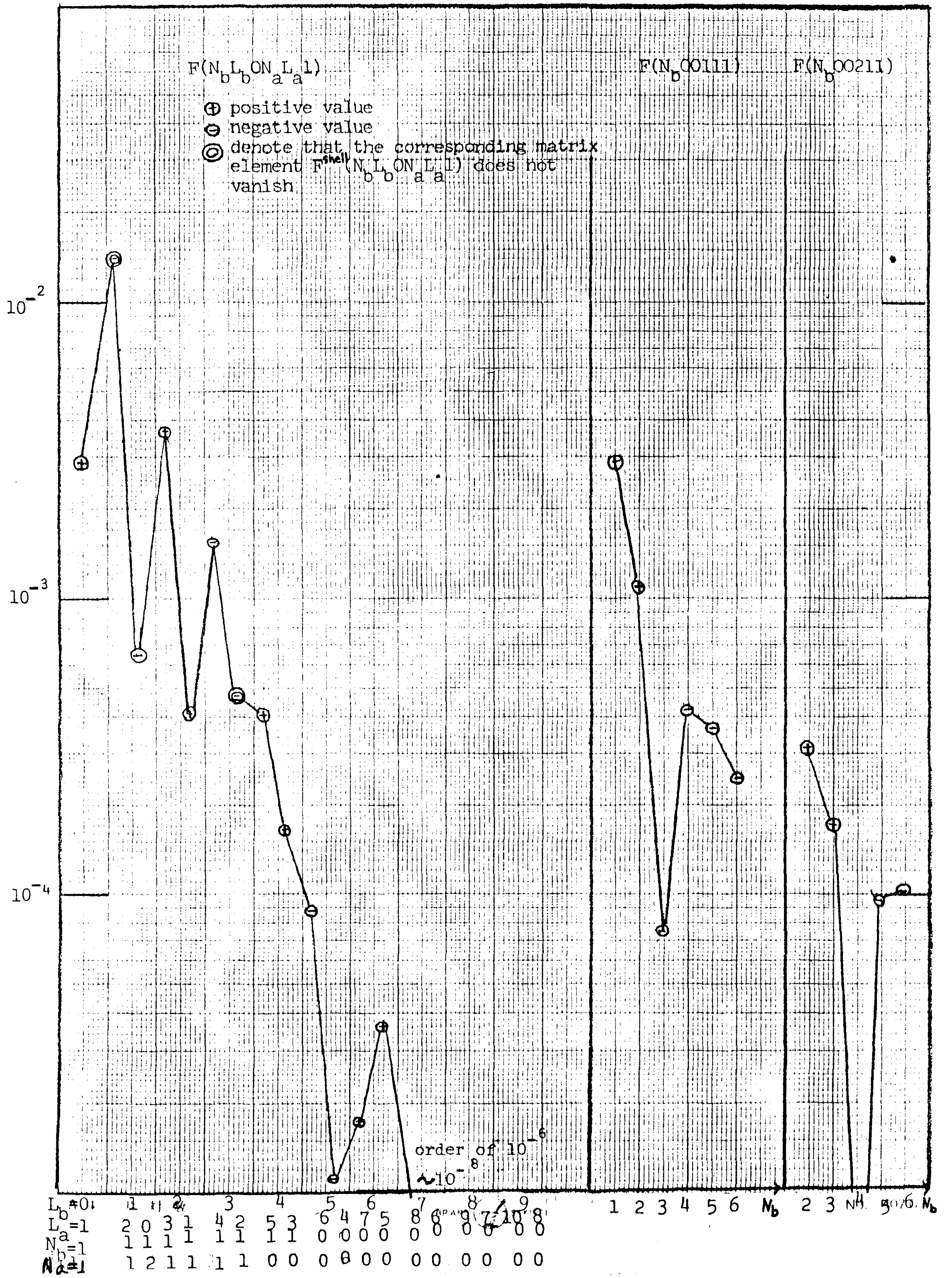


Fig. 8 Comparison between macroscopic and