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Osaka University
The method of the microscopic calculation
of the cross section in the distorted wave Born approximation

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Abstract:

We develop a new method for the calculation of the cross section in the distorted wave Born approximation (DWBA). The form factor in the transition matrix element is expanded with harmonic oscillator wave functions. This method is convenient not only for the microscopic calculation using shell model wave functions, but also for the study of the recoil effect of the target nucleus. This can be conveniently applied to the investigation about the term which is assumed to be zero in the usual DWBA calculation and about the heavy particle stripping term.

The numerical calculations are performed for the heavy particle stripping term on the reaction $^{11}\text{B}(d,n)^{12}\text{C}$. We found that the damping of the matrix elements with increase of the nodes of harmonic oscillator wave functions is very rapid. We also found that the recoil effect plays an important role about the angular distribution of the calculated cross section. The result obtained using the reasonable interaction is much smaller than that of the usual stripping process. We can conclude that even if we calculate the heavy particle stripping term microscopically, the heavy particle stripping contribution on the reaction of $^{11}\text{B}(d,n)^{12}\text{C}$ is very small contrary to the result of Owen and Madansky.
1. Introduction

The theory of the deuteron stripping reaction given by Butler\(^1\) has had remarkable success in explaining forward peaks of angular distributions. For the stripping reaction \(A(a,b)B\), the main feature of the theory is shown in Fig. 1a.

![Fig. 1a](image)

This reaction is induced by the interaction between the particles \(b\) and \(x\) written with the solid line in the final channel, or by the interaction between the particle \(x\) and \(A\) written with the dotted line in the initial channel. However, some experiments extending the measurement of the angular distribution to the backward angle show a large component in the backward direction. To explain these results, Owen, Madansky\(^2\) and others\(^3\) have taken into account of the heavy particle stripping mechanism in addition to the usual stripping process. The feature of the heavy particle stripping mechanism is shown in Fig. 1b.

![Fig. 1b](image)

This reaction is induced by the interaction between the particles \(b\) and \(C\) written with the solid line in the final channel, or by the interaction between the particles \(a\) and \(C\) written with the dotted line in the initial channel. Owen and
Madansky calculated the cross sections of the reaction $^{11}\text{B}(\text{d,n})^{12}\text{C}$ in the plane wave Born approximation and obtained fairly good fits to the experimental results shown in Fig. 2. But to obtain good results, they had to use the parameter $\Lambda_2/\Lambda_1$ describing how much the heavy particle stripping process contributes to the cross sections.

On the other hand, the analysis of deuteron stripping reactions using the distorted wave Born approximation (DWBA) have obtained more remarkable success than the analysis using the plane wave Born approximation and have explained experimental results not only qualitatively but also quantitatively. In spite of such brilliant success of stripping reaction using DWBA, there have been only a few calculations of the heavy particle stripping mechanism using DWBA. The reasons are as follows:

a) In D.W.B.A. calculations, distortion effects reduce the heavy particle stripping cross section.

b) In the actual calculations of the heavy particle stripping term, if we use the plane wave Born approximation, the six-dimensional integration of the transition matrix can be factorized into the product of two three-dimensional integrations. But if we use D.W.B.A., this factorization can't be done. Furthermore, the calculation of the heavy particle stripping term is more laborious than that of the usual stripping owing to the complicated form factor of the heavy particle stripping term.

Almost all the calculations done up to now are restricted by the assumption that $^{10}\text{B}$ in the calculation of the heavy particle stripping term on the reaction $^{11}\text{B}(\text{d,n})^{12}\text{C}$ is a single cluster and that interaction inducing reactions only depend on the relative distance between the center-of-mass of $^{10}\text{B}$ and the neutron. We call the calculation made by the above assumption as the macroscopic calculation, while we call the calculation without the above assumption, that is, considering $^{10}\text{B}$ as not a single cluster but assembly of nucleons as the microscopic calculation.
At the stage in the development of the study of the heavy particle stripping term, it is appropriate to present calculations of it using DWBA as consistently as possible. We develop a new method for the calculation of the cross section for this purpose. In §2, we give a treatment of the heavy particle stripping term introducing the discussion of Rodberg, and also give our numerical calculation on the reaction $^{11}$B($d,n)^{12}$C which Owen and Madansky have treated. We give a description of the cross section using form factors expanded with harmonic oscillator wave functions in §3. This method is applied to the calculation of the heavy particle stripping term on the reaction $^{11}$B($d,n)^{12}$C. Results and discussions are given in §4. Concluding remarks are given in §5.

2. Treatment of the heavy particle stripping process

2.1 Derivation of $T^{DWBA}$ matrix

The reaction

$$A(a,b)B$$ (2.1)

is considered. We write the Hamiltonian of the total system $H$ as

$$H_a = H_a + K_a + V_{aA},$$ (2.2a)

$$H_b = H_b + K_b + V_{bB},$$ (2.2b)

where $H_a$ is the intrinsic Hamiltonian of the particle $a$, $K_{aA}$ the kinetic energy operator between particles $a$ and $A$, and $V_{aA}$ the interaction between particles $a$ and $A$. We define the intrinsic state of the particle $a$ by $|\Phi_a\rangle$, the intrinsic state of the nucleus $A$ by $|\Phi_A\rangle$ and the plane wave of the relative motion between particles $a$ and $A$ by $|\psi\rangle$. If we write the total energy by $E$, we obtain

$$\langle H_a + K_{aA} - E \rangle \Phi_a |\Phi_A\rangle = 0,$$ (2.3a)

$$\langle H_b + K_{bB} - E \rangle \Phi_b |\psi\rangle = 0.$$ (2.3b)
The transition matrix $T_{ba}$ can be written as

$$T_{ba} = \langle \phi_b \Phi_b \Psi_B | V_{bb} | \Psi_B \Phi_A \phi_A \rangle$$

where

$$\Omega_a^{(\psi)} = 1 + \frac{1}{E - H + i\epsilon} V_a A,$$

$$\Omega_a^{(+)} = \Omega_a^{(\psi)} | \phi_a \Phi_a \Psi_A \rangle,$$

We define the operator $\omega_a^{(+)}$ and the distorted wave $\chi_a^{(+)}$ as

$$\omega_a^{(+)} = 1 + \frac{1}{E - H_a - H_b - K_{bb} - U_{bb} + i\epsilon} U_{bb}^{-1},$$

$$\chi_a^{(+)} = \omega_a^{(+)} | \phi_a \Phi_a \Psi_A \rangle,$$

$$\omega_b^{(+)} = 1 + \frac{1}{E - H_b - H_b - K_{bb} - U_{bb} + i\epsilon} U_{bb}^{-1},$$

$$\chi_b^{(+)} = \omega_b^{(+)} | \phi_b \Phi_b \Psi_B \rangle.$$

From Eq. (2.7a), we obtain the equality as

$$V_{bb} \Omega_a^{(+)} = (\omega_b^{(\ast)} (V_{bb} - U_{bb}) + V_{bb} (1 - \frac{1}{E - H_b - H_b - K_{bb} - U_{bb} + i\epsilon} (V_{bb} - U_{bb})) \Omega_a^{(\psi)}$$

If we use the equality

$$\frac{1}{E - H_b - H_b - K_{bb} - U_{bb} + i\epsilon} - \frac{1}{E - H + i\epsilon} = -\frac{1}{E - H - H_b - K_{bb} - U_{bb} + i\epsilon} (V_{bb} - U_{bb}) \frac{1}{E - H + i\epsilon},$$

the second term of Eq. (2.8) becomes

$$V_{bb} (1 - \frac{1}{E - H_b - H_b - K_{bb} - U_{bb} + i\epsilon} (V_{bb} - U_{bb})) \Omega_a^{(+)}$$

$$= V_{bb} \frac{1}{E - H_b - H_b - K_{bb} - U_{bb} + i\epsilon} (E - H + i\epsilon) (1 + \frac{1}{E - H + i\epsilon} V_a A)$$

$$= V_{bb} \frac{1}{E - H_b - H_b - K_{bb} - U_{bb} + i\epsilon} (E - H_a - H_b - K_{bb} + i\epsilon)$$

$$= (\omega_b^{(\ast)} - 1) \epsilon.$$
Under the condition
\[ \lim_{\epsilon \to 0} \epsilon (\omega_{0}^{*} - 1) = 0, \]  
we obtain \( T_{ba} \) as
\[ T_{ba} = \langle \chi_{b}^{a} | (V_{bb} - U_{bb}) | \Phi_{a}^{0} \rangle. \]  

If we use the relation obtained from Eqs. (2.5a) and (2.6a)
\[ \Omega_{a}^{(+)} = (1 + \frac{1}{E - H + U_{a}}) \Omega_{a}^{(+)} \]  
we obtain the amplitude of DWBA as
\[ T_{ba}^{DWBA} = \langle \chi_{b}^{a} | (V_{bb} - U_{bb}) | \Phi_{a}^{0} \rangle. \]  

2.2 Exchange effect

We follow the antisymmetric treatment of the transition matrix made by Tobocman \(^6\). The state \( \Phi_{a}^{(+)} \) is the solution of the total Hamiltonian with the boundary condition that in the channel a there are the incoming wave and the outgoing scattered wave and in other open channels b there are outgoing scattered waves. We can write the asymptotic behavior of \( \Phi_{a}^{(+)} \) using transition matrices \( T_{ba} \) in Eq. (2.4) as
\[ \tilde{\Phi}_{a}^{(+)} = \Phi_{a}^{0} - \sum_{b} T_{ba} \frac{\mu_{b}}{2\pi \hbar^{2}} \Phi_{b}^{0} \frac{e^{ik_{b}r_{b}}}{T_{sb}}, \]  

where \( \mu_{b} \) is the reduced mass of the channel b. This is not antisymmetric wave function. From Eq. (2.15), we obtain the antisymmetric solution of the total Hamiltonian \( H \) as
\[ \tilde{\Phi}_{a}^{(+)} = N^{-\frac{1}{2}} \sum_{\kappa} \epsilon_{n} p_{n} \Phi_{a}^{(+)} \]  

\[ \]
where $N$ is the number of identical nucleons, $P_n$ is the permutation operator and $\epsilon_n$ is its parity. The sum over $n$ extends all possible permutations of $N$ particles. The asymptotic behavior of Eq. (2.16) is

$$\Phi_n \sim \Phi_n N^{-\frac{1}{2}} \sum_n \epsilon_n P_n \left( \Phi_a \Phi_b \right)$$

$$= \sum_n N^{-\frac{1}{2}} \epsilon_n T_{ba} \frac{\mu_b}{2\pi R} P_n \left( \Phi_a \Phi_b \right) \left( \frac{e^{ik_{b}\Phi B}}{R_{b}} \right)$$

Since the sum over $b$ includes all possible permutations, we can replace $b$ by $P_{n_{b}}^{-1}$ in the summand for each value of $n$. Thus

$$\Phi_n \sim \Phi_n N^{-\frac{1}{2}} \sum_n \epsilon_n P_n \left( \Phi_a \Phi_b \right) - \sum_{n_{b}} T_{ba} \frac{\mu_b}{2\pi R} \Phi_b \Phi_B \left( \frac{e^{ik_{b}\Phi B}}{R_{b}} \right)$$

$$= \Phi_n N^{-\frac{1}{2}} \sum_n \epsilon_n P_n \left( \Phi_a \Phi_b \right) - \sum_{n_{b}} T_{b\alpha} \frac{\mu_b}{2\pi R} \Phi_b \Phi_B \left( \frac{e^{ik_{b}\Phi B}}{R_{b}} \right)$$

(2.18)

where

$$T_{b\alpha} = \sum_n N^{-\frac{1}{2}} \epsilon_n T_{bR\alpha}.$$  

(2.19)

The differential cross section $\frac{d\sigma_{\alpha}}{d\Omega}$ can be written as

$$\frac{d\sigma_{\alpha}}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi \hbar)^2} \frac{k_{b}}{k_{a}} \sum_{n} \left| T_{bR\alpha} \right|^2 = \frac{\mu_a \mu_b}{(2\pi \hbar)^2} \frac{k_{b}}{k_{a}} \sum_{n} \left| T_{bR\alpha} \right|^2$$

$$= \frac{\mu_a \mu_b}{(2\pi \hbar)^2} \frac{k_{b}}{k_{a}} \sum_{n} \epsilon_n T_{bR\alpha}$$

(2.20)

In Eq. (2.20), we sum the cross section over $P_{n_{b}}$ since we cannot distinguish between $\Phi_b \Phi_B$ and $P_n \left( \Phi_b \Phi_B \right)$.