microscopic results

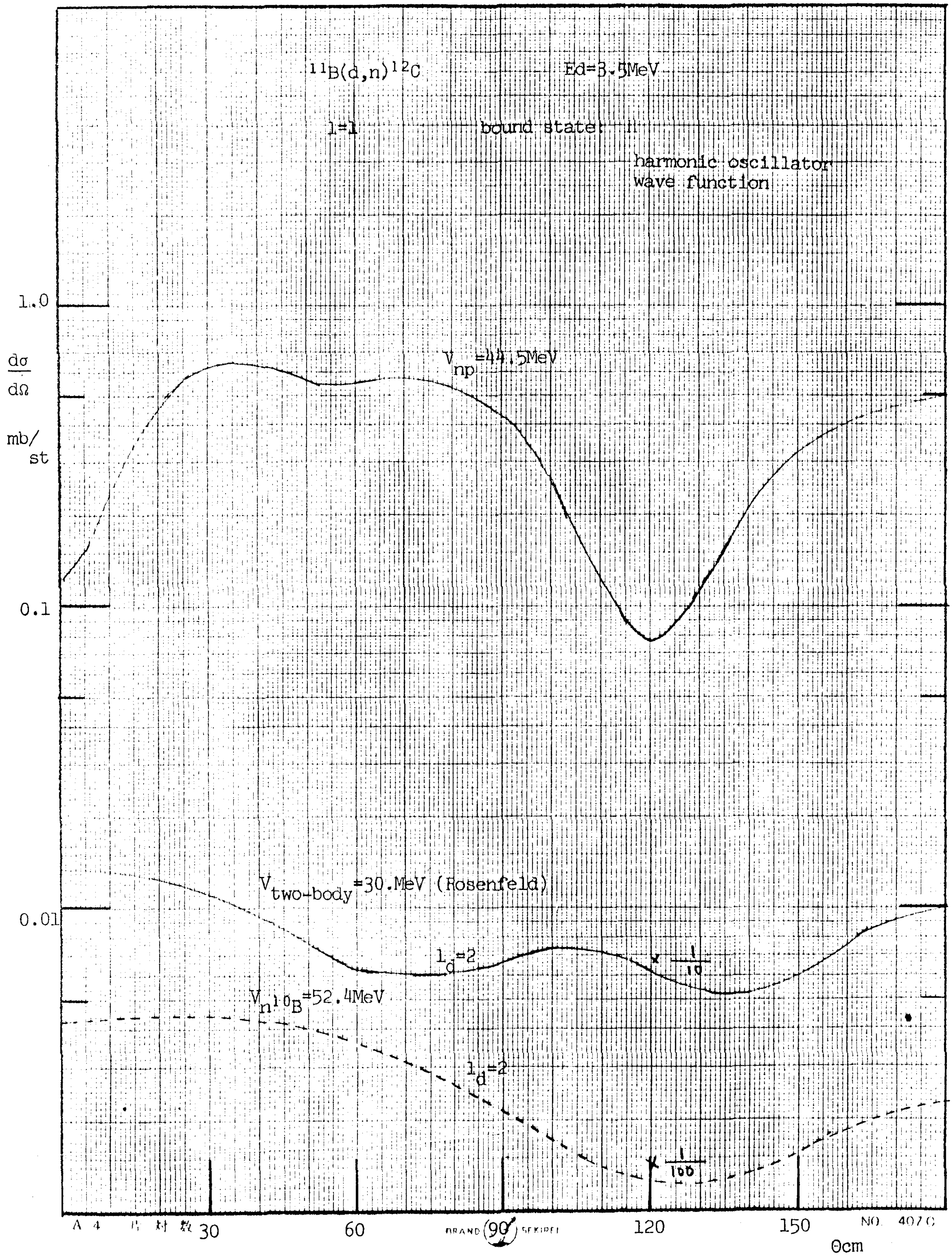
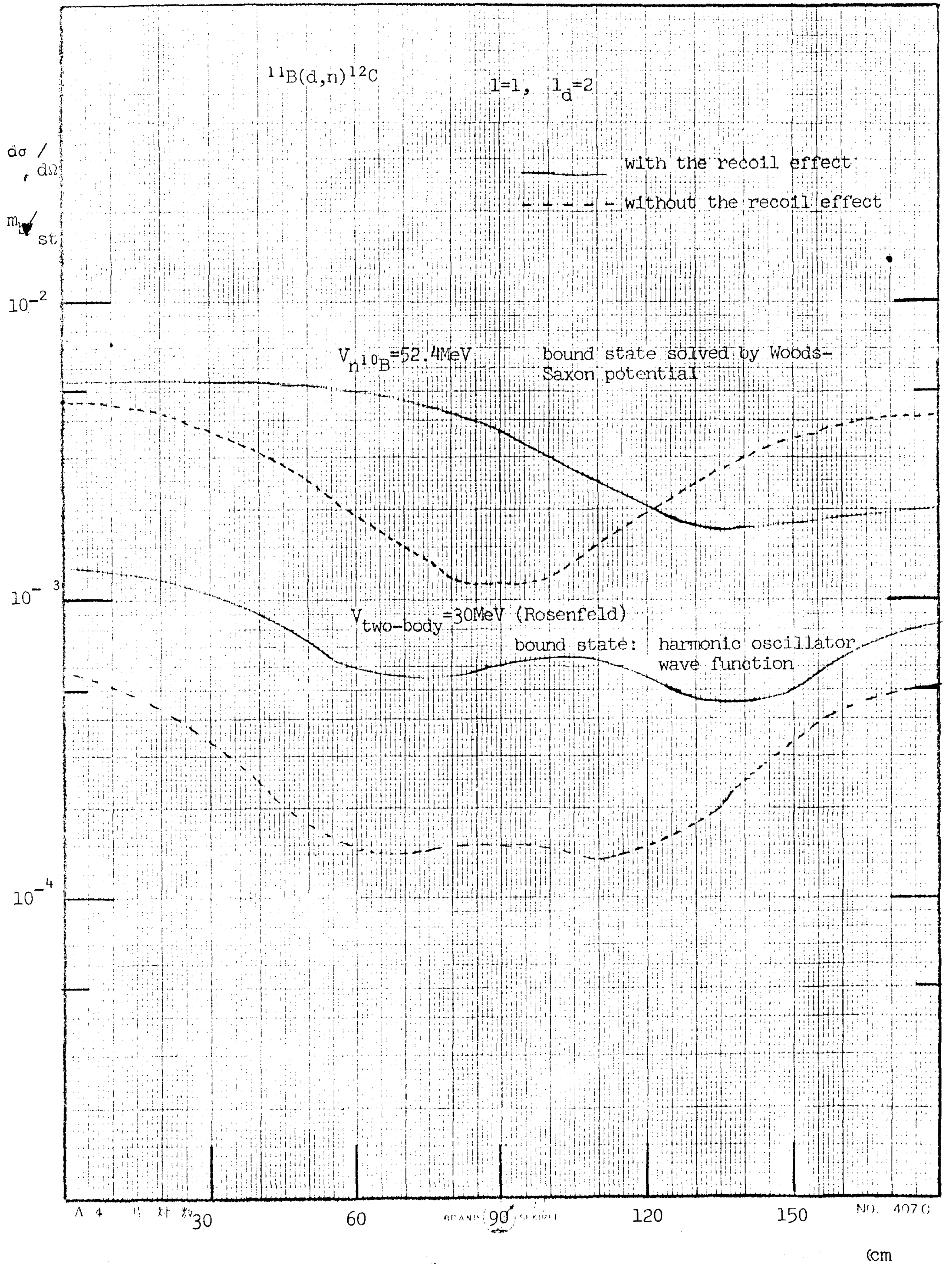


Fig. 9 Recoil effect



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