2.3 Application to (d,n) (or (d,p)) reaction

Tobocman gave explicit expression for the (d,p) reaction antisymmetrizing protons and neutrons separately. Here we give explicit expression for the (d,n)
reaction treating protons and neutrons as the same for the following calculations. For simplicity, we write the operator $\sum_n \varepsilon_n P_n$ as

$$\mathcal{P} = \sum_n \varepsilon_n P_n. \quad (2.21)$$

Then we may write $\sum_n \varepsilon_n T_{b_i b_j}^{\text{DWBA}}$ explicitly as

$$\sum_n \varepsilon_n T_{b_i b_j}^{\text{DWBA}} = \left< \prod_{i=1}^n \tilde{\Phi}_b(b_1) \tilde{\Phi}_B(b_3, b_4, \ldots, b_N) \right| \left( \sum_i V_{b_i b_1} - U_{b_1 b_i} \right) \mathcal{P}$$

$$\left| \tilde{\Phi}_A(b_3, b_4, \ldots, b_N) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \right>$$

where $b_1, b_2, \ldots, b_N$ in the wave function $\tilde{\Phi}_B$ and etc. are $N$ equivalent nucleons. $V_{b_i b_1}$ represents the two-body interaction between particles $b_i$ and $b_1$, and $U_{b_1 b_i}$ represents the distorting potential which the particle $b_1$ feels. We operate the $\mathcal{P}$ to the initial channel states, and use the relation $\mathcal{P} = \frac{\mathcal{P}^a}{N!}$

$$\mathcal{P} \left< \tilde{\Phi}_A(b_3, b_4, \ldots, b_N) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \right>$$

$$= \frac{\mathcal{P}^a}{N! \cdot (n-2)!} \left| \tilde{\Phi}_A(b_3, b_4, \ldots, b_N) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \right> \quad (2.23)$$

where $\tilde{\Phi}_A(b_3, \ldots, b_N)$ is the normalized antisymmetrized wave function of $\tilde{\Phi}_A(b_3, \ldots, b_N)$. Next we operate the $\mathcal{P}$ to the interaction and the final state.

$$\sum_n \varepsilon_n T_{b_i b_j}^{\text{DWBA}} = \frac{\left< \prod\right.}{2} \left( \prod \right)$$

$$\left| \tilde{\Phi}_A(b_3, b_4, \ldots, b_N) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \right>$$

$$= \left< \prod_{i=1}^n V_{b_i b_1} - U_{b_1 b_i} \right> \left| \tilde{\Phi}_A(b_3, b_4, \ldots, b_N) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \right>$$

$$+ \left( n-2 \right) \chi_a^+ (b_3) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \left| \tilde{\Phi}_A(b_3, b_4, \ldots, b_N) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \right>$$

$$+ \left( n-2 \right) \chi_a^+ (b_3) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \left| \tilde{\Phi}_A(b_3, b_4, \ldots, b_N) \tilde{\Phi}_a(b_1, b_2) \chi_a^+ (b_1, b_2) \right>$$

$$\left( 2.24 \right)$$

If we drop the suffix $b_i$ which distinguishes between $N$ nucleons, Eq. (2.24) becomes.
where \( x \) is the captured particle in the particle \( a \), and \( C \) is the remainder obtained by removing the particle \( b \) from the nucleus \( A \). Thus the transition amplitude is seen to consist of three direct terms and three exchange terms. The first term of the direct amplitude is called the stripping term, the first term of the exchange amplitude is called the knock on term and the second term of the exchange amplitude is called the heavy particle stripping term.

### 2.4 Treatment of the heavy particle stripping term in DWBA

As we have mentioned initially, one of the reason why there have been very few calculations in DWBA is the cancelation of the heavy particle stripping term and the distorting optical potential. We introduce the treatment of Rodberg et al.\(^5\) Their conclusion is that the heavy particle stripping contribution to the cross section is of order \( \frac{1}{A} \) compared with the usual stripping contribution. We explain it in detail. They derived the amplitude of DWBA as

\[
T_{ba}^{\text{DWBA}} = \left< \chi_b^{(-)} \Phi_{b} \mathcal{U}_{b} B \left| V_{bx} + V_{bA} - U_{bB} \mathcal{U}_{A} \Phi_{a} \chi_{a}^{(+) \prime} \right> \quad (2.14)
\]

in the reaction

\[
a + A \rightarrow (b+x)+A \rightarrow b+(x+A) \rightarrow b+B \quad (2.26)
\]

They call the transition amplitude \( \left< \chi_b^{(-)} \Phi_{b} \mathcal{U}_{b} B \left| V_{bA} - U_{bB} \mathcal{U}_{A} \Phi_{a} \chi_{a}^{(+) \prime} \right> \) in \( T_{ba}^{\text{DWBA}} \) of Eq. (2.14) the heavy particle stripping term. If the interaction \( V_{bA} \) is assumed as a one-body Woods-Saxon potential with the strength \( V_0 \) equal to that of the optical potential \( U_{bB} \)

\[
V_{bA} = \frac{V_0}{1 + e^{\left( \Gamma_{bA} - \Gamma_{bA}^{(A)} \right) / \hbar}} \quad (2.27)
\]

and the distorting optical potential \( U_{bB} \) is assumed as

\[
U_{bB} = \frac{V_0}{1 + e^{\left( \Gamma_{bB} - \Gamma_{bB}^{(A+B)} \right) / \hbar}}
\]
then their heavy particle stripping term is

\[
\frac{\langle V_{BA} - U_{BB} \rangle}{A} \approx \frac{V}{A} \frac{e^{i\frac{q_0 - r_0(A+1)^2}{\hbar^2} / \lambda}}{1 + e^{i\frac{q_0 - r_0(A+1)^2}{\hbar^2} / \lambda}} \left( \frac{2\in\mathcal{P}_A - \mathcal{P}_b - \frac{1}{3} r_0(A+1)^2}{\mathcal{P}_b \mathcal{P}_A - r_0 \mathcal{P}_b} \right).
\]

The residual interaction in Eq. (2.29) increases with the increasing target radius. These introduce into the cross section a factor proportional to the volume and thus to A. Combined with the factor of \( \frac{1}{A^2} \) coming from the residual interaction, this causes the resulting heavy particle stripping cross section to be of order \( \frac{1}{A} \) compared with the deuteron stripping cross section. We remark that the heavy particle stripping term of their definition is the contribution of the second and the third direct terms of Eq. (2.25), while the usual heavy particle stripping term is the contribution of the second and third exchange terms of Eq. (2.25). Of course the same discussion can be applied to the usual heavy particle stripping calculation under the assumption that the interaction \( V_{BC} \) is a one-body Woods-Saxon potential with the same strength of the optical potential \( U_{BB} \).

But the approximation of replacing the interaction between complex particles by one-body interaction has several question.

a. state dependence:

The potential seen in the bound state cannot be the same as that seen in the scattering state.

b. energy dependence of the optical potential:

Even in the same scattering states, the potentials seen by the particle with different energies are different. To avoid the energy dependence of optical potential we must use the nonlocal optical potential and introduce the imaginary part of the optical potential. It is not correct to assume that the interaction between a nucleon and a complex nucleus is a one-body potential having the same strength of the optical potential. The reason is as follows: If we assume that the interaction \( V_{bB} = V_{bx} + V_{bA} \) in Eq. (2.14) is one-body Woods-Saxon
potential having the same strength of the optical potential \( U_{bb} \), we obtain

\[
\frac{DWBA}{T_{ba}} = 0,
\]

while \( T_{ba}^{DWBA} = \langle \chi_b^{(-)} \Phi_b^* \Psi_B^* | V_{bx} | \phi_A \Phi_a \chi_a^{(+)} \rangle \) can explain many experimental results. In such case, it is important to examine how much the second direct term or the second exchange term contribute to the total cross section without assuming that the interaction between a nucleon and a complex nucleus is a one-body optical potential. For this propose our method of the calculation is very useful. So we calculate the cross section of the second exchange term i.e. the usual heavy particle stripping term on the reaction \(^{11}\text{B}(d,n)^{12}\text{C} \).

2.5 Macroscopic calculation 7)

If we dare to assume the interaction to be a one-body type, it is reasonable to determine the interaction type according to its role inducing the reaction.

a. The interaction \( V_{bx} \) combines particles \( b \) and \( x \), and makes the bound state \( a \). We assume that the interaction is a Gaussian form and that its strength is adjusted to give the binding energy of the particle \( a \).

b. The interaction \( V_{bc} \) combined particles \( b \) and \( C \) and makes the bound state \( A \). It is replaced by a one-body Woods-Saxon potentials, and its strength is chosen to give a right seperation energy of the particle \( b \) from the nucleus \( A \).

c. The interaction \( V_{bA} \) is the interaction between particles which are not bound, and then it is reasonable to replace the interaction \( V_{bA} \) by the one-body Woods-Saxon potential having the strength equal to that of the optical potential \( U_{bb} \).

d. The interaction \( V_{ba} \) is the interaction between particles which are not bound. But as the number of nucleons in the particle \( a \) is much smaller than that of the nucleus \( B \), we cannot treat this interaction like the interaction
that

\[ V_{ba}. \]  We assume that the interaction \( V_{ba} \) is Gaussian type and its magnitude is equal to that of the interaction \( V_{bx} \). From the above consideration, the calculation of the second and the third direct terms of Eq. (2.25) is the same as that of Rodberg et al, neglecting the contribution of the imaginary part of the optical potential. In the calculation of the exchange amplitude of Eq. (2.25), the interaction \( V_{bc} \) and \( U_{bb} \) are the same Woods-Saxon potentials, but the strength of them may be different.

Next we introduce the outline of our macroscopic calculations. The differential cross section is derived from the transition matrix in Eq. (2.25) and can be written as

\[
\frac{d\sigma_{\text{ex}}}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi)^2} \frac{\hbar^4}{K_a} \frac{a^3 J_b + 1}{(2J_a + 1)(2S + 1)} \sum_{J_a} A_{J_a J_a}^{(D)} \left( \beta_{J_a m}(\theta) + \beta_{J_a m}(\theta) - \beta_{J_a m}(\theta) \right) + \sum_{E_{J_a J_a}} A_{E_{J_a J_a}}^{(E)} \left( \beta_{E_{m}}(\theta) + \beta_{E_{m}}(\theta) - \beta_{E_{m}}(\theta) \right) \right]^2
\]

where \( J_A \) and \( J_B \) are the spins of the target and the residual nuclei respectively and \( s_a \) is the spin of the incident nucleon a. The quantities \( \mu_a \) and \( \mu_b \) are the spectroscopic amplitudes of the direct and exchange processes respectively depending upon the internal nuclear structure, where \( l_b \) and \( l_a \) specify the orbital angular momentum of the bound nucleon in the initial and final states respectively. The spectroscopic amplitudes may be defined as

\[ A_{i s_j}^{(D)} = \frac{i^\ell}{\sqrt{2s_a+1}} \frac{2s+1}{2s+1} a(s) J_{BA}(lsj), \quad (2.31a) \]

\[ A_{i s_j}^{(E)} = \frac{i^\ell}{\sqrt{2s_a+1}} \frac{2s+1}{2s+1} (-)^{s} J_{BC}(l_{a} s_{a} J_{a}) J_{AC}(l_{b} s_{b} J_{b}) (-)^{s} J_{C}^{s} J_{A}^{s} J_{A}^{s} \]

\[ x[(2J+1)(2l+1)(2S+1)(2J_b+1)(2J_a+1)]^{\frac{1}{2}} \]

\[ x \left( \begin{array}{c} J_A J_B J_{a} J_{b} J_{C} \end{array} \right) x \left( \begin{array}{c} s \ s_a \ s_b \end{array} \right) \]

\[ J_{BA}(lsj) \] and \( a(s) \) in Eqs. (2.31) are the reduced widths and may be defined in the form
\begin{equation}
\tilde{F}_m^{BA}(\xi, \eta, \sigma) = \sum_{ij \mu \nu} (f_{ij}^{A} m A M_{M} m_{M} m_{M} m) (l S m l d l d M e m) \times \int_{BA} (\xi, \eta) \gamma_{i}^{B}(r) \gamma_{j}^{M}(l) \gamma_{\nu}^{M}(\sigma) \tilde{F}_{j a M}^{A}(\tilde{r}) , \tag{2.32a}
\end{equation}

where the wave function \( \gamma_{i}^{B}(r) \) is the radial wave function of the particles \( x \),

\begin{equation}
\gamma_{S a M}^{a} (r_{BA}, \sigma_{BA}, \sigma_{BA}) = \sum_{S b s M b M a M a} (S b S m b m \mid S a M a M a) \phi (\rho_{BA} y_{S b M b} (\sigma_{BA}) y_{S a M a} (\sigma_{BA}) \), \tag{2.32b}
\end{equation}

\[ \text{where } \gamma_{i}^{B}(r) \text{ is the radial wave function of the particle } x, \gamma_{S a M}^{a} (r_{BA}, \sigma_{BA}, \sigma_{BA}) \text{ is the spin wave function of the particle } x, \phi \text{ is the spatial part of the intrinsic wave function of the particle } a, \sigma \text{ is the spin coordinate and } \xi \text{ is the intrinsic coordinate of the nucleus } A. \] In Eq. (2.30), \( \theta_{l m}^{(a)} \) is written as

\begin{equation}
(2l + 1)^{2} \mu \bar{l} \bar{E}^{(a)} f_{l m}^{(a)} = \hat{J}^{(a)} \int d \pi_{A} \int d \pi_{B} \gamma_{i}^{B}(r) \chi_{i}^{B}(l) f_{l m}(\pi_{BA}, \pi_{A}) \chi_{l m}(\pi_{B}, \pi_{A}), \tag{2.33}
\end{equation}

where \( J^{(D)} \) and \( J^{(E)} \) are the Jacobians in the transformation of the coordinates of \( \pi_{A} \) and \( \pi_{B} \) to \( \pi_{BA} \) and \( \pi_{BA} \) respectively. They are expressed as

\begin{equation}
J^{(D)} = \left( \begin{array}{c}
\frac{aB}{x(A+a)}
\end{array} \right)^{3}, \tag{2.34a}
\end{equation}

\begin{equation}
J^{(E)} = \left( \begin{array}{c}
\frac{aB}{C(B+b)}
\end{array} \right)^{3}, \tag{2.34b}
\end{equation}

where \( a, b, x, A, B, \) and \( C \) are the masses of corresponding particles \( a, b, x, A, B \) and \( C \). In Eq. (2.33), \( f_{l m}^{(a)} (\pi_{BA}, \pi_{A}) \) may be written as

\begin{equation}
f_{l m}^{(bA)} (\pi_{BA}, \pi_{A}) = \gamma_{l}^{B}(r_{BA}) \gamma_{l m}^{A} (\pi_{BA}) \gamma_{l m}^{B} (\pi_{BA}), \tag{2.35a}
\end{equation}

\begin{equation}
f_{l m}^{(bB)} (\pi_{BA}, \pi_{A}) = \gamma_{l}^{B}(r_{BA}) \gamma_{l m}^{B} (\pi_{BA}) \gamma_{l m}^{B} (\pi_{BA}), \tag{2.35b}
\end{equation}

\begin{equation}
f_{l m}^{(bC)} (\pi_{BA}, \pi_{A}) = \gamma_{l}^{B}(r_{BA}) \gamma_{l m}^{C} (\pi_{BA}) \gamma_{l m}^{B} (\pi_{BA}). \tag{2.35c}
\end{equation}
From Eq. (2.30), we can write for the cross section of each term for simplicity as

$$\frac{d\sigma^{(a)}}{d\Omega} = C \left( |\psi^{(a)}|^2 \right) \sum_{m} |\rho_{lm}^{(a)}|^2,$$

(2.36)

if we neglect the interference term. The notation $c$ is the factor independent on $\alpha$, and $n^{(a)}$ is the factor due to the equivalent nucleons.

From Eq. (2.25), $n^{(D)}$ is equal to $2\sqrt{\frac{N-1}{2}}$ and $n^{(E)}$ is equal to $(N-2)\sqrt{\frac{N-2}{2}}$.

We calculate the cross sections of Eq. (2.36) on the reaction $^{11}$B ($d,n$)$^{12}$C. The bound states $\psi_1(r)$ of Eqs. (2.35) are solved in the Woods-Saxon well by giving right separation energies, and the bound state $\psi_{oo}(r)$ of Eqs. (2.35) is solved in the Gaussian well by giving the binding energy of the particle $a$.

(in this case $a$ is the deuteron). The spectroscopic factors are calculated from the assumed configuration $^{8)}$:

$^{11}$B_gnd: $-0.672 \left[ \Psi_P(4^3S) \right]^2 \psi_2 + 0.74 \left[ \Psi_D(4^3S) \right]^2 \psi_2$,  

(2.37a)

$^{12}$C_gnd: $\left[ \Psi_S(4^5S) \right] \psi_0$,  

(2.37b)

where $\psi_L[f]$ is the spatial part of the wave function with the symmetry $[f]$ and the angular momentum $L$, and $2T+12S+1$ is the spin-isospin part of the wave function with the symmetry $[T]$ which is dual to the symmetry $[F]$, the spin
angular momentum $S$ and the isospin angular momentum $T$. The suffix under the right side of the bracket is the total angular momentum. We show the result in Table 2 and Fig. 3. From Table 2, we can see that because of the very small spectroscopic amplitudes of exchange terms, the contribution of the heavy particle stripping terms is very small.

This result seems to be contrary to the result of Owen and Madansky. But as we mentioned in §1, they used the parameter $\frac{n_y}{n_z}$ which correspond to $\frac{n(\frac{1}{2})}{n(\frac{3}{2})}$ in our calculation. According to their calculation $\frac{n_y}{n_z}$ is set to be of order unit, but in our calculation $\frac{n(\frac{1}{2})}{n(\frac{3}{2})}$ is of order $\frac{1}{10}$ for the capture of the deuteron into D state and is of order $\frac{1}{100}$ for the capture of the deuteron into S state. This difference arises from the following reasons:

a. The wave functions in their calculation are only the products of orbital parts and spin parts, and have not the total angular momentum. Then the recoupling factors such as Racah coefficients and $X$-coefficients in Eq. (2.31b) do not appear in the exchange term.

b. Their symmetric orbital wave function for two nucleons in the P-shell is described as

$$\psi (n_1, n_2) = \frac{1}{\sqrt{2}} (\psi_p (n_1) + \psi_p (n_2)),$$

but usually it must be described

$$\psi (n_1, n_2) = \psi_p (n_1) \psi_p (n_2).$$

Using the latter type wave function, the spectroscopic amplitude of the exchange term reduces more rapidly than that of the direct term.

We see from table 2 that the values of $\beta_{1m} (\frac{1}{2}^f)$ vary with the change of optical parameters but that the ratio $\frac{\beta_{1m} (\frac{1}{2}^f)}{\beta_{1m} (\frac{3}{2}^f)}$ does not seem to be more larger than the value of order 10 in the $^{11}\text{B}(d,n)^{12}\text{C}$. From Fig. 3, the angular pattern of the heavy particle stripping term has not backward peak in DWBA because of the distortion effect. So the main component of the reaction may be the usual stripping
term and it may have the possibility of explaining the large backward peak due to the distortion effect. The angular patterns of the stripping and heavy particle stripping terms calculated by using harmonic oscillator bound state wave functions are shown in Fig. 4. We carry out this calculation to compare with the microscopic calculation of §4. From Fig. 4 we see that the results obtained by using harmonic oscillator bound states are similar to those obtained by using Woods-Saxon bound state about the angular pattern. However the heavy particle stripping cross section obtained by using harmonic oscillator bound states are much more reduced. This tends to make the ratio \( \frac{\rho_{(b)} \Omega_{(b)}}{\rho_{(a)} \Omega_{(a)}} \) smaller.

### Table 1. Optical parameter

<table>
<thead>
<tr>
<th>incident channel</th>
<th>exit channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>v</td>
<td>50.0</td>
</tr>
<tr>
<td>w</td>
<td>36.0</td>
</tr>
<tr>
<td>( r_0 )</td>
<td>1.5</td>
</tr>
<tr>
<td>( a_R )</td>
<td>0.65</td>
</tr>
<tr>
<td>( a_I )</td>
<td>0.65</td>
</tr>
<tr>
<td>b</td>
<td>---</td>
</tr>
</tbody>
</table>

The notations \( v, w, a_R, a_I \) and \( b \) are defined by Eq. (3.13) and \( r_0 \) is defined by \( R = r_0 A^{1/3} \), where \( A \) is the mass number of the target nucleus. The deuteron optical parameters of the type (a) are given by Hodgson\(^{14}\) and those of the type (b) are given by Nelson et al\(^{15}\).
Table 2. Cross sections of each process

<table>
<thead>
<tr>
<th>inducing interaction</th>
<th>strength</th>
<th>$n\ \text{Aesj}^2$</th>
<th>$\text{lm}^2$</th>
<th>magnitude of cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
</tr>
<tr>
<td>Direct $V_{bx}$</td>
<td>44.5</td>
<td>3.936</td>
<td>21.8</td>
<td>110</td>
</tr>
<tr>
<td>$V_{ba}$</td>
<td>43</td>
<td>3.936</td>
<td>11.1</td>
<td>230</td>
</tr>
<tr>
<td>$U_{bb}$</td>
<td>43</td>
<td>3.936</td>
<td>14.5</td>
<td>263</td>
</tr>
<tr>
<td>$iU_{bb}$</td>
<td>12</td>
<td>3.936</td>
<td>16.0</td>
<td>188</td>
</tr>
<tr>
<td>Exchange $V_{ba}$ capture of $V_{bc}$</td>
<td>44.5</td>
<td>$2.557\times10^{-3}$</td>
<td>2.31</td>
<td>15.5</td>
</tr>
<tr>
<td>of deuteron into $U_{bb}$</td>
<td>52.4</td>
<td>$2.557\times10^{-3}$</td>
<td>16.6</td>
<td>15.6</td>
</tr>
<tr>
<td>S-state $U_{bb}$</td>
<td>43</td>
<td>$2.557\times10^{-3}$</td>
<td>15.7</td>
<td>13.3</td>
</tr>
<tr>
<td>$iU_{bb}$</td>
<td>12</td>
<td>$2.557\times10^{-3}$</td>
<td>7.57</td>
<td>9.84</td>
</tr>
<tr>
<td>Exchange $V_{ba}$ capture of $V_{bc}$</td>
<td>44.5</td>
<td>$1.679\times10^{-1}$</td>
<td>0.811</td>
<td>6.05</td>
</tr>
<tr>
<td>of deuteron into D-state $U_{bb}$</td>
<td>52.4</td>
<td>$1.679\times10^{-1}$</td>
<td>4.93</td>
<td>6.06</td>
</tr>
<tr>
<td>$iU_{bb}$</td>
<td>43</td>
<td>$1.679\times10^{-1}$</td>
<td>4.90</td>
<td>5.48</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>$1.779\times10^{-1}$</td>
<td>1.53</td>
<td>2.91</td>
</tr>
</tbody>
</table>

The results (a) and (b) correspond to the optical parameters (a) and (b) respectively.
Fig. 2 Experimental values and calculated values of the cross sections

$^{11}\text{B}(d,n)^{12}\text{C}$

$l=1$, $s=1/2$, $j=3/2$

$V_{np}=4.5\text{MeV}$

$E_d=1.7\text{MeV}$

$E_d=3.8\text{MeV}$

$E_d=3.5\text{MeV}$

$0^\circ \text{c.m.}$
Fig. 3a  $^{11}\text{B}(d,n)^{12}\text{C}$

$E_d = 3.5\text{MeV} \quad l = 1, \quad S = 1/2, \quad J = 3/2$
Fig. 3b

$^{11}$B(d,n)$^{12}$C

$E_d = 3.5$ MeV    $l = 1$, $s = 1/2$, $j = 3/2$

mb/st
3. Method of the calculation

3.1 Method

The reaction \( A(a,b)B \) is considered. The corresponding transition amplitude can be written as

\[
T_{ba} = \int d\mu_a \int d\mu_b \chi_b^{\pm}(\mu_b, \nu_b) \langle J_b M_b S_b M_b | V | J_A M_A S_a M_a \rangle \chi_a^{\pm}(\mu_a, \nu_a), \tag{3.1}
\]

where \( J_A, J_B, S_a \) and \( S_b \) are the spins of the particles, and \( M_A, M_B, m_a \) and \( m_b \) are their corresponding \( Z \)-components, \( \chi_b^{\pm} \) and \( \chi_a^{\pm} \) are the distorted incoming and outgoing waves respectively and \( H \) is the Jacobian defined in Eqs. (2.24a) and (2.34b).

If we use the completeness of the harmonic oscillator wave functions \( \psi_{NLM}(\nu, \mu) \), the transition amplitude (3.1) becomes \( ^9 ) \)

\[
T_{ba} = J \sum_{N_a, L_a M_a} \int d\mu_b \int d\mu_a \chi_b^{\pm}(\mu_b, \nu_b) \chi_a^{\pm}(\mu_a, \nu_a) \psi_{N_a L_a M_a}(\frac{a b}{B + b}, \nu_a, \mu_a) \psi_{N_b L_b M_b}(\frac{b b}{B + b}, \nu_b, \mu_b) \langle J_b M_b S_b M_b | V | J_A M_A S_a M_a \rangle \psi_{N_a L_a M_a}(\frac{a a + a}{A + a}, \nu_a, \mu_a) \psi_{N_b L_b M_b}(\frac{b b}{B + b}, \nu_b, \mu_b) \tag{3.2}
\]

The harmonic oscillator wave functions \( \psi_{NLM}(\nu, \mu) \) can be written as

\[
\psi_{NLM}(\nu, \mu) = \psi_{NL}(\nu, \mu) \chi_{LM}(\hat{\mu}),
\]

\[
\psi_{NL}(\nu, \mu) = \left[ \frac{2L-N+2}{N!} \frac{(2L+2N+1)!!}{(2L+1)!!} \right]^{1/2} \frac{\mu L}{2} \pi^{1/2} e^{-\frac{1}{2} \nu \mu^2} \left\{ \sum_{k=1}^{2L+1} \left( \frac{2L+1}{(2L+2k+1)!} \right) (-2 \nu \mu^2)^k \right\}.
\tag{3.3}
\]
We expand the form factor using multipole terms which correspond to the transfer to the target with definite angular momenta \( j, l \) and \( s \).

\[
\mathbf{j} = \mathbf{j}_B - \mathbf{j}_A, \quad \mathbf{s} = \mathbf{s}_a - \mathbf{s}_b, \quad \mathbf{j} = l + s.
\]

(3.4)

The transition amplitude becomes

\[
T_{ba} = \mathbf{j} \sum_{N_a, L_a M_a} \int d^4 \mathbf{k}_a \mathbf{X}_{l}^{(+)}(\mathbf{k}_a) \sum_{l', j} \left( \mathbf{j}_a \mathbf{j}_b \mathbf{m}_j | \mathbf{J}_{1} \mathbf{M}_{1} \right) (l s m m | j m j)
\]

\[
\times \left( S_a S_b m_a - m_b | S m_s \right) (-S_b - m_b \mathbf{e}_j \mathbf{e}_j) \mathbf{X}_s^{(+)}(S_m, S_m)
\]

(3.5)

Using the orthogonality of Clebsh-Gordan coefficients, we obtain the inverted form as

\[
G_{lj} \mathbf{e}_j (N_a L_a M_a, N_b L_b M_b) = \mathbf{e}_j \mathbf{e}_j (N_a L_a M_a, N_b L_b M_b)
\]

(3.6)

where

\[
\left( \mathbf{L}_a \mathbf{S}_b \mathbf{J}_a \mathbf{J}_b \right) \text{ I M} | \text{ I M} >
\]

\[
= \sum_{M_a, M_b} \left( \mathbf{J}_a \mathbf{M}_a \mathbf{M}_b \mathbf{M}_b \mathbf{M}_b \right) \left( \mathbf{L}_a \mathbf{M}_b \mathbf{S}_b \mathbf{M}_b \mathbf{M}_b \mathbf{M}_b \mathbf{M}_b \right)
\]

The information of the reaction type is included in this matrix element. If we sum over Z-components in Eq. (3.6) and write \( G_{lj} \mathbf{e}_j (N_a L_a M_a, N_b L_b M_b) = \mathbf{e}_j \mathbf{e}_j (N_a L_a M_a, N_b L_b M_b) \) as the product of two factors
\[
G_{ij}^{(N_{ib}M_{ib}N_{a}L_{a}M_{a})}(\mathbf{r}_{ib}, \mathbf{r}_{aA}) = A_{ij}^{(N_{ib}M_{ib}N_{a}L_{a}M_{a})}(\mathbf{r}_{ib}, \mathbf{r}_{aA}),
\]

(3.7)

we obtain

\[
A_{ij}^{(N_{ib}M_{ib}N_{a}L_{a}M_{a})} = \sum_{\ell} \sum_{j_{a}j_{b}j_{B}} i^{\ell} (-)^{j_{a}+j_{b}-j_{B}+\ell} \sqrt{\frac{(2\ell+1)(2S+1)(2j_{a}+1)}{2J_{B}+1}} \langle j_{a}^{(l_{a})} j_{b}^{(l_{b})} j_{B}^{(l_{B})} | \mathbf{V}(\mathbf{r}) | \mathbf{V}(\mathbf{r}) \rangle \chi_{4}^{(L_{a}M_{a}L_{b}M_{b})} \chi_{3}^{(L_{b}M_{b})}.
\]

(3.8)

defining the form factor

\[
\chi_{3}^{(L_{a}M_{a}L_{b}M_{b})} = \frac{(N_{ib}M_{ib}N_{a}L_{a}M_{a})}{(n_{ib}, n_{aA})} = \langle L_{a} - M_{a} | L_{b} M_{b} | l_{m} \rangle \langle N_{ib} L_{b} - M_{ib} | -b_{b} \rangle \langle n_{ib}, n_{aA} \rangle \chi_{3}^{(L_{a}M_{a})} - \frac{\delta_{A}}{2} \chi_{3}^{(L_{a}M_{a})}.
\]

(3.9)

We neglect the spin-orbit coupling in the distorted wave and define the partial amplitude by

\[
(2\ell+1)^{\ell} \tilde{\rho}_{km}^{(N_{ib}M_{ib}N_{a}L_{a}M_{a})}(\theta) = \sum_{n_{ib}n_{aA}} i^{\ell} \tilde{\rho}_{km}^{(N_{ib}M_{ib}N_{a}L_{a}M_{a})}(\theta) \chi_{3}^{(L_{a}M_{a}L_{b}M_{b})}(n_{ib}, n_{aA}) \chi_{3}^{(L_{a}M_{a})}(n_{ib}, n_{aA}).
\]

(3.10)

To calculate Eq. (3.10), the distorted wave \(\chi_{3}^{(L_{a}M_{a})}(\mathbf{r})\) is expanded in partial waves as

\[
\chi_{3}^{(L_{a}M_{a})}(\mathbf{r}) = \sum_{L_{a}M_{a}} i^{L_{a}} \chi_{3}^{(L_{a}M_{a})}(\mathbf{r}) \chi_{3}^{(L_{a}M_{a})}(\mathbf{r}),
\]

(3.11a)

\[
\chi_{3}^{(L_{a}M_{a})}(\mathbf{r}) = \sum_{L_{a}M_{a}} i^{L_{a}} \chi_{3}^{(L_{a}M_{a})}(\mathbf{r}) \chi_{3}^{(L_{a}M_{a})}(\mathbf{r}),
\]

(3.11b)

where \(\hat{\mathbf{r}}\) and \(\hat{\mathbf{k}}\) denote the polar angles of the vectors \(\mathbf{r}\) and \(\mathbf{k}\). The function \(\chi_{3}^{(L_{a}M_{a})}(\hat{\mathbf{r}})\) is the spherical harmonics. The partial-distorted wave \(\chi_{3}^{(L_{a}M_{a})}(\mathbf{r})\) is the solution of a radial Schrödinger equation with a optical potential \(U(\mathbf{r})\)

\[
\left[ \frac{d^{2}}{dr^{2}} + k^{2} - \frac{2\eta}{\mathbf{r}} + \frac{2LEU(\mathbf{r}) - L(L+1)}{\mathbf{r}^{2}} \right] \chi_{3}^{(L_{a}M_{a})}(\mathbf{r}) = 0,
\]

(3.12)

where \(\eta\) is the Coulomb parameter \(\eta = \frac{Z_{2}Z_{2}^{*}e^{2}}{2\mathbf{r}}\). For \(U(\mathbf{r})\) we take a Gaussian absorption form or a Woods-Saxon absorption form,
\[ U(r) = \frac{V}{1 + \exp((r-R)/a_R)} + i \theta \exp\left(-\frac{(r-R)^2}{b} \right) \]  
\[ \text{or} \]
\[ U(r) = \frac{V}{1 + \exp((r-R)/a_R)} + i \frac{\theta}{1 + \exp((r-R)/a_R)} \]  
\hspace{5cm} \text{(3.13a)}
\hspace{5cm} \text{(3.13b)}

Inserting Eqs. (3.9) and (3.11) into Eq. (3.10) and performing the angular part of the integration, we obtain

\[ \beta_{lm}^{(N_bN_aN_sN_t)}(\theta) = \frac{4\pi}{k_a k_b} \left( \frac{L_b + l - L_m + m}{2} \right)^2 \left( \frac{2L_b + 1}{2L_m + 1} \right)^{L_b} I_{l+m}^{(N_bN_a)} I_{l-m}^{(N_sN_t)} \]
\hspace{5cm} \text{(3.14)}

where

\[ I_{l+m}^{(N_bN_a)} = \int_0^{\infty} \delta_{l+m} g_c g_b \chi_{N_b}(k_a \tau_a) \chi_{N_a}(k_b \tau_b) \left( \frac{L_b}{B+b} \right)^{L_b} \left( \frac{1}{B+b} \right)^{L_a} \]
\[ \text{The function } \chi_{A}^{(n)}(r) \text{ is the radial part of the harmonic oscillator wave function defined by Eq. (3.3).} \]

If we use the relation

\[ \langle \ell_1 j_1 \ell_2 j_2 \ell_3 j_3; J, j \rangle = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)(2J_3+1)}{(2J_1+1)(2J_2+1)(2J_3+1)}} \left\{ \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ j_1 & j_2 & j_3 \\ J_1 & J_2 & J_3 \\ j_3 & J_3 & j_3 \end{array} \right\} \]
\[ \ell \leq s_1 \ell_2 j, j_3 \]
we obtain for the spectroscopic amplitude \( A_{1s} \) in L-S representation the following equation

\[ A^{(N_bN_aN_sN_t)}_{1s} = \sum' \langle J_B | L_B S_B | J_A | L_A S_A \rangle i^{l}(-)^{J_A} + J_A - I + L_A + 1 \]
\[ \times (2I+1)(2A+1)(2J_1+1)(2J_2+1)(2J_3+1)(2j_3+1) \]
\[ \times w(J_A, J_A, J_B; I, I) \]
\[ \times \langle L\ell_s a' a' | L\ell a' a' | L\ell' b' b' \rangle \]
\[ \langle L\ell_s b' b' | L\ell a' a' | L\ell' B' B' \rangle \]
\[ \langle L\ell a' a' | L\ell' B' B' \rangle \]
\[ \times \langle L B \rangle \mid \langle S \rangle \rangle \langle I S | V | (L\ell_A a') \langle \ell' S' A' | v' I M \rangle \rangle. \]
\hspace{5cm} \text{(3.16)}

The notation \( \langle J_B | L_B S_B \rangle \) is defined by the equation
The matrix element \( \langle L_b L_b \rangle_{L_b} (S_b S_b)_{s_b} I M I V \mid (L_A L_A)_{L_A} (S_a S_a)_{s_a} I M \rangle \) can be written as

\[
\langle L_b L_b \rangle_{L_b} (S_b S_b)_{s_b} I M I V \mid (L_A L_A)_{L_A} (S_a S_a)_{s_a} I M \rangle
= \langle t_b t_b M_{t_b} M_{t_b} \mid \Lambda_T M_{\Lambda_T} \rangle \langle t_a t_a M_{t_a} M_{t_a} \mid \Lambda_T M_{\Lambda_T} \rangle
\langle L_b L_b \rangle_{L_b} (S_b S_b)_{s_b} I M I V \mid (L_A L_A)_{L_A} (S_a S_a)_{s_a} I M \rangle,
\]

(3.17)

where \( t_a, t_b, \Lambda_A \), and \( \Lambda_B \) are the isospins of the particle \( a, b, A, \) and \( B \) respectively, and \( m_{t_a}, m_{t_b}, M_{\Lambda_A} \), and \( M_{\Lambda_B} \) their \( Z \)-components respectively. The differential cross section is then given by

\[
\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar)^2} \frac{k_b}{k_a} \frac{2J_b+1}{2J_a+1} \sum \frac{1}{l_{2J_a+1} l_{2J_b+1}} \left| \sum_{N_{\Lambda_a},l_{2J_a+1}} A_{l_{2J_a+1}} \right|^2 \left( \frac{\beta_{l_{2J_a+1}}^{(N_{\Lambda_a}, l_{2J_a+1})}}{1} \right)^2,
\]

(3.18)

where \( \mu_a \) is the reduced mass between particles \( a \) and \( A \).

3.2 Recoil effect

It is difficult to calculate the matrix element in Eq. (3.17) numerically.

We define \( F_{\text{shell}}(N_b L_b M_b : N_A L_A M_A) \) by

\[
F_{\text{shell}}(N_b L_b M_b : N_A L_A M_A) = \sum (L_b L_b M_b L_b M_b) (S_b S_b M_b M_b) (L_A L_A M_A) (S_a S_a M_a M_a) \times \langle L_b L_b \rangle_{L_b} (S_b S_b)_{s_b} I M I V \mid (L_A L_A)_{L_A} (S_a S_a)_{s_a} I M \rangle_{\text{shell}},
\]

(3.19)

and define \( F(n_b L_b M_b : n_A L_A M_A) \) by
The wave functions in the matrix element \(<(l_{0} L_{0})_{A}(s_{0} S_{0})_{A}s_{0}(l_{0} L_{0})_{B}(s_{0} S_{0})_{B}s_{0})_{B}>\) are intrinsic wave functions \(\Psi_{A}^{\text{shell}}\) and \(\Psi_{B}^{\text{shell}}\). If we assume that:

\[
\Psi_{A}^{\text{shell}} = \gamma_{A00}(\Lambda_{A}, \Lambda_{A}) \Psi_{A}, \tag{3.21a}
\]

\[
\Psi_{B}^{\text{shell}} = \gamma_{B00}(\Lambda_{B}, \Lambda_{B}) \Psi_{B}, \tag{3.21b}
\]

and perform the integration over the coordinate of the center-of-mass of the total particles, we obtain the relation

\[
F^{\text{shell}}(m_{A}, m_{B}; l_{A}, l_{B}) = \sum_{m_{A}, m_{B}} \left\langle l_{A} m_{A} l_{B} m_{B} | l_{A} m_{A} l_{B} m_{B} \right\rangle \left\langle l_{A} m_{A} l_{B} m_{B} | l_{A} m_{A} l_{B} m_{B} \right\rangle
\]

\[
\times \left( N_{A} A_{00} L_{A} | A_{00} L_{A} \right) \left( N_{B} B_{00} L_{B} | B_{00} L_{B} \right) F(m_{A}, m_{B}; m_{A}, m_{B}). \tag{3.22}
\]

The notation \((m_{A}, l_{A}; l_{A}) \mid m_{B}, l_{B} \mid N_{A} C_{A} N_{B} C_{B} L_{A} L_{B}\) is the Talmi coefficient.

The validity of the approximation written by Eqs. (3.21) was discussed by Elliot et al.\(^{10}\). We insert Eqs. (3.22) and (3.20) into the inverted form of Eq. (3.19) and we finally obtain the equation

\[
\left\langle (l_{0} L_{0})_{A}(s_{0} S_{0})_{A}s_{0}(l_{0} L_{0})_{B}(s_{0} S_{0})_{B}s_{0})_{B} \right\rangle
\]

\[
= \sqrt{(2L_{A}+1)(2L_{B}+1)} \sum_{m_{A}, m_{B}} (2L_{M}+1) W(l_{A} L_{A} L_{B}; L_{M}) W(l_{B} L_{B} L_{A}; L_{M})
\]

\[
\times \left( N_{A} A_{00} L_{A} | A_{00} L_{A} \right) \left( N_{B} B_{00} L_{B} | B_{00} L_{B} \right) \left( N_{A} L_{00} A_{00} L_{A} \right) \left( N_{B} L_{00} B_{00} L_{B} \right)
\]

\[
\times \left\langle l_{0} L_{0} l_{0} L_{0} (s_{0} S_{0})_{A}s_{0}(l_{0} L_{0})_{B}(s_{0} S_{0})_{B}s_{0})_{B} \right\rangle. \tag{3.23}
\]
Using shell model wave functions for the target and residual nucleus states, we can calculate matrices \( \langle (L_{b} L_{b})_{A} (S_{b} S_{b})_{S} (t_{b} T_{b})_{T} | V_{1} (L_{a} L_{a})_{A} (S_{a} S_{a})_{S} (t_{a} T_{a})_{T} \rangle \) and obtain matrices \( \langle (L_{b} L_{b})_{A} (S_{b} S_{b})_{S} (t_{b} T_{b})_{T} | V_{1} (L_{a} L_{a})_{A} (S_{a} S_{a})_{S} (t_{a} T_{a})_{T} \rangle \) in Eq. (3.16). If we take only one term \( N=1 \) in the summation of Eq. (3.23), we obtain the equation

\[
\langle (L_{b} L_{b})_{A} (S_{b} S_{b})_{S} (t_{b} T_{b})_{T} | V_{1} (L_{a} L_{a})_{A} (S_{a} S_{a})_{S} (t_{a} T_{a})_{T} \rangle
\]

\[
= \left( \frac{a+A}{A} \right)^{2 \frac{2N_{b}+1}{2}} \left( \frac{b+B}{B} \right)^{2 \frac{2N_{a}+1}{2}} \langle (L_{b} L_{b})_{A} (S_{b} S_{b})_{S} (t_{b} T_{b})_{T} | V_{1} (L_{a} L_{a})_{A} (S_{a} S_{a})_{S} (t_{a} T_{a})_{T} \rangle \rangle
\]

(3.23a)

The removal of the center-of-mass motion of the microscopic calculation is similar to that of the center-of-mass motion in the calculation of the reduced width for nucleon clusters in the shell model. We show the numerical results for the heavy particle stripping term of the reaction \(^{11}\text{B}(d,n)^{12}\text{C}\) in §4.

3.3 Comparison our method with the usual calculation

We show that our method is identical to the usual calculation in the stripping reaction. We consider the reaction

\[
a+A + (b+x) + A+ b + (b+A) + b+B
\]

The matrix element \( \langle((L_{b} S_{b})_{J_{b}} J_{B}) | V_{b x} | ((L_{a} S_{a})_{J_{a}} J_{A}) \rangle \) in Eq. (3.8) can be written as

\[
\langle((L_{b} S_{b})_{J_{b}} J_{B}) | V_{b x} | ((L_{a} S_{a})_{J_{a}} J_{A}) \rangle
\]

\[
= (-)^{j_{a}+j_{A}-1} J_{BA} (l_{b}, s_{b}, j_{b}) \sqrt{(2J_{B}+1)(2J_{b}+1)(2j_{a}+1)(2j_{a}+1)(2L_{a}+1)(2S_{a}+1)}
\]

\[
\times W(j_{a} J_{a}, j_{b} J_{b}, J_{j_{a}} J_{j_{b}}) \frac{1}{(L_{b} S_{b})_{J_{b}} (L_{a} S_{a} j_{a})} <\left( N_{b b} \left( \frac{b-B}{B-b} V_{b b} \right) N_{b} (V_{b} J_{A}) \right)_{L_{a}} | V_{b b} | \rho_{b} - \langle a_{a} A \rangle \rangle
\]

(3.24)
where \( \psi_{N_a L_a} \left( \frac{bB}{B+b}, V \right) \) is the harmonic oscillator wave function including the spherical harmonics of rank \( L_b \) and \( \psi_{I_b} \) is the intrinsic spatial wave function of the particle \( a \). Substituting Eq. (3.24) into Eq. (3.8) yields the result as

\[
A_{ij} = i^l (-1)^{L_a + L_b + L_{tot} + 1} \frac{(2L_b + 1)(2s + 1)}{N(2l + 1)(2s + 1)} \left\langle J_{BA} \left( \ell_{ij} \right) \psi_{I_b} \left( \left[ \psi_{N_b L_b} \left( \frac{bB}{B+b}, V \right) \psi_{I_b} \right]_{L_{tot}} \left[ \psi_{N_a L_a} \left( \frac{aA}{A+a}, V \right) \Phi_{N_b L_b} \right]_{L_{tot}} \right) \right\rangle_{s}.
\]

(3.25)

From the Eqs. (3.9) (3.10) and (3.25), the quantity \( \sum_{N_{a_L a}} A_{ij} \) becomes

\[
\sum_{N_{a_L a}} A_{ij} \rho_{L_m} \rho_{L_{tot}}
\]

in Eq. (3.18) becomes

\[
\]

\[
\]

\[
\]

(3.26)
where we use the completeness of the harmonic oscillator wave function and the relation

\[
\langle \left[ \frac{\hbar b}{b+b} \right]_{N_Lb} | V_{\text{ex}} | \frac{\hbar b}{b+b} \rangle = \sum_{\lambda \lambda_L} \langle \left[ \frac{\hbar b}{b+b} \right]_{N_Lb} | V_{\text{ex}} | \frac{\hbar b}{b+b} \rangle
\]

3.4 Calculation of the matrix elements in the form factor

We use the notations \( \langle (bB)_{A\lambda} | V_{1} | (aA)_{A} \rangle \) and \( \langle \left[ \frac{\hbar b}{b+b} \right]_{N_Lb} | V_{\text{ex}} | \frac{\hbar b}{b+b} \rangle \) as the simplified form and the explicit form of the matrix elements \( \langle (bB)_{A \lambda} | V_{1} | (aA)_{A} \rangle \) respectively. The notation \( \Phi_{LB} \) is the orbital part of the wave function of the nucleus B and \( \Phi_{LB} \) is the spin-isospin part of the wave function of the nucleus B. \( \Phi_{b}(Y_b) \) is the orbital part of the internal wave function of the particle b. In our calculation \( \Phi_{b}(Y_b) \) is assumed as follows:

\[
\Phi_{b}(Y_b) = \frac{Y_{000}(Y_b, 0)}{\text{wave function of center-of-mass motion of particle b}} \quad (3.27)
\]

where \( Y_{000}(Y_b, 0) \) is the harmonic oscillator wave function defined by Eq. (3.3). For example, if the particle b is composed of three nucleons

\[
\Phi_{b}(Y_b) = \frac{Y_{000}(Y_b, 0)}{\text{wave function of center-of-mass motion of particle b}} \quad (3.28a)
\]

where \( r_{1-3} \) is the distance between the particle 1 and the center-of-mass of the particles 2 and 3, and \( r_{2-3} \) is the distance between particles 2 and 3. If the particle b is composed of four nucleons
The matrix element \( \langle (bB)\Lambda | V | (aA)\Lambda \rangle \) written here is the matrix element \( \langle (bB)\Lambda | V | (aA)\Lambda \rangle \) in Eq. (3.27), but we remove the suffix shell for simplicity. We give the expression for the matrix element \( \langle (bB)\Lambda | V | (aA)\Lambda \rangle \) according to the six terms in Eq. (2.25)

\[ a. \ V = V_{bx}; \ \text{stripping term} \]

We transform the orbital part of the wave function of the particle \( a \) as

\[
\Psi_{N\Lambda m\Lambda}(a\nu, \hbar \alpha) \phi_b(\nu_b) = \sum_{\alpha\betax} \alpha_{\betax}(n) \langle \nu_{b\betax} \rho_{\betax} \rho_{\betax} | N\Lambda m\Lambda \rangle (\Psi_{N\Lambda m\Lambda}(b\nu, \hbar \alpha) \Psi_{N\Lambda m\Lambda}(x\nu, \hbar \alpha) \phi_b(\nu_b)) \]  

\[ \text{La} m \Lambda, \]

(3.29)

where

\[
\alpha_{\betax}(n) = \int \Psi_{N\Lambda m\Lambda}(a\nu, \hbar \alpha) \Psi_{N\Lambda m\Lambda}(x\nu, \hbar \alpha) \alpha_{\betax} \]  

(3.30)

and \( \alpha \) is determined by Eqs. (3.28). Next we transform the wave function of the nucleus \( B \) as

\[
\Phi_L^{2T_B+1/2 S_B+1/2} = \sum_{A \chi^{'}} < B | A \chi^{'}, x' > [ \Phi_L^{2T_B+1/2 S_B+1/2}, \Phi_L^{2T_B+1/2 S_B+1/2} ] LBSB T_B ,
\]

Here \( < B | A \chi^{'}, x' > \) is the coefficient of fractional parcentage and it is explicitly written as

\[
< B | A \chi^{'}, x' > = < \alpha_{\betax} LBSB T_B | \alpha_{\alpha A L\Lambda S\Lambda T\Lambda} \alpha \chi^{'} x^{'} x^{'} x^{'} >.
\]

(3.31)

The notation \( \alpha_B \) denotes the quantum number except for the angular momentum.
quantum numbers. By using these equations, we obtain the matrix element

\[ \langle b \lambda' a' | V_{bx} | b \lambda a \rangle \]

\[ = \sum_{n \nu' \lambda} a_{b \lambda}(n) b \nu' x \lambda ; L_b | M_b : M_d ; N_d ; n_0 ; L_0 ; L_a \rangle \langle \Gamma_A | \langle V_{bx} | b \lambda' a' \rangle \langle b \lambda a | V_{bx} | b \lambda' a' \rangle \]

The recoupling factor in Eq. (3.32) is given by the following equation:

\[ \langle b \lambda' A' | V_{bx} | b \lambda A \rangle \]

\[ = \langle b \lambda' A' | V_{bx} | b \lambda A \rangle \]

\[ = \langle L_b (L_a x) L_b ; L \lambda (L_d x) L \lambda A ; L \lambda A | S_b (S_a x) S_b ; L \lambda (L_d x) S_a S_b ; L \lambda A \rangle \langle T_b (T_a x) T_b ; L \lambda T_a | T_b (T_a x) T_b ; L \lambda T_a \rangle . \]

The letters a, b, ... in Eqs. (3.31), (3.32) and (3.33) denote all the quantum numbers of the particles a, b, .... The matrix element \( \langle b \lambda' a | V_{bx} | b \lambda a \rangle \) in Eq. (3.32) is the two-body matrix element in the reaction \((d, p)\) and \((d, n)\).

If the particles x and b are composed of more than two nucleons, we take out one nucleon from the particles x and b using the coefficients of fractional parantage and the Talmi coefficients. The matrix element \( \langle b \lambda' a | V_{bx} | b \lambda a \rangle \) becomes as follows

\[ \langle b \lambda' a | V_{bx} | b \lambda a \rangle \]

\[ = \sum_{n \nu' \lambda} b_{b \nu' x} (n) \langle n_0 L_b n_0 L_b ; L_b | M_b : M_d ; N_d ; n_0 L_0 \lambda' 0 ; L_a \rangle \langle S_b t_b | S_b t_b ; \nu \lambda \nu \lambda \rangle \]

\[ \times \langle b b a \lambda a | b \lambda (b x) a | b \lambda (b x) a \rangle \langle b (b \lambda x \lambda) \rangle a | b \lambda (b x) a | b \lambda (b x) a \rangle \langle \lambda' | x \lambda \rangle \]
where

\[ b_{bb_2}(n) = \int 4 \pi n_{00} (\beta V, 1_{bb_2}) \varphi_{000} (\beta V, 1_{bb_2}) \, d\Omega_{bb_2} \]  

(3.35a)

\[ a_{bb_2}(n) = \int 4 \pi n_{00} (\beta V, 1_{bb_2}) \varphi_{000} (\beta V, 1_{bb_2}) \, d\Omega_{bb_2} \]  

(3.35b)

\[ a_{x_2}(n) = \int 4 \pi n_{00} (\delta V, 1_{x_2}) \varphi_{000} (\delta V, 1_{x_2}) \, d\Omega_{x_2} \]  

(3.35c)

The parameters \( B \) and \( y \) is determined from Eqs. (3.28) and \( \rho_A \) is the parameter adjusted to the separation energy of the particle \( x \) from the nucleus \( B \). The notations \( n(B) \) and \( n(x) \) is the nucleon numbers contained in the particle \( b \) and \( x \) respectively. We take the two body interaction as

\[ V = f(t_{1/2}) \left( W_{bb_2} \rho - H_{bb_2} \rho \right), \quad W + B + H + M = 1. \]

b. \( V = V_{bA} \)

We transform the states of the particles \( B, a \) and \( A \) into the following forms

\[ \mathcal{L}_{bA}^{2Ta+125b+1} = \sum_{\lambda' \lambda} \langle B || A' \lambda' \rangle \left( \mathcal{L}_A^{2Ta+125a+1} \mathcal{L}_b^{2Ta+125b+1} \right) \mathcal{L}_{bA}^{2Ta+125a+1} \]  

(3.36a)
By using these equations, we obtain

\[ \langle (bB) \Lambda | V_{bA} | (aA) \rangle \]

\[ = \sum_{nA'x'B'} \langle b(A'x)B'; A|x (bA')B; A'B'C_y > \langle b(C_y) A'; B'| (b'y)d C; B' \rangle \]

\[ \times \langle A_{xb} (n) | (n \times x \times y \times z) ; La | M_x ; M_b | N \Lambda L \Lambda o ; La \rangle \langle \text{SATA} | 5x 5y 5z \rangle \]

\[ \times \langle (x'y') A; A|x (x'y') B'; A'y < b' (C_y') A'; B' | (b'y') d C; B' > \rangle \langle A'y | C_y' \rangle \]

\[ \times \langle \langle N_{nLb} (b'y') \phi_b (b') 2_{T+1} 2_{S+1} | 7 \rangle \langle b'y' | 2 \rangle \rangle \langle \text{SATA} | V_{b'y' | d} \rangle \]

\[ = \sum_{n' \Lambda n' L'} \langle b_{n'\Lambda} (n') | (n' \times x \times y \times z) ; La | M_{n' \Lambda} ; M_b | N_{n' \Lambda} L \Lambda o ; Lb \rangle \langle \text{SATA} | 5x 5y 5z \rangle \]

The matrix element \( \langle (b'y) d | V_{b'y} | (b'y') d \rangle \) in Eq. (3.37) is a two-body matrix element if the particle \( b \) is a single nucleon, but if the particle \( b \) is composed of more than two nucleons, we take out the single nucleon \( b_2 \) from the particle \( b \). The matrix element \( \langle (b'y) d | V_{b'y} | (b'y') d \rangle \) then becomes \( \langle (b) d | V_{b} | (b') d \rangle \).
\[ x < (b, b_z) b y ; d | b, (b_z) e ; d > \alpha_b b_z (n') (N_b b, N_z b ; l_b ; l' b | l_b | N_b b' N_z b' l_b') \]

\[ x < S b_t b_t (S_b t_b, y_d y_d) < (b, b_z) b y ; d | b, (b_z) e ; d > \]

\[ x \left( W + (-)^{s_e+1} B + (-)^{s_e} H + (-)^{s_e+1} M \right) \]

\[ \chi \left( \frac{1}{\text{d}b} V b \chi \left( \frac{1}{\text{d}b} V b \chi \right) \right). \]  

(c. \( V = U(r_{bB}) \).

In the numerical calculation, we set \( V = 1 \) and multiply the final channel distorted wave function \( \chi_{d,(k,\rho)} \) by the optical potential \( U(r_{bB}) \). Then we obtain

\[ \chi (b, b) A \chi (a, A) A \]

\[ = \sum \alpha_b x (n) (N_b b N_z b ; l_b ; l_b N_a A n o j l_a) \chi_{a x} \{ l, s o, s t x \} \chi \left( \frac{1}{\text{d}b} \chi \left( \frac{1}{\text{d}b} \chi \right) \right). \]

(d. \( V = V_{ba} \); knock on term

The matrix element \( (b, b) A \chi (a, A) A \) can be written in the following form:

\[ \langle (b, b) | V a | (a, A) \rangle \]

\[ = \sum \langle b | (c, b) (c, b) B d b (a, a) d A \rangle \langle A l | V a \rangle \langle a, (A, A) d b (c, c) d A \rangle \chi \left( \frac{1}{\text{d}b} \chi \left( \frac{1}{\text{d}b} \chi \right) \right). \]

\[ \left( \frac{1}{\text{d}b} \chi \left( \frac{1}{\text{d}b} \chi \right) \right) \chi \left( \frac{1}{\text{d}b} \chi \left( \frac{1}{\text{d}b} \chi \right) \right). \]  

(3.40)
If the particle a is a single nucleon, we set $\phi_a(x) = 1$. It is also the same about the particle b. If it is not a single nucleon, the matrix element in Eq. (3.40) can be written as the linear combination of two-body elements

$$
\langle (b'a')d | V_{ba} | (a'b)d \rangle = \sum_{a_1 a_2 b_1 b_2} \sum_{e c} \langle (b_1 b_2) b a ; d | b_1 (b_2 a) e ; d \rangle 
\times a_1 a_2 (a_2) (Na_1 Na_2 La ; La | Ma_1' 1 | Na_2 '0 ; La) \langle Sa_1 | Sa_1 \{ \chi_1 \} \chi_2 \rangle 
\times \langle b_2 (a_1 a_2) a ; e | (b_1 a_2) C a_1 ; e \rangle 
\times \langle b_1 b_2 (a_1 a_2) (b_1 b_2) a ; d | (b_1 a_2) e ; d \rangle \langle a_1 | a_1 \} \langle b_2 (a_1 a_2) a ; e | (b_2 a_2) C a_1 ; e \rangle 
\times \eta (b) \eta (a) (W + (-)^{2c+1} B + (-)^{tc} H + (-)^{2c+tc+1} M ) 
\times \left[ 4nb_2 l_{b_2} (\nu, \nu b_2) 4na_2 l_{a_2} (\nu, \nu a_2) l_{c} | V_{ba} | 4nb_2 l_{b_2} (\nu, \nu b_2) 4na_2 l_{a_2} (\nu, \nu a_2) l_{c} \right] 
\times \int \hat{\epsilon}_{b_1}^{*} \hat{\epsilon}_{b_2} (\nu b, \nu b_1) \phi_{b_1}^{*} (\nu b) d\nu b_1 d\nu b \int \hat{\epsilon}_{a_1}^{*} \hat{\epsilon}_{a_2} (\nu a, \nu a_2) \phi_{a_2} (\nu a) d\nu a_2 d\nu a_1. 
$$
\hspace{1cm} (3.41)

e. V = V_{bc} \ \ \text{heavy particle stripping term}

The matrix element can be written as
If the particle $b$ is composed of more than two nucleons the matrix element
\[
\langle (yb)d | V_{yb} | (yb')d \rangle
\]
is expressed using two-body matrix elements.

\[
\langle (yb)d | V_{yb} | (yb')d \rangle = \sum b_{ib_2} (n) (n_{b_1} b_1 n_{b_2} b_2) | L_b | M_b : I | N_{b} L_{b} n_{0} J_{0} L_{0} \rangle \langle 9b \sb{b_1} L_{b_1} b_1 b_2 y_{b_2} >
\]

\[
\times \left( (4y_{b_1} b_{b_2} (b_{b_1} b_{b_2}) b_{b_1} b_{b_2} \langle b_{b_1} b_{b_2} b_{b_1} b_{b_2} | (yb_1 c b_1) \rangle \langle y_2 c b_1 b_2 | (yb_2 c b_1 b_2) \rangle \right) \left( (4y_{b_1} b_{b_2} (b_{b_1} b_{b_2}) b_{b_1} b_{b_2} | (yb_{b_1} b_{b_2} c b_1 b_2) \rangle \langle y_2 c b_1 b_2 | (yb_{b_1} b_{b_2} c b_1 b_2) \rangle \right).
\]

(3.43)

\[ f. \quad V = U(r_{BB}) \]

As in the case $c$, we multiply the final channel distorted wave function
\[
\chi_{L_{b}} (r_{BB}) \text{ by the optical potential } U(r_{BB}). \text{ The overlap integral } \langle (bB)\Lambda | (aA)\Lambda \rangle \text{ in the exchange configuration becomes}
\]
\[ \langle (bB)\Lambda | (aA)\Lambda \rangle \]

\[ = \sum_{c,d} \langle B|c_0 \rangle \langle b|(c_0)B\Lambda | (ba)\Lambda | (ba)d c_0 \Lambda | (ba)\Lambda | (ba)d c_0 \Lambda \rangle \times \int \frac{\phi^*_\Lambda (pb) \phi (pb) \overline{\Phi}_\Lambda \, dp_b \, dp_b \, ds_b \int \overline{\Phi}_\Lambda (N_a A_a) \phi_a (pb) \, dp_b \, da \, ds_a. \]

(3.34)

4. Numerical Calculation of the heavy particle stripping term on the reaction $^{11}\text{B}(d,n)\text{C}$ and discussion

4.1 Numerical calculation of the matrix element in the form factor

As we have mentioned in §1, the heavy particle stripping mechanism is shown in Fig. 2. In the case of the reaction $^{11}\text{B}(d,n)\text{C}$, we take $a$ to be a deuteron, $b$ to be a neutron, $A$ to be $^{11}\text{B}$, $B$ to be $^{12}\text{C}$ and $C$ to be $^{10}\text{B}$. Then wave functions describing $^{11}\text{B}$ and $^{12}\text{C}$ for this calculation are given in Eqs. (2.27). Since the core $C$ is composed of four $S$-shell nucleons and six $P$-shell nucleons, the matrix element in Eq. (3.23) can be written as

\[ \langle (L_b L_b)\Lambda_L (S_b S_b)\Lambda_S (T_b T_b) \Lambda_T | V_{bC} | (L_a L_a)\Lambda_L (S_a S_a)\Lambda_S (T_a T_a) \Lambda_T \rangle \]

\[ = 4 \langle (L_b L_b)\Lambda_L (S_b S_b)\Lambda_S (T_b T_b) \Lambda_T | V_{bC} | (L_a L_a)\Lambda_L (S_a S_a)\Lambda_S (T_a T_a) \Lambda_T \rangle \]

\[ \langle (N_a A_a) \phi_{00} (\frac{1}{2} \Lambda_\alpha, \alpha) | 11\{0\}^{(6)} \Phi_{11}^{(6)} | (L_a L_a)\Lambda_L (S_a S_a)\Lambda_S (T_a T_a) \Lambda_T \rangle \]
where $V_{sb}$ is the interaction between particle $b$ and a nucleon in the S-shell, $V_{pb}$ the interaction between the particle $b$ and a nucleon in the P-shell, and $\mathbf{r}$ the coordinate of the internal motion of the deuteron. The parameter $\nu_d$ represents the extension of the intrinsic state of the deuteron and should be adjusted to the binding energy of the deuteron. The notation $|LsLs0\rangle$ denotes the state constructed by coupling the two states to have the total spatial angular momentum $L$, the total spin $S_z$ and the total iso-spin $T_z$. To simplify the notation, we write the first term of Eq. (4.1) as $P_{shell}^{sb} (N_b L_b L_b N A A a a A)$ and the second term as $P_{shell}^{pb} (N_b L_b N L L L a a A)$. Using the table of Jahm and Wierigern (12), we obtain for the decomposition the following expressions:

$$1^2 C \rightarrow 1^0 B + d:$$

$$\Psi_s (\frac{1}{2} \frac{1}{2}) \Gamma (\frac{1}{2}) = \frac{1}{2} \sqrt{7} \left( \Psi_5 (\frac{1}{2}) \Gamma (\frac{1}{2}) \Psi_5 (\frac{1}{2}) \Gamma (\frac{1}{2}) \right)_{000}$$

$$+ \frac{1}{2(247)} \left( \Psi_5 (\frac{1}{2}) \Gamma (\frac{1}{2}) \Psi_5 (\frac{1}{2}) \Gamma (\frac{1}{2}) \right)_{000} + \frac{1}{2(247)} \left( \Psi_5 (\frac{1}{2}) \Gamma (\frac{1}{2}) \Psi_5 (\frac{1}{2}) \Gamma (\frac{1}{2}) \right)_{000}$$

$$1^1 B \rightarrow 1^0 B + n$$

$$\Psi_p (\frac{1}{2} \frac{1}{2}) \Gamma (\frac{1}{2}) = \frac{1}{2} \sqrt{7} \left( \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \right)_{1\frac{1}{2} \frac{1}{2}}$$

$$+ \frac{1}{2(247)} \left( \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \right)_{1\frac{1}{2} \frac{1}{2}} + \frac{1}{2(247)} \left( \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \right)_{1\frac{1}{2} \frac{1}{2}}$$

$$- \frac{1}{2(247)} \left( \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \Psi_p (\frac{1}{2}) \Gamma (\frac{1}{2}) \right)_{1\frac{1}{2} \frac{1}{2}}$$
We recouple the final channel wave function $|b(a'C)B;A> \rightarrow |a'(bC)A;A>$ and separate the spatial part of the wave function of the particle $a'$ into the center-of-mass motion and the internal motion using the Talmi coefficient. The letters $a'$, $b$, $A$ and $C$ in the wave function $|b(a'C)B;A>$ represent the total quantum numbers of each particle. The non-vanishing matrix elements in Eq. (4.1) can be written for $L_A = 1$ in the following form,

$$\mathcal{F}_{\text{shell}}(N_b,10N_0)_{\frac{a}{a'}} = \frac{1}{3\sqrt{7}} \langle \Psi_{N_b}^{22} \Gamma_{S(42)}^{13} \Gamma_{43} \rangle_{1/2} \langle \Phi_A \Gamma_{P(42)}^{22} \Gamma_{43} \rangle_{1/2} V_{bf} \langle \Phi_{P(43)}^{22} \Gamma_{43} \rangle_{1/2} \rangle_{1/2}$$
\[ F_{\text{shell}}(N_{b,10N_a}^{21}) \]

\[ = \left\{ \frac{1}{4 \cdot 3.1} \left\langle \left[ \left(4_{Na1}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{1/2} \right]_{1/2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ \times \left\{ \left[4_{Na2}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{1/2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ \times \left\{ \left[4_{Na0}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{1/2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ (4.3a) \]

\[ F_{\text{shell}}(N_{b,30N_a}^{21}) \]

\[ = \left\{ \frac{1}{4 \cdot 3.1} \left\langle \left[ \left(4_{Na3}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{1/2} \right]_{1/2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ \times \left\{ \left[4_{Na2}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{1/2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ \times \left\{ \left[4_{Na0}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{1/2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ (4.3b) \]

\[ F_{\text{shell}}(N_{b,10N_a}^{22}) \]

\[ = \left\{ \frac{1}{4 \cdot 3.1} \left\langle \left[ \left(4_{Na1}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{2} \right]_{1/2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ (4.3c) \]

and for \( L_a = 2 \)

\[ F_{\text{shell}}(N_{b,10N_a}^{22}) \]

\[ = \left\{ \frac{1}{4 \cdot 3.1} \left\langle \left[ \left(4_{Na1}^{22} \Gamma \mid \Psi_{D}(43)^{13} \Gamma_{43} \right)_{2} \right]_{2} \left| V_{p} \right| \Psi_{P}(43)^{22} \Gamma_{43} \right\rangle \right\} \]

\[ (4.3d) \]
The matrix element $F_{shell}^{\text{shell}}(N_b, L_b, N_b, L_B, N_a, L_A)$ can be obtained by substituting $V_{pb}$ for the interaction $V_{sb}$ in the expression $F_{shell}^{\text{shell}}(N_b, L_b, N_b, L_B, N_a, L_A)$. We use the notation $E_p(N_b, L_b, N_b, L_B, N_a, L_A)$ for the remainder obtained by removing the overlap integrals from the matrix element $F_{shell}^{\text{shell}}(N_b, L_b, N_b, L_B, N_a, L_A)$ in Eqs. (4.3). If we insert Eqs. (4.2b) and (4.2c) into Eqs. (4.3), the matrix element $E_p(N_b, L_b, N_b, L_B, N_a, L_A)$ can be written as a linear combination of the matrix elements of the following type.
We may use the simple notation $<(\text{tC})A|V_{\text{bp}}|(\text{b'C'})A>$ for the above matrix element.

We separate the matrix element $E_p(N_{b'b'B}N_{a'a'A})$ into two parts. One of them contains only the matrix elements of the type as $<(\text{b'C})A|V_{\text{bp}}|(\text{b'C})A>$ and is denoted by $E_p^{(ND)}(N_{b'b'B}N_{a'a'A})$ and the other contains the matrix elements of the type as $<(\text{b'C})A|V_{\text{bp}}|(\text{b'C'})A>$ ($\text{C'\neq C}$) and is denoted by $E_p^{(D)}(N_{b'b'B}N_{a'a'A})$. If we take the interaction depending only on the relative distance $r_{\text{bC}}$, as we made in the macroscopic calculation of § 2, the term $E_p^{(ND)}(N_{b'b'B}N_{a'a'A})$ vanishes. But as we shall see later (see Table 6), according to this microscopic calculation using the antisymmetrized wave functions for the target and residual nuclei and the two-body interaction of Rosenfeld type, the term $E_p^{(D)}(N_{b'b'B}N_{a'a'A})$ vanishes. If we express the matrix elements $<(\text{b'C})A|V_{\text{bp}}|(\text{b'C})A>$ using two-body matrix elements, we obtain the final results such as Eq. (3.42) expressed by the linear combination of two-body matrix elements for the term $E_p(N_{b'b'B}N_{a'a'A})$.

In the step of this calculation, we show the intermediate states appearing in this calculation in Fig. 5. The matrix elements $E_p(N_{b'b'B}N_{a'a'A})$ are expressed in terms of the two-body matrix elements in Table 3.
Fig. 5 Intermediate States

<table>
<thead>
<tr>
<th>B → C+a</th>
<th>A → C'+b</th>
<th>C → D+ one nucleon</th>
<th>C' → D+ one nucleon</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>D</td>
<td>C'</td>
<td></td>
</tr>
<tr>
<td>[42]^{13}S</td>
<td>[41]^{22}p</td>
<td>[42]^{13}S</td>
<td></td>
</tr>
<tr>
<td>[42]^{13}D_I</td>
<td>[32]^{22}p</td>
<td>[42]^{31}S</td>
<td></td>
</tr>
<tr>
<td>[42]^{13}D_{II}</td>
<td>[32]^{24}p</td>
<td>[42]^{13}D_{I}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[41]^{22}D</td>
<td>[42]^{31}D_{I}</td>
<td></td>
</tr>
<tr>
<td>[42]^{13}D_{II}</td>
<td>[32]^{22}D</td>
<td>[42]^{31}D_{II}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[32]^{24}D</td>
<td>[33]^{11}p</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[41]^{22}F</td>
<td>[33]^{33}p</td>
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<td>[42]^{22}F</td>
<td>[33]^{11}p</td>
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<td></td>
<td>[32]^{24}F</td>
<td>[33]^{33}F</td>
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<tr>
<td></td>
<td>[42]^{22}F</td>
<td>[42]^{13}F</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[32]^{24}F</td>
<td>[42]^{31}F</td>
<td></td>
</tr>
</tbody>
</table>
Table 3  Representation of Ep(NbLbLaNaLa) by two-body matrix elements

<table>
<thead>
<tr>
<th></th>
<th>$E_{p}(Nb10Na10)$</th>
<th>$E_{p}(Nb10Na21)$</th>
<th>$E_{p}(Nb10Na22)$</th>
<th>$E_{p}(Nb30Na21)$</th>
<th>$E_{p}(Nb30Na22)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-20</td>
<td>-22</td>
<td>2932</td>
<td>620</td>
<td>2420</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-60</td>
<td>33</td>
<td>-4943</td>
<td>22482</td>
<td>6270</td>
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<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-100</td>
<td>-11</td>
<td>-6254</td>
<td>-1822</td>
<td>9310</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-16</td>
<td>40</td>
<td>-1856</td>
<td>-64</td>
<td>2080</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-48</td>
<td>-60</td>
<td>-6792</td>
<td>-1416</td>
<td>5880</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-80</td>
<td>20</td>
<td>-2296</td>
<td>1480</td>
<td>6440</td>
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<table>
<thead>
<tr>
<th></th>
<th>$E_{p}(Nb10Na10)$</th>
<th>$E_{p}(Nb10Na21)$</th>
<th>$E_{p}(Nb10Na22)$</th>
<th>$E_{p}(Nb30Na21)$</th>
<th>$E_{p}(Nb30Na22)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>18</td>
<td>-1440</td>
<td>2052</td>
<td>-612</td>
<td>2160</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>27</td>
<td>-2160</td>
<td>2700</td>
<td>-540</td>
<td>1610</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-45</td>
<td>-27</td>
<td>-2052</td>
<td>612</td>
<td>-540</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-18</td>
<td>1440</td>
<td>-2448</td>
<td>-540</td>
<td>1610</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>72</td>
<td>-5760</td>
<td>8208</td>
<td>-10800</td>
<td>2160</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>108</td>
<td>8640</td>
<td>-2448</td>
<td>2160</td>
<td>10800</td>
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<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-180</td>
<td>-5760</td>
<td>8208</td>
<td>-10800</td>
<td>2160</td>
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common factor 1

<table>
<thead>
<tr>
<th></th>
<th>$E_{p}(Nb30Na21)$</th>
<th>$E_{p}(Nb30Na22)$</th>
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</thead>
<tbody>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-262</td>
<td>1170</td>
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<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>312</td>
<td>2280</td>
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<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>234</td>
<td>1710</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>-234</td>
<td>-1710</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>936</td>
<td>6840</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>418</td>
<td>-30</td>
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</tbody>
</table>

common factor 9

<table>
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<th>$E_{p}(Nb30Na21)$</th>
<th>$E_{p}(Nb30Na22)$</th>
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</thead>
<tbody>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>152</td>
<td>1610</td>
</tr>
<tr>
<td>$(4n_{e}1y_{o1})<em>{L} L</em>{B} (4n_{e}1y_{o1})_{L}$</td>
<td>218</td>
<td>2770</td>
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</tbody>
</table>
In Table 3, the spin-isospin wave functions are defined by

\begin{align*}
\eta_1 &= \frac{1}{4} \left( \gamma^6 - \sqrt{3} \gamma^3 \gamma + \sqrt{3} \gamma^3 \gamma - 3 \gamma^3 \gamma \right), \\
\eta_2 &= \frac{1}{2} \left( \gamma^3 \gamma - \sqrt{3} \gamma^3 \gamma \right), \\
\eta_3 &= \frac{1}{4} \left( \gamma^3 \gamma - \sqrt{3} \gamma^3 \gamma + \sqrt{3} \gamma^3 \gamma - 3 \gamma^3 \gamma \right), \\
\eta_4 &= \frac{1}{4} \left( \gamma^4 - \sqrt{3} \gamma^3 \gamma + \sqrt{3} \gamma^3 \gamma - 3 \gamma^3 \gamma \right), \\
\eta_5 &= \frac{1}{4} \left( \gamma^5 + \sqrt{3} \gamma^3 \gamma + \sqrt{3} \gamma^3 \gamma + 3 \gamma^3 \gamma \right), \\
\eta_6 &= \frac{1}{2} \left( \gamma^6 + \sqrt{3} \gamma^3 \gamma \right).
\end{align*}

For example, \( E_p^{(D)}(N_b10N_a01) \) is defined by

\[ E_p^{(N)}(N_b10N_a01) = \frac{1}{243} \left( -20 \langle (4N_b101)_0 | V | (4N_b101)_0 \rangle \eta_1 \right) \]

\[ - 60 \langle (4N_b101)_1 | V | (4N_b101)_1 \rangle \eta_1 \rangle - 60 \langle (4N_b101)_2 | V | (4N_b101)_2 \rangle \eta_1 \rangle \]

\[ - 16 \langle (4N_b101)_1 | V | (4N_b101)_2 \rangle \eta_1 \rangle - 48 \langle (4N_b101)_1 | V | (4N_b101)_2 \rangle \eta_1 \rangle \]

\[ - 80 \langle (4N_b101)_2 | V | (4N_b101)_2 \rangle \eta_1 \rangle. \]

We check on this step of the calculation in the following way.

a. We have used the relation that the matrix elements \( \langle (bC)A|V|(b'C')A \rangle \) are expressed by the linear combinations of the two-body interaction. If we make the interaction \( V_{pb} \) equal to unity and the parameter \( v \) equal to \( v_A \), we obtain

\[ \langle (4N_b101)_{22} \gamma^2 \gamma \gamma \gamma | \gamma \gamma \gamma \gamma | (4N_a101)_{22} \gamma^2 \gamma \gamma \gamma \rangle \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \ga
To examine whether above relation is correct or not, we set $V_{pb} = 1$. Then the above equation is examined to be correct, as its right side is unity.

b. We express the matrix element $E_p(N_b L_b N_a L_a)$ by the two-body matrix elements of the type

$$\langle (4_{01} 4_{01})_L \left| V_{pb} \right| (4_{01} 4_{01})_L \rangle^{2T+1 2S+1} \rangle$$

Owing to the symmetry of the wave function $(4_{01} 4_{01})_L^{2T+1 2S+1}$, the two-body matrix elements such as

$$\langle (4_{01} 4_{01})_0 \left| V_{pb} \right| (4_{01} 4_{01})_0 \rangle^{11} \rangle \quad \text{and} \quad \langle (4_{01} 4_{01})_1 \left| V_{pb} \right| (4_{01} 4_{01})_1 \rangle^{13} \rangle$$

are not contained in the matrix element $E_p(N_b L_b N_a L_a)$.

Finally, if we take the two-body interaction

$$V(r) = V_W + V_S (\hat{S}_1 \cdot \hat{S}_2) + V_T (\hat{T}_1 \cdot \hat{T}_2) + V_{ST} (\hat{S}_1 \cdot \hat{T}_1) (\hat{S}_2 \cdot \hat{T}_2) f(r)$$

$$f(r) = \left( V_0 e^{-\frac{r^2}{\rho^2}} \right)$$

the matrix element $F_{shell}^{p}(N_b L_b N_a L_a)$ becomes as follows:
\[
F_{(d)\text{shell}}^{(1)} (N_{b10}, N_{a01})
= \frac{1}{2.4313} \left\{ (-36 V_{w} + 24 V_{s}) S_{1} + (-108 + 92 V_{s}) P_{1} + (-180 V_{w} + 120 V_{s}) D_{1} \right\}
\times \int \left[ \left( \frac{4.90}{(2\nu, 1, 1)} \right) y_{10} (2\nu, 1, 1) \right] d\nu d\sigma d\nu d\sigma \\
- \int \left[ \left( \frac{4.90}{(2\nu, 1, 0)} \right) y_{10} (2\nu, 1, 0) \right] d\nu d\sigma d\nu d\sigma
\right\}
\right)
\]

(4.6a)

\[
F_{(d)\text{shell}}^{(2)} (N_{b10}, N_{a01})
= \frac{1}{2.4313} \left\{ (18 V_{w} + 66 V_{s} - 54 V_{r} + 54 V_{r} \omega) S_{1} + (-27 V_{w} - 153 V_{s} - 81 V_{r} - 243 V_{r} \omega) P_{1} \\
+ (9 V_{w} + 87 V_{s} + 135 V_{r} + 513 V_{r} \omega) D_{1} \right\}
\times \int \left[ \left( \frac{4.90}{(2\nu, 1, 1)} \right) y_{10} (2\nu, 1, 1) \right] d\nu d\sigma d\nu d\sigma \\
- \int \left[ \left( \frac{4.90}{(2\nu, 1, 0)} \right) y_{10} (2\nu, 1, 0) \right] d\nu d\sigma d\nu d\sigma
\right\}
\right)
\]

(4.6b)

\[
F_{(d)\text{shell}}^{(3)} (N_{b10}, N_{a21})
= \frac{1}{2.4313} \left\{ (-4,788 V_{w} + 4,008 V_{s}) S_{1} + (-11,286 V_{w} + 2,194 V_{s}) P_{1} \\
+ (-8,550 V_{w} + 1,022 V_{s}) D_{1} \right\}
\times \int \left[ \left( \frac{4.90}{(2\nu, 1, 1)} \right) y_{02} (2\nu, 1, 1) \right] d\nu d\sigma d\nu d\sigma \\
- \int \left[ \left( \frac{4.90}{(2\nu, 1, 0)} \right) y_{02} (2\nu, 1, 0) \right] d\nu d\sigma d\nu d\sigma
\right\}
\right)
\]

(4.6c)
\[ F_p^{(ND)\text{shell} (N_{b10} Na_{21})} \]

\[ = \frac{1}{64 \sqrt{2} 29313 \frac{13}{15} \pi} \left\{ \begin{array}{l}
(-684 V_{W} + 2,616 V_{\sigma} + 4,320 V_{T} + 18,416 V_{\phi T}) \ S_{1}^{l} \\
+ (1,026 V_{W} - 3,352 V_{\sigma} - 6,156 V_{T} - 18,486 V_{\phi T}) \ P_{1}^{l} + (-342 V_{W} + 5736 V_{\sigma} + 6,36 V_{T} + 17,171 V_{\phi T}) \ D_{1}^{l} \\
\end{array} \right\} \]

\[ \times \int \text{ \text{4Na}_2 (2\nu, \text{1l}_{\alpha} \nu_{a}) \ 402 (2\nu_{a}, \nu_{a}) \ d
\]

\[ \text{Na}_2 \cdot \int \text{ \text{400} (1, \nu_{a}, \nu_{a}) \ 400 (1, \nu_{a}, \nu_{a}) \ d} \]

\[ \text{I}_{1} \]

\[ (4.6a) \]

\[ F_p^{(ND)\text{shell} (N_{b30} Na_{21})} \]

\[ = \frac{1}{16 \sqrt{2} 81,15 \frac{71}{77}} \left\{ \begin{array}{l}
(500 V_{W} + 602 V_{\sigma} - 705 V_{T} - 146 V_{\phi T}) \ D_{3}^{l} \\
\end{array} \right\} \]

\[ \times \int \text{ \text{4Na}_2 (2\nu, \text{1l}_{\alpha} \nu_{a}) \ 402 (2\nu_{a}, \nu_{a}) \ d
\]

\[ \text{Na}_2 \cdot \int \text{ \text{400} (1, \nu_{a}, \nu_{a}) \ 400 (1, \nu_{a}, \nu_{a}) \ d} \]

\[ \text{I}_{1} \]

\[ (4.6e) \]

\[ F_p^{(ND)\text{shell} (N_{b10} Na_{22})} \]

\[ = \frac{1}{64 \sqrt{2} 29313 \frac{51,17}{15} \pi} \left\{ \begin{array}{l}
(4,500 V_{W} - 2,760 V_{\sigma}) \ S_{1}^{l} \\
+ (12, 150 V_{W} - 6,160 V_{\sigma}) \ P_{1}^{l} + (15, 750 V_{W} - 12,180 V_{\phi T}) \ D_{1}^{l} \\
\end{array} \right\} \]

\[ \times \int \text{ \text{4Na}_2 (2\nu, \text{1l}_{\alpha} \nu_{a}) \ 402 (2\nu_{a}, \nu_{a}) \ d
\]

\[ \text{Na}_2 \cdot \int \text{ \text{400} (1, \nu_{a}, \nu_{a}) \ 400 (1, \nu_{a}, \nu_{a}) \ d} \]

\[ \text{I}_{1} \]

\[ (4.6f) \]

\[ F_p^{(ND)\text{shell} (N_{b10} Na_{22})} \]

\[ = \frac{1}{64 \sqrt{2} 29313 \frac{51,17}{15} \pi} \left\{ \begin{array}{l}
(-900 V_{W} + 2,040 V_{\sigma} - 6,480 V_{T} - 10,800 V_{\phi T}) \ S_{1}^{l} \\
\end{array} \right\} \]

\[ (4.6g) \]
\[ + (1350 V_{W} + 14,760 V_{S} + 8,100 V_{T} + 24,300 V_{ST}) P_{i}^{\prime} + (-450 V_{W} - 11,800 V_{S} - 1,620 V_{T} - 45,900 V_{ST}) D_{i}^{\prime} \]
\[ \times \int_{\ast}^{\ast} y_{Na2}^{K} (2\nu, 1\alpha) y_{01}^{L} (2\nu_{A}, 1\alpha_{A}) d^{2}l_{A} \int y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) d^{2}l_{\Gamma}, \]
\[ f_{P}^{\text{shell}} (N_{0} 30 Na 22) = \frac{1}{16 \frac{\pi}{2} 27 \frac{\pi}{15} 5_{1}^{4} \frac{\pi}{49} (3,450 V_{W} - 1,770 V_{S} - 5,180 V_{T} - 10,350 V_{ST}) D_{i}^{\prime} \]
\[ \times \int_{\ast}^{\ast} y_{Na2}^{K} (2\nu, 1\alpha) y_{02}^{L} (2\nu_{A}, 1\alpha_{A}) d^{2}l_{A} \int y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) d^{2}l_{\Gamma}, \]
\[ f_{S}^{\text{shell}} (N_{0} 10 Na 01) = (-) \frac{1}{3} V_{W} \frac{3_{1}^{4} \pi}{3_{1}^{4} \pi} P_{i}^{\prime} \{ \int y_{Na0}^{K} (2\nu, 1\alpha) y_{10}^{L} (2\nu_{A}, 1\alpha_{A}) d^{2}l_{A} \int y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) d^{2}l_{\Gamma} \}
\[ f_{S}^{\text{shell}} (N_{0} 10 Na 01) = (-) \frac{1}{3} V_{W} \frac{3_{1}^{4} \pi}{3_{1}^{4} \pi} P_{i}^{\prime} \{ \int y_{Na0}^{K} (2\nu, 1\alpha) y_{02}^{L} (2\nu_{A}, 1\alpha_{A}) d^{2}l_{A} \int y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) d^{2}l_{\Gamma} \}
\[ f_{S}^{\text{shell}} (N_{0} 10 Na 02) = (-) \frac{1}{3} V_{W} \frac{3_{1}^{4} \pi}{3_{1}^{4} \pi} P_{i}^{\prime} \{ \int y_{Na0}^{K} (2\nu, 1\alpha) y_{02}^{L} (2\nu_{A}, 1\alpha_{A}) d^{2}l_{A} \int y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) y_{00}^{0} (1\nu_{D}, 1\nu_{\Gamma}) d^{2}l_{\Gamma} \}
\]
where we use the notation for the two-body matrix element as follows:
\[ L_{\nu} = \left< \left< y_{\nu m}^{K} (\nu_{A}, 1\alpha_{A}) y_{\nu_{D} \nu_{\Gamma}}^{0} (1\nu_{D}, 1\nu_{\Gamma}) \right| f_{i}^{(1\alpha)} \left| y_{\nu_{D} \nu_{\Gamma}}^{0} (1\nu_{D}, 1\nu_{\Gamma}) y_{\nu_{A} \nu_{D}}^{0} (1\nu_{A}, 1\nu_{D}) \right> \right> \]
These equations are the explicit expression of Eq. (3.42) for the reaction $^{11}\text{B}(d,n)^{12}\text{C}$. If we put $V_{sb}=V_{pb}=1$, $v_{d}=v_{A}=v$, we obtain the non-vanishing overlap integrals. From Eqs. (4.6a), (4.6b) and (4.6i)

$$F_{\text{shell}}(101011)=F_{\text{shell}}(101010)=-\frac{19}{15} \times 10^{-5} = (-)3888 \times 10^{-5},$$

from Eqs. (4.6c), (4.6d) and (4.6j)

$$F_{\text{shell}}(101021)=F_{\text{shell}}(101020)=\frac{5}{17} \times 10^{-5} = (-)1429 \times 10^{-5},$$

from Eq. (4.6f), (4.6g) and (4.6k)

$$F_{\text{shell}}(101022)=F_{\text{shell}}(101020)=\frac{5}{17} \times 10^{-5} = 2875 \times 10^{-5}.$$

We give the parameters used for the calculation in Table 4. Table 5 shows the values of $V_{\omega}$, $V_{\sigma}$, $V_{\tau}$ and $V_{\sigma\tau}$ for the four types of the interaction.

4.2 Discussion about the matrix elements in the form factor and the cross section

In Table 6, we give the numerical results of the matrix elements $6F_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ plus $4F_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ for four types of the nuclear force such as Wigner, Serber, Rosenfeld and Gillet types. The matrix elements $F_{\text{p}}(D)_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ for the Rosenfeld interaction vanish, although the matrix elements $F_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ have the same trend for four types of the interaction. To see the damping of the matrix elements $F_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ with the increase of the node $N_{b}$, we show the feature of $F_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ for the Rosenfeld force by the solid line of Fig. 6. We also calculated the matrix elements for $\lambda=0.75$ and 0.80. The results obtained for these values of $\lambda$ are similar to those for $\lambda$ equal to 0.7.

From Table 6, the matrix elements $4F_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ are of the same order as the matrix elements $6F_{\text{shell}}(N_{b}L_{b}N_{L}L_{b},N_{b}L_{b}N_{L}L_{b})$ in the case of the Wigner and Serber forces. But in our calculations, this S shell effect is considered to be small implicitly. If the effect of nucleons in the S shell is large, we must take into account the neutron going out from S shell in addition to from P-shell. In other words we must
take as the antisymmetric wave functions for the description of the target \(^{11}\text{B}\) (antisymmetric wave function of four nucleons in the S shell and seven nucleons in the P shell) in stead of (antisymmetric wave function of four nucleons in the S shell) \(\times\) (antisymmetric wave function of seven nucleons in the P shell).

The contribution of the S shell in the case of the Rosenfeld type happens to vanish due to the fact that it has only \(V_w\) term as can be seen from Eqs. (4.6i) (4.6j) and (4.6k). We compare the result for the Rosenfeld interaction with that of the macroscopic calculation.

We show the results of the microscopic and the macroscopic calculation in Fig. 8. The result of the microscopic calculation with the reasonable strength of the two-body Rosenfeld interaction is larger than that of the macroscopic calculation by the factor 10. But it is much smaller than that of the usual stripping calculation. The orders of the experimental results are few milli-barns and the results of the usual stripping calculations are of the same order as the experimental results. We can conclude that even if we calculate the heavy particle stripping term microscopically, the heavy particle stripping contribution on the reaction of \(^{11}\text{B} (d,n)\,^{12}\text{C}\) is very small.

We also find the following results. The cross section of the capture of the deuteron into S state is as large as that into D state. The contribution of the D state in the wave function of the \(^{11}\text{B}\) is very small as in the case of the macroscopic calculation. This result is due to the recoupling factors of Eq. (3.16) in spite of the fairly large matrix elements shown in Table 6.

### 4.3 Recoil effect

To obtain \(F(N_b L_b L_b N_a L_a)\), we solve Eq. (3.23) by using the electric computer HITAC-5020. The results are shown by the dotted line in Fig. 6 for the Rosenfeld force. In Fig. 6, we show only the matrix elements \(F(N_b L_b L_b N_a L_a)\). But in
addition to the above matrix elements, many matrix elements with the same order of values appear, for example, about 100 matrix elements in the case $L_A=1$. The main contributions of these matrix elements for the Rosenfeld force are shown in Fig. 7. These matrix elements play an important role to affect on the pattern of the angular distribution of the cross section. In Fig. 9, we show the cross sections calculated with and without the recoil effect. In the macroscopic calculation without the recoil effect, the form factor is as follows

\[
\hat{F}_{lm}(N_b L_b N_a L_a) = \sum_{l_a l_b \mu_a \mu_b} (\lambda_a \lambda_b \eta_a (\eta_b, \lambda_a) Y_{l_a \lambda_a}^* Y_{l_b \lambda_b}) V(l_a l_b) Y_{l_a \mu_a} Y_{l_b \mu_b} \text{in stead of Eq. (2.35e).}
\]

We calculated the matrix elements for Eq. (3.23a) and obtained nearly the same values as the exact solutions concerning the matrix elements $F(N_{lb} L_{lb} N_{a} L_{a})_{b,a,a'}$ having the same quantum numbers as the non-vanishing matrix elements $F_{\text{shell}}(N_{lb} L_{lb} N_{a} L_{a})_{b,a,a'}$. But we can obtain no matrix elements having other quantum numbers by solving Eq. (3.23a).

From this discussion, we may say the following conclusions about the recoil effect.

a. The recoil effect makes the value of the cross section large by the factor about $(a+A) x (b+B) = 1.5$ (in this case) because the main contribution to the cross section comes from the matrix element $F(010021)$. This can be seen from Fig. 9.

b. The exact recoil effect change the angular pattern because of the large contribution of the matrix elements such as $F(N_b 0011)$ in Fig. 7.

5. Concluding remarks

The main work of this paper is to develop the method of the DWBA microscopic calculation. We remark as follows:
Application:

a. This method can be conveniently applied to the investigation about the term which is assumed to be zero in the usual DWBA calculation and about the heavy particle stripping term.
b. This method can also be applied to the microscopic calculation on the reaction (t,p) and (d,t) etc.

Problem of the convergence:
c. We find that, in the numerical calculation on \( ^{11}\text{B}(d,n)^{12}\text{C} \), the matrix elements in the form factor decrease very rapidly with the increase of the nodes of the harmonic oscillator wave functions used for the expansion of the form factor.

d. This method is very convenient to include the recoil effect in the microscopic calculation. This effect makes the magnitude of the cross section large, and varies the angular pattern.

Cross section of the heavy particle stripping process on \( ^{11}\text{B}(d,n)^{12}\text{C} \):
e. The value of the cross section of the heavy particle stripping process is much smaller than that of the usual stripping process. From Fig. 2, the usual stripping calculations with more reasonable optical parameters may explain the backward peak of the experimental results due to the distortion effect.

Acknowledgment

The author is very grateful to Professor S. Yoshida for valuable suggestions concerning the method of this calculation and Dr. H. Yoshida for useful discussions and encouragement throughout this work. He is also grateful to Professor M. Muraoka for reading the manuscript and giving valuable suggestions, to other members of the nuclear theory group, Osaka University for useful discussions and to Miss K. Matsuura for typing the manuscript.
### Table 4 Parameter of the harmonic oscillator wave function

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
<th>Validity of the value of the parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>the force range of the Gaussian interaction</td>
<td>2.0fm</td>
<td>From the Kuruth's calculation (13) about the energy of P-shell nuclei, the range of the parameter ( \lambda ) extends from 0.69 to 0.77. In our calculation, we take ( \lambda = 0.7 ) so ( v_A = \frac{2\lambda^2}{\delta^2} = 0.24 )</td>
</tr>
<tr>
<td>( v_A )</td>
<td>the parameter of the bound harmonic oscillator wave function of P-shell nucleon in ( ^{12}\text{C} ) and ( ^{11}\text{B} )</td>
<td>0.24fm(^{-2})</td>
<td>C. L. Lin and S. Yoshida have used the parameter ( v_d = 0.213 \text{fm}^{-2} ) from the electron scattering. But to simplify our calculation, we have taken ( v_d = 0.24 \text{fm}^{-2} )</td>
</tr>
<tr>
<td>( v_d )</td>
<td>the parameter of the bound state harmonic oscillator wave function describing the internal bound state of the deuteron</td>
<td>0.24fm(^{-2})</td>
<td>To simplify the calculation we have taken ( v = v_A )</td>
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<tr>
<td>( v )</td>
<td>the parameter of the harmonic oscillator wave function used to expand the form factor</td>
<td>0.24fm(^{-2})</td>
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### Table 5 Type of the force used for our calculation

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<th>( v_\sigma )</th>
<th>( v_T )</th>
<th>( v_{\sigma T} )</th>
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Table 6  Value of the expansion coefficient of the form factor

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<th>( 6F_{p}^{(ND)}(N_{b}10101) )</th>
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<tr>
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</table>

| \( N_b \) | \( Wigner \) | \( Serber \) | \( Rosenfeld \) | \( Gillet \) |
|---|---|---|---|
| 0 | -1739 | 0 | -377.5 |
| 1 | -413.6 | 0 | -127.7 |
| 2 | -105.7 | 0 | -24.46 |
| 3 | -9.971 | 0 | 3.2 |
| 4 | 9.263 | 0 | 6.473 |
| 5 | 8.589 | 0 | 4.474 |

| \( N_b \) | \( Wigner \) | \( Serber \) | \( Rosenfeld \) | \( Gillet \) |
|---|---|---|---|
| 0 | -1857 | -1087 | -1088 |
| 1 | -573.4 | -270.6 | -270.9 |
| 2 | -75.33 | 15.71 | 15.83 |
| 3 | 41.71 | 62.40 | 62.48 |
| 4 | 45.49 | 46.54 | 46.59 |
| 5 | 28.55 | 26.24 | 26.26 |

<p>| ( N_b ) | ( Wigner ) | ( Serber ) | ( Rosenfeld ) | ( Gillet ) |
|---|---|---|---|
| 0 | -16.12 | -33.81 | -34.17 |
| 1 | -11.48 | -24.07 | -24.33 |
| 2 | -6.385 | -13.38 | -13.53 |
| 3 | -3.156 | -6.616 | -6.688 |
| 4 | -1.450 | -3.640 | -3.73 |
| 5 | -0.6349 | -1.331 | -1.345 |</p>
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$4P_s(N_d 10101)$

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$4P_s(N_d 10022)$
Fig. 6  Value of $F(NbLbL'_{3}NaLa_{A}) = 6F_{1}(NbLbL'_{3}NaLa_{A}) + 4F_{s}(NbLbL'_{3}NaLa_{A})$
Fig. 7 Value of $F(N_{b}L_{b}L_{a}N_{L}L_{a}) = 6F_{b}(N_{b}L_{b}N_{L}L_{a}) + 4F_{s}(N_{b}L_{b}N_{L}L_{a})$

appearing by solving Eq. (3.23). Rosenfeld force
Fig. 8 Comparison between macroscopic and microscopic results.
Fig. 9 Recoil effect

\[ ^{11}\text{B}(d,n)^{12}\text{C} \]

\( l=1, l_d=2 \)

- - - - with the recoil effect
- - - - without the recoil effect

\( V_{n^{10}\text{B}}=52.4\text{MeV} \)
bound state solved by Woods-Saxon potential

\( V_{\text{two-body}}=30\text{MeV} \) (Rosenfeld)
bound state: harmonic oscillator wave function
Reference


